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The Averaging Mechanism

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Abstract

In two-person meta-bargaining theory we consider two-person bargaining games and we assume that the agents want to apply two different bargaining solutions. A mechanism is a function which assigns to every meta-bargaining game an allocation depending on the two bargaining solutions supported by the agents. We define a mechanism which can be justified by the rationale of the underlying step-by-step bargaining procedure and list some properties which are only satisfied by our mechanism. For every bargaining game we define a correponding non-cooperative game in which agents can choose bargaining solutions as strategies and the outcome is determined by the mechanism. We show that each agent has a unique dominant strategy, namely his respective dictatorial solution. The function which assigns to every bargaining game the equilibirium outcome of the corresponding non-cooperative game is the discrete Raiffa solution. Hence, we give a non-cooperative foundation of the discrete Raiffa solution.

Keywords: Meta-bargaining, Step-by-step bargaining, Mechanism, Raiffa solution, Non-cooperative foundations

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1 Introduction

1.1 Motivation

The work we will present in the following is concerned with two-person bargaining theory. In this theory we deal with situations in which by collaborating the agents can reach certain agreements which benefit both. If the collaboration fails nobody can affect the well-being of the other agent. As long as there is only one agreement which is most preferred by both agents, collaboration will be easy. Problems arise if there are several agreements and the agents prefer different ones. Typical examples of such situations are bargaining situations between two agents like 'dividing the heritage between two heirs', 'trading between two nations', or 'negotiations between a labor union and an employer'. The analysis of such bargaining situations is the aim of bargaining theory. In order to carry out this analysis often two-person bargaining games as formalized by Nash (1950) are considered. A (two-person) bargaining game (S, d) consists of a set $S \subseteq \mathbb{R}^2$ of feasible utility allocations for two agents and the disagreement point d which is contained in the set S. If the agents reach an agreement about the final outcome they receive the corresponding utility levels, otherwise they receive the outcome of the disagreement point d. In order to investigate which agreements the agents may reach we often make use of a bargaining solution. A bargaining solution is a function which proposes an agreement for every bargaining game of a certain class. A bargaining solution may arise from an analysis motivated either by cooperative or by non-cooperative considerations. In the cooperative set-up we formulate properties which we consider to mirror agents' ideas about equity and fairness and look for functions which are compatible with these requirements. The seminal work in this area is Nash (1950). In the non-cooperative set-up a non-cooperative game resembling some general underlying structure of the bargaining process is formulated for any bargaining game. Then we look for the equilibria (Nash equilibria or some refinements) of these non-cooperative games. If for every game there exists a unique equilibrium we have created a bargaining solution. This idea of non-cooperative foundation of a bargaining solution was initiated in Nash (1953) and is often referred to as the "Nash program". Another fundamental work in this line of research is Rubinstein's sequential bargaining model (cf. Rubinstein (1982)).

Now let us assume that agents always want to apply a certain bargaining solution; maybe because they have attended a course in bargaining theory or maybe because they have some intuitive ideas about which point to select in every bargaining game. Problems arise if the agents want to apply two different bargaining solutions and for a particular bargaining game they are involved in the allocations proposed by these solutions do not coincide. In this case they have two possibilities. Either they accept the payoff of the disagreement point, d, or they agree on a compromise in dependence of their preferred solutions. This sort of bargaining is called meta-bargaining. Formally, a (two-person) meta-bargaining game is a tuple $[(S,d);f,g]$ where (S,d) is a bargaining game and f and g are bargaining solutions supported by agent 1 and agent 2, respectively. To analyze how agents in a meta-bargaining game may reach a compromise depending on the bargaining game and their respective preferred bargaining solutions, we will employ an analogous method as for the analysis of bargaining games introducing meta-bargaining solutions which will be called mechanisms. A mechanism is a function which proposes an agreement for every meta-bargaining game of a certain class of meta-bargaining games. Meta-bargaining games may be analyzed from a cooperative point of view. In this cooperative approach we formulate properties which mirror agents' ideas about fair compromises and look for functions which are compatible with these properties. A first work in this line is Anbarci and Yi (1992). Marco, Peris, and Subiza (1995) criticize the previous paper and propose a mechanism which is very similar to Anbarci and Yi's mechanism but expunges the shortcomings of their mechanism. They characterize it uniquely by a formalized system of axioms (properties). A non-cooperative approach to meta-bargaining theory was initiated by van Damme (1986). He starts with the definition of a mechanism. Then he investigates whether the mechanism is robust against strategic considerations, i. e., he defines for any bargaining game a non-cooperative game in which agents can choose bargaining solutions as strategies and the payoff is determined by the outcome of the mechanism. He analyzes the Nash equilibria of these games. Looking at the intersection of the sets of Nash equilibria of all games he proves that under his mechanism only the Nash solution (cf. Nash (1950, 1953)) is optimal in the sense that only both agents choosing the Nash solution constitutes an equilibrium in every non-cooperative game. By modifying van Damme's mechanism Chun (1985) concludes that the Kalai-Smorodinsky solution (cf. Kalai and Smorodinsky (1975)) is optimal. In Naeve-Steinweg (1997) the non-cooperative approach is supplemented by cooperative considerations. It is shown that van Damme's and Chun's mechanism fail to satisfy some desirable properties like Pareto-optimality. Both mechanisms are modified in such a manner that while the non-cooperative analysis still yield the same conclusions they now satisfy some properties like Pareto-optimality the original mechanisms do not satisfy.

In this paper we apply both approaches. We introduce a new mechanism which simulates a step-by-step bargaining process. The underlying rational is quite close to what we can observe people do in order to find a compromise between their differing preferred outcomes. Very often agents toss a coin and let the decision which outcome will be realized made by chance. Our mechanism suggests that the agents accept the expected value of this experiment as a new disagreement point and go on bargaining in the same way until they reach a Pareto-optimal outcome. We list some properties a mechanism may satisfy and show that our mechanism satisfies all these properties. In addition, we give a set of axioms which characterize our mechanism on a subset of meta-bargaining games. Then, by analyzing for every bargaining game the corresponding non-cooperative game we show that each agent has a unique dominant strategy, namely his respective dictatorial solution. Thus, every non-cooperative game has a unique Nash-equilibrium. The function assigning to every bargaining game the unique equilibrium outcome of the corresponding non-cooperative game is the discrete Raiffa solution. Hence, we obtain a non-cooperative foundation of the discrete Raiffa solution.

This paper is organized as follows. In Subsection 1.2 will state the notation we will make use off and the terms of bargaining theory we will employ in this paper. Then we introduce the notions of meta-bargaining games and mechanisms. In Section 2 we define our mechanism. Then we characterize the mechanism by some properties and finish Section 2 by analyzing the associated non-cooperative games which leads to the non-cooperative foundation of the discrete Raiffa solution. Section 3 which contains some concluding remarks finishes this paper.

1.2 Basic definitions and notation

In this subsection we introduce the notation, basic definitions and notions of bargaining and meta-bargaining theory we will use in the following. Large parts of the presentation follow Thomson (1994).

Let $A \neq \emptyset$ be a subset of $\mathbb{R}^2$ and let x, y, and d be elements of $\mathbb{R}^2$.

- Vector inequalities are denoted by $x \geq y$, if $x_i \geq y_i$, for $i = 1, 2$, $x \gneq y$, if $x_i \geq y_i$, for $i = 1, 2$, and $x \neq y$, and $x > y$, if $x_i > y_i$, for $i = 1, 2$.
- $\Delta := \{x \in \mathbb{R}^2 \,|\, x_1 = x_2\}$ denotes the diagonal in $\mathbb{R}^2$.
- $\mathcal{N}_\epsilon(d) := \{x \in \mathbb{R}^2 \,|\, \mid x - d \mid < \epsilon\}$ denotes the open ϵ-neighborhood of d.
- inter (A) denotes the interior and ∂A denotes the boundary of the set A.
- $A_{\geq d} := \{x \in A \,|\, x \geq d\}$. For simplifying notation we define $A_+ := A_{\geq 0}$.
- The convex hull of A, cvh (A), is defined by

  $$\text{cvh}\,(A) := \{x \in \mathbb{R}^2 \,|\, x = \lambda a + (1 - \lambda)b,\ \lambda \in [0, 1],\ a, b \in A\}.$$
- A is called d-comprehensive if $x \in A_{\geq d}$, $x \geq y \geq d$, implies $y \in A$.
- For a set $A \subseteq \mathbb{R}^2_{\geq d}$ the d-comprehensive hull of A, d-comh (A), is defined by

  $$d\text{-comh}\,(A) := \{y \in \mathbb{R}^2_{\geq d} \,|\, \exists a \in A \text{ such that } a \geq y \geq d\}.$$
- For a set $A \subseteq \mathbb{R}^2_{\geq d}$ the d-comprehensive and convex hull of A, d-ccvh (A), is defined by

  $$d\text{-ccvh}\,(A) := d\text{-comh}\,(\text{cvh}\,(A)).$$
- The set of (strongly) Pareto-optimal allocations (also called Pareto-boundary) is defined by $PO(A) = \{\, x \in A \,|\, \{y \in A | y \gneq x\} = \emptyset \,\}$ and the set of weakly Pareto-optimal allocations is defined by $WPO(A) = \{\, x \in A \,|\, \{y \in A | y > x\} = \emptyset \,\}$.

A (two-person) bargaining game is a tuple (S, d) where $S \subseteq \mathbb{R}^2$ denotes the set of feasible utility allocations and $d \in S$ denotes the disagreement point. If the agents unanimously agree on a point $x \in S$, i. e., if they sign a binding contract that $x \in S$ is the final outcome, they obtain x. Otherwise they obtain d. The bargaining games we will consider in the following are elements of the set

$$\Sigma := \{(S, d) \,|\, d \in S \subseteq \mathbb{R}^2,\ S \text{ is compact, convex, } d\text{-comprehensive}\}.$$

For a bargaining game (S, d) the utopia point is defined by

$$u_i(S, d) = \max\{x_1 \geq d_1 \,|\, \exists x_2 \geq d_2 : (x_1, x_2) \in S\}$$

and

$$u_2(S,d) = \max\{x_2 \geq d_2 \,|\, \exists x_1 \geq d_1 : (x_1, x_2) \in S\}.$$

Given a set of bargaining games under consideration, Σ, a bargaining solution is a function $f : \Sigma \longrightarrow \mathbb{R}^2$ which proposes for every bargaining game (S,d) in Σ a final outcome $f(S,d) \in S$ the agents should agree on. A bargaining solution $f : \Sigma \longrightarrow \mathbb{R}^2$ is said to satisfy the property

- Pareto-optimality (PO)
  if for all $(S,d) \in \Sigma$: $f(S,d) \in PO(S)$;
- weak Pareto-optimality (WPO)
  if for all $(S,d) \in \Sigma : f(S,d) \in WPO(S)$;
- individual rationality (IR)
  if for all $(S,d) \in \Sigma : f(S,d) \geq d$;
- independence of individually irrational allocations (IIA)
  if for all $(S,d) \in \Sigma$ and all $(S',d) \in \Sigma$ with $S' \subseteq S$ and $f(S,d) \in S'$ we have $f(S',d) = f(S,d)$;
- cutting (CUT)
  if for all $(S,d) \in \Sigma$ and all $(S',d) \in \Sigma$ such that $S' \subseteq S$ and $x \in S \backslash S'$ implies $[x_i \geq f_i(S,d)$ and $x_j \leq f_j(S,d)]$, we have $f_i(S',d) \leq f_i(S,d)$;
- individual monotonicity (IMON)
  if for all $i \in \{1,2\}$ and for all games (S,d), $(S',d) \in \Sigma$ satisfying $S \subseteq S'$ and $u_i(S,d) = u_i(S',d)$, we have $f_j(S,d) \leq f_j(S',d)$, $j \in \{1,2\}$, $j \neq i$;
- disagreement point monotonicity (d-MON)
  if for all games (S,d), $(S,d') \in \Sigma$ such $d'_i > d_i$ and $d'_j = d_j$, $i,j \in \{1,2\}$, $i \neq j$, we have $f_i(S,d') \geq f_i(S,d)$;
- strong disagreement point monotonicity (ST-d-MON)
  if for all games (S,d), $(S,d') \in \Sigma$ such $d'_i > d_i$ and $d'_j = d_j$, $i,j \in \{1,2\}$, $i \neq j$, we have $f_j(S,d') \leq f_j(S,d)$;
- no transfer paradox (NTP)
  if for all games (S,d), $(S,d') \in \Sigma$ such $d'_i > d_i$ and $d'_j < d_j$, $i,j \in \{1,2\}$, $i \neq j$, we have $f_i(S,d') \geq f_i(S,d)$ and $f_j(S,d') \leq f_j(S,d)$;
- continuity (CONT)
  if for all $(S,d) \in \Sigma$ and all sequences $((S^k, d^k))_{k \in \mathbb{N}}$ such that $(S^k, d^k) \xrightarrow{k} (S,d)$ (in the Hausdorff–metric, for a definition see for example Jansen and Tijs (1983))

and $(S^k, d^k) \in \Sigma$, $\forall k$, then $f(S^k, d^k) \xrightarrow{k} f(S, d)$;

- continuity with respect to the disagreement point (d-CONT) if for all $(S, d) \in \Sigma$ and all sequences $(d^k)_{k \in \mathbb{N}}$ such that $d^k \xrightarrow{k} d$ and $(S, d^k) \in \Sigma$, $\forall k$, then $f(S, d^k) \xrightarrow{k} f(S, d)$;
- continuity with respect to the feasible set (S-CONT) if for all $(S, d) \in \Sigma$ and all sequences $(S^k)_{k \in \mathbb{N}}$ such that $S^k \xrightarrow{k} S$ and $(S^k, d) \in \Sigma$, $\forall k$, then $f(S^k, d) \xrightarrow{k} f(S, d)$.

The sets of bargaining solutions we will consider in the sequel are contained in the set $\mathcal{F} := \{f : \Sigma \longrightarrow \mathbb{R}^2 \,|\, f \text{ satisfies IR }\}$.

Some well-known bargaining solutions are listed in the following Consider a bargaining game $(S, d) \in \Sigma$. The Nash solution f^N (cf. Nash (1950, 1953)) is defined by

$$f^N(S, d) := \operatorname{argmax} \{\Pi_{i=1}^2 (x_i - d_i) | x \in S_{\geq d}\}$$

if $d \notin WPO(S)$ and $f^N(S, d) := \bar{x} \in PO(S_{\geq d})$ if $d \in WPO(S)$. The Kalai-Smorodinsky solution f^{KS} (cf. Kalai and Smorodinsky (1975)) is defined by

$$f^{KS}(S, d) := \bar{x} \in PO(S) \cap \text{cvh}(\{d, u(S, d)\})$$

if $d \notin WPO(S)$ and $f^{KS}(S, d) := \bar{x} \in PO(S_{\geq d})$ if $d \in WPO(S)$. The egalitarian solution f^E (cf. Kalai (1977)) is defined by

$$f^E(S, d) := \bar{x} \in WPO(S) \cap (\{d\} + \Delta_+)$$

if $d \notin WPO(S)$ and $f^E(S, d) := d$ if $d \in WPO(S)$. The discrete Raiffa solution (cf. Raiffa (1953) and Luce and Raiffa (1957)) is defined by

$$f^R(S, d) := \lim_{t \to \infty} d^t$$

where $d^1 := d$ and $d^t := \frac{1}{2}\Big[f^{D,1}(S, d^{t-1}) + f^{D,2}(S, d^{t-1})\Big]$, $\forall t \geq 2$. The dictatorial solutions $f^{D,i}$, $i = 1, 2$ (they cannot be traced to a particular source) are defined by

$$f^{D,1}(S, d) := (u_1(S, d), d_2) \quad \text{and} \quad f^{D,2}(S, d) := (d_1, u_2(S, d)),$$

respectively. The lexicographic dictatorial solutions $f^{D,i*}$, $i = 1, 2$, are defined by

$$f^{D,i*}(S, d) := \bar{x} \in PO(S) \cap S_{\geq f^{D,i}(S,d)}.$$

After stating the basic notions of bargaining theory we will make use of we now give the definitions of meta-bargaining games and mechanisms. These definitions as well as some definitions we will use later are taken from Marco, Peris, and Subiza (1995).

Definition 1

Given a set of (two-person) bargaining games, Σ, and a set of bargaining solutions, $\mathcal{F}$, a (two-person) meta-bargaining game is a triple $[(S,d);f,g]$, where (S,d) is a bargaining game in Σ and $f,g \in \mathcal{F}$ are two solutions for bargaining games supported by agent 1 and agent 2, respectively.
$\Sigma_{\mathcal{F}} := \{[(S,d);f,g] | (S,d) \in \Sigma, f,g \in \mathcal{F}\}$ *denotes the class of two-person meta-bargaining games under consideration.*

If the agents reach an agreement about the final outcome of the meta-bargaining game they receive the corresponding utility levels, otherwise they receive the outcome of the disagreement point. To analyze which agreements the agents may reach it is often made use of a mechanism.

Definition 2

Given a set of (two-person) meta-bargaining games under consideration, $\Sigma_{\mathcal{F}}$, a mechanism is a function $M : \Sigma_{\mathcal{F}} \longrightarrow \mathbb{R}^2$ which assigns to every meta-bargaining game $[(S,d);f,g]$ in $\Sigma_{\mathcal{F}}$ an allocation $M[(S,d);f,g] \in S$.

Analogously to a bargaining solution which proposes for every bargaining game in a certain class of bargaining games an agreement the agents should agree on if their ideas about fairness are the same, a mechanism, i. e., a meta-bargaining solution proposes for every meta-bargaining game a final outcome the agents should agree on in order to solve their conflict arising from their different ideas about fairness. Alternatively, we can interpret a mechanism as an arbitration rule used by an arbitrator who is asked by the agents to solve their conflict.

Remark 1

Let $\Sigma_{\mathcal{F}}$ be a class of meta-bargaining games. We create a new bargaining solution $M[\,\cdot\,;f,g] : \Sigma \longrightarrow \mathbb{R}^2$ by fixing a pair $f,g \in \mathcal{F}$ of bargaining solutions and a mechanism $M : \Sigma_{\mathcal{F}} \longrightarrow \mathbb{R}^2$.

2 The Averaging Mechanism

In this subsection we introduce a mechanism which can be justified by the rationale of the underlying step-by-step bargaining procedure. It seems that what this step-by-step bargaining process does is quite close to what agents sometimes do if they are involved in a conflict in which their ideas about the final outcome are different but they both agree that disagreeing is not desirable. Agents in such a situation could think of tossing a fair coin and let the decision about which outcome will be the final one be made by chance. The expected value of this experiment is represented by the midpoint of the line connecting the allocations corresponding to the proposed outcomes of their respective supported solutions. If the expected value, i. e., if this midpoint is a (weakly) Pareto-optimal allocation every risk-averse and even every risk-neutral agent will consider this allocation as a reasonable outcome of their meta-bargaining game and they will agree on this allocation. In case that the midpoint is in the interior of the set of all possible utility allocations they will realize that considering this allocation as a new disagreement point and starting bargaining about allocations which give to each agent as least as much as this disagreement point is quite rational and therefore exactly what they think they should do. This procedure will be repeated by the agents until they reach a (weakly) Pareto-optimal allocation. We start our considerations by marking out the framework and defining the mechanism. We list some properties the mechanism satisfies and we show that our mechanism is uniquely characterized by some of these properties. Then we define for every bargaining game a corresponding non-cooperative game. By analyzing these non-cooperative games we show that choosing his respective dictatorial solution is the unique dominant strategy for each agent. The function which assigns to each bargaining game the unique equilibrium outcome of the corresponding non-cooperative game is the discrete Raiffa solution. Hence, in the line of the "Nash program" we give a non-cooperative foundation of the discrete Raiffa solution.

As we have already pointed out we restrict attention to agents who are not risk-loving. To do so, we state the following postulate.

Postulate 1

Agents are risk-averse or risk-neutral.

Nevertheless, it is not clear that a risk-loving agent will not support the mechanism defined below. This may depend on the grade of his risk-loving, on his opponent, and the solutions they are supporting. Recall, that we consider bargaining games $(S, d) \in \Sigma$ where the bargaining set is non-empty, compact, convex, and d-comprehensive and where the disagreement point might be an element of the boundary and bargaining solutions $f \in \mathcal{F}$ which satisfy individual rationality. $\Sigma_{\mathcal{F}}$ denotes the set of meta-bargaining games we will consider in the following.

In the following we define the averaging mechanism as the function which assigns to a meta-bargaining game the limit of a sequence of disagreement points which we obtain by simulating a hypothetical step-by-step bargaining procedure in which agents toss a coin in order to determine the final outcome and consider the expected value of this experiment as a new disagreement point.

Definition 3

The mechanism $U : \Sigma_{\mathcal{F}} \longrightarrow \mathbb{R}^2$ *is defined by*

$$U[(S,d); f, g] = \lim_{t \to \infty} d^t,$$

where the sequence of disagreement points $(d^t)_{t \in \mathbb{N}}$ *is defined by*

$$d^1 = d, \quad \textit{and} \quad d^t = \frac{1}{2}\Big(f(S, d^{t-1}) + g(S, d^{t-1})\Big), \quad t > 1.$$

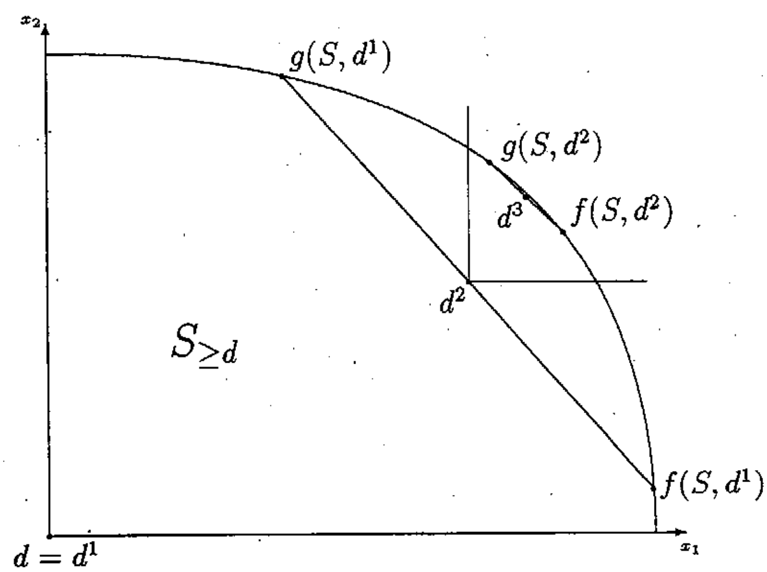

Figure 1

Proposition 1

On the class $\Sigma_{\mathcal{F}}$ U is a mechanism.

Proof:

Consider a meta-bargaining game $[(S,d);f,g] \in \Sigma_{\mathcal{F}}$. f and g satisfying individual rationality implies $f(S,d) \geq d = d^1$ and $g(S,d) \geq d = d^1$. Thus,

$$d^2 = \frac{1}{2}f(S,d^1) + \frac{1}{2}g(S,d^1) \geq d^1$$

holds. Clearly, (S,d^2) is an element of Σ. By repeating the argument we see that $(d^t)_{t\geq 1}$ is a monotonic non-decreasing sequence in $S_{\geq d}$. Since S is bounded from above this sequence converges. Hence, $\lim_{t\to\infty} d^t = U[(S,d);f,g]$ exists and it is an element of $S_{\geq d}$. q.e.d.

Analogously to bargaining theory we can formulate some reasonable and desirable properties a mechanism may satisfy. There are two kinds of properties. Since $M[\,\cdot\,;f,g]$ can be seen as a bargaining solution induced by the mechanism M and the bargaining solutions f and g we can adopt some classical properties for bargaining solutions. But there is also a need for a new kind of properties which capture a notion of meta-fairness and meta-equity, i. e., properties which say something about how agents should be treated when they have different ideas about a fair solution of a bargaining game. All but the last three properties are taken from Marco, Peris, and Subiza (1995).

Definition 4

Given the class of meta-bargaining games, $\Sigma_{\mathcal{F}}$, a meta-bargaining mechanism M is said to satisfy

- *Pareto-optimality* (**PO**)
  if $\forall f,g \in \mathcal{F}$, $M[\,\cdot\,;f,g]$ *satisfies PO;*
- *weak Pareto-optimality* (**WPO**)
  if $\forall f,g \in \mathcal{F}$, $M[\,\cdot\,;f,g]$ *satisfies WPO;*
- *individual rationality* (**IR**)
  if $\forall f,g \in \mathcal{F}$, $M[\,\cdot\,;f,g]$ *satisfies IR;*
- *continuity with respect to the disagreement point* (d-**CONT**)
  if $\forall f,g \in \mathcal{F}$, $M[\,\cdot\,;f,g]$ *satisfies d-CONT;*

- *impartiality* (**IM**)
  if $\forall\,[(S,d);f,g] \in \Sigma_{\mathcal{F}}$, $M[(S,d);f,g] = M[(S,d);g,f]$ *holds;*
- *unanimity* (**UN**)
  if $\forall\,[(S,d);f,g] \in \Sigma_{\mathcal{F}}$, $f(S,d) = g(S,d)$ *implies* $M[(S,d);f,g] = f(S,d)$*;*
- *mediating* (**ME**)
  if $\forall\,[(S,d);f,g]$ *in* $\Sigma_{\mathcal{F}}$ *we have*
  $$M_1[(S,d);f,g] \in [\min\{f_1(S,d), g_1(S,d)\}, \max\{f_1(S,d), g_1(S,d)\}],$$
  $$M_2[(S,d);f,g] \in [\min\{f_2(S,d), g_2(S,d)\}, \max\{f_2(S,d), g_2(S,d)\}].$$
- *generalized midpoint domination* (**GMD**)
  if $\forall\,[(S,d);f,g]$ *in* $\Sigma_{\mathcal{F}}$ *we have* $M[(S,d);f,g] \geq \frac{1}{2}f(S,d) + \frac{1}{2}g(S,d)$*;*
- *step-by-step bargaining* (**STEP**)
  if $\forall\,[(S,d);f,g]$ *in* $\Sigma_{\mathcal{F}}$ *for which there exists a game* $(\tilde{S},d) \in \Sigma$ *satisfying* $\tilde{S} \subseteq S$, $f(\tilde{S},d) = f(S,d)$, $g(\tilde{S},d) = g(S,d)$, *and* $\text{cvh}\,(\{f(\tilde{S},d), g(\tilde{S},d)\}) \in PO(\tilde{S})$, *then we have* $M[(S, M[(\tilde{S},d);f,g]);f,g] = M[(S,d);f,g]$.

Pareto-optimality (**PO**), weak Pareto-optimality (**WPO**), individual rationality (**IR**), and continuity with respect to the disagreement point (d-**CONT**) are very well-known properties in bargaining theory. Impartiality (**IM**) says that the outcome of the mechanism should not depend on which agent supports which solution. At fist glance this axiom looks a little bit dubious. But we have to recall that in meta-bargaining theory an agent always supports the same bargaining solution. Which solution an agent supports does not depend on whether he is affected by a bargaining game as agent 1 or agent 2. In the same spirit impartiality assures that the outcome chosen by a mechanism does not depend on which agent supports which bargaining solution. Unanimity (**UN**) states that as soon as the agents have reached an agreement for the bargaining game, in particular if they want to apply the same bargaining solution the mechanism should respect this.

Mediating (**ME**) means that the final outcome lies "in between" the two allocations proposed by the solutions. **ME** implies **UN** and **ME** implies **IR** if f and g satisfy IR. Generalized midpoint domination (**GMD**) means that a minimal amount of cooperation should enable the agents to reach at least the average of their preferred outcomes. **GMD** implies **IR** if f and g satisfy IR. **GMD** implies **ME** if f and g satisfy PO. The property step-by-step bargaining (**STEP**) is very similar to the property step-by-step negotiations which has been introduced by Kalai

(1977) for bargaining games and also used by Ponsati and Watson (1994) in their characterization of the Nash solution, but it is less demanding. **STEP** restricts the set of admissible mechanisms to the ones which are invariant under a certain decomposition. Suppose that for a given meta-bargaining game $[(S,d);f,g]$ there exists the "very special smaller bargaining game" $(\tilde{S},d)$, where $\tilde{S}$ is a subset of S, the bargaining solution of each agent assigns to both bargaining games the same allocation, and the Pareto-boundary of the set $\tilde{S}$ contains the straight line between the two solution outcomes. For both agents both bargaining games look alike in the sense that giving them their will, that is, applying their respective preferred bargaining solution the outcome would be the same. In the situation in which the agents support two different solutions, **STEP** offers the agents the possibility either to solve the original problem at one shot or to solve first the "easy" problem $(\tilde{S},d)$ where the relevant part of the Pareto-boundary is a straight line and hence, utility is transferable at a constant rate and then to bargain about the allocations which dominate the mechanism outcome of the smaller game. Both ways of proceeding lead to the same final outcome.

Let us assume, a mechanism M is asked to satisfy **PO**, **ME**, and d-**CONT** (which are quite harmless and sensible conditions) and let us consider a meta-bargaining game $[(S,d);f,g]$ and the bargaining game $(\tilde{S},d)$ satisfying the conditions of **STEP**. M satisfying **PO** and **ME** implies that on the straight line $\mathrm{cvh}(\{f(\tilde{S},d),g(\tilde{S},d)\})$ $=\mathrm{cvh}(\{f(S,d),g(S,d)\})$ there exists an allocation $\bar{m}$ which is the mechanism outcome of the bargaining game $[(\tilde{S},d);f,g]$. On the other hand, we can consider the meta-bargaining games $[(S,f(S,d));f,g]$ where $M[(S,f(S,d));f,g]=f(S,d)$ and $[(S,g(S,d));f,g]$ where $M[(S,g(S,d));f,g]=g(S,d)$. M satisfying **PO**, **ME**, and d-**CONT** implies that on the straight line $\mathrm{cvh}(\{f(S,d),g(S,d)\})$ there exists an allocation $\bar{d}$ with the property that the mechanism assigns the same allocation to the meta-bargaining games $[(S,d);f,g]$ and $[(S,\bar{d});f,g]$. The property **STEP** is fulfilled if both allocations, $\bar{d}$ and $\bar{m}$, coincide.

Proposition 2

On the class $\Sigma_{\mathcal{F}}$, the mechanism U satisfies **IR**, **IM**, **STEP**, *and* **GMD**.

Proof:
Let $[(S,d);f,g]$ be a meta-bargaining game in $\Sigma_{\mathcal{F}}$. As a by product of the proof to Proposition 1 we obtain the result that U satisfies **IR** and **GMD**.

The calculation

$$\begin{aligned} U[(S,d);f,g] &= \lim_{t\to\infty} \frac{1}{2}\Big(f(S,d^{t-1}) + g(S,d^{t-1})\Big) \\ &= \lim_{t\to\infty} \frac{1}{2}\Big(g(S,d^{t-1}) + f(S,d^{t-1})\Big) \\ &= U[(S,d);g,f] \end{aligned}$$

shows that U satisfies **IM**.

Suppose that there exists a subset $\tilde{S} \subseteq S$ such that $(\tilde{S},d) \in \Sigma$, $f(\tilde{S},d) = f(S,d)$, $g(\tilde{S},d) = g(S,d)$, and $\operatorname{cvh}(\{f(\tilde{S},d), g(\tilde{S},d)\}) \in PO(\tilde{S})$ is satisfied. According to the definition of U it is clear that

$$U[(\tilde{S},d);f,g] = \frac{1}{2}f(\tilde{S},d) + \frac{1}{2}g(\tilde{S},d) = \frac{1}{2}f(S,d) + \frac{1}{2}g(S,d)$$

holds since f and g satisfy IR. In addition, we have

$$U[(S,d);f,g] = \lim_{t\to\infty} \frac{1}{2}\Big(f(S,d^{t-1}) + g(S,d^{t-1})\Big)$$

where d^2 equals $\frac{1}{2}f(S,d) + \frac{1}{2}g(S,d)$. Hence, the equation

$$U[(S,U[(\tilde{S},d);f,g]);f,g] = U[(S,d);f,g]$$

holds which means that U satisfies **STEP**. q.e.d.

Define $\Sigma^0 := \{(S,d) \in \Sigma \,|\, d \notin WPO(S)\}$ as the class of bargaining games in Σ where the disagreement point is not an element of the weak Pareto-boundary. To obtain a characterization result we have to restrict attention to bargaining solutions which satisfy PO, IR, and d-CONT on Σ and CUT and S-CONT on Σ^0, that is, we define

$$\mathcal{F}_1 \;:=\; \left\{ f \in \mathcal{F} \;\middle|\; \begin{array}{l} \text{f satisfies IR, PO, and } d\text{-CONT on } \Sigma, \\ \text{f satisfies CUT and } S\text{-CONT on } \Sigma^0 \end{array} \right\}.$$

$\Sigma_{\mathcal{F}_1}$ denotes the set of meta-bargaining games. With a slight abuse of notation, we denote the restriction of U on the set $\Sigma_{\mathcal{F}_1}$, $U|_{\Sigma_{\mathcal{F}_1}}$, by U.

Only considering bargaining solutions which satisfy PO and IR is quite sensible. To assure that bargaining solutions are well-defined for bargaining games (S, d) where $d \in \partial S$ we require continuity with respect to the disagreement point. d-CONT and S-CONT state that neither small changes in the feasible set nor small changes of the disagreement point have disastrous effects on the solution outcome. CUT states that a bargaining solution is sensitive to changes in the feasible set in the sense that if we cut away alternatives which an agent prefers to the solution outcome the bargaining solution favours the other agent. Among others, the Nash solution and the Kalai-Smorodinsky solution are elements of $\mathcal{F}_1$ (the Nash solution and the Kalai-Smorodinsky solution satisfy CUT, since CUT is implied by IIA as well as by PO and IMON together (cf. Thomson (1994), page 1258); for a proof that the Nash solution and the Kalai-Smorodinsky solution satisfy d-CONT on Σ and S-CONT on Σ^0 consult Jansen and Tijs (1983), Corollary 3.1, page 98, and Proposition 3.1, page 96).

Proposition 3

On the class $\Sigma_{\mathcal{F}_1}$, U is a mechanism which satisfies in addition to the properties **IR**, **IM**, **STEP**, *and* **GMD** *also the properties* **PO**, ***d*-CONT**, **UN**, *and* **ME**.

Proof:

Let $[(S, d); f, g]$ be an arbitrary meta-bargaining game in $\Sigma_{\mathcal{F}_1}$. Since we do not have imposed any further restrictions on the admissible bargaining sets, it is clear that every meta-bargaining game $[(S, d^t); f, g]$, $t \geq 1$, induced by the function U is an element of $\Sigma_{\mathcal{F}_1}$. Hence, U is a mechanism on $\Sigma_{\mathcal{F}_1}$.

We only have to show that U satsifies **PO**, ***d*-CONT**, **UN**, and **ME**. If $d \in WPO(S)$ holds it is easily seen that U satisfies all the properties. Therefore, we assume $d \notin WPO(S)$.

To see that U satisfies **PO** we calculate

$$\begin{aligned} U[(S, d); f, g] &= \lim_{t \to \infty} d^t \\ &= \lim_{t \to \infty} \frac{1}{2}\Big(f(S, d^{t-1}) + g(S, d^{t-1})\Big) \end{aligned}$$

$$= \frac{1}{2}\Big(f(S, \lim_{t\to\infty} d^{t-1}) + g(S, \lim_{t\to\infty} d^{t-1})\Big)$$
$$= \frac{1}{2}\Big(f(S, \lim_{t\to\infty} d^{t}) + g(S, \lim_{t\to\infty} d^{t})\Big).$$

Since f and g satisfy PO and IR this implies

$$\lim_{t\to\infty} d^t = f(S, \lim_{t\to\infty} d^t) = g(S, \lim_{t\to\infty} d^t) \in PO(S)$$

and furthermore $U[(S,d); f, g] \in PO(S)$.

To see that U satisfies **d-CONT** we consider a sequence of disagreement points $(d_n)_{n\in\mathbb{N}}$ such that $[(S,d_n); f, g]) \in \Sigma_{\mathcal{F}_1}$, for all $n \in \mathbb{N}$, and $\lim_{n\to\infty} d_n = d$ holds. Each meta-bargaining game $[(S,d_n); f, g]$, $n \in \mathbb{N}$, generates a non-decreasing sequence of disagreement points, $(d_n^t)_{t\geq 1}$. We write

$$d_n^* := \lim_{t\to\infty} d_n^t = U[(S, d_n); f, g].$$

In the same way the meta-bargaining game $[(S,d); f, g]$ generates a non-decreasing sequence of disagreement points $(d^t)_{t\geq 1}$ and we write

$$d^* := \lim_{t\to\infty} d^t = U[(S,d); f, g].$$

Since f and g satisfy d-CONT we can calculate

$$\begin{aligned}
d^2 &= \frac{1}{2}\Big(f(S,d^1) + g(S,d^1)\Big)\\
&\overset{Def.}{=} \frac{1}{2}\Big(f(S, \lim_{n\to\infty} d_n^1) + g(S, \lim_{n\to\infty} d_n^1)\Big)\\
&\overset{d-CONT}{=} \frac{1}{2}\Big(\lim_{n\to\infty} f(S, d_n^1) + \lim_{n\to\infty} g(S, d_n^1)\Big)\\
&= \lim_{n\to\infty} \frac{1}{2}\Big(f(S,d_n^1) + g(S,d_n^1)\Big)\\
&= \lim_{n\to\infty} d_n^2.
\end{aligned}$$

Starting with the sequence $(d_n^2)_{n\in\mathbb{N}}$ which converges to d^2 we can repeat the arguments and show that $(d_n^3)_{n\in\mathbb{N}}$ converges to d^3. Thus, we can conclude $\lim_{n\to\infty} d_n^t = d^t$, for all $t \geq 1$, and furthermore

$$d^* = U[(S,d); f, g] = \lim_{t\to\infty} d^t = \lim_{t\to\infty}(\lim_{n\to\infty} d_n^t).$$

In the following we have to show

$$d^* = U[(S,d); f, g] = \lim_{n\to\infty} U[(S,d_n); f, g] = \lim_{n\to\infty} d_n^* = \lim_{n\to\infty}(\lim_{t\to\infty} d_n^t).$$

Let $\varepsilon_1 > 0$ be an arbitrary positive number and let $a \in \mathcal{N}_{\varepsilon_1}(d^*)$, $a < d^*$, be such that

$$((a + \mathbb{R}^2_+) \cap S) \subseteq (\mathcal{N}_{\varepsilon_1}(d^*) \cap S)$$

holds. (Note, that for the existence of a we need the fact that d^* is a Pareto-optimal allocation. Furthermore, a needs not to be individually rational so that in the case $d_1 = d_1^*$ and $d_2 = d_2^*$, respectively, our construction goes through.)

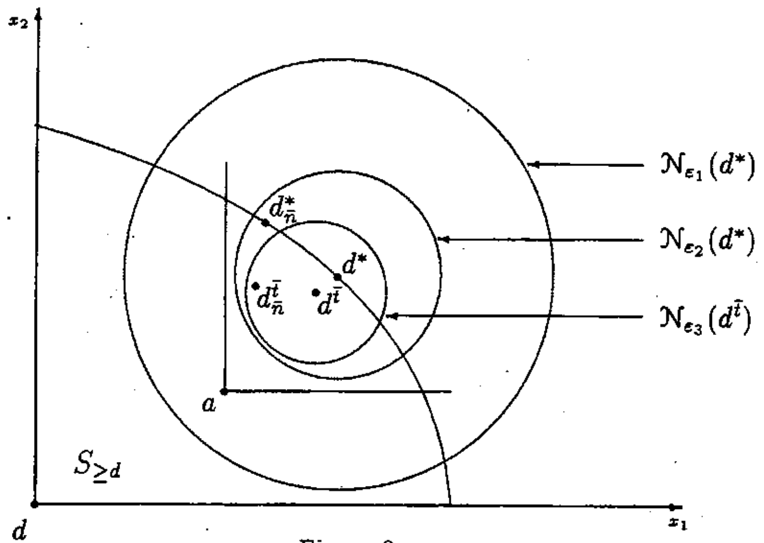

Figure 2

We choose $\varepsilon_1 \geq \varepsilon_2 > 0$ such that

$$\mathcal{N}_{\varepsilon_2}(d^*) \subseteq (a + \mathbb{R}^2_+)$$

is satisfied. For ε_2 there exists $\bar{t} \geq 1$ such that

$$d^t \in \mathcal{N}_{\varepsilon_2}(d^*), \ \forall t \geq \bar{t},$$

holds. We consider now $d^{\bar{t}} = \lim_{n\to\infty} d^{\bar{t}}_n$. For $\varepsilon_3 > 0$ which is so small that

$$\mathcal{N}_{\varepsilon_3}(d^{\bar{t}}) \subseteq \mathcal{N}_{\varepsilon_2}(d^*)$$

holds there exists $\bar{n}$ such that

$$d^{\bar{t}}_n \in \mathcal{N}_{\varepsilon_3}(d^{\bar{t}}), \ \forall n \geq \bar{n},$$

is satisfied. Based on the monotonicity of the sequences $(d^t_n)_{t\geq 1}$, $n \in \mathbb{N}$, we can conclude for all $n \geq \bar{n}$

$$d_n^* = \lim_{t\to\infty} d_n^t \geq d_n^{\bar{t}} \geq a.$$

Hence, for an arbitrary $\varepsilon_1 > 0$ we have found an $\bar{n}$ such that $d_n^* \in \mathcal{N}_{\varepsilon_1}(d^*)$ holds for all $n \geq \bar{n}$ which is equivalent to

$$d^* = U[(S,d);f,g] = \lim_{n\to\infty} d_n^* = \lim_{n\to\infty} U[(S,d_n);f,g].$$

Thus, U satisfies d-**CONT**.

To see that U also satisfies **ME** consider the following. Since $(d^t)_{t\geq 1}$ is a monotonic non-decreasing sequence and

$$d^2 = \frac{1}{2}\Big(f(S,d) + g(S,d)\Big)$$

holds it follows

$$U_i[(S,d);f,g] = \lim_{t\to\infty} d_i^t \geq d_i^2 \geq \min\{f_i(S,d), g_i(S,d)\}, \quad i = 1,2.$$

f, g, and U satisfying Pareto-optimality yields the desired result that U satisfies **ME**.

Since U satisfies **ME** it also satisfies **UN**. q.e.d.

Before we show that the averaging mechanism is the only mechanism on $\Sigma_{\mathcal{F}_1}$ satisfying **STEP** and **GMD** we prove the following lemma.

Lemma 1

For every meta-bargaining game $[(S,d);f,g]$ in $\Sigma_{\mathcal{F}_1}$ there exists a bargaining game $(\tilde{S},d)$ in Σ satisfying the requirements of the property **STEP**.

Proof:

Let $[(S,d);f,g]$ be an element of $\Sigma_{\mathcal{F}_1}$. We have to show that there exists a bargaining game $(\tilde{S},d) \in \Sigma$ such that $\tilde{S} \subseteq S$, $f(\tilde{S},d) = f(S,d)$, $g(\tilde{S},d) = g(S,d)$, and $\text{cvh}(\{f(\tilde{S},d), g(\tilde{S},d)\}) \in PO(\tilde{S})$ hold. By construction, $[(\tilde{S},d);f,g]$ is an element of $\Sigma_{\mathcal{F}_1}$.

If $f(S,d) = g(S,d)$ then $\tilde{S} = d\text{-ccvh}(\{f(S,d)\})$ fulfills the requirements of **STEP**. Let us now consider the case $f(S,d) \neq g(S,d)$. We begin with considering the bargaining games (S',d) and (S'',d) in Σ where the set

$$S' := S \backslash \{x \in S \mid \exists y \in \text{cvh}(\{f(S,d), g(S,d)\}):\ x > y\} \subseteq S$$

arises from S by cutting away all elements which "lie above the straight line connecting the two solution outcomes" and where

$$S'' := d\text{-ccvh}(\{f(S,d), g(S,d)\})$$

is just the d-comprehensive and convex hull of the two solution outcomes. $S'' \subseteq S$ holds because S is d-comprehensive. It is clear that $S' \in \Sigma$ and $S'' \in \Sigma$ holds. $f(S,d) \neq g(S,d)$ and f and g satisfying PO and IR assures $d \notin WPO(S')$ and $d \notin WPO(S'')$. This implies $S' \in \Sigma^0$ and $S'' \in \Sigma^0$. W. l. o. g. we assume $f_1(S,d) \leq g_1(S,d)$ as depicted in Figure 3.

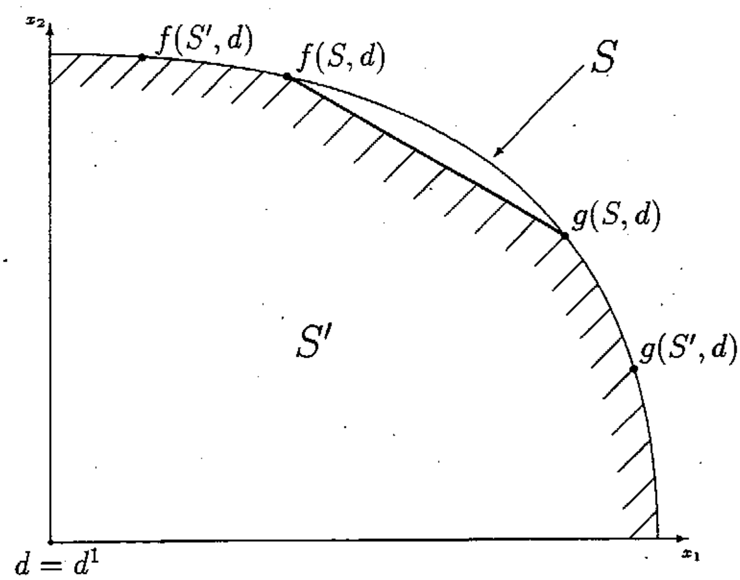

Figure 3

f and g satisfying CUT and PO implies

$$f_2(S',d) \geq f_2(S,d) \quad \text{and} \quad g_1(S,d) \leq g_1(S',d)$$

and

$$f_2(S,d) \geq f_2(S'',d) \quad \text{and} \quad g_1(S'',d) \leq g_1(S,d).$$

Let $\mathcal{S}$ be the set of all bargaining sets "between" S'' and S', that is, define

$$\mathcal{S} := \{R \subseteq \mathbb{R}^2 \,|\, (R,d) \in \Sigma^0,\ S'' \subseteq R \subseteq S'\}.$$

f and g satisfying CUT, S-CONT, and PO implies that there exists the bargaining set $\tilde{S} \in \mathcal{S}$ such that

$$g(\tilde{S},d) = g(S,d) \quad \text{and } f(\tilde{S},d) = f(S,d)$$

holds. To see this we apply the following procedure. We define two transformation functions

$$T^1 : [g_1(S,d), u_1(S,d)] \times \mathcal{S} \longrightarrow \mathcal{S},\ \ T^1(\mu_1, S) := \{x \in S \,|\, x_1 \leq \mu_1\}$$

and

$$T^2 : [f_2(S,d), u_2(S,d)] \times \mathcal{S} \longrightarrow \mathcal{S},\ \ T^2(\mu_2, S) := \{x \in S \,|\, x_2 \leq \mu_2\}.$$

For $\mu_2 = u_2(S,d)$ we have

$$T^2(u_2(S,d), S') = S'.$$

Because f and g satisfy CUT and PO decreasing μ_2 implies that $f_1(T^2(\mu_2, S'), d)$ and $g_1(T^2(\mu_2, S'), d)$ increase, that is, that $f(T^2(\mu_2, S'), d)$ and $g(T^2(\mu_2, S'), d)$ are moving to the right. Let $\bar{\mu}_2$ be such that

$$f(T^2(\bar{\mu}_2, S'), d) = f(S,d).$$

Consider now $S'^1 := T^2(\bar{\mu}_2, S')$. For $\mu_1 = u_1(S,d)$ we have

$$T^1(u_1(S,d), S'^1) = S'^1.$$

Since f and g satisfy CUT and PO decreasing μ_1 implies that $f_2(T^1(\mu_1, S'^1), d)$ and $g_2(T^1(\mu_1, S'^1), d)$ increase, that is, that $f(T^1(\mu_1, S'^1), d)$ and $g(T^1(\mu_1, S'^1), d)$ are moving to the left. Let $\bar{\mu}_1$ be such that

$$g(T^1(\bar{\mu}_1, S'^1), d) = g(S,d).$$

Consider now $S'^2 := T^1(\bar{\mu}_1, S'^1)$. For $\mu_2 = \bar{\mu}_2$ we have

$$T^2(\bar{\mu}_2, S'^2)) = S'^2.$$

By repeating the procedure we find the set $\tilde{S}$. q.e.d.

Proposition 4

The averaging mechanism U is the only mechanism on $\Sigma_{\mathcal{F}_1}$ satisfying **GMD** *and* **STEP**.

Proof:

We consider a meta-bargaining game $[(S,d);f,g]$ in $\Sigma_{\mathcal{F}_1}$. Let M be another mechanism satisfying **GMD** and **STEP**. If d is a weakly Pareto-optimal allocation, f and g satisfying PO and M satisfying **GMD** immediately implies $M[(S,d);f,g] = U[(S,d);f,g]$. Therefore, we assume in the following $d \notin WPO(S)$.

For the meta-bargaining game $[(S,d);f,g]$ the mechanism M induces a sequence of disagreement points $(d^t)_{t\in\mathbb{N}}$ in the following way. We start by defining $d = d^1$. In the lemma above we have seen that there exists a meta-bargaining game $[(\tilde{S},d);f,g] \in \Sigma_{\mathcal{F}_1}$ such that $\tilde{S} \subseteq S$, $f(\tilde{S},d) = f(S,d)$, $g(\tilde{S},d) = g(S,d)$, and $\mathrm{cvh}(\{f(\tilde{S},d), g(\tilde{S},d)\}) \in PO(\tilde{S})$ holds. Since M satisfies **STEP** we obtain

$$M[(S, M[(\tilde{S},d);f,g]);f,g] = M[(S,d);f,g].$$

We define $d^2 := M[(\tilde{S},d);f,g]);f,g]$. Since $\frac{1}{2}(f(\tilde{S},d)+g(\tilde{S},d))$ is a Pareto-optimal allocation and M satisfies **GMD** it follows

$$\begin{aligned} d^2 &= M[(\tilde{S},d);f,g]);f,g] \\ &\overset{\mathbf{GMD}}{=} \frac{1}{2}f(\tilde{S},d)+\frac{1}{2}g(\tilde{S},d) \\ &\overset{Def.}{=} \frac{1}{2}f(S,d)+\frac{1}{2}g(S,d) \\ &\overset{IR}{\geq} d. \end{aligned}$$

By considering $[(S,d^2);f,g]$ and repeating the arguments we obtain d^3 and successively the monotonic non-decreasing sequence $(d^t)_{t\in\mathbb{N}}$ satisfying

$$M[(S,d);f,g] = M[(S,d^t);f,g] \overset{\mathbf{GMD}}{\geq} \frac{1}{2}f(S,d^t)+\frac{1}{2}g(S,d^t) \overset{IR}{\geq} d^t,$$

for all $t \geq 1$. In addition, the limit $\lim_{t\to\infty} d^t$ exists because S is compact. Since $(M[(S,d^t);f,g])_{t\in\mathbb{N}}$ is a constant sequence we can conclude

$$M[(S,d);f,g] = \lim_{t\to\infty} M[(S,d^t);f,g] \geq \lim_{t\to\infty} d^t.$$

f and g satisfying PO, IR, and d-CONT implies

$$\begin{aligned}\lim_{t\to\infty} d^t &= \lim_{t\to\infty} \frac{1}{2}\Big(f(S,d^{t-1}) + g(S,d^{t-1})\Big) \\ &= \lim_{t\to\infty} \frac{1}{2}\Big(f(S,d^t) + g(S,d^t)\Big) \\ &= \frac{1}{2}\Big(f(S,\lim_{t\to\infty} d^t) + g(S,\lim_{t\to\infty} d^t)\Big)\end{aligned}$$

and hence

$$\lim_{t\to\infty} d^t = f(S,\lim_{t\to\infty} d^t) = g(S,\lim_{t\to\infty} d^t) \in PO(S).$$

Therefore,

$$M[(S,d);f,g] = \lim_{t\to\infty} d^t$$

and by the definition of the mechanism U

$$M[(S,d);f,g] = U[(S,d);f,g].$$

q.e.d.

Remark 2
The axioms are independent. The following mechanisms satisfy **GMD** but not **STEP**.

1. $A : \Sigma_{\mathcal{F}_1} \longrightarrow \mathbb{R}^2,\ [(S,d);f,g] \longmapsto \frac{1}{2}\Big(f(S,d) + g(S,d)\Big) \in S.$
2. $B_h : \Sigma_{\mathcal{F}_1} \longrightarrow \mathbb{R}^2,\ [(S,d);f,g] \longmapsto h\Big(S, \frac{1}{2}(f(S,d) + g(S,d))\Big) \in S,$
   where h is an arbitrary bargaining solution on Σ satisfying IR.
3. $C : \Sigma_{\mathcal{F}_1} \longrightarrow \mathbb{R}^2,\ [(S,d);f,g] \longmapsto d + t_{S,d,f,g} \cdot \Big(\frac{1}{2}(f(S,d) + g(S,d)) - d\Big) \in S,$
   where

   $$t_{S,d,f,g} = \max\left\{ t \in \mathbb{R}_+ \;\middle|\; d + t \cdot \left(\frac{1}{2}(f(S,d) + g(S,d)) - d\right) \in S \right\}.$$

   Note, that this function is a generalization of the bargaining solution G which has been introduced by Salonen (1985). If the framework is appropriate defined, $G(\,\cdot\,) = C[\,\cdot\,; f^{D,1*}, f^{D,2*}]$ holds.

Analogously, the following mechanisms satisfy **STEP** but not **GMD**.

1. $D : \Sigma_{\mathcal{F}_1} \longrightarrow \mathbb{R}^2,\ [(S,d); f, g] \longmapsto d \in S.$

2. $E_p : \Sigma_{\mathcal{F}_1} \longrightarrow \mathbb{R}^2,\ [(S,d); f, g] \longmapsto d + t_{S,d,p} \cdot p \in S,$
   where

   $$t_{S,d,p} = \max\{t \in \mathbb{R}_+ \,|\, d + t \cdot p \in S\}.$$

   and $p \in \{x \in \mathbb{R}^2_+ \,|\, x_1 + x_2 = 1\}$.
   $\{E_p \,|\, p \geq 0,\ p_1 + p_2 = 1\}$ is the family of "proportional mechanisms". It contains the adopted versions of Kalai's proportional bargaining solutions, in particular the egalitarian solution, f^E, and the dictatorial solutions, $f^{D,1}$ and $f^{D,2}$.

3. $F_k : \Sigma_{\mathcal{F}_1} \longrightarrow \mathbb{R}^2,\ [(S,d); f, g] \longmapsto \lim d^t \in S,$
   where

   $$d^1 = d$$

   and

   $$d^t = k \cdot f(S, d^{t-1}) + (1 - k) \cdot g(S, d^{t-1}), \quad \forall t \geq 2,$$

   and $k \in [0,1]$.
   $\{F_k \,|\, k \in [0,1]\}$ is the family of the asymmetric versions of the averaging mechanism U. ◀

In the following we want to enlarge the family of admissible bargaining solutions by admitting weakly Pareto-optimal bargaining solutions. To be able to obtain a characterization we have to require that those bargaining solutions satisfy IIA instead of the weaker property CUT. We define

$$\mathcal{F}_2 = \mathcal{F}_1 \bigcup \left\{ f \in \mathcal{F} \;\middle|\; \begin{array}{l} \text{f satisfies WPO, IR, and } d\text{-CONT on } \Sigma, \\ \text{f satisfies IIA and } S\text{-CONT on } \Sigma^0 \end{array} \right\}.$$

$\Sigma_{\mathcal{F}_2}$ denotes the set of meta-bargaining games. With a slight abuse of notation, we denote the restriction of U on the set $\Sigma_{\mathcal{F}_2}$, $U|_{\Sigma_{\mathcal{F}_2}}$, by U.

Consider the meta-bargaining games $[(S,0); f^E, f^{D,2}]$ and $[(S,0); f^N, f^{D,2}]$ in $\Sigma_{\mathcal{F}_2}$ where $S = 0\text{-ccvh}\,((1,1))$. It holds

$$f^E(S,0) = f^N(S,0) = (1,1) \qquad f^{D,2}(S,0) = (0,1)$$

and furthermore

$$U[(S,0); f^E, f^{D,2}] = \left(0, \frac{1}{2}\right) \quad \text{and} \quad U[(S,0); f^N, f^{D,2}] = (1,1).$$

Let M be a mechanism satisfying **GMD** and **STEP**. **GMD** yields

$$M[(S,0); f^E, f^{D,2}] \geq \left(0, \frac{1}{2}\right) \quad \text{and} \quad M[(S,0); f^N, f^{D,2}] \geq \left(0, \frac{1}{2}\right).$$

Since we cannot apply **STEP** for neither meta-bargaining game we cannot conclude that M and U coincide. Thus, for obtaining a characterization result, enlarging the family of bargaining solutions is at the cost of unanimity. That is, we only can characterize the lexicographic extension U^* of the averaging mechanism, U, defined by

$$U^* : \Sigma_{\mathcal{F}_2} \longrightarrow \mathbb{R}^2, \quad [(S,d); f,g] \longmapsto x \in PO(S_{\geq U[(S,d);f,g]}) \in S.$$

Proposition 5

On the class $\Sigma_{\mathcal{F}_2}$, the function U is a mechanism which satisfies **WPO**, **IR**, **IM**, **STEP**, *and* **GMD** *and the function U^* is a mechanism which satisfies* **PO**, **IR**, **IM**, **STEP**, *and* **GMD**.

Proof:

Let $[(S,d); f,g]$ be an arbitrary meta-bargaining game in $\Sigma_{\mathcal{F}_2}$. The mechanisms U and U^* induce the same monotonic non-decreasing sequence of disagreement points. Therefore, it is clear that every meta-bargaining game $[(S,d^t); f,g]$, $t \geq 1$, induced by the function U^* is an element of $\Sigma_{\mathcal{F}_2}$. To see that U^* is well-defined we first calculate

$$U[(S,d); f,g] = \lim_{t\to\infty} d^t = \frac{1}{2}\Big(f(S, \lim_{t\to\infty} d^t) + g(S, \lim_{t\to\infty} d^t)\Big).$$

Since f and g satisfy WPO and IR this implies

$$U[(S,d); f,g] = \lim_{t\to\infty} d^t = f(S, \lim_{t\to\infty} d^t) = g(S, \lim_{t\to\infty} d^t) \in WPO(S).$$

Consequently, $PO(S_{\geq U[(S,d);f,g]})$ has only a single element x which, by definition equals $U^*[(S,d);f,g]$.

As a byproduct we obtain the result that U satisfies **WPO** and U^* satisfies **PO**. In Proposition 2 we have already shown that U also satisfies **IR**, **IM**, **STEP**, and **GMD**. Immediately, we can conclude that also U^* satisfies these properties. q.e.d.

Before we show that the averaging mechanism is the only mechanism on $\Sigma_{\mathcal{F}_2}$ satisfying **PO**, **STEP**, and **GMD** we prove the following lemma.

Lemma 2

For every meta-bargaining game $[(S,d);f,g]$ in $\Sigma_{\mathcal{F}_2}$ for which either

$$f_1(S,d) < g_1(S,d) \quad \text{and} \quad f_2(S,d) > g_2(S,d)$$

or

$$f_1(S,d) > g_1(S,d) \quad \text{and} \quad f_2(S,d) < g_2(S,d)$$

or

$$f(S,d) = g(S,d)$$

holds there exists a bargaining game $(\tilde{S},d)$ satisfying the requirements of the property **STEP**.

Proof:

Consider two bargaining solutions f and g in $\mathcal{F}_2$. For a meta-bargaining game $[(S,d);f,g]$ in $\Sigma_{\mathcal{F}_2}$ we have already shown the existence of the set $\tilde{S}$ in case that both, f and g, satisfy PO on $\mathcal{F}_2$. It remains to show the existence of the set in case that either one of the bargaining solutions or both bargaining solutions do not satisfy PO on Σ. W.l.o.g. we assume that g satisfies WPO but not PO on $\Sigma_{\mathcal{F}_2}$. By definition, g also satisfies IIA.

If $f(S,d)$ and $g(S,d)$ coincide then the set $\tilde{S} = d\text{-ccvh}(\{f(S,d)\})$ satisfies the requirements of **STEP**.

Let us suppose in the following that f and g are such that $f_1(S,d) < g_1(S,d)$ and

$f_2(S,d) > g_2(S,d)$ holds. If also f satisfies WPO and IIA the set $\tilde{S}$ is given by

$$\tilde{S} := d\text{-ccvh}\left(\{f(S,d), g(S,d)\}\right).$$

Of course, $[(\tilde{S},d); f,g]$ is an element of $\Sigma_{\mathcal{F}_2}$. However, if f satisfies PO and CUT we find the set $\tilde{S} \in \Sigma$ with the help of the set $S' \in \Sigma$ defined by

$$S' := S \backslash \{x \in S \mid \exists y \in \text{cvh}\left(\{f(S,d), g(S,d)\}\right): \ x > y\} \subseteq S$$

(such that $g(S',d) = g(S,d)$ and $f_1(S',d) \leq f_1(S,d)$ holds) and the help of the transformation function $T_{[(S,d);f,g]}$ defined by

$$T_{[(S,d);f,g]} : [f_2(S,d), u_2(S,d)] \longrightarrow \{R \subseteq \mathbb{R}^2 \mid (R,d) \in \Sigma^0\},$$

$$T_{[(S,d);f,g]}(\mu) := \{x \in S' \mid x_2 \leq \mu\}.$$

Again, $[(\tilde{S},d); f,g]$ is an element of $\Sigma_{\mathcal{F}_2}$.

The case $f_1(S,d) > g_1(S,d)$ and $f_2(S,d) < g_2(S,d)$ is treated analogously. q.e.d.

Proposition 6
The mechanism U^* is the only mechanism on $\Sigma_{\mathcal{F}_2}$ satisfying **PO**, **GMD**, *and* **STEP**.

Proof:
We consider a meta-bargaining game $[(S,d); f,g]$ in $\Sigma_{\mathcal{F}_2}$. Let M be another mechanism satisfying **PO**, **GMD**, and **STEP**. For the meta-bargaining game $[(S,d); f,g]$ the mechanism M induces a (maybe finite) sequence of disagreement points $(d^t)_{t \in N \subseteq \mathbb{N}}$ in the following way. We start by defining $d = d^1$. If $f(S,d)$ and $g(S,d)$ are such that there does not exist a meta-bargaining game $[(\tilde{S},d); f,g] \in \Sigma_{\mathcal{F}_2}$ satisfying the requirements of **STEP**, then we have found the last member of the sequence. Otherwise, we can add one more in the following way. $f(S,d)$ and $g(S,d)$ are such that there exists a meta-bargaining game $[(\tilde{S},d); f,g] \in \Sigma_{\mathcal{F}_1}$ satisfying $\tilde{S} \subseteq S$, $f(\tilde{S},d) = f(S,d)$, $g(\tilde{S},d) = g(S,d)$, and $\text{cvh}\left(\{f(\tilde{S},d), g(\tilde{S},d)\}\right) \in PO(\tilde{S})$. Since M satisfies **STEP** we obtain

$$M[(S, M[(\tilde{S},d); f,g]); f,g] = M[(S,d); f,g].$$

We define $d^2 := M[(\tilde{S}, d); f, g]); f, g]$. Since $\frac{1}{2}(f(\tilde{S}, d) + g(\tilde{S}, d))$ is a Pareto-optimal allocation and M satisfies **GMD** it follows

$$\begin{aligned} d^2 &= M[(\tilde{S}, d); f, g]); f, g] \\ &\overset{\textbf{GMD}}{=} \frac{1}{2}f(\tilde{S}, d) + \frac{1}{2}g(\tilde{S}, d) \\ &\overset{Def.}{=} \frac{1}{2}f(S, d) + \frac{1}{2}g(S, d) \\ &\overset{IR}{\geq} d \end{aligned}$$

and

$$M[(S, d); f, g] \overset{\textbf{STEP}}{=} M[(S, d^2); f, g] \overset{\textbf{GMD}, IR}{\geq} d^2.$$

By considering $[(S, d^2); f, g]$ and repeating the arguments we either obtain d^3 or we have already found the last member of the sequence. By iterating this process, successively we obtain a monotonic non-decreasing sequence $(d^t)_{t \in N \subseteq \mathbb{N}}$ satisfying

$$M[(S, d); f, g] = M[(S, d^t); f, g] \geq d^t,$$

for all $t \in N$, $t \geq 1$.

If $N = \mathbb{N}$ then the limit $\lim_{t\to\infty} d^t$ exists. Since $M[(S, d^t); f, g]$ is a constant sequence we can conclude

$$M[(S, d); f, g] = \lim_{t\to\infty} M[(S, d^t); f, g] \geq \lim_{t\to\infty} d^t, \quad \forall t \in \mathbb{N}.$$

f and g satisfying WPO, IR and d-CONT implies

$$\begin{aligned} \lim_{t\to\infty} d^t &= \lim_{t\to\infty} \frac{1}{2}\Big(f(S, d^{t-1}) + g(S, d^{t-1})\Big) \\ &= \frac{1}{2}\Big(f(S, \lim_{t\to\infty} d^t) + g(S, \lim_{t\to\infty} d^t)\Big) \end{aligned}$$

and hence

$$\lim_{t\to\infty} d^t = f(S, \lim_{t\to\infty} d^t) = g(S, \lim_{t\to\infty} d^t) = U[(S, d); f, g] \in WPO(S).$$

Since M satisfies **PO** it follows

$$M[(S, d); f, g] = x \in PO(S_{\geq U[(S,d);f,g]})$$

and by the definition of the mechanism U^*

$$M[(S, d); f, g] = U^*[(S, d); f, g].$$

Finally, we have to consider the case that N is a finite subset of $\mathbb{N}$, that is $N = \{1, \ldots, \bar{t}\}$, $\bar{t} \in \mathbb{N}$. We know that for all meta-bargaining games $[(S, d^t); f, g]$, $t < \bar{t}$, there exists a set $\tilde{S}$ satisfying the requirements of **STEP**. For $[(S, d^{\bar{t}}); f, g]$ the set $\tilde{S}$ does not exist. According to our construction, $d^{\bar{t}} = M[(S, d^{\bar{t}-1}); f, g]$ satisfies

$$M[(S, d); f, g] = M[(S, d^{\bar{t}}); f, g].$$

In addition, we can conclude that

$$\text{cvh}\,(\{f(S, d^{\bar{t}}), g(S, d^{\bar{t}})\}) \in WPO(S) \backslash PO(S)$$

holds because otherwise the set $\tilde{S}$ would exist. M satisfying **GMD** yields

$$M[(S, d); f, g] = M[(S, d^{\bar{t}}); f, g] \geq \frac{1}{2}(f(S, d^{\bar{t}}) + g(S, d^{\bar{t}})) =: \bar{d} \in WPO(S).$$

M satisfying **PO** yields

$$M[(S, d); f, g] = x \in PO(S_{\geq \bar{d}})$$

and thus, by the definition of u and U^*

$$M[(S, d); f, g] = U^*[(S, d); f, g].$$

q.e.d.

Remark 3

The axioms are independent. The mechanism U satisfies **STEP** and **GMD** but not **PO**. The mechanism B_h[1] where h is an arbitrary bargaining solution on Σ satisfying IR and PO, and the mechanism C^*, the lexicographic extension of the mechanism C, satisfy **PO** and **GMD** but not **STEP**. The mechanisms E_p^* and F_k^*, the lexicographic extensions of the mechanisms E_p (where $p \in \mathbb{R}_+^2$, $p_1 + p_2 = 1$) and F_k (where $k \in [0, 1]$), respectively, satisfy **PO** and **STEP** but not **GMD**. ◀

Remark 4

Note, that by adopting the properties we have used to characterize the averaging mechanism to the context of bargaining theory we obtain a characterization of the Raiffa solution. So far only the continuous Raiffa solution has been characterized (cf. Bronisz and Kruś (1986), Livne (1989), and Peters and van Damme (1991)). ◀

[1] For the definitions consult Remark 2.

Let us consider bargaining games in Σ and two particular bargaining solutions in $\mathcal{F}_2$, namely $f^{D,1}$ and $f^{D,2}$. By definition, the Raiffa solution[2] is the function induced by the mechanism U if both agents support their respective dictatorial solutions, that is,

$$f^R(\cdot) = U[\,\cdot\,; f^{D,1}, f^{D,2}]: \Sigma \longrightarrow \mathbb{R}^2, \quad f^R(S,d) = U[(S,d); f^{D,1}, f^{D,2}].$$

In the proof to Proposition 5 we have shown that the equality

$$U[(S,d); f, g] = \lim_{t\to\infty} d^t = f(S, \lim_{t\to\infty} d^t) = g(S, \lim_{t\to\infty} d^t) \in WPO(S)$$

holds for all meta-bargaining games $[(S,d); f, g]$ in $\Sigma_{\mathcal{F}_2}$. In particular, we have

$$U[(S,d); f^{D,1}, f^{D,2}] = \lim_{t\to\infty} d^t = f^{D,1}(S, \lim_{t\to\infty} d^t) = f^{D,2}(S, \lim_{t\to\infty} d^t)$$

for every bargaining game (S,d) in Σ. However, the solution outcomes of $f^{D,1}$ and $f^{D,2}$ only coincide if the disagreement point is a Pareto-optimal allocation. Thus, $U[(S,d); f^{D,1}, f^{D,2}]$ is a Pareto-optimal allocation which means that the equality

$$U[(S,d); f^{D,1}, f^{D,2}] = U^*[(S,d); f^{D,1}, f^{D,2}]$$

holds for every bargaining game (S,d) in Σ.

In the sequel we consider bargaining solutions in $\mathcal{F}_2$ which also satisfy the property d-MON, that is, we consider bargaining solutions in

$$\mathcal{F}_3 := \{f \in \mathcal{F}_2 \,|\, f \text{ satisfies } d\text{-MON}\}.$$

d-MON suggests that a solution behaves according to the intuitive idea that an increase of agent i's "fallback position", d_i, induces an increase of his final outcome. Thomson (1987, pp. 53 – 54) states that on Σ^0 the Nash solution, the Kalai-Smorodinsky solution, and the egalitarian solution satisfy d-MON. It is obvious that on Σ^0 the dictatorial solutions satisfy d-MON, too. In addition, Thomson (1987, page 55) states that on the class Σ^0 every bargaining solution f that satisfies WPO, CONT, and d-MON also satisfies the properties ST-d-MON and NTP. It is not difficult to see that the bargaining solutions in $\mathcal{F}_3$ satisfy d-MON, ST-d-MON and NTP not only on Σ^0 but also on Σ since the bargaining solutions in $\Sigma_{\mathcal{F}_3}$ satisfy WPO, IR, d-CONT, and d-MON not only on Σ^0 but also on Σ.

[2]For the rest of this section we refer to the discrete Raiffa solution by the term Raiffa solution.

To analyze the mechanism in a non-cooperative set-up we define three sequences of disagreement points. To do so, let (S,d) be a bargaining game in Σ and let f and g be two bargaining in $\mathcal{F}_3$.

- $(d^t)_{t\in\mathbb{N}}$ defined by $d^1 = d$ and $d^t = \frac{1}{2}(f(S,d^{t-1}) + g(S,d^{t-1}))$, $t \geq 2$, as the sequence of disagreement points induced by the mechanism U if $[(S,d);f,g]$ is the meta-bargaining game we start with,
- $(\tilde{d}^t)_{t\in\mathbb{N}}$ defined by $\tilde{d}^1 = d$ and $\tilde{d}^t = \frac{1}{2}(f^{D,1}(S,\tilde{d}^{t-1}) + g(S,\tilde{d}^{t-1}))$, $t \geq 2$, as the sequence of disagreement points induced by the mechanism U if $[(S,d);f^{D,1},g]$ is the meta-bargaining game we start with, and
- $(\bar{d}^t)_{t\in\mathbb{N}}$ defined by $\bar{d}^1 = d$ and $\bar{d}^t = \frac{1}{2}(f^{D,1}(S,d^{t-1}) + g(S,d^{t-1}))$, $t \geq 2$.

Comparing d^t with $\tilde{d}^t$ gives us some information about which effects agent 1's deviation from the bargaining solution f to the his respective dictatorial solution has. Comparing d^t and $\tilde{d}^t$ with $\bar{d}^t$ gives us some information about how the total effect of the deviation can be split into the effect of deviation at stage t and deviation from the very beginning.

Like van Damme (1986) we define for every bargaining game $(S,d) \in \Sigma$ a non-cooperative game

$$\Gamma_U(S,d) := \Big(\mathcal{F},\mathcal{F}; U_1[(S,d),\cdot,\cdot], U_2[(S,d),\cdot,\cdot]\Big)$$

in which agents can choose bargaining solutions as strategies and the payoffs are determined by the mechanism U.

Proposition 7
For every game $(S,d) \in \Sigma$ each agent has a unique dominant strategy, namely his dictatorial solution, that is, for every game $(S,d) \in \Sigma$, only the pair $(f^{D,1}, f^{D,2})$ is a Nash equilibrium in $\Gamma_U(S,d)$.

Proof:
Consider the meta-bargaining games $U[(S,d);f,g]$ and $U[(S,d);f^{D,1},g]$ in $\Sigma_{\mathcal{F}_3}$. We will make use of the sequences $(d^t)_{t\in\mathbb{N}}$, $(\tilde{d}^t)_{t\in\mathbb{N}}$, and $(\bar{d}^t)_{t\in\mathbb{N}}$ we have defined above. It holds for every $f \in \mathcal{F}_3$

$$f_1(S,d') \leq f_1^{D,1}(S,d') \quad \text{and} \quad f_2(S,d') \geq f_2^{D,1}(S,d'), \quad \forall d' \in S,\ d' \geq d.$$

It follows immediately

$$d_1^2 = \frac{1}{2}(f_1(S,d^1) + g_1(S,d^1)) \le \bar{d}_1^2 = \frac{1}{2}(f_1^{D,1}(S,d^1) + g_1(S,d^1)) = \tilde{d}_1^2$$

and

$$d_2^2 = \frac{1}{2}(f_2(S,d^1) + g_2(S,d^1)) \ge \bar{d}_2^2 = \frac{1}{2}(f_2^{D,1}(S,d^1) + g_2(S,d^1)) = \tilde{d}_2^2$$

That is, either d^2 equals $\tilde{d}^2$ which means that we can proceed in the same way or d^2 and $\tilde{d}^2$ are such that we can either apply the properties d-MON and ST-d-MON, respectively, or the property NTP. Let us assume $d^2 \neq \tilde{d}^2$. Then we have

$$\begin{aligned} d_1^3 &= \frac{1}{2}(f_1(S,d^2) + g_1(S,d^2)) \\ &\le \bar{d}_1^3 = \frac{1}{2}(f_1^{D,1}(S,d^2) + g_1(S,d^2)) \\ &\le \tilde{d}_1^3 = \frac{1}{2}(f_1^{D,1}(S,\tilde{d}^2) + g_1(S,\tilde{d}^2)) \end{aligned}$$

and

$$\begin{aligned} d_2^3 &= \frac{1}{2}(f_2(S,d^2) + g_2(S,d^2)) \\ &\ge \bar{d}_2^3 = \frac{1}{2}(f_2^{D,1}(S,d^2) + g_2(S,d^2)) \\ &\ge \tilde{d}_2^3 = \frac{1}{2}(f_2^{D,1}(S,\tilde{d}^2) + g_2(S,\tilde{d}^2)). \end{aligned}$$

By repeating the arguments we can show

$$d_1^t \le \tilde{d}_1^t, \quad \forall t \ge 1, \quad \text{and} \quad d_2^t \ge \tilde{d}_2^t, \quad \forall t \ge 1,$$

which is equivalent to

$$U_1[(S,d);f,g] = \lim_{t\to\infty} d_1^t \le U_1[(S,d);f^{D,1},g] = \lim_{t\to\infty} \tilde{d}_1^t$$

and

$$U_2[(S,d);f,g] = \lim_{t\to\infty} d_2^t \ge U_2[(S,d);f^{D,1},g] = \lim_{t\to\infty} \tilde{d}_2^t.$$

Hence, it is a dominant strategy for agent 1 to play $f^{D,1}$. Analogously, it is a dominant strategy for agent 2 to play $f^{D,2}$. In addition, since the bargaining solutions $f^{D,i*}$, $i \in \{1,2\}$, are not elements of the set $\mathcal{F}_3$ (they do not satisfy S-CONT) it is obvious that no agent has another dominant strategy and no pair of strategies other than $(f^{D,1}, f^{D,2})$ can form an equilibrium in $\Gamma_U(S,d)$. q.e.d.

Remark 5
As we have seen, the function which assigns to every bargaining game $(S,d) \in \Sigma$ the unique equilibrium outcome $U[(S,d); f^{D,1}, f^{D,2}]$ of the corresponding non-cooperative game is exactly the Raiffa solution. Therefore, the Raiffa solution appears in a different light in the sense that each solution outcome can be very well interpreted as the unique equilibrium outcome of a non-cooperative game in which agents choose their unique dominant strategy. But this means that we have provided a non-cooperative foundation of the Raiffa solution. Moreover, by interpreting a mechanism as an arbitration rule and showing that the Raiffa solution is induced by the averaging mechanism we recover Raiffa's original interpretation of the solution. ◄

Remark 6
It seems to us that with a certain effort we can also obtain an implementation result. However, clarifying this question is beyond the scope of this work since this would mean that we have to present completely the current controversial discussions in the literature on implementation theory. ◄

3 Concluding Remarks

In this paper we have introduced a mechanism whose underlying rationale is quite close to what we can observe people are doing in order to find a compromise between their differing preferred outcomes. Often agents toss a coin and leave the decision which outcome will be realized to chance. Our mechanism suggests that the agents accept the expected value of this experiment as a new disagreement point and go on bargaining in the same way until they reach a Pareto-optimal outcome. By varying the class of admissible bargaining solutions we have obtained a characterization of the averaging mechanism and the lexicographic extension of the averaging mechanism. Obtaining a characterization of the averaging mechanism for a larger class of bargaining solutions will be a nice result of future research.

By considering the special case in which agents choose their respective dictatorial solutions we obtain the Raiffa solution. We have seen that in every bargaining situation it is the unique dominant strategy for each agent to choose his respective dictatorial solution. Hence, for every bargaining game each agent choosing his respective dictatorial solution constitutes the unique Nash equilibrium in the associated non-cooperative game. The function assigning to every bargaining game the unique equilibrium outcome of the corresponding non-cooperative game is exactly the Raiffa solution. Hence, in the line of the "Nash program" and "non-cooperative foundations" we have obtained a non-cooperative foundation of the Raiffa solution. Also, it seems to us that with a certain effort we can also obtain an implementation result. For future research it might be an interesting task to clarify the relations between meta-bargaining theory, the theory of non-cooperative foundations, and implementation theory.

The observation that the Raiffa solution arises from fixing a certain pair of bargaining solutions immediately leads to the question if in such a manner bargaining theory can be embedded into meta-bargaining theory or, to put the question the other way round, if it is possible to interpret meta-bargaining theory as a generalization of bargaining theory. It will be a nice result of future research if the embedding can be performed in such a way that we obtain analogous results as above.

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