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Equilibrium Selection in 2x2 Bimatrix Games with Preplay Communication

by

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Abstract

Models describing the selection of an equilibrium in 2x2 bimatrix games with two strong equilibria are the risk dominance (J.C. Harsanyi, R. Selten, 1988) and the Nash-criterion (J.F. Nash, 1950b and 1953) applied to the selection in 2x2 bimatrix games. Two new models based on the theory of prominence (W. Albers, G. Albers, 1983, W. Albers, 1997) which describes the perception of numbers (especially payoffs) are presented. One of the models is connected to risk dominance and the other to the Nash-criterion. Experiments using the strategy method were performed to test the predictions of the different models. Results of these experiments are that the predictions of the models based on the theory of prominence are in a better agreement with the data than the other models. The predictions of the model based on the theory of prominence and related to risk dominance cannot be rejected by these experiments.

1. Introduction

2x2 bimatrix games represent the simplest form of an interpersonal conflict. 2 persons, the players, can choose between two alternatives, their strategies. The strategy combination leads to a payoff. A classification of all 2x2 bimatrix games based on the ordinal ordering of the payoffs was performed by A. Rapoport and M. Guyer (A. Rapoport and M. Guyer, 1966).

In a mixed extension of a 2x2 bimatrix game players can play mixed strategies. A mixed strategy of a player is a probability distribution on his pure strategies. The payoff is according to the expected value of the mixed strategy.

A solution concept for noncooperative games, especially 2x2 bimatrix games, is the Nash equilibrium (J.F. Nash, 1950a and 1951). An existence theorem can be proved for the mixed extension of all matrix games. In non degenerated 2x2 bimatrix games there are three possible cases: there is one equilibrium point in pure strategies, or there is one equilibrium point in mixed strategies, or there are three equilibrium points (two in pure strategies and one in mixed strategies).

In 2x2 bimatix games in which more than one equilibrium exists the equilibrium selection is a problem. A solution of this problem is proposed by the concept of risk dominance (J.C. Harsanyi, R.Selten, 1988). For 2x2 bimatrix games this solution is given by an axiomatic approach. Another concept that can be applied to the equilibrium selection is the Nash-Zeuthen bargaining model (J. Nash 1950b and 1953, F. Zeuthen, 1930). A discussion of these concepts is given in the second part of the paper.

Two new models (model I and model II) are introduced. Model I is based on the Nash-Zeuthen bargaining and Model II is based on risk dominance.

To test the predictions of the models experiments using the strategy method were performed. These are described in the third part of the paper. In these experiments communication was possible. The arguments given by the subjects for one of two possible equilibrium points are collected and interpreted.

A basic concept for the new models (model I and model II) is the theory of prominence (W. Albers, G. Albers, 1983; W. Albers, 1997) which is used for the description of the perception of payoffs.

Some notations, basic definitions and basic theorems about 2x2 bimatrix games:

The normal form of a game G is G(N,St,a).

N={1,..,n} is the set of players.

The strategies of player i ($i \in N$) are denoted by s_i and the strategy set of a player i is denoted by St_i ($s_i \in St_i$).

A strategy combination is denoted by s: $s = s_1 \times s_2 \times .. \times s_n$.

The strategy combination set is $St = St_1 \times St_2 \times .. \times St_n$.

a is the payoff function: a: $St \rightarrow \mathbf{R}^n$.

A 2-person game (N={1,2}) with m strategies of player 1 and n strategies of player 2 (m and n are countable numbers) is called an mxn bimatrix game. It can be represented by two mxn matrices $A = (a_{ij})$ and $B = (b_{ij})$ or the mxn bimatrix (A,B) of the pairs (a_{ij}, b_{ij}). For m=n=2 the game is called a 2x2 bimatrix game. In the following parts of the paper the notations of figure 1 will be used for a 2x2 bimatrix game. A 2x2 bimatrix game is non degenerated if the two matrices are not degenerated.

Figure 1 : The normal form of a 2x2 bimatrix game

	U_2	V_2
U_1	a_{11}, b_{11}	a_{12}, b_{12}
V_1	a_{21}, b_{21}	a_{22}, b_{22}

In the following parts of the paper only games with finite strategy sets of each player are considered.

The mixed extension of a game G(N,St,a) with finite strategy sets of each player i is the game $G(N,\overline{St},\overline{a})$.

The set of players is N={1,..,n}.

A mixed strategy σ_i of player i is defined as: σ_i: $St_i \rightarrow [0,1]$
with $\sigma_i \geq 0$ and $\sum_{s_i \in St_i} \sigma_i(s_i) = 1$ for all players i.

A mixed strategy combination is denoted by $\sigma = \sigma_1 \times \sigma_2 \times .. \times \sigma_n$.
The mixed strategy combination set is $\overline{St} = \overline{St_1} \times \overline{St_2} \times .. \times \overline{St_n}$.
The payoff function is: $\overline{a}(\sigma) = \sum_{s \in St} \sigma_1(s_1) * \sigma_2(s_2) * .. * \sigma_n(s_n) * a(s)$.

A pair (σ_1^*, σ_2^*) of mixed strategies of the bimatrix game (A,B) is in a (Nash) equilibrium, if for all mixed strategies σ_1' and σ_2'
$A(\sigma_1', \sigma_2^*) \leq A(\sigma_1^*, \sigma_2^*)$
$B(\sigma_1^*, \sigma_2') \leq B(\sigma_1^*, \sigma_2^*)$ holds.

A pair (σ_1^*, σ_2^*) of mixed strategies of the bimatrix game (A,B) is in a strong (Nash) equilibrium, if for all mixed strategies σ_1' and σ_2'
$A(\sigma_1', \sigma_2^*) < A(\sigma_1^*, \sigma_2^*)$
$B(\sigma_1^*, \sigma_2') < B(\sigma_1^*, \sigma_2^*)$ holds.

An equilibrium is called an equilibrium in pure strategies, if in the equilibrium for all players it holds $\sigma_i(s_i)=1$ for one pure strategy s_i of player i. An equilibrium is called an equilibrium in mixed strategies in all other cases.

The following well known theorems may be mentioned.

- Theorem (J.F. Nash, 1950a and 1951):
  The mixed extension of a bimatrix game has at least one equilibrium point.

- Theorem (C.E. Lemke, J.T. Howson, 1964):
  The mixed extension of a non degenerated bimatrix game has an odd number of equilibrium points.

- Theorem (see P. Borm 1990, p. 60):

  The mixed extension of a non degenerated 2x2 bimatrix game has 3 equilibrium points.

  There are 3 possible cases:
  one equilibrium point in pure strategies.
  one equilibrium point in mixed strategies.
  two equilibrium points in pure strategies and one in mixed strategies.

2. Models of the equilibrium selection

In this part of the paper models describing the equilibrium selection in 2x2 bimatrix games are discussed. The results of these models are the hypotheses tested in the experiment. In the following sections only 2x2 games with 2 equilibrium points in pure strategies (and one in mixed strategies) are considered. It is assumed that one player prefers one equilibrium point and the other player the other equilibrium point. None of the two equilibrium points dominates the other one in payoffs, i.e in none of the two equilibrium points the payoff for both players is higher than in the other one. Using the notations of figure 1 (if without loss of generality $U=U_1U_2$ and $V=V_1V_2$ are the equilibrium points) it holds:

$u_1= a_{11}-a_{21}>0$, $u_2=b_{11}-b_{12}>0$, $v_1=a_{22}-a_{12}>0$, $v_2=b_{22}-b_{21}>0$; with ($a_{11}>a_{22}$ and $b_{22}>b_{11}$) or ($a_{22}>a_{11}$ and $b_{11}>b_{22}$).

In the following parts of the paper it is also assumed without loss of generality that $a_{22}>a_{11}$ and $b_{11}>b_{22}$. In this situation player 1 prefers to select the strategy V_1, because he gets the highest payoff a_{22} for the strategy combination $V=V_1V_2$. Player 2 prefers to play U_2, because his payoff is maximal for the strategy combination $U=U_1U_2$. The conflict case occurs if both players insist on playing their preferred strategies which leads to a strategy combination of V_1U_2. The strategy combination U_1V_2 occurs in the case of miscoordination: both players select the non preferred strategies. Then the payoffs in the bimatrix can be denoted according to figure 2. The maximal payoff is denoted by a_{max} and b_{max}, the second highest equilibrium payoff by a_{alt} and b_{alt}, the conflict payoff by a_{min} and b_{min} and the miscoordination payoff by a_{mis} and b_{mis}. This notation will be used in the following parts of the paper.

Figure 2 : The notations used for a 2x2 bimatrix game

	U_2	V_2
U_1	a_{alt}, b_{max}	a_{mis}, b_{mis}
V_1	a_{min}, b_{min}	a_{max}, b_{alt}

2.1 The Nash-criterion

For the selection between two equilibria of 2x2 bimatrix games the Nash-criterion (J.F. Nash, 1950b and 1953, Zeuthen, 1930) can be considered. It is bargained about pairs of payoffs (a,b). A threat payoff (which can be interpreted as conflict payoff) is given by (a_{min}, b_{min}). The Nash-criterion selects that payoff (a^*, b^*) for which the product $(a^* - a_{min}) * (b^* - b_{min})$ is maximal.

This idea can be applied to the selection between the equilibrium points of 2x2 bimatrix games. The two equilibrium points are the alternatives. The threat point is given by the conflict payoff. The equilibrium preferred by the Nash-criterion is selected. Using the notations of figure 2 the selection critierion is:

V dominates U [U dominates V] iff

$(a_{max} - a_{min}) * (b_{alt} - b_{min}) > [<] \; (a_{alt} - a_{min}) * (b_{max} - b_{min})$

This criterion is equivalent to the criterion:

V dominates U [U dominates V] iff

$(a_{max} - a_{alt}) / (a_{alt} - a_{min}) \quad > [<] \; (b_{max} - b_{alt}) / (b_{alt} - b_{min})$

2.2 Risk Dominance

J.C. Harsanyi and R. Selten (J.C. Harsanyi, R.Selten, 1988) developed the concept of risk dominance for the equilibrium selection in all games. For the class of games considered here they give an axiomatic characterization. Using the notation of figure 2 the criterion of risk dominance is:

V dominates U [U dominates V] iff
$(a_{max}-a_{mis})*(b_{alt}-b_{min}) > [<] \ (a_{alt}-a_{min})*(b_{max}-b_{mis})$.

An equivalent formulation is:
V dominates U [U dominates V] iff
$(a_{max}-a_{mis})/(a_{alt}-a_{min}) > [<] \ (b_{max}-b_{mis})/(b_{alt}-b_{min})$.

2.3 Models using the Theory of Prominence

Before describing the models that use the theory of prominence (W. Albers, G. Albers, 1983 and W. Albers, 1997) the results of the theory of prominence used in the models is described. One result of the theory of prominence is that some numbers are easier accessible than others. The numbers that are most easily accessible are the prominent numbers P:

$P=\{z*10^n | n\in \mathbf{Z}, z\in\{1,2,5\}\}=\{...,0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100,....\}$.

If the perception is spontaneous the so called spontaneous numbers S are the numbers that are accessible. These are:

$S=\{z*10^n | n\in \mathbf{Z}, z\in\{-7, -5, -3, -2, -1.5, -1, 0, 1, 1.5, 2, 3, 5, 7\}\}$.

The spontaneous numbers include the prominent numbers and one additional number between any two neighboured prominent numbers.

The perception of numbers (also payoffs) is described as difference of steps between spontaneous numbers. The difference between two prominent numbers (ordered according to their size) is 1 step and between two spontaneous numbers (ordered according to their size) 1/2 step.

Another important empirical observation is that the perception is limited for small numbers. There is a smallest unit that can be perceived. In the theory of prominence this is modeled by assuming a smallest full step money unit Δ which permits to define a perception function P_Δ mapping monetary payoffs to the perception space. Table 1 gives the function for Δ=10, for Δ=20 and the spontaneous numbers between -150 and +150 (which are relevant for the experiment).

Table 1: Transformation of the spontaneous numbers between -150 and 150 by the P-function for Δ=10 and Δ=20.

number:	-150,	-100,	-70,	-50,	-30,	-20,	-15,	-10,	-5,	0,
P_{10} :	-4.5,	-4,	-3.5,	-3,	-2.5,	-2,	-1.5,	-1,	-0.5,	0,
P_{20} :	-3.5,	-3,	-2.5,	-2,	-1.5,	-1,	-0.75,	-0.5,	-0.25,	0,

number:	5,	10,	15,	20,	30,	50,	70,	100,	150
P_{10} :	0.5,	1,	1.5,	2,	2.5,	3,	3.5,	4,	4.5
P_{20} :	0.25,	0.5,	0.75,	1,	1,5,	2,	2,5,	3,	3.5

The exact numbers are a further refinement of the spontaneous numbers. The exact numbers between 0 and 100 and the P_Δ-functions are given for Δ=10 and Δ=20 in table 2 (which are relevant for the experiment).

Table 2: The exact numbers between 0 and 100 and the P_Δ-function for $\Delta=10$ and $\Delta=20$.

$\Delta=10$:

	0	2,3	5	7,8	10	12,13	15	17,18	20	25	30	35,40	50	60	70	80	100
P_{10}:	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4

$\Delta=20$:

	0	5	10	15	20	25	30	35,40	50	60	70	80	100
P_{20}:	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3

For numbers higher than Δ the function $P_\Delta(s)$ is (nearly) equal to $3*\log(s/\Delta)+1$[1]. Below the smallest unit Δ the function is linear (compare table 1).

This description of the perception is similar to the Weber-Fechner Law (for example in G.T. Fechner, 1968) which describes the perception of stimuli in psychophysics. The perception is proportional to a logarithmic function above a smallest unit. Comparisons between stimuli are performed by forming differences (not quotients). This seems to be plausible, since the stimulus has been transformed by a function proportional to the logarithm.

In the experimental data the smallest unit has to be determined. This is performed by means of a rule of the theory of prominence. Before formulating the rule some definitions are given below (compare W. Albers, 1997).

A presentation of a number is its presentation as a sum of prominent numbers, where each prominent number occurs at most once, and all coefficients are either +1, -1, or 0. The exactness of a presentation is the smallest prominent number with coefficient

[1] For numbers $x \geq \Delta$ it holds: $|P_\Delta(x)/(3*\log(x/\Delta)+1)-1| \leq 7\%$

unequal zero. The exactness of a number is the crudest exactness over all presentations of the number. The relative exactness of a number x is the exactness devided by |x|[2]. A number has level of [relative] exactness r, if its [relative] exactness is cruder or equal to r. A set of data has [level of] relative exactness r, if at least 75% of the data have this [level of] relative exactness[3].- A scale S(r,a) is the set of 0 and all numbers with (1) relative exactness $\geq$r and (2) exactness $\geq$a.

Rule for the scale S(r,a) and the smallest unit Δ of a set of data:
A set of data is on the scale S(r,a) if the set of data has [level of] relative exactness of r and at least 90% of the data have an exactness of a[4]. The smallest unit of a set of data is:

Δ=a for the scales S(100,a) ("scales with prominent numbers"),
Δ=[2*a] for the scales S(26,a) ("scales with spontaneous numbers"),
Δ=[4*a] for the scales S(10,a) ("scales with exact numbers").

[2]0 has relative exactness of 100% and absolute exactness of ∞.

[3]r has to be maximal.

[4]a has to be maximal.

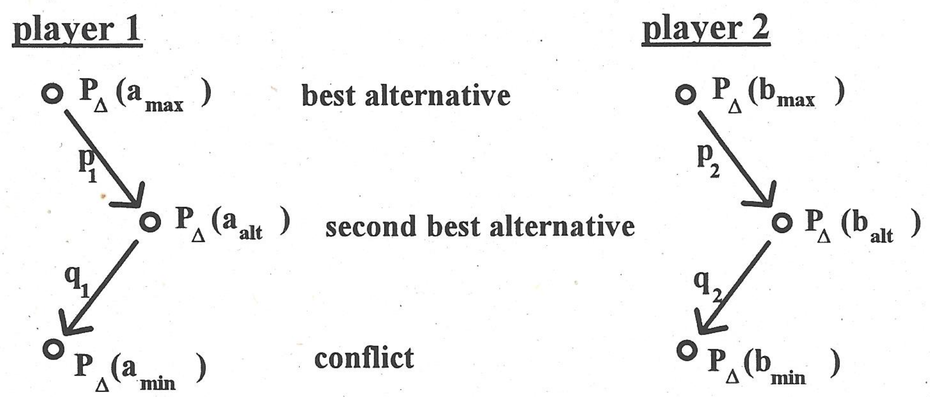
2.3.1 Model I

All payoffs are transformed by the P_Δ-function:

P_Δ: $a_{..} \rightarrow P_\Delta(a_{..})$ and
P_Δ: $b_{..} \rightarrow P_\Delta(b_{..})$, where the index .. can be max, min, alt and mis.

The selection criterion is obtained by a comparison of differences as shown schematically in figure 3.

Figure 3: Schematic presentation of the comparisons of model I



In this model the differences $p_1=P_\Delta(a_{max})-P_\Delta(a_{alt})$ and $p_2=P_\Delta(b_{max})-P_\Delta(b_{alt})$ between the best and second best alternatives are compared with the differences $q_1=P_\Delta(a_{alt})-P_\Delta(a_{min})$ and $q_2=P_\Delta(b_{alt})-P_\Delta(b_{min})$ between the second best alternative and the conflict payoff. The selection criterion is:

V dominates U [U dominates V] iff
p_1-q_1 >[<] p_2-q_2.

Replacing p and q results in
V dominates U [U dominates V] iff

$$(P_\Delta(a_{max})-P_\Delta(a_{alt})) - (P_\Delta(a_{alt})-P_\Delta(a_{min})) > [<]$$
$$(P_\Delta(b_{max})-P_\Delta(b_{alt})) - (P_\Delta(b_{alt})-P_\Delta(b_{min}))$$

Interpretation: For each player the incentive to deviate from the second best alternative (for example $(P_\Delta(a_{max})-P_\Delta(a_{alt})$ for player 1) is compared with the possible loss, if the game ends in a conflict $((P_\Delta(a_{alt})-P_\Delta(a_{min})$ for player 1). The player for whom the difference between the incentive to deviate from the second best alternative and possible loss is bigger has "more arguments" for the equilibrium point favored by him. This point will be selected.

2.3.1.1 Relations between model I and the Nash-criterion

For a comparison the criterion of Model I and the Nash-criterion are presented in a way that the same terms are at the same places.

V dominates U [U dominates V] iff

Nash: $(a_{max} - a_{alt}) / (a_{alt} - a_{min}) > [<]$

Nash: $(b_{max} - b_{alt}) / (b_{alt} - b_{min})$

Model I: $(P_\Delta(a_{max})-P_\Delta(a_{alt})) - (P_\Delta(a_{alt})- P_\Delta(a_{min})) > [<]$

Model I: $(P_\Delta(b_{max})-P_\Delta(b_{alt})) - (P_\Delta(b_{alt})- P_\Delta(b_{min}))$

The difference between the two models is that in model I payoffs are transformed by the P_Δ-function and that after the transformation quotients in the Nash-criterion correspond to differences in model I. This is compatible with the fact that after a transformation by the P_Δ-function (which is for values bigger than Δ a logarithmic function) the quotients are transformed into differences.

Another difference of these approaches is that the P_Δ-function operates on payoffs and not on differences of payoffs. For example the difference $(a_{alt} - a_{min})$ is transformed into $P_\Delta(a_{alt}) - P_\Delta(a_{min})$ and not into $P_\Delta(a_{alt} - a_{min})$. This will be discussed in section 2.3.2.1.

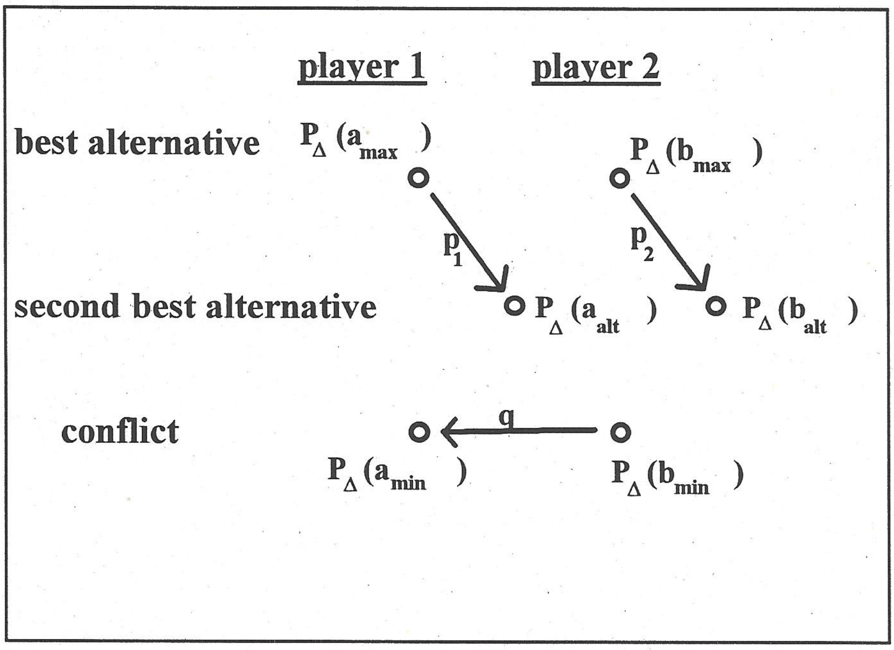
2.3.2 Model II

All payoffs are transformed by the P_Δ-function:

P_Δ: $a_{..} \rightarrow P_\Delta(a_{..})$ and
P_Δ: $b_{..} \rightarrow P_\Delta(b_{..})$, where the index .. can be max, min, alt and mis.

The selection criterion is obtained by comparison of differences as shown schematically in figure 4.

Figure 4: Schematic presentation of the comparisons in model II



In this model the differences $p_1 = P_\Delta(a_{max}) - P_\Delta(a_{alt})$ and $p_2 = P_\Delta(b_{max}) - P_\Delta(b_{alt})$ between best and second best alternative for each player are compared with one another. The difference p_1-p_2 is then compared with the difference $q=P_\Delta(b_{min})-P_\Delta(a_{min})$) between the conflict payoffs. The selection criterion is:

V dominates U [U dominates V] iff

$$p_1 - p_2 > [<] \; q.$$

Replacing p and q leads to the criterion:

V dominates U [U dominates V] iff

$$(P_\Delta(a_{max})-P_\Delta(a_{alt}))-(P_\Delta(b_{max})-P_\Delta(b_{alt})) >[<] (P_\Delta(b_{min})-P_\Delta(a_{min}))$$[5]

Interpretation: in this model an advantage (higher transformed payoffs) in comparing the equilibrium payoffs can be compensated by a disadvantage in comparing the conflict payoffs. The equilibrium point is selected if its advantage is bigger than its disadvantage.

A different approch to the criterion is given by the following consideration. Every transformed payoff can be seen as an "argument" supporting one equilibrium point or the other. Giving a weight of +1 and -1 for supporting one or the other one, respectively, a sum of "arguments" can be calculated. That equilibrium is selected for which the sum is greater than for the other. The "arguments" of player 1 for the equilibrium point V are:

$+P_\Delta(a_{max})$, $-P_\Delta(a_{alt})$ and $+ P_\Delta(a_{min})$.

The "arguments" of player 2 for the equilibrium point U are:

$+P_\Delta(b_{max})$, $-P_\Delta(b_{alt})$ and $+ P_\Delta(b_{min})$.

The weight for the maximal and minimal tranformed payoffs of the players is +1 because a high maximal and minimal payoff support the preferred equilibrium point. The weight for the second best alternative is -1 because a high payoff for the second best alternative supports the other equilibrium point and not the preferred one. The selection criterion is given by a comparison of the sums of arguments:

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[5]It is assumed that $a_{mis}=b_{mis}=0$.

V dominates U [U dominates V] iff
$P_\Delta(a_{max})-P_\Delta(a_{alt})+P_\Delta(a_{min})>[<]$
$P_\Delta(b_{max})-P_\Delta(b_{alt})+P_\Delta(b_{min})$

This is the same selection criterion as the one given above.

Another presentation of the selection criterion is:
V dominates U [U dominates V] iff
$P_\Delta(a_{max})+P_\Delta(b_{alt})+P_\Delta(a_{min})>[<] P_\Delta(a_{alt})+P_\Delta(b_{max})+P_\Delta(b_{min})$

Here the criterion can be interpreted as the sum of the two criteria:

$P_\Delta(a_{max})+P_\Delta(b_{alt}) >[<] P_\Delta(a_{alt})+P_\Delta(b_{max})$ **"Nash-sum"**

\+ $P_\Delta(a_{min}) >[<] P_\Delta(b_{min})$ **"Conflict"**

= $P_\Delta(a_{max})+P_\Delta(b_{alt})+P_\Delta(a_{min})>[<] P_\Delta(a_{alt})+P_\Delta(b_{max})+P_\Delta(b_{min})$

The first criterion is called "Nash-sum" because it corresponds to the Nash-product without threats[6], as it can be seen below:

if $P_\Delta(x)=\log(x)$ then

$\log(a_{max})+\log(b_{alt}) >[<] \log(a_{alt})+\log(b_{max})$,

$\Leftrightarrow$

$\log(a_{max}*b_{alt}) >[<] \log(a_{alt}*b_{max})$,

$\Leftrightarrow$

$a_{max}*b_{alt} >[<] a_{alt}*b_{max}$ ("Nash-product").

The second part is a standardform of the comparison of the conflict payoffs.

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[6] if the payoffs are higher than 0.

It is possible to consider an anchor point for the comparison of the conflict payoffs in model II. Anchor point means that all conflict payoffs are evaluated by their differences to the anchor point. If the highest conflict payoff is an anchor point for the conflict payoffs, the difference between the conflict payoffs is not $P_\Delta(b_{min})$-$P_\Delta(a_{min})$ as described above. If for example a_{min} is the anchor point the difference between the conflict payoffs is $P_\Delta(b_{min}-a_{min})-P_\Delta(a_{min}-a_{min})=P_\Delta(b_{min}-a_{min})$.

If b_{min} is the anchor point one gets the corresponding result.

2.3.2.1 The comparison between model II and risk dominance

To compare model II and risk dominace the selection criteria are written below one another (Again the criteria of risk dominance and model II are taken in a form equivalent to the original ones).

V dominates U [U dominates V] iff

Risk Dominance:	$(a_{max} - a_{mis}) / (a_{alt} - a_{min})$		> [<]
Risk Dominance:	$(b_{max} - b_{mis}) / (b_{alt} - b_{min})$		
Model II:	$P_\Delta(a_{max})$	$- (P_\Delta(a_{alt}) - P_\Delta(a_{min}))$	> [<]
Model II:	$P_\Delta(b_{max})$	$- (P_\Delta(b_{alt}) - P_\Delta(b_{min}))$	

One difference between the 2 models is again the transformation by the P_Δ-function and the correspondence of quotients (in the model of risk dominance) and differences (in model II). The reasons for this have been discussed above, i.e. the P_Δ-function is for values bigger than Δ a logarithmic function and therefore quotients are transformed into differences.

Another difference is that the payoffs for miscoordination are not taken into account in model II. A reason of this is that in the experiment preplay communication is possible and therefore miscoordination might not occur.

An extension of the risk dominance by considering the perception of payoffs and the correspondence of quotients and differences leads to the following criterion:

V dominates U [U dominates V] iff

$$(P_\Delta(a_{max}) - P_\Delta(a_{mis})) - (P_\Delta(a_{alt}) - P_\Delta(a_{min})) > [<]$$
$$(P_\Delta(b_{max}) - P_\Delta(b_{mis})) - (P_\Delta(b_{alt}) - P_\Delta(b_{min}))$$

Besides the differences discussed above the main difference is that the P_Δ-function operates on the payoffs and not on differences of payoffs (as risk dominance does). For example the difference (a_{alt} - a_{min}) is transformed into $P_\Delta(a_{alt}) - P_\Delta(a_{min})$ and not into $P_\Delta(a_{alt} - a_{min})$. An explanation for this is that the strategic equivalence does not hold for the percepted payoffs. It must be replaced by another form of equivalence for perceived payoffs (B. Vogt, 1997).

3. The experiment

In the experiment 2x2 bimatrix games were played by means of the strategy method. Figure 5 shows the corresponding bimatrices. Every bimatrix corresponds to a set of games for which the value q is a parameter (with $q<b_{alt}$).

The two equilibria in pure strategies are U_1U_2 and V_1V_2 for all the bimatrices. U_1U_2 was preferred by the column player and V_1V_2 by the row player. The conflict payoff occurred for V_1U_2.

Every bimatrix in figure 5 defines a set of games depending on q. This set was played with the strategy method. The players had to give their strategy selection for every value of q. This was done by giving a value q^*, such that for $q>q^*$ U_i is selected and for $q\leq q^*$ V_i (i=1,2).

For the payoffs one game (characterized by a value q') was chosen at random. For this special game the payoffs were according to the selected strategies.

Figure 5: The bimatrix games played

x0:

10, 30	0, 0
0, q	30, 0

x5:

10, 30	0, 0
0, q	30, 5

a0:

10, 30	0, 0
0, q	15, 0

a5:

10, 30	0, 0
0, q	15, 5

a10:

10, 30	0, 0
0, q	15, 10

a15:

10, 30	0, 0
0, q	15, 15

X0:

10, 30	0, 0
-10, q	30, 0

X5:

10, 30	0, 0
-10, q	30, 5

A0:

10, 30	0, 0
-10, q	15, 0

A5:

10, 30	0, 0
-10, q	15, 5

A10:

10, 30	0, 0
-10, q	15, 10

A15:

10, 30	0, 0
-10, q	15, 15

3.1 The payoffs

The players received points as their payoffs. The worth of 1 point was 10 DM (~$7). For every game the payoff of a player was the difference to the mean value of the other players in his group having the same role (row or column player, only the games in which the player did not participate were taken for the calculation of the mean value). Losses up to 100DM (~$70) had to be payed by the subjects. For the losses above 100 DM the subjects could choose whether to pay or to work at 15 DM (~$10) per hour.

3.2 The subjects

The subjects were 32 students of economics and business administration after their "Vordiplom". They were divided in 4 groups of 8 subjects.

3.3 Communication

Free communication via terminals was possible.

3.4 The experimental procedure

The experiment was separated into two parts (part A and B).
In part A the games were played in 4 groups of 8 subjects by means of the strategy method. Every set of games (represented by one bimatrix) was played in the following stages:

1) Free communication via terminals.
2) Simultaneous announcement and exchange of the planned strategies.
3) Simultaneous selection (and exchange) of the strategies.
4) Announcement of the drawn game (q-value).

In part B the subjects had to select their strategies for every bimatrix without knowing the other player or having communicated with another player. 3 games per group were paid. The players and games were chosen by chance.

4. Results and discussion

In the following part the results of the experiment are presented and discussed.
Before discussing the tests of the predictions the prominence structure of the q-values responded by the subjects in part B is considered.

For the predictions of the models based on the theory of prominence the smallest unit Δ has to be known. The smallest unit of the responded q-values is determined by means of the rule given in section 2.3. It is given for every game in table 3.

Table 3: The smallest unit Δ:

game	x0	x5	a0	a5	a10	a15	X0	X5	A0	A5	A10	A15
Δ	10	10	20	10	10	10	10	10	10	10	10	10

Another result is that for the games with conflict payoff $(-10,q^*)$ -10 is an anchor point for q^*[7].

4.1 Predictions of the models

The predictions of the critical value q^* for which indifference between the two equilibrium points is obtained are given for games with conflict payoff $(0,q^*)$ in figure 6 and for games with conflict payoff $(-10,q^*)$ in figure 7. For model I and model II the smallest unit Δ and the anchor point are taken into account.

[7] x^o is an anchor point for a set of responses x if the payoffs $x-x^o$ are considered instead of x. That x^o is an anchor point can be confirmed by the fact that the exactnesses of the payoffs $(x-x^o)$ are essentially bigger than the exactnesses of the payoffs x which are to fine.

Figure 6: The predictions of the indifference value q^* for games with conflict payoff $(0,q^*)$:

game	Nash	Risk dom.	model I	model II
x0	-15	-10	-20	-10
x5	-7,5	-5	-10	-5
a0	-60	-20	-40	-25
a5	-45	-15	-20	-15
a10	-30	-10	-10	-10
a15	-15	-5	0	-5

Figure 7: The predictions of the indifference value q^* for games with conflict payoff $(-10,q^*)$:

game	Nash	Risk dom.	model I	model II
X0	-30	-20	-50	-20
X5	-20	-15	-20	-15
A0	-120	-40	-100	-50(-30)
A5	-95	-35	-50	-30(-25)
A10	-70	-30	-20	-20
A15	-45	-25	-10	-15

The numbers in brackets indicate the predictions of model II by assuming an anchor point of -10, i.e. how much less than -10 has q to be.

4.2 Test of the predictions of the models

The predictions of the models as given in figure 6 and figure 7 are tested by means of a binomial-test. The test was performed for two different assumptions, namely assumption a that there are only 4 groups of 8 subjects that are independent and assumption b that the responded q-values of all subjects are independent.

The medians of the 4 groups (given in figure 8) are the data for the test under assumption a.

Figure 8: The medians of the responded q^* for all groups and games.

group	game x0	game x5	game a0	game a5	game a10	game a15
1	-17,5	-12,5	-22,5	-15	-10	-5
2	-25	-15	-25	-19	-13,5	-9,5
3	-15	-7,5	-50	-27,5	-13,5	-7
4	-9,5	-11	-32,5	-19	-14,5	-9,5
	game X0	game X5	game A0	game A5	game A10	game A15
1	-22,5	-16,5	-30	-25	-19	-13,5
2	-25	-18	-37,5	-26	-20,5	-16
3	-19	-13,5	-38,5	-29	-18,5	-12,5
4	-17,5	-12,5	-26	-19,5	-13,5	-10

The results of the test are given in figure 9. In the figure it is indicated by the symbol "-" if the hypothesis "the prediction of the model is true" is rejected by the data of a game. "+" indicates that the hypothesis is not rejected by the data.

Figure 9: The results of the test (assumption of 4 independent groups)

	Nash	Risk dominance	model I	model II	model II plus ref. point
x0	+	+	+	+	+
x5	+	-	+	-	-
a0	-	-	+	+	+
a5	-	+	+	+	+
a10	-	+	+	+	+
a15	-	+	-	+	+
X0	-	+	-	+	+
X5	-	+	-	+	+
A0	-	-	-	-	+
A5	-	-	-	-	+
A10	-	-	+	+	+
A15	-	-	+	+	+

α=12,5%

The predictions of the Nash-criterion are rejected in 10 of 12 cases and show the worst agreement with the data. The predictions of the risk dominance are rejected for 6 of 12 cases. The agreement improves for the concepts based on the theory of prominence. For model I (connected to the Nash-criterion) the predictions are rejected in 5 of 12 cases compared to 10 of 12 for the Nash-criterion. Model II with an anchor point shows the best agreement with the data. The predictions are rejected in 1 of 12 cases compared to 6 of 12 for the risk dominance (to which it is connected).

The results of the test assuming that all responded q-values are independent (assumption b) are given in figure 10.

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Figure 10: The results of the test (assumption b)

	Nash	Risk dom.	model I	model II	model II, ref. point is -10
x0	+	+	+	+	+
x5	-	-	+	-	-
a0	-	+	+	+	+
a5	-	+	+	+	+
a10	-	+	+	+	+
a15	-	+	-	+	+
X0	-	+	-	+	+
X5	-	+	-	+	+
A0	-	-	-	-	+
A5	-	-	-	-	+
A10	-	-	+	+	+
A15	-	-	+	+	+

α=5%

The result of the test is for almost all games the same as for the first assumption.

These results indicate that the models not based on the theory of prominence do not give good predictors for the data. The models based on the theory of prominence are in better agreement. The best agreement is obtained for model II.

4.3 Payoffs and comparisons of payoffs mentioned in the bargaining process

Besides testing the predictons of the models the arguments given in the preplay communication in part a of the experiments (two person bargaining about q) are examined. All payoffs and differences of payoffs mentioned in the bargaining rounds are listed in the first column of figure 11. Since it was not possible to decide whether the linear or logarithmic difference between two payoffs was ment when the difference was mentioned in the bargaining process, the signs "-" are replaced by ⊖ and the signs "+" by ⊕ in the figure. An example for the formulation of differences is: "I improve from a_{alt} to a_{max}, while you get only b_{alt} instead of b_{max}" (with the numbers of the matrix played).

In the second column of the figure the frequencies of the mentioned payoffs and differences of payoffs are given. In the other columns it is indicated whether the payoffs and differences of payoffs mentioned are compatible with the payoffs and differences of payoffs that form the selection criterion of the different models. For example in the second row all payoffs and differences of payoffs of the selection criterion of model II are mentioned. This is indicated by a cross. Sometimes not all differences are mentioned, but only parts of them. This is indicated by a cross in the corresponding row.

Most of the payoffs and differences of payoffs are compatible with model II (65 of 85 cases). In 37 cases all payoffs and differences of payoffs of model II are mentioned. Some differences are in agreement with model I, but only in 5 cases all differences are mentioned. Few payoffs and differences of payoffs are in agreement with the Nash-criterion and risk dominance. In no case all differences were mentioned. Although it is not known which mentioned payoffs or differences of payoffs caused the decision, figure 11 supports the impression that the selection criterion of model II is near to the used selection criteria.

Figure 11: Payoffs and comparisons of payoffs

payoffs and comparisons of payoffs	freq.	Nash	Risk dom.	model I	model II
$a_{max} \ominus a_{alt}$; $b_{max} \ominus b_{alt}$; a_{min}, q^*	37				X
$(a_{alt} \oplus b_{max}) \ominus (a_{max} \oplus b_{alt})$; a_{min}, q^*	5				X
$a_{max} \ominus a_{alt}$ or $b_{max} \ominus b_{alt}$; a_{min}, q^*	12				X
$a_{max} \ominus a_{alt}$; $b_{max} \ominus b_{alt}$	9			X	X
$a_{max} \ominus a_{alt}$	2			X	X
$a_{max} \ominus a_{alt}$; $b_{max} \ominus b_{alt}$; $a_{alt} \ominus a_{min}$; $b_{alt} \ominus q^*$	5			X	
$a_{alt} \ominus a_{min}$; $b_{alt} \ominus q^*$	2	X	X	X	
$a_{alt} \ominus a_{min}$	3	X	X	X	
$a_{max} \ominus a_{min}$; $b_{max} \ominus q^*$	6	X			
$a_{alt} \ominus a_{min}$; $a_{max} \ominus a_{min}$	1	X			
$a_{max} \ominus a_{min}$; $b_{max} \ominus q^*$	2	X			
$a_{max} \ominus a_{alt}$; $b_{max} \ominus q^*$	1				
total	85	14	5	21	65

5 Conclusions

In this paper the equilibrium selection between two strong equilibria of 2x2 bimatrix games is analysed. Besides existing concepts like the Nash-criterion (J.F. Nash, 1950b and 1953) and the risk dominance (J.C. Harsanyi, R. Selten, 1988) two new models based on the theory of prominence (W. Albers, 1997) are introduced. One is based on the Nash-criterion (model I) and the other one on risk dominance (model II).

The results of the experiments using the strategy method with 32 subjects show no good agreement with the predictions of the Nash-criterion and of risk dominance. Better agreement is obtained for model I. The best agreement exists between data and model II. The idea of risk dominance together with the perception of numbers as described by the theory of prominence yields the best result.

The analysis of the payoffs and differences of payoffs mentioned in the bargaining process between two players supports also that the selection criterion of model II is near to the used selection criteria.

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