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The Selection of Mixed Strategies in 2x2 Bimatrix Games

by

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Abstract

In 2x2 bimatrix games with 2 equilibria in pure strategies and 1 in mixed strategies existing models like the risk dominance (J.C. Harsanyi, R. Selten 1988) or the Nash-criterion (J.F. Nash, 1950b and 1953) applied to the selection in 2x2 bimatrix games predict the selection of one of the 2 equilibria in pure strategies. The equilibrium in mixed strategies is never selected. A new model based on the theory of prominence (W. Albers, G. Albers, 1983, W. Albers, 1997) which describes the perception of numbers (especially payoffs) predicts the selection of a mixed equilibrium point for some games. An experiment using the strategy method was performed to test the predictions of the different models. A result of the experiment is that the equilibrium point in mixed strategies is selected by the subjects for several games. The predictions of the models based on the theory of prominence are in a better agreement with the data than the other models. The predictions of the model based on the theory of prominence and related to risk dominance cannot be rejected by this experiment.

1. Introduction

A solution concept for non-cooperative games is the Nash equilibrium (J.F. Nash, 1950a and 1951). An existence theorem can be proved for the mixed extension of all bimatrix games. In non degenerated 2x2 bimatrix games there are three possible cases: there is one equilibrium point in pure strategies, or there is one equilibrium point in mixed strategies, or there are three equilibrium points (two in pure strategies and one in mixed strategies).

In 2x2 bimatix games in which more than one equilibrium exists the equilibrium selection is a problem. A solution of this problem is proposed by the model of risk dominance (J.C. Harsanyi, R.Selten, 1988). For 2x2 bimatrix games this solution is given by an axiomatic approach. Another model that can be applied to the equilibrium selection is the Nash-Zeuthen bargaining model (J. Nash, 1950b and 1953, F. Zeuthen, 1930). All of these models select equilibria in pure strategies and none in mixed strategies.

Two models (model I and model II) select the equilibrium in mixed strategies for some games. Both models use the theory of prominence (W. Albers, G. Albers, 1983 and W. Albers, 1997) which models the perception of numbers. Model I is based on the Nash-Zeuthen bargaining model and model II is related to risk dominance.

An experiment in which players can select between pure and mixed equilibrium points should be a test of the models. It should also be a test of the "instability" of mixed equilibria (in 2x2 bimatrix games) which are often not perfect.

In this paper 2x2 bimatrix games with 2 equilibria in pure strategies and 1 in mixed strategies are examined. In the first part some considerations concerning the principal structure of these games are described. In the second part models describing the selection between the equilibria are presented (compare B. Vogt, W. Albers, 1997). In the third part the experiment is described in which 399 games were played by means of the strategy method to test the predictions of the models.

Some notations, basic definitions and basic theorems about 2x2 bimatrix games:

The normal form of a game G is G(N,St,a).
N={1,..,n} is the set of players.
The strategies of player i ($i \in N$) are denoted by s_i and the strategy set of a player i is denoted by St_i ($s_i \in St_i$).
A strategy combination is denoted by s: $s=s_1 \times s_2 \times .. \times s_n$.
The strategy combination set is $St = St_1 \times St_2 \times .. \times St_n$.
a is the payoff function: a: $St \rightarrow \mathbf{R}^n$.

A 2-person game (N={1,2}) with m strategies of player 1 and n strategies of player 2 (m and n are countable numbers) is called an mxn bimatrix game. It can be represented by two mxn matrices $A = (a_{ij})$ and $B = (b_{ij})$ or the mxn bimatrix (A,B) of the pairs (a_{ij},b_{ij}). For m=n=2 the game is called a 2x2 bimatrix game. In the following parts of the paper the notations of figure 1.1 will be used for a 2x2 bimatrix game. A 2x2 bimatrix game is non degenerated if the two matrices are not degenerated.

Figure 1.1 : The normal form of a 2x2 bimatrix game

	U_2	V_2
U_1	a_{11},b_{11}	a_{12},b_{12}
V_1	a_{21},b_{21}	a_{22},b_{22}

In the following parts of the paper only games with finite strategy sets of each player are considered.

The mixed extension of a game G(N,St,a) with finite strategy sets of each player i is the game $G(N,\overline{St},\overline{a})$.

The set of players is N={1,..,n}.

A mixed strategy σ_i of player i is defined as: σ_i: $St_i \rightarrow [0,1]$

with $\sigma_i \geq 0$ and $\sum_{s_i \in St_i} \sigma_i(s_i) = 1$ for all players i.

A mixed strategy combination is denoted by $\sigma = \sigma_1 \times \sigma_2 \times .. \times \sigma_n$.

The mixed strategy combination set is $\overline{St} = \overline{St_1} \times \overline{St_2} \times .. \times \overline{St_n}$.

The payoff function is: $\overline{a}(\sigma) = \sum_{s \in St} \sigma_1(s_1)*\sigma_2(s_2)*..*\sigma_n(s_n)*a(s)$.

A pair (σ_1^*,σ_2^*) of mixed strategies of the bimatrix game (A,B) is in a (Nash) equilibrium, if for all mixed strategies σ_1' and σ_2'

$A(\sigma_1',\sigma_2^*) \leq A(\sigma_1^*,\sigma_2^*)$

$B(\sigma_1^*,\sigma_2') \leq B(\sigma_1^*,\sigma_2^*)$ holds.

A pair (σ_1^*,σ_2^*) of mixed strategies of the bimatrix game (A,B) is in a strong (Nash) equilibrium, if for all mixed strategies σ_1' and σ_2'

$A(\sigma_1',\sigma_2^*) < A(\sigma_1^*,\sigma_2^*)$

$B(\sigma_1^*,\sigma_2') < B(\sigma_1^*,\sigma_2^*)$ holds.

An equilibrium is called an equilibrium in pure strategies, if in the equilibrium for all players it holds $\sigma_i(s_i)=1$ for one pure strategy s_i of player i. An equilibrium is called an equilibrium in mixed strategies in all other cases.

The following well known theorems may be mentioned.

- Theorem (J.F. Nash, 1950a and 1951):

  The mixed extension of a bimatrix game has at least one equilibrium point.

- Theorem (C.E. Lemke, J.T. Howson, 1964):

  The mixed extension of a non degenerated bimatrix game has an odd number of equilibrium points.

- Theorem (see P. Borm 1990, p. 60):

  The mixed extension of a non degenerated 2x2 bimatrix game has 3 equilibrium points.

  There are 3 possible cases:
  one equilibrium point in pure strategies.
  one equilibrium point in mixed strategies.
  two equilibrium points in pure strategies and one in mixed strategies.

2. Models of the equilibrium selection

In this part of the paper models describing the equilibrium selection in 2x2 bimatrix games are discussed. As a result of the comparison between 2 equilibrium points one equilibrium point is selected. In this part of the paper the models which describe the selection between 2 equilibrium points are described. It is assumed that one player prefers one equilibrium point and the other player prefers the other equilibrium point. None of the equilibrium points dominates the other one in payoffs, i.e. in none of the two equilibrium points the payoff for both players is higher than in the other one. Using the notations of figure 1.1 (if without loss of generality $V=V_1U_2$ and $U=U_1V_2$ are the equilibrium points) it holds:

$a_{21}-a_{11}>0$, $b_{21}-b_{22}>0$, $a_{12}-a_{22}>0$, $b_{12}-b_{11}>0$; with
($a_{21}>a_{12}$ and $b_{12}>b_{21}$) or ($a_{12}>a_{21}$ and $b_{21}>b_{12}$.).

In the following parts of the paper it is also assumed without loss of generality that $a_{21}>a_{12}$ and $b_{12}>b_{21}$. In this situation player 1 prefers to select the strategy V_1, because he gets the highest payoff a_{21} for the strategy combination $V=V_1U_2$. Player 2 prefers to play V_2, because his payoff is maximal for the strategy combination $U=U_1V_2$. The conflict case occurs if both players insist on playing their preferred strategies which leads to a strategy combination of V_1V_2. The strategy combination U_1U_2 occurs in the case of miscoordination: both players select the non preferred strategies. Then the payoffs in the bimatrix can be denoted according to figure 2.1. The maximal payoff is denoted by a_{max} and b_{max}, the second highest equilibrium payoff by a_{alt} and b_{alt}, the conflict payoff by a_{min} and b_{min} and the miscoordination payoff by a_{mis} and b_{mis}. This notation will be used in the following parts of the paper.

Figure 2.1 : The notations used for a 2x2 bimatrix game

	U_2	V_2
U_1	a_{mis}, b_{mis}	a_{alt}, b_{max}
V_1	a_{max}, b_{alt}	a_{min}, b_{min}

2.1 The Nash-criterion

For the selection between two equilibria of 2x2 bimatrix games the Nash-criterion (J.F. Nash, 1950b and 1953, Zeuthen, 1930) can be considered. It is bargained about pairs of payoffs (a,b). A threat payoff (which can be interpreted as conflict payoff) is given by (a_{min}, b_{min}). The Nash-criterion selects that payoff (a^*, b^*) for which the product $(a^* - a_{min}) * (b^* - b_{min})$ is maximal.

This idea can be applied to the selection between the equilibrium points of 2x2 bimatrix games. The two equilibrium points are the alternatives. The threat point is given by the conflict payoff. The equilibrium preferred by the Nash-criterion is selected. Using the notations of figure 2.1 the selection critierion is:

V dominates U [U dominates V] iff

$(a_{max} - a_{min}) * (b_{alt} - b_{min}) > [<] \ (a_{alt} - a_{min}) * (b_{max} - b_{min})$

This criterion is equivalent to the criterion:

V dominates U [U dominates V] iff

$(a_{max} - a_{alt}) / (a_{alt} - a_{min}) \quad > [<] \ (b_{max} - b_{alt}) / (b_{alt} - b_{min})$

2.2 Risk Dominance

J.C. Harsanyi and R. Selten (J.C. Harsanyi, R.Selten, 1988) developed the concept of risk dominance for the equilibrium selection in all games. For the class of games considered here they give an axiomatic characterization. Using the notation of figure 2.1 the criterion of risk dominance is:

V dominates U [U dominates V] iff
$(a_{max}-a_{mis})*(b_{alt}-b_{min}) > [<] \ (a_{alt}-a_{min})*(b_{max}-b_{mis})$.

An equivalent formulation is:
V dominates U [U dominates V] iff
$(a_{max}-a_{mis})/(a_{alt}-a_{min}) > [<] \ (b_{max}-b_{mis})/(b_{alt}-b_{min})$.

2.3 Models using the Theory of Prominence

Before describing the models that use the theory of prominence (W. Albers, G. Albers, 1983 and W. Albers, 1997) the results of the theory of prominence used in the models is described. One result of the theory of prominence is that some numbers are easier accessible than others. The numbers that are most easily accessible are the prominent numbers P:

$P=\{z*10^n|n\in\mathbf{Z}, z\in\{1,2,5\}\}=\{...,0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100,....\}$.

If the perception is spontaneous the so called spontaneous numbers S are the numbers that are accessible. These are:

$S=\{z*10^n|n\in\mathbf{Z}, z\in\{-7, -5, -3, -2, -1.5, -1, 0, 1, 1.5, 2, 3, 5, 7\}\}$.

The spontaneous numbers include the prominent numbers and one additional number between any two neighboured prominent numbers.

The perception of numbers (also payoffs) is described as difference of steps between spontaneous numbers. The difference between two prominent numbers (ordered according to their size) is 1 step and between two spontaneous numbers (ordered according to their size) 1/2 step.

Another important empirical observation is that the perception is limited for small numbers. There is a smallest unit that can be perceived. In the theory of prominence this is modeled by assuming a smallest full step money unit Δ which permits to define a perception function P_Δ mapping monetary payoffs to the perception space. Table 2.1 gives the function for Δ=10, for Δ=20 and the spontaneous numbers between -150 and +150 (which are relevant for the experiment).

Table 2.1: Transformation of the spontaneous numbers between -150 and 150 by the P-function for Δ=10 and Δ=20.

number:	-150,	-100,	-70,	-50,	-30,	-20,	-15,	-10,	-5,	0,
P_{10} :	-4.5,	-4,	-3.5,	-3,	-2.5,	-2,	-1.5,	-1,	-0.5,	0,
P_{20} :	-3.5,	-3,	-2.5,	-2,	-1.5,	-1,	-0.75,	-0.5,	-0.25,	0,

number:	5,	10,	15,	20,	30,	50,	70,	100,	150
P_{10} :	0.5,	1,	1.5,	2,	2.5,	3,	3.5,	4,	4.5
P_{20} :	0.25,	0.5,	0.75,	1,	1,5,	2,	2,5,	3,	3.5

The exact numbers are a further refinement of the spontaneous numbers. The exact numbers between 0 and 100 and the P_Δ-functions are given for Δ=10 and Δ=20 in table 2.2 (which are relevant for the experiment).

Table 2.2: The exact numbers between 0 and 100 and the P_Δ-function for Δ=10 and Δ=20.

Δ=10:

	0		5		10		15		20		30		50		70		100
		2,3		7,8		12,13		17,18		25		35,40		60		80	
P_{10}:	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4

Δ=20:

	0		10		20		30		50		70		100
		5		15		25		35,40		60		80	
P_{20}:	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3

For numbers higher than Δ the function $P_\Delta(s)$ is (nearly) equal to $3*\log(s/\Delta)+1$[1]. Below the smallest unit Δ the function is linear (compare table 2.1).

This description of the perception is similar to the Weber-Fechner Law (for example in G.T. Fechner, 1968) which describes the perception of stimuli in psychophysics. The perception is proportional to a logarithmic function above a smallest unit. Comparisons between stimuli are performed by forming differences (not quotients). This seems to be plausible, since the stimulus has been transformed by a function proportional to the logarithm.

[1] For numbers $x \geq \Delta$ it holds: $|P_\Delta(x)/(3*\log(x/\Delta)+1)-1| \leq 7\%$

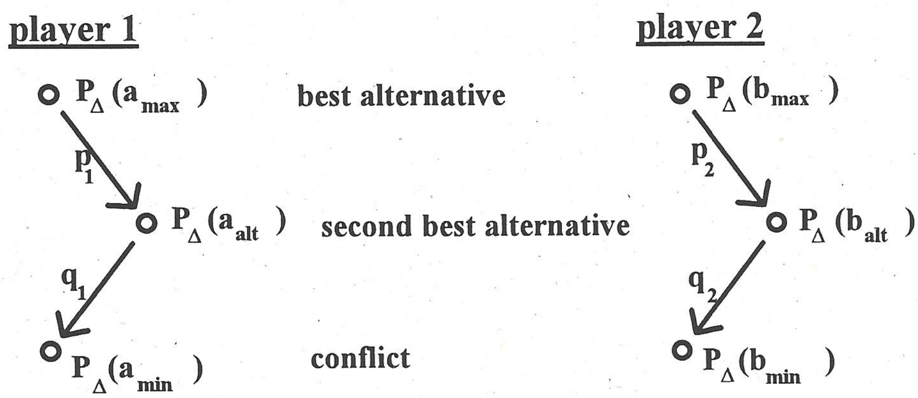
2.3 Model I

All payoffs are transformed by the P_Δ-function:

P_Δ: $a_{..} \rightarrow P_\Delta(a_{..})$ and
P_Δ: $b_{..} \rightarrow P_\Delta(b_{..})$, where the index .. can be max, min, alt and mis.

The selection criterion is obtained by a comparison of differences as shown schematically in figure 2.2.

Figure 2.2: Schematic presentation of the comparisons of model I



In this model the differences $p_1=P_\Delta(a_{max})-P_\Delta(a_{alt})$ and $p_2=P_\Delta(b_{max})-P_\Delta(b_{alt})$ between the best and second best alternatives are compared with the differences $q_1=P_\Delta(a_{alt})-P_\Delta(a_{min})$ and $q_2=P_\Delta(b_{alt})-P_\Delta(b_{min})$ between the second best alternative and the conflict payoff. The selection criterion is:

V dominates U [U dominates V] iff
$p_1-q_1 >[<] p_2-q_2$.

Replacing p and q results in
V dominates U [U dominates V] iff

$(P_\Delta(a_{max})-P_\Delta(a_{alt})) - (P_\Delta(a_{alt})-P_\Delta(a_{min})) > [<]$
$(P_\Delta(b_{max})-P_\Delta(b_{alt})) - (P_\Delta(b_{alt})-P_\Delta(b_{min}))$

Interpretation: For each player the incentive to deviate from the second best alternative (for example $(P_\Delta(a_{max})-P_\Delta(a_{alt})$ for player 1) is compared with the possible loss, if the game ends in a conflict $((P_\Delta(a_{alt})-P_\Delta(a_{min})$ for player 1). The player for whom the difference between the incentive to deviate from the second best alternative and possible loss is bigger has "more arguments" for the equilibrium point favored by him. This point will be selected.

2.3.1 Relations between model I and the Nash-criterion

For a comparison the criterion of Model I and the Nash-criterion are presented in a way that the same terms are at the same places.

V dominates U [U dominates V] iff
Nash: $(a_{max} - a_{alt}) / (a_{alt} - a_{min}) > [<]$
Nash: $(b_{max} - b_{alt}) / (b_{alt} - b_{min})$
Model I: $(P_\Delta(a_{max})-P_\Delta(a_{alt})) - (P_\Delta(a_{alt})-P_\Delta(a_{min})) > [<]$
Model I: $(P_\Delta(b_{max})-P_\Delta(b_{alt})) - (P_\Delta(b_{alt})-P_\Delta(b_{min}))$

The difference between the two models is that in model I payoffs are transformed by the P_Δ-function and that after the transformation quotients in the Nash-criterion correspond to differences in model I. This is compatible with the fact that after a transformation by the P_Δ-function (which is for values bigger than Δ a logarithmic function) the quotients are transformed into differences.

Another difference of these approaches is that the P_Δ-function operates on payoffs and not on differences of payoffs. For example the difference $(a_{alt} - a_{min})$ is transformed into $P_\Delta(a_{alt}) - P_\Delta(a_{min})$ and not into $P_\Delta(a_{alt} - a_{min})$. This will be discussed in chapter 2.3.2.1.

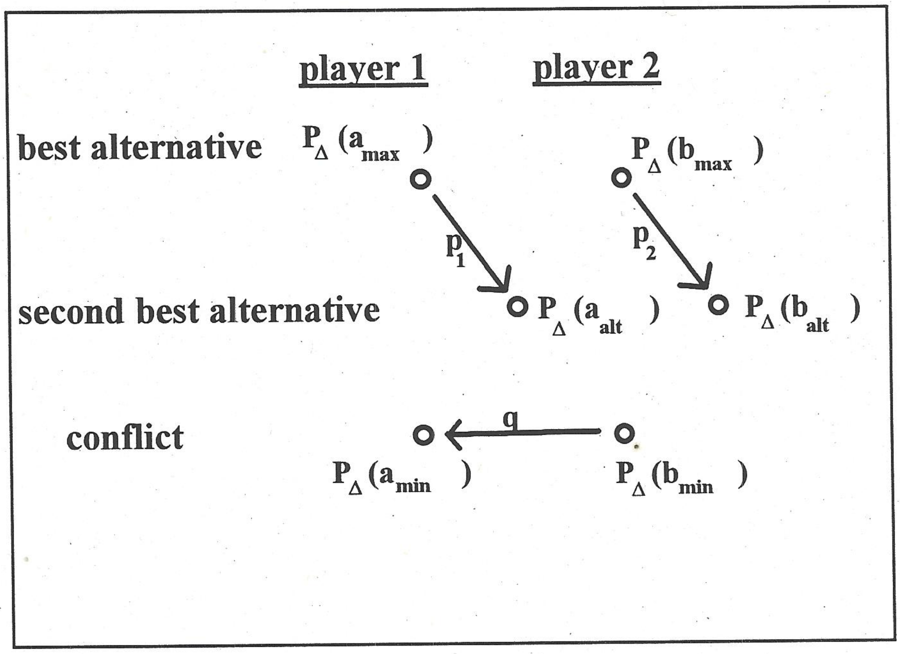
2.4 Model II

All payoffs are transformed by the P_Δ-function:

P_Δ: $a_{..} \rightarrow P_\Delta(a_{..})$ and
P_Δ: $b_{..} \rightarrow P_\Delta(b_{..})$, where the index .. can be max, min, alt and mis.

The selection criterion is obtained by comparison of differences as shown schematically in figure 2.3.

Figure 2.3: Schematic presentation of the comparisons in model II



In this model the differences $p_1=P_\Delta(a_{max})-P_\Delta(a_{alt})$ and $p_2=P_\Delta(b_{max})-P_\Delta(b_{alt})$ between best and second best alternative for each player are compared with one another. The difference p_1-p_2 is then compared with the difference $q=P_\Delta(b_{min})-P_\Delta(a_{min}))$ between the conflict payoffs. The selection criterion is:

V dominates U [U dominates V] iff

$$p_1 - p_2 > [<] \; q.$$

Replacing p and q leads to the criterion:

V dominates U [U dominates V] iff

$$(P_\Delta(a_{max})-P_\Delta(a_{alt}))-(P_\Delta(b_{max})-P_\Delta(b_{alt})) >[<] \; (P_\Delta(b_{min})-P_\Delta(a_{min}))^2$$

Interpretation: in this model an advantage (higher transformed payoffs) in comparing the equilibrium payoffs can be compensated by a disadvantage in comparing the conflict payoffs. The equilibrium point is selected if its advantage is bigger than its disadvantage.

A different approch to the criterion is given by the following consideration. Every transformed payoff can be seen as an "argument" supporting one equilibrium point or the other. Giving a weight of +1 and -1 for supporting one or the other one, respectively, a sum of "arguments" can be calculated. That equilibrium is selected for which the sum is greater than for the other. The "arguments" of player 1 for the equilibrium point V are:

$+P_\Delta(a_{max})$, $-P_\Delta(a_{alt})$ and $+ P_\Delta(a_{min})$.

The "arguments" of player 2 for the equilibrium point U are:

$+P_\Delta(b_{max})$, $-P_\Delta(b_{alt})$ and $+ P_\Delta(b_{min})$.

The weight for the maximal and minimal tranformed payoffs of the players is +1 because a high maximal and minimal payoff support the preferred equilibrium point. The weight for the second best alternative is -1 because a high payoff for the second best alternative supports the other equilibrium point and not the preferred one. The selection criterion is given by a comparison of the sums of arguments:

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[2]It is assumed that $a_{mis}=b_{mis}=0$.

V dominates U [U dominates V] iff
$P_\Delta(a_{max})-P_\Delta(a_{alt})+P_\Delta(a_{min})>[<]$
$P_\Delta(b_{max})-P_\Delta(b_{alt})+P_\Delta(b_{min})$

This is the same selection criterion as the one given above.

Another presentation of the selection criterion is:

V dominates U [U dominates V] iff
$P_\Delta(a_{max})+P_\Delta(b_{alt})+P_\Delta(a_{min})>[<]\ P_\Delta(a_{alt})+P_\Delta(b_{max})+P_\Delta(b_{min})$

Here the criterion can be interpreted as the sum of the two criteria:

$$P_\Delta(a_{max})+P_\Delta(b_{alt}) >[<]\ P_\Delta(a_{alt})+P_\Delta(b_{max}) \qquad \textbf{"Nash-sum"}$$

$$+ \qquad P_\Delta(a_{min}) >[<]\ P_\Delta(b_{min}) \qquad \textbf{"Conflict"}$$

$$= \qquad P_\Delta(a_{max})+P_\Delta(b_{alt})+P_\Delta(a_{min})>[<]\ P_\Delta(a_{alt})+P_\Delta(b_{max})+P_\Delta(b_{min})$$

The first criterion is called "Nash-sum" because it corresponds to the Nash-product without threats[3] , as it can be seen below:

if $P_\Delta(x)=\log(x)$ then

$\log(a_{max})+\log(b_{alt}) >[<] \log(a_{alt})+\log(b_{max})$,

$\Leftrightarrow$

$\log(a_{max}*b_{alt}) >[<] \log(a_{alt}*b_{max})$,

$\Leftrightarrow$

$a_{max}*b_{alt} >[<] a_{alt}*b_{max}$ ("Nash-product").

The second part is a standardform of the comparison of the conflict payoffs.

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[3]if the payoffs are higher than 0.

2.4.1 Model I and Model II if threats are not used

In Model II an agreement can be obtained without using threats. This agreement is obtained by the criterion that the "Nash-sum" is maximal. The conflict payoffs are not compared. The resulting model is called "model II, 2 cases". In the first case an agreement can be obtained without using threats according to the maximal "Nash-sum". In the second case (if the first case does not apply) the selection criterion using the conflict payoffs is applied. Whether case 1 is applied in games depends on the players and the negotiation. A discussion of such examples will be given in the experimental part (compare part 5.2). Model II including the conflict payoffs is the regular case.

Similar to model II model I can be extended to 2 cases. In the first case the selection of an equilibrium is according to the maximal "Nash-sum" without taking conflict payoffs into account. The 2. case is as described in part 2.3.

2.4.2 The comparison between model II and risk dominance

To compare model II and risk dominace the selection criteria are written below one another (Again the criteria of risk dominance and model II are taken in a form equivalent to the original ones).

V dominates U [U dominates V] iff

Risk Dominance:	$(a_{max} - a_{mis}) / (a_{alt} - a_{min})$	$>$ [$<$]
Risk Dominance:	$(b_{max} - b_{mis}) / (b_{alt} - b_{min})$	
Model II:	$P_\Delta(a_{max}) - (P_\Delta(a_{alt}) - P_\Delta(a_{min}))$	$>$ [$<$]
Model II:	$P_\Delta(b_{max}) - (P_\Delta(b_{alt}) - P_\Delta(b_{min}))$	

One difference between the 2 models is again the transformation by the P_Δ-function and the correspondence of quotients (in the model of risk dominance) and differences (in model II). The reasons for this have been discussed above, i.e. the P_Δ-function is for values bigger than Δ a logarithmic function and therefore quotients are transformed into differences.

Another difference is that the payoffs for miscoordination are not taken into account in model II. A reason of this is that in the experiment preplay communication is possible and therefore miscoordination might not occur.

An extension of the risk dominance by considering the perception of payoffs and the correspondence of quotients and differences leads to the following criterion:

V dominates U [U dominates V] iff

$$(P_\Delta(a_{max}) - P_\Delta(a_{mis})) - (P_\Delta(a_{alt}) - P_\Delta(a_{min})) > [<]$$
$$(P_\Delta(b_{max}) - P_\Delta(b_{mis})) - (P_\Delta(b_{alt}) - P_\Delta(b_{min}))$$

Besides the differences discussed above the main difference is that the P_Δ-function operates on the payoffs and not on differences of payoffs (as risk dominance does). For example the difference $(a_{alt} - a_{min})$ is transformed into $P_\Delta(a_{alt}) - P_\Delta(a_{min})$ and not into $P_\Delta(a_{alt} - a_{min})$. An explanation for this is that the strategic equivalence does not hold for the percepted payoffs. It must be replaced by another form of equivalence for perceived payoffs (B. Vogt, 1997).

3. The Experiment

The experiment was performed by means of the strategy method. In the experiment the subjects had to select strategies for 399 bimatrix games simultaneously. The games were of different types: with one equilibrium point, or with three equilibrium points.

The 399 different games were constructed by means of the symmetric 2x2 bimatrix game shown in figure 3.1. The equilibrium points in pure strategies are given by the strategy combinations $V=V_1U_2$ and $U=U_1V_2$. The conflict payoff is obtained for the strategy combination V_1V_2.

Figure 3.1: The bimatrix game used for the construction of all games.

	U_2	V_2
U_1	42.0, 42.0	-34.0, 118.0
V_1	118.0, -34.0	-138.5, -138.5

In the mixed extension of this bimatrix game with the payoff matrices M for player 1 and N for player 2 every player (i=1,2) has the opportunity to "mix" his pure strategies by playing them with probabilities $\sigma_i(U_i)$ and $\sigma_i(V_i)$ with $\sigma_i(U_i)+\sigma_i(V_i)=1$. In the experiment the players could choose between 20 probabilities $\sigma_i(U_i)$.

The row player (player 1) and the column player (player 2) could select between their mixed strategies in steps of 1/19. This results in 20 steps of mixed strategies: $(\sigma_i(U_i)=0/19,\sigma_i(V_i)=19/19)$, $(\sigma_i(U_i)=1/19,\sigma_i(V_i)=18/19)$, ..., $(\sigma_i(U_i)=19/19,\sigma_i(V_i)=0/19)$. with the corresponding payoffs:

$\sigma_1 M(\sigma_2)^t$ for the row player and
$\sigma_1 N(\sigma_2)^t$ for the column player.

The calculation of the payoffs of all 20x20 strategy combinations results in the two 20x20 matrices A and B with the payoffs a_{kl} for the row player and b_{kl} for the column player:

$$a_{kl} = \frac{19-k}{19} * (\frac{19-l}{19} * 42.0 + \frac{l}{19} * (-34.0)) + \frac{k}{19} * (\frac{19-l}{19} * 118.0 + \frac{l}{19} * (-138.5))$$

$$b_{kl} = \frac{19-l}{19} * (\frac{19-k}{19} * 42.0 + \frac{k}{19} * (-34.0)) + \frac{l}{19} * (\frac{19-k}{19} * 118.0 + \frac{k}{19} * (-138.5))$$

The resulting 20x20 bimatrix is presented in figure 3.2.

Figure 3.2: The 20x20 bimatrix

	0	1	2	3	4	5	6	7	8	9
0	42,0 / 42,0	38,0 / 46,0	34,0 / 50,0	30,0 / 54,0	26,0 / 58,0	22,0 / 62,0	18,0 / 66,0	14,0 / 70,0	10,0 / 74,0	6,0 / 78,0
1	46,0 / 38,0	41,5 / 41,5	37,0 / 45,0	32,5 / 48,5	28,0 / 52,0	23,5 / 55,5	19,0 / 59,0	14,5 / 62,5	10,0 / 66,0	5,5 / 69,5
2	50,0 / 34,0	45,0 / 37,0	40,0 / 40,0	35,0 / 43,0	30,0 / 46,0	25,0 / 49,0	20,0 / 52,0	15,0 / 55,0	10,0 / 58,0	5,0 / 61,0
3	54,0 / 30,0	48,5 / 32,5	43,0 / 35,0	37,5 / 37,5	32,0 / 40,0	26,5 / 42,5	21,0 / 45,0	15,5 / 47,5	10,0 / 50,0	4,5 / 52,5
4	58,0 / 26,0	52,0 / 28,0	46,0 / 30,0	40,0 / 32,0	34,0 / 34,0	28,0 / 36,0	22,0 / 38,0	16,0 / 40,0	10,0 / 42,0	4,0 / 44,0
5	62,0 / 22,0	55,5 / 23,5	49,0 / 25,0	42,5 / 26,5	36,0 / 28,0	29,5 / 29,5	23,0 / 31,0	16,5 / 32,5	10,0 / 34,0	3,5 / 35,5
6	66,0 / 18,0	59,0 / 19,0	52,0 / 20,0	45,0 / 21,0	38,0 / 22,0	31,0 / 23,0	24,0 / 24,0	17,0 / 25,0	10,0 / 26,0	3,0 / 27,0
7	70,0 / 14,0	62,5 / 14,5	55,0 / 15,0	47,5 / 15,5	40,0 / 16,0	32,5 / 16,5	25,0 / 17,0	17,5 / 17,5	10,0 / 18,0	2,5 / 18,5
8	74,0 / 10,0	66,0 / 10,0	58,0 / 10,0	50,0 / 10,0	42,0 / 10,0	34,0 / 10,0	26,0 / 10,0	18,0 / 10,0	10,0 / 10,0	2,0 / 10,0
9	78,0 / 6,0	69,5 / 5,5	61,0 / 5,0	52,5 / 4,5	44,0 / 4,0	35,5 / 3;5	27,0 / 3,0	18,5 / 2,5	10,0 / 2,0	1,5 / 1,5
10	82,0 / 2,0	73,0 / 1,0	64,0 / 0,0	55,0 / -1,0	46,0 / -2,0	37,0 / -3,0	28,0 / -4,0	19,0 / -5,0	10,0 / -6,0	1,0 / -7,0
11	86,0 / -2,0	76,5 / -3,5	67,0 / -5,0	57,5 / -6,5	48,0 / -8,0	38,5 / -9,5	29,0 / -11,0	19,5 / -12,5	10,0 / -14,0	0,5 / -15,5
12	90,0 / -6,0	80,0 / -8,0	70,0 / -10,0	60,0 / -12,0	50,0 / -14,0	40,0 / -16,0	30,0 / -18,0	20,0 / -20,0	10,0 / -22,0	0,0 / -24,0
13	94,0 / -10,0	83,5 / -12,5	73,0 / -15,0	62,5 / -17,5	52,0 / -20,0	41,5 / -22,5	31,0 / -25,0	20,5 / -27,5	10,0 / -30,0	-0,5 / -32,5
14	98,0 / -14,0	87,0 / -17,0	76,0 / -20,0	65,0 / -23,0	54,0 / -26,0	43,0 / -29,0	32,0 / -32,0	21,0 / -35,0	10,0 / -38,0	-1,0 / -41,0
15	102,0 / -18,0	90,5 / -21,5	79,0 / -25,0	67,5 / -28,5	56,0 / -32,0	44,5 / -35,5	33,0 / -39,0	21,5 / -42,5	10,0 / -46,0	-1,5 / -49,5
16	106,0 / -22,0	94,0 / -26,0	82,0 / -30,0	70,0 / -34,0	58,0 / -38,0	46,0 / -42,0	34,0 / -46,0	22,0 / -50,0	10,0 / -54,0	-2,0 / -58,0
17	110,0 / -26,0	97,5 / -30,5	85,0 / -35,0	72,5 / -39,5	60,0 / -44,0	47,5 / -48,5	35,0 / -53,0	22,5 / -57,5	10,0 / -62,0	-2,5 / -66,5
18	114,0 / -30,0	101,0 / -35,0	88,0 / -40,0	75,0 / -45,0	62,0 / -50,0	49,0 / -55,0	36,0 / -60,0	23,0 / -65,0	10,0 / -70,0	-3,0 / -75,0
19	118,0 / -34,0	104,5 / -39,5	91,0 / -45,0	77,5 / -50,5	64,0 / -56,0	50,5 / -61,5	37,0 / -67,0	23,5 / -72,5	10,0 / -78,0	-3,5 / -83,5
	0	1	2	3	4	5	6	7	8	9

	10	11	12	13	14	15	16	17	18	19	
0	2,0 / 82,0	-2,0 / 86,0	-6,0 / 90,0	-10,0 / 94,0	-14,0 / 98,0	-18,0 / 102,0	-22,0 / 106,0	-26,0 / 110,0	-30,0 / 114,0	-34,0 / 118,0	0
1	1,0 / 73,0	-3,5 / 76,5	-8,0 / 80,0	-12,5 / 83,5	-17,0 / 87,0	-21,5 / 90,5	-26,0 / 94,0	-30,5 / 97,5	-35,0 / 101,0	-39,5 / 104,5	1
2	0,0 / 64,0	-5,0 / 67,0	-10,0 / 70,0	-15,0 / 73,0	-20,0 / 76,0	-25,0 / 79,0	-30,0 / 82,0	-35,0 / 85,0	-40,0 / 88,0	-45,0 / 91,0	2
3	-1,0 / 55,0	-6,5 / 57,5	-12,0 / 60,0	-17,5 / 62,5	-23,0 / 65,0	-28,5 / 67,5	-34,0 / 70,0	-39,5 / 72,5	-45,0 / 75,0	-50,5 / 77,5	3
4	-2,0 / 46,0	-8,0 / 48,0	-14,0 / 50,0	-20,0 / 52,0	-26,0 / 54,0	-32,0 / 56,0	-38,0 / 58,0	-44,0 / 60,0	-50,0 / 62,0	-56,0 / 64,0	4
5	-3,0 / 37,0	-9,5 / 38,5	-16,0 / 40,0	-22,5 / 41,5	-29,0 / 43,0	-35,5 / 44,5	-42,0 / 46,0	-48,5 / 47,5	-55,0 / 49,0	-61,5 / 50,5	5
6	-4,0 / 28,0	-11,0 / 29,0	-18,0 / 30,0	-25,0 / 31,0	-32,0 / 32,0	-39,0 / 33,0	-46,0 / 34,0	-53,0 / 35,0	-60,0 / 36,0	-67,0 / 37,0	6
7	-5,0 / 19,0	-12,5 / 19,5	-20,0 / 20,0	-27,5 / 20,5	-35,0 / 21,0	-42,5 / 21,5	-50,0 / 22,0	-57,5 / 22,5	-65,0 / 23,0	-72,5 / 23,5	7
8	-6,0 / 10,0	-14,0 / 10,0	-22,0 / 10,0	-30,0 / 10,0	-38,0 / 10,0	-46,0 / 10,0	-54,0 / 10,0	-62,0 / 10,0	-70,0 / 10,0	-78,0 / 10,0	8
9	-7,0 / 1,0	-15,5 / 0,5	-24,0 / 0,0	-32,5 / -0,5	-41,0 / -1,0	-49,5 / -1,5	-58,0 / -2,0	-66,5 / -2,5	-75,0 / -3,0	-83,5 / -3,5	9
10	-8,0 / -8,0	-17,0 / -9,0	-26,0 / -10,0	-35,0 / -11,0	-44,0 / -12,0	-53,0 / -13,0	-62,0 / -14,0	-71,0 / -15,0	-80,0 / -16,0	-89,0 / -17,0	10
11	-9,0 / -17,0	-18,5 / -18,5	-28,0 / -20,0	-37,5 / -21,5	-47,0 / -23,0	-56,5 / -24,5	-66,0 / -26,0	-75,5 / -27,5	-85,0 / -29,0	-94,5 / -30,5	11
12	-10,0 / -26,0	-20,0 / -28,0	-30,0 / -30,0	-40,0 / -32,0	-50,0 / -34,0	-60,0 / -36,0	-70,0 / -38,0	-80,0 / -40,0	-90,0 / -42,0	-100,0 / -44,0	12
13	-11,0 / -35,0	-21,5 / -37,5	-32,0 / -40,0	-42,5 / -42,5	-53,0 / -45,0	-63,5 / -47,5	-74,0 / -50,0	-84,5 / -52,5	-95,0 / -55,0	-105,5 / -57,5	13
14	-12,0 / -44,0	-23,0 / -47,0	-34,0 / -50,0	-45,0 / -53,0	-56,0 / -56,0	-67,0 / -59,0	-78,0 / -62,0	-89,0 / -65,0	-100,0 / -68,0	-111,0 / -71,0	14
15	-13,0 / -53,0	-24,5 / -56,5	-36,0 / -60,0	-47,5 / -63,5	-59,0 / -67,0	-70,5 / -70,5	-82,0 / -74,0	-93,5 / -77,5	-105,0 / -81,0	-116,5 / -84,5	15
16	-14,0 / -62,0	-26,0 / -66,0	-38,0 / -70,0	-50,0 / -74,0	-62,0 / -78,0	-74,0 / -82,0	-86,0 / -86,0	-98,0 / -90,0	-110,0 / -94,0	-122,0 / -98,0	16
17	-15,0 / -71,0	-27,5 / -75,5	-40,0 / -80,0	-52,5 / -84,5	-65,0 / -89,0	-77,5 / -93,5	-90,0 / -98,0	-102,5 / -102,5	-115,0 / -107,0	-127,5 / -111,5	17
18	-16,0 / -80,0	-29,0 / -85,0	-42,0 / -90,0	-55,0 / -95,0	-68,0 / -100,0	-81,0 / -105,0	-94,0 / -110,0	-107,0 / -115,0	-120,0 / -120,0	-133,0 / -125,0	18
19	-17,0 / -89,0	-30,5 / -94,5	-44,0 / -100,0	-57,5 / -105,5	-71,0 / -111,0	-84,5 / -116,5	-98,0 / -122,0	-111,5 / -127,5	-125,0 / -133,0	-138,5 / -138,5	19
	10	11	12	13	14	15	16	17	18	19	

In this bimatrix game both players can select between 20 strategies: the rows and columns denoted as 0,...,19. The payoffs for the strategy combination (row k, column l) are a_{kl} for the row player and b_{kl} for the column player. The three equilibrium points are the two strong equilibrium points 19,0 and 0,19 and the mixed equilibrium point 8,8.

For every value Z ($0 \leq Z \leq 19$) and S ($0 \leq S \leq 19$) we consider the bimatrix game G(Z,S) obtained by restricting the strategies of player 1 to {0,..,Z} and the strategies of player 2 to {0,..,S}. The bimatrix representing the game G(Z,S) is obtained by the reduction of the 20x20 bimatrix to its first Z+1 rows and S+1 columns. This results in a (Z+1)x(S+1) bimatrix. Figure 3.3 shows the result for Z=7 and S=5, the lighted area of the 20x20 bimatrix represents the game G(7,5). It may be remarked that the obtained game is the mixed extension of the 2x2 bimatrix game deduced from the four corners in which the steps in which the strategies can be mixed are restricted.

Figure 3.3: The construction of the game G(7,5) from the 20x20 bimatrix.

	0	1	2	3	4	S=5	6	7	8	9
0	42,0 / 42,0	38,0 / 46,0	34,0 / 50,0	30,0 / 54,0	26,0 / 58,0	22,0 / 62,0	18,0 / 66,0	14,0 / 70,0	10,0 / 74,0	6,0 / 78,0
1	46,0 / 38,0	41,5 / 41,5	37,0 / 45,0	32,5 / 48,5	28,0 / 52,0	23,5 / 55,5	19,0 / 59,0	14,5 / 62,5	10,0 / 66,0	5,5 / 69,5
2	50,0 / 34,0	45,0 / 37,0	40,0 / 40,0	35,0 / 43,0	30,0 / 46,0	25,0 / 49,0	20,0 / 52,0	15,0 / 55,0	10,0 / 58,0	5,0 / 61,0
3	54,0 / 30,0	48,5 / 32,5	43,0 / 35,0	37,5 / 37,5	32,0 / 40,0	26,5 / 42,5	21,0 / 45,0	15,5 / 47,5	10,0 / 50,0	4,5 / 52,5
4	58,0 / 26,0	52,0 / 28,0	46,0 / 30,0	40,0 / 32,0	34,0 / 34,0	28,0 / 36,0	22,0 / 38,0	16,0 / 40,0	10,0 / 42,0	4,0 / 44,0
5	62,0 / 22,0	55,5 / 23,5	49,0 / 25,0	42,5 / 26,5	36,0 / 28,0	29,5 / 29,5	23,0 / 31,0	16,5 / 32,5	10,0 / 34,0	3,5 / 35,5
6	66,0 / 18,0	59,0 / 19,0	52,0 / 20,0	45,0 / 21,0	38,0 / 22,0	31,0 / 23,0	24,0 / 24,0	17,0 / 25,0	10,0 / 26,0	3,0 / 27,0
Z=7 7	70,0 / 14,0	62,5 / 14,5	55,0 / 15,0	47,5 / 15,5	40,0 / 16,0	32,5 / 16,5	25,0 / 17,0	17,5 / 17,5	10,0 / 18,0	2,5 / 18,5
8	74,0 / 10,0	66,0 / 10,0	58,0 / 10,0	50,0 / 10,0	42,0 / 10,0	34,0 / 10,0	26,0 / 10,0	18,0 / 10,0	10,0 / 10,0	2,0 / 10,0
9	78,0 / 6,0	69,5 / 5,5	61,0 / 5,0	52,5 / 4,5	44,0 / 4,0	35,5 / 3,5	27,0 / 3,0	18,5 / 2,5	10,0 / 2,0	1,5 / 1,5
10	82,0 / 2,0	73,0 / 1,0	64,0 / 0,0	55,0 / -1,0	46,0 / -2,0	37,0 / -3,0	28,0 / -4,0	19,0 / -5,0	10,0 / -6,0	1,0 / -7,0
11	86,0 / -2,0	76,5 / -3,5	67,0 / -5,0	57,5 / -6,5	48,0 / -8,0	38,5 / -9,5	29,0 / -11,0	19,5 / -12,5	10,0 / -14,0	0,5 / -15,5
12	90,0 / -6,0	80,0 / -8,0	70,0 / -10,0	60,0 / -12,0	50,0 / -14,0	40,0 / -16,0	30,0 / -18,0	20,0 / -20,0	10,0 / -22,0	0,0 / -24,0
13	94,0 / -10,0	83,5 / -12,5	73,0 / -15,0	62,5 / -17,5	52,0 / -20,0	41,5 / -22,5	31,0 / -25,0	20,5 / -27,5	10,0 / -30,0	-0,5 / -32,5
14	98,0 / -14,0	87,0 / -17,0	76,0 / -20,0	65,0 / -23,0	54,0 / -26,0	43,0 / -29,0	32,0 / -32,0	21,0 / -35,0	10,0 / -38,0	-1,0 / -41,0
15	102,0 / -18,0	90,5 / -21,5	79,0 / -25,0	67,5 / -28,5	56,0 / -32,0	44,5 / -35,5	33,0 / -39,0	21,5 / -42,5	10,0 / -46,0	-1,5 / -49,5
16	106,0 / -22,0	94,0 / -26,0	82,0 / -30,0	70,0 / -34,0	58,0 / -38,0	46,0 / -42,0	34,0 / -46,0	22,0 / -50,0	10,0 / -54,0	-2,0 / -58,0
17	110,0 / -26,0	97,5 / -30,5	85,0 / -35,0	72,5 / -39,5	60,0 / -44,0	47,5 / -48,5	35,0 / -53,0	22,5 / -57,5	10,0 / -62,0	-2,5 / -66,5
18	114,0 / -30,0	101,0 / -35,0	88,0 / -40,0	75,0 / -45,0	62,0 / -50,0	49,0 / -55,0	36,0 / -60,0	23,0 / -65,0	10,0 / -70,0	-3,0 / -75,0
19	118,0 / -34,0	104,5 / -39,5	91,0 / -45,0	77,5 / -50,5	64,0 / -56,0	50,5 / -61,5	37,0 / -67,0	23,5 / -72,5	10,0 / -78,0	-3,5 / -83,5
	0	1	2	3	4	5	6	7	8	9

	10	11	12	13	14	15	16	17	18	19	
0	2,0 / 82,0	-2,0 / 86,0	-6,0 / 90,0	-10,0 / 94,0	-14,0 / 98,0	-18,0 / 102,0	-22,0 / 106,0	-26,0 / 110,0	-30,0 / 114,0	-34,0 / 118,0	0
1	1,0 / 73,0	-3,5 / 76,5	-8,0 / 80,0	-12,5 / 83,5	-17,0 / 87,0	-21,5 / 90,5	-26,0 / 94,0	-30,5 / 97,5	-35,0 / 101,0	-39,5 / 104,5	1
2	0,0 / 64,0	-5,0 / 67,0	-10,0 / 70,0	-15,0 / 73,0	-20,0 / 76,0	-25,0 / 79,0	-30,0 / 82,0	-35,0 / 85,0	-40,0 / 88,0	-45,0 / 91,0	2
3	-1,0 / 55,0	-6,5 / 57,5	-12,0 / 60,0	-17,5 / 62,5	-23,0 / 65,0	-28,5 / 67,5	-34,0 / 70,0	-39,5 / 72,5	-45,0 / 75,0	-50,5 / 77,5	3
4	-2,0 / 46,0	-8,0 / 48,0	-14,0 / 50,0	-20,0 / 52,0	-26,0 / 54,0	-32,0 / 56,0	-38,0 / 58,0	-44,0 / 60,0	-50,0 / 62,0	-56,0 / 64,0	4
5	-3,0 / 37,0	-9,5 / 38,5	-16,0 / 40,0	-22,5 / 41,5	-29,0 / 43,0	-35,5 / 44,5	-42,0 / 46,0	-48,5 / 47,5	-55,0 / 49,0	-61,5 / 50,5	5
6	-4,0 / 28,0	-11,0 / 29,0	-18,0 / 30,0	-25,0 / 31,0	-32,0 / 32,0	-39,0 / 33,0	-46,0 / 34,0	-53,0 / 35,0	-60,0 / 36,0	-67,0 / 37,0	6
Z=7 7	-5,0 / 19,0	-12,5 / 19,5	-20,0 / 20,0	-27,5 / 20,5	-35,0 / 21,0	-42,5 / 21,5	-50,0 / 22,0	-57,5 / 22,5	-65,0 / 23,0	-72,5 / 23,5	7
8	-6,0 / 10,0	-14,0 / 10,0	-22,0 / 10,0	-30,0 / 10,0	-38,0 / 10,0	-46,0 / 10,0	-54,0 / 10,0	-62,0 / 10,0	-70,0 / 10,0	-78,0 / 10,0	8
9	-7,0 / 1,0	-15,5 / 0,5	-24,0 / 0,0	-32,5 / -0,5	-41,0 / -1,0	-49,5 / -1,5	-58,0 / -2,0	-66,5 / -2,5	-75,0 / -3,0	-83,5 / -3,5	9
10	-8,0 / -8,0	-17,0 / -9,0	-26,0 / -10,0	-35,0 / -11,0	-44,0 / -12,0	-53,0 / -13,0	-62,0 / -14,0	-71,0 / -15,0	-80,0 / -16,0	-89,0 / -17,0	10
11	-9,0 / -17,0	-18,5 / -18,5	-28,0 / -20,0	-37,5 / -21,5	-47,0 / -23,0	-56,5 / -24,5	-66,0 / -26,0	-75,5 / -27,5	-85,0 / -29,0	-94,5 / -30,5	11
12	-10,0 / -26,0	-20,0 / -28,0	-30,0 / -30,0	-40,0 / -32,0	-50,0 / -34,0	-60,0 / -36,0	-70,0 / -38,0	-80,0 / -40,0	-90,0 / -42,0	-100,0 / -44,0	12
13	-11,0 / -35,0	-21,5 / -37,5	-32,0 / -40,0	-42,5 / -42,5	-53,0 / -45,0	-63,5 / -47,5	-74,0 / -50,0	-84,5 / -52,5	-95,0 / -55,0	-105,5 / -57,5	13
14	-12,0 / -44,0	-23,0 / -47,0	-34,0 / -50,0	-45,0 / -53,0	-56,0 / -56,0	-67,0 / -59,0	-78,0 / -62,0	-89,0 / -65,0	-100,0 / -68,0	-111,0 / -71,0	14
15	-13,0 / -53,0	-24,5 / -56,5	-36,0 / -60,0	-47,5 / -63,5	-59,0 / -67,0	-70,5 / -70,5	-82,0 / -74,0	-93,5 / -77,5	-105,0 / -81,0	-116,5 / -84,5	15
16	-14,0 / -62,0	-26,0 / -66,0	-38,0 / -70,0	-50,0 / -74,0	-62,0 / -78,0	-74,0 / -82,0	-86,0 / -86,0	-98,0 / -90,0	-110,0 / -94,0	-122,0 / -98,0	16
17	-15,0 / -71,0	-27,5 / -75,5	-40,0 / -80,0	-52,5 / -84,5	-65,0 / -89,0	-77,5 / -93,5	-90,0 / -98,0	-102,5 / -102,5	-115,0 / -107,0	-127,5 / -111,5	17
18	-16,0 / -80,0	-29,0 / -85,0	-42,0 / -90,0	-55,0 / -95,0	-68,0 / -100,0	-81,0 / -105,0	-94,0 / -110,0	-107,0 / -115,0	-120,0 / -120,0	-133,0 / -125,0	18
19	-17,0 / -89,0	-30,5 / -94,5	-44,0 / -100,0	-57,5 / -105,5	-71,0 / -111,0	-84,5 / -116,5	-98,0 / -122,0	-111,5 / -127,5	-125,0 / -133,0	-138,5 / -138,5	19
	10	11	12	13	14	15	16	17	18	19	

399 different games G(Z,S) can be constructed by this procedure. These games have different strategic structures:

Type 1: The games G(Z,S) with ($0 \leq Z \leq 8$, $0 \leq S \leq 8$) are of the type of the prisoner's dilemma.

Type 2: The games G(Z,S) with ($8 < Z \leq 19$, $0 \leq S \leq 8$) have one equilibrium point, (Z,0). The games G(Z,S) with ($0 < Z \leq 8$, $8 < S \leq 19$) have one equilibrium point, (0,S).

Type 3: The games G(Z,S) with ($8 < Z \leq 19$, $8 < S \leq 19$) have two strong equilibrium points, namely (Z,0) and (0,S), and one equilibrium point in mixed strategies, namely 8,8. The conflict payoff is given by Z,S.

Every game G(Z,S) is obtained from the game G(S,Z) by exchanging the players.

3.1. The payoffs

The marginal worth of 1 point of the matrices was 0.5 DM (~ \$0.35). The payoff of a player was determined as difference of the obtained result to the mean result of the other players in the same position in the same game (only those games were considered in this mean of the others in which the player did not participate in any position). One of the 399 games was selected by chance and paid. Losses up to 100 DM had to be payed by the subjects. If the losses were higher than 100 DM the subjects could choose whether to pay or to work for 15 DM per hour.

3.2. The subjects

The subjects were 13 students of economics and business administration after their "Vordiplom".

3.3. The communication

The communication was free and anonymous via computer terminals. The subjects could also fill their planned strategy choices into a 20x20 matrix. For every game

G(Z,S) the strategy choice (i.e. the selected strategy (i.e. the selected row or column)) was written into the position (Z,S) of the matrix. This matrix was submitted anonymously to the other player

3.4. The experimental procedure

The players were matched in pairs. Each pair played the complete set of 399 games. Such a set of 399 games played by two players is denoted as a "399 fold strategy game". A set of games was played according to the following 4 stages:

1) [4] Free communication via computer terminals and exchange of the planned strategies.
2) Simultaneous announcement and exchange of the planned strategies for all games.
3) Simultaneous selection and exchange of the finally selected strategies for all 399 games.
4) A random selection of one of the 399 games. The payoffs were according to the strategies selected for this game.

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[4] The announcements in stage 1 and 2 were not binding.

4. The predictions of the models

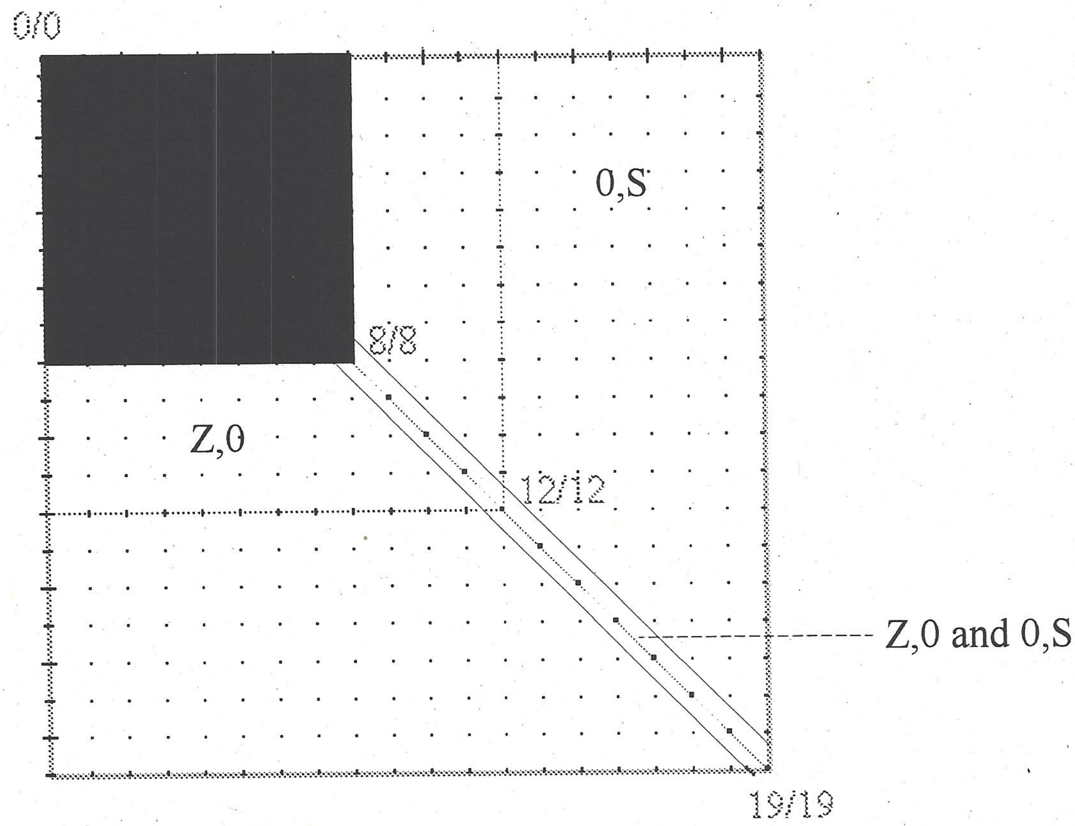
The predictions of the different models are described by means of a scheme like in figure 4.1. Every field of the matrix corresponds to a game G(Z,S). The prediction for a game G(Z,S) is given in the corresponding field (Z,S) by the row and column numbers selected by the model. Games with identical predictions are presented as an area in the matrix.

For games with one equilibrium point (type 2) this equilibrium point is predicted by all models.

4.1. The predictions of the Nash-Zeuthen model

Figure 4.1 shows the predictions of the Nash-Zeuthen model (shortly the Nash model) applied to the selection between any two equilibrium points. For games G(Z,S) with Z>S the equilibrium point Z,0 is selected by this model and for games G(Z,S) with Z<S the equilibrium point 0,S is selected. For games G(Z,S) with Z=S the equilibrium points Z,0 and 0,S are selected. The equilibrium in mixed strategies 8,8 is not selected in any case.

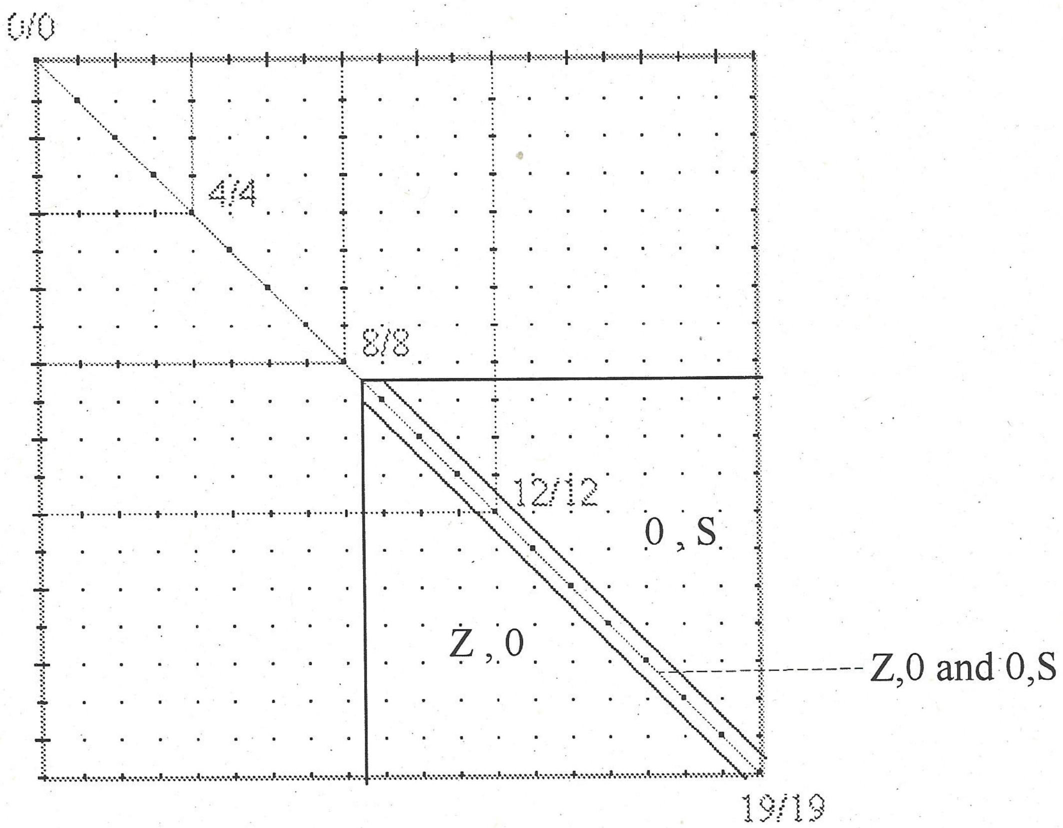
Figure 4.1: The predictions of the Nash-Zeuthen model



4.2 The predictions of the risk dominance

The predictions of the risk dominance are shown in figure 4.2. They are the same as the ones of the Nash-Zeuthen model. This model does not select the equilibrium in mixed strategies 8,8 in any case.

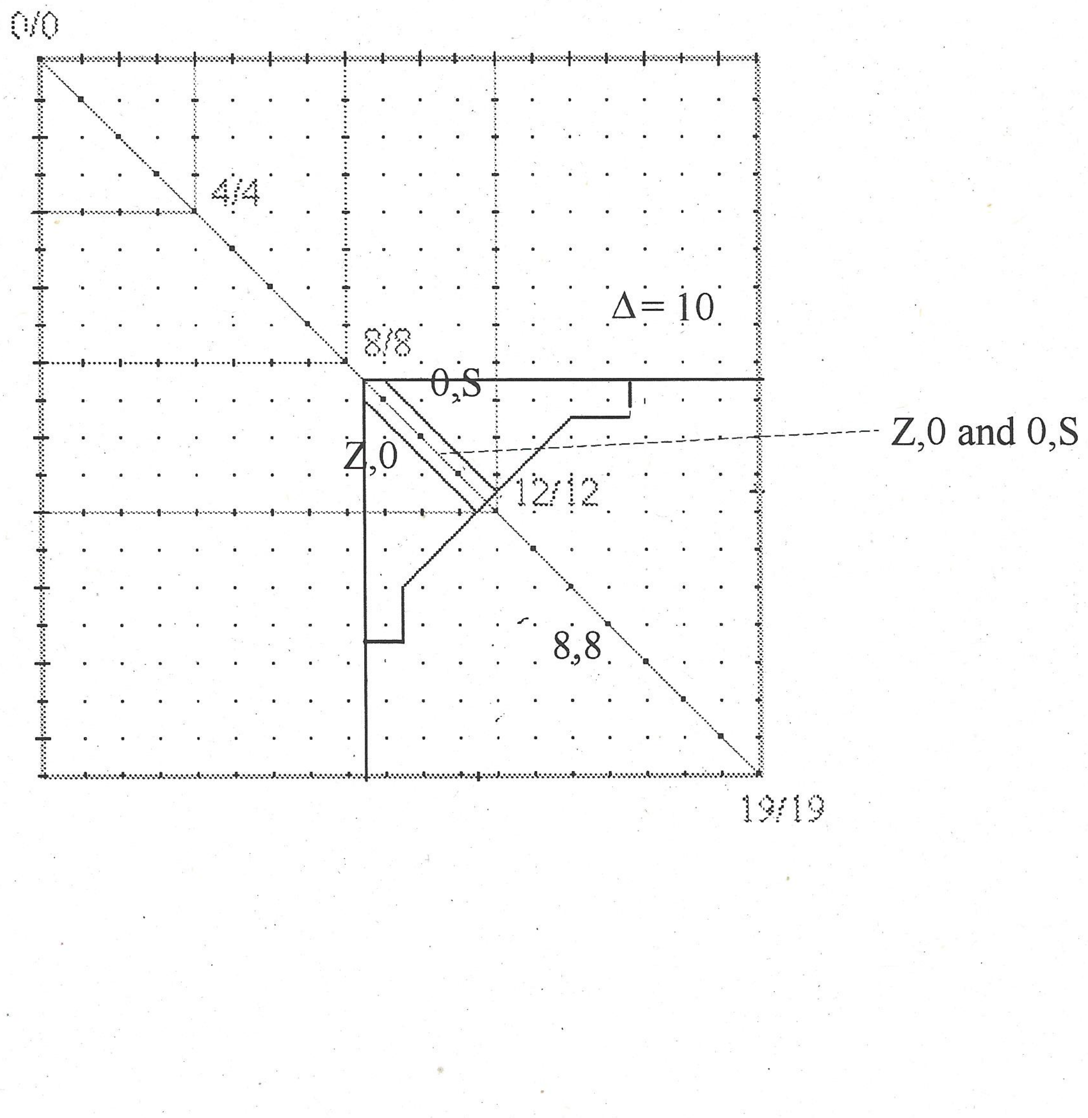
Figure 4.2: The predictions of the risk dominance



4.3. The predictions of model I

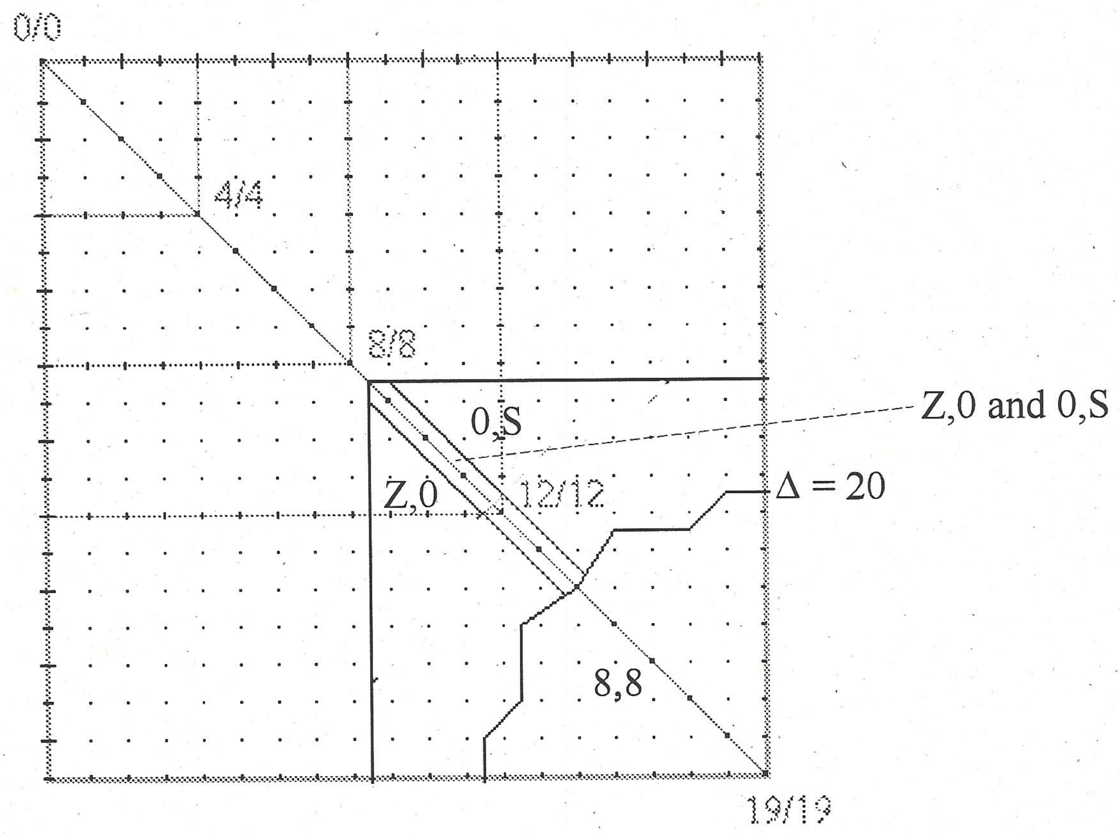
Figures 4.3a and 4.3b give the predictions of model I for the smallest units of Δ=10 and Δ=20, respectively.[5] In contrast to the Nash-Zeuthen model and the risk dominance the mixed equilibrium point 8,8 is selected for some games. Depending on the smallest unit one obtains different predictions for 48 of the 121 games with 3 equilibrium points.

Figure 4.3a: The predictions of model I for a smallest unit of Δ=10



[5]The smallest units of Δ=10 and Δ=20 are chosen for the predictions of the models using prominence theory according to the rules for the smallest unit used for the evaluation of a prospect (W. Albers, 1997). The smallest unit is between 10% and 20% of the maximal payoff. If one takes the prominent numbers near to 10% or 20% of the maximal payoff of this bimatrix this results in the smallest units of Δ=10 and Δ=20.

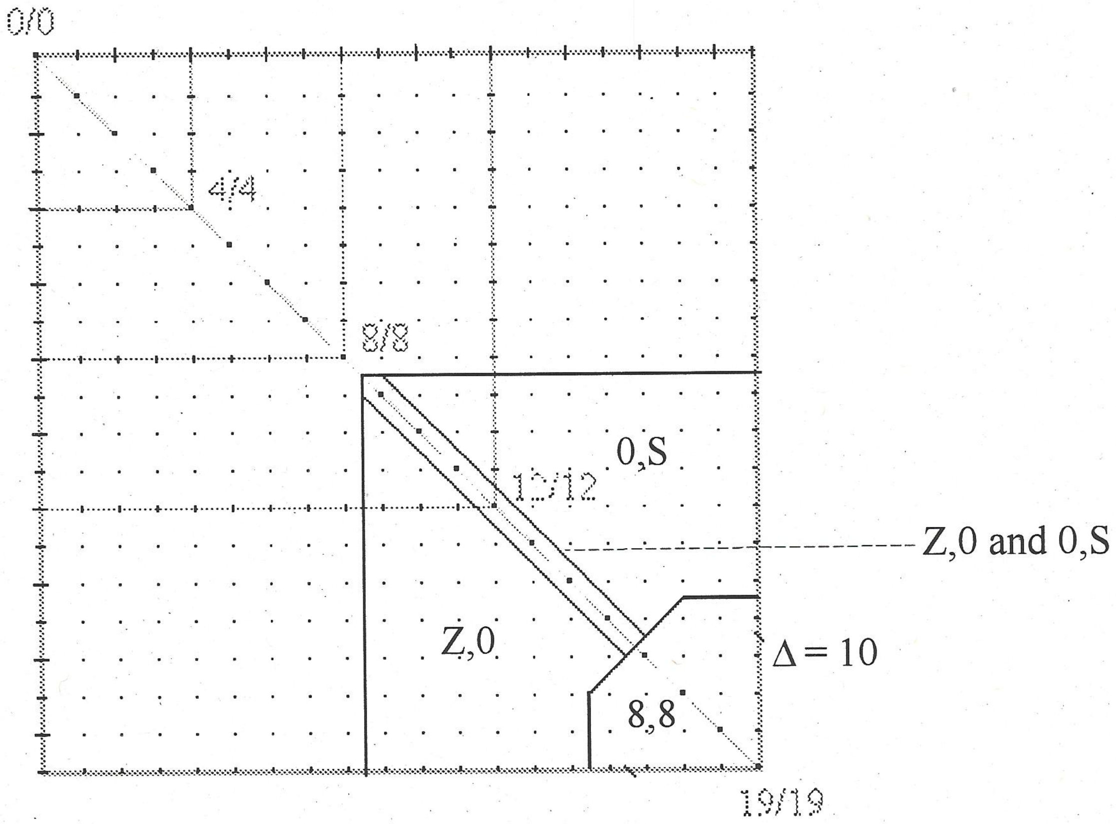
Figure 4.3b: The predictions of model I for a smallest unit of Δ=20



4.4. The predictions of model II

Figure 4.4 presents the predictions of model II for a smallest unit of Δ=10.

Figure 4.4: The predictions of model II for a smallest unit of Δ=10

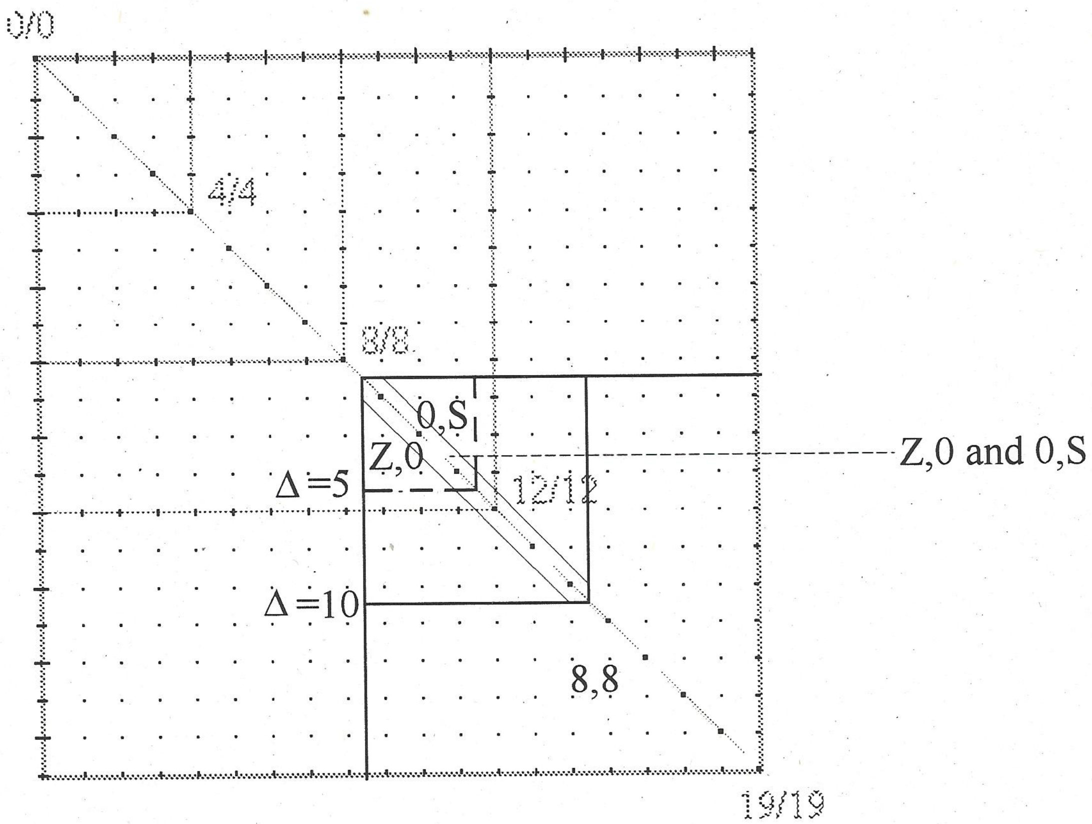


For the games considered and a smallest unit of Δ=20 the predictions of this model are the same as the predictions of the risk dominance .

4.4.1 The predictions of model I and II without threats

Model I and model II permit to select strategies without using threats. In this case the equilibrium point with the highest "Nash-sum" is selected (compare part 2.4.1). Whether in certain games threats are used or not is discussed in the analysis of these games. The prediction for this case is given in figure 4.5.

Figure 4.5: The predictions of model I and model II without threats for smallest units of Δ=5 and Δ=10



In these cases 8,8 is selected for most of the games with 3 equilibrium points.

For a smallest unit of 20 the equilibrium Z,0 [0,S][6] is selected for all games.

[6]In the following parts the predictions for symmetric games are given in brackets [ ].

5. Experimental results

In this section the predictions of the models are tested by means of the experimental data. A binomial-test is used. The hypothesis:" the prediction is wrong." is rejected on a 5% level. Because the payoffs of some games are very similar only 16 games of the games with 3 equilibrium points are used as data for the experimental test. These games are given in figure 5.1 with the results of these games as the experimental data.

5.1. The test of the predictions of the models

Figure 5.1 shows a comparison of the predictions with the experimental data. G-1 to G-6 denote the 6 different 399 fold strategy games of the 20x20 bimatrix. For each of the 399 fold strategy games the hypothesis " the prediction is wrong" is rejected for one 399 fold strategy game if the predictions are correct for 12 or more games of the 16. The total number and the total percentage of correct predictions are given in the last columns of the figure.

Except for the 399 fold strategy game G-4 the predictions of the Nash model do not agree[7] with the results. In 2 of 6 of the 399 fold strategy games an agreement between the predicitions and the experimental data is not even obtained for at least one game. The total percentage of correct predictions is below 50%. The predictions of the Nash model show the worst agreement with the experimental data among these models.

For these games the predictions of the risk dominance are the same as those of the Nash model.

The predictions of model I with a smallest money unit of 20 coincide with the data of the 399 fold strategy games G-1, G-3 and G-6. The predictions of model I with a smallest unit of 10 coincide with the experimental data of the 399 fold strategy games G-2 and G-5. No agreement is obtained for the 399 fold strategy game G-4. Permitting

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[7]In this section it will be denoted as agreement with the experimental data if 12 or more correct predictions occur.

Figure 5.1: The result of the test of the predictions of the models

i,j of the G(i,j)

399 fold strategy games:	9/9	9/12	9/16	9/19	12/9	12/12	12/16	12/19	16/9	16/12	16/16	16/19	19/9	19/12	19/16	19/19
G-1	G	R	R	R	R	G	R	R	R	R	G	G	R	R	G	G
G-2	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G
G-3	G	R	R	R	R	G	R	R	R	R	G	G	R	R	G	G
G-4	G	R	R	R	R	G	R	R	R	R	G	R	R	R	R	G
G-5	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G
G-6	G	R	R	R	R	G	R	R	R	R	G	G	R	R	G	G

The number of correct predictions

Models:	9/9	9/12	9/16	9/19	12/9	12/12	12/16	12/19	16/9	16/12	16/16	16/19	19/9	19/12	19/16	19/19	G-1	G-2	G-3	G-4	G-5	G-6	total	%
Nash	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	10	0	10	12	0	10	42	44
Risk dominance	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	10	0	10	12	0	10	42	44
Model I, smallest unit: 10	R	R	G	G	R	G	G	G	G	G	G	G	G	G	G	G	7	13	7	5	13	7	52	54
Model I, smallest unit: 20	R	R	R	R	R	R	R	G	R	R	G	G	R	G	G	G	12	6	12	10	6	12	58	60
Model II, smallest unit: 10	R	R	R	R	R	R	R	R	R	R	G	G	R	R	G	G	14	4	14	12	4	14	62	65
Model II, smallest unit: 20	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	10	0	10	12	0	10	42	44
Model I+II, no threats, Δ=10	R	R	G	G	R	R	G	G	G	G	G	G	G	G	G	G	6	12	6	4	12	6	46	48
Model II, 2-cases, Δ=10																	14	12	14	12	12	14	78	81

R - Equilibrium point in pure strategies is selected

G - Equilibrium point in mixed strategies is selected

- "The model is not rejected".

solutions of the case without using threats alternatively does not improve the result of the test of model I. The total percentage of agreements is 75% (72 of 96 cases).

The predictions of model II with a smallest unit of 10 coincide with the data for the 399 fold strategy games G-1, G-3, G-4 and G-6. Permitting solutions without using threats alternatively (model II, 2 cases) leads to the result that the 399 fold strategy games G-2 and G-5 agree also with the model. Reasons that support the assumption that players did not use threats in the 399 fold strategy games G-2 and G-5 are given in the next section.

Permitting solutions without using threats alternatively (model II, 2 cases) and using a smallest money unit of 10 is in agreement with all 6 strategy games. The total percentage of cases in which the predictions and the experimental data agree is 81%. No agreement is obtained for symmetric games and for some games of the 399 fold strategy games G-2, G-4 and G-5. This will be discussed in the next section.

A result of the test is that the predictions of the models using the theory of prominence are in better agreement with the data than risk dominance and the Nash model. One reason of this is that the equilibrium point in mixed strategies is selected very often in the experiment .

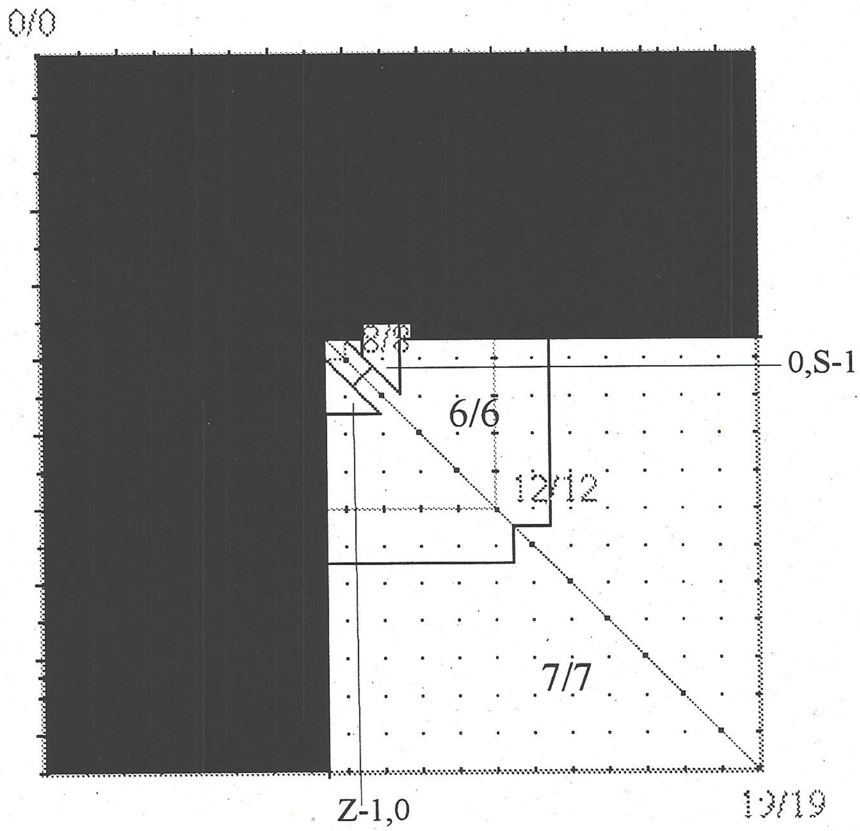
5.2. Discussion of the results

First it will be discussed whether an agreement without threats was obtained in the 399 fold strategy games G-2 and G-5. In these 399 fold strategy games the equilibria in pure strategies Z,0 and 0,S are not selected in any case. Figure 5.2 shows the result of G-5 which is typical for this behavior[8]. In the games with three equilibrium points either the strategy combination 7,7 is selected instead of the mixed equilibrium 8,8, namely for the games G(Z,S) with Z>13 or S>13 (and (G(13,13)) or the strategy combination 6,6 is selected, namely for the games G(Z,S) with Z≤13 and S≤13 (except

[8]The result of G-2 differs only slightly from the one of G-5

of G(13,13), G(9,8) and G(8,9)). These are not equilibrium points[9]. These strategy choices can be explained within a model using the theory of prominence with a smallest unit of 10 and in which threats are not used. The selected strategy combinations correspond to payoffs with maximal "Nash-sum". For 7,7 the payoffs are (17.5, 17.5) and for 6,6 the payoffs are (24,24). The "Nash-sum" is 3.5 for 7,7 and it is >4.25 for 6,6. A comparison with the payoffs for Z,0 [0,S] shows that the corresponding "Nash-sums" are smaller than 4.25 for the games G(Z,S) with Z<13 or S<13 and that the corresponding "Nash-sums" are smaller than 3.5 for the other games. The selection of 7,7 and 6,6 follows from this as in the experiment if the criterion of a maximal "Nash-sum" is used.

Figure 5.2: The strategies selected in the 399 fold strategy game G-5.



[9]In the test these selections are used as a selection of 8,8, because these strategy selections are deviations from 8,8.

The strategy combinations 6,6 and 7,7 are not equilibrium points. Both players get higher payoffs for these combinations than in the equilibrium point 8,8. If one player would deviate he would increase his payoff, but no player deviates from his strategies[10]. This supports again that threats are not used in these 399 fold strategy games.

A separation of the games of the 20x20 bimatrix in symmetric games G(Z,Z) which correspond to the diagonal of the 20x20 bimatrix and non symmetric games G(Z,S) with $Z \neq S$ leads to the following results for the other 399 fold strategy games G-1, G-3, G-4 and G-6.

In the 399 fold strategy game G-4 the result of the non symmetric games is that Z,0 or S,0 are selected which are the equilibria in pure strategies. This corresponds to the predictions of the Nash model, the risk dominance and model II with a smallest unit of 20.

The experimental results of the non symmetric games of the 399 fold strategy games G-1, G-3 and G-6 agree with the predictions of model II with a smallest unit of 10 for all games.

For all 399 fold strategy games the result of the symmetric games of the 20x20 bimatrix is that 8,8 (or 7,7 or 6,6) is selected and not Z,0 or 0,S. One explanation of this fact might be that it is easier to obtain an agreement for symmetric games. Another explanation could be that the players were undecided between Z,0 and 0,S and therefore selected 8,8 (8,8 was also selected in the 399 fold strategy game G-4 in which for non symmetric games only Z,0 and 0,S were selected).

A result of this discussion is that the predictions of model II with 2 cases (with and without threats) show the best agreement with the data. If the symmetric games which have a special structure are excluded model II predicts correctly the outcomes of all games.

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[10]The stability of these combinations seems also be due to the fact that the incentive to deviate is small (normally < 5 which is less than $\Delta/2$).

5.3. The games for which the subjects were indifferent whether to play the equilibrium in pure strategies or the mixed equilibrium

After the experiment the subjects were asked for which of the games they would change their equilibrium selection. The games for which they decided to play 8,8 instead of Z,0 [0,S] if threats were used are given in figure 5.3. The answers were given for the games G(13,S) and G(19,S) (with $8 \leq S \leq Z$; these are in row 13 and 19 of the bimatrix).

Figure 5.3: Change of the equilibrium selection between the equilibria in pure strategies Z,0 [0,S] and the equilibrium in mixed strategies 8,8.

row	column of the equilibrium change	number of subjects
13	12 , 13	13 (all)
19	12 , 13	2
19	13 , 14	0
19	14 , 15	5
19	15 , 16	2
19	16 , 17	2
19	17 , 18	2
19	18 , 19	0

For all games G(13,S) all players preferred to play Z,0 up to S=12.

The results for the games G(19,S) are given in figure 5.3. The median of the S-values where the subjects changed their selected equilibrium was between S=14 and S=15.

A comparison with the predictions of the models leads to the following result. The Nash model and the risk dominance do always predict Z,0. No agreement between prediction and data is obtained.

For a smallest unit of 10 model I predicts a change in the equilibrium selected between S=8 and S=9 (compare figure 4.3) or a change between S=11 and S=12 (for a smallest unit of 20). 2 subjects give a response of S=12/S=13. This is near to the prediction of model I with a smallest unit of 10. The agreement between data and predictions is rather bad.

The prediction of model II (with a smallest unit of 10) are given in figure 4.4. The figure shows a change between S=14 and S=15. This is the same as the median of the responses. 5 subjects give the response S=14/S=15. 2 subjects answer S=15/S=16 and two more subjects answer S=16/S=17 which is near to the prediction of model II. The difference to the prediction might be due to rounding in the numerical perception, i.e. the perception of 118 might be different for these subjects (varying from $P_{10}(118)=4$ to $P_{10}(118)=4.25$). Permitting all these possibilities of perception would include also the responses of these additional 4 subjects.

6. Conclusions

In this paper the selection of mixed strategies in 2x2 bimatrix games is analysed. The risk dominance and the Nash-criterion applied to the selection between equilibrium points predict only the selection of pure strategy equilibria. Additional models using the theory of prominence (model I and model II) predict also the selection of the mixed equilibrium for certain games. In these models an agreement without using threats is possible. The prediction for this case is the selection of the mixed equilibrium for most of the games played.

An experimental test of the predictions was performed by using the strategy method. 2 subjects selected their strategies for the 399 mixed extensions of the 2x2 bimatrix games in one 399 fold strategy game. The basic game used for the construction of the 399 games was the mixed extension of a symmetric 2x2 bimatrix game with 20 possible mixed strategies of each player. By reducing the strategies of each player to 19,18,..,1 and combining these strategies with all reduced strategies of the other player new mixed extensions of 2x2 bimatrix games were constructed. The players played all of these games simultaneously.

A result of the experiment is that the equilibrium in mixed strategies is selected in certain games. Model II based on the theory of prominence and related to risk dominance shows the best agreement with the data. Playing the mixed equilibrium for most of the games was also observed. This behavior can be explained by a model based on the theory of prominence in which threats are not used.

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