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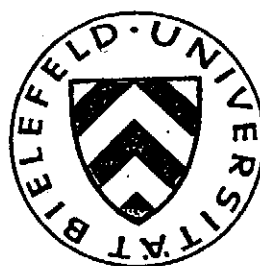
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where Local Governments Provide Industrial
Public Goods

by

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Abstract

This paper investigates different games of interjurisdictional competition where local governments provide public goods that benefit industry. Governments play either a game in tax rates on mobile industrial capital or in public expenditures. Although the literature suggests that competition in public expenditures is always 'more competitive' than in tax rates, this is not necessarily true in the case of industrial public goods. The reason is that capital taxation distorts the use of capital twofold. We show that, in two polar cases, public services are underprovided under tax competition but overprovided under expenditure competition.

Keywords: interjurisdictional competition, different competition regimes, taxation of mobile capital, industrial public goods.

(JEL: H21, H42, H73, R50)

1 Introduction

About ten years ago ZODROW AND MIESZKOWSKI (1986) set up a simple model of interjurisdictional tax competition. This work has been proven as a useful benchmark for the analysis of endogenous provision of local public goods in an interregional setting. Several models have been developed that could be viewed as deliberate extensions of the ZODROW AND MIESZKOWSKI (1986) approach. For example, WILSON (1985), sec. 6, introduces labor mobility. Other tax instruments than a source-based tax on capital income are investigated by GORDON (1986), BUCOVETSKY AND WILSON (1991), and HOYT (1991A). It has been well recognized that the outcome of interjurisdictional competition depends crucially on the number of competing regions (see HOYT (1991B)). While their number becomes small,[1] strategic interactions between local governments play an important role. For the special case of two regions where the spatial distribution of residents is fixed, BUCOVETSKY (1991) and WILSON (1991) investigate asymmetric tax competition between a small and a large region. In a multi-period setting, debt becomes, besides a capital tax rate, an available strategic policy tool. This instrument is analyzed by JENSEN AND TOMA (1991) considering a two-stage game of two regions. However, it has been overlooked for some time that, even in the simple setting where public goods are financed exclusively through a single tax rate, governments face two strategic variables: regions compete either in tax rates or in public spendings. WILDASIN (1988) shows that, if local governments provide local public goods that benefit residents (residential public goods), both equilibria do not coincide, in general; more precisely, *competition in public expenditures is 'more competitive' than in tax rates.*

These and many other extensions of the basic fiscal competition model are discussed in the literature. However, one aspect, though considered in the fundamental work of ZODROW AND MIESZKOWSKI (1986), is widely ignored: the provision of public goods that benefit industry – not residents. Empirically, the provision costs of industrial public services represent a significant part of public expenditures. Keeping this in mind, the neglect of endogenous provision of industrial public goods is, cautiously spoken, inappropriate. One reason that the analy-

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[1]The major branch of the literature deals with the large-number case where regions engage in *perfect* competition. See, for example, WILSON (1985), (1986), ZODROW AND MIESZKOWSKI (1986), and OATES AND SCHWAB (1988).

sis of industrial public services plays a minor role in economic literature might be that many authors expect both types of public goods to be (strategically), more or less, equivalent. WILDASIN (1988), for example, presumes (p. 237) that his finding – competition in public expenditures to be 'more competitive' than in tax rates – remains also true if we consider industrial public goods: *"If local public goods yield benefits to capital owners, [... this] might give the T-equilibrium [equilibrium in tax rates] a character closer to that of a Z-equilibrium [equilibrium in public expenditures]."* However, we should be careful to apply results stemming from models of residential public goods in cases of industrial public goods. The reason is that industrial services improving production possibilities promote economic activities. This in turn, contrary to the case of residential public goods, affects residents' profit income (dividends) as well as public revenues. If more industrial public services attract more capital, public spendings that favor industry induce a second-order effect on public revenues, i.e., revenues increase indirectly. Therefore, the welfare effects that result from the provision of industrial and residential public goods do not coincide. This makes us presume that interjurisdictional competition where governments provide residential public goods differs significantly from the case where they provide industrial public goods. To prove this, we present a model of *imperfect* interjurisdictional competition, where local governments provide *industrial* public goods that are financed exclusively by capital taxation. We show that the equilibrium allocations of both competition regimes – the game in capital tax rates respectively in public expenditures – differ considerably from their residential counterparts, i.e., from the corresponding games where governments provide residential public goods. Consequently, competition in public expenditures is no longer necessarily 'more competitive' than competition in tax rates. In the special case of identical regions, it may occur that public services are underprovided under tax competition but overprovided under expenditure competition.

The plan of this paper is as follows. The second section sets up the model. In section three, two games of interjurisdictional competition are introduced: local governments compete either in tax rates or in expenditure levels. For both games, each region's equilibrium strategies are presented and contrasted with the optimal policy, where all policy tools are available. Since analysis becomes rather arduous in the general case, we focus on identical regions and a symmetric equilibrium in the remainder of section three. The last section summarizes the main results.

2 The Model

We model a federal state that is composed of a small fixed number of jurisdictions, n, each consisting of homogeneous residents. For analytical simplicity, assume that in each region there is only one consumer. In addition, in each region there is one firm. Every local government provides a public good that benefits the resident firm (e.g., infrastructure) and is financed by taxation of mobile industrial capital, exclusively. Since no other sources of public funds are available, each local government can either fix independently its tax rate or its provision level of the public good. Accordingly, we consider two different policy regimes: either local governments compete in tax rates or in public expenditures. In both cases each government's objective is to maximize its resident's utility.

Local firms behaving perfectly competitive produce a homogeneous output good – the unique consumer good – taking the provision level of industrial public services as given. Since our analysis focuses on taxation of industrial capital, all private inputs, except capital, are assumed to be fixed in production. In this case, we can write the production function of the firm residing in region j as a function of capital, K^j, and the provision level of industrial public services, P^j, exclusively, $F^j(K^j, P^j) \; \forall j = 1, \ldots, n$. The production function F^j is assumed to be monotonously increasing and strictly concave in all variables. In addition, the marginal product of capital is the higher the more public services are provided, $F^j_{kp} > 0$. Using the private good as numéraire, local firm's profits are given by

$$\Pi^j = F^j(K^j, P^j) - K^j p^j_k \qquad \forall j = 1, \ldots, n,$$

where p^j_k denotes the after-tax price of capital in region j.

Local consumer's preferences are represented by a twice continuously differentiable, strictly concave utility function $\mathcal{U}^j(X^j)$ where X^j denotes her consumption level of the unique private good. Since the price of this item is normalized to unity, private expenditures are equal to X^j. Let each local resident hold all shares of the corresponding local firm, i.e., the consumer of region j receives full profits of that firm residing in the same region, but she does not receive any income from outside firms.[2] In addition, she receives rent income from her portion, $\theta^j \in [0, 1]$, of the

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[2]Alternatively, we may assume that the production process uses some locationally fixed input, e.g., labor or land, which is owned exclusively by local residents.

nationally fixed capital stock, $\bar{K}$, where $\sum_j \theta^j \equiv 1$. Denoting the nation-wide net return rate of capital by ρ, the private budget constraint of the consumer residing in region j is given by

$$X^j = F^j(K^j, P^j) - K^j p_k^j + \theta^j \rho \bar{K} \qquad \forall j.$$

Assume that local governments produce one unit of the public good by giving up one unit of the private good. Thus, we may view the provision of local public goods as publicly supplied private goods.[3] Since the government is in effect able to transform public revenues into public services, the public budget constraint of region j is given by

$$P^j = \tau^j K^j \qquad \forall j, \tag{1}$$

where local government's tax rate on capital, τ^j, is measured in units of the private good.

Since we are dealing with a small number of regions, a variation of the local capital tax rate does not only affect *local* capital demand but also the nation-wide allocation of capital. Thus, each local government has some impact on the equilibrium (pre-tax) price of capital, ρ. Recognizing this, local governments do no longer behave as price-takers but act strategically. To calculate the effect of τ^j on ρ, the local government takes into account the market clearing condition for capital.

Assume that firms behave perfectly competitive on the national capital market and that capital is freely mobile. In this case, the equilibrium net return to capital, ρ, is the same in all jurisdictions, i.e., we have

$$F_k^j(K^j, P^j) - \tau^j = \rho \qquad \forall j. \tag{2}$$

Since in each region capital is paid by its marginal product, $F_k^j(K^j, P^j)$, resident's capital income is equal to $F^j(K^j, P^j) - K^j F_k^j(K^j, P^j)$. Because capital supply is fixed by assumption, capital demand must exactly meet $\bar{K}$, in equilibrium,

$$\sum_{j=1}^{n} K^j(\rho + \tau^j, P^j) = \bar{K}. \tag{3}$$

The capital market clearing condition, (3), and the n equations given by (2) determine simultaneously the equilibrium values of $K^1, \ldots, K^n$, and ρ. Using

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[3]Alternatively, we may interpret public expenditures as a measure (or proxy variable) for the provision of public goods to avoid evaluation and aggregation problems.

these conditions, that characterize a *perfect* national capital market, each local government is able to compute its impact on the nation-wide net return to capital, $\partial \rho / \partial \tau^j$.

3 Two Games of Interjurisdictional Competition

In this section, we compare two different strategic governmental variables and the resulting equilibria. Either local governments engage in interjurisdictional competition in tax rates or they compete in public expenditures. In the general case, two different equilibria emerge. But before turning to the equilibrium behavior of the local government, we characterize a single region's optimal allocation.

3.1 Region's Optimal Allocation

As a benchmark, first we consider the (constrained) Pareto-efficient allocation of one region. This allows us to investigate what is the best a single region can do given its initial endowment and the behavior of the remainder of the nation. The optimal allocation of region j, defined in this sense, is derived by maximizing resident's utility with respect to (w.r.t.) K^j and P^j directly.[4] If the local government is free to choose K^j and P^j, the optimal solution is characterized by

$$F_p^j = 1, \qquad (4)$$

$$F_k^j = \left(1 + \epsilon_K^j(\rho)\right)\rho - \theta^j \bar{K} \frac{d\rho}{dK^j}, \qquad (5)$$

where $\epsilon_K^j(\rho) := \frac{d\rho}{dK^j}\frac{K^j}{\rho}$ denotes the elasticity of ρ w.r.t. K^j. Note that since τ_k^j is equal to zero, $\epsilon_K^j(\rho)$ is also equal to the elasticity of inverse capital demand.

Since the marginal rate of technical transformation between the private and the public good is equal to one, efficiency requires that the marginal product of public services in private production is also equal to one. This is guaranteed by condition (4). Local government should provide industrial services up to that

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[4]Equivalently, we may assume that, in addition to capital taxation, the local government is entitled to use head taxes (lump sum transfers) on any desired level to raise (or redistribute) public funds.

point where their marginal product in the production process of the private good is equal to unity.

Condition (5) is the usual first order condition (f.o.c.) for a monopsony corrected for an additional income term. The first term of the right hand side, the monopsony part, $(1 + \epsilon_K^j(\rho))\rho$, states that the marginal product of any input, namely capital, should be equal to one plus its elasticity of the inverse demand function times its factor price, $p_k = \rho$. The second term of the right hand side of (5) accounts for the income effect resulting from resident's initial capital endowment. Since local capital demand has a positive impact on the equilibrium price of capital, one additional unit of capital used in local production increases resident's interest income. This marginal income gain lowers region's 'effective social price of capital' – the right hand side of equation (5).

To obtain an alternative perspective, rewrite (5) as

$$F_k^j = \left(K^j - \theta^j \bar{K}\right) \frac{d\rho}{dK^j} + \rho. \tag{6}$$

The extent of local capital excess supply (or demand) plays a crucial role in determining the optimal use of capital. The distortion from the efficiency rule – marginal product of capital equal to factor price – increases with $|K^j - \theta^j \bar{K}|$. The more local capital demand exceeds its supply in absolute values, the more F_k differs from ρ. Thus, a region's optimal allocation is inefficient, in general. The reason is that region's effective social price of capital is not the efficient price. Only if local capital demand meets its local supply, $K^j = \theta^j \bar{K}$, or if local capital demand does not affect the nation-wide net return to capital, $d\rho/dK^j = 0$ (small region), capital is used at its efficient level. However, while $d\rho/dK^j > 0$, a capital net-exporting region uses capital beyond that point where its marginal product equals ρ. The reason is that additional capital demand drives up ρ and thereby capital owners' income. The contrary is true for a net-importing region.

3.2 The Nash Equilibrium in Tax Rates

If local governments compete in *tax rates*, each government maximizes the utility of its resident w.r.t. its capital tax rate, τ^j, given the tax rates of the other governments.

Define the vector of local tax rates by $\vec{\tau} := (\tau^1, \ldots, \tau^n)'$. Since local capital demand depends on the local (gross) price of capital and on the provision level

of local public services, the public respectively the private budget constraint can be written as

$$P^j = \tau^j K^j \left(\tau^j + \rho(\vec{\tau}), P^j \right), \tag{7}$$

$$X^j = F^j \left(K^j \left(\tau^j + \rho(\vec{\tau}), P^j \right), P^j \right)$$
$$- K^j \left(\tau^j + \rho(\vec{\tau}), P^j \right) \left(\tau^j + \rho(\vec{\tau}) \right) + \theta^j \bar{K} \rho(\vec{\tau}). \tag{8}$$

Before characterizing local government's equilibrium policy, we need to know how capital demand varies in response to a change of local tax rates. W.l.o.g. assume that, at some given vector of tax rates, region j increases its tax rate and adjusts its provision level appropriately. If this policy measure reduces local capital demand, dismissed capital has to allocate outside of the region, for the nation-wide supply of capital is assumed to be fixed. This influx of capital perceived by other regions causes their tax revenues to increase even though tax rates are held constant; and their provision levels change correspondingly. (The reverse is true if an increase of τ^j induces a capital movement towards region j, which may happen as will become clear soon.) Thus, in either case, local governments create fiscal externalities[5] through their tax rates.

To see how local capital demand varies if other governments do not retort in tax rates, differentiate local factor demand w.r.t. τ^j yielding

$$\frac{dK^j}{d\tau^j} = K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j} \right) + K_2^j \frac{dP^j}{d\tau^j}, \tag{9}$$

$$\frac{dK^i}{d\tau^j} = K_1^i \frac{\partial \rho}{\partial \tau^j} + K_2^i \frac{dP^i}{d\tau^j} \qquad \forall i \neq j. \tag{10}$$

Since each local government anticipates the extent of capital flight (or influx) which is triggered off by its policy, the sensitivity of capital demand w.r.t. local tax rates, characterized by (9) and (10), plays a crucial role in determining equilibrium policies.

Differentiating the public and the private budget constraint, (7) and (8), w.r.t. τ^j, we can calculate the change of the provision level of the public good and the

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[5]We follow the classical distinction, first introduced by VINER (1931), between *technological* and *pecuniar (or fiscal)* externalities; where the first type generates a shift of technological relations (e.g., production or utility functions), the second, a group of pseudo-externalities, affect individual's financial basics.

change of private consumption for each region. Using (9) and (10) we have

$$\frac{dP^j}{d\tau^j} = \frac{K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)}{1 - \tau^j K_2^j}, \tag{11}$$

$$\frac{dP^i}{d\tau^j} = \frac{\tau^i K_1^i \frac{\partial \rho}{\partial \tau^j}}{1 - \tau^i K_2^i} \qquad \forall i \neq j, \tag{12}$$

$$\frac{dX^j}{d\tau^j} = -K^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right) + F_p^j \frac{dP^j}{d\tau^j} + \theta^j \bar{K} \frac{\partial \rho}{\partial \tau^j}, \tag{13}$$

$$\frac{dX^i}{d\tau^j} = -K^i \frac{\partial \rho}{\partial \tau^j} + F_p^i \frac{dP^i}{d\tau^j} + \theta^i \bar{K} \frac{\partial \rho}{\partial \tau^j} \qquad \forall i \neq j. \tag{14}$$

Dividing equation (13) by (11) yields the net marginal change in the consumption of the private good in terms of the marginal change of the public good. Setting this derivative equal to zero, gives us local government's f.o.c.:

$$-\frac{dX^j}{dP^j} = \frac{K^j + \frac{\partial \rho}{\partial \tau^j}\left(K^j - \theta^j \bar{K}\right)}{K^j + \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)\tau^j K_1^j} \left(1 - \tau^j K_2^j\right) - F_p^j \overset{!}{=} 0. \tag{15}$$

Condition (15) states that the local government should adjust its tax rate such that the marginal provision cost of the public good net of its marginal benefit are equal zero. Or in other words, the social marginal rate of transformation between the private and the public good should be equal to the marginal product of public services in private production.

The fraction term accounts for those units of the private good that have to be sacrificed to raise public revenues, sufficient to provide one additional unit of industrial public services. The numerator represents the marginal impact of τ_k on private consumption: for a fixed provision level of the public good, a marginal increase of τ_k reduces resident's profit income (dividends) by $K^j + \frac{\partial \rho}{\partial \tau^j} K^j$ and her rent income by $-\frac{\partial \rho}{\partial \tau^j} \theta^j \bar{K}$. The denominator represents the marginal impact of τ_k on public revenues given the provision level of the public good. Thus, we may interpret this fraction as the marginal rate of transformation between private income and public funds for a given provision level of public services. Roughly speaking, this term reflects the 'social marginal value of income' using governmental income as numéraire.

Now consider the additional factor that stems from the fact that we deal with industrial public goods, $(1 - \tau^j K_2^j)$. If K_2^j is positive, this bracket term is smaller than one. In this case, the provision of the public good is extended proportional to the partial derivative of factor demand w.r.t. the public good. The reason is that,

ceteris paribus, the provision of industrial services attracts additional capital. This broadens the tax basis of capital taxation thereby reducing the provision costs of industrial public goods itself. We call this indirect effect the *fiscal 'feed-back effect'* of industrial public goods. Since this bracket term measures the (effective) marginal costs of industrial public services, the first part of (15) represents the social value of the provision costs of one additional unit of the industrial public good, or the social marginal rate of transformation between the private and the public good.

Since public services improve the production technology of the private sector, the marginal rate of transformation should be equal to F_p^j. Note that residents do not benefit directly from the provision of industrial public services. Therefore, the impact of P^j on the production function, F^j, already represents total marginal social benefit that accrues from the provision of the public good.

Evidently, local government's f.o.c., given by (15), and region's optimality rule, given by (5) and (4), do not coincide, in general. This divergency shows that the local regulator is not able to pursue region's optimal policy by means of a single tax rate on capital. While the industrial public good is not provided at that level where its marginal product, F_p^j, is equal to one (the technical marginal rate of transformation of the public sector), it is provided inefficiently. Namely, if in equation (15) F_p^j is greater (smaller) than one, public services are underprovided (overprovided).

To gain further insight into the capital market, we evaluate local government's impact on the equilibrium net return to capital, $\partial \rho / \partial \tau^j$. Differentiating the market clearing condition, (3), w.r.t. τ^j, yields

$$\sum_{i \neq j} \left(K_1^i \frac{\partial \rho}{\partial \tau^j} + K_2^i \frac{dP^i}{d\tau^j} \right) + K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j} \right) + K_2^j \frac{dP^j}{d\tau^j} = 0.$$

Using (11) and (12) and solving for $\partial \rho / \partial \tau^j$ gives

$$\frac{\partial \rho}{\partial \tau^j} = -\frac{K_1^j + K_2^j \frac{\tau^j K_1^j}{1 - \tau^j K_2^j}}{\sum_{i=1}^n K_1^i + K_2^i \frac{\tau^i K_1^i}{1 - \tau^i K_2^i}} - \frac{K_2^j \frac{K^j}{1 - \tau^j K_2^j}}{\sum_{i=1}^n K_1^i + K_2^i \frac{\tau^i K_1^i}{1 - \tau^i K_2^i}}. \tag{16}$$

To evaluate (16) we need the following lemma.

Lemma 3.1 *Equilibrium capital demand falls if, ceteris paribus, the gross-price of capital increases; it rises if, ceteris paribus, the provision of public services*

increases:

$$K_1^j = \frac{1}{F_{kk}^j} < 0, \tag{17}$$

$$K_2^j = -\frac{F_{kp}^j}{F_{kk}^j} > 0. \tag{18}$$

Proof: Differentiate firm's f.o.c.

$$F_k^j(K^j, P^j) = \underbrace{p_k^j}_{=\rho^j + \rho} \tag{19}$$

w.r.t. p_k^j and P^j, holding P^j respectively p_k^j constant. Using the concavity of F^j and $F_{kp}^j > 0$, this procedure proves Lemma 3.1. #

Since K_2^j is positive by Lemma 3.1, the only term whose sign is still ambiguous is $1 - \tau^j K_2^j$. Unfortunately, we are not able to decide whether this sign is positive or negative, in general. This depends on the curvature of the production function. As a benchmark of distinction, we use the following condition.[6]

Condition 3.1 *Evaluated in the neighborhood of the equilibrium, the derivatives of the production function F^j satisfy*

$$0 < \tau^j F_{kp}^j < -F_{kk}^j.$$

Condition 3.1 enables us to determine the sign of $1 - \tau^j K_2^j$.

Lemma 3.2 *The effective marginal costs of industrial public services,*

$$1 - \tau^j K_2^j,$$

are positive if and only if Condition 3.1 holds.

Proof: Follows from Condition 3.1. #

By Lemma 3.2, we can evaluate the sign of equation (16) which gives us our first proposition.

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[6]Apparently, this is a somehow uncomfortable 'joint condition' that is not clearly fulfilled or violated ex ante, for it depends on the equilibrium outcome. However, this condition, as well as two others of the subsequent analysis, are not that questionable as it might appear; they do not represent assumptions but serve as benchmarks for case distinction.

To give some intuition under which circumstances Condition 3.1 is fulfilled, two examples are given in the Appendix.

Proposition 3.1 *Under Condition 3.1 regions' average impact on the net return to capital is larger than* $-1/n$.

Proof: Define $\delta^j := -\dfrac{K_2^j \frac{K^j}{1-\tau^j K_2^j}}{\sum_{i=1}^n K_1^i + K_2^i \frac{\tau^i K_1^i}{1-\tau^i K_2^i}}$. Since $K_2^j > 0$ and $1 - \tau^j K_2^j > 0$ by Lemma 3.2, δ^j is positive for all j. Summing (16) over all j and dividing by n yields

$$\overline{\frac{\partial \rho}{\partial \tau}} := \frac{1}{n} \sum_{j=1}^n \frac{\partial \rho}{\partial \tau^j} = -\frac{1}{n} + \bar{\delta} > -\frac{1}{n},$$

where $\bar{\delta} := \frac{1}{n} \sum_{i=1}^n \delta^j > 0$.   #

Contrary to the standard model of interjurisdictional competition in tax rates, Proposition 3.1 tells us that $\partial \rho / \partial \tau^j \in (-1, 0)$ is not necessarily true any longer. More precisely, for local government's impact on the nation-wide net return to capital we have $\partial \rho / \partial \tau^j \in (-1 + \delta^j, \delta^j)$. If local governments provide *industrial* public goods, their impact on the nation-wide net return to capital is higher (smaller in absolute values if $\partial \rho / \tau^j$ is negative) than in the case where local governments provide *residential* public goods. The reason is that capital taxation not only drives out capital but improves the production possibilities and thus attracts new capital. Since revenues from capital taxation are spent in favor of the industry, the extent of capital flight is reduced ('feed-back effect'). Therefore, the marginal impact of the local tax rate on ρ is larger, i.e., less negative, if governments provide industrial public goods.

Note that the impact of the local tax rate on the net return to capital may even be positive. Since $\partial \rho / \partial \tau^j$ may lie in $(0, \delta^j)$, we cannot exclude the possibility that enhancing the provision of public goods increases local capital demand. From the literature we know that this does never occur if governments provide residential public goods. If, however, firms benefit from public services, a higher provision level might attract capital by more than a higher tax rate deters it. In this case, the 'feed-back effect' is that strong that local capital demand is increased resulting in a higher net return to capital.

Before turning to the competition regime where, instead of setting tax rates, local governments determine public spendings directly, we wish to consider the special case of a single region. If the whole nation consists of just one jurisdiction ($n = 1$ and $\theta = 1$), K is equal to $\bar{K}$, for 'local' capital demand must meet 'local'

capital supply. Moreover, from (16) we have

$$\frac{\partial \rho}{\partial \tau} = -\left(1 + K \frac{K_2}{K_1}\right).$$

Substituting this into (15) gives us the f.o.c. for the single government,

$$\frac{K(1 - \tau K_2)}{K + \tau K_1 - \tau(K_1 + KK_2)} = 1 \overset{!}{=} F_p.$$

Thus, as we have expected, for $n = 1$, public goods are provided efficiently. If the whole nation consists only of one region, all induced effects from capital movement are internalized. The reason is that capital has no opportunity to avoid a higher tax rate, i.e., the government need not to be afraid of capital flight. In this case, capital taxation is in effect non-distortionary.

3.3 The Nash Equilibrium in Public Expenditures

Let us now consider the case where local governments compete in public expenditures. In this case, their strategic variables are the P^js. Each local government determines its level of public expenditures and uno actu its provision level of public goods, given the (equilibrium) provision levels of the other regions. From these provision levels those capital tax rates can be calculated that cover the corresponding equilibrium provision costs.

Let $\vec{P} := (P^1, \ldots, P^n)'$ denote the vector of provision levels of local public goods. Since local governments compete in public services, the public budget constraint of region j takes the form

$$P^j = \tau^j(\vec{P}) K^j \left(\tau^j(\vec{P}) + \rho(\vec{\tau}(\vec{P})), P^j\right). \tag{20}$$

Differentiating the corresponding private budget constraint

$$\begin{aligned} X^j &= F^j\left(K^j\left(\tau^j(\vec{P}) + \rho(\vec{\tau}(\vec{P})), P^j\right), P^j\right) \\ &\quad - K^j\left(\tau^j(\vec{P}) + \rho(\vec{\tau}(\vec{P})), P^j\right)\left[\tau^j + \rho(\vec{\tau}(\vec{P}))\right] + \theta^j \bar{K} \rho(\vec{\tau}(\vec{P})) \end{aligned}$$

w.r.t. P^j yields the social marginal rate of transformation between the private and the public good net of its marginal product in private production,

$$-\frac{dX^j}{dP^j} = -F_p^j + K^j \frac{\partial \tau^j}{\partial P^j} + \left(K^j - \theta^j \bar{K}\right) \frac{d\rho}{d\vec{\tau}} \frac{\partial \vec{\tau}}{\partial P^j}. \tag{21}$$

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Setting the right hand side equal to zero determines implicitly the equilibrium provision level.

To evaluate (21) we have to calculate the terms $d\rho/d\bar{\tau}$ and $\partial\bar{\tau}/\partial P^j$. Since $d\rho/d\bar{\tau}$ follows from (16), it remains to calculate $\partial\tau^j/\partial P^j$ and $\partial\tau^i/dP^j$ $\forall i \neq j$.

Differentiating the public budget constraint (20) w.r.t. P^j yields a system of n equations that has to be solved for the n unknown derivatives. Using $dK^j/dP^j = \frac{\partial K^j}{\partial P^j} + \frac{dK^j}{d\bar{\tau}}\frac{\partial\bar{\tau}}{\partial P^j}$ where $\frac{dK^j}{d\bar{\tau}}$ is given by (9) and (10), this procedure yields

$$\frac{\partial\tau^j}{\partial P^j}\left[K^j + \tau^j K_1^j\left(1 + \frac{\partial\rho}{\partial\tau^j}\right)\right] + \tau^j K_1^j \sum_{l \neq j}\frac{\partial\rho}{\partial\tau^l}\frac{\partial\tau^l}{\partial P^j} + \tau^j K_2^j = 1, \quad (22)$$

$$\frac{\partial\tau^i}{\partial P^j}\left[K^i + \tau^i K_1^i\left(1 + \frac{\partial\rho}{\partial\tau^i}\right)\right] + \tau^i K_1^i \sum_{\substack{l \neq j \\ l \neq i}}\frac{\partial\rho}{\partial\tau^l}\frac{\partial\tau^l}{\partial P^j} + \tau^i K_1^i\frac{\partial\rho}{\partial\tau^j}\frac{\partial\tau^j}{\partial P^j} = 0. \quad (23)$$

Assuming weak symmetry, $\partial\rho/\partial\tau^i = \partial\rho/\partial\tau^j$ $\forall i,j$ and $\partial\tau^i/\partial P^j = \partial\tau^l/\partial P^j$ $\forall i, l \neq j$, we can solve (23) for $\partial\tau^i/\partial P^j$,

$$\frac{\partial\tau^i}{\partial P^j} = -\frac{\tau^i K_1^i\frac{\partial\rho}{\partial\tau^j}}{K^i + \tau^i K_1^i\left(1 + \frac{\partial\rho}{\partial\tau^i}\right) + (n-2)\tau^i K_1^i\frac{\partial\rho}{\partial\tau^i}}\frac{\partial\tau^j}{\partial P^j}. \quad (24)$$

Substituting this result into (22) yields

$$\frac{\partial\tau^j}{\partial P^j} = \frac{\Delta_i\left(1 - \tau^j K_2^j\right)}{\Delta_i\left(K^j + \tau^j K_1^j\left(1 + \frac{\partial\rho}{\partial\tau^j}\right)\right) - (n-1)\tau^j\tau^i K_1^j K_1^i\frac{\partial\rho}{\partial\tau^j}\frac{\partial\rho}{\partial\tau^i}}, \quad (25)$$

where

$$\Delta_i := K^i + \tau^i K_1^i\left(1 + \frac{\partial\rho}{\partial\tau^i}\right) + (n-2)\tau^i K_1^i\frac{\partial\rho}{\partial\tau^i}.$$

Re-substituting (25) into (24) gives $\partial\tau^i/\partial P^j$. Using these results we can, at least in principle, compute local government's equilibrium provision level. Unfortunately, formulae become rather inconvenient and calculations are tedious. To gain further insight, we make some simplifying assumptions in the subsequent analysis. At the moment, all we can say is that the tax and the expenditure equilibrium do not coincide, in general. Only if, evaluated at the corresponding equilibrium variables, (15) and (21) hold simultaneously, both equilibria coincide. However, there is no inherent reason to expect this.[7]

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[7]Note also that, under both competition regimes, the equilibrium allocations differ significantly from the corresponding equilibria where governments provide industrial local public goods. (Compare equations (8.1) and (8.2) of WILDASIN (1988) with (15) and (21), respectively.)

3.4 Identical Jurisdictions

Since (24) and (25) are hard to handle, it is instructive to consider the case of identical jurisdictions. Before comparing the marginal rates of transformation under both competition regimes, we investigate equilibrium capital demand and the nation-wide net return to capital, first. In the special case of identical regions, local government's impact on ρ reduces to

$$\frac{\partial \rho}{\partial \tau^j} = -\frac{1}{n}\left(1 + K^j \frac{K_2^j}{K_1^j}\right) \qquad \forall j. \tag{26}$$

Even under the simplifying assumption of identical regions, we are not able to determine the sign of $\partial \rho / \partial \tau^j$ unambiguously, in general. This depends on the elasticities of capital demand. Let $\varepsilon_{P^j}^j := P^j K_2^j / K^j$ and $\varepsilon_{\rho+\tau^j}^j := P^j K_1^j / K^j$ denote the elasticities of capital demand w.r.t. the public good and w.r.t. the (after-tax) price of capital, respectively. If equilibrium capital demand is 'not too elastic' w.r.t. the provision level of the public good, $\partial \rho / \partial \tau^j$ is negative. We use the following condition as a distinctive mark.[8]

Condition 3.2 *Evaluated in the neighborhood of the equilibrium, capital demand satisfies*

$$-\varepsilon_{\rho+\tau^j}^j \frac{\tau^j}{\rho + \tau^j} > \varepsilon_{P^j}^j.$$

Evaluating equation (26) under Condition 3.2, we have the following lemma.

Lemma 3.3 *For $n < \infty$, the nation-wide net return to capital, ρ, depends negatively (positively) on the local tax rate on capital, τ^j, if and only if Condition 3.2 holds (does not hold).*

Proof: By definition of $\varepsilon_{\rho+\tau^j}^j$ and $\varepsilon_{P^j}^j$, Condition 3.2 is equivalent to $K_1^j + K^j K_2^j < 0$. $\#$

Using $F_{kp}^j > 0$, we also have that, under Condition 3.2, an increase of τ^j causes the pre-tax and the after-tax price of capital to move in opposite directions.

Lemma 3.4 *Let Condition 3.2 hold. The after-tax price of capital in region j, $p_k^j := \rho + \tau^j$, rises if the capital tax rate of region j, τ^j, is increased.*

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[8]For an example, see also the Appendix.

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Proof: Using (26), $1 + \frac{\partial \rho}{\partial \tau^j}$ is equal to $\frac{n-1}{n} - \frac{K^j}{n} \frac{K_2^j}{K_1^j}$. Since K_2^j is positive, this term is also positive (for all $n > 1$). #

So far, we have analyzed how ρ and p_k^j respond to changes of τ^j. But actually, we are more interested in the variations of capital demand. Namely, a fundamental question is, does capital taxation induce capital flight or possibly an influx of capital? To see how capital demand changes, substitute $dP^j/d\tau^j = \tau^j \, dK^j/d\tau^j + K^j$ into (9) yielding

$$\frac{dK^j}{d\tau^j} = \frac{K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right) + K^j K_2^j}{1 - \tau^j K_2^j} = \frac{n-1}{n} \frac{K_1^j + K^j K_2^j}{1 - \tau^j K_2^j},$$

where we have made use of (26) for the last equality. We see that, under Condition 3.1, $dK^j/d\tau^j$ being be positive is equivalent to $K_1^j + K^j K_2^j > 0$. This gives us the next proposition.

Proposition 3.2 *Let Conditions 3.1 and 3.2 hold. If a local government increases its tax rate on capital, local capital demand decreases.*

Thus, if Condition 3.1 and 3.2 hold, a rise of the local tax rate induces capital flight and consequently a decrease of ρ. This is the 'regular' case that we already know from traditional theory of interjurisdictional competition.[9]

Note, however, that Condition 3.2 is equivalent to $F_{kp}^j < 1/K^j$. If F_{kp}^j satisfies Condition 3.1, there is some range where Condition 3.1 is fulfilled but Condition 3.2 is violated:[10] $0 < 1/K^j < F_{kp}^j < -F_{kk}^j/\tau^j$. Violation of Condition 3.2, however, implies $-\varepsilon_{\rho+\tau^j}^j \frac{\tau^j}{\rho+\tau^j} < \varepsilon_{P^j}^j$. In this case, capital is that elastic w.r.t. the public services that the provision effect dominates the price effect. The net return to capital and local capital demand *increase* even though capital is taxed more severely. The reason is that P^j affects the marginal product of capital, F_k^j, to that large extent that an increase of P^j enhances local capital demand. Clearly, if more capital is used as the government raises its tax rate, the net return to capital, ρ, and its after-tax price, p_k^j, increase. Thus, contrary to the traditional tax competition literature, taxation of capital *may* attract new capital if production benefits sufficiently from provision of public goods. If this phenomenon occurs,

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[9]The same is true if Condition 3.1 and 3.2 are violated.

[10]In general, four distinct cases might emerge, since both terms, $\partial \rho/\partial \tau^j$ and $\partial K^j/\partial \tau^j$, could be of either sign. However, for the moment, we assume that Condition 3.1 holds.

the 'feed-back effect' of industrial public goods is sufficiently large to induce an influx of capital.

Corollary 3.1 *Let Condition 3.1 hold, and let Condition 3.2 be violated. An increase of the local tax rate on capital results in a rise of local capital demand (capital influx) and of the net-return to capital.*

Proof: The proof of the first part is obvious from previous analysis, whereas the second part is a direct consequence of Lemma 3.3.   #

Now let us turn to local government's equilibrium behavior under both competition regimes. After having derived the marginal rates of transformation in both cases, we will compare the corresponding equilibrium taxation rules. This comparison allows us to judge which regime leads to 'more competitive' behavior.

Competition in Tax Rates

If local governments compete in tax rates on mobile industrial capital, equilibrium behavior is characterized by equation (15). Since all regions are identical ($\theta^j = 1/n \ \forall j$), in each jurisdiction equilibrium local capital demand must exactly meet local capital supply, i.e., we have $K^j = \theta^j \bar{K}$. Taking equation (26) into account, the equilibrium local tax rate is determined implicitly by

$$\frac{K^j \left(1 - \tau K_2^j\right)}{K^j + \left(1 + \frac{\partial \rho}{\partial \tau^j}\right) \tau K_1^j} = F_p^j, \tag{27}$$

or equivalently

$$\frac{K^j \left(1 - \tau K_2^j\right)}{K^j \left(1 - \tau K_2^j\right) - (n-1)\tau K_1^j \frac{\partial \rho}{\partial \tau^j}} = F_p^j. \tag{28}$$

Since $1 - \tau K_2^j > 0$ by Lemma 3.2, we see from (28) that the marginal rate of transformation between the private and the public good is greater than unity if and only if $(n-1)\partial \rho / \partial \tau^j$ is negative. Using (26), this is equivalent to $K_1^j + K^j K_2^j < 0$.

Proposition 3.3 *Let local governments engage in competition in tax rates on capital where Condition 3.1 holds. In the case of identical regions (with $n \geq 2$), public services are underprovided (overprovided) if and only if Condition 3.2 holds (is violated).*

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Since the crucial term determining the provision level of public goods, $K_1^j + K^j K_2^j < 0$, also determines $\partial \rho / \partial \tau^j$ (see Lemma 3.3), we can re-formulate Proposition 3.3 as follows. Provided that n is small, local public goods are underprovided (overprovided) if and only if $\partial \rho / \partial \tau^j$ is negative (positive). In the 'standard' case of interjurisdictional competition where $\partial \rho / \partial \tau^j < 0$, industrial public services are underprovided.

Thus, for $1 < n << +\infty$, public services are provided efficiently if the elasticity of capital demand w.r.t. the public good is equal to minus the elasticity w.r.t. the gross-price of capital times the percentage tax rate on capital (based on its gross price):

$$\varepsilon_P^j = -\frac{\tau^j}{\rho + \tau^j} \varepsilon_{\rho + \tau}^j .$$

The important feature of Proposition 3.3 is that, in the case of identical regions, overprovision as well as efficient provision of local public goods *may* occur. Clearly, the 'probability' of efficiency as equilibrium outcome is rather low, for it requires that the 'feed-back effect' is sufficiently strong (see above). However, this finding is new and contrasts with the result of the traditional theory of interjurisdictional competition dealing with provision of local *residential* public goods: If local governments provide public goods that benefit residents, their provision levels are always(!) too low at the symmetric equilibrium of identical regions. Moreover, in the traditional literature, even for heterogeneous jurisdictions residential public goods are underprovided unless local capital excess demand is 'too high'. In the case of industrial public goods, however, efficient provision and overprovision of local public goods as equilibrium outcome may emerge; namely, it occurs if Condition 3.2 does not hold.

Note that perfect competition of local governments does not ensure efficiency. The reason is that, although n tends to infinity, implying $\partial \rho / \partial \tau^j \to 0$, the term $(n-1) \partial \rho / \partial \tau^j$ does not approach to zero, as long as $K_1^j + K^j K_2^j \neq 0$. Thus, perfect competition is neither necessary nor sufficient to ensure efficiency, even though regions are identical.

Competition in Public Expenditures

Next consider interjurisdictional competition in public expenditures. Focusing on a symmetric Nash equilibrium, (25) reduces to

$$\frac{\partial \tau^j}{\partial P^j} = \frac{\left(K^j + \tau^j K_1^j + (n-1)\tau^j K_1^j \frac{\partial \rho}{\partial \tau^j}\right)\left(1 - \tau^j K_2^j\right)}{\left(K^j + \tau^j K_1^j + (n-1)\tau^j K_1^j \frac{\partial \rho}{\partial \tau^j}\right)\left(K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)\right) - (n-1)\left(\tau^j K_1^j \frac{\partial \rho}{\partial \tau^j}\right)^2}.$$

Using (26), this further simplifies to

$$\frac{\partial \tau^j}{\partial P^j} = \frac{nK^j + \tau^j K_1^j + \tau^j K_2^j K^j - n\tau^j K_2^j K^j}{nK^j \left(K^j + \tau^j K_1^j\right)}. \tag{29}$$

Substituting (29) into (24) and using $\tau^i = \tau^j \ \forall i, j$ gives

$$\frac{\partial \tau^i}{\partial P^j} = \frac{\tau^j \left(K_1^j + K_2^j K^j\right)}{nK^j \left(K^j + \tau^j K_1^j\right)} \qquad \forall i \neq j. \tag{30}$$

To evaluate the marginal rate of transformation, we have to compute dK^j/dP^j and dK^i/dP^j, given by

$$\frac{dK^j}{dP^j} = K_1^j \left(\frac{d\rho}{d\bar{\tau}} \frac{\partial \bar{\tau}}{\partial P^j} + \frac{\partial \tau^j}{\partial P^j}\right) + K_2^j,$$

$$\frac{dK^i}{dP^j} = K_1^i \left(\frac{d\rho}{d\bar{\tau}} \frac{\partial \bar{\tau}}{\partial P^j} + \frac{\partial \tau^i}{\partial P^j}\right) \qquad \forall i \neq j.$$

Using (26), (29), and (30), dK^j/dP^j and dK^i/dP^j reduce to

$$\frac{dK^j}{dP^j} = \frac{n-1}{n} \frac{K_1^j + K_2^j K^j}{K^j + \tau^j K_1^j},$$

$$\frac{dK^i}{dP^j} = -\frac{1}{n} \frac{K_1^i + K_2^i K^i}{K^i + \tau^i K_1^i} \qquad \forall i \neq j.$$

Substituting $\partial \tau^j/dP^j$, given by (29), into government's f.o.c., (21), and using $K^j = \theta^j \bar{K}$, we get

$$1 - \tau^j \frac{n-1}{n} \frac{K_1^j + K_2^j K^j}{K^j + \tau^j K_1^j} = F_p^j. \tag{31}$$

The marginal provision costs of public services can be decomposed into two parts: a non-distortionary part and a distortionary part. The first one represents the provision costs under efficiency which are equal to unity. The latter part incorporates the change in capital demand (flight or influx) as the local government varies its provision level. According to (31), the government of region j should fix its P^j

such that the efficient provision costs minus marginal revenues, resulting from the induced change of capital demand, equal the marginal product of public services in private production. To evaluate (31), we use the following condition.[11]

Condition 3.3 *Evaluated in the neighborhood of the equilibrium, public revenues increase in the local tax rate, if the provision level of public services, P^j, and the net-return to capital are held constant:*

$$K^j + \tau^j K_1^j > 0.$$

Loosely speaking, Condition 3.3 says that, evaluated at the equilibrium, any government faces an increasing part of the Laffer-curve. Under this condition, the provision level of local public services hinges crucially on the term $K_1^j + K_2^j K^j$.

A comparison of our three stated conditions gives us the following lemmata.

Lemma 3.5 *Let Condition 3.1 hold. Then Condition 3.3 is satisfied if Condition 3.2 is violated.*

Proof: Condition 3.1 implies $1 - \tau^j K_2^j > 0$, which is equivalent to $\frac{1}{F_{kp}^j} + \tau^j K_1^j > 0$. If Condition 3.2 does not hold, $\frac{1}{F_{kp}^j} < K^j$ is true. Compounding both inequalities gives $K^j + \tau^j K_1^j > 0$. #

Lemma 3.6 *If Conditions 3.2 and 3.3 hold, Condition 3.1 is also satisfied*

Proof: Obvious from $1/F_{kp}^j > K^j$ and $K^j > -\tau^j K_1^j$.

Analogous to the case of competition in tax rates (see Proposition 3.3), we get by means of Condition 3.3:

Proposition 3.4 *Let local governments engage in competition in public expenditures where Condition 3.3 holds. In the case of identical regions (with $n \geq 2$), public services are underprovided (overprovided) if and only if Condition 3.2 holds (does not hold).*

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[11]For an example, see also the Appendix.

Thus, given $n << +\infty$, underprovision of public good occurs if and only if $\partial\rho/\partial\tau^j$ is negative (positive). Again, if $K_1^j + K^j K_2^j = 0$, local public goods are provided efficiently; whereas perfect competition $(n \to +\infty)$ does not lead to efficiency.

It is instructive to decompose the marginal provision costs of public services into a 'standard' part and a 'feed-back' part. Therefore, rewrite equation (31) as

$$\frac{1 + \frac{1}{n}\frac{\tau^j}{\rho+\tau^j}\varepsilon_{\rho+\tau^j}^j}{1 + \frac{\tau^j}{\rho+\tau^j}\varepsilon_{\rho+\tau^j}^j} - \frac{n-1}{n}\frac{1}{1 + \frac{\tau^j}{\rho+\tau^j}\varepsilon_{\rho+\tau^j}^j}\tau^j K_2^j = F_p^j. \qquad (32)$$

The marginal provision cost of the public good, the left hand side, consists of two terms: the direct and the indirect provision cost, where the latter stems from the 'feed-back effect' of industrial public services. The first term equals to the equilibrium marginal rate of substitution between the private and the public good when local governments provide *residential* public goods and all capital is held by domestic consumers, i.e., $\theta^j = 1/n \, \forall j$. If, however, all capital is owned by foreigners, i.e., $\theta^j = 0 \, \forall j$, local government fixes its provision of residential public goods where the marginal rate of substitution is equal to[12]

$$\frac{n-1}{n}\frac{1}{1 + \frac{\tau^j}{\rho+\tau^j}\varepsilon_{\rho+\tau^j}^j}.$$

Denoting these terms, by $\text{MRS}^j|_{\theta^j=1/n}$ respectively $\text{MRS}^j|_{\theta^j=0}$, local governments optimality condition (32) can be written as

$$1 - \tau^j\frac{dK^j}{d\tau^j} = \text{MRS}^j|_{\theta^j=1/n} - \tau^j K_2^j \, \text{MRS}^j|_{\theta^j=0} = F_p^j.$$

$\text{MRS}^j|_{\theta^j=1/n}$ represents the marginal costs of residential public goods in standard policy rules. The second part, however, $\tau^j K_2^j \, \text{MRS}^j|_{\theta^j=0}$, is proportional to the 'feed-back effect'. This term does not appear in models of residential public goods. The 'feed-back effect' lowers the provision cost of industrial public services compared to those of residential public goods.

A Comparison of both Types of Equilibria

WILDASIN (1988) shows that local governments behave 'more aggressively' under competition in public expenditures compared to competition in tax rates. However, his analysis is confined to the case of residential public goods. Since the

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[12]See WILDASIN (1988) eq. (10.1).

provision of residential and industrial public goods are strategically not equivalent, it is an open question whether WILDASIN'S result is also valid if governments provide industrial services. We show that, in the case of identical regions, competition in tax rates may be 'more competitive' than competition in public expenditures, although the opposite seems to be 'more likely'.

To investigate which form of interjurisdictional competition leads to 'more aggressive' behavior, we have to compare the corresponding f.o.c.s; i.e., we weigh the taxation formulae, given by (28) and (31), against each other. If the marginal rate of transformation is larger in expenditure competition than in tax competition, we call the first one 'more competitive' or 'more aggressive' (and vice versa).

Before deriving our result, we have to prove the following lemma.

Lemma 3.7 *Let Condition 3.2 and 3.3 hold. Public revenues increase in the local tax rate, if the provision level of public services, P^j, is held constant:*

$$K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j} \right) > 0.$$

Proof: We have

$$K^j + \left(1 + \frac{\partial \rho}{\partial \tau^j} \right) \tau^j K_1^j > 0$$

$$\Leftrightarrow \quad K^j + \tau^j K_1^j > \frac{1}{n} \tau^j \left(K^j K_2^j + K_1^j \right).$$

Since the left hand side is positive by Condition 3.3 and the right hand side is negative by Condition 3.2, this inequality is always true.   #

Roughly speaking, Lemma 3.7 says that, whatever the value of $\partial \rho / \partial \tau^j$ may be, public revenues increase if the local government raises its tax rate holding P^j constant, $\frac{\partial Rev^j}{\partial \tau^j}|_{P^j = \bar{P}^j} > 0$. In this sense, Lemma 3.7 generalizes Condition 3.3.

Now we are well prepared to state our proposition.

Proposition 3.5 *Let Condition 3.2 and 3.3 hold. In equilibrium, the marginal rate of transformation between the private and the public good is larger under competition in public expenditures than under competition in tax rates.*

Proof: (Omitting regional superscripts.) From (28) and (31) we know that if the marginal rate of transformation is larger under expenditure than under tax

competition,[13]

$$\frac{K\left(1 - \tau K_2\right)}{K\left(1 - \tau K_2\right) - (n-1)\tau K_1 \frac{\partial \rho}{\partial \tau}} < \frac{K + \tau K_1 - \tau \frac{n-1}{n}\left(K_1 + K_2 K\right)}{K + \tau K_1} \qquad (33)$$

must be true. Using (26), Condition 3.3 and Lemma 3.7, unequality (33) is equivalent to

$$
\begin{aligned}
n^2 \left(K - \tau K K_2\right)\left(K + \tau K_1\right) \; &< \; \left(nK + \tau K_1 - (n-1)\tau K K_2\right) \\
&\quad \times \left(nK + (n-1)\tau K_1 - \tau K K_2\right) \\
\Leftrightarrow \quad \left(K_1 + K K_2\right)^2 \left(1-n\right)\tau^2 \; &< \; 0,
\end{aligned}
$$

which is true for all $n > 1$. #

Applying Lemma 3.6, we see by inspection of Proposition 3.3 and 3.4 that, under the assumptions of Proposition 3.5, public services are underprovided under both competition regimes.

Corollary 3.2 *Let Condition 3.2 and 3.3 hold. In equilibrium, public goods are underprovided under competition in public expenditures as well as under competition in tax rates.*

These results show that the marginal rate of transformation between the private and the public good is larger under competition in public spendings than under competition in tax rates. Thus, the extent of underprovision of local public goods is larger when governments engage in competition in public spendings. This implies that governments engaging in expenditure competition tend to tax capital less heavily resulting in lower public revenues and thus in lower provision levels. In this sense, interjurisdictional competition is 'more competitive' if governments' strategic variables are public expenditures.

However, this result is subject to Condition 3.2 and 3.3. If one of them is violated, the contrary may be true. From the proof of Proposition 3.5 it should be clear that tax competition is 'more competitive' than expenditure competition if one (and only one) of the denominators of (33) is negative, i.e.,

$$\left(K^j + \tau^j K_1^j\right)\left(K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)\right) < 0. \qquad (34)$$

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[13]Since Proposition 3.5 compares optimality *rules*, not values, we have to evaluate both at the same point, namely around the equilibrium.

There are two cases in which this is true. In the first case, the first bracket term is negative; the second term, positive; i.e., $K^j + \tau^j K_1^j < 0$ and $K^j + \tau^j K_1^j > \frac{1}{n}\tau^j \left(K^j K_2^j + K_1^j\right)$. This implies that Condition 3.3 is violated, whereas Condition 3.2 holds, and F_{kp}^j is 'sufficiently small', in the sense that[14]

$$F_{kp}^j \; < \; \frac{1}{K^j} - n\left(\frac{1}{K^j} + \frac{F_{kk}^j}{\tau^j}\right). \tag{35}$$

On the contrary, in the second case, $0 < K^j + \tau^j K_1^j < \frac{1}{n}\tau^j \left(K^j K_2^j + K_1^j\right)$ must be true; i.e., Condition 3.3 holds, whereas Condition 3.2 does not, and F_{kp}^j is 'sufficiently large', in the sense that the reverse of (35) holds.

To sum up, if either Conditions 3.3 does not hold and F_{kp}^j is sufficiently small or Condition 3.2 does not hold and F_{kp}^j is sufficiently large, inequality (34) is fulfilled. In other words, if a local government faces a *decreasing* branch of the Laffer curve (either in the sense of Condition 3.2 or of 3.3) and public services have a very large or very little impact on the marginal product of capital, tax competition is 'more competitive' than expenditure competition.

There is an alternative view of condition (34). Solving (34) for $\partial \rho / \partial \tau^j$ yields

$$\frac{\partial \rho}{\partial \tau^j} \mathop{<}_{(>)} -\frac{1}{\tau^j K_1^j}\left(K^j + \tau^j K_1^j\right), \tag{36}$$

if $K^j + \tau^j K_1^j$ is negative (positive). In the first (second) case, a rise of τ^j induces that much capital flight (influx) such that the nation-wide net return to capital decreases (increases) considerably. Clearly, local capital demand decreases if the 'feed-back effect' does not countervail potential capital flight; it increases if the 'feed-back effect' is sufficiently strong to cause an influx of capital. Therefore, to affect ρ to that large extent as given by (36), the 'feed-back effect' has to be sufficiently (in)significant. To see this more concretely, consider equation (27). Since, evaluated at the equilibrium, the terms $K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)$ and $1 - \tau^j K_2^j$ have the same signs, the latter one is positive in the first (negative in the second) case. This in turn, implies that Condition 3.1 is also satisfied (violated). Thus, if F_{kp}^j is too small (large) an increase of the tax rate induces extensive capital flight (influx). In both cases, Proposition 3.5 is reversed.

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[14]Note that the right hand side may even be negative. In this case, (35) is never fulfilled while we assume $F_{kp}^j > 0$.

Proposition 3.6 *Competition in tax rates is 'more competitive' than competition in public expenditures if in equilibrium either (a) or (b) is true:*

(a) (i) The marginal impact of P^j on the marginal product of capital is 'too small', in the sense that $K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)$ is positive, (ii) Conditions 3.1 and 3.2 hold, whereas (iii) Condition 3.3 does not.

(b) (i) The marginal impact of P^j on the marginal product of capital is 'too large', in the sense that $K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)$ is negative, (ii) Condition 3.3 holds, whereas (iii) 3.1 and 3.2 do not.

Corollary 3.3 characterizes the capital market in both cases.

Corollary 3.3 *Under condition (a) respectively (b) of Proposition 3.6 we have*

(a) $\partial \rho / \partial \tau^j < 0$ and $dK^j / d\tau^j < 0$.

(b) $\partial \rho / \partial \tau^j > 0$ and $dK^j / d\tau^j > 0$.

Recalling our previous results, namely Propositions 3.3 and 3.4, we see that cases (a) and (b) are characterized as follows.

Proposition 3.7 *Under condition (a) or (b) of Proposition 3.6, local public goods are underprovided if regions engage in tax competition and overprovided if they engage in expenditure competition.*

Proof: By inspection of (28) respectively (31).   #

If either condition (a) or (b) holds, the provision level of local public goods is not only higher under expenditure competition than under tax competition, but even more we have overprovision under the first and underprovision under the second competition regime. Hence, although there are some good reasons to believe that WILDASIN'S (1988) result is also valid if local governments provide industrial public goods, this is not necessarily true. Propositions 3.6 and 3.7 show that he is wrong in expecting competition in public expenditures always to be 'more competitive' than competition in tax rates. If, at a margin, private production gains considerably or very little (if any) from provision of public goods, this presumption may be wrong, depending on the slope of the Laffer curve.

In the first case of Proposition 3.6, the intuition behind our result is the following. Capital demand is relatively more sensitive w.r.t. the tax rate than w.r.t. the provision level of public services. The tax rate on capital has reached such a high level that, holding P^j constant, government is faced by a decreasing sector of the Laffer curve, i.e., that revenues fall if τ^j is raised, $K^j + \tau^j K_1^j < 0$. Since F_{kp}^j is very small, production does not benefit much from the provision of public services. This induces extensive capital flight and a significant decline of the net return to capital. ρ does even fall to that large extent that $K^j + \tau^j K_1^j \left(1 + \frac{\partial \rho}{\partial \tau^j}\right)$ is positive, although public revenues fall if ρ is fixed.

The perhaps more relevant and more interesting case is that one where F_{kp}^j is sufficiently large, case (b). Here government faces an increasing branch of the Laffer-curve, $K^j + \tau^j K_1^j > 0$, if ρ is fixed. But taking into account the change of ρ, public revenues decrease. The reason is that improved local production possibilities attract additional capital thereby increasing the equilibrium net return to capital. In this case, the 'feed-back effect' is that large that $\partial \rho / \partial \tau^j$ is positive, raising the after-tax price of capital overproportionally.

Note that if n tends to infinity, the difference between competition in tax rates and competition in public expenditures disappears.

Proposition 3.8 *If the number of competing region tends to infinity, competition in tax rates and competition in public expenditures coincide.*

Proof: First of all, recall that for $n \longrightarrow +\infty$, $\partial \rho / \partial \tau^j$ approaches to zero. (See equation (26).) Using this fact, the social marginal provision costs under competition in tax rates, given by equation (27), tend to $K^j (1 - \tau^j K_2^j) / (K^j + \tau^j K_1^j)$. On the other hand, the marginal provision costs under competition in public expenditures are also equal to this term. #

Even though both regimes of interjurisdictional competition differ, in general, for the large number case, competition in tax rates and in public expenditures are strategically equivalent.

4 Conclusion

We have considered a model of interjurisdictional competition where local governments provide local public goods that benefit industry. Required public expen-

ditures are financed through distortionary taxation of mobile industrial capital. We have investigated two competition regimes: local governments fix either tax rates or expenditure levels. Since both regimes are strategically not equivalent, the corresponding taxation formulae differ and two distinct equilibria occur.

A similar model was analyzed by WILDASIN (1988) where governments provide residential public goods. He found that, in the case of identical regions, interjurisdictional competition in public expenditures is 'more competitive' than in tax rates. We show that analogous is not necessarily true if governments provide local public goods that benefit industry. This non-equivalence is due to the fact that industrial services affect production possibilities. In the regular case where production is improved, capital flight is at least mitigated through the provision of public goods. Thus, the provision of industrial public goods causes a second-order effect on public revenues lowering the marginal social cost of public services. We call this effect 'feed-back effect' of public goods. If this 'feed-back effect' is sufficiently strong, raising the tax rate on capital and thereby the provision level of the industrial public good induces an influx of capital. This does never occur if we deal with residential public goods.

While the marginal impact of public services on the productivity of capital is moderate, interjurisdictional competition in public expenditures is 'more competitive' than competition in tax rates. In this case, public goods are underprovided under both competition regimes. If, however, this effect is either sufficiently low or high, the result is reversed: interjurisdictional competition in tax rates is 'more competitive' than competition in public expenditures; moreover, public goods are underprovided under the tax regime but overprovided under the expenditure regime. Only in the limiting case where the number of regions tends to infinity, both competition regimes are strategically equivalent.

A Appendix

This appendix presents two examples of interjurisdictional competition. The first one represents the WILDASIN-case where competition in expenditures is 'more competitive' than in tax rates. The second example shows that the opposite may also emerge without imposing restrictive, or even less plausible, assumptions.

Assume that all regions are identical and that their total number, n, equals 10. Let total capital supply be $\bar{K} = 1000$. By symmetry, we know that in each region equilibrium capital demand amounts to 100.

Throughout both examples, local production possibilities can be represented by a Cobb-Douglas production function:

$$F(K, P) = \alpha_0 K_1^{\alpha} P_2^{\alpha},$$

with $\alpha_0 = 10$, $0 < \alpha_1 < 1$, and $0 < \alpha_2 < 1$.

For a given supply of the public good, P, firm's factor demand is equal to

$$K = \left(\frac{\alpha_0 \alpha_1}{\rho + \tau} P^{\alpha_2} \right)^{1/(1-\alpha_1)} \quad (= 100),$$

and its first partial derivatives are given by

$$K_1 = \frac{1}{F_{kk}} = \frac{\alpha_1}{\alpha_1 - 1} \frac{F(K, P)}{(\rho + \tau)^2} \tag{A.1}$$

$$K_2 = -\frac{F_{kp}}{F_{kk}} = -\frac{\alpha_2}{(\alpha_1 - 1)\tau}, \tag{A.2}$$

where we have made use of the governmental budget constraint, $P = \tau K$. Applying these results, we can calculate the crucial terms

$$\begin{aligned} t_1 &:= 1 - \tau K_2 \\ t_2 &:= K_1 + K K_2 \\ t_3 &:= K + \tau K_1 \end{aligned}$$

that correspond to our Conditions 3.1, 3.2, and 3.3, respectively.

Let us consider two different specifications of the production function.

A.1 Example 1

Let $\alpha_1 = 0.5$ and $\alpha_2 = 0.25$. Using (A.1), (A.2), and $K = 100$, we have

$$
\begin{aligned}
F_k &= 1.5811\tau^{0.25} & F_p &= 0.7906\tau^{-0.75} \\
F_{kk} &= -0.0079\tau^{0.25} & F_{pp} &= -0.0059\tau^{-1.75} \\
F_{kp} &= 0.0040\tau^{-0.75} \\
K_1 &= -316.228\tau^{0.25}(\rho + \tau)^{-2} & K_2 &= 0.5\tau^{-1}
\end{aligned}
$$

Substituting this into t_1, t_2, and t_3 gives

$$
\begin{aligned}
t_1 &= 0.5 \\
t_2 &= 50\tau^{-1} - 126.491\tau^{-0.25} \\
t_3 &= 100 - 126.491\tau^{0.75}.
\end{aligned}
$$

This enables us to compute the equilibrium tax rates and expenditure levels of both competition regimes.

Tax competition: $\tau^{tc} = 0.4359$ and $P = 43.59$.

Expenditure competition: $\tau^{ec} = 0.4324$ and $P = 43.24$.

Thus, we have WILDASIN'S (1988) result that competition in public expenditures is 'more competitive' than competition in tax rates. Substituting the tax rates into t_2 and t_3, we see, as we have expected from our previous analysis, that Conditions 3.1, 3.2, and 3.3 are fulfilled.

A.2 Example 2

If, however, we parameterize the production with $\alpha_1 = 0.7$ and $\alpha_2 = 0.4$, results change. Using this specification, we have

$$
\begin{aligned}
F_k &= 11.0943\tau^{0.4} & F_p &= 6.3396\tau^{-0.6} \\
F_{kk} &= -0.0333\tau^{0.4} & F_{pp} &= -0.0380\tau^{-1.6} \\
F_{kp} &= 0.0444\tau^{-0.6} \\
K_1 &= -3698.03\tau^{0.4}(\rho + \tau)^{-2} & K_2 &= 1.3333\tau^{-1}
\end{aligned}
$$

and

$$
\begin{aligned}
t_1 &= -0.3333 \\
t_2 &= 133.33\tau^{-1} - 30.0456\tau^{-0.4} \\
t_3 &= 100 - 30.0456\tau^{0.6}
\end{aligned}
$$

Using these results, the equilibrium policies are the following.

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Tax competition: $\tau^{tc} = 9.9898$ and $P = 998.98$.

Expenditure competition: $\tau^{ec} = 10.0830$ and $P = 1008.30$.

In this case, WILDASIN'S (1988) result is reversed, i.e., competition in public expenditures is 'less competitive' than competition in tax rates. The reason is that the production function exhibits increasing returns to scale and that the marginal impact of the public good on the productivity of capital, F_{kp}, is sufficiently large.

References

[1] ABE, KENZO, 1992, *Tariff Reform in a Small Open Economy with Public Production*, International Economic Review, 33, 209 - 222.

[2] BUCOVETSKY, SAM, 1991, *Asymmetric Tax Competition*, Journal of Urban Economics, 30, 167 - 181.

[3] BUCOVETSKY, SAM AND JOHN D. WILSON, 1991, *Tax Competition with Two Tax Instruments*, Regional Science and Urban Economics, 21, 333 - 350.

[4] BURBRIDGE, JOHN B. AND GORDON M. MYERS, 1994, *Population Mobility and Capital Tax Competition*, Regional Science and Urban Economics, 24, 441 - 459.

[5] GORDON, ROGER H., 1986, *Taxation of Investment and Savings in a World Economy*, American Economic Review, 76, 1086 - 1102.

[6] HOYT, WILLIAM H., 1991A, *Competitive Jurisdictions, Congestion, and the Henry George Theorem*, Regional Science and Urban Economics, 21, 351 - 370.

[7] — , 1991B, *Property Taxation, Nash Equilibrium, and Market Power*, Journal of Urban Economics, 30, 123 - 131.

[8] JENSEN, RICHARD AND EUGENIA F. TOMA, 1991, *Debt in a Model of Tax Competition*, Regional Science and Urban Economics, 21, 371 - 392.

[9] MINTZ, JACK AND HENRY TULKENS, 1986, *Commodity Tax Competition Between Member States of a Federation: Equilibrium and Efficiency*, Journal of Public Economics, 29, 133 - 172.

[10] OATES, WALLACE E. AND ROBERT M. SCHWAB, 1988, *Economic Competition Among Jurisdictions: Efficiency Enhancing or Distortion Inducing?*, Journal of Public Economics, 35, 333 - 354.

[11] RICHTER, WOLFRAM F. AND DIETMAR WELLISCH 1993, *Allokative Theorie des interregionalen Finanzausgleichs bei unvollständiger Landrentenabsorption*, Finanzarchiv, 50, 433 - 457.

[12] VINER, JACOB, 1931, *Cost Curves and Supply Curves*, Zeitschrift für Nationalökonomie, 111, 23 - 46.

[13] WILDASIN, DAVID E., 1988, *Nash Equilibria in Models of Fiscal Competition*, Journal of Public Economics, 35, 229 - 240.

[14] WILSON, JOHN D., 1985, *Optimal Property Taxation in the Presence of Interregional Capital Mobility*, Journal of Urban Economics, 17, 73 - 89.

[15] — , 1986, *A Theory of Interregional Tax Competition*, Journal of Urban Economics, 19, 296 - 315.

[16] — , 1991, *Tax Competition with Interregional Differences in Factor Endowments*, Regional Science and Urban Economics, 21, 423 - 451.

[17] ZODROW, GEORGE R. AND PETER MIESZKOWSKI, 1986, *Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods*, Journal of Urban Economics, 19, 356 - 370.