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A Model of the Concession Behavior in the Sequence of Offers of the German Electronic Stock Exchange Trading Market (IBIS) Based on the Prominence Structure of the Bid Ask Spread

by

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Abstract

Theoretical models of bidding in double auctions or markets permit arbitrary real numbers as bids and asks. However, in reality the exactness is restricted. Two phenomena can be observed. 1. Agents analyse markets with a given level of graneness of judgement. 2. The multilateral bidding process happening in the auction refines the structure obtained by the graneness of judgement to a finer, but again limited level of graneness. The paper analyses the structure of bids and asks of the most frequently traded German stock (Volkswagen) for the complete order book of the German electronic stock exchange market (IBIS) during a period of three months. It gives a first detailed analysis of the structure of offers induced by the prominence structure of numerical perception.

The main result is that the concession behavior is determined by the prominence structure of the present best offer on the concession maker's side, and on the extension of the spread. Specificly, it is identical when the spread is shifted in multiples of .50, and it is identical for bid- and ask-concessions when the corresponding steps of multiples of .50 have the same distance from the respective best offer on the concession maker's side.

1 Graininess of Judgement

The theory of prominence (ALBERS 1997) says that the mental construction of numerical responses can be modelled such that the response is constructed as a sum of the prominent numbers $\ldots, 1, 2, 5, 10, 20, 50, \ldots$, where every prominent number is used at most once, and is either subtracted or added. For instance $17 = 20 - 5 + 2$. The corresponding mental decision process creating the response can be modelled as starting with a sufficiently large prominent number and thereafter deciding sequentially in descending order for every smaller prominent number whether to add or subtract it to the respective obtained preliminary result. The process stops when the exactness of analysis of the responder does not permit a further refinement of the decision.

The theory of prominence makes a prediction of the exactness of the response (i.e. the finest prominent number used in the response) for situations with unprecise information. Basic determinant of the numerical response is the 'range of reasonable alternatives', i.e. the range between lowest and highest reasonable response, where a response is denoted as reasonable, when it is not rejected by the decision maker. The prediction of the theory is that the described process of refinement of the response stops at a certain finest prominent number p, which is such that there are at least 3 and at most 5 numbers that are in the range of reasonable alternatives and that can be constructed as responses using no finer number than p ('exactness selection rule'). For instance, if the range of reasonable alternatives is 30 to 50, then the exactness is $p = 10$ which permits the responses 30, 40, 50, if 50 is excluded as a reasonable alternative, then $p = 5$ which permits the responses 30, 35, 40, 45. Experience from empirical results indicates that in situations where the degree of reasonability can be described by a density function, the range of reasonable alternatives is the set of numbers obtained by cutting of the 10%-tails at both ends.

The best experiment that could be made to support and illustrate the exactness selection rule was the bearing experiment (VOGTS, ALBERS 1992). The subject was in front of a screen. On top of the screen was a horizontal scale, with marks at multiples of 10 (200, 210, ..., 390). In a given distance from the scale was a parallel line, one point of this line was marked. The task was to start with the point on the parallel line, bear vertically upward, thereby identify some may be unprecise image, and characterize the obtained position on the scale by a number ('give the response you are most content with'). The positions of the points on the parallel line were varied in a way that the numbers 0, 1, 2, ..., 9 occured with the same frequencies. The 'diffuseness' of the signal could be varied by selecting different distances of the parallel line from the scale. When the parallel line was on the bottom of the screen, the responses were multiples of 10, for half the distance most subjects responded multiples of 5, for 1/8 of the screen all integer responses occured. The experiment illustrates, how the exactness of the responses is increased with increasing precision of the image of the signal obtained by bearing. (The diffuseness of the signal could be controlled by the errors made in the responses.)

2 Special Aspects of Information Processing in Double Auction Markets

Different from the situation in the bearing experiment a decision maker on the stock market gets his information about the reasonable price of a stock from different sources including the respective current bids and asks on the market of the given and other stocks. This information has to be transformed and aggregated to obtain the diffuse numerical information about a reasonable market price, and reasonable bids and asks. This rough idea can be transformed to a numerical response in a way as described above.

There is an interesting idea following from this model: if market agents follow the model and construct their responses according to the rules described above, this would result in comparatively crude prices. Now assume that the whole level of the market prices is raised by some positive signal for say 0.5%, and idealistically assume that all diffuse signals are also raised by 0.5%. Then some of the prices change, others not, depending on the step structure of possible prices as predicted by the exactness selection rule. This changes the system of relative prices of the stocks. A market analyst knowing the system of relative prices would benefit from selling stocks that are overevaluated by the present step structure, and buying stocks that are underevaluated. Therefore it seems reasonable that experienced agents also develop a system of relative prices (price ratios between different stocks, or ratios to given indices as DAX) which produces essentially more precision than the evaluation of a single stock.

Our impression is that the precision reached for the VW-stock was usually 0.50 DM. This is five times the smallest unit 0.10 DM that was allowed to be traded on the IBIS market at that time.

3 Aspects of Multiperson Bargaining

Watching the movement of best bids and asks on the electronic stock market, suggests the idea of two-person bilateral negotiations on the price. While in fact it nearly never happens (below 0.1% of the cases) that one agent gives two subsequent offers. Nevertheless, the structure of respective best offers is similar as concession making in bilateral bargaining. These kinds of offers reach an exactness of 0.10 DM.

4 The Concession Behavior

The observed negotiation process is clearly dominated by three types of concessions:

1. Concessions of 0.10 DM (above best bid or below best ask)
2. Concessions of 0.20 DM (above best bid or below best ask)
3. Concessions to the next multiple of 0.50 DM

4. Accept best offer

The interesting result is that the respective best bid or ask serves as an anchor point only for concessions of 0.10 or 0.20 DM. Larger concessions follow the grid of multiples of 0.50 DM, or just accept the best offer.

Concessions of Type 4 (accept opposite side of spread)

We first consider the proportion of concessions of type 4 as proportion of the total sum of concessions. This proportion depends on how many steps of the grid $.50 * i$ (i integer) are passed. To give a formula, let b the number of numbers of type $.50 * i$ (i integer) within the open interval between the lower and upper bound of the spread (excluding the boundaries of the spread). Then

> the proportion of concessions that accept the other side of the spread is 3/4 for $b = 0$, and $(1/4)^b$ otherwise.

The probability of acception reduces by 1/4 whenever a value of the 0.50 DM grid is crossed. The probability of concession only depends on the value that is obtained by rounding the concession value to the next multiple of 0.50 DM. (So to say: concessions of parts of .50 count as concessions of .50.)

Table 1 of the appendix shows the probabilities of concessions of Type 4. (The table shows that the strict step structure as given in the model is 'replaced' by a more continuous reaction in the actual behavior at the border $j = 5, 6$. But the deviations are comparatively low, and therefore not included in the model.)

In the following we consider the proportions of concessions of Type 1 - 3 as proportion of the total number of concessions of Type 1 - 3:

Concessions of Type 2 (.20 from anchor point)

The proportion of concessions of Type 2 (compared to all concessions of Types 1 - 3) is roughly constant at 20%.

Concessions of Type 1 (.10 from best present bid)

We first consider the proportion of concessions for fixed length of the spread (measured before the concession is made). The proportion of bid-concessions of Type 1 depends on the numerical structure of the best bid bb (lower bound of the spread) in the moment when the concession is made. This means that the proportion is roughly identical for situations where the preceding best bids are identical up to a multiple of 0.50 DM. The same property holds for the ask side, where the numerical structure of the best ask, ba, determines the proportions. Moreover, the proportions for bid and ask side are identical when the sum of best bid and best ask, $bb + ba$, is integer. Accordingly, we have to distinguish 5 classes of cases described by the parameter i ($i = 0, 1, 2, 3, 4$):

class i: concession on bid side, best bid $X + i * .10$
concession on ask side, best ask $Y - i * .10$
(where X, Y are adequate integer multiples of .50)

These cases are such that (for given length of the current spread) the proportions of concessions are equal for all situations belonging to the same parameter i. Moreover, the proportion depends on the length $n * .10$ of the current spread (difference of best bid and best ask). Regression analysis gave the formula

$$p(concession = .10) = 7 * i + 5 * i^2 + 6 * n \text{ (for } i = 0, \ldots, 3)^1 \ (R^2 = .82)$$

The result is that the proportion of concessions of Type 1 does not depend on the numerical structure of the other side of the spread, but essentially depends on the structure of the own border point of the spread, which serves as the reference point of the concessions. The dependence of concessions of Type 1 on the size of the spread could be expected.
The result modelled by the regression is presented in Figures 2.1 and 2.2, where in the first the value of i, in the second the value of n (= length of spread) is used as a parameter. (Figure 1 gives the probability as a function of n (= length of spread) for the different classes $i = 1, 2, 3, 4$; in this figure we distinguished inside a class i the cases (a) bid side, best bid is $integer + i * .10$, (b) bid side, best bid is $integer + .50 + i * .10$, (c) ask side, best ask is $integer + i * .10$, (d) ask side, best ask is $integer + .50 + i * .10 + integer$.)

Concessions of Type 3 (.20 from best present bid)

The proportion of concessions of Type 3 is predicted as .8 minus the proportion of concessions of Type 1.

Which side concedes

The question which side concedes depends on the best present bid $x = X + i * .10$, and the best present ask $y = Y - j * .10$ (X, Y multiples of .50, $0 < i, j < 5$). We consider the proportion of concessions of one side relative to the total number of concessions. The results are:

- the proportions of concessions only depend on i and j
- the proportions of concessions of the bid side for $i = \hat{i}$, $j = \hat{j}$ equals the proportion of concessions of the ask side for $i = \hat{j}$, $j = \hat{i}$.
- proportions of concessions are equal for $i = 1, 2, 3, 4$ (and $j = 1, 2, 3, 4$)

The specific values are:

- if $i = 0$ and $j =$ (1 or 2 or 3 or 4) then the proportion of concessions of the bid side is 3/7, of the ask side is 4/7
- if $j = 0$ and $i =$ (1 or 2 or 3 or 4) then the proportion of concessions of the ask side is 3/7, of the bid side is 4/7
- in all other cases both sides have equal proportions of concessions.

[1] for $i = 4$, concessions of Type 1 and 3 coincide, the joint proportion is 80%.

This means that (for each of the values i, j) two grades of prominence are distinguished: i (or j) a multiple of .50, and i (or j) not a multiple of .50. In case that one side is at a multiple of .50, and the other side not, the side with the cruder number concedes less frequently (namely in 3 of 7) cases. In all other cases the frequencies of concessions are equal.

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Appendix

Table 1: Probability of Choosing Opposite Side of Spread as a Function of Own Position (i=column) and Other Side of Spread (j=row), and Corresponding Frequencies of Matchings and Concessions of Bidder in Column Position *)

i=\j=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
probabilities of choosing opposit side of gap [%]:																								
0	87-	-87	75	73	61	49	32	26	28	33	11	20	13	24	24	50	11	.	23	12				
1		74	77	71	58	40	26	20	24	24	16	10	26	17	12	.	33	33	25	.	.			
2			85	85	72	27	27	24	27	27	.	15	13	37	14	.	.	.	.	52	.	56		
3				67	75	38	25	20	19	24	19	14	11	10	13	.	33	.	.	.	.	.	.	
4					68	44	46	23	24	24	.	4	5	11	28	.	.	.	.	.	.	.	.	.
frequencies of matchings+concessions:																								
0	53	414	761	836	3053	153	415	456	426	918	23	55	68	56	222	4	10	7	10	166				
1		43	174	234	1239	100	247	301	256	531	20	37	55	34	80	1	6	5	7	19	4			
2			20	130	925	108	316	316	267	625	9	46	50	55	84	4	9	8	4	23	.	25		
3				3	543	77	263	283	236	461	14	37	45	24	53	4	6	1	2	6	1	.	2	
4					88	29	84	108	80	194	1	16	16	16	32	1	3	1	2	.	1	1	1	.
	<----- box 1 ----->					<----- box 2 ----->					<----- box 3 ----->					<----- box 4 ----->					<----- box 5 --			

*) values a,b,c,d aggregated

Figure 2: proportion of concession by .10 DM (as function of n= length of spread, and i= class of best bid/ask of own side, for conditions a,b,c,d)

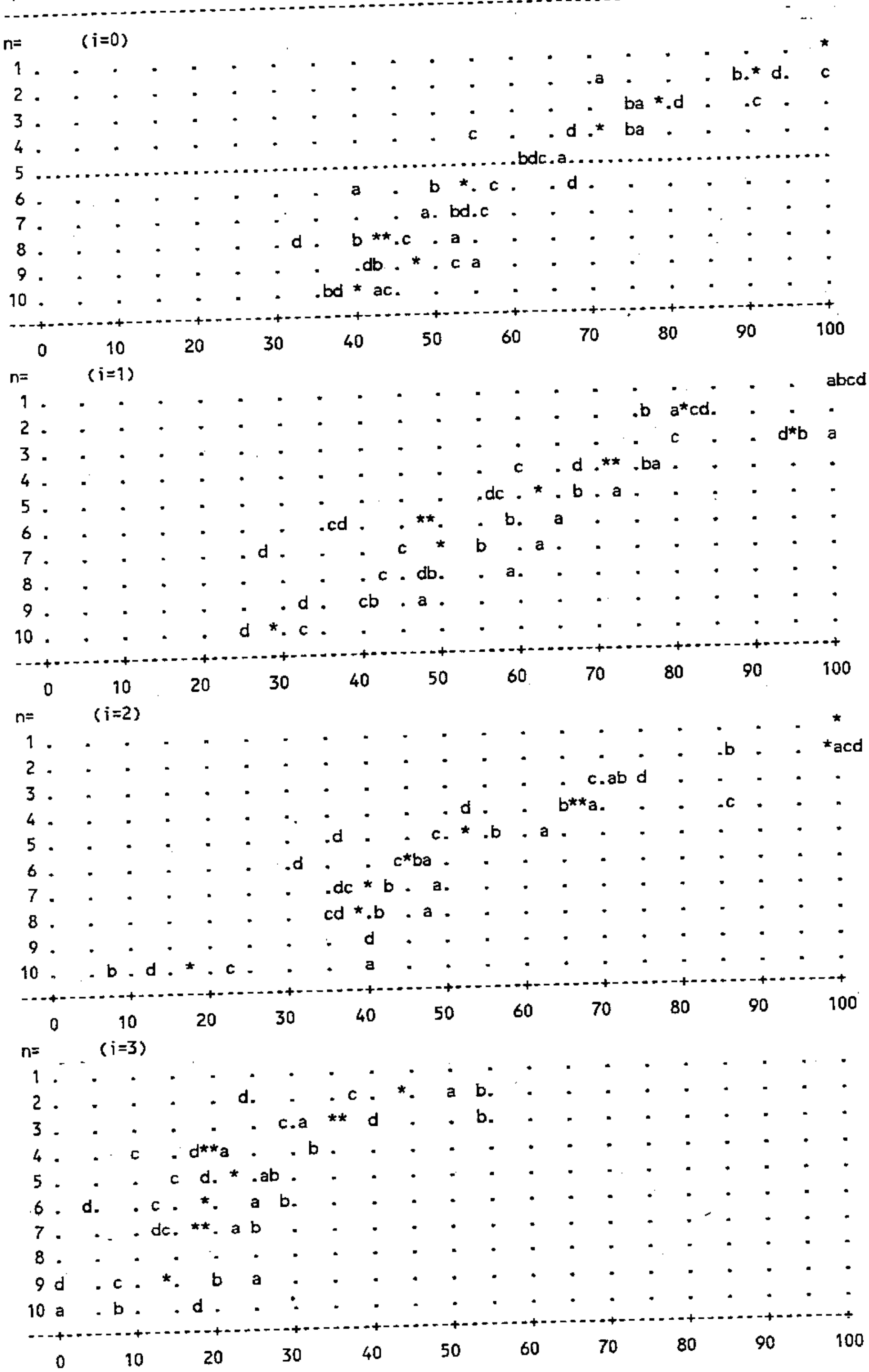

Figure 3.1: Sensitivity of Reactions:
probability of concession by .10 DM for different values of i and n
(i= class of own side of spread, n= length of spread) *)

```
--------------------------------------------------------------------------------
i=   (n= parameter: respective points marked by n)
0,5 .  .  .  .  .  .  .  :  .  .  9 8 . 7 6  .  5.  .4  .  . 3 .  .2  .  .
    .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
    .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
1,6 .  .  .  .  .  .  .  .  . 9.  8  7  6.  . 5 .  .4  .  (3) .  2  .  .
    .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
    .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
2,7 .  .  .  .  .  . (9)  .8  7  6  . 5 .  .  4  .  3  . (2)  .  .  .
    .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
    .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
3,8 .  .  . 9 8 7 6 5 .4  .  3  . 2.  .  .  .  .  .  .  .  .  .  .  .
----+-------+-------+-------+-------+-------+-------+-------+-------+-------+-------+
    0      10      20      30      40      50      60      70      80      90     100 [%]
--------------------------------------------------------------------------------
```

*) values for given i corrected by "regression" of 'probability' on 'n'
(all values are means over a,b,c,d) (values in brackets by crude regression)

Figure 3.2: (same situation as in Figure 2, but i selected as parameter)

```
--------------------------------------------------------------------------------
n=          (i= parameter, respective points marked by i)
 1 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 2 .  .  .  .  .  .  .  .  .  3.  .  .  .  .  .  .  .  .  2  .10 .  .
 3 .  .  .  .  .  .  .  3.  .  .  .  .  .  .  .  2 .  . 10.  .  .  .
 4 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 5 .  .  .  .  .  3.  .  .  .  .  .  2.  . 10.  .  .  .  .  .  .  .
 6 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 7 .  .  .  . 3.  .  .  .  2  .  1 0 .  .  .  .  .  .  .  .  .  .
 8 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 9 .  .  . 3.  .  .  .2  .  10  .  .  .  .  .  .  .  .  .  .  .  .
10 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
---+-------+-------+-------+-------+-------+-------+-------+-------+-------+-------+
   0      10      20      30      40      50      60      70      80      90     100 [%]
```