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The Boundedly Rational Decision Process Creating Probability Responses Empirical Results Confirming the Theory of Prominence

by

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Abstract

A central idea of the theory of prominence in the decimal system is that every number is presented as a sum of full step numbers (i.e. the numbers $a * 10^i$ with $a = 1, 2,$ or 5, and i integer), where every full step number appears at most once in the presentation with sign +1 or -1. For example $42 = 50 - 10 + 2$. The question is whether subjects really perceive numbers this way, and if they create numerical responses in a way that creates presentations of numbers as sums of full step numbers of decreasing amounts. The answer is not only interesting for further analysis of boundedly rational numerical decision processing, but also for the theory of prominence.

To approach the problem, we asked 30 subjects first to answer a set of questions by numerical responses, and thereafter to describe how they came to their responses, and – if possible – to give numbers they considered during the process of finding the numerical response. We did not consider reports where the subjects could not remember any number but their final response, and we restricted the analysis to questions which asked for probabilities.

The observed processes have a clear structure. They consist of three phases. In phase 1 an anchor point $A = 0$, 50, or 100 is selected. In phase 2 the anchor point is refined by adding or subtracting a full step number F. This phase can be repeatedly applied until the obtained result $A + F$ or $A - F$ cannot be improved. In this phase new values of F replace the old ones. In phase 3 the result is stepwise refined by adding or subtracting full step numbers of decreasing order, until the decision maker reaches the limit of her ability of judgement.

Moreover there are several rules and properties which are fulfilled by the process, and which suggest that numerical decision processing has quite strict rules as the grammar of a language. Finally a detailed process model is given which describes 127 of the 134 considered reports in a reasonable way.[1]

[1]I thank Andras Güntzel who made the interviews of this investigation as part of his diploma thesis. He analysed the data under different aspects (see Güntzel 1993).

1 The Problem

The theory of prominence predicts that subjects create decimal numerical responses as sums of full step numbers where every full step number appears at most once and the coefficients are +1, -1 or 0 (example: $17 = 20-5+2$). The numerical response to a question consists of two parts: 1. to find the 'correct' idea about the magnitude of the response, and 2. to transform this idea into a numerical response. Empirical results indicate that these two processes are mixed. It seems that subjects construct a numerical response as a sum of full step numbers, but during the process replace the respective last component of the response if it does not pass an internal control process.

The purpose of this paper is to get information about the structure of such a numerical response process, in order to check whether or how the process fits to results of the theory of prominence.

Although the obtained model describes the general structure built up in the reconstruction process rather than the structure of the original decision process, it nevertheless permits insight into the structure of boundedly rational numerical decision processing.

2 The Investigation

30 subjects were asked for their responses to a selection of 22 questions. They were asked to give those responses with which they felt most content. We tried to avoid to mention numbers within the questions, since we were concerned that otherwise subjects could be influenced in their decision process, for instance to imitate this number, or to use it as a reference point. Some of the questions induced responses which were not related to the prominence structure of the decimal system: for instance the number of spokes of the wheel of a bicycle was in many responses a multiple of 12 (the correct number is 36), and the responses to the amount of time that a letter needs from Germany to Australia are related to the seven days of a week. Most of the questions had no correct answers that were available, except for questions 2, 5, 8, 9, 10 and 12 with the correct answers 9%, 6 Millions, 36 spokes, men on the moon, and 6 cm. Other questions sometimes induced calculations as to some up the two components, for instance most subjects subdivided the task to estimate the distance between Bielefeld and Salzburg into two parts, namely to estimate th distance between Bielefeld and Munich, and the distance between Munich and Salzburg, and give the sum of the two numbers as the response.

We omitted those questions from the investigation, where subjects could expect that experimentor knew the correct answer, and could compare the subject's answer with the correct value. Thereby we excluded the motive to hit the correct answer by guessing a very precise number.

To be sure that the responses were selected from a decimal scale, we restricted the analysis to those questions which asked for probabilities, or for a response between 0 and 100.

Part of the subjects were students, but there were also subjects who had no education at the university. Subjects 1 to 15 answered only questions 1 to 13, the other subjects answered all questions.

After the subjects had been asked for their responses for all questions they were question by question asked to explain how they had come to their respective responses. They were asked to report about what they had been thinking when they came to their responses, to report their thoughts and perhaps other numbers they had considered during the response process as precisely as possible, but only as far as they could remember. In most cases the subjects started their reports with statements that involved no numbers. In these cases the subjects were asked why they came just to their response. It was avoided to ask for additional information too intensively to avoid that subjects gave informations they had not really considered during their decision.

Problem of the approach is that subjects have to reconstruct their processes that led to the decision of the numerical response. From an investigation on decision processes concerning the choice of a holiday trip of one or two weeks, where we compared post decision reports that were picked up months after the decision with think aloud protocols of decision processes, we know that ex post reports are more 'stream lined', and clearly tend to omit errors in the description of the process. Nevertheless, we decided not to use the think aloud method, since we expected that numerical decision processes are essentially more subconscious than holiday decision, and could be easily disturbed ore modified by thinking aloud. Moreover, there was only a short time between the immediate answer and the question for the process.

3 Remarks Concerning Numerical Decision Processing

A report is the sequence of numbers that a subject gives when she is asked how she came to her response. The theory of prominence suggests that subjects give there responses in such-a-way that a presentation of the number of the response is obtained.

In our opinion numerical responses are created as follows: There is an unconscious part of the brain that creates proposals of numerical responses, this part learns unconsciously as a neural net, it selects inputs and transforms them to outputs as a learning black box. It seems that in numerical decision processing it creates one output, which is then checked in the conscious part of the brain by different kinds of reasoning, this reasoning can involve logical consistency, consistency with data of experience, consistency with parameters of conscious decision and learning processing as aspiration levels. Schemes of reasoning are developed by experience, learning and education. Another tool of the conscious control process is to check whether a given decision fits to the subjects feelings and emotions.

In the related subprocesses subjects can ask 'an internal emotional decision generator' whether it prefers one of two alternatives, whether he likes a given alternative or not. In this part of decision processing emotional components as love, hate fear, hunger can enter the decision process. It is not clear if and how but seems reasonable that repeatedly applied mechanisms of reasoning by iterated application enter the black box (or neural net) of uncontrolled decision processing. (This also raises the further question how genetic evolution can influence decision processing, for instance imbed attitudes of social behavior. Our idea is that emotional preferences can be genetically anchored, and enter decision processing via the mentioned control mechanisms.) The actually selected/applied arguments of the conscious reasoning process are (or can be) also used as inputs of the unconscious part of the process. – We are long way apart from being able to 'prove' that this model is correct (or better to make the model sufficiently plausible). But it seems to be helpful to have a model in mind when details of the explorative research concerning the creation of numerical responses are developed.

Open questions of the present investigation are if it is possible that subjects memorise intermediate steps of the construction of responses. Surely the memory can at best give those proposals which entered the control process. How many proposals are these, might the probability of memorizing be related to the intensity or diffulty of rejection? Is a subject able to reconstruct pieces of the process (as the sequence of events of an accident is usually reconstructed using detectivistic methods). If it is possible to reconstruct, this indicates that there is some kind of logic that constructs the proposals made by the 'black box'. Which principles are available that permit subjects to fill gaps? How conscious are such principles of reconstruction? – The research here cannot go into these questions but rather tries to extract general principles that seem to be used in the construction of numerical responses, where we presently do not ask, whether the principles are generated in the conscious or the unconscious part of the decision processing. It may even be that the reports given by the subjects are only enforced by the question how they came to their response, what induced a separate reasoning process which did not really happen in the decision itself (think aloud studies could help in this respect, but involve still the same problem, namely if the think aloud procedure enforces a higher weight of the conscious part of the process.) – Any way we are here interested in the question if there is a kind of reasoning proces – if ex post or not – supports the idea that numerical responses are constructed in a way that supports the theory of prominence.

4 First Results

We distinguish between reports that involved only one number (the final response), and reports that give several numbers. Of course, only the second kind of report can be used to conclude on the structure of the process. In the majority of cases subjects could only report one number. (The high proportion of 1-number reports illustrates the difficulties that subjects had when they tried to report on the process.)

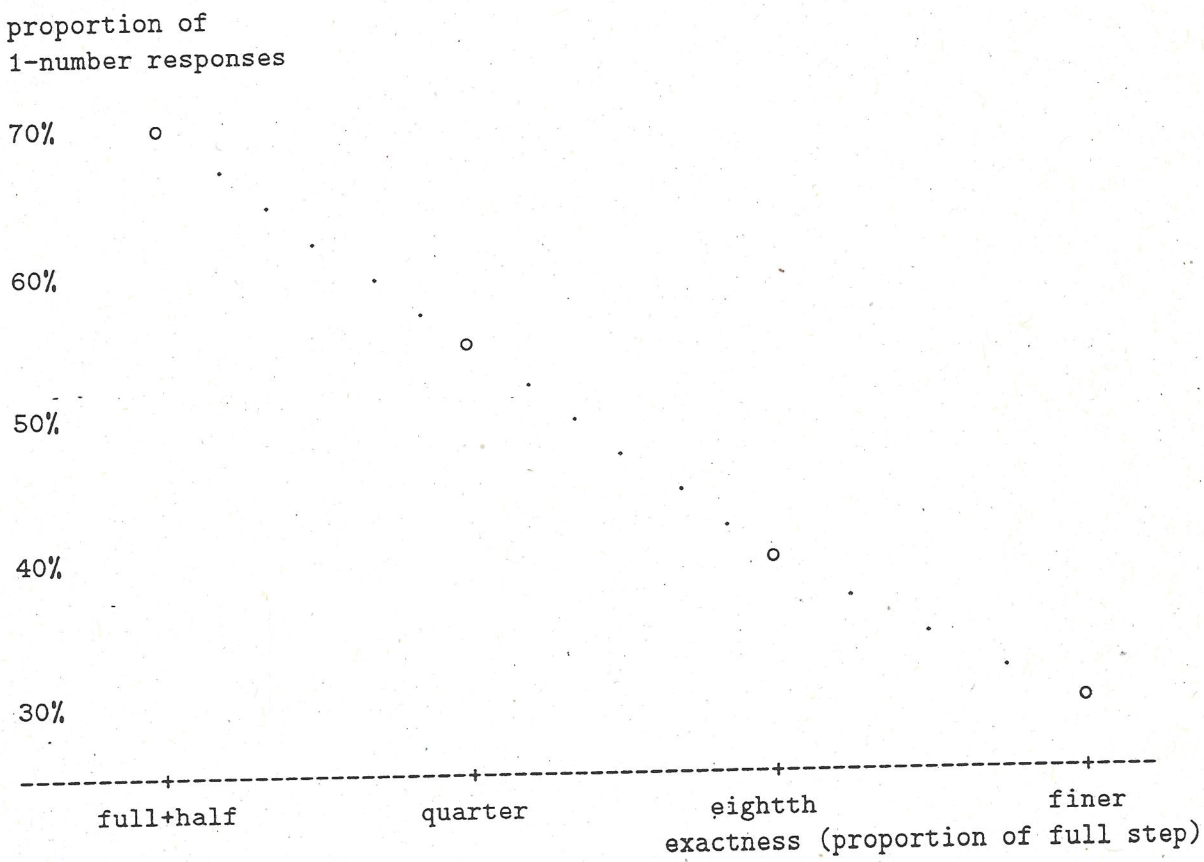
Table 1 shows the frequencies of the numerical results. It can be seen that the proportion of cases where more than one number is reported, increases with the 'preciseness' of the final response (for 0, 20, 25, 30, 50, 70, 80, 100 the proportion is 38/124, for 10, 40, 60, 90 it is 33/72, for 5, 15, 35, 45, 55, 65, 85, 95 it is 32/56, and for the finer numbers it is 20/28, all differences are significant on the 5% level by Fisher's exact).

Table 1: Frequencies of the Numerical Responses

# numbers in report	response 0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	SUM
1 number	-	2	8	1	10	10	14	5	14	2	15	2	13	5	17	10	10	4	4	3	-	149
>1 number	-	1	3	6	5	2	9	5	8	8	2	3	14	3	5	7	8	4	8	2	-	103

# numbers in report	response 2	6	9	12	17	18	37	42	43	48	49	52	72	78	79	89	93	97	99	SUM	TOTAL
1 number	-	-	-	2	2	-	-	-	1	-	-	1	-	-	1	-	1	-	-	8	157
>1 number	1	1	1	1	-	2	1	1	1	1	2	-	1	1	-	1	1	1	3	20	123

Figure 2: Proportion of 1-Number Responses as a Function of Relative Exactness of the Responded Number



Since we are interested in the structure of the decision processing that leads subjects to their responses, we are not interested in the responses that do not inform about steps of the decision processing. We will therefore only consider reports that contain at least 2 different numbers.

The following Table B shows that in the reports of subjects (with 2 exceptions which occured in one report) all intermediate steps are multiples of 10 or cruder (note that in the intermediate steps 25 in fact has the character of a full step number). This is a first clear sign that the expected structure with decreasing level of exactness – similar as in the construction of presentations – can be found in the reports.

Table 3: Frequencies of Numbers as Final Results and Intermediate Steps of Reports

	0	1	2	3	5	6	7	8	9	10	12	15	18	20	22	25	28	30	32	35	37	40	42	43	45	48	49	50
or	100	99	98	97	95	94	93	92	91	90	88	85	82	80	78	75	72	70	68	65	63	60	58	57	55	52	51	50
final	-	1	1	1	2	1	1		1	3	1	6	1	8		7	1	7		5	1	7	1	1	5	2	1	1
inter	24				1		1			10				10		4		12				7						27

5 Principles of the Construction of Reports

A detailed analysis of the reports suggested to distinguish the following three phases:

(pha) the three phases of a report

part 1: selection of an anchor point $A = 0$, 50 or 100

part 2: first refinement of anchor point by adding or subtracting a full step number. phase f can be applied repeatedly, new values of F replace preceding ones (F-values after the last selected F-value are not reported)

part 3: refinement of obtained preliminary result by adding or subtracting a full step number. phase r can be applied repeatedly, R-values are reported only if they are really added or subtracted.

Example: A subject reports the numbers $50 - 70 - 60 - 62$. The anchor point is $A = 50$. The first attempt of refinement is made by adding $F_1 = +20$ what gives the next component $A + F_1 = 50 + 20 = 70$ of the report. $F_2 = +10$ replaces F_1 and gives $A + F_2 = 50 + 10 = 60$. Thereafter follows the refinement $R = +2$, which gives the final result $A + F_2 + R = 50 + 10 + 2 = 62$. This report can be described by its sequence of responded numbers $50 - 70 - 60 - 72$, or by its AFR-values $A = 50$, $F_1 = +20$, $F_2 = +10$, $R = +2$. The AFR-type of the report is the sequence $AFFR$ denoting that the report consists of one A-value, 2 F-values, and one R-value. It can happen that subjects forget to report components. As an example consider the report '$50 - 70 - 62$'. Assuming that subjects only select full step numbers, we conclude that the report omitted the 60 after 70, i.e. $F = 10$ after $F = 20$. Added components are inserted in brackets, as in '$A = 50$, $F = 20$, ($F = 10$), $R = -2$', or indicated by dots as in '50-70-..-62', or in the AFR-type '$AF.R$'.

We debriefed the reports in such a way that – if possible – the given structure is obtained by using full step numbers for all F- and R-values. The question was whether we could expect that subjects did not forget one or the other number in their report:

(omi) components of a report that are omitted

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phase 1: the anchor point may be omitted
phase 2, 3: the second last component of a report may be omitted, but only in cases where thereby $2+1$, $5+2$, or $10+5$ is added or subtracted in one instead of two steps

For reports with at least 2 components (after omissions) $A = 0$ is omitted in 14 of 22 cases, $A = 50$ in 15 of 70 cases, and $A = 100$ in 7 of 33 cases. – The addition or subtraction of $2+1$, $5+2$, $10+5$ in one step happens for $2+1$ and $5+2$ in all 5 cases, for $10+5$ in 5 of 12 cases where the omission principle could be applied. – The indicate that components seem not to be forgotten by error, but rather following certain principles. That the anchor point is not mentioned so frequently suggests that many subjects are not consciously aware that their considerations are based in an anchor point.

The two principles (pha) and (omi) essentially restrict possible reports. Nevertheless they describe possible reports surprisingly well.

Predictive power: 129 of the 135 reports with more than one component (in the response) fulfill conditions (pha) and (omi).

6 Further Principles of the Construction of Reports

Another question is whether mistakes that are made during the search process are reported or not. (These mistakes can help to explain whether a certain choice is made, and might be therefore mentioned by the subjects in order to explain why they came to their decision.) The result of the investigation is that errors are reported only in phase 2:

(err) errors that are mentioned in the report

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mistakes in F-values are reported
mistakes in R-values are not reported

In 39 of the 125 reports more than one F-value is given. In all cases the last of the F-values was the finally selected one. – In 30 of the 31 reports that involve R-values, all given R-values are needed to construct the final result.

The theory of prominence suggests that subjects create numerical responses as sums of full step numbers, where they stepwise decrease the fineness of analysis until the boundary of the person's sensitivity is reached. Accordingly we expect that reports are such that the response becomes finer in every step:

(ord) restrictions of order
the absolute amounts of the R-values are less than the absolute amount of the last (=selected) F-value
the R-values are ordered with decreasing absolute amounts

Both properties are always fulfilled: There are 32 reports involving R-values, in all of them the absolute amount of the first R-value is less than that of the last F-value. There are 5 cases where more than one R-value is mentioned, in all of them the R-values are decreasing.

The principles (err) and (ord) are generally valid and may be evaluated as general properties of reports.

The following properties indicate that there are further rules for the construction of reports, but we are not sure whether these properties are induced by the special kind of problems, or if they are general properties of the response behavior.

(num) number of components
subjects report at most four components
subjects report at most two F-values
subjects report at most two R-values

There is no report in our data with more than 4 components. Of course, several of the reports are not complete and omit the anchor point or the second last R-value. Table A1 shows that the proportion of omissions increases with the length of the complete report. It seems that the problems of reconstructing the complete report increase with the 'complexity' of the report. – There are no reports that involve more than two R-values.

Table 4: Frequencies of Omitted Components by Length of Complete Report

length of complete report		3	4	5	>5
omitted components	0	16	1	-	-
	1	24	15	2	-
	2	//	2	2	-
	3		//	-	-
	4			//	-

*) // refers to cases where only one component is reported, these cases are not considered here

There is only one report that involves three F-values. This result permits one of the following conclusions. Either subjects need not compare the quality of the selected F-value with that of other (neighboured) F-values in order to identify a maximum. Or the

F-values that have been considered after the subjects have found the optimal F-value are not reported, i.e. only those F-values enter a report which were treated under the assumption that they are best responses.

(mon) 'monotonicity' of choices of F-values

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if there are 2 F-values (different from $F = 0$), then the second is either the next lower or next higher full step number

This rule is fulfilled in 28 of the 33 reports with two F-values.

An interesting question is, in which way very fine responses are created. The refinement of the response creating the last digit (of the decimal presentation) is generally performed in one (!) step (one exception, namely $6 = 5 + 2 - 1$). Table A2 shows the frequencies of these responses. The surprising results are: (1) except for responses ending with 9, no responses are given that have exactness 1, and (2) in all 4 cases where 3 or 7 are responded, the response is not created in two steps (as $3 = 2 + 1$, or $7 = 5 + 2$), but in one step, as if the numbers 3 and 7 as last digits are immediately available, and do not need a two-step presentation.

Table 5: Frequencies of Presentations of Last Digit

last digit	1	2	3		4	5	6	7		8	9	0
poss.presentat.	1	2	3	,X-7	4	5	6	7	,X-3	X-2	X-1	
frequency 1)	-	3	1	1	-	18	-*)	-	2	4	7	42(+27)

1) the report 100-3/4-72 is not contained, since the structure breaks from fraction to decimal presentation

*) once 6 was presented as 5+2-1

These observations permit to formulate the rule

(fin) fineness

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there is at most one component of the report that is finer than 10.

This statement is true in 54 of the 55 cases where results are obtained that have a finer exactness than 10.

7 Shortness

From a theoretical point of view it seems reasonable that the decision process should be developed such that the obtained results are 'short'. Obviously it only makes sense to define shortness for 'complete' reports:

Definition: A report is complete, if no component is omitted.

There are two reasonable definitions of shortness.

Definition 1: A report is short, if there is no report with less components that gives the same number.

Definition 2: A report $X = (x_1, \ldots, x_r)$ is short* if there is a number x, such that $|x_r - x|, \ldots, |x_1 - x|$ is lexicographically minimal under all reports $(y_1, \ldots y_s)$ with $|y_s - x| \leq |x_r - x|$.

It can be shown that all reports that are short* are also short. – Examples: $15 = 20-10+5$ is neither short nor short*. $17 = 10+5+2$ and $17 = 20-5+2$ are short but not short*, $17 = 20-2-1$ is short and short*.

(sho) shortness

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reports are short and short*

8 reports are not short*, all of them are also not short. The exceptions are presented in Table 6. The table illustrates that there are reasonable explanations that can explain the deviations from shortness. The explanations assume that reports can contain errors of F-values, or false selections of anchor points that are corrected afterwards. Although we do not want to extend the model in a way to permit such errors, the reader may nevertheless see that violations of shortness only occur because the given model strictly assumes that subjects either do not make errors when they produce their reports, or do not mention these errors in their reports.

Table 6: Reports that Violate the Shortness Property

report	given presentation		short presentation	re-interpretation *) (permitting errors)
one false component reported:				
(100)-80-70-75	(100)-20-10+5	-->	(100)-20-5	(100)-80-\70-75
(50)-70-80-75	(50)+20+10-5	-->	(50)+20-5	(50)-70-\80-75
(0)-20v30-25	(0)+20+10-5	-->	(0)+20+5	(0)-20-\30-25
(50)- 0-30-20	(50)-20-10	-->	20	0 -\30-20
new anchor point found during the process:				
(50)-70v100-80	(50)+50-20	-->	100 -20	\(50)-\70-100-80
100 -75-50-60	100 -50+10	-->	50 +10	\100 -\75-50-60
(100)-90-50-70	(100)-50+20	-->	50 +20	\(100)-\90-50-20
no re-interpretation:				
(50)-70-80-78	(50)+20+10-2	-->	100 -20-2	no re-interpretation

*) re-interpretation by assuming that errors are sometimes reported (errors denoted by \..); in reports 5,6,7 the 'correct' anchor point is found in the third component

Although there are only a few exceptions from shortness, the result is that the reports tend to contain errors that are induced by 'wrong' selection of anchor points, or reports of false components. This suggests that responses are more likely to be constructed by a decision process. A model that describes this process is given in the next section.

8 A Process Model for the Construction of Numerical Responses

The properties collected so far permit to construct a precise model that describes the process creating numerical responses (see Table below). The process uses the following notations

> Notation: $dir(signal|A)$ is +1 if the signal is greater than A, -1 if the signal is less than A, and 0, when the decision maker cannot decide.
>
> Notation: '$x/2'$', '$x*2'$' denote the nearest full step number to $x/2$, $x*2$, ('50/2' may also be selected as 25 instead of 20).

We assume that the information of the decision maker which serves to identify the optimal response can be used to make the following judgements:

> 'direction': The decision maker can decide whether a numerical response is greater or less than the signal, or if she is indifferent.

The operator $dir(signal|X)$ gives +1, -1, or 0 if the signal is above X, below X, or she cannot decide.

> 'preference': Given two numerical responses, the decision maker can decide whether one of the alternatives is nearer to the signal than the other, or if she is indifferent.

In cases where the direction response is 'indifferent', the procedure is stopped since the response is found.

Using the notations and abilities the process model can be formulated:

```
 A Process Creating Numerical Responses 1)
+----------------------------------------------------------------+
| (phase 1:  select anchor point)                                |
|   select  A= 0, 50, or 100                                     |choice
+----------------------------------------------------------------+
| (phase 2:  determine first refinement)                         |
|   <if dir(signal|A)=0 then X=A, goto step 3>                   |dir=0?
|   select a full step number F                                  |choice
| step 1:                                                        |
|   dir=dir(signal|A), X=A+F*dir                                 |definition
|   <if dir(signal|X)=0 then goto step 3>                        |dir=0?
|   if dir<>dir(signal|X) then G='F/2' else G='F*2'              |definition
|   Y=A+G*dir                                                    |definition
|   if Y nearer to signal than X then replace F by G, goto step 1 |preference
+----------------------------------------------------------------+
| (phase 3:  determine further refinements)                      |
|   R=F                                                          |
| step 2:                                                        |
|   <if dir(signal|X)=0 then goto step 3>                        |dir=0?
|   R='R/2', Y=X+R*dir(signal|X)                                 |definition
|   if X nearer to signal than Y then goto step 2                |preference
|   S='R/2', Z=X+S*dir(signal|X)                                 |definition
|   if Z nearer to signal than Y then X=Z, R=S, goto step 2      |preference
|   if Y nearer to signal than Z then X=Y, goto step 2           |preference
+----------------------------------------------------------------+
| (give response)                                                |
| step 3:                                                        |
|   respond X                                                    |response
+----------------------------------------------------------------+
  1)  'F/2','F*2' denote the nearest full step numbers to F/2, F*2
      dir(signal|X)= direction of signal with respect to X
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Notice that the process does not only create a response, but also a corresponding report, which depends on the selection of A and the first value of F. Compared with short responses this approach permits errors in the selection of the parameters which are thereafter compensated by the system.

(Notice that the S-part of Phase 3 ensures shortness in the R-phase, and avoids that the procedure stops when – by chance – x and y are equally near to the signal, but the limit of the abilities of judgement is not yet reached.)

To evaluate the quality of the given model we give the following definitions:

Definition: A preference is called (t, s)-preference, if there is a 'true value' t and a 'sensitivity level' s such that
x nearer to the signal than $y \leftrightarrow |x-t| - |y-t| < s$
$dir(signal|x) = +1 \leftrightarrow x - t > s$

$dir(signal|x) = -1 \leftrightarrow x - t < s$
$dir(signal|x) = 0 \leftrightarrow -s \leq t - x \leq s$

The idea of this definition is that the signal is somehow similar as a distribution with a midpoint in t, that the quality of two responses cannot be distinguished when their distances to t differ by less than s. – We do not assume that subjects preferences are always (t, s)-prefernces, but we will see below that subjects reports are as if there preferences are (t, s)-preferences. This permits to judge the quality of the process model more easily:

> Definition: A report is called strongly compatible with the process model, if there is a (t, s)-preference, such that the report follows the process model (if one selects A and the first F-value as in the report).
>
> Definition: A report is called weakly compatible with the process model, if there is a strongly compatible report from which the given report is obtained by omitting components.

Experimental result:

> 127 of the 134 given reports of the subjects are weakly compatible with the process model.

(2 of the 7 exceptions mention values $F = 0$ in the report. One report is obtained by starting with 1/3 and then rounding to 30%. Remain 4 unexplained deviations, of which 3 do not seem to follow any system.)

Table 7 presents the frequencies of different types of reports, where the types are given by the process model (for the respective A- and first F-values that gave the maximal fit). From our point of view it seems very interesting that the total number of components of the respective complete reports never exceeds 5, and that the remembered number of components is always below 4.

Table 7: Types of Reports and Their Frequencies 1) 2)

AF	.F	AFF	.FF		AFFF	.FFF	AF'F	.F'F			
65	//	8	7		-	2	1	2			
		AFR	.FR	A.R	AFFR	.FFR	AF.R		AFFFR	.FFFR	.F'FR
		7	8	8	2	8	1		-	-	1
					AFRR	.FRR	AF:R		AFFRR	.FFRR	AFF:R
					-	4	2		-	1	1

1) A= anchor point, F= first component, R= refinement, .= omitted A or F, := omitted R, '= missing F-values by process model

2) the 7 reports that are not explained by the theory are not included

9 Concluding Remarks

The paper illustrates that the theory of prominence permits to describe numerical responses. This holds for the rather general approach given by the phase model (pha) and the rules (omi) of omitting components. Moreover, the observed responses enable us to identify general principles of construction, concerning which errors are mentioned in the report (err), the restrictions in order (ord), and the number of components (num), and the monotonicity of choices of F- and R-values.

It could be shown that the theoretical construct of shortness is frequently fulfilled, however the best fit is obtained by the process model of Section 8.

The reader may remember that the paper only deels with responses on the percent scale. We expect that the general sructure of numerical responses is similar, but will probably not select an anchor point in phase 1. In the investigation here we failed to obtain usable results for general numerical responses, since we had not sufficiently cared to restrict the questions such that only decimal responses are given, such that results are not obtained as sums, and such that responses did not have a priori correct answers.

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Appendix 1

Table A, Part 1: Reports with at least Three Components

freq	report 1)	A, F-, and R-values 2)			type 3)	comment 4)
exactness finer than 5:						
3	100-99	100	-1		AF	c
.	100-..-97	100	(-2	-1h)	A(.R)	c
.	100-..-93	100	(-5	-2h)	A(.R)	c
.	100-10C-89	100	-10	-1e	AFR	c
.	100-3/4-..-72	100	-1/4	(-2 -1)	AF(.R)	c (fraction)
.	..-70-80-78	(50)	+20	+10h -2Q	.FRR	c
.	50-52	50	+2		AF	c
2	50-49	50	-1		AF	c
.	50-48	50	-2		AF	c
.	50-40-..-43	50	-10	(+2Q +1h)	AF(.R)	c
.	50-40-42	50	-10	+2Q	AFR	c
.	50-30v40-..-37	50	-20 -10H	(-2Q -1h)	AFF(.R)	c
2	.-20-18	(0)	+20	-2E	.FR	c
.	.-10-9	(0)	+10	-1E	.FR	c
.	.-10-5-7-6	(0)	+10 +5H	+2H -1H	.FFRR	c
.	0-2	0	+2		AF	c
exactness 5:						
2	100-95	100	-5		AF	c
2	100-90-85	100	-10	-5h	AFR	c
.	..-90-85	(100)	-10	-5h	.FR	c
.	..-80-90-85	(100)	-20 -10H	-5H	.FFR	c
.	..-80-70-75	(100)	-20	-10h +5H	.FRR	c
.	..-70-80-75	(50)	+20	+10h -5H	.FRR	c
3	50-..-65	50	(+10	+5h)	A(.R)	c
3	50-55	50	+5		AF	c
5	50-45	50	-5		AF	c
.	50v40-45	50	-10 -5H		AFF	c
.	50-30-..-45	50	-20(-10H)-5H		AF.F	c
.	..-30-..-45	(50)	-20(-10H)-5H		.F.F	c
.	..-30-40-35	(50)	-20 -10H	-5H	.FFR	c
.	50-..-35	50	(-10	-5h)	A(.R)	c
.	50-40-35	50	-10	-5h	AFR	c
.	..-30-25	(50)	-20	-5q	.FR	c
.	.-20v30-25	(0)	+20	+10h -5H	.FRR	c
.	.-20-15	(0)	+20	-5Q	.FR	c
.	.-20-10-15	(0)	+20 +10H	+5H	.FFR	c
2	.-10v20-15	(0)	+10 +20d	-5H	.FFR	c
.	.-10-20-15	(0)	+10 +20d	-5H	.FFR	c
.	0-..-15	0	(+10	+5h)	A(.R)	c
.	0-5	0	+5		AF	c

1) frequency 0 denoted by dot

2) d,h,H,Q,E = absolute amount is roughly double, half, quarter, eighth of preceding absolute amount

3) (.R) denotes 3, 7, or 15 when given in one component

4) c denotes reports that are weakly consistent with the process model.
\ denotes false responses

Table A, Part 2: Reports with at least Three Components

exactness 10:

freq	report 1)	A	F-values	R-values	type 2)	comment 3)
8	100-90	100	-10		AF	c
.	..-90-80	(100)	-10 -20d		.FF	c
.	100-75-50-60	100	-25 -50d	+10H	AFFR	c
.	..-70v100-80	(50)	+20 +50d	-20H	.FFR	c
2	..-100-..-60	(50)	+50(+20)+10		.F.F	c
.	..-3/4-60	(50)	+25 +10H		.FF	c
11	50-60	50	+10		AF	c
6	50-40	50	-10		AF	c
.	50v30-40	50	-20 -10H		AF	c
2	50-25-35	50	-25	+10H	AFR	c (fraction)
.	50-40-30	50	-10 -20d		AFF	c
.	..-40-50-30	(50)	-10 0 -20d		AFOF	- (F=0 mentioned)
.	..- 0-30-20	(50)	-50 -20H	-10H	.FFR	c
.	0-100-..-40	0	+100(+50H)	-10	AF.R	c
.	.-50-20-30	(0)	+50 +20H	+10H	AFFR	c
2	.-10-50-20	(0)	+10 +50 +20H		.FFF	c
.	0-10-20	0	+10 +20d		AFF	c
.	.-20-10	(0)	+20 +10H		.FF	c
2	0-10	0	+10		AF	c

exactness 20 or 25:

freq	report 1)	A	F-values	R-values	type 2)	comment 3)
4	100-80	100	-20		AF	c
2	..-50-80	(100)	-50 -20H		.FF	c
.	100-75	100	-25		AF	c
.	..-90-..-50-70	(100)	-10(-20)-50d	+20H	.F.FR	c
.	50-100-75	50	+50 +25H		AFF	c
.	50v100-75	50	+50 +25H		AFF	c
.	50-1/4C-75	50	+25		AF	c
.	50-75	50	+25		AF	c
.	50-100-70	50	+50 +20H		AFF	c
.	50v100-70	50	+50 +20H		AFF	c
3	50-70	50	+20		AF	c
.	..-30-50	(50)	-20 0		AFO	- (F=0 mentioned)
5	50-30	50	-20		AF	c
2	..- 0-30	(50)	-50 -20		.FR	c
.	0-20	0	+20		AF	c

exactness 50:

freq	report 1)	A	F-values	R-values	type 2)	comment 3)
.	100-50	100	-50		AF	c
.	0v100-50	0	+100 +50H		AFF	c

reports that are not explained by the approach:

freq	report 1)	A	F-values	R-values	type 2)	comment 3)
.	40-70	(50)	-10 +20		.FF	- (F posit & negat)
.	0-70Cv80C-30	(50)	-50 -20H	\-10h	.FF\R	- (\R mentioned)
.	15-20-25	???				- (steps of 5)
.	0v50-36	???				- ??
.	80-40	???				- ??

rounding fractions to decimals:

freq	report 1)	A	F-values	R-values	type 2)	comment 3)
.	0-(1/8)-12	0	(+10)	+2	A.R	(c)(1/8 --> 12)
.	1/3-30					- (1/3 --> 30)

Appendix 2

Table A1: List of Questions

(1) How would you evaluate the taste of coke on a scale from 0 to 100?
(2) What do you think, how many percent of the Italians are catholic?
(3) What do you think, how many percent of the cars in West-Germany have a catalyst?
(4) What do you think, how many percent of the students eat in the mensa for at least three times a week?
(5) What do you think, which might be the number of inhabitants of Peking?
(6) What do you think, how many percent of the inhabitants of West-Germany are smokers?
(7) What do you think, how many percent of the grown-up population of West-Germany drink coffee regulary?
(8) What do you think, how many spokes has a wheel of a bicycle?
(9) What do you think, how many kilometers of highways are in West- Germany?
(10) What do you think, how many men have been on the moon?
(11) The inhabitants of West-Germany go on holiday between one and two times a year. What do you think is the exact value (mean)?
(12) What do you think, how long is a filter cigarette (in cm)?
(13) What do you think, how long needs an air-mail letter to Australia?
(14) What do you think, how many households in West-Germany are connect with a cable net?
(15) What do you think, how many german households have at least one pet?
(16) What do you think, how many percent of employed persons listen to radio while working?
(17) What do you think, how many percent of the German inhabitants wear glasses?
(18) What do you think, how many percent of the grown-up persons have a PC?
(19) What do you think, how many percent of the new born children of the last year were female?
(20) What do you think, how many percent of the West-German households have at least one video-recorder?
(21) What do you think, how many percent of the West-German children at elementary school age have pizza as favorite dish?
(22) What do you think is the probability of an armed intervention of the US in Iraque within the next 10 years?[2]

[2]In January/February 1993 there were some small military interventions of the USA in the Iraque. The investigation was completed far before the military intervention of the US in the Iraque.