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**Pigouvian Taxes May Fail
Even in a Perfect World**

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Abstract

The paper demonstrates that Pigouvian taxes on externalities usually fail to decentralize Pareto optimal allocations even under perfect competition, perfect information, and uniqueness of Walrasian equilibrium prices if there are different firms with linear technologies.

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1 Introduction

Pigouvian taxes are known to be powerful tools in order to internalize real (or technical) externalities. If all markets are competitive, a Pareto optimum can be achieved by charging a tax on externalities which is equal to marginal damage of the externality. In a general equilibrium framework, the marginal damage can be expressed as a sum of consumers' marginal rates of substitution plus the sum of producers' marginal rates of transformation between the externality and a certain private consumption commodity. This is all well understood.

However, there may be a problem with Pigouvian taxes even under perfect competition and perfect information. If the competitive equilibrium is not unique, the economy might end up in the wrong equilibrium even under a "correct" tax. This problem is widely been neglected in the literature on environmental economics (see e.g. BAUMOL and OATES and many others not cited here). LAFFONT [1989], however, more carefully writes in his chapter on externalities, (pages 21/22):

"However, there is a difficulty with this policy prescription [Pigouvian taxation (remark by the author of this article)]. For taxes given by t^ . . . , other equilibria that are not Pareto optimal may exist. Decentralization of the Pareto optimum requires the announcement of the correct prices as well."*

Whereas the first two statements are undoubtedly correct, the last statement suggests that announcing the correct prices is also *sufficient* to achieve a Pareto efficient allocation. The aim of this note is to show that even announcing the correct prices is *not* sufficient in order to decentralize a Pareto optimal allocation by means of Pigouvian taxes. I will present a simple economy which has several (a continuum of) Walrasian equilibria even though the Walrasian prices and the optimal Pigouvian tax are unique. Yet, the equilibrium allocation of production among the firms is *not* unique. Among all these equilibria, however, only one equilibrium is efficient.

2 The Model

Consider a simple economy with one consumer and two firms $i=1,2$. There are two consumption commodities X and Y and one pollutant. Commodity Y also serves as the only input for the firms to produce commodity X . The firms have constant return to scale technologies. Firm i needs c_i units of commodity Y to produce one unit of commodity X . Moreover, one unit of output generates d_i units of pollution. Denote firm i 's output by x_i , total output by $x = x_1 + x_2$, and total pollution by $E = d_1x_1 + d_2x_2$. Without loss of generality we assume $c_1 < c_2$, and $d_1 > d_2$. So firm 1 has the lower private cost but at the same time is the worse polluter.

The consumer draws utility from the two commodities and suffers from the pollution such that her utility is given by $U(x, y, E)$. She also owns an initial endowment w of commodity Y such that $y = w - c_1x_1 - c_2x_2$.¹ The utility function U is assumed to be strictly quasi-concave, moreover, $U_X > 0$, $U_Y > 0$, and $U_E < 0$.

3 The Social Optimum

If the externality is not internalized, perfect competition would lead to $p = c_1$. Only firm 1 produces, firm 2 is out of business.² In contrast, the *socially optimal* allocation is the unique solution of

$$\max_{x, y, x_1, x_2, E} U(x, y, E) \quad (3.1)$$

s.t. $x = x_1 + x_2$, $y = w - c_1x_1 - c_2x_2$, $E = d_1x_1 + d_2x_2$, and $x_i \geq 0$ for $i = 1, 2$. Or by substituting these equations into (3.1):

$$\max_{x_1, x_2} U(x_1 + x_2, w - c_1x_1 - c_2x_2, d_1x_1 + d_2x_2) . \quad (3.2)$$

Denote by

$$P(x, y, E) := \frac{U_X(x, y, E)}{U_Y(x, y, E)}$$

the consumer's marginal willingness to pay for commodity X in terms of commodity Y (the marginal rate of substitution between X and Y), and by

$$MD(x, y, E) := -\frac{U_E(x, y, E)}{U_Y(x, y, E)}$$

the marginal disutility of pollution in terms of commodity Y .

Suppose first that the pollution very harmless, i.e., U_E and U_{EE} are small or even zero. Then clearly the higher cost firm 2 should not produce and we have $x_2 = 0$. The (unique) solution x_1^* to (3.2) is then given by:

$$P(x_1^*, w - c_1x_1^*, d_1x_1^*) = c_1 + d_1MD(x_1^*, w - c_1x_1^*, d_1x_1^*) . \quad (3.3)$$

If in contrast the damage from pollution is sufficiently severe, then, since $d_2 < d_1$, only the cleaner firm 2 should produce, leading to $x_1 = 0$. The (unique) solution x_2^* to (3.2) is then given by:

$$P(x_2^*, w - c_2x_2^*, d_2x_2^*) = c_2 + d_2MD(x_2^*, w - c_2x_2^*, d_2x_2^*) . \quad (3.4)$$

¹ w is sufficiently large to always guarantee interior solutions.

²Of course, perfect competition is not reasonable with only one low cost firm, and one could argue that Bertrand competition leads to $p = c_2 - \varepsilon$. However, this is not the point here. To restore perfect competition we could assume $n_i > 1$ firms of each type. For the sake of notational convenience, however, we assume one firm of each type only. For an analysis of a Bertrand model see REQUATE [1993].

Let us now consider the interesting case where it is optimal to employ *both* firms. The social optimum is then given by the 5 equations

$$\begin{aligned} P(x, y, E) &= c_i + d_i MD(x, y, E), & i = 1, 2, \\ x &= x_1 + x_2, \\ y &= w - c_1 x_1 - c_2 x_2, \\ E &= d_1 x_1 + d_2 x_2. \end{aligned}$$

Since $c_1 < c_2$ and $d_1 > d_2$, and since U is strictly quasi-concave, the system is non-singular and there is a *unique* solution, in particular for x_1 and x_2 .

4 Optimal Pigouvian Tax

Let us now consider a private ownership economy, where the consumer holds all shares of the firms. The price of commodity Y is normalized to 1 and p denotes the price of commodity X . Moreover, a well informed regulator is entitled to charge a Pigouvian tax τ on emissions. The tax revenues are redistributed lump sum to the consumer who maximizes

$$\max_{x, y} U(x, y, E) \quad \text{s.t. } px + y = w + \tau(d_1 x_1 + d_2 x_2).$$

Under the tax firm i 's marginal cost amounts to $c_i + d_i \tau$. If, in a social optimum, only one type of firm is supposed to be active, everything is fine and the optimal tax is given by $\tau^* = MD(x_i^*, w - c_i x_i^*, d_i x_i^*)$. Hence assume that it is socially optimal to employ both firms. Then a necessary condition for the optimal tax is that the marginal costs of the two firms be equal — that is, $c_1 + \tau^* d_1 = c_2 + \tau^* d_2$, which is the case for

$$\tau^* = \tau^{ec} := \frac{c_2 - c_1}{d_1 - d_2}.$$

We call this the equal cost tax. Perfect competition requires

$$p = c_1 + \tau^{ec} d_1 = c_2 + \tau^{ec} d_2 = (d_1 c_2 - d_2 c_1) / (d_1 - d_2) =: \tilde{p}. \quad (4.5)$$

Utility maximization leads to

$$P(x, y, E) = \tilde{p}.$$

By Walras' law, market clearing on both markets — that is, $x = x_1 + x_2$ and $y = w - c_1 x_1 - c_2 x_2$ — implies that the consumer's budget constraint is satisfied.

But now the **Walrasian equilibrium with a tax on emissions** is given by a single equation:

$$P(x_1 + x_2, w - c_1 x_1 - c_2 x_2, d_1 x_1 + d_2 x_2) = \tilde{p}, \quad (4.6)$$

which yields several solutions in x_1 and x_2 . Thus, despite a unique equilibrium price, the *allocation* of production among the firms is *not* unique. Rather, there is a continuum of Walrasian equilibrium allocations (x_1, x_2) among the firms which satisfy (4.6). For example one solution is given by $x_2 = 0$ and $x_1 = \bar{x}$, where \bar{x} solves $P(x, w - c_1x, d_1x) = \tilde{p}$. Another solution is given by $x_1 = 0$ and $x_2 = \underline{x}$, where \underline{x} solves $P(x, w - c_2x, d_2x) = \tilde{p}$. For each aggregate output $x \in [\underline{x}, \bar{x}]$ we can find (x_1, x_2) which satisfy (4.6) and $x_1 + x_2 = x$. Different aggregate allocations (x_1, x_2) , however, lead to different pollution levels $E \in [d_2\underline{x}, d_1\bar{x}]$! On the other hand, the social optimum has a *unique* solution if the consumer's utility function is strictly quasi-concave. Thus at most one allocation among the continuum of Walrasian equilibrium allocations under tax $\tau = \tau^{ec}$ is efficient.³ The non-uniqueness of equilibrium is *not* due to different *equilibrium prices*. It is rather due to non-uniqueness of *equilibrium allocations*!

The crucial reason for the result is that the supply correspondences of the firms are not single valued if the price for the output commodity equals marginal cost. The firms can supply any amount between zero and infinity. In contrast to increasing marginal costs, equation (4.5) does not determine the firms' output uniquely. (Typically, constant marginal costs is a standard assumption made in almost every IO-paper.)

So, decentralized decision making under perfect competition in general does *not* lead to first best allocations even if a well informed regulator charges the correct tax, and a benevolent Walrasian auctioneer announces the correct prices!

5 Emission Permits

The problem of non-uniqueness of equilibrium does not arise if instead of charging taxes, pollution permits were issued. Clearly, the optimal number of permits to be issued, say L , has to be equal to optimal total pollution — that is, $L = E^*$. One can easily show that for all $L \in [d_2\underline{x}, d_1\bar{x}]$ the market price for permits must be equal to $(c_2 - c_1)/(d_1 - d_2) = \tau^{ec}$.⁴ Also in this case, the supply of the firms on the output market, and thus the *demand* for permits on the permits market are not single valued. However, since *supply* for permits is single valued there must be a *unique* equilibrium on the permits market leading to a *unique* Walrasian allocation.

³One can indeed show that there exists one efficient Walrasian equilibrium.

⁴If the market price for permits were lower than $(c_2 - c_1)/(d_1 - d_2)$, only firm 1 would demand permits. But then, due to perfect competition, the market price for commodity X would be lower than $\tilde{p} = (d_1c_2 - d_2c_1)/(d_1 - d_2)$. This would lead to demand for the commodity higher than \bar{x} , and thus would create demand for permits higher than $d_1\bar{x}$. But then the permits market would not be in equilibrium since $L \leq d_1\bar{x}$. By a similar argument one shows that the market price for permits cannot exceed $(c_2 - c_1)/(d_1 - d_2)$.

6 Conclusions

We have demonstrated that optimal Pigouvian taxation may fail to decentralize Pareto efficient allocations even if equilibrium prices are unique. Despite a unique equilibrium price, the equilibrium allocation among the firms may fail to be unique. Different equilibrium allocations, in turn, may lead to different levels of externalities among which only a particular level is socially optimal. The problem arises when firms have linear technologies and their supply correspondences are not single valued. The problem does not arise if the firms' technologies are *strictly* convex and the number of firms is exogenous. But the problem again arises if the number of firms is endogenous even if the firms' supply correspondences are single valued (see REQUATE [1995]⁵).

Our example shows that decentralization of Pareto efficient allocations in the presence of externalities by means of Pigouvian taxes may cause serious problems even under seemingly perfect conditions such as perfect competition, perfect information, and uniqueness of Walrasian equilibrium prices!

References

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⁵REQUATE [1995] considers a model with two types of firms which have convex technologies up to fixed costs. By the fixed costs the number of firms can be endogenized by zero profits. Although the socially optimal number of firms of each type is unique, the *equilibrium* number of firms is not uniquely determined under a Pigouvian tax.