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The Strength of Reciprocity in a Reciprocity Game

by

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Abstract

In a reciprocity game the dependence of reciprocal behavior on the improvement of one subject and the deterioration of another subject in case of cooperation have been studied. In this 2-person reciprocity game player 2 decides between two alternatives with payoffs to player 1 and player 2. Before the decision of player 2 player 1 could give a side-payment to player 2 to make player 2 select the pareto-optimal alternative favored by player 1, but not by player 2. Player 2 receives this side-payment independently of the alternative he selects afterwards. In an experiment this game was played with different parameters in order to obtain the strength of reciprocity depending on these parameters. The best predictor of the payment of player 1 is the criterion based on modeling the numerical perception according to the theory of prominence and an equal division of the perceived improvement of player 1 and the perceived deterioration of player 2 in case of cooperation.

Introduction

Reciprocal behavior has been observed in many experimental situations, for example in the gift exchange experiment (J. Berg, J. Dickhaut, K.A. McCabe, 1995) or similar games (W. Güth, P. Ockenfels, M. Wendel, 1994; F. Bolle 1995; M. Dufwenberg, U. Gneezy, 1996; E. Jacobsen, A. Sadrieh, 1996), comparable games in the labor market context (E. Fehr, G. Kirchsteiger, A. Riedl, 1993) or games in extensive form (K.A. McCabe, S.J. Rasenti and V.L. Smith, 1996). In these experiments most of the deviations from rational behavior are attributed to reciprocity.

Reciprocity is characterized (K.A. McCabe, S.J. Rasenti and V.L. Smith, 1996) as "a specialized mental algorithm (L. Cosmides 1985; L. Cosmides, J. Tooby, 1992) in which long term self interest is best served by promoting an image both to others and yourself that cheating on cooperative social exchange (either explicit or implicit) is punished (negative reciprocity), and initiation of cooperative social exchange is rewarded (positive reciprocity)." This behavior is also observed in single games. The number of players that initiate cooperation increases if it is possible to punish and their own cost of punishment is not too high.

In this context one question is how do subjects interpret an action to be the initiation of cooperation. In the gift exchange game subjects had to make the right transfer (all they had) to initiate cooperation (E. Jacobsen, A. Sadrieh, 1996), lower transfers were less sufficient to initiate cooperation. More specifically, a situation is considered in which cooperation causes improvement of one player (who initiates), a deterioration of the other player (who is expected to react reciprocally) and an overall improvement. Which amount of money has to be transfered from the improving player to the player who deteriorates so that this amount is interpreted as an initiation of cooperation and reciprocal behavior can be expected. What is the fairness criterion that determines the overall division of the gains in case of cooperation? Are preferences about equality in payoffs important or do persons care for the payoff of another person?

Another point of discussion is which possibilities increase the number of persons initiating cooperation and reacting cooperatively. One point is a possible punishment after the non reciprocal behavior of the other person. More general even the possibility that being non cooperative might not pay always could increase the number of persons showing reciprocal behavior and increase the number of persons initiating cooperation although punishment is not possible. To answer these questions the games SR and DSR described in the next part have been examined experimentally.

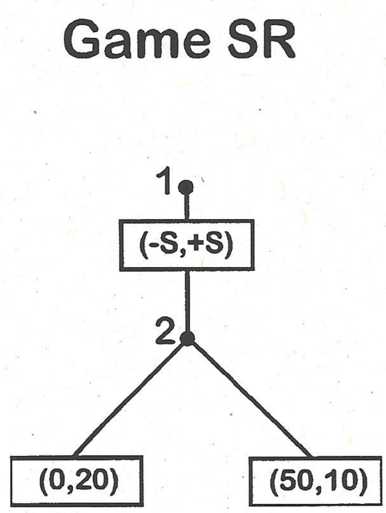
The paper is organized as follows. First the games are described. Then the models for the behavior in the reciprocity games are given. In the next part the experiment is described and the predictions of the models for the experiment are shown. Finally the results are presented and discussed.

Games

In figure 1 a reciprocity game is shown. This game is called an SR (side-payment - reciprocity) game. In this game player 1 can give a side-payment S ($0 \leq S$) to player 2. Player 2 decides afterwards between two alternatives with payoffs to player 1 and player 2[1]. In this game the total payoff pairs are (-S+0,+S+20) or (-S+50,+S+10) depending on the selection of player 2.

[1] In all games the payoff of the players is the sum of all payoffs given by the vectors in the rectangles along the path determined by their decisions different to the extensive form of a game.

Figure 1: The game SR (side-payment - reciprocity).



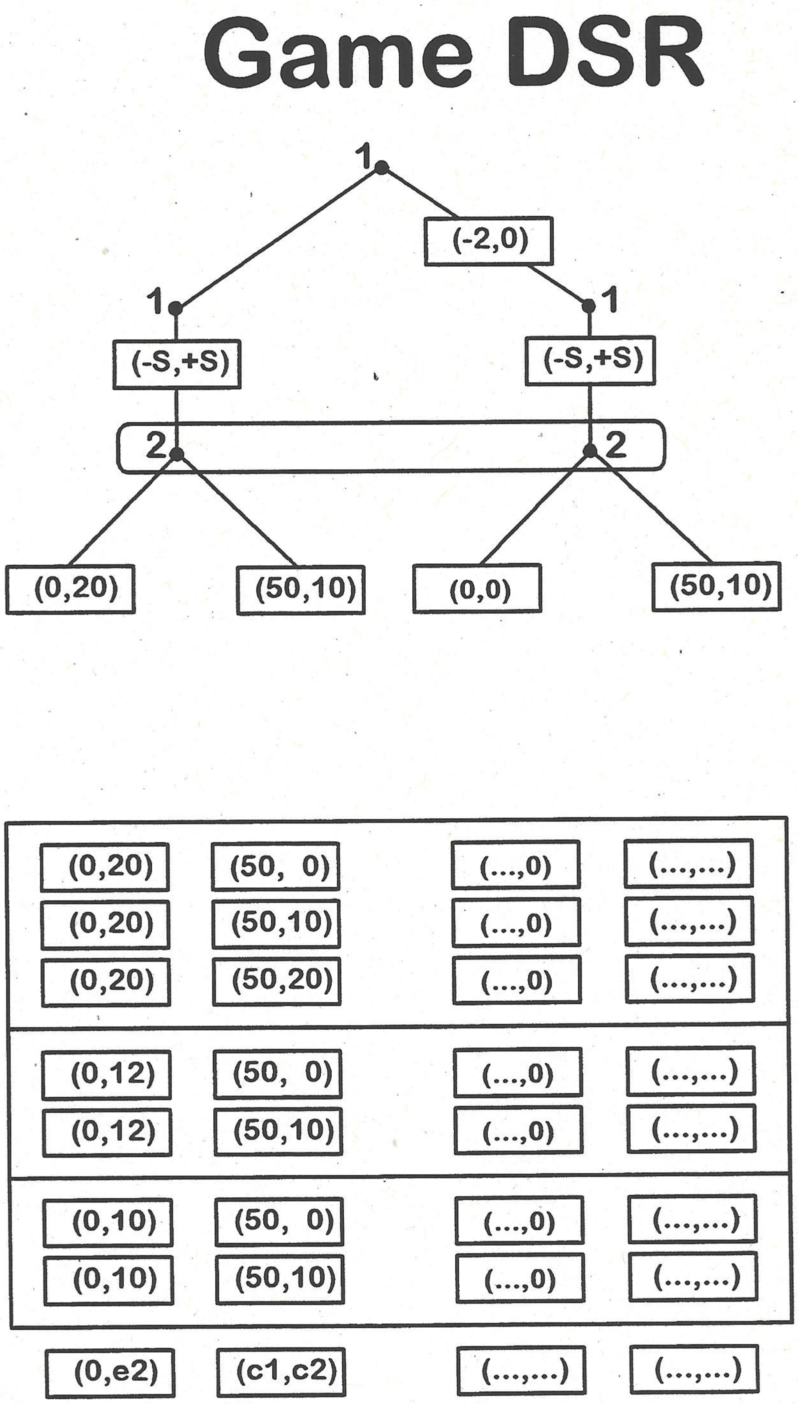
Player 1 prefers the alternative with the payoffs (50,10), because his payoff is higher than in the alternative with the payoffs (0,20). In the subgame perfect equilibrium player 2 selects the alternative with payoffs (0,20) which is denoted as the egoistic alternative independently of the side-payment of player 1, because he receives the side-payment independently of the alternative he selects and his payoff is higher if he selects the egoistic alternative. Therefore player 1 selects S=0 in the subgame perfect equilibrium. Selecting the alternative (50,10) which is denoted as the cooperative alternative with a higher payoff sum might be possible if player 1 induces reciprocity by giving the "right side-payment S" and player 2 reacts reciprocally. Neglecting the side-payment this kind of cooperation causes an improvement in the payoff of player 1 from 0 to 50 and a deterioration in payoff of player 2 from 20 to 10 (which is also the incentive to deviate from the reciprocal behavior). The side-payment has to account for these differences.

The inclusion of a destruction move as the first move of player 1 results in the game DSR (destruction move - side-payment - reciprocity) given in figure 2. Player 1 can decide to modify the payoffs of player 2 in the egoistic alternative from 20 to 0. The cost k of this modification is k=-2. Player 2 does not know whether player 1 has modified the game when receiving the side-payment and when deciding between the two alternatives. Selecting the egoistic alternative which is non reciprocal in game SR and game DSR might have negative consequences (might result in a lower payoff).

One question in both games is which is the criterion that determines the side-payment. Therefore the game is played for several payoffs in the cooperative alternative (c_1,c_2) and the egoistic alternative $(0,e_2)$ which are given in the lower part of figure 2. In addition in the DSR game the use of the destruction move (for different cost k=-2 or k=-10) and the reaction on this is subject of the experiment.

The subgame perfect equilibrium of this game is that player 1 selects k=0 and S=0 and that player 2 selects the egoistic alternative $(0,e_2)$. Player 1 can always improve his payoff by not modifying, because modifying effects only the payoff of player 2. If player 1 does not modify the side-payment is S=0 and player 2 selects the alternative $(0,e_2)$. The cooperative choice is (c_1,c_2).

Figure 2: The game DSR (destruction move - side-payment - reciprocity)



Models

The theory of prominence

Before describing possible criteria which could determine the side-payment a short introduction to the theory of prominence in the decimal system (W. Albers, 1997) is given, because it is fundamental to some models.

One result of the theory of prominence is that some numbers are easier accessible than others. The numbers that are most easily accessible are the prominent numbers P:

$P=\{n*10^z | z \in \mathbf{Z}, n \in \{1,2,5\}\}=\{...,0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100,....\}$.

If the perception is spontaneous the so called spontaneous numbers S are the numbers that are accessible. These are:

$S=\{n*10^z | z \in \mathbf{Z}, n \in \{-7, -5, -3, -2, -1.5, -1, 0, 1, 1.5, 2, 3, 5, 7\}\}$.

The spontaneous numbers include the prominent numbers and one additional number between any two neighbored prominent numbers.

The perception of numbers (including payoffs) is described as differences of steps between prominent and spontaneous numbers. The difference between two neighbored prominent numbers (ordered according to their size) is 1 step and the difference between two neighbored spontaneous numbers (ordered according to their size) is 1/2 step.

The perception is limited for small numbers (due to the problem). In the theory of prominence this is modeled by assuming a finest perceived full step unit Δ which permits to define a perception function v_Δ (by $v_\Delta(\Delta)=1$) mapping monetary payoffs to the perception space. Table 1 gives the function for $\Delta=10$, for $\Delta=20$ and the spontaneous numbers between -150 and +150 (which are relevant for the experiment).

Table 1: Transformation of the spontaneous numbers between -150 and 150 by the v_Δ-function for Δ=10 and Δ=20.

number:	-150	-100	-70	-50	-30	-20	-15	-10	-5	0	5	10	15	20	30	50	70	100	150
v_{10} :	-4.5	-4	-3.5	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
v_{20} :	-3.5	-3	-2.5	-2	-1.5	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1	1.5	2	2.5	3	3.5

Further refinements of the perception (for example the exact numbers) are described in W. Albers (1997). 1/4 steps are assigned to the exact numbers as given in table 2.

Table 2: The exact numbers between 0 and 100 and the v_Δ-function for Δ=10 and Δ=20.

numbers	0		5		10		15		20		30		50		70		100
		2,3		7,8		12,13		17,18		25		35,40		60		80	
v_{10}	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4
v_{20}	0		0.25		0.5		0.75		1	1.25	1.5	1.75	2	2.25	2.5	2.75	3

For numbers $x > \Delta$ the function $v_\Delta(x)$ is (nearly) equal to $3*\log(x/\Delta)+1$[2]. Below the smallest unit Δ the function is linear (compare table 1).

This description of the perception is similar to the Weber-Fechner law (for example in G.T. Fechner, 1968) which describes the perception of stimuli in psychophysics. The perception is proportional to a logarithmic function above a smallest unit. Comparisons between stimuli are performed by forming differences (not quotients). This seems to be plausible, since the stimulus has been transformed by a function proportional to the logarithm.

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2 For numbers $x \geq \Delta$ it holds: $|v_\Delta(x)/(3*\log(x/\Delta)+1)-1| \leq 7\%$

In the experimental data the smallest unit depends on the problem (the game) and has to be determined. This is performed by means of a **rule** of the theory of prominence: Δ is the prominent number 2 steps below the smallest prominent number greater than the maximal payoff in the game.

Criteria for the side-payment

Using the notations of figure 1 and figure 2 several criteria are given below which result in different side-payments.

The first criterion is that the payoff sum in case of cooperation is divided equally.

Equal division of the payoff sum in case of cooperation (ED).

The side-payment is **$S=(c_1-c_2)/2$** .
This results in a total payoff of $((c_1+c_2)/2,(c_1+c_2)/2)$

The second criterion includes the deterioration of one player in case of cooperation. It is also equally divided.

Equal division of the improvement of player 1 and the deterioration of player 2 in case of cooperation (EID).

The improvement of player 1 is c_1-0 and the deterioration of player 2 is e_2-c_2.
This results in a side-payment of **$S=(c_1-0+e_2-c_2)/2$**

The third criterion is given by including the theory of prominence in the criterion EID. All payoffs are transformed by the v_Δ-function on the perception level. On this level the comparisons are performed as in the EID criterion.

Equal division of the improvement of player 1 and the deterioration of player 2 in case of cooperation with the inclusion of the theory of prominence (EIDP).

This results in a perceived side-payment of:

$v_\Delta(S)=(v_\Delta(c_1)-v_\Delta(0)+v_\Delta(e_2)-v_\Delta(c_2))/2$.

A comparison of the criteria is given below.

ED : $S=(c_1 - c_2)/2$

EID : $S=(c_1 - 0 + e_2 - c_2)/2$

EIDP: $v_\Delta(S)=(v_\Delta(c_1)-v_\Delta(0)+v_\Delta(e_2)-v_\Delta(c_2))/2$

It can be seen that the ED criterion does not consider all payoffs. This is the case in the EID criterion. In the EIDP criterion the comparisons are performed on a perception level. The improvement and the deterioration are valued as in the equilibrium selection between two payoff pairs (c_1,c_2) and $(0,e_2)$ in 2x2 bimatrix games (B.Vogt, W. Albers 1997).

The payoffs of both players are perceived on the same scale. No additional weight for the own payoff and the payoff of the other one or utility of an equal payoff is introduced. A deviation from linear perception of payoffs is due to the logarithmic perception described by the theory of prominence in the decimal system. The fairness criterion itself is an equal division of the improvement and the deterioration.

Because Δ depends on the problem it is determined in advance and not fitted by means of the data. Therefore none of these 3 criteria has free parameters.

Experiment

The payoffs

The players received points as their payoffs. The worth of 1 point was 0.5 DM (~$.33). Losses up to 100DM (~$66) had to be paid by the subjects.

The subjects

The subjects were 32 students. They were divided in 4 groups of 8 subjects.

Communication

Free preplay communication via terminals was possible.

Experimental performance

In part 1 of the experiment single games were played in 4 groups of 8 subjects with free preplay communication between 2 players[3].

In part 2 a strategy game was played. All subjects selected their strategies for all games and all roles (player 1 and player 2). One game was paid per type of game and per person. Subjects were assigned to each other randomly.

[3] In the single games the payoff of one subject was the difference of his payoff to the mean payoff of the other subjects not in his group and playing the same role.

The strategies for the games were given as:

Game SR: as **player 1**: a side-payment S_p.
as **player 2**: a minimal side-payment S_{min} necessary to select (c_1,c_2).

Game DSR: as **player 1**: **k=0 or k=-2**
as **player 1**: a side-payment S_p.
as **player 2**: a minimal side-payment S_{min} necessary to select (c_1,c_2).

Predictions and hypotheses

The predictions for the reciprocity game are given in figure 3. The predictions of the side-payment S do not depend on the destruction move and are the same for the games SR and DSR. For the model EIDP based on the theory of prominence Δ has to be determined. Δ is 2 steps below the smallest prominent number greater than the maximal payoff (according to the rules of the theory of prominence). The maximal payoff is the prominent number 50. This results in Δ=10 (thus Δ is not a free parameter). These predictions are the hypotheses tested.

Figure 3: Predictions of the side-payment S for all games

Game SR with payoffs $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
ED	25	25	25	20	20	20	15
EID	30	31	35	25	26	30	25
EIDP	20	22	30	15	16	20	15

Game DSR (k=-2) with payoffs $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
ED	25	25	25	20	20	20	15
EID	30	31	35	25	26	30	25
EIDP	20	22	30	15	16	20	15

Game DSR (k=-10) with payoffs $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
ED	25	25	25	20	20	20	15
EID	30	31	35	25	26	30	25
EIDP	20	22	30	15	16	20	15

Results of the strategy game

In this part the results of the strategy game are given and discussed. In figure 4 the strategies selected by the subjects are shown as:
strategy of player 1 / strategy of player 2. For example ($S=0$ / (c_1,c_2) for $S_{min}<50$) describes the strategy that this subject gives a side-payment $S=0$ as player 1 and selects (c_1,c_2) as player 2 if he receives a side-payment $S \geq S_{min}$ with $S_{min}<50$.

Figure 4: The strategies selected by the subjects for all games with $e_2>c_2$.

	$S>0$/ (c_1,c_2) for $S_{min}<50$	$S=0$/ (c_1,c_2) for $S_{min}<50$	$S>0$/ $(0,e_2)$	$S=0$/ $(0,e_2)$
game SR	23	0	0	9
game DSR (k=-2)	32	0	0	0
game DSR (k=-10)	23	0	0	9

The strategy selection of S=0/(0,e2) is a subgame perfect equilibrium (plus k=0 for player 1). The selection of S>0/((c1,c2) for $S_{min}<50$) corresponds to reciprocal behavior. These are the only observed selections. All other mixed selections do not occur. Two patterns of behavior are observed over all games.

The pattern of behavior 1 is to select S=0/(0,e2). This selection is dependent on the destruction move: if the destruction move is not possible or the cost is too high (k=-10, i.e. $|k|$ is equal to the smallest perceived money unit Δ) 9 subjects show this pattern of behavior.

The **pattern of behavior 2** is $S>0/(c_1,c_2)$ for $S_{min}<50$. This pattern is independent of the destruction move for 23 subjects. 9 subjects show this pattern if the cost for the destruction move is not too high.

An interpretation of this behavior is that 23 subjects show reciprocal behavior and 9 subjects imitate reciprocal behavior if destruction of the advantage of being non reciprocal is possible and likely to occur (if the cost is not too high).

The results of the side-payments in the strategy game are given in figure 5 for all conditions. The medians of the four groups are shown.

Figure 5: Medians of minimal side-payment to cooperate (to select (c_1,c_2)) in the reciprocity games

Game SR with payoffs $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
group 1	22	23.75	30	15	18.5	22.5	15
group 2	20	22	25	17.5	20	20	10
group 3	20	21.5	25	12.75	13.25	17.5	9.75
group 4	23	23	30	15	18	20	15

Game DSR (k=-2) with payoffs $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
group 1	20	22	30	12.5	19.25	20	15.75
group 2	17.5	18	23.5	15	15	15	8.5
group 3	15	16	20	7.75	8	13.25	5.75
group 4	20	23	27.5	16.5	18.5	20	15

Game DSR (k=-10) with payoffs $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
group 1	21.25	22	30	13.75	18.25	18.75	15.5
group 2	17.5	20	30	15	15	20	10
group 3	13.25	15	22	8.75	9.25	13	8.75
group 4	21	22	21.5	15	19	20	16

These are the data which are used for a test of the predictions given in figure 3. Assuming independence of these four groups a binomial test[4] is performed. The results are shown in figure 6. A "-" indicates the prediction is rejected and a "+" indicates if this is not the case.

Figure 6: Results of the test for all criteria and games

Game SR $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
ED	-	-	+	-	+	+	+
EID	-	-	-	-	-	-	-
EIDP	+	+	+	+	+	+	+

Game DSR (k=-2) $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
ED	-	-	+	-	-	+	+
EID	-	-	-	-	-	-	-
EIDP	+	+	+	+	+	+	+

Game DSR (k=-10) $(0,e_2)/(c_1,c_2)$

	(0,10)/ (50,0)	(0,12)/ (50,0)	(0,20)/ (50,0)	(0,10)/ (50,10)	(0,12)/ (50,10)	(0,20)/ (50,10)	(0,20)/ (50,20)
ED	-	-	+	-	-	+	+
EID	-	-	-	-	-	-	-
EIDP	+	+	+	+	+	+	+

[4] For all tests: α=12,5% assuming the independence of the four groups and
α=5% assuming independence of the strategies of all subjects.
This leads to the same result of the binomial test.

The concept ED (equal division in case of cooperation) is rejected in 3 or 4 of 7 cases (depending on whether the destruction move is possible or not and on the cost of the destruction move). The concept EID (equal division of the improvement of player 1 and the deterioration of player 2) is rejected in all cases. The concept incorporating the theory of prominence in the decimal system EIDP is rejected in 0 of 7 cases under all conditions and is the best predictor for the experimental data.

In a binomial test with α=30% the concept EIDP is not rejected. This is certainly not a test, but gives hints for the validity of the result.

An interpretation of this result is that the criterion of reciprocal behavior is given by EIDP and that it does not depend on the destruction move. Some subjects do not apply this criterion depending on the destruction move as it is observed in the analysis of the strategies selected, but if reciprocal behavior is observed the criterion seems to be applied.

Conclusions

In this paper a 2-person reciprocity game has been analyzed. In this game player 2 decides between two alternatives with payoffs to player 1 and player 2. Before the decision of player 2 player 1 could give a side-payment to player 2 to make player 2 choose the pareto-optimal alternative favored by player 1, but not by player 2. Player 2 received this side-payment independently of the alternative he selected afterwards.

In the reciprocity game several criteria which determine a payment inducing reciprocity depending on the improvement of one player and the deterioration of the other player in case of cooperation have been tested. The criterion predicting an equal division of the payoff sum in case of cooperation is rejected in ≈50% of the games. The criterion based on the theory of prominence and an equal division of the perceived improvement of one player and the perceived deterioration of the other player in case of cooperation is the best predictor for the experimental data. The inclusion of a destruction move for non reciprocal behavior does not change the criterion for reciprocal behavior, but if being non reciprocal can not have negative consequence or these consequences are not likely to occur some subjects behave non reciprocal. If the consequences are likely to occur these subjects behave reciprocally.

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