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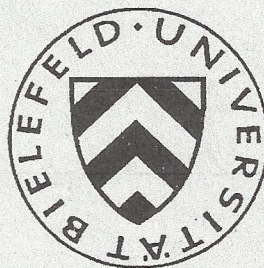
No. 272

## Selection between pareto-optimal outcomes in 2-person bargaining

by

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### **Abstract**

Models describing the selection between pareto-optimal outcomes in 2 person bargaining with different status quo points are the Nash-criterion (J.F. Nash 1950 and 1953), the Kalai-Smorodinky criterion (E. Kalai, M. Smorodinskiy 1975 and E. Kalai 1977), equal absolute payoff above the status quo point and equal absolute payoff below the Bliss point. A new model based on the theory of prominence (W. Albers, G. Albers 1983, W. Albers 1997) which describes the perception of numbers (especially payoffs) and related to the Kalai-Smorodinky criterion is presented. Experiments using the strategy method were performed to test the predictions of the different models. Results of these experiments are that the model based on prominence theory is a better predictor for the data than the other models and its predictions are not rejected by the experimental data.

# 1 Introduction

A classical bargaining problem is the 2-person bargaining about payoffs  $(a, b) \in S$  (the payoff of player 1 is denoted by  $a$  and  $b$  denotes the one of player 2,  $S \subset \mathbf{R}$  is a non-empty closed convex set). It is modeled by Nash (J.F. Nash 1950 and 1953) and an axiomatic solution is given. Other possible solutions of this problem are equal payoffs for both persons above a status quo point or equal (absolute or relative (E. Kalai, M. Smorodinsky 1975, E. Kalai 1977)) payoffs below the Bliss point. In addition to these models a new model (model I) is given in this paper. This model uses the Kalai-Smorodinsky approach (E. Kalai, M. Smorodinsky 1975; E. Kalai 1977), but models the perception of numerical outcomes according to the theory of prominence (W. Albers, G. Albers 1983, W. Albers 1997).

To test the predictions of the models experiments using the strategy method were performed. This will be described in the third part of the paper. In these experiments communication was possible.

The selection problem examined in this paper can be described by means of figure 1. The convex set  $S$  is characterized by the following conditions for the payoff  $(a, b)$ :

$$(a_{min}, b_{min}) \leq (a, b) \leq (a_{max}, b_{max})$$

and

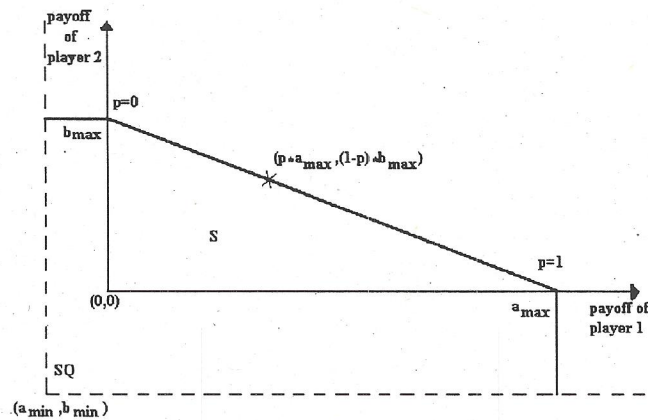
$$\frac{a}{a_{max}} + \frac{b}{b_{max}} \leq 1$$

with constants

$$a_{max}, b_{max}, a_{min}, b_{min} \in \mathbf{R} \text{ and } a_{max} > 0, b_{max} > 0, a_{max} > a_{min}, b_{max} > b_{min}$$

The payoffs on the pareto-line (with  $(a, b) \geq (0, 0)$ ) can be characterized by a parameter  $p$  ( $0 \leq p \leq 1$ ). The payoffs on the pareto-line are  $(a, b) = (p \cdot a_{max}, (1-p) \cdot b_{max})$ .

Figure 1: The considered 2-person bargaining situation

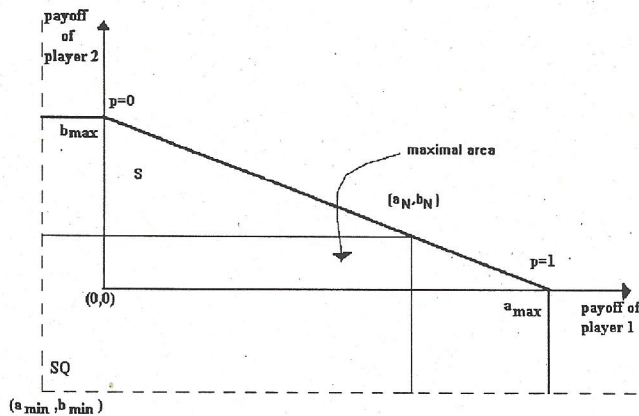


## 2 Models for the selection between pareto-optimal outcomes

### 2.1 Maximal Nash-product

J.F. Nash (J.F. Nash 1950 and 1953) predicts that the payoff vector  $(a_N, b_N)$  is selected for which  $(a - a_{min}) * (b - b_{min})$  is maximal. For the examined problem the solution can be obtained by determining analytically the parameter  $p$  for which the area  $(p * a_{max} - a_{min}) * ((1 - p) * b_{max} - b_{min})$  is maximal (compare figure 2).

Figure 2: Maximal Nash-product



It is  $p_N = (1 + \frac{a_{min}}{a_{max}} - \frac{b_{min}}{b_{max}}) * \frac{1}{2}$ <sup>1</sup>. The corresponding payoffs are:

$$\text{Nash: } (a_N, b_N) = ((1 + \frac{a_{min}}{a_{max}} - \frac{b_{min}}{b_{max}}) * \frac{a_{max}}{2}, (1 - \frac{a_{min}}{a_{max}} + \frac{b_{min}}{b_{max}}) * \frac{b_{max}}{2})$$

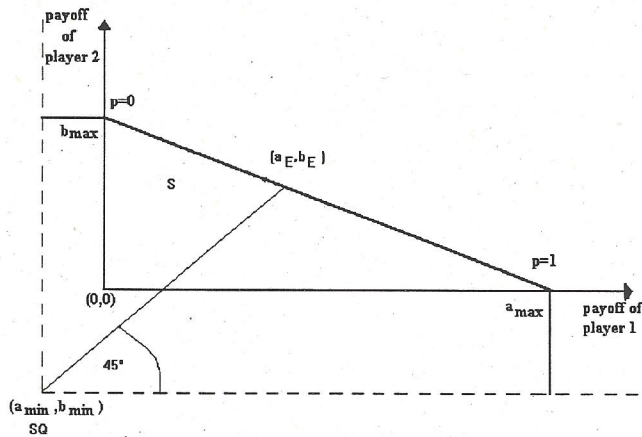
<sup>1</sup>The correct solution is  $p' = \min(1, \max(0, p))$ , where  $p$  denotes the  $p$  given in the formula. For reason of a better reading only the value of  $p$  is given for all models.



## 2.2 Equal payoffs above the status quo

Equal payoffs above the status quo point  $(a_E, b_E)$  are determined by the intersection of the pareto-line with the line through  $(a_{min}, b_{min})$  with gradient 1 (compare figure 3).

Figure 3: Equal payoffs above the status quo point



This approach gives  $p_E = \left( \frac{b_{max} + a_{min} - b_{min}}{a_{max} + b_{max}} \right)$ . The corresponding payoffs are:

Equal improvement:

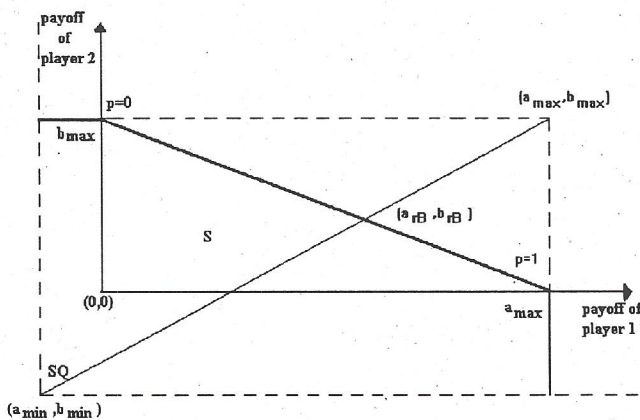
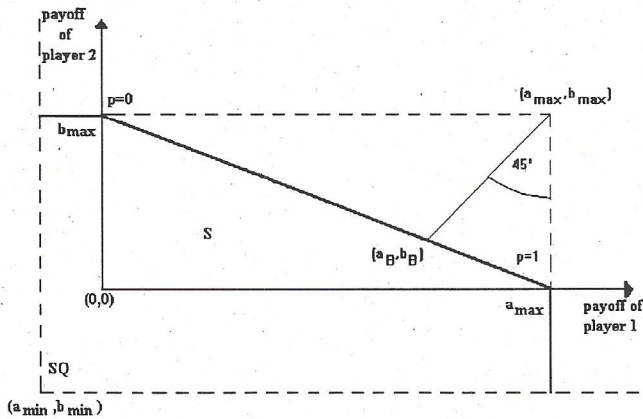
$$(a_E, b_E) = \left( \left( \frac{b_{max} + a_{min} - b_{min}}{a_{max} + b_{max}} \right) * a_{max}, \left( \frac{a_{max} + b_{min} - a_{min}}{a_{max} + b_{max}} \right) * b_{max} \right)$$



### 2.3 Equal payoffs below the Bliss point

Equal payoffs below the Bliss point  $(a_{max}, b_{max})$  are determined as shown in figure 4.

Figure 4: Equal payoffs below the Bliss point: upper part: absolute  $(a_B, b_B)$ ; lower part: relative  $(a_{rB}, b_{rB})$



Equal absolute payoffs below the Bliss point  $(a_B, b_B)$  are determined by the intersection of the pareto-line with the line through  $(a_{max}, b_{max})$  with gradient 1 (compare figure 4).

This gives  $p_B = \left(\frac{a_{max}}{a_{max} + b_{max}}\right)$ . The corresponding payoffs for equal absolute payoffs below the Bliss point are:

$$(a_B, b_B) = \left(\left(\frac{a_{max}}{a_{max} + b_{max}}\right) * a_{max}, \left(\frac{b_{max}}{a_{max} + b_{max}}\right) * b_{max}\right).$$

Equal relative payoffs below the Bliss point  $(a_{rB}, b_{rB})$  are predicted by the Kalai-Smorodinsky model (E. Kalai, M. Smorodinsky 1975, E. Kalai 1977)). The payoffs are determined by



the intersection of the pareto-line with the line through  $(a_{max}, b_{max})$  and  $(a_{min}, b_{min})$  (compare figure 4). For this point it holds:  $\frac{a_{max}-a_{rB}}{a_{rB}-a_{min}} = \frac{b_{max}-b_{rB}}{b_{rB}-b_{min}}$ .

This results in  $p = \frac{(b_{max}-b_{min}) * a_{max}}{(b_{max}-b_{min}) * a_{max} + (a_{max}-a_{min}) * b_{max}}$ .

The corresponding payoffs for the Kalai-Smorodinsky model are:

$$(a_{rB}, b_{rB}) = \left( \frac{(b_{max}-b_{min}) * a_{max}}{(b_{max}-b_{min}) * a_{max} + (a_{max}-a_{min}) * b_{max}} * a_{max}, \frac{(a_{max}-a_{min}) * b_{max}}{(b_{max}-b_{min}) * a_{max} + (a_{max}-a_{min}) * b_{max}} * b_{max} \right).$$

## 2.4 A Model using the theory of prominence

### 2.4.1 The theory of prominence

The theory of prominence (W. Albers, G. Albers 1983, W. Albers 1997) describes the structure and perception of numbers in the decimal system. In this part the perception function  $P_{\Delta}$  which is a result of the theory of prominence is described. It is needed to describe the perception of payoffs in model I. - A short introduction into the theory of prominence is given in this part. A foundation of the theory of prominence is given in (W. Albers 1997).

Prominent numbers are:

$$P = \{z * 10^n | n \in \mathbf{Z}, z \in \{1, 2, 5\}\} = \{\dots, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, \dots\}.$$

The  $P_{\Delta}$ -function ( $\Delta \in P$ ) which is the perception-function is defined on the prominent numbers:

- (1)  $P_{\Delta}: P \rightarrow \mathbf{R}$ .
- (2)  $P_{\Delta}(0)=0$ .
- (3)  $P_{\Delta}(\Delta)=1$ .
- (4)  $P_{\Delta}(q)=P_{\Delta}(p)+1$  for 2 prominent numbers  $p < q$  with  $p, q \geq \Delta$  and  $p$  and  $q$  such that no prominent number  $r$  exists with  $p < r < q$ .
- (5)  $P_{\Delta}(-|x|)=-P_{\Delta}(|x|)$  for  $|x| \in P$ .

Interpretation: Differences between prominent numbers are perceived as constant steps (1 step).  $P_{\Delta}(0)$  is defined as  $P_{\Delta}(0)=0$ . Assuming that  $\Delta$  is the smallest perceived prominent number one gets  $P_{\Delta}(\Delta)=1$ . The values of negative numbers  $x$  are the negative values of the corresponding positive numbers  $|x|$ . For  $|x| > \Delta$  it holds  $|\frac{P_{\Delta}(x)}{3 * \log_{10}(\frac{x}{\Delta}) + 1} - 1| \leq 7\%$ .

For positive stimuli this is similar to the Weber-Fechner law of psychophysics (G.T. Fechner 1968) that describes for example the perception of optical and acoustical signals. Stimuli below a certain intensity are not perceived. Above this value the perceived intensity of a signal  $i$  is a logarithmic function of the intensity of the original signal  $s$ :  $i = const.1 * \log(s) + const.2$ .

The  $P_{\Delta}$ -function is also defined for other sets of numbers called the spontaneous (S) and the exact numbers (E) which are refinements of the prominent numbers by introducing half and quarter steps, i.e. one additional spontaneous number between any two neighbored prominent numbers and one additional exact number between any two neighbored spontaneous numbers (for details see the sets  $M(1, \Delta)$ ,  $M(2, \Delta)$  in W. Albers 1997, part I). The  $P_{\Delta}$ -function is given in table 1 for the prominent, the spontaneous and the exact numbers



in the range that is relevant for the experiment (the  $P_{\Delta}$ -values of negative numbers are the negative  $P_{\Delta}$ -values of the corresponding positive numbers). It may be noted that the smallest perceived unit  $\Delta$  is determined on the full step level and thereby can only be a prominent number.

Table 1: The  $P_{\Delta}$ -values for  $\Delta=10$  and  $\Delta=20$  for the exact numbers (quarter steps) between 0 and 100.

|                     |   |      |     |      |    |       |     |       |   |      |     |      |   |      |     |      |   |
|---------------------|---|------|-----|------|----|-------|-----|-------|---|------|-----|------|---|------|-----|------|---|
| $\Delta = 10$       |   |      |     |      |    |       |     |       |   |      |     |      |   |      |     |      |   |
| full steps          | 0 |      |     |      | 10 |       |     |       |   | 20   |     |      |   | 50   |     | 100  |   |
| half steps          |   |      | 5   |      |    |       |     | 15    |   |      |     | 30   |   |      | 70  |      |   |
| quarter steps       |   | 2/3  |     | 7/8  |    | 12/13 |     | 17/18 |   | 25   |     | 40   |   | 60   |     | 80   |   |
| $P_{\Delta}$ -value | 0 | 0.25 | 0.5 | 0.75 | 1  | 1.25  | 1.5 | 1.75  | 2 | 2.25 | 2.5 | 2.75 | 3 | 3.25 | 3.5 | 3.75 | 4 |
| $\Delta = 20$       |   |      |     |      |    |       |     |       |   |      |     |      |   |      |     |      |   |
| full steps          | 0 |      |     |      | 20 |       |     |       |   | 50   |     |      |   | 100  |     |      |   |
| half steps          |   |      | 10  |      |    |       |     | 30    |   |      |     | 70   |   |      |     |      |   |
| quarter steps       |   | 5    |     | 15   |    | 25    |     | 40    |   | 60   |     | 80   |   |      |     |      |   |
| $P_{\Delta}$ -value | 0 | 0.25 | 0.5 | 0.75 | 1  | 1.25  | 1.5 | 1.75  | 2 | 2.25 | 2.5 | 2.75 | 3 |      |     |      |   |

In the experimental data the smallest unit has to be determined. This is performed by means of a rule of the theory of prominence. Before formulating the rule some definitions are given below (compare W. Albers 1997).

A presentation of a number is its presentation as a sum of prominent numbers, where each prominent number occurs at most once, and all coefficients are either +1, -1, or 0. The exactness of a presentation is the smallest prominent number with coefficient unequal zero. The exactness of a number is the crudest exactness over all presentations of the number. The relative exactness of a number  $x$  is the exactness divided by  $|x|$ . A number has level of [relative] exactness  $r$ , if its [relative] exactness is cruder or equal to  $r$ . A set of data has [level of] relative exactness  $r$ , if at least 75% of the data have this [level of] relative exactness<sup>2</sup>. - A scale  $S(r,a)$  is the set of 0 and all numbers with (1) relative exactness  $\geq r$  and (2) exactness  $\geq a$ .

Rule for the scale  $S(r,a)$  and the smallest unit  $\Delta$  of a set of data:

A set of data is on the scale  $S(r,a)$  if the set of data has [level of] relative exactness of  $r$  and at least 90% of the data have an exactness of  $a$ <sup>3</sup>. The smallest unit of a set of data is:

- $\Delta=a$  for the scales  $S(100,a)$  ("scales with prominent numbers"),
- $\Delta=[2*a]$  for the scales  $S(26,a)$  ("scales with spontaneous numbers"),
- $\Delta=[4*a]$  for the scales  $S(100,a)$  ("scales with exact numbers")

<sup>2</sup> $r$  has to be maximal

<sup>3</sup> $a$  has to be maximal.



### 2.4.2 Model I

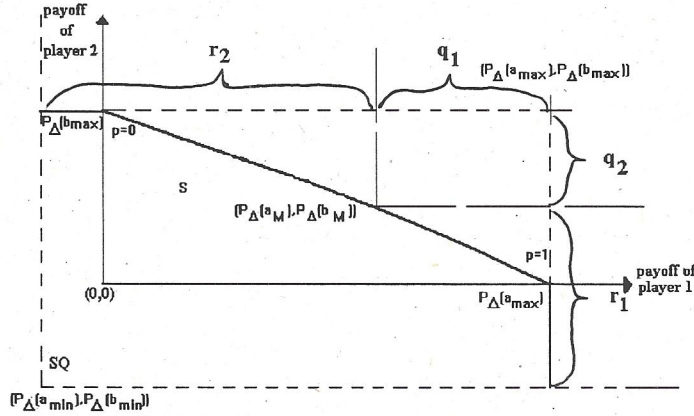
All payoffs are transformed by the  $P_{\Delta}$ -function:

$P_{\Delta} : a_{..} \rightarrow P_{\Delta}(a_{..})$  and  $P_{\Delta} : b_{..} \rightarrow P_{\Delta}(b_{..})$ .

The selection criterion for the payoff  $(a_M, b_M)$  is obtained by a comparison of concessions.

This is shown schematically in figure 5.

Figure 5: Schematic presentation of the comparisons in model I



In this model the difference  $q_1 = P_{\Delta}(a_{max}) - P_{\Delta}(a_M)$  is the concession of player 1 and  $q_2 = P_{\Delta}(b_{max}) - P_{\Delta}(b_M)$  is the concession of player 2 relative to the Bliss point. These differences indicate how much both players have lowered their demands. The other concessions are given by  $r_1 = P_{\Delta}(b_M) - P_{\Delta}(b_{min})$  which is a concession of player 1 giving player 2 a certain payoff above the conflict payoff of player 2 and  $r_2 = P_{\Delta}(a_M) - P_{\Delta}(a_{min})$  which is a concession of player 2 giving player 1 a certain payoff above the conflict payoff of player 1. The selection criterion is that the sum of concession of both players have to be equal:

The pareto-optimal payoff  $(a_M, b_M)$  is selected, for which  $q_1 + r_1 = q_2 + r_2$ .

Replacing  $q$  and  $r$  results in:

The pareto-optimal payoff  $(a_M, b_M)$  is selected, for which

$$(P_{\Delta}(a_{max}) - P_{\Delta}(a_M)) + (P_{\Delta}(b_M) - P_{\Delta}(b_{min})) = (P_{\Delta}(b_{max}) - P_{\Delta}(b_M)) + (P_{\Delta}(a_M) - P_{\Delta}(a_{min})) \text{ or}$$

The pareto-optimal payoff  $(a_M, b_M)$  is selected, for which

$$(P_{\Delta}(a_{max}) - P_{\Delta}(a_M)) - (P_{\Delta}(a_M) - P_{\Delta}(a_{min})) = (P_{\Delta}(b_{max}) - P_{\Delta}(b_M)) - (P_{\Delta}(b_M) - P_{\Delta}(b_{min})).$$

A different interpretation is obtained by measuring the concessions on each dimension separately, i.e.  $q_1 - r_2 = q_2 - r_1$ . Here  $q_1 - r_2$  can be interpreted as the "value of concessions of player 1", where  $q_1$  gives the concessions below the Bliss point, while  $r_2$  gives



the concessions not made and is therefore evaluated with a negative sign. The analogous result is obtained for player 2.

### 2.4.3 Relation between model I and the Kalai-Smorodinsky model

For a comparison of the two models the criteria of model I and the Kalai-Smorodinsky model (K-S) are written below one another.

$$\begin{aligned} \text{K-S :} & \quad (a_{max} - a_{rB}) / (a_{rB} - a_{min}) = \\ \text{K-S:} & \quad (b_{max} - b_{rB}) / (b_{rB} - b_{min}) \\ \text{Model I:} & \quad (P_{\Delta}(a_{max}) - P_{\Delta}(a_M)) - (P_{\Delta}(a_M) - P_{\Delta}(a_{min})) = \\ \text{Model I:} & \quad (P_{\Delta}(b_{max}) - P_{\Delta}(b_M)) - (P_{\Delta}(b_M) - P_{\Delta}(b_{min})) \end{aligned}$$

The differences between these two models are that in model I the payoffs are transformed by the  $P_{\Delta}$ -function and that after the transformation quotients in the Kalai-Smorodinsky criterion correspond to differences in model I. This is compatible with the fact that after a transformation by the  $P_{\Delta}$ -function (which is for values higher than  $\Delta$  proportional to a "logarithmic function") the quotients are transformed into differences.

Another difference is that the  $P_{\Delta}$ -function operates on the payoffs and not on differences of payoffs. For example the difference  $(a_{rB} - a_{min})$  is transformed into  $P_{\Delta}(a_M) - P_{\Delta}(a_{min})$  and not into  $P_{\Delta}(a_{rB} - a_{min})$ . An explanation for this is that the strategic equivalence does not hold for the perceived payoffs, it must be replaced by another form of equivalence for perceived payoffs (B. Vogt 1997).

## 3 The Experiment

In the experiment the bargaining game discussed above was played for several values of  $a_{max}$ ,  $a_{min}$ ,  $b_{max}$  and  $b_{min}$ . Figure 6 shows the values of these parameters.

In each game the players bargained about the outcome of the game by means of a parameter  $\mathbf{p}$  ( $0 \leq \mathbf{p} \leq 100$ ). The payoffs corresponding to  $\mathbf{p}$  were  $(\mathbf{p}/100) * a_{max}$  for player 1 and  $((100 - \mathbf{p})/100) * b_{max}$  for player 2. By doing this every point on the pareto-line with payoffs higher or equal to 0 for both players can be the outcome of the game.

The players gave their strategies in the role of player 1 as a proposal  $\mathbf{p}'_1$  and in the role of player 2 as a proposal  $\mathbf{p}'_2$ . Given the proposals  $\mathbf{p}'_1$  and  $\mathbf{p}'_2$  the payoff was the mean of the proposals, if  $\mathbf{p}'_1 \leq \mathbf{p}'_2$ , and it was the status quo payoff, if  $\mathbf{p}'_1 > \mathbf{p}'_2$ .

Figure 6: Values of the parameters in the different types of games

| game | $a_{\max}$ | $a_{\min}$ | $b_{\max}$ | $b_{\min}$ |
|------|------------|------------|------------|------------|
| 1    | 100        | -100       | 10         | 0          |
| 2    | 100        | -40        | 10         | 0          |
| 3    | 100        | -20        | 10         | 0          |
| 4    | 100        | -10        | 10         | 0          |
| 5    | 100        | 0          | 10         | 0          |
| 6    | 100        | 0          | 10         | -10        |
| 7    | 100        | 0          | 10         | -20        |
| 8    | 100        | 0          | 10         | -40        |
| 9    | 100        | -100       | 20         | 0          |
| 10   | 100        | -40        | 20         | 0          |
| 11   | 100        | -20        | 20         | 0          |
| 12   | 100        | -10        | 20         | 0          |
| 13   | 100        | 0          | 20         | 0          |
| 14   | 100        | 0          | 20         | -10        |
| 15   | 100        | 0          | 20         | -20        |
| 16   | 100        | 0          | 20         | -40        |
| 17   | 100        | -100       | 40         | 0          |
| 18   | 100        | -40        | 40         | 0          |
| 19   | 100        | -20        | 40         | 0          |
| 20   | 100        | -10        | 40         | 0          |
| 21   | 100        | 0          | 40         | 0          |
| 22   | 100        | 0          | 40         | -10        |
| 23   | 100        | 0          | 40         | -20        |
| 24   | 100        | 0          | 40         | -40        |
| 25   | 100        | -100       | 100        | 0          |
| 26   | 100        | -40        | 100        | 0          |
| 27   | 100        | -20        | 100        | 0          |
| 28   | 100        | -10        | 100        | 0          |
| 29   | 100        | 0          | 100        | 0          |
| 30   | 100        | 0          | 100        | -10        |
| 31   | 100        | 0          | 100        | -20        |
| 32   | 100        | 0          | 100        | -40        |

### 3.1 The payoffs

The worth of 1 point was 2 DM ( $\sim$  \$1.4). For every game the payoff of a player was the difference to the mean value of the other players in his group having the same role (player 1 or 2, only games in which the player did not participate were considered for the calculation of the mean value). Losses up to 100 DM ( $\sim$  \$70) had to be payed by the subjects. For the losses above 100 DM the subjects could choose whether to pay or to work at 15 DM ( $\sim$  \$ 10) per hour.



### 3.2 The subjects

The subjects were 32 students of economics and business administration after their "Vordiplom".

### 3.3 Communication

Free communication via terminals was possible.

### 3.4 The experimental procedure

The experiment was divided into two parts (Part A and B).

In Part A the subjects were subdivided into 4 groups of 8 subjects. Within every group the players were matched in pairs, they were rematched in every round. Every pair played one of the 32 games (shown in figure 6). Every game was played in the following stages:

1. Free communication via terminals.
2. Simultaneous announcement (and exchange) of the planned strategies.
3. Simultaneous selection (and exchange) of the strategies.

In part B all games were played by means of the strategy method. The subjects had to select their strategies for all 32 games and for both roles (player 1 or 2) without knowing the other player or having communicated with another player. Three games per group were paid. The players and games were chosen by chance.

## 4 Predictions

For the predictions of the model based on the theory of prominence the smallest unit  $\Delta$  has to be known. It can be determined from the p-values given by the subjects by means of the theory of prominence by the rule described in section 2.4.1. After determining the smallest unit  $\Delta$  the predictions of model I are determined. The predictions for the models described in part 2 are given in figure 7 .

Figure 7: Predicted values of p (in %) by the different solution concepts.

| game | Nash | Equal payoff<br>above the SQ | Equal payoff | below Bliss | Model I |
|------|------|------------------------------|--------------|-------------|---------|
|      |      |                              | (absolute)   | (relative)  |         |
| 1    | 0    | 0                            | 90.91        | 33.33       | 5       |
| 2    | 30   | 0                            | 90.91        | 41.67       | 7       |
| 3    | 40   | 0                            | 90.91        | 45.45       | 10      |
| 4    | 45   | 0                            | 90.91        | 47.62       | 17      |
| 5    | 50   | 9.09                         | 90.91        | 50          | 25      |
| 6    | 100  | 18.18                        | 90.91        | 66.67       | 40      |
| 7    | 100  | 27.27                        | 90.91        | 75          | 50      |
| 8    | 100  | 45.45                        | 90.91        | 83.33       | 60      |
| 9    | 0    | 0                            | 83.33        | 33.33       | 7       |
| 10   | 30   | 0                            | 83.33        | 41.67       | 10      |
| 11   | 40   | 0                            | 83.33        | 45.45       | 12      |
| 12   | 45   | 8.33                         | 83.33        | 47.62       | 17      |
| 13   | 50   | 16.67                        | 83.33        | 50          | 30      |
| 14   | 75   | 25                           | 83.33        | 60          | 40      |
| 15   | 100  | 33.33                        | 83.33        | 66.67       | 50      |
| 16   | 100  | 50                           | 83.33        | 75          | 70      |
| 17   | 0    | 0                            | 71.43        | 33.33       | 8       |
| 18   | 30   | 0                            | 71.43        | 41.67       | 17      |
| 19   | 40   | 14.29                        | 71.43        | 45.45       | 25      |
| 20   | 45   | 21.43                        | 71.43        | 47.62       | 30      |
| 21   | 50   | 28.57                        | 71.43        | 50          | 40      |
| 22   | 62.5 | 35.71                        | 71.43        | 55.56       | 50      |
| 23   | 75   | 42.86                        | 71.43        | 60          | 60      |
| 24   | 100  | 57.14                        | 71.43        | 66.67       | 75      |
| 25   | 0    | 0                            | 50           | 33.33       | 12      |
| 26   | 30   | 30                           | 50           | 41.67       | 17      |
| 27   | 40   | 40                           | 50           | 45.45       | 30      |
| 28   | 45   | 45                           | 50           | 47.62       | 40      |
| 29   | 50   | 50                           | 50           | 50          | 50      |
| 30   | 55   | 55                           | 50           | 52.38       | 60      |
| 31   | 60   | 60                           | 50           | 54.55       | 70      |
| 32   | 70   | 70                           | 50           | 58.33       | 80      |

## 5 Results and discussion

### 5.1 Test of the predictions of the models

The predictions of the models given in figure 7 are tested by means of a binomial-test. The test was performed for two assumptions. In one approach it is assumed that there are only 4 groups of 8 subjects that are independent (assumption A) and in the other



approach it is assumed that the p-values of all subjects are independent (assumption B). The medians of the 4 groups given in figure 8 are the data for the test assuming only the independence of the 4 groups.

Figure 8: Medians of the values  $p^* = (p_1^* + p_2^*)/2$  responded for all groups and games.

| game | group 1 | group 2 | group 3 | group 4 |
|------|---------|---------|---------|---------|
| 1    | 5.25    | 5       | 0       | 0.5     |
| 2    | 13.75   | 13.75   | 1.5     | 3.75    |
| 3    | 20      | 21.25   | 3.75    | 6       |
| 4    | 25      | 27.5    | 7.5     | 12.75   |
| 5    | 40      | 42.5    | 9.5     | 21.25   |
| 6    | 52.5    | 56.25   | 25      | 47.75   |
| 7    | 58.75   | 65      | 46.25   | 72.75   |
| 8    | 60      | 81.25   | 84.5    | 89.25   |
| 9    | 7.5     | 7.5     | 0.75    | 1.75    |
| 10   | 15      | 15      | 2.75    | 7       |
| 11   | 22.5    | 25      | 7       | 11.75   |
| 12   | 30      | 32.5    | 11.75   | 16.75   |
| 13   | 43.75   | 42.5    | 19.75   | 33.25   |
| 14   | 53.75   | 52.5    | 31.75   | 55      |
| 15   | 63.75   | 63.75   | 50      | 71.25   |
| 16   | 70      | 78.75   | 89      | 89.75   |
| 17   | 11.25   | 11.25   | 2.5     | 2.5     |
| 18   | 20      | 20      | 5       | 11.25   |
| 19   | 26.25   | 27.5    | 12.5    | 18.75   |
| 20   | 31.25   | 36.25   | 23      | 25.5    |
| 21   | 41.25   | 45      | 30      | 39.25   |
| 22   | 52.5    | 51.25   | 43.75   | 52.5    |
| 23   | 63.75   | 58.75   | 70      | 66.25   |
| 24   | 77.5    | 71.25   | 91.75   | 88      |
| 25   | 16.25   | 22.25   | 3.75    | 5       |
| 26   | 18.75   | 23.75   | 18      | 15      |
| 27   | 26.25   | 32.5    | 34.5    | 22.5    |
| 28   | 37.5    | 42.5    | 42.5    | 39      |
| 29   | 50      | 50      | 50      | 50      |
| 30   | 60      | 60      | 60      | 61.25   |
| 31   | 67.5    | 70      | 67.25   | 70      |
| 32   | 76.25   | 78.75   | 83.25   | 86.25   |

The results of the test are given in figure 9. In the figure it is indicated by the symbol "–" if the hypothesis "the prediction of the model is true" is rejected by the data of a game. A "+" indicates that the hypothesis is not rejected by the data.

Figure 9: Results of the test (assumption of 4 independent groups).

| game | Nash | Equal payoff above the SQ | Equal payoff below Bliss |            | Model I |
|------|------|---------------------------|--------------------------|------------|---------|
|      |      |                           | (absolute)               | (relative) |         |
| 1    | +    | +                         | -                        | -          | +       |
| 2    | -    | -                         | -                        | -          | +       |
| 3    | -    | -                         | -                        | -          | +       |
| 4    | -    | -                         | -                        | -          | +       |
| 5    | -    | -                         | -                        | -          | +       |
| 6    | -    | -                         | -                        | -          | +       |
| 7    | -    | -                         | -                        | -          | +       |
| 8    | -    | -                         | -                        | +          | +       |
| 9    | -    | -                         | -                        | -          | +       |
| 10   | -    | -                         | -                        | -          | +       |
| 11   | -    | -                         | -                        | -          | +       |
| 12   | -    | -                         | -                        | -          | +       |
| 13   | -    | -                         | -                        | -          | +       |
| 14   | -    | -                         | -                        | -          | +       |
| 15   | -    | -                         | -                        | +          | +       |
| 16   | -    | -                         | +                        | +          | +       |
| 17   | -    | -                         | -                        | -          | +       |
| 18   | -    | -                         | -                        | -          | +       |
| 19   | -    | +                         | -                        | -          | +       |
| 20   | -    | -                         | -                        | -          | +       |
| 21   | -    | -                         | -                        | -          | +       |
| 22   | -    | -                         | -                        | -          | +       |
| 23   | -    | -                         | -                        | +          | +       |
| 24   | -    | -                         | +                        | -          | +       |
| 25   | -    | -                         | -                        | -          | +       |
| 26   | -    | -                         | -                        | -          | +       |
| 27   | -    | -                         | -                        | -          | +       |
| 28   | -    | -                         | -                        | -          | +       |
| 29   | +    | +                         | +                        | +          | +       |
| 30   | -    | -                         | -                        | -          | +       |
| 31   | -    | -                         | -                        | -          | +       |
| 32   | -    | -                         | -                        | -          | +       |

$$\alpha = 12.5\%$$

The predictions of the Nash-criterion are rejected in 30 of 32 cases, the predictions of the models "equal absolute payoff below Bliss" and "equal absolute payoff above the SQ" are rejected in 29 of 32 cases and show the worst agreement with the data. The predictions of the Kalai-Smorodinsky model ("equal relative payoff below Bliss") are rejected in 27 of 32 cases and show bad agreement with the data.

Model I shows clearly the best agreement with the data. The predictions are rejected in 0 of 32 cases.



The results of the test assuming that all responded p-values are independent are given in figure10.

Figure 10: Results of the test (assumption B).

| game | Nash | Equal payoff<br>above the SQ | Equal payoff | below Bliss | Model I |
|------|------|------------------------------|--------------|-------------|---------|
|      |      |                              | (absolute)   | (relative)  |         |
| 1    | -    | -                            | -            | -           | +       |
| 2    | -    | -                            | -            | -           | +       |
| 3    | -    | -                            | -            | -           | +       |
| 4    | -    | -                            | -            | -           | +       |
| 5    | -    | -                            | -            | -           | +       |
| 6    | -    | -                            | -            | -           | +       |
| 7    | -    | -                            | -            | -           | +       |
| 8    | -    | -                            | -            | +           | +       |
| 9    | -    | -                            | -            | -           | +       |
| 10   | -    | -                            | -            | -           | +       |
| 11   | -    | -                            | -            | -           | +       |
| 12   | -    | -                            | -            | -           | +       |
| 13   | -    | -                            | -            | -           | +       |
| 14   | -    | -                            | -            | +           | +       |
| 15   | -    | -                            | -            | +           | +       |
| 16   | -    | -                            | +            | +           | +       |
| 17   | -    | -                            | -            | -           | +       |
| 18   | -    | -                            | -            | -           | +       |
| 19   | -    | -                            | -            | -           | +       |
| 20   | -    | -                            | -            | -           | +       |
| 21   | -    | -                            | -            | -           | +       |
| 22   | -    | -                            | -            | +           | +       |
| 23   | -    | -                            | +            | +           | +       |
| 24   | -    | -                            | +            | -           | +       |
| 25   | -    | -                            | -            | -           | +       |
| 26   | -    | -                            | -            | -           | +       |
| 27   | -    | -                            | -            | -           | +       |
| 28   | -    | -                            | -            | -           | +       |
| 29   | +    | +                            | +            | +           | +       |
| 30   | -    | -                            | -            | -           | +       |
| 31   | -    | -                            | -            | -           | +       |
| 32   | -    | -                            | -            | -           | +       |

$$\alpha = 5\%$$

The result of the test is for almost all games the same as for the first assumption. These results indicate that the models that are not based on the theory of prominence do not give good predictors for the data while the model based on the theory of prominence predicts the data surprisingly well.

## 6 Conclusions

In this paper the selection between pareto-optimal outcomes in 2-person bargaining was analysed. Besides existing concepts as the Nash-criterion (J.F. Nash 1950 and 1953), equal payoff above the status quo point and equal (absolute and relative (E. Kalai, M. Smorodinsky 1975, E. Kalai 1977)) payoff below the Bliss point a new model (model I) based on the theory of prominence (W. Albers 1997) is introduced. It is obtained by the idea of an equal sum of concessions of both players and is related to the Kalai-Smorodinsky model.

The results of the experiment conducted by means of the strategy method with 32 subjects do not show good agreement with the predictions of the Nash-criterion, equal payoff above the status quo point and equal (absolute and relative) payoff below the Bliss point. The best predictor is given by model I.

The idea of an equal sum of concessions assuming that the perception of numbers is described by the theory of prominence yields the best result. The main difference between model I and the Kalai-Smorodinsky model is that the linear perception of numbers has to be replaced in model I because of the perception of payoffs according to the theory of prominence.



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