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Criteria for Fair Divisions in Ultimatum Games

by

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Abstract

An ultimatum game and a modified ultimatum game were studied. In the ultimatum game 100 could be divided. In the modified ultimatum game player 1 could propose a pair of payoffs $(p, (100-p)/10)$ with a parameter p between 0 and 100. If player 2 accepted the proposal player 1 received a payoff of p and player 2 received a payoff of $(100-p)/10$. Otherwise both players received 0. In the experiment player 1 received less than 50% of his maximal payoff (which was 100) and player 2 received more than 50 % of his maximal payoff (which was 10). A predictor for the experimental data is a model of boundedly rational behavior based on the numerical perception as modeled in the theory of prominence and interpreting perceived payoffs as arguments for a proposal. The modified ultimatum game is compared with the game in which both players had to agree on a pair of payoffs $(p, (100-p)/10)$, but player 1 did not have the right to propose a pair of payoffs.

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Introduction

In ultimatum bargaining situations (for a review of ultimatum bargaining and experiments on ultimatum bargaining see W. Güth, R. Tietz 1990; C. Camerer, R.H. Thaler 1995; A.E. Roth 1995) it is well known that the subgame perfect equilibrium is not a good predictor for the outcome. Many explanations of the observed behavior have been proposed. These explanations have been experimentally tested under a lot of conditions. The main point of the discussion is: Why do people deviate from the subgame perfect equilibrium or is the subgame perfect equilibrium a prediction for the experiments, i.e. why does the responder reject a positive amount of money in laboratory experiments? Some reasons for the observed behavior are deduced from psychological considerations. In a recent study (K. Abbink, G.E. Bolton, A. Sadrieh, Fang-Fang Tang 1996) it was tested whether responders react only on their own payoff (by adaptive learning) or also on the payoff of the proposer (by punishment). It was concluded that both payoffs influence the behavior of the responder.

In this paper the main focus is on the influence of numerical perception on the results of an ultimatum game. A fairness criterion for the average division in ultimatum games is given which connects the payoffs of both players. The criterion is based on modeling boundedly rational behavior. In this criterion the numerical perception is modeled as described by the theory of prominence (W. Albers 1997). All payoffs are transformed by a perception function. The criterion is obtained by modeling the transformed payoffs as numerical arguments which both players have to support a proposal. The sum of these arguments is equal for both players. The resulting criterion is compared with the modified Kalai-Smorodinsky criterion (B. Vogt, W. Albers 1997) which is the best predictor for the experimental data in the games in which 2 players can agree on a payoff pair and none of the players has the right to propose a payoff pair.

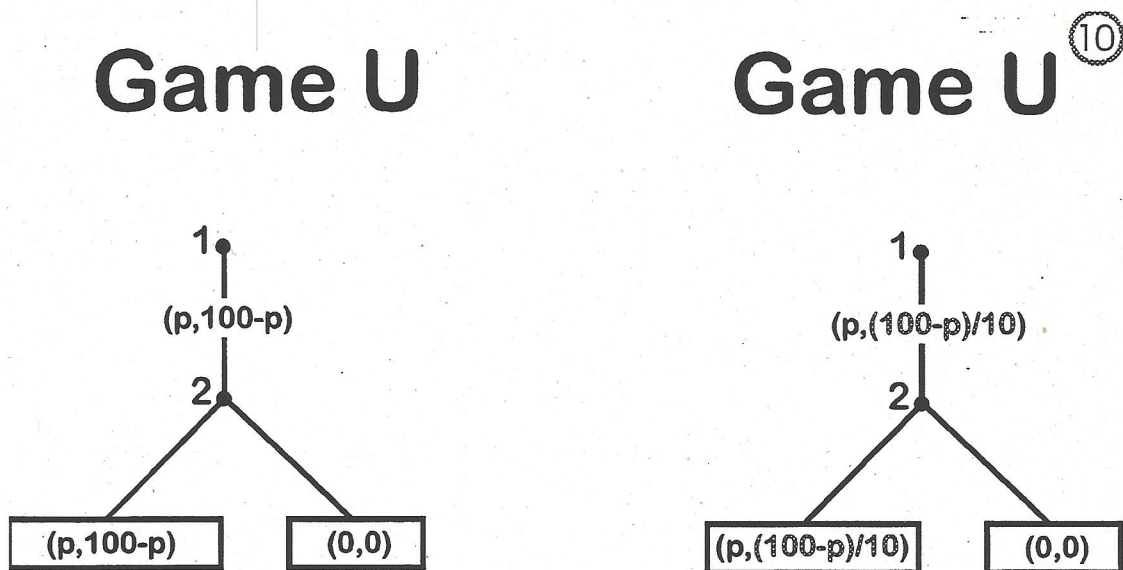
The criterion was tested by an experiment in which an ultimatum game (in which 100 could be divided) and a modified ultimatum game were played. In the modified ultimatum game player 1 could propose a pair of payoffs $(p, (100-p)/10)$ with a

parameter p between 0 and 100. If player 2 accepted the proposal player 1 received a payoff of p and player 2 received a payoff of $(100-p)/10$. Otherwise both players received 0. In the modified game player 2 receives $1/10$ of the amount player 1 receives less. The cost of rejecting a proposal is also reduced, because it is only $1/10$ for a proposal p compared with game U.

The games

The situation considered is shown in figure 1. In the figure payoffs are given in rectangles and proposals in brackets. Player 1 proposes a payoff pair and player 2 can accept or reject it. In case of rejection both players receive 0. In the left part of figure 1 the standard ultimatum game U is shown. Player 1 proposes a division of 100 and player 2 can accept (this is the move to the left) or reject (the move to the right) this proposal. In the right part of the figure the modified ultimatum game U^{10} is shown. In this game player 1 proposes a pair of payoffs $(p, (100-p)/10)$ (with $0 \leq p \leq 100$) and player 2 can accept (the move to the left) or reject (the move to the right) this proposal. In both games in the subgame perfect equilibrium player 2 receives virtually nothing (denoted as ε). In game U^{10} the symmetry between the players is further reduced: the transfer rate of money is $1/10$.

Two different kinds of reasoning may guide player 2 in game U^{10} . The first one is: Player 2 can increase the payoff of player 1 by giving up $1/10$ of the amount he gets. If player 2 cares for the payoff sum, the results of this game should be close to the game theoretic prediction (player 1 gets $100-\varepsilon$ and player 2 gets ε). The second consideration is: If player 2 rejects a proposal, he does not give up as much money as in game U. Therefore it is easier to reject a proposal. This would result in a higher relative payoff of player 2 (the proportion of the maximal amount that player 2 can obtain) in game U^{10} than in game U.

Figure 1: Game U and U^{10} 

Models

The theory of prominence

Before describing the criterion for the division a short introduction in the theory of prominence in the decimal system (W. Albers, 1997) is given, because it is fundamental to the model.

One result of the theory of prominence is that some numbers are easier accessible than others. The numbers that are most easily accessible are the prominent numbers P :

$$P = \{n \cdot 10^z \mid z \in \mathbb{Z}, n \in \{1, 2, 5\}\} = \{\dots, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, \dots\}.$$

If the perception is spontaneous the so called spontaneous numbers S are the numbers that are accessible. These are:

$$S = \{n \cdot 10^z \mid z \in \mathbb{Z}, n \in \{-7, -5, -3, -2, -1.5, -1, 0, 1, 1.5, 2, 3, 5, 7\}\}.$$

The spontaneous numbers include the prominent numbers and one additional number between any two neighbored prominent numbers.

The perception of numbers (including payoffs) is described as differences of steps between prominent and spontaneous numbers. The difference between two neighbored prominent numbers (ordered according to their size) is 1 step and the difference between two neighbored spontaneous numbers (ordered according to their size) is 1/2 step.

The perception is limited for small numbers (due to the problem). In the theory of prominence this is modeled by assuming a finest perceived full step unit Δ which permits to define a perception function v_Δ (by $v_\Delta(\Delta) = 1$) mapping monetary payoffs to the perception space. Table 1 gives the function for $\Delta = 10$, for $\Delta = 20$ and the spontaneous numbers between -150 and +150 (which are relevant for the experiment).

Table 1: Transformation of the spontaneous numbers between -150 and 150 by the v_Δ -function for $\Delta = 10$ and $\Delta = 20$.

number:	-150,	-100,	-70,	-50,	-30,	-20,	-15,	-10,	-5,	0,	5,	10,	15,	20,	30,	50,	70,	100,	150
v_{10} :	-4.5,	-4,	-3.5,	-3,	-2.5,	-2,	-1.5,	-1,	-0.5,	0,	0.5,	1,	1.5,	2,	2.5,	3,	3.5,	4,	4.5
v_{20} :	-3.5,	-3,	-2.5,	-2,	-1.5,	-1,	-0.75,	-0.5,	-0.25,	0,	0.25,	0.5,	0.75,	1,	1.5,	2,	2.5,	3,	3.5

Further refinements of the perception (for example the exact numbers) are described in W. Albers (1997). 1/4 steps are assigned to the exact numbers as given in table 2.

Table 2: The exact numbers between 0 and 100 and the v_{Δ} -function
for $\Delta=10$ and $\Delta=20$.

numbers	0	5	10	15	20	30	50	70	100								
		2,3	7,8	12,13	17,18	25	35,40	60	80								
v_{10}	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4
v_{20}	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3				

For numbers $x > \Delta$ the function $v_{\Delta}(x)$ is (nearly) equal to $3 \cdot \log(x/\Delta) + 1$ ¹. Below the smallest unit Δ the function is linear (compare table 1).

This description of the perception is similar to the Weber-Fechner law (for example in G.T. Fechner, 1968) which describes the perception of stimuli in psychophysics. The perception is proportional to a logarithmic function above a smallest unit. Comparisons between stimuli are performed by forming differences (not quotients). This seems to be plausible, since the stimulus has been transformed by a function proportional to the logarithm.

In the experimental data the smallest unit depends on the problem (the game) and has to be determined. This is performed by means of a rule of the theory of prominence: Δ is the prominent number 2 steps below the smallest prominent number greater than the maximal payoff in the game.

¹ For numbers $x \geq \Delta$ it holds: $|v_{\Delta}(x)/(3 \cdot \log(x/\Delta) + 1) - 1| \leq 7\%$.

Criteria for the accepted proposal

The boundedly rational behavior in the games with the structure described in figure 1 is modeled. The resulting model consists of 3 main parts. These parts are denoted as the perception of payoff, the advantage of player 1, equal sum of the steps of concession.

The perception of payoffs: The payoffs are perceived according to the theory of prominence. All payoffs are transformed by the v_{Δ} -function.

The advantage of player 1: The proposal of player 1 is such that he gets at least the payoff of player 2. It is taken into account that player 1 has the right to propose a division which gives him a stronger position than player 2.

Equal sum of the steps of concession: starting from the maximal payoffs they can obtain both players make equal steps of concessions on the scale defined by the transformed payoffs to get the proposed and accepted payoff pair $(x_L, y_L)^2$.

In the following this is applied to the games studied in this paper.

The modeling of the advantage of player 1 determines the maximal payoffs: The maximal payoff of player 1 is $x_B=100$. The maximal payoff of player 2 is $y_B=50$ in game U (in this case player 1 gets 50 which is his lowest payoff according to the condition that he gets at least the payoff player 2) and $y_B=9.1$ in game U^{10} (player 1 would also get 9.1).

Equal sum of the steps of concession: This part of the model is explained in table 3: The possible steps of concession of both players to reach an agreement (x_L, y_L) are given in the table.

² In the following the payoffs of player 1 are denoted as x and the payoffs of player 2 as y .

Table 3: The steps of concession of both players

	player 1	player 2
concession below the maximal payoff	$v_{\Delta}(x_B)-v_{\Delta}(x_L)$	$v_{\Delta}(y_B)-v_{\Delta}(y_L)$
concession above the minimal payoff (0,0)	$v_{\Delta}(y_L)-v_{\Delta}(0)$	$v_{\Delta}(x_L)-v_{\Delta}(0)$
sum	$v_{\Delta}(x_B)-v_{\Delta}(x_L)$ + $v_{\Delta}(y_L)-v_{\Delta}(0)$	$v_{\Delta}(y_B)-v_{\Delta}(y_L)$ + $v_{\Delta}(x_L)-v_{\Delta}(0)$

$v_{\Delta}(x_B)-v_{\Delta}(x_L)$ is the concession player 1 makes if he lowers his payoff in steps below his maximal payoff. $v_{\Delta}(y_L)-v_{\Delta}(0)$ is the concession he makes by giving player 2 a certain amount above player 2's conflict payoff 0. All these concessions are measured in steps. The analogous consideration is shown for player 2 in table 3.

The equal sum of concessions results in the criterion for the proposed and accepted payoff pair (x_L, y_L) , it is:

$$v_{\Delta}(x_B)-v_{\Delta}(x_L) + (v_{\Delta}(y_L)-v_{\Delta}(0)) = v_{\Delta}(y_B)-v_{\Delta}(y_L) + (v_{\Delta}(x_L)-v_{\Delta}(0))$$

or

The payoff pair (x_L, y_L) is proposed and accepted for which it holds:

$$v_{\Delta}(x_B)-v_{\Delta}(x_L) - (v_{\Delta}(x_L)-v_{\Delta}(0)) = v_{\Delta}(y_B)-v_{\Delta}(y_L) - (v_{\Delta}(y_L)-v_{\Delta}(0))$$

Interpretation of the criterion

The perception of payoffs is described as modeled in the theory of prominence: numbers are perceived in steps resulting in a logarithmic perception function for numbers $x > \Delta$. The advantage of player 1 to make a proposal results in a higher

maximal payoff he can claim. The steps of concession a player makes can be interpreted as an argument supporting a proposal. At the end the sum of arguments is equal for both players.

In the resulting criterion the payoffs of both players are perceived on one scale (without additional weights for one's own and the other's payoff or utility of equal share). After modeling the perception of numbers the criterion is obtained by considering the arguments that are steps of concession.

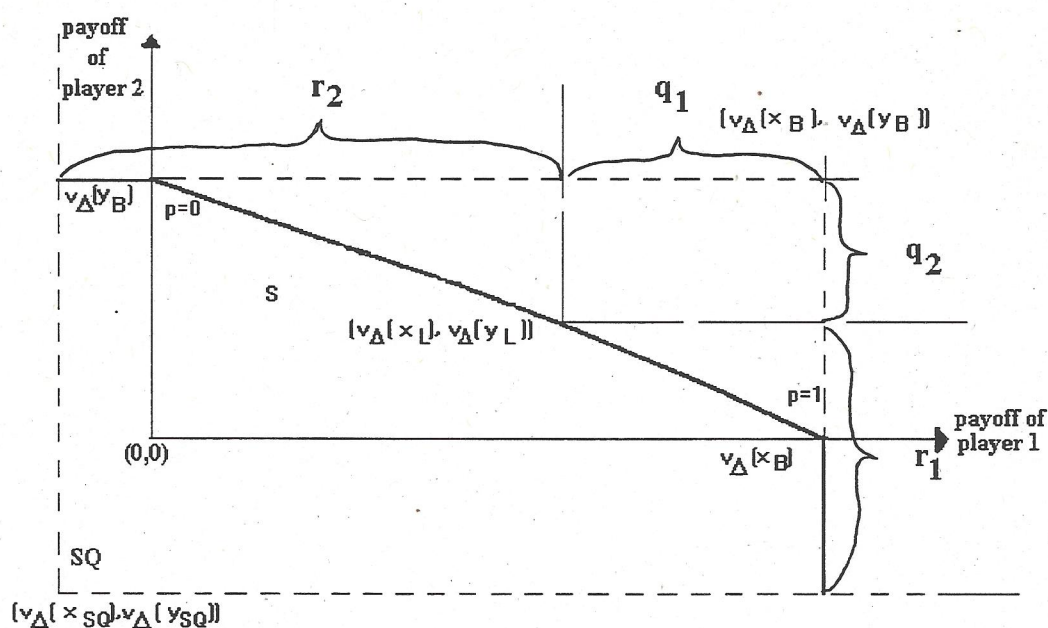
Comparison with the modified Kalai-Smorodinsky criterion

Starting point of this comparison is the selection of payoff pairs in which player 1 does not have the right to make a proposal. Game U is compared with the game in which two players bargain about a payoff pair $(p, 100-p)$. If they do not agree on a payoff pair the status quo point is paid which is $(0,0)$ in this game, but might be different in general. Game U^{10} is compared with the analogous situation without the right of player 1 to make a proposal, i.e. two players bargain about a payoff pair $(p, (100-p)/10)$. If they do not agree the status quo is paid. This situation has been analyzed in (B. Vogt, W. Albers 1997). Several criteria like the Nash-criterion (J.F. Nash 1950 and 1953) or the Kalai-Smorodinsky criterion (E. Kalai, M. Smorodinsky 1975, E. Kalai 1977) have been experimentally tested. The best predictor was the developed model called the modified Kalai-Smorodinsky criterion.

The criterion is explained by means of figure 2. The payoffs are already transformed according to the theory of prominence. The transformed maximal payoff of player 1 and player 2 are denoted as $v_{\Delta}(x_B)$ and $v_{\Delta}(y_B)$, respectively. The Bliss point is $(v_{\Delta}(x_B), v_{\Delta}(y_B))$. The status quo is given by $(v_{\Delta}(x_{SQ}), v_{\Delta}(y_{SQ}))$. The line connecting $(v_{\Delta}(x_B), 0)$ and $(0, v_{\Delta}(y_B))$ gives the pareto-optimal payoff pairs on which the players can agree. Player 1 makes steps of concession if he lowers his payoff in steps below his maximal payoff (this is denoted as q_1) and if he makes steps of concession by giving

player 2 a certain amount above player 2's conflict payoff $v_{\Delta}(x_{SQ})$ (this is denoted as r_1). The analogous steps of concession of player 2 are denoted by r_2 and q_2 . They are added up to a sum of concession and these two sums are equal for the selected payoff pair.

Figure 2: 2-person bargaining about pareto-optimal payoff pairs.



The modified Kalai-Smorodinsky selection criterion is:

The Pareto-optimal payoff pair (x_L, y_L) is selected according to:

$$q_1 + r_1 = q_2 + r_2$$

or $q_1 - r_2 = q_2 - r_1$ resulting in (if the status quo is $(v_{\Delta}(0), v_{\Delta}(0))$):

The Pareto-optimal payoff pair (x_L, y_L) is selected for which it holds:

$$(v_{\Delta}(x_B) - v_{\Delta}(x_L)) - (v_{\Delta}(x_L) - v_{\Delta}(0)) = (v_{\Delta}(y_B) - v_{\Delta}(y_L)) - (v_{\Delta}(y_L) - v_{\Delta}(0))$$

If one compares this criterion with the one for the ultimatum games the difference is the magnitude of the payoff in the Bliss point. In this game the maximal payoffs of

player 1 and 2 are the payoffs in the Bliss point. In the ultimatum game situation the Bliss point is obtained by considering the advantage of player 1 to make a proposal in addition: The proposal of player 1 is such that he gets at least the payoff of player 2.

For example in the game with payoff pairs $(p, 100-p)$ and without the right to make a proposal the Bliss point (x_B, y_B) is $(100, 100)$. In the corresponding game U it is $(100, 50)$.

Experiment

The payoffs

The players received points as their payoffs. The worth of 1 point was 0.5 DM (~\$.33). Losses up to 100 DM (~\$66) had to be paid by the subjects.

The subjects

The subjects were 32 students. They were divided in 4 groups of 8 subjects.

Communication

Free preplay communication via terminals was possible.

Experimental performance

In part 1 of the experiment single games were played in 4 groups of 8 subjects with free preplay communication between 2 players³.

In part 2 a strategy game was played. All subjects selected their strategies for all games and all roles (player 1 and player 2). One game was paid per type of game and per person. Subjects were assigned to each other randomly.

The strategies for the games were given as:

Game U and U¹⁰: as player 1: a proposal p_p .

as player 2: a minimal $(100-p)_a$ accepted.

Predictions

For the predictions Δ has to be determined. According to the theory of prominence it is 2 steps below the maximal payoff which is 100. This results in $\Delta=20$ in U and in U¹⁰.

The resulting predictions are:

$(x_L, y_L) = (60, 40)$ in U and

$(x_L, y_L) = (30, 7)$ in U¹⁰.⁴

³ In the single games the payoff of one subject was the difference of his payoff to the mean payoff of the other subjects not in his group and playing the same role.

⁴ 30 is the exact number which is nearest to the exact result. Due to the limits of perception (the numbers selected by the subjects were not finer than the exact numbers in this experiment or in other experiments) 30 is the prediction.

Results

The results of the strategy experiment are given in figure 3 and 4. In figure 3 the direction in which subjects change their proposals as player 1 and their minimal amount accepted as player 2 from game U to U^{10} are shown.

Figure 3: The changes in the strategies selected by the subjects from game U to U^{10}

	As player 2		As player 1	
	$100-p_a$ in U < $100-p_a$ in U^{10}	$100-p_a$ in U > $100-p_a$ in U^{10}	p_p in U > p_p in U^{10}	p_p in U < p_p in U^{10}
number of subjects	32	0	32	0

All subjects reduce their proposals p_p as player 1 from game U to U^{10} and increase their minimal amounts accepted as player 2. The possibility of player 2 to reject a proposal at "lower cost" (he gives up less money) rises the minimal amount accepted and reduces the proposals p_p . An interpretation of this result is that the cost of rejection has a stronger influence on the result than the possibility to obtain a maximal payoff sum.

For a test of the predictions of the model the medians of the proposals p_p as player 1 and the minimal amounts $(100-p)_a$ accepted as player 2 are given in figure 4 for the 4 groups.

Figure 4: Medians of $(p_p, (100-p)_a)$ in game U and $(p_p, (100-p)_a/10)$ in game U¹⁰

	game U	game U ¹⁰
group 1	(65, 32.5)	(25, 7.1)
group 2	(60, 40)	(30, 7)
group 3	(55, 47.5)	(30, 6)
group 4	(60.5, 40)	(40, 6)
Prediction	(60, 40)	(30, 7)

The difference between the experimental data and the prediction is very small. These data are used for a test of the predictions. A binomial test was used to indicate the quality of the predictions. The test is performed for a significance level of $\alpha=35\%$. This use of the test is certainly not a test, but gives hints for the validity of the results. The result of this procedure is that the predictions are not rejected on this significance level.

The same procedure is applied assuming independence of the strategies of all 32 subjects. This leads to the same result of the procedure. Summarizing the criterion of the model of boundedly rational behavior is a good predictor for the data.

Conclusion

In this paper a criterion for the proposals accepted in the ultimatum game is given and tested by an experiment in which an ultimatum game and a modified ultimatum game were played. Especially the modified ultimatum game is a test of the criterion because the maximal payoffs of both players differ by a factor of 10.

The model of boundedly rational behavior resulting in a criterion for a proposal that is accepted consists of 3 parts. Part 1 is the **perception of payoffs**: All payoffs are transformed by the perception function (v_{Δ} -function) described by the theory of prominence. In part 2 the **advantage of player 1** is modeled: the proposal of player 1 is such that he gets at least the payoff of player 2. In part 3 the selection criterion is given by an **equal sum of the steps of concession**: starting from the maximal payoffs both players can obtain they make equal steps of concession on the scale defined by the transformed payoffs to get the proposed and accepted payoff pair (x_L, y_L) . This leads to the criterion without free parameters.

A comparison with the modified Kalai-Smorodinsky criterion (B. Vogt, W. Albers 1997) leads to the result that the modified Kalai-Smorodinsky criterion and the adaptation of the Bliss point to the ultimatum bargaining situation result in the same selection criterion.

The experimental result is that the model of boundedly behavior is a good predictor of the experimental data for the ultimatum and the modified ultimatum game.

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