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The Price Response Function and Logarithmic Perception of Prices and Quantities

by

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Abstract

The relation between the price response function and numerical perception is studied. In the theory of prominence the perception of numbers is modeled. The resulting perception function is applied to the perception of prices and quantities to obtain a price response function. For this price response function a relative change of the price results in a relative change of the quantity. This reaction is based on a logarithmic perception of prices and quantities. The gradient of the resulting function is -1. A price response function obtained from the scanner-data of all fruit and vegetables sold in a consumer market within 2 months shows the predicted reaction of a logarithmic change of the quantity if a logarithmic change of the price occurs.

Introduction

A lot of theoretical and empirical work (for example see references in G.J. Tellis 1988 or in H. Simon 1995) has been spend on price response functions. Four main types of functions have been discussed: the linear function, the multiplicative function, the attraction function and the Gutenberg function. Each of them was supported by empirical studies. It is concluded in (H. Simon 1995) that in general in empirical studies it could not be decided which one is the best predictor for the market data. This is attributed to many factors that influence the price response function. Reasoning about other factors and examining them is proposed to give further information. During this process many factors that influence the price response function have been recognized.

In this paper we concentrate on the price response function only caused by numerical perception. This is an attempt to isolate the reaction only caused by numerical perception. The resulting function neglects other possible factors that might have an influence. In the theoretical part the perception of numbers as modeled in the theory of prominence (W. Albers 1997) is used to obtain a price response function. The modeling of the numerical perception in the theory of prominence is similar to the Weber-Fechner law (for example in G.T. Fechner 1968) of psychophysics which models the perception of physical stimuli (like the perception of light). It is well known that the perception of prices might be described by this law. In the theory of prominence this is extended to numerical perception (not only for money, but also for quantities). After dividing the prices and quantities by their averages one function is obtained for the products. A general version of this function depends on one parameter. A special case of the prediction is a price response function without free parameters which is also compared with the empirical data.

In the empirical part of this study the price response function of all fruit and vegetables sold in a consumer market within 2 months is estimated using the scanner-data of the market. Fruit and vegetables were selected, because the influence of other possible factors was expected to be small. The short time of 2 months was selected to avoid general changes in the opinion of consumers towards fruit and vegetables. The influence of brands and advertising does not seem to be very important. For slight price changes of fruit and vegetables consumers will not change the consumer market where they do their weekly shopping. They will react by changing the bought quantity. Other factors that influence the price response function surely exist, but their influence is expected to be small.

The perception of stimuli in psychophysics.

In psychophysics the perception of physical stimuli (like light, etc.) by persons is described by the Weber-Fechner laws (for example in G.T. Fechner 1968).
Weber (1834) observed the following two rules for the psychophysical responses:

Rule 1: Constant perceived relative difference: the finest perceived relative difference between two stimuli is a constant which only depends on the variable (kind of the stimulus) and the person. This is written as:
$\Delta I = k * I$, with I: intensity of a stimulus

Rule 2: Smallest perceived absolute value: there is a smallest physical stimulus that can be perceived. It is a constant which only depends on the variable (kind of the stimulus) and the person.

The Fechner law is given by rule 3
Rule 3: Logarithmic perception: a relative increase of the magnitude of a stimulus causes a linear increase of the perceived intensity P above a smallest perceived intensity:
$P = a * \log I + c$
a and c are constants.

Perception of numbers in the theory of prominence

In the theory of prominence (W. Albers 1997) the numerical perception is modeled. Numerical stimuli that are numbers x are considered instead of psychophysical stimuli. In addition to the laws of psychophysics the smallest absolute unit that is perceived depends on the problem (the numbers in the problem). A smallest perceived full step unit Δ is defined. Δ is 20% of the maximal number in a problem. Several perception functions v_Δ are obtained. These functions have the general form:

$$v_\Delta = a * \log x - \log(\Delta) + 1 \text{ for } x > \Delta .$$

For the determination of the price response function the influence of the smallest perceived full step is taken into account by dividing prices and quantities by the average price and quantity, respectively, which is around 5*Δ. Thus the additive constant in the v_Δ-function is determined by v_Δ(average price)=1 and v_Δ(average quantity)=1 and is denoted as v-function in the following parts. In determining a price response function this fixing of the additive constant is known as considering the reference price: prices are considered around a reference price (for example the average competitive price). Here the average price of a product is used. According to the theory of prominence the perception function has the same shape for all products after the division by the average price or the average quantity independent of the price level (.1 DM or 100 DM) and the level of the quantity.

The price response function and numerical perception

Changes in the quantity caused by price changes can be due to the fact that consumers buy less of the quantity of a product or that a certain number of consumers do not buy the product at all. If consumers react by buying less quantity of the product the reaction should occur on a logarithmic scale and the laws of numerical perception can be applied. If the reaction is such that a certain number of consumers do not buy the product anymore the situation is a little different. But on the aggregate level this can have the same effect, because buying or not buying a product occurs several times and on average the sum of the decisions should reflect the logarithmic reduction of the quantity.

In this part the evaluation of a price/quantity combination is described. It is the situation considered in which the amount consumers spend on a certain product is constant. This situation is denoted as constant budget spending. This results in a constant evaluation of a product in which price changes are only compensated by quantity changes. The perceived quantity (q) and price (p) are interpreted as being proportional to the evaluation of the quantity and of the price, respectively. When adding up the single evaluations weights for the evaluation of the price ($1>\alpha>0$) and the quantity ($1-\alpha$) summing up to 1 are introduced. Adding up the single evaluations leads to a total evaluation of:

$u(p,q)= \alpha*v(p)+(1-\alpha)*v(q) +c$

with c denoting a constant.

$v(\bar{p})=1$ and $v(\bar{q})=1$ as described in the last section[1]. In this formula no interference term v(p,q) is taken into account, because quantity and price are the two independent dimensions on which the perception occurs.

For constant budget spending the price response function is obtained for $u=\bar{u}$=constant, resulting in:

$v(q)= -\alpha/(1-\alpha)*v(p)+(\bar{u}-c)/(1-\alpha)$ or

$v(q)=\beta*v(p)+c'$ (with $\beta=-\alpha/(1-\alpha)$ and $c'=(\bar{u}-c)/(1-\alpha)$.[2]

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[1] Thus $v = a*\log\frac{p}{\bar{p}}+1$ and $v = a*\log\frac{q}{\bar{q}}+1$ around the average price $\bar{p}$ and the average quantity $\bar{q}$, respectively.

[2] For further considerations the absolute constant c' is neglected. In the data analysis only operations are performed that change c', but not β

For prices and quantities around the average values of p and q[3] the functions v(p) and v(q) can be replaced by the logarithm. This leads to:
$\log(q)=\beta*\log(p)+c_1$ (with c_1 a constant).

This is a linear function on a logarithmic scale, i.e. relative price changes correspond to relative changes in the quantity. This is different from the attraction and the Gutenberg function which predict a complex reaction on changes of the price and it is different from the linear price response function which predicts a linear change of the quantities if a linear change of the prices occurs.

Considering the following transformation of the evaluation function: $u'=\exp(u)$. This leads to the evaluation function: $u'=c_2*p^{\alpha}*q^{1-\alpha}$, because $v\approx\log$. This is a Cobb-Douglas function. For $u'=\overline{u'}$=constant a multiplicative price response function is obtained:
$q=(\overline{u'}/c_2)^{1/(\alpha-1)}*p^{\alpha/(\alpha-1)}=c_2'*p^{\beta}$ (with $c_2'=(\overline{u'}/c_2)^{1/(\alpha-1)}$).

This multiplicative price response function has the constant price elasticity of β. The interpretation of this function is that relative price changes result in relative changes of the quantity. This is due to the fact that the original evaluation u is given by $u=\log(u')$ and therefore the perception of the original quantities is logarithmic.

A special case of this price response function is given by $\beta=-1$. This prediction seems to be a natural one, because the theory of prominence does not predict a difference in the perception of prices and quantities in this case. Therefore the weights for the evaluation should be equal $\alpha=(1-\alpha)$ and therefore $\beta=\alpha/(\alpha-1)=-1$.

Deviations from this might be caused by consumers who do not consider to buy at other markets if the price of some fruit is raised, because they believe that the prices in this market are in general lower than in other markets. It might be also due to consumers who do not want to buy less quantity of a product, because they need a certain amount. It might also be too costly to go to another market to buy fruit and

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[3] Around the average denotes a factor of 5.

vegetables. Therefore higher quantities than expected will be bought. This will have small influence on the price response function resulting in a price response function of the form: $v(q)=\beta*v(p)+c_1$ with $\beta<1$, because an increase of the price of one product will not fully be compensated by a change of the quantity.

The data

The empirical data were scanner data of all fruit and vegetables sold in a German consumer market.

The time period

The data were collected from 04/09/1996 (after Easter) until 06/19/1996 (before summer holiday) to avoid seasonal influences.

The products

For this study all products of the group fruit and vegetables for which the prices were changed in the period are considered. The products are pineapples, cauliflower, salad (3 kinds of salad), cucumbers, kiwis, carrots (2 types), oranges (2 sizes), radishes (2 kinds), lemons, onions (2 types), kohlrabis, avocados, potatoes and grapefruit.

The predicted price response function

The prediction is:

$$\beta \cdot \log \frac{Q_{it}}{\bar{Q}_i} + c = \log \frac{P_{it}}{\bar{P}_i}$$

with:

$\bar{Q}_i$: average quantity sold of the product i on a day

$\bar{P}_i$: average price of the product i in a certain period

Q_{it} : quantity sold of product i on day t

P_{it} : price of product i on day t

c : a constant

For the special case the prediction is $\beta=-1$.

The calibration of the variables

The sold quantities depend on the day of the week. For example the sold quantities are much higher on Thursdays than on Mondays. To avoid to observe the influence of the day of the week on the data the quantities of product i are measured as proportion of the sold quantity of the product group of fruit and vegetables on the day t denoted as PPG_{it}.

$$PPG_{it} = \frac{Q_{it}}{Q_{PGt}}$$

with

$$Q_{PGt} = \sum_i Q_{it}$$

The average quantity sold is replaced by a factor S representing the percentage of product i in the group of fruit and vegetables.

$$S_i = \frac{\sum_t Q_{it}}{\sum_t Q_{1t}}$$

The resulting calibrated proportion of the sold quantity of the product group of fruit and vegetables of product i at day t (PPG_{it}^{cal}) is given by

$$PPG_{it}^{cal} = \frac{Q_{it}}{Q_{PGt} \cdot S_i}.$$

PPG_{it}^{cal} will be denoted as the calibrated quantity in the following parts

These operations do not change the gradient of log(q)=β*log(p)+c, but the same constant c_1 is added for all products resulting in log(q)=β*log(p)+ c_2 (with $c_2=c+c_1$).

The calibrated prices P_{it}^{cal} are obtained by:

$$P_{it}^{cal} = \frac{P_{it}}{\bar{P}_i}.$$

with

$$\bar{P}_i = \frac{\sum_t P_{it}}{T}$$

P_{it}: price of product i on day t

T: length of the time period (in days)

P_{it}^{cal} will be denoted as the calibrated price in the following parts.

After performing this operations the prediction of a price response function is:

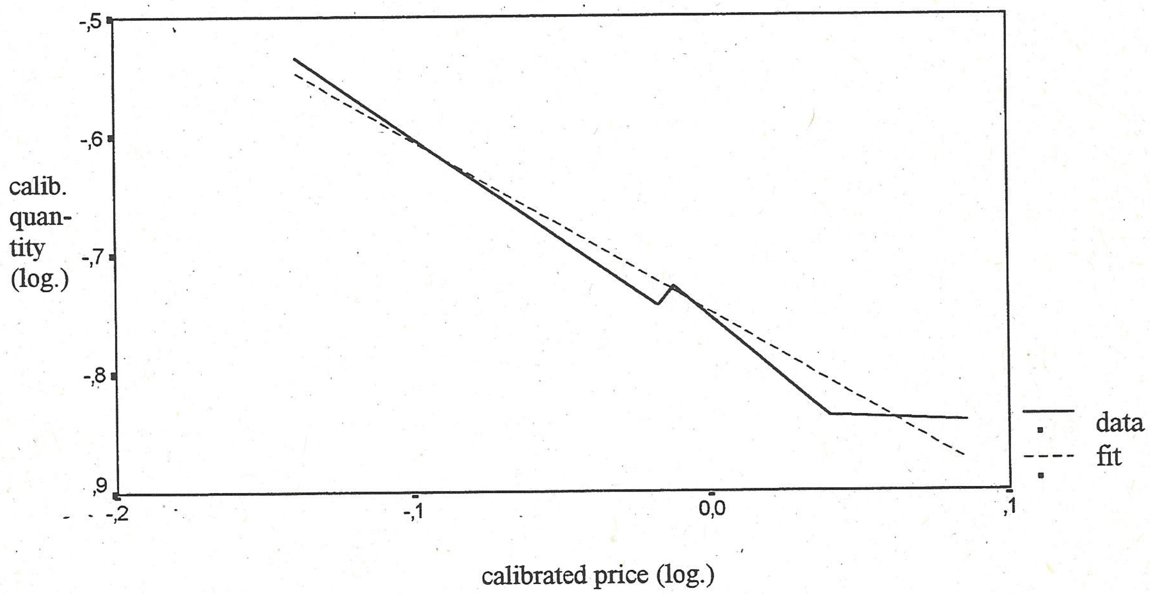
$$\log(PPG_{it}^{cal}) = \beta * \log(P_{it}^{cal}) + \text{const.}$$

The price response function

The analysis of the price response function will be performed in several steps. First the logarithmic dependence of the quantity on the price will be examined (allowing β to be different from 1). Then the difference of β from 1 will be examined.

In figure 1 the price response function for kohlrabi is shown as an example. On the axes the logarithms of the calibrated prices and the calibrated quantities are plotted.

Figure 1: The price response function for kohlrabis

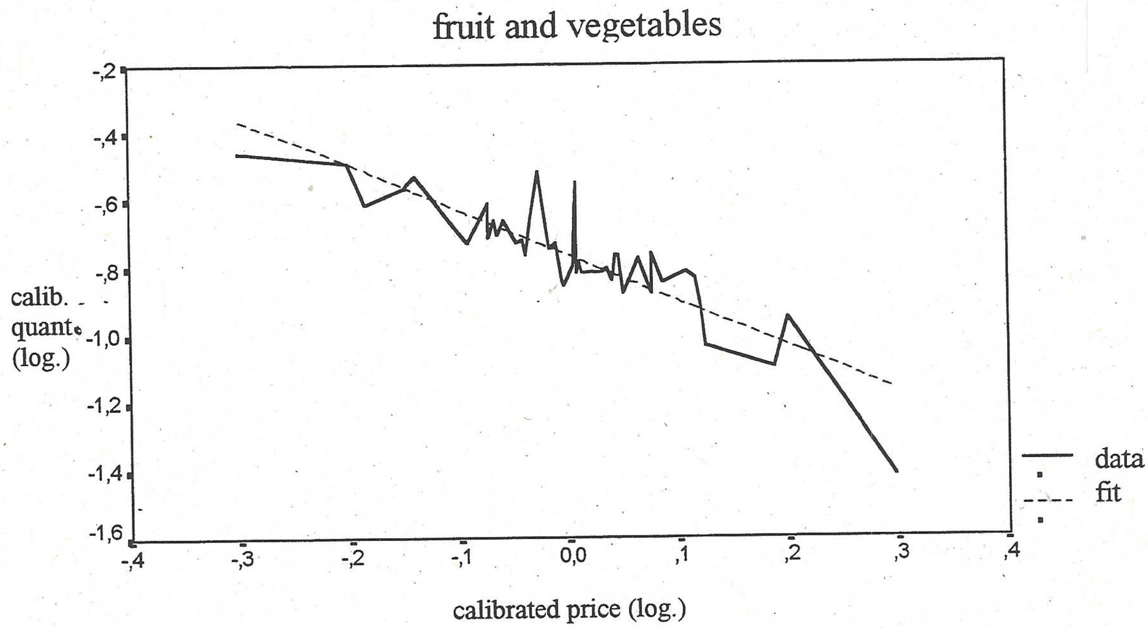


The least square fit (r^2=0,96) of the data is given by the dashed line. This is the fit with the best r^2 of all least square fits of all other simple dependencies (liner-log. or linear-linear).

For the analysis of all data only the products for which an increase in price results in a decrease in quantity are considered. The relation between price and quantity for the products for which this normal reaction is not observed is not considered to be describable by a price response function of this kind. Other factors must have a great influence. The products with a normal reaction on price changes are characterized by the number of permutations that are necessary to get a normal reaction. A permutation occurs if prices have to be exchanged to get a ordering such that higher prices correspond to higher quantities. If less than 25% of all prices are permutations a normal reaction is assumed. This results in the following products: pineapples, cauliflower, kiwis, carrots (both types), oranges (both sizes), radishes, onions, salad, kohlrabis, avocados, potatoes and grapefruit.

If one plots all price changes and all changes of the quantities into one diagram figure 2 is obtained.

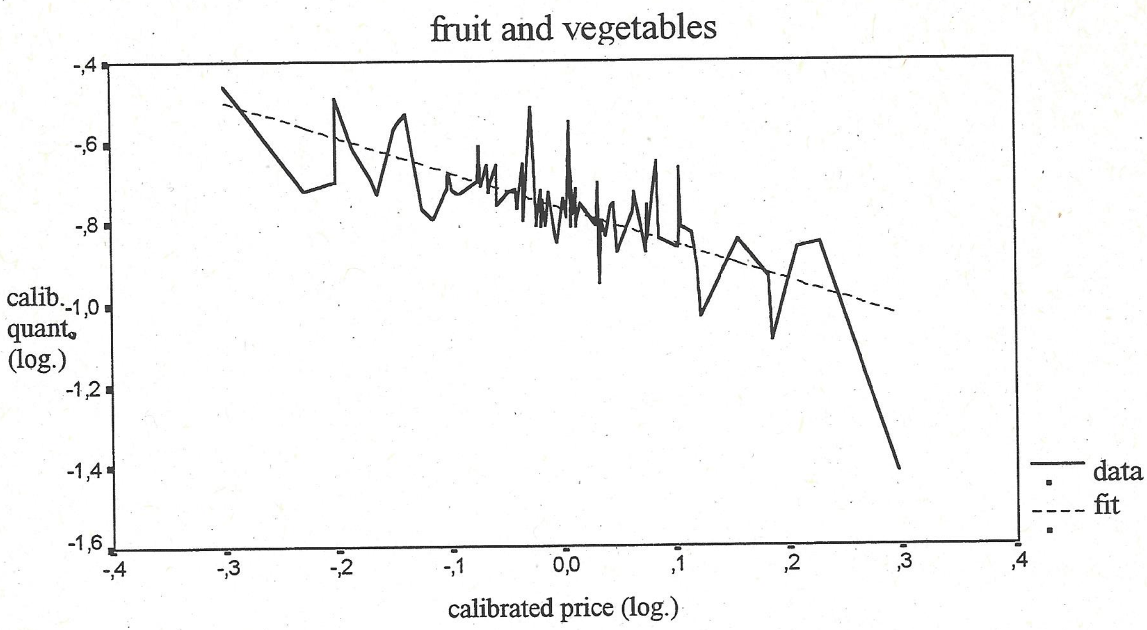
Figure 2: The price response function for fruit and vegetables.



The least square fit (r^2=0,76) is given by the dashed line. Again all other simple dependencies (liner-log. or linear-linear) result in a smaller r^2.

Considering all products gives the plot in figure 3

Figure 3: The price response function for all kinds of fruit and vegetables



Summarizing a price response function showing linear behavior, if prices and quantities are perceived on a logarithmic scale is supported by the empirical data.

The β-coefficient

In this part the β-coefficients for all products with normal price reaction (measured by the permutations necessary to obtain monotonicity) and with more than 2 price changes are considered. The β-coefficients for the different products are shown in figure 4.

Figure 4: The β-coefficients for different products

product	β-coefficient
cauliflower	-0.956967
kiwis	-0.975889
radishes	-0.947106
salad	-0.957370
kohlrabis	-0.979322
avocados	-0.920439
oranges	-0.852712

The β-coefficients are very close to -1 as predicted. In a binomial test the hypothesis that β has equal probability to be in the interval [-0.9,-1] or in any interval not including [-0.9,-1], for example the interval [-0.8,-0.9], is rejected on a 7% level[4].

The deviations from -1 were expected, because for small price changes consumers will not change the consumer market to buy their fruit and vegetables, because the cost of doing this is higher than the savings. Perhaps consumers do also assume that the prices

[4] In this test it is not taken into account that the interval [-0.9,-1] is small. Considering this would lead to better test results

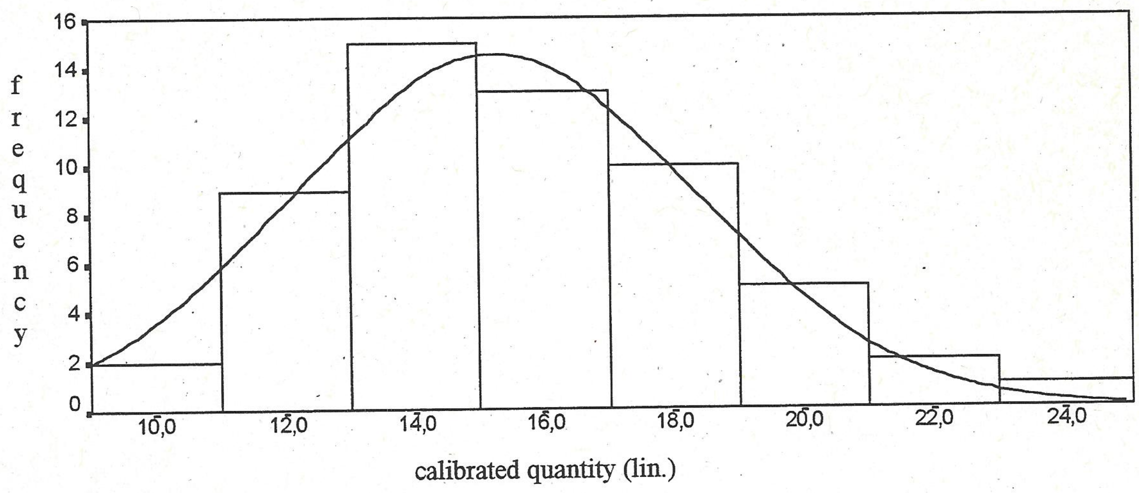
of the consumer market will be the lowest. Therefore not all price changes result in a change of the quantities.

The empirical data support a logarithmic increase in quantity for a logarithmic increase of the price as predicted in the theoretical part. After dividing the quantity by the average quantity and the price by the average price the price response functions of different products are very similar (β is nearly the same) and its gradient is close to -1 (as predicted by the price response function without free parameters).

Further support of the logarithmic perception of quantities

The perception of quantities is further examined by plotting the frequencies with which certain quantities are bought. The quantities are again measured as calibrated quantities. This is a continuous measure. It is divided into few equidistant classes (7 to 10). The histogram (frequencies versus proportion of the product group) is given for a product for which the normal distribution of prices is a good approximation to the empirical data. Here salad is chosen and the results are shown in figure 5.

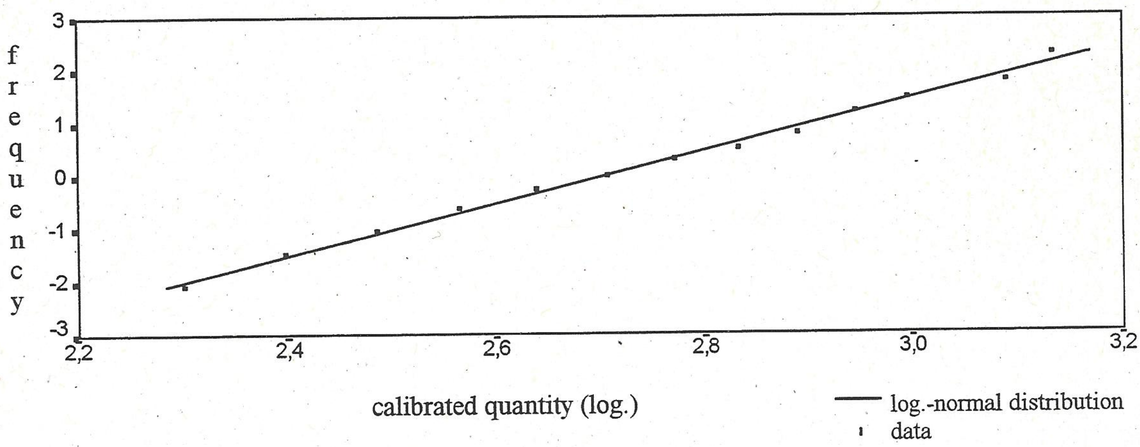
Figure 5: Frequencies versus the calibrated quantities of the sales of salad



The fit by a normal distribution shows some discrepancies: the maximum does not fit. Therefore figure 6 is considered.

In figure 6 the frequencies of the logarithm of the calibrated quantities are compared with the expected frequencies if the distribution was a normal distribution on the logarithmic scale. The log-normal distribution gives a good fit of the data.

Figure 6: Frequencies versus the logarithm of the calibrated quantities of the sales of salad



This plot supports the hypotheses that quantities are perceived on a logarithmic scale. This is similar to findings on the aggregate consumption and income level (A. Kneip, W. Hildenbrand, 1995).

Conclusion

The price response function of fruit and vegetables has been examined. A theoretical function based on logarithmic perception of prices and quantities as predicted by the theory of prominence has been deduced. This function predicts a relative change of quantities if a relative change of the price occurs. After calibrating according to a smallest unit this function has the same shape for all products and a gradient of -1. It has a constant price elasticity like the multiplicative price response function for all products.

The scanner data of a consumer market over a period of 2 months were used to estimate the price response function. Fruit and vegetables were selected, because other factors like advertising, brands, etc. were expected to have only small influence on the price response function. A result of the analysis of the empirical data is that the characteristics of the empirical price response function are as predicted. Because of this the obtained price response function might be seen as modeling the numerical perception and should be subtracted from other complex econometrically estimated functions to study other factors that influence the price response function.

The logarithmic perception of quantities was also observable in the log-normal distribution of the frequencies with which certain quantities were bought.

References

W. Albers (1997): Foundations of a theory of prominence in the decimal system, part I-V, IMW working papers 265, 266, 269, 270, 271

W. Albers, G. Albers (1983): On the prominence structure in the decimal system, in Decision making under uncertainty, Ed. U. Schulz, Elsevier Science Publishers B.V. (North Holland), p. 271-287

P. Dickson, A.G. Sawyer (1990): The price knowledge and search of supermarket shoppers, Journal of Marketing 54, p. 42-53

Fechner G. T. (1968): In Sachen der Psychophysik, Amsterdam: E.J. Bonset

A. Gabor, C. Granger (1964): Price sensitivity of consumers, Journal of Advertising Research 4, p. 40-44

J.M. Kamen, R.J. Toman (1971): Psychophysics of prices. A Reaffirmation, Journal of Marketing Research 8, p. 252-257

A. Kneip, W. Hildenbrand (1995): Aggregate consumption expenditure and income distribution effects, Working paper SFB 303 Universität Bonn, Bonn

H. Simon (1995): Preismanagement kompakt, Probleme und Methoden des modernen Pricing, Gäbler, Wiesbaden or H. Simon (1989): Price management, North Holland, Amsterdam

G.J. Tellis (1988): The price elasticity of selective demand: A meta-analysis of econometric models of sales, Journal of Marketing Research 25, p 331-341

R. Thaler (1985): Mental accounting and consumer choice, Marketing Science 4, p. 199-214

V. Zeithaml (1988): Consumer perceptions of price, quality and value: A means-end model and synthesis of evidence, Journal of Marketing 52, p. 2-22