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Evaluation of Lotteries with Two Alternatives by the Theory of Prominence
A Normative Benchmark of Risk Neutrality that Predicts Median Behavior of Subjects

by

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Abstract

The purpose of this paper is, to show that the theory of prominence is an adequate tool to make predictions for subjects' money equivalents of lotteries of type $[x(p), y(q)]$. The evaluation principle is similar to that of KAHNEMAN and TVERSKY (1989 and 1992), however the evaluation functions for money and probabilities are not obtained by econometric curve fitting, but by behavioral principles described by the theory of prominence. Moreover, money and probability space are clearly distinguished from the perception space, specificly the result of the evaluation in the perception space is remapped to the money space to obtain the money equivalent.

The surprising result is that a parameterfree boundedly rational model using the insights of the theory of prominence concerning the evaluation of numerical stimuli (in the decimal system) adequately describes median behavior of subjects. The model is very simple, has no parameters that have to be adjusted. It seems that this instrument can be used as a benchmark which predicts how median subjects behave, i.e. how subjects behave, who have – compared with other subjects – no special motives to take or to avoid risk. From this point of view the model defines a benchmark of 'risk neutral' behavior, where preferences are only induced by the principles of perception as defined by the theory of prominence.[1]

[1] I thank Bodo Vogt for many help for discussions and him, Ralf Sievert, Andreas Uphaus, and Axel Wöller for their assistance to run the experiments.

1 Introduction

It is the intention of this paper to extend the theory of prominence in a way that it can be applied for the evaluation of prospects.

Problem 1 – the 'pure' effect of numerical perception: Our aim was to separate what may be called 'pure effect of numerical perception' from other emotional influences. From pre-experiments we knew that subjects show a more risk loving behavior for low amounts of money, and a risk averse behavior for very high amounts. Our impression was that subjects' behavior (students) is nearest to the ideal pattern we want to model for money amounts that are not too low (over DM 500), and not too high (below DM 100000). The region roughly conforms with that selected by KAHNEMAN and TVERSKY in their studies (1989, 1992).

Problem 2 – selection of numbers involved in the tasks: The theory of prominence models perception in steps and assumes linear interpolation between steps. The evaluation of a prospect as $[-1000(90\%), +5000(10\%)]$ needs to map the money amounts -1000, and 5000 into the perception space, form a kind of weighted mean between the two, where the 'emotional' weights of 90% and 10% have to be determined, and then to give a response on the money scale that creates the same value in the perception space. This is not an easy task. It is solved by try and error with different tentative responses, for each of which the subject checks whether it is possible to interprete the response as her correct answer to the problem. Such an evaluation task (in which we asked for a lower and upper bound of equivalence) usually took the subjects several minutes, the time could not be reduced by experience. Our impression is that numbers are perceived via step-wise refinement, as for example $7000 = 5000 + 2000$, $6800 = 5000 + 2000 - 200$. If the task involves such unround numbers, it creates the additional problem to perceive the unround numbers by interpolation, rounding, or other methods. To avoid the additional problem to map unround numbers into the perception space, and also to avoid noise that might be created when some subjects do not interpolate linearly, or round or truncate, we restricted the analysis to (positive and negative) full step numbers. For probabilies we selected the values 1%, 10%, 20% and 50%, and the corresponding counterprobabilities.

Main problem was the identification of a rule that predicts the finest perceived full steps on the money scale (FPFM), and on the probability scale (FPFP) as a function of the given prospect. From pre-studies we knew that FPFM would be mostly at about '2 full steps below the largest absolute amount of the payoffs involved in the task'. Pre-studies showed also that there seemed to be special cases where a '3 steps' rule performed better. In these pre-studies we did not pick up the whole range of indifference, but only asked for one response per prospect, which may have been on different ends of the indifference range, depending on the situation. For instance it may be more on the side of 'buying' (= low value), if a person spontaneously does not want to have the lottery, since she puts herself into the position of having the money, and it may be on the side of 'selling' (= high value) if the subject prefers to have the lottery, since she puts herself in the position of the seller. (We did so in the second part of the first version of ALBERS.1997.V.) We therefore asked the extreme cases at both ends in this study. The result is that we only need one prediction, the 20% rule. (The evaluations of the money scale conform with the 'normal case' of the other study. The evaluations of the probability scale also do not need any subrules, as the preceding approach needed for some 80% subcases.) The obtained result of this study are very simple rules to determine FPFM, and FPFP.

Our impression is that the obtained result can be interpreted as a benchmark of risk neutral behavior. The interesting point is that this benchmark can be produced by a parameter free boundedly rational behavioral model based on the theory of prominence.

2 The Questionnaire

20 subjects (students of business administration and economics) were asked for their money equivalents for lotteries $[x(p), y(q)]$ with

> payoffs $(x, y) = (10,5), (10,0), (10,1), (10,.5); (10,-10), (10,-5), (10,-1), (10,-.5); (-10,5), (-10,1), (-10,.5); (-10,-5), (-10,-1), (-10,-.5)$; (all values in Thousand DM), and $(1000,500)$,$(1000,0)$,$(1000,-1000)$,$(1000,-500)$,$(-1000,500)$, $(-1000,-500)$.
>
> probabilities $(p, q) = (90\%, 10\%)$,$(10\%, 90\%)$,$(50\%, 50\%)$,$(80\%, 20\%)$,$(20\%, 80\%)$, $(99\%, 1\%)$,$(1\%, 99\%)$.

The lotteries were presented on different sheets of paper, where every sheet contained questions with identical probabilities, in the order given above. On every sheet, the questions were aggregated in groups of four or three according to the first number x, and the sign of the second number y (see the semicolons in the presentation of the values). The seperation was done by vertical lines. The subjects were asked to answer every question isolatedly, not to care for consistency of their responses, and to interrupt shortly at each of the vertical lines.

For every lottery, the upper and lower money equivalent were asked by the following 'indifference method':

> Imagine, the lottery ticket is in some distance in front of you on a table (the experimentor takes a piece of paper and puts it on his desk in front of the class). On the other side of the table is a file of money, or – in case of negative payoffs – a debenture. Go through different money amounts, and decide whether you prefer to get the money (debenture)or if you prefer to get the lottery ticket. Please give the lowest amount (a), and the highest amount (b), for which you are indifferent. – To be indifferent means that you leave the decision which of the alternatives is taken to somebody else. To be sure that you are really indifferent, imagine that there is a bad fairy, who can look into your heart, and will select that alternative which you do not prefer to be selected.

As the Becker-de Groot-Marschak procedure, this question addresses money that the subjects get from the decision maker. Different from that procedure, we obtain two values, concerning the two borders of a range of indifference. Experiments comparing the two procedures showed that some subjects answer the lower, others the upper bound of the range of indifference. The main reason why we selected our procedure was that it can be used to evaluate lotteries with positive and negative values. The betting mechanism of the BdGM procedure would make it necessary to endow the subjects with money so that they can loose money in the lottery. But we strongly suggest that external endowments may shift the anchor point of the considerations.

According to the description, we asked for 22 x 7 = 154 lotteries and thereby received 308 responses. The subjects needed between 4 and 5 ours to give their answers. They were motivated to give correct responses, since they afterwards wanted to investigate on the obtained data themselves within the seminar.

After the subjects had answered the questionnaire, it could be observed that several of them underestimated the small probability 1%. Some of them even neglected payoffs that occured with low probabilities. Therefore we confronted them the next day with the offer to play the lottery [1.000(99%), −10.000(1%)] at the price they had filled into the questionnaire. And we spoke about possibilities to illustrate 1% by its expectation to select one out of 100 persons, which were about 4 times as many persons that were at that moment present in their room. Thereafter we again raised the questions with 99% and 1% probabilities.

3 Predictions by the Theory of Prominence

The general idea of perception of numbers is that differences of numbers are perceived via a full step scale $S(1, FPF)$[1] where the distance of any two neighboured full steps of the scale is 1, and FPF is the finest perceived full step. From this scale the perception function $per(..|FPF) : R \longrightarrow R$ is obtained by defining $per(0|FPF) = 0$, measuring the distances of full step numbers in steps, and by linear interpolation between full step numbers. (Example: For the $FPF = 10$ the scale $S(1.FPF)$ is given by $\ldots, -100, -50, -20, -10, 0, 10, 20, 50, 100, \ldots$, and for $x = 60$, one obtains $f(x|10) = 3.2$.)

The perception of money has the additional aspect that steps in the range of negative payoffs count double. Denoting the finest perceived full step of money by $FPFM$ we obtain the perception function

$$\begin{aligned} f(x|FPFM) &:= per(x|FPFM) \text{if } x \geq 0, \\ f(x|FPFM) &:= 2 * per(x|FPFM) \text{if } x < 0. \end{aligned}$$

Perception of probabilities distinguishes probabilities (between 0 and 50), and counterprobabilities (between 50 and 100). We assume that the finest perceived fulls step of

[1] For the definitions see the appendix.

probabilites ($FPFP$), is the same for probabilities and counterprobabilities. For prominent numbers (of probabilities or counterprobabilities) differences are measured in steps, as above. The two scales are stitched in the 50%-point. Using interpolation one obtains a potential. According to usual notations we define for the perceived value of probability 0 by $\pi(0|FPFP) = 0$. (Example: If $FPFP = 1$, the scale of probabilities and counterprobabilities is 0, 1, 2, 5, 10, 20, 50, 80, 90, 95, 98, 99, 100. The value of 92% is $\pi(92\%|1) = 8.4$.)

The money equivalent of a lottery is predicted by

$$V[x(p), y(q)] := v^{-1}(v(x) * \pi(p) + v(y) * \pi(q))$$

where $v(\cdot) = v(\cdot|FPFM)$ with

> $FPFM$ is the largest full step number that is more than one step below the maximum of $|x|$ and $|y|$,

and $\pi(\cdot) = \pi(\cdot|FPFP)$ with

> $FPFP$ is the largest full step number that is finer or equal to the minimum of p and q.

The prediction concerns median behavior. There is no doubt, that individual preferences for risk can and do essentially deviate from the the median. It is not the purpose of the model to predict individual deviations from the median. In fact, this cannot be expected since the theory does not imply any individual measures of risk attitudes. The models does not use any parameter.

The prediction of the theory of prominence describes that part of risk pattern that is induced by the numeric perception per se. It permits to define 'risk neutral' behavior from a completely new point of view: While traditional theories define 'risk neutrality' as applying linear perception functions for money and utility (according to the traditional approach modeling perception as linear, and deviations from linearity as errors), the new approach describes a new benchmark of 'risk neutrality', by that decision behavior that is induced by the perception of numerical stimuli as defined by the theory of prominence (as a normative model). For example, the (nearly) logarithmic curvature of the perception function of values that are sufficiently far from the zero point is part of this natural perception.[2]

> The theory of prominence presents a parameterfree predictor of responses of subjects which are – compared to other subjects – 'risk neutral'. It models that part of observed risk attitudes which are induced (or can be explained) by the natural nonlinear shape of the perception functions.

It is the purpose of this paper to support the thesis that this natural shape adequately models subjects behavior.

[2] As nobody would have the idea to describe perception of loudness as driven by 'risk seeking behavior', it also can only disturb, to define natural responses induced by the mathematical structure of a problem not as 'risk neutral'.

4 Results

Table A shows the predictions of the theory and the median (a)- and (b)-values for the first 15 (x, y) values with maximal absolute payoffs of 10.000 DM. For the obtained 15 x 7 = 105 lotteries, in 65 cases the predicted values are in the range of observed median responses. In 15 of the other cases the distance from the range is below 2, in 16 of the remaining cases, it is below 5. Substantial deviations (distance to the range between 7 and 11) occur in 6 cases, which can be identified as belonging to similar situations (see the squares in Table A). The lotteries that leed to major errors are

lottery	pred.	(a)	(b)
$[-10.000(10\%), 5.000(90\%)]$	1300:	2000	2500
$[-10.000(10\%), 1.000(90\%)]$	-600:	100	200
$[-10.000(10\%), 500(90\%)]$	-800:	000	100
$[-10.000(20\%), 5.000(80\%)]$	0:	700	1000
$[-10.000(20\%), 1.000(80\%)]$	-1100:	0	-100
$[-10.000(20\%), 500(80\%)]$	-1300:	0	-200

These are the 6 cases where the high negative payoff is obtained with low probability 10% or 20%, and the other payoff is comparatively low and positive. The result suggests that the subjects have difficulties to integrate positive and negative payoffs, when the result is near to zero. The impression is that there is a 'resistance' to going from the value with higher probability (this is the value in which 18 of 20 subjects start there considerations) in the direction of the negative payoffs. Subjects seem to evaluate steps 'on the other side of the zero point' not as high as the theory predicts.

The predicted results are essentially better than by the usual expected values which are given in Table B (far below the point .1% level of significance $\chi^2 = 46$, one degree of freedom).

The repetition of the questions concerning the probabilities 99%-1% and 1%-99% caused a significant improvement of the result compared to the predictions.

An interesting result mentioned by the subjects is that the best fit of the predictions seems to be obtained for the cases with 80%-20% and 20%-80%, although the subjects reported, that the decisions were most difficult in these cases.

In 23 of the 32 cases of the corrected version, in which a deviation from the theory could be observed, are such that the observed responses are nearer to the expected value. Only 9 of these values are in the other direction. This supports the assumption that most deviations from the concept are caused by the calculations of expected values. In fact these calculations are easy for 50%-50% lotteries, and suggest themselves for lotteries with 1%, since it may be hard to imagine 1% emotionally. After the additional question on the next day, and after telling the subjects that 1% probability selects in the mean 1 out of 100 persons, the imaginative power of the subjects may have improved so that they thereafter had more trust into their emotional judgements and gave responses that were less near to the expected values.

5 Final Remarks

The data clearly confirm the theory of prominence as a tool to model median behavior of subjects. It seems to make sense to use the predictions of the theory of prominence as a neutral measure to characterize from standard behavior.

The model developed thus far is checked for the case, where only prominent numbers are presented, which are probably easily identified by the subjects. Additional investigations on unround numbers will follow to check how calculations are performed in general cases.

Another restriction of the analysis is that - up to now - we only considered lotteries with two alternatives. We did not yet aply the theory in the case with three and more alternatives.

In this paper we only analysed lotteries with maximal absolute payoff 10.000. The corresponding payoffs are clearly substantial. For lower amounts, we expect that the *FPFM* seems to become cruder in relation to largest absolute payoff. From other investigations we also know, that the motive to get 'tension' by selecting risk becomes increasingly important for low payoffs (see ALBERS, POPE, SELTEN 1998). In this investigation 'median subjects' selected the following additional lotteries $[X(50\%), -X(50\%)]$, when they received the amounts S for sure:

$S =$	10	100	1000	10000	100000
$X =$	10	50	200	1000	5000

The table shows that for low payoffs tension can be the predominant motive of the evaluation of lotteries. Of course, the concept here gives predictions only for that part of motivation which is not effected by tension.

Appendix: Notations

The full step numbers are the numbers $\{a * 10^i : a \in \{1,2,5\}, i \text{integer}\}$. Let a a full step number. The full step scale $S(1,a)$ is the set of all full step numbers that are greaterequal a, their negatives, and 0. For two full step numbers x, y we write $x \lhd y$, if $x < y$, and there is no full step number between x and y. The perception function induced by the full step scale $S(1,a)$ is the function $f(x|a)$ with (1) $f(0|a) = 0$, (2) if $x \lhd y$ in $S(1,a)$ then $f(y|a) = f(x|a) + 1$, (3) if $x \lhd y$ in $S(1,a)$, and z is a real number with $x < z < y$ then $f(z|a) = f(x|a) + |z|/|y - x|$.

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Table A: Predictions by Theory of Prominence and Money-Equivalents of Lotteries [x(p),y(q)] by Medians of (a)- and (b)-Values 1) 2)

x:y	full steps of predicted scale	probability (proportion equivalent of probability) 50%(1/2) -a--b-	90%(5/6) -a--b-	10%(1/6) -a--b-	80%(3/4) -a--b-	20%(1/4) -a--b-	99%(11/12) -a--b-	4) (first) -a--b-	1%(1/12) -a--b-	4) (first) -a--b-	deviations from pred. + : .
10:5 3)	5---6---7---8---9---X	75: 70 75+	87: 81 90+	58: 55 60+	80: 76 82+	63: 59 64+	93: 95 98:	(95 98:)	54: 53 55+	(52 53:)	6 1
10:0	0---2---5---X	35: 39 45.	75: 66 76+	10: 9 10+	63: 57 67+	15: 10 18+	88: 89 91:	(90 95:)	5: 1 2.	(1 1.)	4 1 2
10:1	.-2---5---X	43: 42 50+	79: 73 80+	18: 20 23:	69: 62 73+	24: 20 28+	90: 90 95+	(90 95+)	14: 12 13:	(10 13:)	5 2
10:.5	.--2---5---X	38: 37 46+	77: 70 80+	14: 14 19+	66: 60 67+	19: 17 23+	89: 90 94:	(90 95:)	10: 6 7.	(6 7.)	5 1 1
10:-10	X---5---2---0---2---5---X	-15:-10-15+	35: 32 50+	-63:-63-73+	15: 20 25.	-46:-50-60.	63: 49 61:	(80 85)	-81:-86-90.	(-87-90)	3 1 3
10:-5	5---2---0---2---5---X	-5: -1 -1.	45: 45 56+	-33:-29-33+	28: 29 35:	-24:-18-25+	71: 57 68.	(85 89)	-41:-41-45+	(-42-45:)	4 1 2
10:-1	.-0---2---5---X	20: 13 23+	67: 61 68+	-4: -1 -2:	50: 45 50+	0: 0 5+	83: 77 85+	(90 93)	-8: -7 -8+	(-7 -8+)	6 1
10:-.5	.0---2---5---X	28: 25 33+	71: 71 76+	3: 0 -1.	56: 53 58+	8: 2 8+	85: 81 90+	(90 93.)	-4: -3 -4+	(-3 -4+)	6 1
-10:5	X---5---2---0---2---5	-20:-20-25+	-67:-57-66:	13: 20 25	-50:-47-58+	0: 7 10	-83:-80-85+	(-86-90.)	30: 30 35+	(40 45)	4 1 >2<
-10:1	X---5---2---0-.	-31:-34-38.	-73:-70-77+	-6: 1 2	-59:-55-65+	-11: 0 -1	-86:-86-90+	(-92-92)	0: 5 6.	(7 8)	3 2 >2<
-10:.5	X---5---2---0.	-33:-35-46:	-70:-70-79+	-8: 0 1	-61:-57-68+	-13: 0 -2	-87:-87-90+	(-92-93.)	-3: 1 2.	(2 3.)	3 1 1 >2<
-10:-5	X---9---8---7---6---5	-75:-60-70.	-87:-77-85:	-58:-55-60+	-80:-70-80+	-63:-58-61:	-93:-90-92:	(-94-95:)	-54:-52-54+	(-50-52:)	4 3
-10: 0	X---5---2---0	-35:-34-40+	-75:-70-80+	-10: -5-10+	-63:-55-63+	-15:-10-15+	-88:-86-90+	(-92-95.)	-5: -1 -2.	(0 -1.)	6 1
-10:-1	X---5---2-.	-43(-31-38).	-79(-67-75).	-18:-16-20+	-69:-65-74+	-24:-20-25+	-90:-88-92+	(-92-95:)	-14:-12-13:	(-10-11.)	4 1 2
-10:-.5	X---5---2--.	-38:-40-45:	-77:-70-78+	-14:-10-14+	-66:-65-70+	-19:-15-20+	-89:-87-91+	(-92-95.)	-10: -6 -7.	(-5 -6.)	5 1 1

sum of deviations from prediction (in column):	50%(1/2)	90%(5/6)	10%(1/6)	80%(3/4)	20%(1/4)	99%(11/12)	(first)	1%(1/12)	(first)	total
			1 ??							
+ (predicted value within range)	9	12	9	13	10	10	(1)	6	(2)	69
: (distance from range <= 2)	2	2	2	1	1	4	(5)	2	(4)	14
. (distance from range <= 5)	4	1	1	1	1	1	(5)	7	(6)	16
(distance from range > 5)			>3<		>3<		(4)		(3)	>6<

1) entries of table, as 92: 81 90+ mean: 9200= predicted value, 8100= (a)-value, 9000= (b)-value, last symbol: += prediction within range, := distance from range lessequal 2, .= distance from range lessequal 5.

2) Interpolation: substructure below 5-10 is 5-6-7-8-10, with linear interpolation within substeps; in all other cases linear interpolation between full steps of the predicted scale (see column 2)

3) all money amounts in left column to be multiplied by 1000 (currency: Deutschmarks)

4) 'first' denotes the responses of the first time, before subjects were additionally informed to amagine that one percent of probability selects in the mean one out of 100 persons, and before they were offered to play the lottery [-10.000(1%),1.000(99%)] by the experimenter

Table B: Money Equivalents of Lotteries [x(p),y(q)] by Usual Expected Value Theory

x:y	full steps of predicted scale	probability (proportion equivalent of probability) 50%(1/2) a b	90%(5/6) a b	10%(1/6) a b	80%(3/4) a b	20%(1/4) a b	99%(11/12) a b	(first) a b	1%(1/12) a b	(first) a b	deviations from pred. +	:	.	
10:5 3)	5---6---7---8---9---X	75: 70 75+	95: 81 90.	55: 55 60+	90: 76 82	60: 59 64+	99: 95 98:	(95 98:)	51: 53 55:	(52 53:)	3	2	1	1
10:0	0---2---5---X	50: 39 45.	90: 66 76	10: 9 10+	80: 57 67	20: 10 18:	99: 89 91	(90 95.)	1: 1 2+	(1 1+)	2	1	1	3
10:1	.-2---5---X	55: 42 50.	91: 73 80	19: 20 23:	82: 62 73	28: 20 28+	99: 90 95.	(90 95.)	11: 12 13:	(10 13+)	1	2	2	2
10:.5	.--2---5---X	52: 37 46	90: 70 80	14: 14 19+	81: 60 67	29: 17 23	99: 90 94	(90 95.)	6: 6 7+	(6 7+)	2			5
10:-10	X---5---2---0---2---5---X	0:-10-15	80: 32 50	-80:-63-73	60: 20 25	-60:-50-60+.	98: 49 61	(80 85)	-98:-86-90	(-87-90)	1			6
10:-5	5---2---0---2---5---X	25: -1 -1	85: 45 56	-35:-29-33:	70: 29 35	-51:-18-25+	98: 57 68	(85 89)	-41:-41-45+	(-42-45:)	2	1		4
10:-1	.-0---2---5---X	45: 13 23	89: 61 68	1: -1 -2:	78: 45 50	12: 0 5	99: 77 85	(90 93)	-9: -7 -8:	(-7 -8:)		2		5
10:-.5	.0---2---5---X	47: 25 33	89: 71 76	6: 0 -1	79: 53 58	21: 2 8	99: 81 90	(90 93)	-4: -3 -4+	(-3 -4+)	1			6
-10:5	X---5---2---0---2---5	-25:-20-25+	-85:-57-66	35: 20 25	-70:-47-58	20: 7 10	-98:-80-85	(-86-90)	41: 30 35	(40 45+)	1			6
-10:1	X---5---2---0-.	-45:-34-38	-89:-70-77	-1: 1 2:	-78:-55-65	-12: 0 -1	-99:-86-90	(-92-92)	9: 5 6.	(7 8:)		1	1	5
-10:.5	X---5---2---0.	-47:-35-46:	-89:-70-79	[illegible]6: 0 1	-79:-57-68	-21: 0 -2	-99:-87-90	(-92-93)	4: 1 2:	(2 3:)		2		5
-10:-5	X---9---8---7---6---5	-75:-60-70.	-95:-77-85	-55:-55-60+	-90:-70-80	-60:-58-61+	-99:-90-92	(-94-95.)	-51:-52-54:	(-50-52+)	2	1	1	3
-10: 0	X---5---2---0	-50:-34-40	-90:-70-80	-10: -5-10+	-80:-55-63	-20:-10-15.	-99:-86-90	(-92-95.)	-1: -1 -2+	(0 -1+)	2		1	4
-10:-1	X---5---2-.	-55(-31-38)	-91(-67-75)	-19:-16-20+	-82:-65-74	-28:-20-25:	-99:-88-92	(-92-95.)	-11:-12-13:	(-10-11+)	1	2		4
-10:-.5	X---5---2--.	-52:-40-45	-90:-70-78	-14:-10-14+	-81:-65-70	-29:-15-20	-99:-87-91	(-92-95.)	-6: -6 -7+	(-5 -6+)	2			5

sum of deviations from prediction (in column):	50%	90%	10%	80%	20%	99%	(first)	1%	(first)				
+ (predicted value within range)	2	-	7	-	5	-	(-)	6	(9)	20			
: (distance from range <= 2)	1	-	4	-	2	1	(1)	6	(5)		14		
. (distance from range <= 5)	3	1	-	-	1	1	(7)	1	( )			7	
(distance from range > 5)	9	14	4	15	7	13	(7)	2	(1)				>64<