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The Complexity of a Number as a Quantitative Predictor of the Frequency of Responses under Decimal Perception A Contribution to the Theory of Prominence

by

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Abstract

It is a well known phenomenon that spontaneous numerical responses preferably select numbers that have a simple structure. The theory of prominence (see ALBERS 1997) permits to analyse this structure, and to define measures of simplicity of numbers. Adequate measures of simplicity should permit to predict frequencies with which numbers are selected as responses, assuming that numbers that 'simpler' numbers are responded more frequently.

This paper gives such a measure, and compares the predictions with an extended set of numerical responses, data which were answers on the question to 'give information about prices of different articles for a person who does not know the prices in Germany'. The data cover a range from pennies (price of a candy) to hundred thousands of marks (price of a house).

According with the theory of prominence it could be shown that the observed frequencies show 'decimal consistency', i.e. the relative frequency of selecting a response x does not depend on the level, but only on the relative structure of the response, and this structure is not changed when a number is multiplied by ten. More precisely, the concept predicts that the logarithm of the frequency of a number is a linear function of the perceived 'complexity' of the number.

The concept is based on the theory of prominence, the idea is that every number is perceived as a sum of the full step numbers $\{a*10^i : a \in \{1,2,5\}, i \text{ integer}\}$, where all coefficients are 0, +1, or -1, and it is not allowed to select $+5-2$ or $+5+2$ as the last two steps of a presentation. The smallest number with coefficient unequal zero is denoted as the exactness of a presentation. The exactness of a number is the crudest exactness over all of its presentations. The complexity of a number is the number divided by its exactness, for instance $115 = 100 + 10 + 5$ has exactness 5, and complexity $115/5 = 23$.

The interesting result of the investigation is that numbers with the same complexity have the same frequency, if they are ending with 2, 5 or 8, which must be multiplied by 2 when they are ending with 1, 3, 4, 6, 7 or 9. The frequency function is a pendent to the price demand function, where the complexity is the price, and the frequency of selection is the demand.

The paper shows that the theory of prominence permits surprisingly clear predictions of the (relative) frequencies of numerical responses. It seems that the approach adequately describes the 'difficulty' related to the construction or finding of numerical responses.

1 Theory

1.1 Numerical Response Systems, Full Step Numbers

The idea of a numerical response system is a system of round numbers, which permits to specify responses in a stepwise refining way, where in each step the precision of the response is doubled, and the process is stopped when a finer response is not possible.

An ideal response system is given by the powers of 2, where the decision maker goes through the powers in decreasing order, and stepwise answers whether to use the respective power with coefficient +1, -1, or 0. In every stage of the process she obtains the respective best response that is possible by using the given and finer powers of two. (Example: $13 = 1*16 + 0*8 - 1*4 + 0*2 + 1*1$.)

Our culture seems to have tried to create a similar system using the decimal structure. But a system of round numbers, which is closed to itered doubling, does not exist. Instead, the set of

'full step numbers' $:= \{a * 10^i : a \in \{1, 2, 5\}, i \text{ integer}\}$

is selected. Similar to the dual system, it also permits the presentation of numbers as sums of full step numbers using (only) the coefficients +1, -1, or 0, as for instance $13 = 1*20 - 1*10 + 1*5 - 1*2$.

1.2 Exactness, Relative Exactness, Complexity

In a given situation, subjects restrict the exactness of their analysis. A simple way to define exactness is:

> Definitions: The exactness (or prominence) of a presentation of a number is the smallest full step number with coefficient unequal 0. – The exactness of a number is the crudest exactness over all possible presentations of the number. The relative exactness is the exactness devided by the number. The complexity of a number is the number devided by its exactness. A number has level l of [relative] exactness, if its [relative] exactness is cruderequal l.[1]

(Example: the presentation $10+5-2$ has exactness 2. The exactness of 13 is 2, since there is no prentation of 13 with a cruder exactness. But there are presentations of 13 with a finer exactness, for instance $13 = 10 + 2 + 1$. The relative exactness of 13 is $2/13 = .15$.)

The idea behind this approach is that a subject that creates a numerical response adjusts her exactness of analysis to the given problem in a way that she cuts off (or better does not select) finer components of a response if she cannot be more precise, if she does not want

[1] In experimental cases subjects can use 25 as a full step number, but only as the last component of a presentation. But this is unimportant for this investigation, although for instance 25 and 125 show slightly higher frequencies than predicted.

to be more precise, or if it does not make sense to be more precise: Budgets of states may be given with an exactness a Billion, private annual income with an exactness of 1000, the (estimated) price of a bottle of wine with an exactness of 1 or .50. From this point of view, 119 (with exactness 1) indicates a possibly finer judgement than 120 (exactness 20). The notation 'exactness' may misleed insofar that the number with the higher value of exactness 20 is obtained on a cruder level of analysis than the number with the lower exactness 1. Cruder analysis induces greater values of exactness. To avoid confusion, we do not speak of higher and lower, but of cruder and finer exactness, as precision with an exactness of .0001 cm is finer than precision with an exactness of .001 cm. The confusion is avoided if one uses the term 'prominence' (in the sense of Schelling 1962) instead of 'exactness', which indicates that a number with higher prominence is somehow more attractive, and will be more likely selected as a response, (as a mountain that is higher than the others, or a person that is better known). Disadvantage of the term 'prominence' is that it is not quite as selfdescribing as 'exactness'. – Words as 'tolerance' or 'graininess' would not have the direction problem, but both terms rather correspond to 'level of exactness' than to 'exactness': A tolerance of .0001 cm permits that a piece has a length of 1 cm, .1 cm, .01 cm. 'Graininess' insofar induces the right idea, that part of decision making is to crack crude pieces of information down to a certain degree of graininess. But in numerical decision processing the result at a given graininess of cracking are numbers on a given level (!) of exactness.

The idea behind the definition of exactness is, that exactness of a number gives the crudest level of exactness on which all components of the presentation can be perceived.

The construction of a number as a response works similarly as the cracking process: the number has to be distinguished from others (mainly its neighbours) on different levels of exactness, and thereby stepwise more specified, until a further decision is not possible. The proportion of very precise numbers in responses should therefore be related to the preciseness of information. This does not only make sense for the construction of a response, i.e. from the sender's point of view, but also for the receiver. She expects high precision of knowledge, when the numerical response(s) are very precise, while unprecise signals suggest the conclusion, that the sender's level of information does not permit a finer precision. Accordingly, it can make sense to develop a culture in which reliable persons do not give more precise signals than their state of information permits to construct. The theory of prominence says that we are in such a culture – although it can be that not all of us are consciously aware. (The theory of prominence even offers an exactness selection rule: the relative level of exactness is selected such that there are between 3 and 5 numbers on this level of exactnes in the range of reasonable alternatives, see ALBERS 1997.I.

1.3 Scales

Relative exactness (or equivalently its reciprocal, the complexity) selects numbers between 10 and 100 in the following order: 100, 50, 20; 30; 15; 70; 40, (80); 25; 12, 60; (13); 35, 70;

17; (18), 45, 90; 11, 22, 55; ....

An interesting feature of the obtained system is, that at some levels it happens that numbers fulfill the criterion of relative exactness (which permits them to be perceived in all components), but can on this level of exactness not be separated from (at least) one number mentioned before. Example: $80 = 100 - 20$ has exactness 20, which says that 80 can be perceived when the exactness of perception permits to identify differences of 20. However, there is also another number that entered the set of responses even earlier, with the same exactness of 20, namely $70 = 50 + 20$. Assuming that finer differences than 20 cannot perceived, it is not possible to separate 70 from 80. Therefore we define 70 and 80 to belong to the same 'step', as long as the fineness of analysis has not reached an exactness of 10.

These considerations leed to the definition of a scale:

> Definitions: Let a a full step number, r a real number (between 0 and 1). A scale $S(r, a)$ is the set of all numbers with relative exactness greater equal r, and absolute exactness greater equal a. – Two numbers a, b of a scale $S(r, a)$ belong to the same step, iff $|a - b| / \min(|a|, |b|) < r$.
>
> Lemma: There are at most two numbers on one step of a scale $S(r, a)$, these are either of type $X + 2 * 10^i$, $X + 3 * 10^i$, or of type $X + 7 * 10^i$, $X + 8 * 10^i$ (where X is cruder 10^i, i integer).

An example of a scale is $S(1, 10)$. It is given by the numbers ..., -100, -50, -20, -10, 0, +10, +20, +50, +100, ...

1.4 The Response Selection Rule

Our empirical results indicate that subjects eliminate the problem that in certain scales two numbers can belong to the same step by giving one of the numbers a higher priority. This section shows how this is done.

Usually two numbers with the same exactness have a distance that is not smaller than the exactness. This holds for numbers with exactness 5, as 5, 15, 25, 35, 45, and for numbers with exactness 10, as 10, 40, 60, 90. However numbers with exactness 20, as 20, 30, 70, 80 do not fulfill the property, the distance of 20 and 30 (and of 70 and 80) is 10. – Under exactness 20 it is not possible to distinguish 20 and 30. Assume, a decision maker has exactness 20 of analysis. Can we predict, which of the numbers 20 or 30 she will select, when she has to give a numerical response? It seems reasonable that she selects the number which is more easy to access. The mental construction of 20 seems easier than $30 = 50 - 20$, so 20 is selected first. The next question is under which precision 30 arises as a response. This can only be when 30 can be distinguished from 20, i.e. when the exactness of analysis is 10. This idea, that 30 is selected when it is separable from 20, can be also modelled by assuming that 30 is presented as $20 + 10$ (instead of $50 - 20$). – Similar arguments hold for the separation of 120 and 130, where 130 is described as

$100 + 20 + 10$. – The other case are the numbers 70 and 80. Which of these numbers is selected more easily? We assume that a presentation is mentally generated by deciding decreasingly (starting with a sufficiently high number) for every full step number whether to add it to or subtract it from the respective present result, and that in every step of the process the decision maker tries to come as near to the true number as possible. Then we obtain the presentations $70 = 100 - 50 + 20$, and $80 = 100 - 20$. Since 70 needs the additional construction of 50 as a reference point, simplicty of presentation ('shortness') decides for 80. Again 70 will only be responded if it can be separated from 80, under exactness 10. As above, the decision is as if 70 is presented as $100 - 20 - 10$. – The same arguments holds for the selection of 170 after 180, which is as if 170 is presented as $200 - 20 - 10$.

In general we model perception by the

> Response Selection Rule: Preference of the choice of numerical responses is ruled by relative exactness (and equivalently by complexity), if within every presentation it is forbidden to use $5*10^i - 2*10^i$ and $5*10^i + 2*10^i$ (i integer).

(The last line says that numbers of type $3 * 10^i$ are presented as $2 * 10^i + 1 * 10^i$, and numbers of type $7 * 10^i$ are presented as $10 * 10^{i+1} - 2 * 10^i - 1 * 10^i$.)

In the following we use the term 'presentation' only for presentations that fulfill the conditions of the response selection rule. Thereby steps of scales to always contain exactly one number.

1.5 Perception Functions

> Definition: Every scale $S(r, a)$ induces a perception function $per(r, a)$ given by the conditions (1) the distance of any two neighboured elements of the scale is one step, and (2) for numbers which are not elements of the scale the value is obtained by linear interpolation.

In the following we consider perception functions only for $r = 1$, and denote them as $per(a) := per(1, a)$.

Perception functions could be succesfully applied to different situations involving judgements of fairness (see ALBERS 1997.III, IV, V).

Another interesting empirical investigation shows that price demand functions are linear if one assumes that perception of prices follows a perception function, and that the reaction on prices is on a logarithmic scale (see FEGEL 1997). This paper was based on scanner data of vegetables and fruits. It may be noted that on this market demand is in general determined by single consumer's decision to buy or not to buy, rather than by decisions for a quantity.

1.6 Decimal Consistency of Frequencies

An interesting feature of relative exactness and complexity is that the 'attractivity' of a number is defined via its decimal presentation in such a way that multiplication of a number with the factor 10 does not change its ranking of attractivity.

If in addition the task, by which the numeric responses have been raised, has the property that (within a given range of responses) all responses are 'a priori equally likely'[2] (except for their decimal attractivity to be selected as a response) then it can be expected that the obtained data show the following property:

> Decimal Consistency (DC):
> If in a data set the numerical responses x, y, and the numerical responses $10 * x$, $10 * y$ are pairwise 'a priori equally likely', then the corresponding frequencies of responses are in identical relations:
> $freq(x)/freq(y) = freq(10 * x)/freq(10 * y)$.

For every article the prior distributuion of reasonable responses may be described by a density function which may have features of a normal distribution. By aggregating over many articles it may be suggested that one obtains ranges where all responses are 'a priori equally likely'. – Concerning our data set we assume that the condition of equal prior probability is given for numbers that are not too different, for instance if their relative distance $\max(x/y, y/x)$ is not greater than 2.

2 The Questionnaire

198 Subjects, students of Business Administration and Economics, answered two questions concerning prices for 60 articles of private consumption. (The data have been picked up in 1987 from the participants of an compulsory course on price theory. The prices do not conform with the standarts of today.) The general instruction was as follows:

> "Assume a person comes to Germany who does not know the prices here. Please inform this person about the size of prices in Germany. If you want, you can give different responses for different qualities or levels of goods. Do not aggregate via means if different responses come into your mind."

The 60 articles ranged between a candy and a house, and thereby covered a range of nearly 7 powers of 10.

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[2]Given a data set of responses of a group of subjects on a collection of signals. For every number x and every person i, let $S(i, x)$ the set of signals for which x is accepted as a reasonable response by person i. Then the prior probability to select x can be defined to be proportional to $p(x) := \sum_i \#S(i, x)$. (A more complicated measure uses density functions $p(x|i, s)$ for the probability that x is accepted as a response by player i for a signal s, and obtains the overall prior probability as the mean of the $p(x|i, s)$ over all i and all s.) – Here we do not want to verify that the assumptions concerning the prior probabilities are correct, nor even that the kind of modelling is correct. Our aim is only, to introduce decimal consistency as a reasonable property, which is worth to be checked.

The articles were asked in by and large increasing order of prices, so that the subjects did not have to make high jumps in the exactness of responses between two subceding responses. In order to avoid that tiredness effects correlated with price level, the articles were separated into three groups A, B, C of 20; the groups were presented to the subjects in different orders A, B, C; B, C, A; C, A, B.

After these questions had been answered, the subjects were asked to go through all articles, and answer the following question:

> "How sure do you feel about your response? Evaluate this by a number between 0 and 100."

In the following we will only analyse the structure of the price.

3 Results

3.1 Decimal Consistency

Table C gives a list of the articles. Table D informs for all articles about the frequencies of selected prices up to a power of ten omitting the prices with rare frequencies. The aggregated data over all articles are presented in Tables A and B which show the frequencies of responses for the ranges $[1 * 10^i, 10 * 10^i], (i = 1, \ldots, 6)$ and the total frequencies. The tables give the spontaneous impression that the relations of the frequencies are by and large identical on different price levels i.

We apply the test for independence of rank orderings[3] to show that the orderings of the frequencies of numbers show decimal consistency. For this purpose we subdivide the considered range of numbers into the intervals $I(1,2,i) := [1 * 10^i, 2 * 10^i]$, $I(2,5,i) := [2 * 10^i, 5 * 10^i]$, and $I(5,10,i) := [5 * 10^i, 10 * 10^i]$ (i integer), see Table A7. For each of these ranges the numbers are ordered according to their complexities. The number of permutations (of neighboured elements) that are necessary to obtain the order by frequencies from the order by complexity is given in the line '# permutat'. (For the given logarithmic scale of frequencies we obtain a relatively high noise for small numbers. We therefore restricted the analysis to numbers with complexity greaterequal 20. As the data show, this characterization of 'noisiness' by and large conforms with another measure of

[3]The test for independence of rank orderings: given a finite set of elements $e_1, \ldots, e_n$, and rank orderings $q = (q_1, \ldots, q_n)$, $r = (r_1, \ldots, r_n)$ (without loss of generality let $q_1 = 1, \ldots, q_n = n$). The set of different rank orderings equals the set of permutations of $1, \ldots, n$. For every permutation r let $pp(r)$ the minimal number of permutations of pairs of neighbours that is necessary to obtain r from $1, 2, \ldots, n$, let $R(\leq pp)$ the set of all permutations with $pp(r) \leq pp$. Then $\alpha(pp) := \#R(\leq pp)/2^{n-1}$ is the probability that a permutation with $pp(r) \leq r$ occurs. Under the 0-hypothesis that the second ordering is randomly drawn, $\alpha(pp)$ gives the probability that a permutation with $pp(r) \leq r$ occurs, i.e. the level of significance on which the 0-hypothesis can be rejected. – For 7 elements, the number of possible pairwise permutations is between 0 and 21. 2 permutations permit to conclude that under the 0-hypothesis that the two orderings are independent the result is under the 1% most unexpected results, i.e. the 0-hypothesis is rejected on the 1% level. 3 permutations correspond to the 3% level, 4 permutations to the 5% level.

noisiness, where the responses to a number are denoted as being in the range of noise, if all but at most two frequencies in the columns 10^i ($i = .01, \ldots, 100000$) are 0 or 1.) The level of significance on which the 0-hypothesis that both orderings are independent (or random) is rejected is indicated by the number of asterisks, j asterisks denote a level of 10^{-j}, one asterisk denotes 5%. It can be easily seen that the 0-hypothesis is rejected on the 1% level for all columns (exception is the range [500000, 1000000], where the frequencies drastically decrease from 41 for 500000 to 2 (!) for 1000000). There is a general tendency that for low numbers the frequencies increase with the amount of the number, while for high numbers they decrease. This can be seen from the ratios of frequencies of the numbers in the ends of the intervals, which have all complexity 1. Surprising is, that decimal consistency can be supported in spite of this unsymmetry in the prior probabilities of frequencies. For the sum-columns the level of significance is always extremely high (below .001%).

The result strongly supports decimal consistency.

3.2 Prediction of the Frequency of Selection via Complexity

To reduce noise, we aggregate the frequencies over all numbers which differ only by multiplication with an integer power of ten (i.e. we aggregate over the lines of Table A6). The obtained data set serves as a quantitative measure to inform on the quality of the frequency prediction.

The frequency of a given number x is given by the

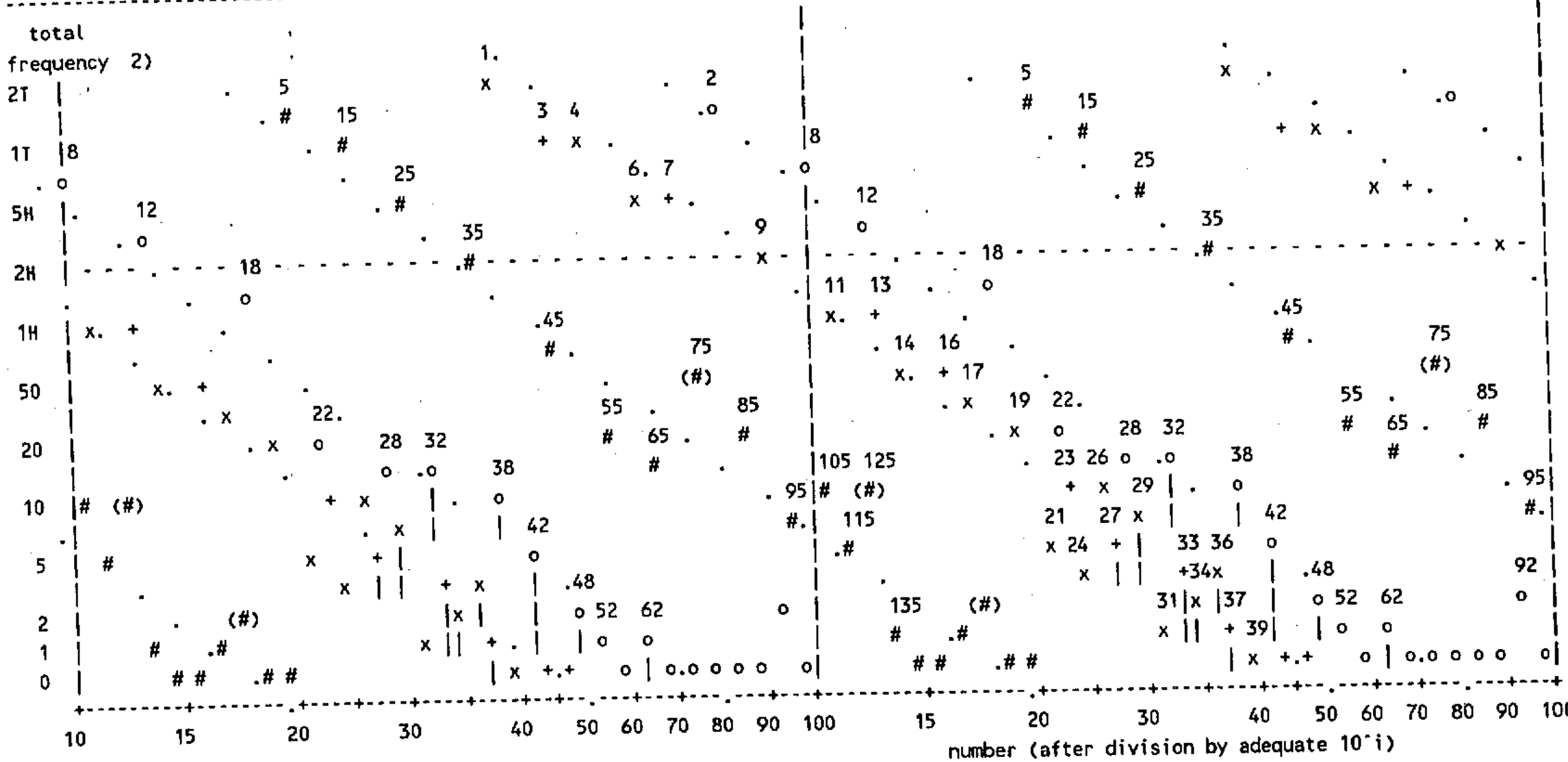
> frequency prediction function:
> $\log(frequency(x)) = (f_0 - a * per(complexity(x))) * b$

f_0 is proportional to the frequency of the data set (it ensures that the total frequency is correct), the multiplier a is such that a change of factor 2 in the complexity gives a change of factor 10 in frequency, for the perception function the finest perceived full step (FPF) (below which the perception is linear) has the value 7 (which is may be a result of aggregation between 5 and 10 where about half of the subjects selected 5, and the others selected 10). The result is multiplied by a factor b, which is 1, if the exactness of x is $2 * 10^i$ or $5 * 10^i$, and is 2 if the exactness of x is $1 * 10^i$ (i an adequate integer).

Figure 2 shows the data of the regression of the frequency prediction function. The result is highly significant ($R^2 = .95$, by linear regression).

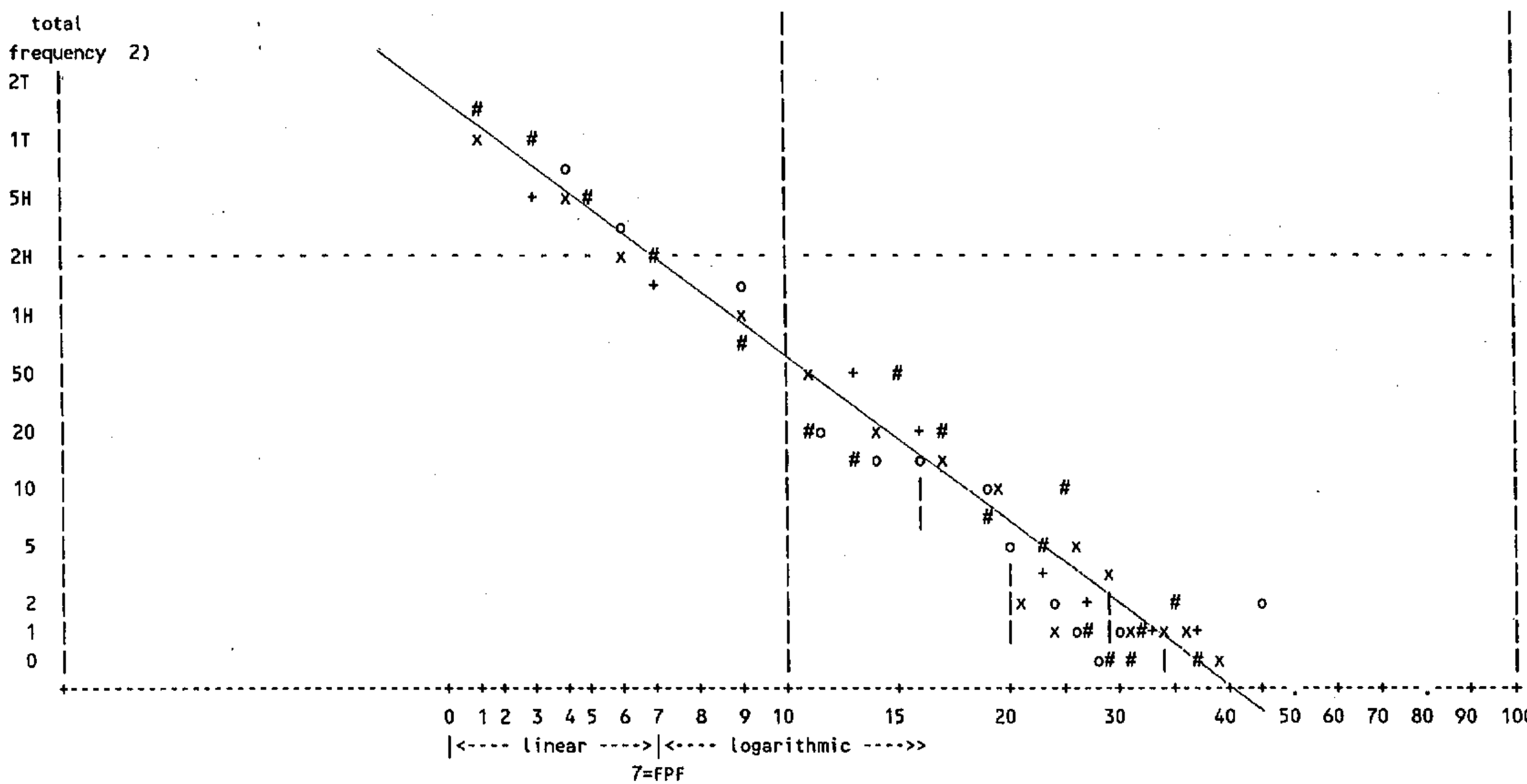
Figure 1 shows the data of the regression before the numbers are transformed to their respective complexities. The figure permits to consider the data of numbers with exactness 1, 2, and 5 separately. (To get separate linear functions for each of the exactness values, we transformed the data with complexity below 7 in such a way, that we multiplied the values of the perception function with 1, 2, or 5.)

Figure 1: Numbers and Their Frequencies 1)



1) to obtain linear function, numbers with complexity below 7 are coded linearly with distances 0--1-2--3--4-5--6--7 (above broken horizontal line)

Figure 2: log(frequency) as a function of per(complexity(x)) under FPF=7



*) x = numbers with exactness 1*10^i, o = numbers with exactness 2*10^i, # = numbers with exactness 5*10^i

3.3 Comments Concerning the Results

This result is similar to that of the price demand function: the complexity of the number corresponds to the price, the frequency of selection correspond to the demand. The relation of the two variables is linear when frequency/demand is considered on a logarithmic scale, and complexity/price is considered under the perception function. This analogy suggests that the complexity (which may be interpreted as a measure for the difficulty to construct the number) creates a kind of costs which subjects 'avoid' by selecting alternatives that are 'less costly'. So far this analogy. We are not sure whether or not this interpretation can serve to understand the motives of the players in their decisions.

Our opinion is rather that the process of giving a numerical response is stopped when the responder cannot be more precise than her present idea of a response. What does it mean that available ideas of prices have the structure as given by the frequency prediction function. Does the probability of having a more precise idea of a number linearly decrease with its complexity?

A remark concerning the linear piece of the perception function: We modelled the evaluation of the complexity via a perception function. The reason, why we did not take just the logarithm, is that log(frequency) changes so much for small values of complexity of x. In fact for values below 7 a linear function is the clearly better predictor. This (compared to logarithmic perception) low sensitivity for small values of complexity may be related to the fact that the numbers with complexity below 7 (i.e. 1, 3, 4, 6, 7; 2, 8, 12; 5, 15, 25, 35) are mentally more easily available, so that another quality of perception causes the linearity of perception. Presently we cannot decide whether there is really some 'perception function as defined in Section 1.5' applied in the construction of the response.

4 Concluding Remarks

There are three surprising results:

1. (presentation) The definition of exactness has to be modified in a way that between 1 and 10 the numbers 2 and 8 are perceived on a cruder level (exactness 2) than $1, 3, 4, 6, 7, 9$, while the other numbers $1, 3, 4, 6, 7, 9$ are on the same level (exactness 1). This aim can be reached by forbidding $5 * 10^i - 2 * 10^i$ and $5 * 10^i + 2 * 10^i$ (i integer) to be used in presentations.
2. (decimal consistency) The frequency structure of responses is by and large the same, if numbers are multiplied by integer powers of 10. This holds over the whole considered interval of prices from .01 to 1 Million DM.
3. (prediction of frequencies via complexity) A quantitative prediction of frequencies is possible, when the prior probability of all numbers is equal, and selection is only influenced by decimal prominence. The relation between the logarithm of the frequency and the value of the complexity under the perception function is linear. (This relation is analogously to the shape of price demand functions.) Within the

logarithmic part of perception, double complexity gives tenfold frequency.

The paper applies the theory of prominence as a useful tool to predict frequencies of numerical choices.

We think that the knowledge of the response pattern permits to refine instruments of statistical analysis in a way that deviations from theoretical responses that are induced by the prominence structure can be anticipated, and need not be interpreted as errors. It should be possible to separate phenomena of prominence from ordinary noise, and thereby refine the statistical instruments of error analysis.

The paper shows that construction (or selection) of numerical responses has clearly logarithmic features. Decimal consistency is a clear indicator that frequency of the choice of a number only depends on its sequence of decimal digits (omitting the zeros after the last digit that is different from zero).

The frequency prediction via the complexity of numbers strongly supports the theory of prominence. The clear linear shape (when measuring frequencies in a logarithmic way, and complexity via the perception function) is very surprising. It might be fruitful to invest further research into the question, which mechanisms produce the linearity of the function. The result is on the same lines as the linearity of the price demand function after measuring demand in a logarithmic way, and prices via the perception function.

Remark 1: A question is whether result 1. concerning the presentation of numbers has general character. Are there other problems or situations where the numbers 3 and 7 have a higher probability to be selected as responses than 1, 4, 6, 9. Are there situations where the presentations $3 = 5 - 2$, and $7 = 5 + 2$ (which are here 'artificially' excluded by a special rule) are not excluded? VOGT, ALBERS (1997) had such results in the bearing experiment, when he introduced a scale with numbers 105, 115, 125, 135, ..., 195 as reference marks to estimate the position of a signal. In this situation, where the 5 is easily accessed, 3 and 7 are more likely to be responded than 2 and 8. A similar result can be observed for probabiities, where the 50 is an a priori given and mentally easily available reference value which permits to construct $70 = 50 + 20$, and $30 = 50 - 20$ easily. (In this context see ALBERS 1997.On the construction of probability responses.)

Remark 2: Another question addresses the use of 25 as a full step number. The first approach of ALBERS and ALBERS (1982) allowed 25 as a full step number. The recent approach of ALBERS (1997) permits 25 as a unit only as last component of a presentation, as in $175 = 200 - 25$, or $425 = 500 - 100 + 25$. The data here show that $25 * 10^i$ is in its frequency right between that of $15 * 10^i$ and $35 * 10^i$. However, $75 * 10^i$ is clearly more frequent than the mean of $65 * 10^i$ and $85 * 10^i$. This indicates that some subjects used 25 as a unit (full step) for the presentation of $75 = 100 - 25$. On the other hand, the frequency of 75 is clearly below that of 80 (and even of 70), what indicates that most of the subjects prefer to use 20 instead of 25 to describe a number between 50 and 100, i.e. they prefer $100 - 20$ and not $100 - 25$. Therefore we decided in the context here, not to model the use of $25 * 10^i$ as a full step number.

Remark 3: It is a special property of this carefully created data set that the relative exactness of analysis selected by the subjects in their responses is widely spread over a long range of possible values. Only this specific feature of the data permitted to obtain the observed linear relation of log-frequencies and per-complexity. It should be mentioned that there are other situations, where the structure of the problem itself suggests (or even forces) subjects to keep the exactness of their analysis on a fixed level. An example for such data are proposals in (certain) n-person characteristic function games. The difference is the following: In the investigation here, the answers are given as estimates of prices. The exactness selection rule (see ALBERS 1997.II) says that the exactness of a response is selected such that there are between 3 and 5 numbers on the selected level of exactness in the 'range of reasonable alternatives'. For price estimates this range can essentially depend on the article, certain persons may have (or may think to have) very precise information for certain articles. For n-person characteristic function games the range of reasonable alternatives is induced by the structure of the game, so that it usually happens that all subjects have always the same level of exactness in their analysis. We think that it is possible to create also other situations, where the subjects' level of relative exactness is fixed to one value throughout the whole experiment. – The result obtained here needs or implies that the range of reasonable alternatives is not restricted by a general rule.

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Table A.1: Frequencies of Responses over all Articles, natural ordering, Part 1 1) 2)

number	multiplier .01	.10	1.00	10	100	1T	10T	100T	SUM	3)	complexity 5)	
1.00	15	149 (1)	410 (3)	496 (2)	674-27(4)	195 (1)	87	58 (1)	2077-27	(12)	1	
1.05	-	-	1	9	-	1	-	-	11			21
1.10	-	-	29 (1)	<64>(1)	19-1	7	2	2	123-1	(2)	11	
1.15	-	-	3	1	1	1	-	-	6			23
1.19	-	-	-	-	2-1	-	-	-	2-1		119	
1.20	-	-	82 (1)	153	105-7 (1)	47	12	17	416-7	(1)		6
1.25	-	-	3	3	3	2	-	-	11			25
1.30	-	-	21	39	30-3	16 (1)	3	2	111-3	(1)	13	
1.35	-	-	1	-	-	-	-	-	1			27
1.39	-	-	-	-	1	-	-	-	1		139	
1.40	-	-	7	29	6	11	-	1	54		14	
1.50	-	15	128-3	268 (3)	325-3	163	45	58	1002-6	(3)		3
1.60	-	-	3	9	14	13	-	2	41		16	
1.65	-	-	-	-	-	-	1	-	1			33
1.69	-	-	-	-	2	-	-	-	2		169	
1.70	-	-	2	6	10	11	4	4	37		17	
1.75	-	-	1	-	-	1	-	-	2			35
1.80	-	-	12-1	45	27 (1)	29 (1)	11	13 (1)	137-1	(3)		9
1.90	-	-	4	3	3	8	-	2	20		19	
1.98	-	-	-	-	1	-	-	-	1			99
2.00	46	62	224-15(1)	363-3(2)	395 (2)	207 (2)	72 (1)	60	1383-18	(8)		1
2.10	-	-	2	2	1	1	-	-	6		21	
2.15	-	-	-	-	-	1	-	-	1			43
2.20	-	-	3	3	2	5	6	3	22			11
2.25	-	-	-	-	-	-	1	-	1			45
2.30	-	-	3-1	2	3	1	3	-	12-1		23	
2.40	-	-	2-1	1	-	-	1	-	4-1		24	
2.50	-	3	72-11	134-2	190	50 (1)	28	40	517-13	(1)		5
2.60	-	-	2-1	6	2	-	1	1	12-1		26	
2.70	-	-	3-1	1	1	-	-	-	5-1		27	
2.75	-	-	1-1	-	-	-	-	1	2-1			55
2.80	-	-	6-2	2	5	2	-	1	15-2			14
2.85	-	-	1-1	-	-	-	-	-	1-1			57
2.90	-	-	5-3	1-1	1	-	-	1	8-4		29	
2.95	-	-	-	-	-	-	-	1	1			59
3.00	15	58	229-48	292-9(1)	311 (4)	141	22	49	1102-57	(5)	3	
3.10	-	-	1	-	-	-	-	-	1		31	
3.20	-	-	8-5	2	2	1	-	-	13-5			16
3.30	-	-	2-2	-	1	-	-	-	3-2		33	
3.40	-	-	2-1	-	-	-	-	-	2-1		34	
3.50	-	2	53-26(2)	41-2	77	23	2	27	226-26	(2)		7
3.60	-	-	2-2	1	-	-	-	1	4-2		36	
3.70	-	-	1-1	-	1	-	-	-	1-1		37	
3.80	-	-	3-2	-	3	-	-	3	9-2			19
4.00	5	26	144-39(1)	246-8(2)	198 (1)	76	9	38 (1)	737-47	(5)	4	
4.10	-	-	1-1	-	1	-	-	-	2-1		41	
4.20	-	-	3-3	-	1	-	-	-	4-3			21
4.25	-	-	2-2	-	-	-	-	-	2-2			85
4.50	-	1	16-1	17-1	22	10	1	7	74-1			9
4.60	-	-	1-1	-	-	-	-	-	1-1		46	
4.80	-	-	1-1	-	-	1	-	-	2-1			24

.) for footnotes see next page

Table A.2: Frequencies of Responses over all Articles, natural ordering, Part 2 1) 2)

number	multiplier .01	.10	1.00	10	100	1T	10T	100T	SUM	3)	complexity 5)	
5.00	86(1)	178	266-10	424-22(2)	192	110 (2)	15	41	1226-32	(5)		1
5.20	-	-	-	-	1	-	-	-	1			26
5.40	-	-	1-1	-	1	-	-	-	2-1			27
5.50	-	-	3	6-2	5	1	-	4	19-2			11
5.60	-	-	1-1	-	-	-	-	-	1-1			28
5.90	-	-	1	1-1	-	-	-	-	2-1		59	
6.00	-	78	157-6(2)	99-16	87	40	10	14	585-22	(2)	6	
6.20	-	-	1-1	-	-	-	-	-	1-1			31
6.50	-	2	3	2-1	4	6	-	1	18-1			13
6.90	-	-	-	1-1	-	-	-	-	1-1		69	
7.00	-	77 (2)	108-1(1)	167-17	35	25	6	3	421-17	(3)	7	
7.50	-	13	7	20-5	5	4	6	1	56-5			15
7.90	-	-	-	1-1	-	-	-	-	1-1		79	
8.00	1	142	198-2	352-43(4)	55	46	39	6	838-45	(4)		4
8.50	-	3	6	5	1	2	1	-	18			17
8.90	-	-	-	2-1	-	-	-	-	2-1		89	
9.00	-	36	74	95-14(2)	21	7	7	1	241-14	(2)	9	
9.25	-	<<2>>f	-	-	-	-	-	-	2			185
9.50	-	3	3	1-1	-	2	-	1	9-1			19
9.80	-	-	-	2-2	-	-	-	-	2-2			49
9.90	-	-	-	1-1	-	-	-	-	1-1		99	
10.00	149(1)	410 (3)	496 (2)	674-27(4)	195 (1)	87	58 (1)	2	1914-27	(12)	1	

1) numbers are separated as products of 'number' and 'multiplier'

2) the reductions (as '674-27') refer to the frequencies obtained by omitting the two articles with extremely fine responses (bread with median 3.50, tennis shoes with median 80.00)

3) the numbers in brackets refer to the number of articles that had the median at this value

5) the left column refers to numbers with exactness 1, the second to number with exactness 2 or 5

Table B.1: Frequencies of Responses over all Articles,
ordering by complexity, Part 1 1) 2)

number	multiplier .01	.10	1.00	10	100	1T	10T	100T	SUM	3)	complexity 5)	
range 1*10^i-2*10^i:												
1.00	15	149 (1)	410 (3)	496 (2)	674-27(4)	195 (1)	87	58 (1)	2077-27	(12)	1	
2.00	46	62	224-15(1)	363-3(2)	395 (2)	207 (2)	72 (1)	60	1383-18	(8)		1
1.50	-	15	128-3	268 (3)	325-3	163	45	58	1002-6	(3)		3
1.20	-	-	82 (1)	153	105-7 (1)	47	12	17	416-7	(1)		6
1.80	-	-	12-1	45	27 (1)	29 (1)	11	13 (1)	137-1	(3)		9
1.10	-	-	29 (1)	<64>(1)	19-1	7	2	2	123-1	(2)	11	
1.30	-	-	21	39	30-3	16 (1)	3	2	111-3	(1)	13	
1.40	-	-	7	29	6	11	-	1	54		14	
1.60	-	-	3	9	14	13	-	2	41		16	
1.70	-	-	2	6	10	11	4	4	37		17	
1.90	-	-	4	3	3	8	-	2	20		19	
# permutat 6)		(0)	4****	1*****	4****	7***	5***	7**	0*****			
within noise: 4)												
1.05 n	-	-	1	9	-	1	-	-	11			21
1.15 n	-	-	3	1	1	1	-	-	6			23
1.25	-	-	3	3	3	2	-	-	11			25
1.35 n	-	-	1	-	-	-	-	-	1			27
1.65 n	-	-	-	-	-	-	1	-	1			33
1.75 n	-	-	1	-	-	1	-	-	2			35
1.19 n	-	-	-	-	2-1	-	-	-	2-1		119	
1.39 n	-	-	-	-	1	-	-	-	1		139	
1.69 n	-	-	-	-	2	-	-	-	2		169	
1.98 n	-	-	-	-	1	-	-	-	1			99

1) numbers are separated as products of 'number' and 'multiplier'
2) the reductions (as '674-27') refer to the frequencies obtained by omitting the two articles with extremely fine responses (bread with median 3.50, tennis shoes with median 80.00)
3) the numbers in brackets refer to the number of articles that had the median at this value
4) a number is classified as 'within noise' if its complexity is greater than 20. Another criterion is that less than 3 frequencies (in the columns) are greater than 1 (these number are denoted by 'n' in Column 2). The categories are identical up to 1.25 and 2.30.
5) the left column refers to numbers with exactness 1, the second to number with exactness 2 or 5
6) number of permutations meeded to obtain observed order (in column) from theoretical order (via complexity) -- the number of stars gives the level of significance, k stars mean 10^(-k).

Table B.2: Frequencies of Responses over all Articles, ordering by complexity, Part 2 1) 2)

number	multiplier .01	.10	1.00	10	100	1T	10T	100T	SUM	3)	complexity 5)	
range 2*10^i-5*10^i:												
2.00	46	62	224-15(1)	363-3(2)	395 (2)	207 (2)	72 (1)	60	1383-18	(8)		1
5.00	86(1)	178	266-10	424-22(2)	192	110 (2)	15	41	1226-32	(5)		1
3.00	15	58	229-48	292-9(1)	311 (4)	141	22	49	1102-57	(5)	3	
4.00	5	26	144-39(1)	246-8(2)	198 (1)	76	9	38 (1)	737-47	(5)	4	
2.50	-	3	72-11	134-2	190	50 (1)	28	40	517-13	(1)		5
3.50	-	2	53-26(2)	41-2	77	23	2	27	226-26	(2)		7
4.50	-	1	16-1	17-1	22	10	1	7	74-1			9
2.20	-	-	3	3	2	5	6	3	22			11
2.80	-	-	6-2	2	5	2	-	1	15-2			14
3.20	-	-	8-5	2	2	1	-	-	13-5			16
3.80	-	-	3-2	-	3	-	-	3	9-2			19
# permutat 6)		(1)	5***	1*****	5***	1*****	6**	4****	0*****			
within noise:		4)										
2.10 n	-	-	2	2	1	1	-	-	6		21	
4.20 n	-	-	3-3	-	1	-	-	-	4-3			21
2.30	-	-	3-1	2	3	1	3	-	12-1		23	
2.40 n	-	-	2-1	1	-	-	1	-	4-1		24	
4.80 n	-	-	1-1	-	-	1	-	-	2-1			24
2.60 n	-	-	2-1	6	2	-	1	1	12-1		26	
2.70 n	-	-	3-1	1	1	-	-	-	5-1		27	
2.90 n	-	-	5-3	1-1	1	-	-	1	8-4		29	
3.10 n	-	-	1	-	-	-	-	-	1		31	
3.30 n	-	-	2-2	-	1	-	-	-	3-2		33	
3.40 n	-	-	2-1	-	-	-	-	-	2-1		34	
3.60 n	-	-	2-2	1	-	-	-	1	4-2		36	
3.70 n	-	-	1-1	-	1	-	-	-	1-1		37	
4.10 n	-	-	1-1	-	1	-	-	-	2-1		41	
4.60 n	-	-	1-1	-	-	-	-	-	1-1		46	
2.15 n	-	-	-	-	-	1	-	-	1			43
2.25 n	-	-	-	-	-	-	1	-	1			45
2.75 n	-	-	1-1	-	-	-	-	1	2-1			55
2.85 n	-	-	1-1	-	-	-	-	-	1-1			57
2.95 n	-	-	-	-	-	-	-	1	1			59
4.25 n	-	-	2-2	-	-	-	-	-	2-2			85

Table B.3: Frequencies of Responses over all Articles, ordering by complexity, Part 3 1) 2)

number	multiplier .01	.10	1.00	10	100	1T	10T	100T	SUM 3)	complexity 5)
range 5*10^i-10*10^i:										
10.00	149(1)	410 (3)	496 (2)	674-27(4)	195 (1)	87	58 (1)	2	1914-27 (12)	1
5.00	86(1)	178	266-10	424-22(2)	192	110 (2)	15	41	1226-32 (5)	1
8.00	1	142	198-2	352-43(4)	55	46	39	6	838-45 (4)	4
6.00	-	78	157-6(2)	99-16	87	40	10	14	585-22 (2)	6
7.00	-	77 (2)	108-1(1)	167-17	35	25	6	3	421-17 (3)	7
9.00	-	36	74	95-14(2)	21	7	7	1	241-14 (2)	9
5.50	-	-	3	6-2	5	1	-	4	19-2	11
6.50	-	2	3	2-1	4	6	-	1	18-1	13
7.50	-	13	7	20-5	5	4	6	1	56-5	15
8.50	-	3	6	5	1	2	1	-	18	17
9.50	-	3	3	1-1	-	2	-	1	9-1	19
# permutat 6)		6***	4***	4****	3****	5***	6**	(9)	2*****	
within noise: 4)										
5.20 n	-	-	-	-	1	-	-	-	1	26
5.40 n	-	-	1-1	-	1	-	-	-	2-1	27
5.60 n	-	-	1-1	-	-	-	-	-	1-1	28
6.20 n	-	-	1-1	-	-	-	-	-	1-1	31
9.80 n	-	-	-	2-2	-	-	-	-	2-2	49
5.90 n	-	-	1	1-1	-	-	-	-	2-1	59
6.90 n	-	-	-	1-1	-	-	-	-	1-1	69
7.90 n	-	-	-	1-1	-	-	-	-	1-1	79
8.90 n	-	-	-	2-1	-	-	-	-	2-1	89
9.90 n	-	-	-	1-1	-	-	-	-	1-1	99
9.25 n	-	<<2>>f	-	-	-	-	-	-	2	185
10.00	149(1)	410 (3)	496 (2)	674-27(4)	195 (1)	87	58 (1)	2	1914-27 (12)	1

Table C: List of Articles

BONB	Bonbon
LOLL	Lolly
DROP	Rolle Drops
KAUG	Kaugummi
OSTE	Osterei
RAGU	Radiergummi
BLEI	Bleistift
SCHO	Tafel Schokolade
SEIF	Stück Seife
ZAHN	Zahnbürste
KUGE	Kugelschreiber
BROT	Brot
STRU	Strumpfhose
SOCK	Paar Socken
PRAL	Schachtel Pralinen
WEIN	Flasche Wein
SEKT	Flasche Sekt
WASC	Waschmittel
KORN	Flasche Korn
EIER	Flasche Eierlikör
KAFF	Packung Kaffee
WEIB	Flasche Weinbrand
FÜLL	Füllhalter
KRAW	Krawatte
ESSE	Essen
SONN	Sonnenbrille
HEMD	Hemd
NICK	Nicki
PULL	Pullover
AKTE	Aktentasche
SAND	Sandale
STEH	Stehlampe
HEIZ	Heizlüfter
KAFM	Kaffeemaschine
RASI	Rasierapparat
TURN	Turnschuhe
SCHU	Schuhe
HOSE	Hose
KLEI	Kleid
KOFF	Koffer
FOTO	Fotoapparat
ANZU	Anzug
BRIL	Brille
RADI	Radio
KASS	Kassettenrecorder
MANT	Mantel
FAHR	Fahrrad
LEDE	Lederjacke
WOHM	Wohnungsmiete
HIFI	Hifi-Anlage
FERN	Fernseher
VIDE	Videorecorder
POLS	Polstergarnitur
ORIE	Orientteppich
GEBR	Gebrauchtwagen
COMP	Computer
KÜCH	Kücheneinrichtung
NEUW	Neuwagwagen
APPA	Appartment
EIGE	Eigenheim
HAUS	Haus

Table D: Frequencies of Main Prices for All Articles (prices up to a power of 10)

	10	11	12	13	14	15	16	17	18	19	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	10	11	12	13	14	15	16	17	18	19	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	10
1 BONB	50<	.	.	.	.	1	.	.	.	.	4	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	15	.	.	.	.	.	.	.	.	.	>44	.	15	.	5	.	60	.	.	.	.	.	1	.	.	.	50
2 LOLL	90	.	.	.	.	14	.	.	.	.	40	2<	8	1	.	.	9	.	.	.	.	.	1	.	.	.	.	.	.	.	.	1	.	.	.	.	3	.	.	.	.	.	>25	.	.	.	.	.	.	.	.	.	90
3 DROP	1	.	.	.	.	.	.	.	.	.	2	.	15	.	>9	.	45	.	15	.	25*	6	28	.	2	.	36<	.	6	.	.	1	.	.	.	.	2	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
4 KAUG	.	.	.	.	.	.	.	.	.	.	.	1	4	.	3	.	>36	.	38	1	35	3	42	2	5	1	22<	1	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5 OSTE	3	.	.	.	.	.	.	.	.	.	8	.	>10	.	3	.	17	.	6	.	1	.	10	.	7	.	46*	1	13	2	1	19	.	.	3	.	18<	6	6	.	1	.	2	.	2	.	.	.	.	.	.	.	3
6 RADI	1	.	.	.	.	.	.	.	.	.	4	.	>12	1	4	1	32	.	7	1	4	1	22	.	2	.	50*	2	9	1	.	24<	1	.	1	.	11	2	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1
7 BLEI	3	.	.	.	.	.	.	.	.	.	3	.	8	.	5	.	>23	.	7	.	7	3	24	.	8	.	67	5	7	2	3	10<	.	.	4	.	5	.	2	.	.	.	.	.	.	.	.	.	.	.	1	.	3
8 SCHO	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	3	1	4	1	>87	17	38	13	1	21<	.	.	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
9 SEIF	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	5	.	4	.	3	.	>11	.	7	.	58	3	8	2	2	19	.	2	.	3	34	6	14<	3	5	.	3	.	.	.	.	.	1	.	.	.	.
10 ZAHN	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	1	.	11	.	1	1	.	>16	2	.	3	1	67	28	32	4<	7	.	7	.	1	.	1	.	1	.	.	.	.
11 KUGE	26	1<	1	.	2	3	.	.	1	.	7	2	3	.	1	.	9	.	.	.	.	.	2	.	.	.	>24	.	.	.	.	10	.	.	.	.	28	7	15	4	2	2	21	.	3	.	3	.	7	.	4	.	26
12 BROT	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	.	.	1	.	15	>11	48	26	39	1	10<	.	6	.	1	.	2	.	.	.	.
13 STRU	15<	.	2	.	.	2	.	.	.	.	2	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	5	.	.	.	.	3	.	.	.	.	>15	6	36	3	23	2	28*	.	14	.	1	.	7	1	1	.	15<
14 SOCK	13<	1	5	.	.	6	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	1	.	.	.	.	12	>5	31	5	19	2	44*	.	13	.	10	2	17	.	2	.	13<
15 PRAL	24	1<	3	.	.	8	.	.	1	.	6	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	4	1	>13	4	15	3	38	1	24	2	16	1	27	.	1	1	24
16 WEIN	13<	.	1	1	.	5	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	.	>17	2	21	2	47	.	38	1	18	.	19	.	6	.	13<
17 SEKT	31	1	7<	3	.	7	1	.	.	.	5	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	8	1	8	>4	35	2	25	.	19	2	27	.	7	.	31
18 WASC	41	2	9	2	3	33	.	.	.	.	26	12	20<	6	4	.	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	2	.	2	1	8	1	>4	.	41
19 KORN	73	8	14	5	1	11<	1	1	8	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	7	.	4	.	>13	.	33	3	16	.	73
20 EIER	55	6	29	11	4	17<	1	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	6	.	7	.	>17	1	25	.	14	.	55
21 KAFF	62	40	47<	6	1	5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	2	.	3	.	1	.	4	1	>14	2	62
22 WEIB	28	3	18	10	16	46	13	4	18	2	20<	7	5	1	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	2	.	1	.	>28
23 FÜLL	37	1	8	.	.	34	.	.	4	.	35	9	15<	.	6	2	3	.	1	.	.	1	2	.	.	.	3	.	.	.	.	.	.	.	.	.	.	.	3	.	2	.	3	.	>13	.	1	.	12	.	1	.	37
24 KRAW	34	.	2	.	1	23	.	.	3	1	62	14	27<	4	4	.	4	1	2	.	.	.	2	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	4	.	.	.	.	.	3	.	1	.	>34
25 ESSE	10	.	4	.	1	>29	1	1	6	.	65	33	14	3	12<	2	5	.	.	.	.	.	1	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	1	.	10
26 SONN	19	.	2	1	.	9	1	.	1	.	33	12	30	7	18	2	19	.	4	.	2<	2	4	.	2	.	7	.	.	.	.	2	.	.	.	.	2	.	.	.	.	.	4	.	1	.	2	.	3	.	.	.	>19
27 HEMD	3	.	1	.	.	9	.	.	2	.	>22	20	33	4	37	2	35	.	8	.	8<	1	8	.	1	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	3
28 NICK	3	.	.	.	.	13	.	.	.	.	>26	4	34	3	30	1	38	.	15<	.	9	1	6	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3
29 PULL	.	.	.	.	.	1	.	.	.	.	8	4	>12	.	10	1	14	.	(34)	.	13	1	26*<	.	6	.	42	1	6<	2	1	10	.	.	.	.	3	2	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.
30 AKTE	.	.	.	.	.	.	.	.	.	.	6	.	11	.	>7	.	39	.	9	.	13	1	18	.	4	.	48	.	11	1	.	17<	1	.	1	.	5	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
31 SAND	5	.	.	.	.	1	.	.	1	.	>15	6	20	1	21	2	41	.	23	.	16	1	15	.	4	.	16<	.	2	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5
32 STEH	1	.	.	.	.	3	.	.	.	.	5	.	6	1	>7	1	12	.	8	.	10	.	16	1	2	.	32	2	6	.	.	37	.	.	6	.	23<	3	9	.	2	.	1	.	.	.	.	.	.	.	.	.	1
33 HEIZ	.	.	.	.	.	.	.	.	.	.	2	3	9	4	>9	1	31	.	11	.	6	.	23	.	3	.	40	1	1	2	1	16	.	.	.	.	14<	4	7	.	1	.	.	.	.	.	.	.	.	.	.	.	.
34 KAFM	.	.	.	.	.	1	.	.	.	.	5	4	>32	3	28	1	53	2	23	.	12	2	12<	.	4	.	13	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
35 RASI	.	.	.	.	.	1	.	.	.	.	2	.	4	1	>18	.	18	1	10	.	8	.	23	.	11	.	58	3	13<	3	3	9	.	.	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
36 TURN	1	.	.	.	.	.	.	.	.	.	3	2	9	2	>8	1	22	2	16	1	17	5	43	.	14	1	27<	1	7	3	.	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1
37 SCHU	.	.	.	.	.	.	.	.	.	.	.	.	2	.	3	1	>17	.	6	.	11	.	32	1	4	.	57	3	24	5	.	15<	3	3	3	.	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
38 HOSE	.	.	.	.	.	.	.	.	.	.	1	.	3	.	7	.	>14	.	10	1	22	.	46	3	20	.	50<	2	5	4	.	4	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
39 KLEI	2	.	.	.	.	.	.	.	.	.	.	.	.	1	2	.	5	.	5	.	3	>2	13	.	7	.	41	3	11	3	.	30	2	2	1	.	27<	5	6	.	.	.	.	.	.	.	.	.	.	.	.	.	2
40 KOFF	.	.	.	.	.	.	.	.	.	.	1	.	.	.	2	.	10	.	5	.	>5	3	17	.	4	.	73	2	5	2	.	41	.	.	.	.	16<	3	4	.	4	.	.	.	.	.	.	.	.	.	.	.	.
41 FOTO	4	.	1	.	.	4	.	.	.	.	1	1	1	.	1	.	3	.	1	.	.	.	6	.	.	.	>19	1	.	1	.	20	.	.	1	.	21	12	20	9	26	1	17	.	15	1	6<	.	4	.	1	.	4
42 ANZU	1	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5	.	4	.	.	>13	1	.	1	1	33	25	41	9	22	2	28<	1	6	.	.	.	.	.	1	.	1
43 BRIL	.	.	.	.	.	.	.	.	.	.	2	.	1	.	3	.	9	.	>5	.	2	.	13	.	1	.	28	.	7	.	.	18	.	3	2	.	39	22	22<	2	3	2	1	.	.	.	.	.	.	.	.	.	.
44 RADI	.	.	.	.	.	1	.	.	.	.	.	1	.	.	2	.	>19	.	1	.	6	.	17	.	4	.	58	.	2	1	.	26	4	.	2	1	30	6<	11	.	2	.	2	1	1	.	.	.	.	.	.	.	.
45 KASS	4	.	.	.	.	.	.	.	.	.	.	.	1	.	2	.	9	.	2	.	2	.	>7	.	3	.	28	.	5	1	1	22	.	.	3	1	32	13	20	6	14<	3	2	.	6	.	.	.	3	.	1	.	4
46 MANT	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	2	.	.	.	.	.	9	.	1	.	.	>21	2	2	6	.	44	23*	41	8	21<	.	9	.	5	.	1	.	1	.	.	.	2
47 FAHR	2	.	1	.	.	.	.	.	.	.	1	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	3	.	1	.	.	7	.	.	1	.	>21	31	45	15	23	5	24<	.	12	.	1	.	2	.	.	.	2
48 LEDE	4	.	.	.	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5	.	.	.	.	8	.	.	.	.	>21	14	38	11	44	3	33	.	7<	.	4	.	1	1	1	.	4
49 WOHM	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	>2-	.	1-	.	1-	3=	.	.	4=-	.	46	23	31	17*	27	4	24	3	12<	1	10	2	3	.	.	.	.
50 HIFI	28	.	4	2	1	29	2	5	6*	3	39	13	15	5<	4	.	8	.	1	.	2	.	1	.	.	.	2	.	.	.	.	.	.	.	.	.	1	.	1	.	2	.	6	.	5	.	.	1	>7	.	3	.	28
51 FERN	33	1	11	4	2	29	2	1	7	2	23<	4	2	.	.	.	2	.	.	.	.	.	.	.	.	.	1	.	.	.	.	2	.	.	.	.	4	.	6	.	1	1	>16	.	7	1	7	1	18	.	2	.	33
52 VIDE	39	5	20	8	4	46	4	2	12	2	14<	5	3	.	.	.	.	.	.	.	.	.	.	.	.	.	3	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	3	.	2	.	1	1	4	.	>9	.	39
53 POLS	12	.	2	2	1	21	.	2	3	1	35	7	41	9	13	1	18<	.	3	.	1	.	3	1	.	.	2	.	.	.	.	.	.	.	.	.	2	.	2	.	3	.	3	.	2	1	1	.	3	.	.	.	>12
54 ORIE	22	1	1	.	.	8	.	.	.	.	33	5	19	1	9	1	13	.	5	.	4	.	6	.	2	.	18=-<		1	.	.	3	.	.	.	.	3	.	3	.	2	.	9	.	1	.	.	.	>7	.	.	.	22
55 GEBR	6	.	.	.	.	6	.	.	.	.	>13	10	21	5	20	3*	31	.	15	1	11	1	14	1	3	1	21<	.	.	.	.	4	.	.	.	.	1	.	2	.	.	.	5	.	.	.	.	.	1	.	.	.	6
56 COMP	35	.	5	.	3	15	4	.	1	.	25	.	21	1	12	1	16	.	4	1	7<	1	3	.	.	.	6	1	1	.	.	.	.	.	.	.	1	.	2	.	1	.	8	.	4	.	4	.	>2	.	3	.	35
57 KÜCH	1	.	1	.	.	3	1	1	.	.	>20	5	20	2	18	4	21*	1	12	4	.	2	17	.	1	1	29	.	5	.	.	12-<	.	.	.	.	7	1	3	.	.	.	2	.	1	.	.	.	.	.	.	.	1
58 NEUW	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	1	.	.	.	.	.	2	.	1	.	>11	1	5	3	.	26	.	4	11	.	58*	25	15<	2	5	1	.	.	2	.	1	.	.	.	.	.	.
59 APPA	36	2	11	.	.	22	.	2	4	1	13-	1	1	3<	2	1	10	.	4	.	1	.	2	.	.	.	1	.	.	.	.	.	.	.	.	.	4	1	2	.	1	.	>10	.	5	.	5	5	33	.	3	.	36*
60 EIGE	18	.	7	2	1	37	1	1	9*	.	39	22	11<	3	3	.	1	.	.	.	.	.	2	.	1	.	.	.	.	.	.	1	.	.	.	.	.	1	1	.	3	.	>5	.	2	.	.	1	6	1	4	.	18
61 HAUS	2	.	.	.	.	.	1	1	.	1	9	>17	37	21	33*	6	32	4	10<	1	2	1	2	.	.	1	2	.	.	.	.	1	.	.	1	.	2	.	1	.	.	.	.	.	1	.	.	.	.	.	.	.	2