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## Full Information, Hidden Action and Hidden Information in Principal-Agent Games

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## ABSTRACT

In principal-agent relations boundedly rational behavior is modeled. For the problem in which the level of activity of the agent is observable, the problem of moral hazard and for the problem of hidden information the models are good predictors of the experimental results. In the problem of moral hazard the principals offer one of the contracts which they proposed for each level of activity if the level was observable and the agents select the corresponding level. In the problem of hidden information all types of agents are selected, but the bad type receives a payoff that is too high.

Keywords: principal-agent theory, moral hazard, hidden information, reciprocity

# 1 INTRODUCTION

In principal-agent theory the contracting between a decision maker (the agent) who effects his welfare and the welfare of another person (the principal) by his decision is studied. Modifications of the agent's actions that are preferred by the principal yield negative utility to the agent. The principal is willing to pay a reward for the agent which should make him select a certain action. Another characteristic of the situation is that the principal cannot observe the action of the agent (hidden action) or the ability of the agent to take an action (hidden information). This asymmetric information excludes a "simple" agreement in the contract on payoffs and actions. One question in principal-agent theory is: how can the principal offer a reward (and intend that the agent takes a certain action which is not observable by the principal) that maximizes the principal's payoffs such that the agent takes the intended action by maximizing his own payoff? This is called the problem of incentive compatibility of a contract. The second problem in this context occurs if the principal faces an agent of whom he does not know the ability to take certain actions. Several types of agents with different abilities are possible as contract partners. What kind of contracts are selected in this situation?

Since the first works (Akerlof (1970), Alchian and Demsetz (1972), Ross (1973), Stiglitz (1974)) a lot of theoretical work has been spent on these questions in the last 30 years. In empirical studies often the question has been raised why do agents receive considerable higher payoffs than predicted by existing models? After several years of modeling this problem and observing empirical facts the problem has recently been attacked experimentally (Keser and Willinger (1997), Güth, Klose, Königstein and Schwalbach (1996), Fehr, Kirchsteiger and Riedl (1993), Fehr, Gächter, Kirchsteiger (1997)). In the experimental studies agents do also receive a higher payoff than predicted by the theory.

A possible reason for the experimental result is that reciprocity causes this deviation. Reciprocal behavior has been observed in many experimental situations, for example in the gift exchange experiment (Berg, Dickhaut and McCabe (1995)) or similar games (Güth, Ockenfels and Wendel (1997), Bolle (1995), Dufwenberg and Gneezy (1996), Jacobsen and Sadrieh (1996)), comparable games in the labor market context (Fehr, Kirchsteiger and Riedl (1993), Fehr, Gächter, Kirchsteiger (1997)) or games in extensive form (McCabe, Rasenti and Smith (1998), Vogt (1998a)). In these experiments most of the deviations from rational behavior are attributed to reciprocity. A quantitative modeling of reciprocity was performed in (M. Rabin (1993)). A further modeling explaining a lot of experimental results can be found in (E. Fehr and K. Schmidt (1998), G.E. Bolton and A. Ockenfels (1998)).

Reciprocity is characterized (McCabe, Rasenti and Smith (1998)) as "a specialized mental algorithm (Cosmides (1985), Cosmides and Tooby (1992)) in which long term self interest is best served by promoting an image both to others and yourself that cheating on cooperative social exchange (either explicit or implicit) is punished (negative reciprocity), and initiation of cooperative social exchange is rewarded (positive reciprocity)." This behavior is also observed in single games. The number of players that initiate cooperation increases if it is possible to punish and their own cost of punishment is not too high.

In the principal-agent relation reciprocity causes too high payments by the principal because he wants to achieve a positive reaction (selection of the intended action) of the agent.

Another explanation of the result is due to the special form of the principal-agent relation. The principal proposes a reward. This is similar to an ultimatum game in which a similar deviation from the rational solution towards higher payoffs of the responder (who corresponds to the agent) are observed (for a review of ultimatum bargaining and experiments on ultimatum bargaining see Güth and Tietz (1990), Güth (1995), Camerer and Thaler (1995),

Roth (1995)). It seems as if fairness criteria influence the principal agent problem. For a reciprocity game and ultimatum games simple criteria based on the numerical perception as described in the theory of prominence (Albers (1997), Albers (1999)) are given in Vogt (1998a) and in Vogt (1998b). The criteria for the principal-agent relation and the relations to ultimatum games and reciprocity are examined in this paper.

The interaction between principal and agent is modeled as a game and not as an optimization problem of the principal (as partly in the literature), because two persons are interacting strategically in this context. The optimization problem is obtained by considering the subgame perfect equilibrium of these games.

First the situation is considered in which the principal knows which action the agent selects. A model of boundedly rational behavior in this situation is given and experimentally tested. Then the problem of hidden action (moral hazard) and the problem of hidden information (if the principal faces agents of different types) are considered. Models based on the model of boundedly rational behavior in the situation with full information are developed and experimentally tested. In these games not only positive payoffs, but also negative payoffs are considered.

The models of boundedly rational behavior consists of several rules. Each of these rules corresponds to a certain substructure of the game. Therefore the problem of moral hazard and hidden information can be modeled by the same principle rules with slight modifications corresponding to the modification of the games. The model predicts a considerable payoff of the agent in the problem of moral hazard. In the hidden information problem the better type of the agent determines the payoff of the agent. In contrast to other models in which the agent of the bad type is selected the good and bad type are selected in this model, but the bad type of

the agent receives a payoff that is too high. The model of boundedly rational behavior is a good predictor for the experimental data.

## 2 THE PRINCIPAL-AGENT GAMES

The situation with full information is denoted as OA (one action) and described by means of Figure 1a. Five states of the world ( $U_1$ ,  $U_2$ ,  $U_3$ ,  $U_4$  and  $U_5$ ) are possible with equal probabilities. The agent decides whether to take an action or not. If he does not take the action<sup>1</sup> the principal and the agent receive a payoff of 0. If he takes the action his cost is 20. Then the payoff of the principal depends on a the state of the world which is realized. The two payoffs 100 and 0 are possible with different probabilities 0.8 for the payoff of 100 and 0.2 for the payoff of 0. Before the agent decides whether to take an action or not the principal can make a contract with the agent. In this contract it is specified that the agent takes the action and receives a money transfer from the principal of  $w_1$  if the principal gets 100 and a transfer of  $w_2$  if the principal gets a payoff of 0.  $w_1=w_2$  correspond to a fixed transfer to the agent, otherwise the transfer depends on the payoff of the principal (the chance move). The question is: which pairs  $(w_1, w_2)$  will be specified in the contract?

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<sup>1</sup> The agent's action corresponds to his level of activity if more actions are possible.

FIGURE 1: — PRINCIPAL-AGENT GAMES WITH FULL INFORMATION

1a: Game OA

Payoff of the principal in					Cost of the Agent
U <sub>1</sub>	U <sub>2</sub>	U <sub>3</sub>	U <sub>4</sub>	U <sub>5</sub>	
100	100	100	100	0	20

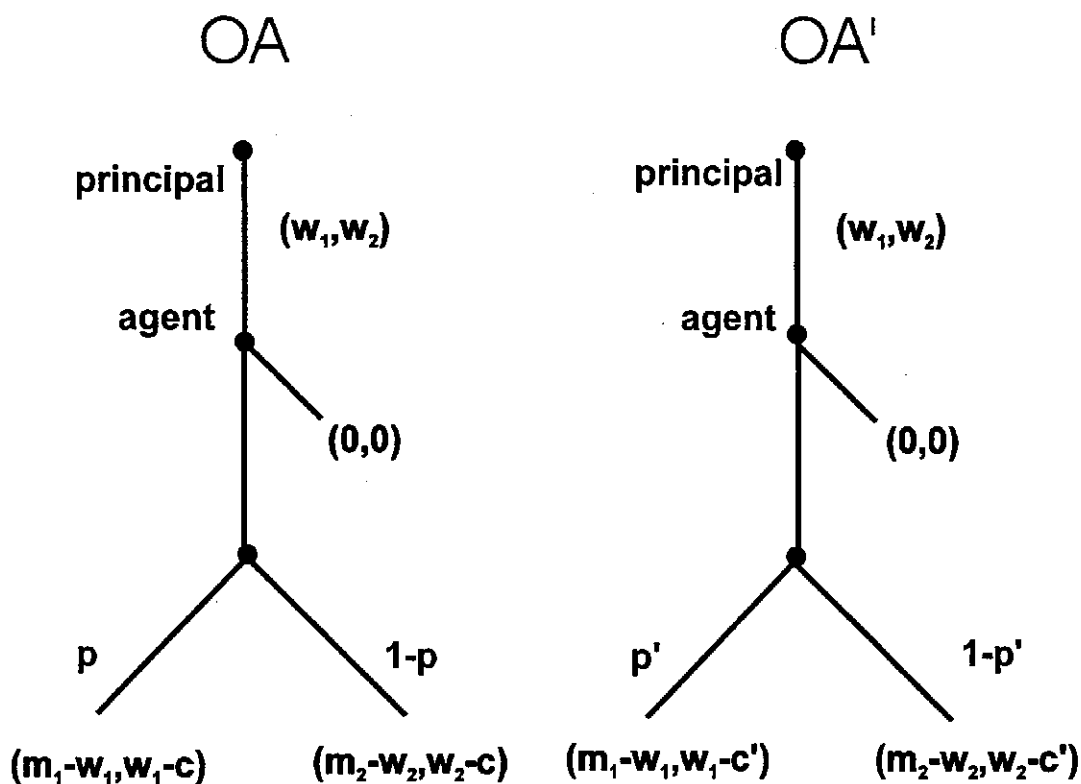
1b: Game OA'

Payoff of the principal in					cost of the agent
U <sub>1</sub>	U <sub>2</sub>	U <sub>3</sub>	U <sub>4</sub>	U <sub>5</sub>	
100	0	0	0	0	0

The same game is played with different probabilities of 0.2 for the payoff 100 and 0.8 for the payoff of 0 and different cost of 0. This game is called OA' and is shown in Figure 1b.

One might also look at the extensive form of games of this type. It is shown in Figure 2. The higher payoff of the principal is denoted as  $m_1$  and the lower payoff as  $m_2$  ( $m_1 > m_2$ ). The cost in game OA is denoted as  $c$  and in game OA' as  $c'$  ( $c > c'$ ). The probability of the payoff  $m_1$  in game OA is denoted as  $p$  and the probability of  $m_2$  in game OA is  $1-p$ . In game OA' the probability of the payoff  $m_1$  is denoted as  $p'$  and the probability of  $m_2$  is  $1-p'$  (with  $p > p'$ ). The principal makes a proposal  $(w_1, w_2)$  to the agent. If the agent rejects the proposal the payoff is  $(0, 0)$ . If he accepts a chance move decides about the payoffs that depend on  $(w_1, w_2)$  and  $m_1, m_2$  and the cost according to the game described above.

FIGURE 2: — THE EXTENSIVE FORM OF THE GAMES OA AND OA'



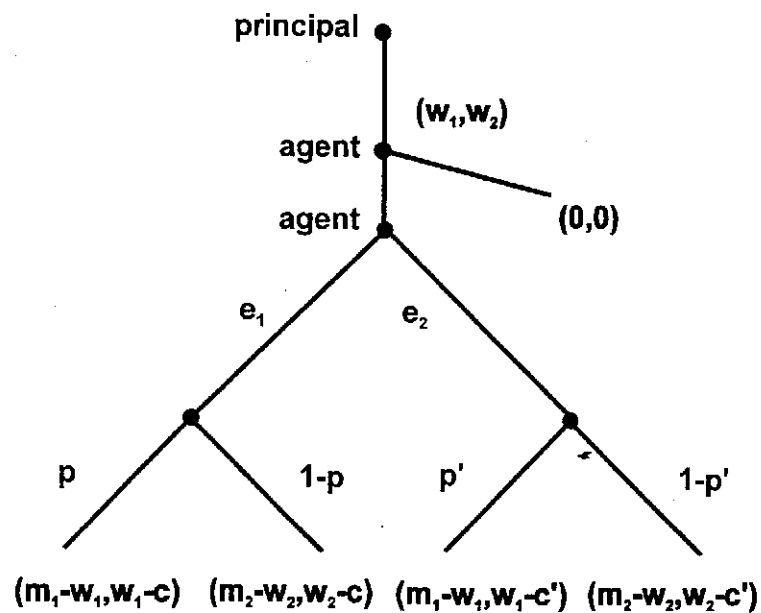
These games could be denoted as ultimatum games with prospects as payoff. The maximal payoff of the principal is a prospect: for example in game OA it is the prospect  $[m_1(p), m_2(1-p)]$  (payoff  $m_1$  with probability  $p$  and payoff  $m_2$  with probability  $1-p$ ). He can make a transfer to the agent to receive this prospect. The transfer proposal he makes is also a prospect: for example in game OA it is  $[(w_1-c)(p), (w_2-c)(1-p)]$  (payoff  $w_1-c$  with probability  $p$  and payoff  $w_2-c$  with probability  $1-p$ ). The expected payoff sum is  $m_1 \cdot p + m_2 \cdot (1-p) - c$ , i.e. the expected maximal payoff of the principal minus the cost of the agent.

The question examined in these games is: On which contracts do principal and agent agree? It is therefore played for different values of the parameters  $m_i$ ,  $c_i$  and  $p_i$ .



Starting with these games two other games are constructed shown in Figures 3 and 4. The difference between the game OA shown in Figure 2 and the game TA (two actions) shown in Figure 3 is that in the game TA the agent can select between two actions  $e_1$  and  $e_2$  after accepting the contract<sup>2</sup>. The action  $e_1$  corresponds to the game OA and  $e_2$  corresponds to game OA'. In addition to the games OA and OA' the incentive compatibility constraint is important in this game: if the principal offers a pair  $(w_1, w_2)$  and intends that the agent selects a certain action  $e_i$  ( $i=1,2$ ), does the agent select the intended action or not?

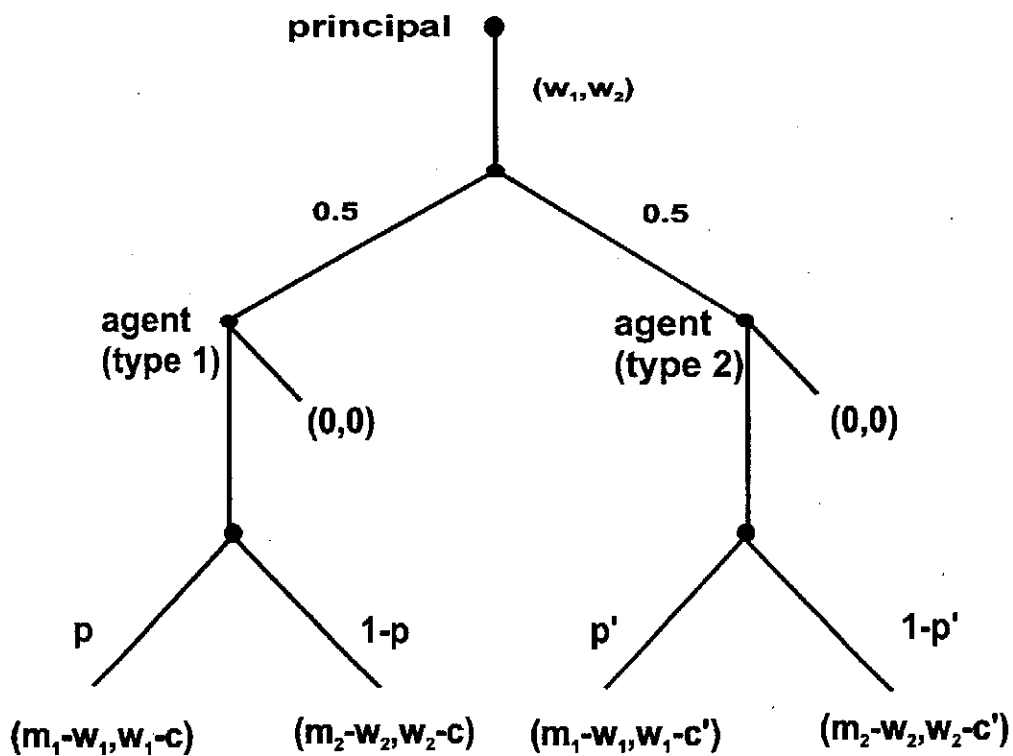
FIGURE 3: — THE EXTENSIVE FORM OF THE GAME TA



<sup>2</sup> In a game of this kind it is modeled that the principal cannot observe the action of the agent.

Another problem is considered in the game TT (two types) in Figure 4. Again the structure of the game TT can be explained by a comparison with the games OA and OA'. The agent in game TT can be of two types. Type 1 corresponds to the game OA and type 2 corresponds to the game OA'. The principal is facing an agent who could be of type 1 with probability 0.5 or of type 2 with probability 0.5. The agent knows his type, but the principal does not. Which payoff pair does the principal propose? How is the behavior of the principal if different types of agents exist?

FIGURE 4: — THE EXTENSIVE FORM OF THE GAME TT



### 3 MODELS

Before modeling the possible solutions of the games a short introduction in the theory of prominence in the decimal system (Albers and Albers (1983), Albers (1997), Albers (1999)) is given, because the perception of payoffs as modeled in this theory is fundamental to some models.

#### 3.1 *The theory of prominence*

The theory of prominence is based on the empirical observation that some numbers are easier accessible than others. The most easily accessible numbers are called the prominent numbers (or full step numbers)  $P$  which are:

$$P = \{n \cdot 10^z \mid z \in \mathbb{Z}, n \in \{1, 2, 5\}\} = \{\dots, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, \dots\}$$

In the theory of prominence the perception of numbers (including payoffs) is described by "steps". By definition the difference between two neighbored prominent numbers (ordered according to their size) is one step. In specific problems (or tasks) different smallest amounts of money are important. For example in the annual budget of a state \$ 1 billion might be perceived as the smallest "important" amount, whereas it might be between \$ 1 to \$ 5 for the price of a dinner. In the theory of prominence this is modeled by assuming a "finest perceived full step" unit  $\Delta$ . The difference between zero and this number is perceived as one step. This defines a perception function  $v_\Delta$  by  $v_\Delta(\Delta) = 1$  for positive payoffs. Between the full steps it is linearly interpolated.

The perception of negative numbers is modeled as for positive numbers with one exception: steps in the range of negative numbers count double similar to the prospect theory

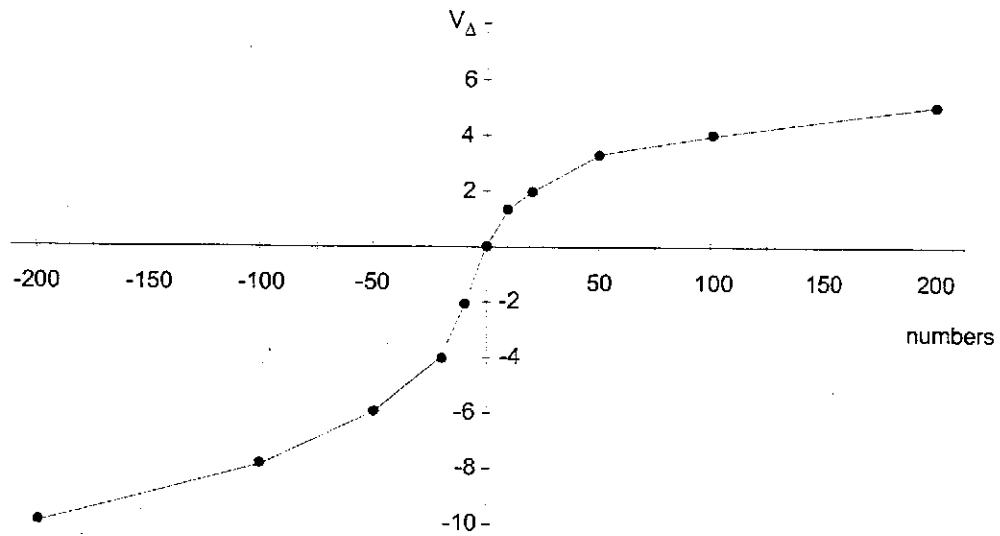
(Kahneman and Tversky, 1979, Tversky and Kahneman, 1992) where the negative payoffs get a higher weight than positive payoffs). This defines a perception function for negative numbers  $-x$ , i.e.  $v_{\Delta}(-x) = -2 \cdot v_{\Delta}(|-x|)$ . More complicated models are discussed in (Albers 1999). These models do not change the results drastically. For reasons of simplicity this model is used.

In a specific task (the experiment, a game) the finest perceived full step  $\Delta$  has to be determined. According to Albers (1997) the best rule which can be deduced from subjects' behavior (in different tasks) is the following rule:

$\Delta$  is the prominent number 2 steps below the smallest prominent number greater than or equal to the maximal absolute payoff in the task.

This results in a perception function as given in Figure 5 for  $\Delta=10$ . For different values of  $\Delta$  one obtains different perception functions. For positive numbers greater than the finest perceived full step  $\Delta$  the perception function has a "logarithmic shape" with linear interpolation between the prominent numbers. This is similar to the Weber-Fechner law of psychophysics (for example reprinted in Fechner (1968)) which predicts a logarithmic perception above a smallest unit. For numbers the smallest unit which is task dependent permits the extension of the perception function to negative numbers.

FIGURE 5: — THE  $V_{\Delta}$ -FUNCTION FOR  $\Delta=10$



### 3.2 Game OA and Game OA'

#### 3.2.1 The linear model

In this model it is assumed that the probabilities and payoffs are perceived linearly.

Game OA:

The game theoretic solution of the game OA is the subgame perfect equilibrium. The principal proposes a pair  $(w_1^*, w_2^*)$  such that the expected payoff of the agent is zero as in ultimatum games (as long as the expected payoff of the principal is positive).

This results in the two conditions:

$$p \cdot (m_1 - w_1^*) + (1-p) \cdot (m_2 - w_2^*) \geq 0 \quad (\text{individual rationality for the principal})$$

and

$$p \cdot (w_1^* - c) + (1-p) \cdot (w_2^* - c) = 0 \quad (\text{individual rationality for the agent; IR})$$

or

$$p \cdot w_1^* + (1-p) \cdot w_2^* = c$$

and

$$p \cdot m_1 + (1-p) \cdot m_2 - c \geq 0.$$

This can be interpreted as: The expected payoff of the agent is equal to his cost and the maximal expected payoff of the principal is equal to the maximal expected payoff minus the cost of the agent.

For game OA' the analogous result is:

$$p' \cdot (m_1 - w_1^*) + (1-p') \cdot (m_2 - w_2^*) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p' \cdot (w_1^* - c') + (1-p') \cdot (w_2^* - c') = 0 \quad (\text{individual rationality for the agent; IR})$$

or

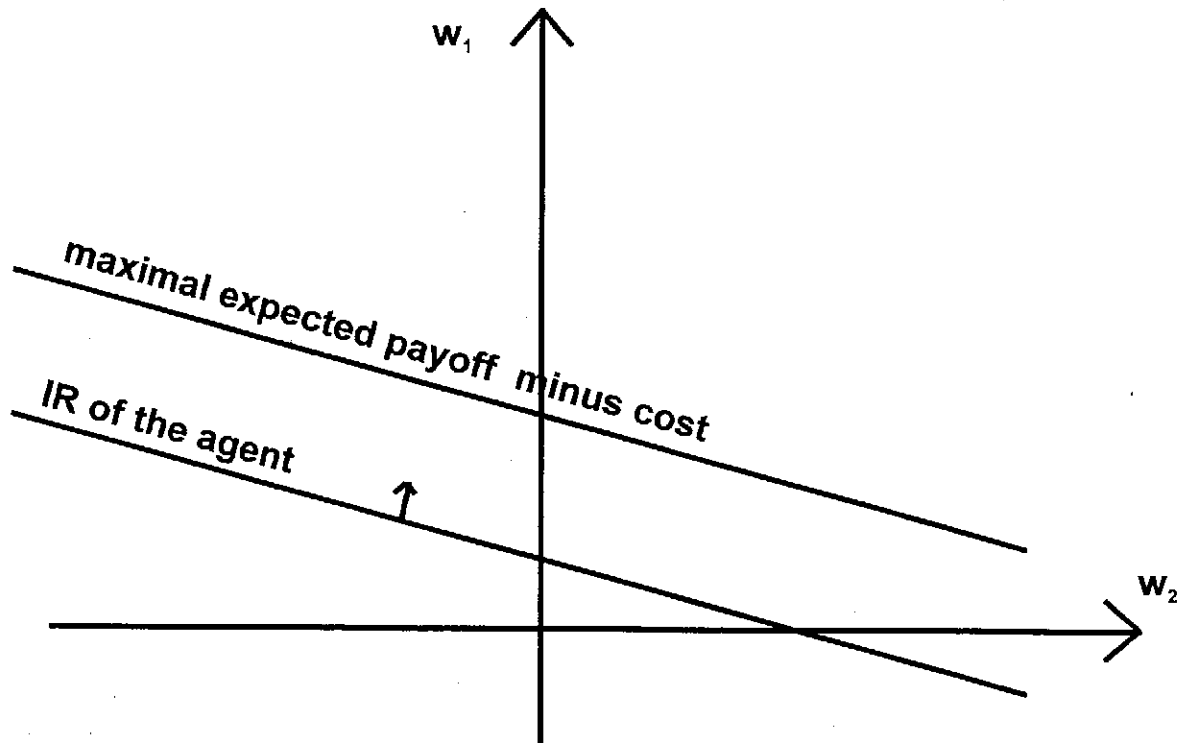
$$p' \cdot w_1^* + (1-p') \cdot w_2^* = c' \text{ and}$$

$$p' \cdot m_1 + (1-p') \cdot m_2 - c' \geq 0.$$

This can be interpreted as in game OA'.

In general a solution of this problem results not in one pair  $(w_1, w_2)$  as solution of this problem, but infinitely many pairs as shown in Figure 6.

FIGURE 6: —  $(w_1^*, w_2^*)$  OF THE SUBGAME PERFECT EQUILIBRIUM IN  
THE GAMES OA AND OA'



In general the payoffs need not represent the utility of the players in an experiment as assumed in the theory.

Replacing  $p$  by weighting functions  $\Pi_P(p)$ ,  $\Pi_A(p)$  and payoffs  $x$  by value functions  $v_P(x)$ ,  $v_A(x)$  for the principal and agent, respectively leads to the following conditions:

Game OA:

$$\Pi_P(p) \cdot v_P(m_1 - w_1^*) + \Pi_P(1-p) \cdot v_P(m_2 - w_2^*) \geq 0 \quad (\text{individual rationality for the principal})$$

$$\Pi_A(p) \cdot v_A(w_1^* - c) + \Pi_A(1-p) \cdot v_A(w_2^* - c) = 0 \quad (\text{individual rationality for the agent})$$

Game OA':

$$\Pi_P(p') \cdot v_P(m_1 - w_1^*) + \Pi_P(1-p') \cdot v_P(m_2 - w_2^*) \geq 0 \quad (\text{individual rationality for the principal})$$

$$\Pi_A(p') \cdot v_A(w_1^* - c') + \Pi_A(1-p') \cdot v_A(w_2^* - c') = 0 \quad (\text{individual rationality for the agent})$$

In principle  $\Pi$ - and  $v$ -function need not be the same for the principle and the agent. In the experiment the values of  $p$  and  $p'$  were selected such that  $\Pi(p_i) \cong p_i$  according to the prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)) and the theory of prominence (Albers (1997), Albers (1999)).

Assuming that the  $v$ -functions are the same for both players which should be the case for the average player according to the theory of prominence the conditions are:

Game OA:

$$p \cdot v(m_1 - w_1^*) + (1-p) \cdot v(m_2 - w_2^*) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p \cdot v(w_1^* - c) + (1-p) \cdot v(w_2^* - c) = 0 \quad (\text{individual rationality for the agent})$$

Game OA':

$$p' \cdot v(m_1 - w_1^*) + (1-p') \cdot v(m_2 - w_2^*) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p' \cdot v(w_1^* - c') + (1-p') \cdot v(w_2^* - c') = 0 \quad (\text{individual rationality for the agent})$$

In general these conditions do not lead to one pair  $(w_1, w_2)$  as solution, because  $w$  is only replaced by  $v(w)$ . For reasons of simplicity only the linear model is considered in the following. Assuming the  $v$ -function of the prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)) and the theory of prominence (Albers (1997), Albers (1999)) the corresponding predictions will be tested by the experimental data.



### 3.2.2 Modeling of boundedly rational behavior

In the following part a model of boundedly rational behavior is introduced. In this model it is taken into account that the games OA and OA' can be regarded as ultimatum games with prospects as payoffs. The main part of the model consists of four rules (rule 0 - rule 3) which determine a fair proposal. By means of rule 4 the non negativity of the payoffs is implemented as a condition.

The rules are:

**Rule 0 (The separation of the cases):** The fair  $w_1^*$  is obtained by playing an ultimatum game between principal and agent for the payoff  $m_1$  and  $w_2^*$  is obtained by playing another ultimatum game between principal and agent for the payoff  $m_2$ .

This rule implies that the pair  $(w_1^*, w_2^*)$  does not depend on the probabilities. An interpretation of this rule is that principal and agent bargain for the good and the bad case separately.

For the determination of the outcome of an ultimatum game the model given and tested in Vogt (1998b) is used. It will be explained by the next three rules. The principal corresponds to the player 1 and the agent corresponds to player 2 in Vogt (1998b).

**Rule 1** is denoted as "the perception of payoff", **rule 2** as "the strength of the principal" and **rule 3** as the "equal sum of concessions".

**Rule 1 (The perception of payoffs):** The payoffs are perceived according to the theory of prominence. All payoffs are transformed by the  $v_{\Delta}$ -function.

**Rule 2 (The strength of the principal):** The proposal of the principal is such that he gets at least the payoff of the agent. This takes into account that the principal has the right to propose a division which gives him a stronger position than the agent.

**Rule 3 (Equal sum of concessions):** Given the maximal and minimal payoffs of both the principal and the agent the proposed and accepted payoff pair  $(x_L, y_L)^3$  is determined via the modified Kalai-Smorodinsky criterion (Vogt and Albers (1997)). This criterion is based on the equality of the sum of concessions measured as differences of the transformed payoffs.

Rule 4 determines whether the perceived payoff for a pair of wages is non negative for both players.

**Rule 4 (Selection of the best alternative):** If the principal or the agent can select between different alternatives, they select the alternative that gives them the highest perceived payoff.

In the following this is applied to the games studied in this paper.

The modeling of **the strength of the principal** determines the maximal payoffs: In game OA the maximal payoff of the principal is  $x_B = m_I - c$  and the maximal payoff of the agent is  $y_B = (m_I - c)/2$  (in this case the principal gets  $(m_I - c)/2$  which is his lowest payoff according to the condition that he gets at least the payoff of the agent).

**Equal sum of concessions:** Given the maximal and minimal payoffs of both the principal and the agent the modified Kalai-Smorodinsky criterion (Vogt and Albers (1997)) which is a

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<sup>3</sup> In the following the payoffs of the principal are denoted as  $x$  and the payoffs of the agent as  $y$ .

good predictor for experimental data in two person bargaining about pareto-optimal payoff pairs is applied<sup>4</sup>. The criterion is explained in Figure 7: The possible concessions of both players to reach an agreement  $(x_L, y_L)$  are given in the figure.

FIGURE 7: — THE CONCESSIONS OF BOTH PLAYERS

	the principal	the agent
concession below the maximal payoff	$v_{\Delta}(x_B) - v_{\Delta}(x_L)$	$v_{\Delta}(y_B) - v_{\Delta}(y_L)$
concession above the minimal payoff (0,0)	$v_{\Delta}(y_L) - v_{\Delta}(0)$	$v_{\Delta}(x_L) - v_{\Delta}(0)$
sum	$v_{\Delta}(x_B) - v_{\Delta}(x_L)$ + $v_{\Delta}(y_L) - v_{\Delta}(0)$	$v_{\Delta}(y_B) - v_{\Delta}(y_L)$ + $v_{\Delta}(x_L) - v_{\Delta}(0)$

$v_{\Delta}(x_B) - v_{\Delta}(x_L)$  is the concession the principal makes if he lowers his payoff below his maximal payoff.  $v_{\Delta}(y_L) - v_{\Delta}(0)$  is the concession he makes by giving the agent a certain amount above the agent's conflict payoff 0. All these concessions are measured in differences of the perceived payoffs. The analogous consideration is shown for the agent in table I.

The equal sum of concessions results in the criterion for the proposed and accepted payoff pair  $(x_L, y_L)$ , it is:

$$v_{\Delta}(x_B) - v_{\Delta}(x_L) + (v_{\Delta}(y_L) - v_{\Delta}(0)) = v_{\Delta}(y_B) - v_{\Delta}(y_L) + (v_{\Delta}(x_L) - v_{\Delta}(0))$$

or

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<sup>4</sup> A detailed description of the criterion and its connection to the Kalai-Smorodinsky criterion is given in Vogt and Albers (1997).

The payoff pair  $(x_L, y_L)$  is proposed and accepted for which it holds:

$$v_{\Delta}(x_B) - v_{\Delta}(x_L) - (v_{\Delta}(x_L) - v_{\Delta}(0)) = v_{\Delta}(y_B) - v_{\Delta}(y_L) - (v_{\Delta}(y_L) - v_{\Delta}(0))$$

Applying this to game OA the following conditions are obtained for  $w_1^{*1}$  and  $w_2^{*1}$ :<sup>5</sup>

$$w_1^{*1}: \quad (v_{\Delta}(m_1 - c) - v_{\Delta}(m_1 - w_1^{*1})) - (v_{\Delta}(m_1 - w_1^{*1}) - v_{\Delta}(0)) = \\ (v_{\Delta}((m_1 - c)/2) - v_{\Delta}(w_1^{*1} - c)) - (v_{\Delta}(w_1^{*1} - c) - v_{\Delta}(0))$$

$$w_2^{*1}: \quad (v_{\Delta}(m_2 - c) - v_{\Delta}(m_2 - w_2^{*1})) - (v_{\Delta}(m_2 - w_2^{*1}) - v_{\Delta}(0)) = \\ (v_{\Delta}((m_2 - c)/2) - v_{\Delta}(w_2^{*1} - c)) - (v_{\Delta}(w_2^{*1} - c) - v_{\Delta}(0))$$

For the game OA' the following conditions are obtained for  $w_1^{*2}$  and  $w_2^{*2}$ :<sup>6</sup>

$$w_1^{*2}: \quad (v_{\Delta}(m_1 - c') - v_{\Delta}(m_1 - w_1^{*2})) - (v_{\Delta}(m_1 - w_1^{*2}) - v_{\Delta}(0)) = \\ (v_{\Delta}((m_1 - c')/2) - v_{\Delta}(w_1^{*2} - c')) - (v_{\Delta}(w_1^{*2} - c') - v_{\Delta}(0))$$

$$w_2^{*2}: \quad (v_{\Delta}(m_2 - c') - v_{\Delta}(m_2 - w_2^{*2})) - (v_{\Delta}(m_2 - w_2^{*2}) - v_{\Delta}(0)) = \\ (v_{\Delta}((m_2 - c')/2) - v_{\Delta}(w_2^{*2} - c')) - (v_{\Delta}(w_2^{*2} - c') - v_{\Delta}(0))$$

By means of rule 4 it is checked whether the evaluation of the payoff of the principal and the agent is non negative for the obtained pair  $(w_1^*, w_2^*)$ . If this is the case the pair  $(w_1^*, w_2^*)$  is the solution, otherwise it is  $(0, 0)$ . The principal and the agent evaluate the payoffs according to the theory of prominence (Albers (1997), Albers (1999)). In game OA the payoff of the principal is a prospect  $[(m_1 - w_1^{*1})(p), (m_2 - w_2^{*1})(1-p)]$  with payoffs  $m_1 - w_1^{*1}$  and  $m_2 - w_2^{*1}$  and the probabilities  $p$  and  $1-p$ , respectively. It is evaluated by:

$$v_{\Delta}([(m_1 - w_1^{*1})(p), (m_2 - w_2^{*1})(1-p)]) = \Pi(p) \cdot v_{\Delta}((m_1 - w_1^{*1})) + \Pi(1-p) \cdot v_{\Delta}((m_2 - w_2^{*1}))$$

<sup>5</sup> These conditions hold for  $m_i - c > 0$  ( $i=1,2$ ). For  $m_i - c < 0$  the maximal payoffs of the agent and the principal have to be exchanged according to the advantage of the principal, because  $(m_i - c)/2 > (m_i - c)$ .

<sup>6</sup> Corresponding to game OA these conditions hold for  $m_i - c' > 0$  ( $i=1,2$ ). For  $m_i - c' < 0$  the maximal payoffs of the agent and the principal have to be exchanged according to the advantage of the principal, because  $(m_i - c')/2 > (m_i - c')$ .

with  $\Pi(p)$  the weighting function of the probabilities and  $v_{\Delta}(m)$  the value function of payoffs. Because  $\Pi(0.8) \cong 0.8$  and  $\Pi(0.2) \cong 0.2$  for the probabilities  $p$  and  $p'$  used in the experiment, the evaluation is:

$$v_{\Delta}([(m_1 - w_1^{*1})(p), (m_2 - w_2^{*1})(1-p)]) = p \cdot v_{\Delta}((m_1 - w_1^{*1})) + (1-p) \cdot v_{\Delta}((m_2 - w_2^{*1}))$$

Analogously the payoff of the agent that is the prospect  $[(w_1^{*1} - c)(p), (w_2^{*1} - c)(1-p)]$  is evaluated by:

$$v_{\Delta}([(w_1^{*1} - c)(p), (w_2^{*1} - c)(1-p)]) = p \cdot v_{\Delta}((w_1^{*1} - c)) + (1-p) \cdot v_{\Delta}((w_2^{*1} - c))$$

If both evaluations are non negative the pair  $(w_1^{*1}, w_2^{*1})$  is the solution of the game. Otherwise  $(0,0)$  is the solution. If for both  $m_i$  it holds:  $m_i - c > 0$  (only gains) the fair proposal itself ensures that the evaluation of the payoffs for the principal and the agent are non negative.

For game OA' the analogous evaluation is performed for the payoff of the the principal and the payoff of the agent:

$$\text{principal: } v_{\Delta}([(m_1 - w_1^{*2})(p'), (m_2 - w_2^{*2})(1-p')]) = p' \cdot v_{\Delta}((m_1 - w_1^{*2})) + (1-p') \cdot v_{\Delta}((m_2 - w_2^{*2}))$$

$$\text{agent: } v_{\Delta}([(w_1^{*2} - c')(p'), (w_2^{*2} - c')(1-p')]) = p' \cdot v_{\Delta}((w_1^{*2} - c')) + (1-p') \cdot v_{\Delta}((w_2^{*2} - c'))$$

Whether  $(w_1^{*2}, w_2^{*2})$  is the solution of the game is determined by the condition of non negativity as in game OA.

### 3.3 Game TA

#### 3.3.1 The linear model

For this game the subgame perfect equilibrium is the solution. In addition to the games OA and OA' the constraint of incentive compatibility (IC) has to be introduced. It is that the agent

selects action  $e_1$  if his payoff is higher for this action than for action  $e_2$  and vice versa. This results in an additional condition:

The agent selects action  $e_1$  if:

$$p \cdot (w_1 - c) + (1 - p) \cdot (w_2 - c) > p' \cdot (w_1 - c') + (1 - p') \cdot (w_2 - c').$$

The condition of individual rationality for action  $e_1$  is the same as for the corresponding game OA and the condition of individual rationality for action  $e_2$  is the same as for the corresponding game OA'.

Action  $e_1$ :

$$p \cdot (m_1 - w_1) + (1 - p) \cdot (m_2 - w_2) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p \cdot (w_1 - c) + (1 - p) \cdot (w_2 - c) = 0 \quad (\text{individual rationality for the agent})$$

or

$$p \cdot w_1 + (1 - p) \cdot w_2 = c \text{ and}$$

$$p \cdot m_1 + (1 - p) \cdot m_2 - c \geq 0.$$

Action  $e_2$ :

$$p' \cdot (m_1 - w_1) + (1 - p') \cdot (m_2 - w_2) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p' \cdot (w_1 - c') + (1 - p') \cdot (w_2 - c') = 0 \quad (\text{individual rationality for the agent})$$

or

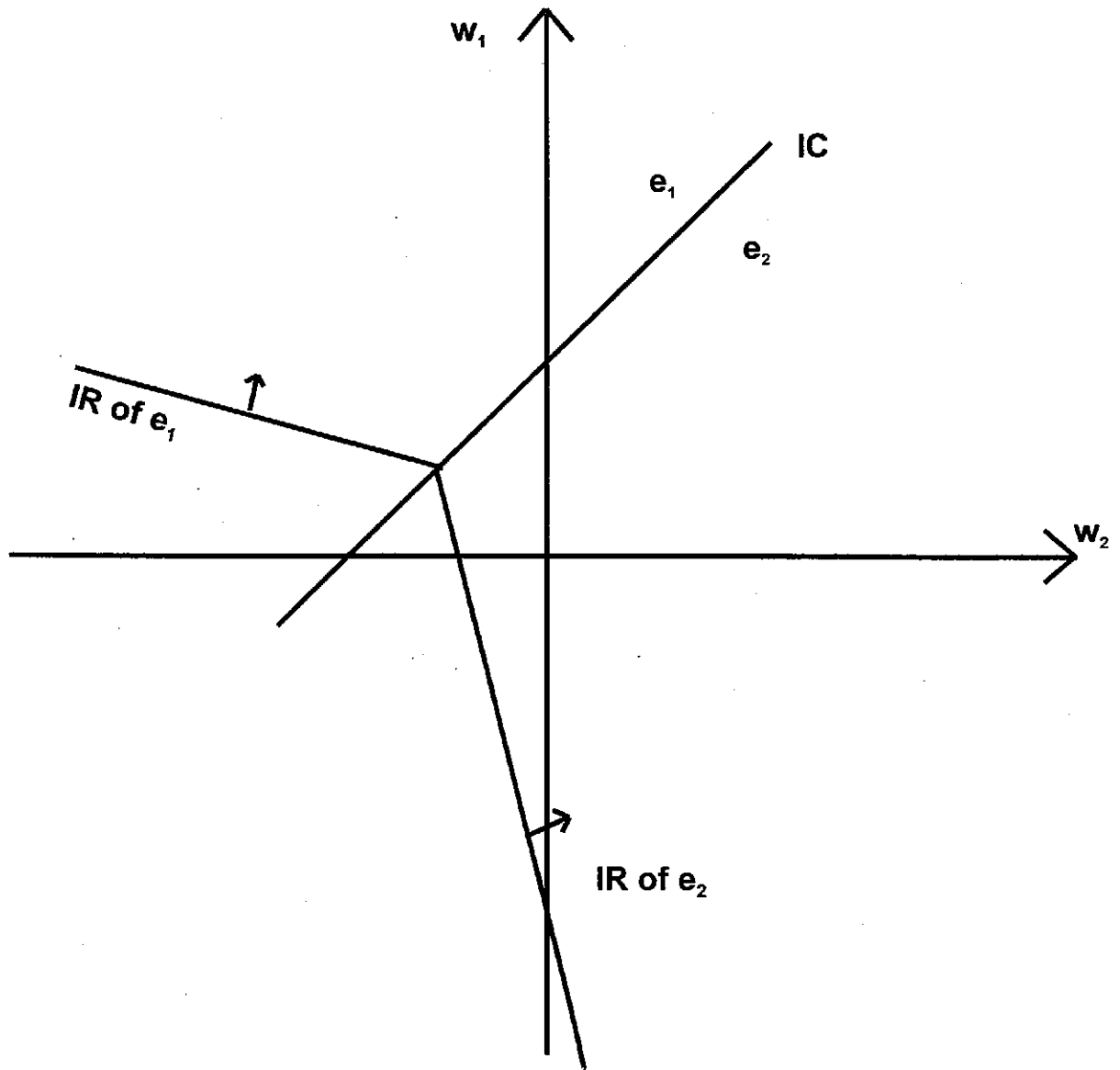
$$p' \cdot w_1 + (1 - p') \cdot w_2 = c' \text{ and}$$

$$p' \cdot m_1 + (1 - p') \cdot m_2 - c' \geq 0.$$

These conditions allow the same interpretation for each action as in game OA or OA': the expected payoff of the agent is equal to his cost and the maximal expected payoff of the principal is equal to the maximal expected payoff minus the cost of the agent for each action.

A solution fulfilling all conditions is characterized by: The principal proposes a pair  $(w_1^*, w_2^*)$  that gives him maximal expected payoff minus cost of the agent and fulfills the corresponding incentive compatibility constraint. The situation is displayed in Figure 8.

FIGURE 8: —  $(w_1^*, w_2^*)$  OF THE SUBGAME PERFECT EQUILIBRIUM IN THE GAME TA





### 3.3.2 Modeling of boundedly rational behavior

In this model the possible proposals for the action  $e_1$  and for the action  $e_2$  are determined via the rules 0 - 3 (given in the model of boundedly rational behavior for the games OA and OA'). By means of rule 4 (given in the model for the games OA and OA') out of the possible proposals the non negative one resulting in the highest perceived payoff is selected as proposal which will be accepted (with the selection of the intended action).

As described in the section games action  $e_1$  corresponds to game OA. Because of this the proposal  $(w_1^{*1}, w_2^{*1})$  in game OA is the same as the proposal for action  $e_1$ . Because action  $e_2$  corresponds to game OA' (compare the section games) the solution  $(w_1^{*2}, w_2^{*2})$  for game OA' is the same as the proposal for action  $e_2$ .

By means of rule 4 one of the pairs  $(w_1^{*1}, w_2^{*1})$  or  $(w_1^{*2}, w_2^{*2})$  for the actions is selected by the principal. The pair is selected according to the evaluation of the corresponding prospects by the principal. The principal evaluates the prospect according to the theory of prominence. (Albers (1997), Albers (1999)). If both evaluations are negative no solution is obtained. If one evaluation is non negative the action connected with the higher evaluation is selected. The pair  $(w_1^{*1}, w_2^{*1})$  for  $e_1$  is selected if the evaluation of the payoff of the principal for this action is higher than or equal to the payoff for  $e_2$ :

$$v_{\Delta}([(m_1 - w_1^{*1})(p), (m_2 - w_2^{*1})(1-p)]) \geq v_{\Delta}([(m_1 - w_1^{*2})(p'), (m_2 - w_2^{*2})(1-p')]),$$

$$\text{i.e. } p \cdot v_{\Delta}((m_1 - w_1^{*1})) + (1-p) \cdot v_{\Delta}((m_2 - w_2^{*1})) \geq p' \cdot v_{\Delta}((m_1 - w_1^{*2})) + (1-p') \cdot v_{\Delta}((m_2 - w_2^{*2})).$$

Otherwise the pair  $(w_1^{*2}, w_2^{*2})$  for  $e_2$  is selected.

The agent accepts the pair and selects the corresponding action.

### 3.4 Game TT

#### 3.4.1 The linear model

As described in the section games type 1 corresponds to the game OA and type 2 to the game OA'. For this game the subgame perfect equilibrium is the solution giving the agent an expected payoff of zero. This results in the following constraints as in the games OA and OA':

type 1:

$$p \cdot (m_1 - w_1) + (1 - p) \cdot (m_2 - w_2) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p \cdot (w_1 - c) + (1 - p) \cdot (w_2 - c) = 0 \quad (\text{individual rationality for the agent})$$

or

$$p \cdot w_1 + (1 - p) \cdot w_2 = c \text{ and}$$

$$p \cdot m_1 + (1 - p) \cdot m_2 - c \geq 0.$$

type 2:

$$p' \cdot (m_1 - w_1) + (1 - p') \cdot (m_2 - w_2) \geq 0 \quad (\text{individual rationality for the principal})$$

$$p' \cdot (w_1 - c') + (1 - p') \cdot (w_2 - c') = 0 \quad (\text{individual rationality for the agent})$$

or

$$p' \cdot w_1 + (1 - p') \cdot w_2 = c' \text{ and}$$

$$p' \cdot m_1 + (1 - p') \cdot m_2 - c' \geq 0.$$

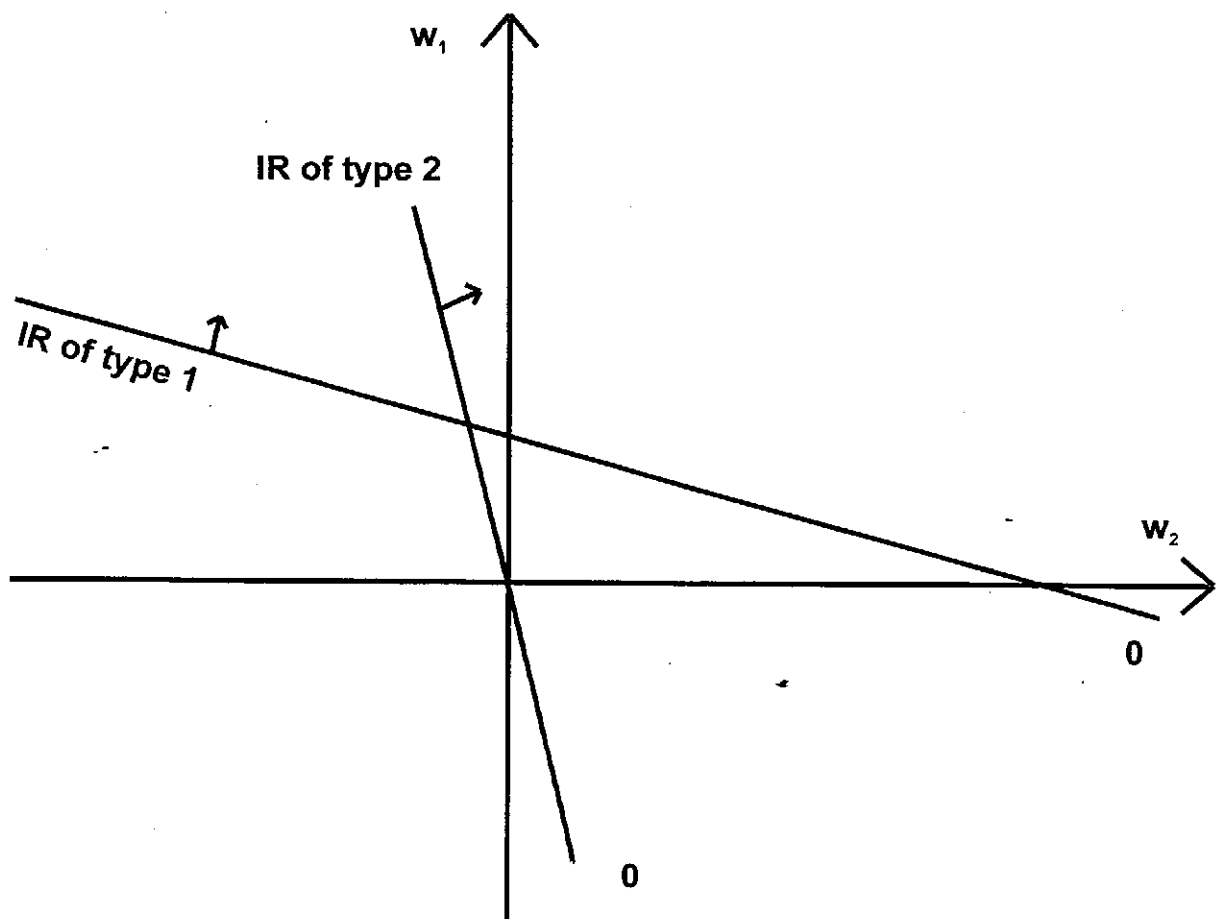
The proposals which are accepted are determined by these individual rationality constraints: If the expected payoff of the principal is positive for both types if the agent

receives 0<sup>7</sup> he proposes a payoff pair  $(w_1^*, w_2^*)$  which is given by the intersection of the lines of individual rationality of the two possible types of the agent.

Both types of agents accept this proposal and the payoff is according to the type.

This is shown in Figure 9.

FIGURE 9: —  $(w_1^*, w_2^*)$  OF THE SUBGAME PERFECT EQUILIBRIUM IN THE GAME TT



<sup>7</sup> Because the expected payoff sum is constant for type 1 (which corresponds to game OA) and it is constant for type 2 (which corresponds to game OA'), the principal has to check if his expected payoff is greater than 0 only for two cases.

If the expected payoff of the principal is positive for type 1 if the agent receives 0 and negative for type 2 he proposes a payoff pair  $(w_1^*, w_2^*)$  given by

type 1:

$$p \cdot (w_1 - c) + (1 - p) \cdot (w_2 - c) = 0$$

and

type 2:

$$p' \cdot (w_1 - c') + (1 - p') \cdot (w_2 - c') < 0.$$

The agent accepts it if he is of type 1 and rejects if he is of type 2.

If the expected payoff of the principal is negative for both types if the agent receives 0 he proposes a payoff pair  $(w_1^*, w_2^*)$  given by

type 1:

$$p \cdot (w_1 - c) + (1 - p) \cdot (w_2 - c) < 0$$

and

type 2:

$$p' \cdot (w_1 - c') + (1 - p') \cdot (w_2 - c') < 0.$$

The agent rejects this proposal independent of his type.

### 3.4.2 Modeling of boundedly rational behavior

In this model the possible proposals for the agents of type 1 and for the agents of type 2 are determined via the rules 0 - 3 (given in the model of boundedly rational behavior for the

games OA and OA'). By means of rule 4 (given in the model for the games OA and OA') out of the possible proposals the non negative one resulting in the highest perceived payoff is selected as proposal which will be accepted. This proposal will be accepted by a type of an agent if his perceived payoff is non negative.

As described in the section games type 1 corresponds to game OA. Because of this the proposal  $(w_1^{*1}, w_2^{*1})$  in game OA is the same as the proposal for type 2. Because type 2 corresponds to game OA' (compare the section games) the proposal  $(w_1^{*2}, w_2^{*2})$  in game OA' is the same as the proposal for type 2.

One of these proposals is selected by means of rule 4 by the principal. According to the determination of the fair proposals it holds:  $w_1^{*1} > w_1^{*2}$  (for the games in the experiment). Both types of agents reject proposals that are below these fair proposals<sup>8</sup> and accept all other proposals. Especially, an agent of type 1 rejects the proposal for type 2 and an agent of type 2 accepts a proposal for type 1. Therefore the principal has to calculate whether his perceived payoff<sup>9</sup> is at least 0 and higher if he makes a proposal that agents of type 1 and type 2 accept than if he makes a proposal that only agents of type 2 accept. Therefore three cases are possible: In the experiment the case is examined in which his evaluated payoff is higher if he makes a proposal that agents of type 1 accept. In these cases the principal should make a proposal for type 1 which is accepted by the types of agents depending on their evaluation of the prospects they receive.

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<sup>8</sup> As in ultimatum games rejecting positive amounts of money by the responders determines the behavior in the game.

<sup>9</sup> All perceived payoffs and prospects are evaluated according to the theory of prominence as it is shown in the section about the modeling of the boundedly rational behavior in the games OA and OA'.

Case 1: The principal selects the proposal  $(w_1^{*1}, w_2^{*1})$  for type 1, if his perceived payoff is higher (and greater zero), if both types of agents accept this proposal, than the payoff for a proposal that only type 2 accepts. This results in the condition:  $(w_1^{*1}, w_2^{*1})$  is proposed, if:

$$0.5 \cdot v_{\Delta}([(m_1 - w_1^{*1})(p), (m_2 - w_2^{*1})(1-p)]) + 0.5 \cdot v_{\Delta}([(m_1 - w_1^{*1})(p'), (m_2 - w_2^{*1})(1-p')]) \geq v_{\Delta}([(m_1 - w_1^{*2})(p'), (m_2 - w_2^{*2})(1-p')]), \text{ i.e.}$$

$$0.5 \cdot (p \cdot v_{\Delta}(m_1 - w_1^{*1}) + (1-p) \cdot v_{\Delta}(m_2 - w_2^{*1})) + 0.5 \cdot (p' \cdot v_{\Delta}(m_1 - w_1^{*1}) + (1-p') \cdot v_{\Delta}(m_2 - w_2^{*1})) \geq p' \cdot v_{\Delta}(m_1 - w_1^{*2}) + (1-p') \cdot v_{\Delta}(m_2 - w_2^{*2}).$$

Case 2: The principal selects the proposal  $(w_1^{*2}, w_2^{*2})$  for type 2, if his perceived payoff is highest (and greater zero), if only type 2 accepts this proposal. This results in the condition:  $(w_1^{*2}, w_2^{*2})$  is proposed, if:

$$0.5 \cdot v_{\Delta}([(m_1 - w_1^{*1})(p), (m_2 - w_2^{*1})(1-p)]) + 0.5 \cdot v_{\Delta}([(m_1 - w_1^{*1})(p'), (m_2 - w_2^{*1})(1-p')]) < v_{\Delta}([(m_1 - w_1^{*2})(p'), (m_2 - w_2^{*2})(1-p')]), \text{ i.e.}$$

$$0.5 \cdot (p \cdot v_{\Delta}(m_1 - w_1^{*1}) + (1-p) \cdot v_{\Delta}(m_2 - w_2^{*1})) + 0.5 \cdot (p' \cdot v_{\Delta}(m_1 - w_1^{*1}) + (1-p') \cdot v_{\Delta}(m_2 - w_2^{*1})) < p' \cdot v_{\Delta}(m_1 - w_1^{*2}) + (1-p') \cdot v_{\Delta}(m_2 - w_2^{*2}).$$

Case 3: The perceived payoff of the principal is negative for both proposals  $(w_1^{*1}, w_2^{*1})$  and  $(w_1^{*2}, w_2^{*2})$ , then a pair is selected that both types of agents reject.

## 4 METHOD

### *4.1 The subjects*

In the games 36 students from the University of Bielefeld participated as subjects. The subjects were recruited by announcements in the university promising monetary reward contingent on performance in a group decision making experiment. The subjects got points as payoffs. 1 point was 0.5 DM (~\$ 0.30). Losses up to 100 DM (~\$ 60) had to be paid by the subjects, for losses above 100 DM the subjects could choose whether to pay or to work at 15 DM (~ \$ 10) per hour, but this did not occur. The average payoff of a subject was 110 DM (~\$ 66).

### *4.2 Experimental Procedure*

The experiment for each game lasted approximately 110 minutes with the first 20 minutes consisting of orientation and instructions. The experiments were conducted at the University of Bielefeld. The experiment started with the instructions (see Appendix) on the structure of the game and a learning phase in which single games were played in 6 groups of 6 subjects<sup>10</sup> with free anonymous preplay communication via computer terminals between 2 randomly matched players<sup>11</sup>. The computer terminals were well separated from one another preventing

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<sup>10</sup> The subjects were not informed about this subdivision.

communication between the subjects. In the experiment the games were played for the following values of the payoffs  $m_1$ ,  $m_2$ , the probabilities  $p$ ,  $p'$ , and the cost  $c$ ,  $c'$  that are shown in Figure 10. Only the payoff  $m_2$  is varied.

FIGURE 10: — VALUES OF THE PARAMETERS IN THE EXPERIMENT

Game OA:

$m_1$	$m_2$	$p$	$C$
100	40	0.8	-20
100	20	0.8	-20
100	0	0.8	-20
100	-20	0.8	-20
100	-40	0.8	-20

Game OA'

$m_1$	$m_2$	$p'$	$c'$
100	40	0.2	0
100	20	0.2	0
100	0	0.2	0
100	-20	0.2	0
100	-40	0.2	0

---

<sup>11</sup> In the learning phase small amounts of money were the payoffs. The subjects received not more than 10% of their total payoff from these games.



Game TA

$m_1$	$m_2$	$p$	$p'$	$c$	$c'$
100	40	0.8	0.2	-20	0
100	20	0.8	0.2	-20	0
100	0	0.8	0.2	-20	0
100	-20	0.8	0.2	-20	0
100	-40	0.8	0.2	-20	0

Game TT

$m_1$	$m_2$	$p$	$p'$	$c$	$c'$
100	40	0.8	0.2	-20	0
100	20	0.8	0.2	-20	0
100	0	0.8	0.2	-20	0
100	-20	0.8	0.2	-20	0
100	-40	0.8	0.2	-20	0

After the learning phase a strategy game was played. All subjects selected their strategies for all games and all roles (the principal and the agent). One game was paid per type of game and per person. Subjects were assigned to each other randomly. The subjects were informed about the procedure.

The strategies for the games were given as:

- Game OA and OA':** as **principal:** a proposal  $(w_1, w_2)_P$ .  
as **agent:** a minimal  $(w_1, w_2)_A$  accepted.<sup>12</sup>
- Game TA:** as **principal:** a proposal  $(w_1, w_2)_P$ .  
as **agent:** a minimal  $(w_1, w_2)_{e_1}$  necessary to select action  $e_1$   
and  
a minimal  $(w_1, w_2)_{e_2}$  necessary to select action  $e_2$
- Game TT:** as **principal:** a proposal  $(w_1, w_2)_P$ .  
as **agent:** a minimal  $(w_1, w_2)_{T1}$  accepted as type 1 and  
a minimal  $(w_1, w_2)_{T2}$  accepted as type 2.

After the experiment was completed each subject was separately paid in cash contingent on his performance and thanked.

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<sup>12</sup> As agents the subjects could give a set or a range of minimal pairs  $(w_1, w_2)_A$  accepted like lines, etc., but this did not occur. In all games the agents could give their strategies like this.

## 5 PREDICTIONS

### 5.1 The linear model

Game OA:

For all payoffs  $m_2$  the lines of individual rationality (IR) in the linear model are given by:

$w_1^* = -0.25 w_2^* + 25$  in game OA and it is

$w_1^* = -4 w_2^*$  in game OA'.

All proposals are accepted except of game OA' for  $m_2 = -40$ .

Game TA:

The lines of individual rationality are for all payoffs  $m_2$ :

$w_1 = -0.25 w_2 + 25$  for action  $e_1$  (as in game OA) and

$w_1 = -4 w_2$  for action  $e_2$  (as in game OA').

For all payoffs  $m_2$  the line of incentive compatibility (IC) is given by:

$w_1 = w_2 + 33.33$ .

The principal always makes a proposal  $(w_1^*, w_2^*)$  for action  $e_1$ . It is always accepted.

Game TT:

The intersection of the two lines of individual rationality (as in game OA and OA') for both types is given by:  $(w_1^*, w_2^*) = (26.66, -6.66)$ . This proposal is accepted in all games and for all types except of the one for type 2 and  $m_2 = -40$ .<sup>13</sup>

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<sup>13</sup> For a test of the linear model a range of 5 around the prediction is taken as hypotheses for all games.

## 5.2 The model of boundedly rational behavior

The predictions of the model of boundedly rational behavior are summarized in Figure 11.

For the predictions first  $\Delta$  is determined by the rule of the theory of prominence: it is 2 steps below the smallest prominent number greater than or equal to the maximal absolute payoff in a game. Then  $(w_1^*, w_2^*)$  is calculated according to the conditions given in the model of boundedly rational behavior.

Games OA and OA':

The predictions are shown in Figure 11.

FIGURE 11: — PREDICTIONS OF THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR  
FOR THE GAMES OA AND OA'

game OA				game OA'			
$m_2 =$	$w_1^{*1}$	$w_2^{*1}$		$m_2 =$	$w_1^{*2}$	$w_2^{*2}$	
	proposal of the principal		agent		proposal of the principal		agent
40	53	28	accepted	40	40 <sup>1</sup>	16	accepted
20	53	20	accepted	20	40	8	accepted
0	53	8	accepted	0	40	0	accepted
-20	53	-4	accepted	-20	40	-13	rejected
-40	53	-17	accepted	-40	40	-24	rejected

<sup>1</sup> Rounded value (with no influence on the results of the tests)

Game TA:

Due to the fact that the principal maximizes his payoff he wants the agent to select action  $e_1$ . Therefore he proposes the same pair  $(w_1^{*1}, w_2^{*1})$  as in game OA, because this payoff pair is regarded as fair. The agent will accept this proposal and select the action  $e_1$ . The minimal amount accepted by the agent (in this case the agent selects action  $e_2$ ) is the pair  $(w_1^{*2}, w_2^{*2})$  if his evaluated payoff is non negative otherwise it is a pair that leads to rejection. The resulting pairs  $(w_1^{*1}, w_2^{*1})$  and  $(w_1^{*2}, w_2^{*2})$  are given in Figure 12.

FIGURE 12: — PREDICTIONS OF THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR  
FOR THE GAME TA

$m_2 =$	$w_1^{*1}$	$w_2^{*1}$	
	proposal of the principal		the agents's action
40	53	28	$e_1$
20	53	20	$e_1$
0	53	8	$e_1$
-20	53	-4	$e_1$
-40	53	-17	$e_1$
minimal amount accepted to select $e_2$ by the agent			
$m_2 =$	$w_1^{*2}$	$w_2^{*2}$	
40	40	16	accepted
20	40	8	accepted
0	40	0	accepted
-20	40	-13	rejected
-40	40	-24	rejected

Game TT:

In this game the hypothesis is that the principal makes a proposal above the minimal amount accepted by type 1 (the better type). The proposals should therefore be equal to the proposals in game TA (hypothesis 1) or game OA. The minimal amounts accepted should be the same as in Game TA. The resulting proposals and minimal amounts accepted are given in Figure 13 and Figure 14.

FIGURE 13: — PREDICTIONS OF THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR  
FOR THE PROPOSALS IN GAME TT

$m_2 =$	$w_1^{*1}$	$w_2^{*1}$
	Proposal of the principal	
40	53	28
20	53	20
0	53	8
-20	53	-4
-40	53	-17

FIGURE 14: — PREDICTIONS OF THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR: THE MINIMAL AMOUNT ACCEPTED BY THE AGENT IN GAME TT

type 1				type 2			
$m_2 =$	$w_1^{*1}$	$w_2^{*1}$		$m_2 =$	$w_1^{*2}$	$w_2^{*2}$	
40	53	28	accepted	40	40	16	accepted
20	53	20	accepted	20	40	8	accepted
0	53	8	accepted	0	40	0	accepted
-20	53	-4	accepted	-20	40	-13	rejected
-40	53	-17	accepted	-40	40	-24	rejected

These are the predictions tested in the experiments.

## 6 RESULTS

The results of the strategy game are presented and discussed in this part.

### 6.1 Games OA and OA'

The medians of  $(w_1, w_2)$  for the 6 groups and all games OA are presented in Figure 15 and Figure 16. In the upper part of the figure the proposals of the subjects as principal are given



and in the lower part the minimal amounts accepted as agent. If either the proposed  $w_1$  or the proposed  $w_2$  is below the corresponding minimal amount accepted the proposal is rejected.

FIGURE 15: — MEDIAN OF THE 6 GROUPS OF THE PROPOSALS AS PRINCIPALS AND THE MINIMAL AMOUNTS ACCEPTED AS AGENTS IN GAME OA

	group 1		group 2		group 3		group 4		group 5		group 6	
as principal	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$
$m_2=$												
40	49	22.5	42.5	20	50	28	60	20	60	20	55	30
20	49	20	42.5	10	45	23	58	20	60	10	55	10
0	49	15	47.5	5	40	10	58	10	60	0	55	5
-20	44	10	45	0	40	2.5	60	0	55	0	50	0
-40	44	-5	45	-5	40	0	60	0	55	0	50	-5
as agent	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$
$m_2=$												
40	42.5	20.5	35	20	40	23	50	20	60	15	53	20
20	42.5	20	35	10	40	20	53	20	58	7.5	53	10
0	37.5	5	37.5	0	40	2.5	53	5	55	0	53	5
-20	37.5	0	35	-5	35	-2	50	-5	50	0	50	0
-40	37.5	-13	30	-10	30	-13	50	-17	45	0	50	-20

FIGURE 16: — MEDIANS OF THE 6 GROUPS OF THE PROPOSALS AS PRINCIPALS AND THE MINIMALLY ACCEPTED AMOUNTS AS AGENTS IN GAME OA'

	group 1		group 2		group 3		group 4		group 5		group 6	
as principal	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>
m <sub>2</sub> =												
40	40	15	42.5	20	42.5	20	50	20	45	20	50	20
20	42.5	7.5	42.5	10	40	10	50	10	45	10	50	10
0	42.5	0	42.5	0	35	5	50	0	48	0	50	0
-20	52.5	-18	47.5	-10	30	-10	35	-10	43	-5	50	-10
-40	62.5	-33	35	-15	30	-20	35	-20	40	-20	50	-20
as agent	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>
m <sub>2</sub> =												
40	35	12.5	10	20	32.5	20	50	20	40	15	40	20
20	35	7.5	10	10	30	10	50	10	40	5	45	10
0	35	0	10	10	30	0	50	0	40	0	40	0
-20	20	0	20	0	30	5	40	0	35	-4.5	50	-8
-40	15	0	20	0	25	2.5	40	-10	23	0	50	-18

These data are used for a test of the predictions. The results of a binomial test of the predictions of the linear model are shown in Figure 17. A "-" indicates that the prediction is rejected, a "+" indicates that it is not rejected.<sup>14</sup> All predictions are rejected on a 5% level except of the predictions for  $m_2 = -40$  in both games.<sup>15</sup>

FIGURE 17: — RESULTS OF THE TEST OF THE LINEAR MODEL ( $\alpha=5\%$ )  
FOR THE GAMES OA AND OA'

Game OA		Game OA'	
$m_2 =$		$m_2 =$	
40	-	40	-
20	-	20	-
0	-	0	-
-20	-	-20	-
-40	+	-40	+

---

<sup>14</sup> A data point is counted as greater than the prediction if the proposal as the principal and the minimal amount accepted as the agent are greater than the prediction. Analogous a data point is counted as smaller than the prediction if the proposal as the principal and the minimal amount accepted as the agent are smaller than the prediction.

<sup>15</sup> This result also holds if one assumes a logarithmic value function for money as in the theory of prominence (Albers (1997)) or the value-function of the prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)) except of a "+" for  $m_2 = -20$  for game OA.

The same test for the predictions of the model of boundedly rational behavior leads to the result that no prediction is rejected. Using the test for  $\alpha=60\%$  which is certainly not a test, but gives hints for the validity of the predictions, gives the result presented in Figure 18. Again no prediction is rejected showing that the model of the boundedly rational behavior is a very good predictor for the data.

FIGURE 18: — RESULTS OF THE TEST THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR  
 $(\alpha=60\%)$  FOR THE GAMES OA AND OA'

Game OA		
$m_2=$	$w_1$	$w_2$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

Game OA'		
$m_2=$	$w_1$	$w_2$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

## 6.2 Game TA

The medians of the 6 groups of the proposals as the principal are shown in Figure 19, the medians of the minimal amounts necessary to select  $e_1$  are given in Figure 20 and the minimal amounts necessary to select  $e_2$  in Figure 21. In the role of the principal all subjects were asked in addition to their proposal which action they wanted to achieve with their proposal. All subjects wanted to achieve that the agent selects action  $e_1$ .

FIGURE 19: — MEDIAN OF THE PROPOSAL AS PRINCIPAL IN THE GAME TA

	group 1		group 2		group 3		group 4		group 5		group 6	
as principal	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>
m <sub>2</sub> =												
40	50	25	57	20	50	28	60	30	45	10	60	15
20	50	20	57	12.5	50	20	55	20	50	0	47.5	10
0	50	15	53	9.5	50	10	50	10	55	0	45	5
-20	50	10	53	4	45	10	50	0	55	-7.5	50	0
-40	50	-5	50	5	45	10	60	-10	57.5	-10	55	-10

FIGURE 20: — MEDIAN OF THE PROPOSAL NECESSARY TO SELECT THE HIGHER EFFORT  $e_1$  AS

AGENT IN THE GAME TA

	group 1		group 2		group 3		group 4		group 5		group 6	
as agent	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$	$w_1$	$w_2$
$m_2 =$												
40	50	20	45	19.5	50	20	50	20	35	20	50	15
20	50	17.5	50	12.5	50	0	50	10	40	10	42.5	5
0	50	5	50	0	40	0	50	10	45.5	0	40	10
-20	50	0	45	-5	40	-10	50	-10	52.5	-7.5	50	-5
-40	50	-12.5	50	-10	30	-17	50	-20	55	-12.5	45	-15

FIGURE 21: — MEDIANS OF THE PROPOSAL NECESSARY TO SELECT THE LOWER EFFORT  $e_2$  AS AGENT (I.E. THE LOWEST PROPOSAL ACCEPTED) IN THE GAME TA

	group 1		group 2		group 3		group 4		group 5		group 6	
as agent	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>
$m_2 =$												
40	50	15	50	19.5	30	15	50	20	30	20	47.5	15
20	50	7.5	45	10	30	0	50	10	37.5	10	45	5
0	50	0	40	0	30	0	50	0	45	0	32.5	5
-20	60	0	52.5	-5	50	-10	50	0	52.5	-7.5	55	0
-40	62.5	-5	57.5	-10	50	-17	50	-10	60	-12.5	62.5	-10

First the predictions of the linear model are tested. The proposals and minimal amounts necessary for action  $e_1$  are taken as data for the test of the predictions for the action  $e_1$  and the minimal amounts necessary for action  $e_2$  are the data to test the predictions for the action  $e_2$ . The results of a binomial test are shown in Figure 22. A rejection of the prediction is indicated

by a "-", a "+" indicates that the prediction is not rejected.<sup>16</sup> All predictions are rejected on a 5% level except of the prediction for  $m_2=-40$  for both actions.<sup>17</sup>

FIGURE 22: — RESULTS OF THE TEST OF THE LINEAR MODEL ( $\alpha=5\%$ ) FOR THE GAME TA

action $e_1$		action $e_2$	
$m_2=$		$m_2=$	
40	-	40	-
20	-	20	-
0	-	0	-
-20	-	-20	-
-40	+	-40	+

The result of the test for the predictions of the model of boundedly rational behavior is that no prediction is rejected. Using the test for  $\alpha=60\%$  which gives hints for the validity of the predictions, gives the result presented in Figure 23. Again no prediction is rejected showing that the model of the boundedly rational behavior is a very good predictor for the data.

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<sup>16</sup> A data point is counted as greater or smaller than the prediction analogously to tests for the games OA and OA'.

<sup>17</sup> Assuming a logarithmic value function for money or the value-function of the prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)) does not change the result except of a "+" for  $m_2=-20$  for action  $e_1$ .



FIGURE 23: — RESULTS OF THE TEST THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR

( $\alpha=60\%$ ) FOR THE GAME TA

action $e_1$			action $e_2$		
$m_2=$	$w_1$	$w_2$	$m_2=$	$w_1$	$w_2$
40	+	+	40	+	+
20	+	+	20	+	+
0	+	+	0	+	+
-20	+	+	-20	+	+
-40	+	+	-40	+	+

### 6.3 Game TT

First the hypotheses that the proposals are the same in game TA and TT, that the minimal amount necessary to select  $e_1$  in the game TA is the same as the minimal amount accepted by the agent of type 1 and that the minimal amount necessary to select  $e_2$  in the game TA is the same as the minimal amount accepted by the agent of type 2 are examined. Again a test for  $\alpha=60\%$  is used. This is not a test, but gives hints for the equality of the proposals, and minimal amounts accepted in game TA and TT. The result is shown in Figure 24. A "+" indicates that the hypothesis is not rejected. The hypotheses are not rejected on these level indicating that the proposals and minimal amounts accepted are the same in both games.

FIGURE 24: — RESULTS OF THE TEST THAT THE PROPOSALS AND THE MINIMAL AMOUNTS

ACCEPTED ARE THE SAME IN THE GAMES TA AND TT ( $\alpha=60\%$ )

a: proposal in game TT = proposal in game TA?

$m_2=$	$w_1(\text{game TA})$ $=w_1(\text{game TT})$	$w_2(\text{game TA})$ $=w_2(\text{game TT})$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

b: minimal amount accepted as type 1

=minimal amount necessary for  $e_1$ ?

$m_2=$	$w_1(\text{game TA})$ $=w_1(\text{game TT})$	$w_2(\text{game TA})$ $=w_2(\text{game TT})$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

c: minimal amount accepted as type 2

=minimal amount necessary for  $e_2$ ?

$m_2=$	$w_1(\text{game TA})$ $=w_1(\text{game TT})$	$w_2(\text{game TA})$ $=w_2(\text{game TT})$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

First the predictions of the linear model are tested. The hypothesis is a pair  $(w_1, w_2) = (26.66, -6.66)$ . The results of a binomial test are shown in Figure 25. All predictions are rejected on a 5% level except of the the prediction for  $m_2 = -40$  for both types.<sup>18</sup>

FIGURE 25: — RESULTS OF THE TEST ( $\alpha=5\%$ ) OF THE PREDICTIONS OF THE LINEAR MODEL  
FOR GAME TT

$m_2 =$	$w_1$	$w_2$
40	-	-
20	-	-
0	-	-
-20	-	-
-40	+	+

---

<sup>18</sup> As in the other tests of the linear model this result also holds if one assumes a logarithmic value function for money as in the theory of prominence (Albers (1997)) or the value-function of the prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)) except of a “+” for  $m_2 = -20$  for type 1.

The same test for the predictions of the model of boundedly rational behavior leads to the result that no prediction is rejected. Using the test for  $\alpha=60\%$  gives the result presented in Figure 26. Again no prediction is rejected showing that the model of the boundedly rational behavior is a very good predictor for the data.

FIGURE 26: — RESULTS OF THE TEST THE MODEL OF BOUNDEDLY RATIONAL BEHAVIOR  
( $\alpha=60\%$ ) FOR GAME TT

proposal and

minimal amount

accepted by type 1

$m_2=$	$w_1$	$w_2$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

minimal amount

accepted by type 2

$m_2=$	$w_1$	$w_2$
40	+	+
20	+	+
0	+	+
-20	+	+
-40	+	+

An interpretation of this result is that the presence of two agents of different types lowers the payoff of the principal, because he has to pay both as if they were of the good type, because of fairness considerations. These considerations do not prevent a too high payoff for an agent of type 2 as it is possible in the linear model.

## 7 CONCLUSION

Several principal-agent games are examined. Models of boundedly rational behavior in these situations are given and experimentally tested. In these games positive and negative payoffs were possible.

In the first principal-agent game the principal knows the action the agent takes, but the payoff depends on a chance move with two outcomes (a good and a bad one). This is similar to an ultimatum game with prospects as payoffs. A model for the boundedly rational behavior in this situation is given and tested. In the model of boundedly rational behavior first the game is decomposed into two ultimatum games: one for the good outcome and one for the bad outcome. Each of these games is solved by the means of the model of boundedly rational behavior in ultimatum games consisting of three rules. Rule 1 is **the perception of payoffs**: All payoffs are transformed by the perception function ( $v_{\Delta}$ -function) described by the theory of prominence. In rule 2 **the strength of the principal** is modeled: the proposal of the principal is such that he gets at least the payoff of the agent. In rule 3 the selection criterion is given by an **equal sum of concessions**: Starting from the maximal payoffs both players can obtain they make equal concessions on the scale defined by the transformed payoffs to get the proposed and accepted payoff pair. The experimental result is that this model is a good predictor for the data.

The problem of moral hazard in which an agent can select between two actions which is not observable by the principal is examined in another game. The experimental result of this part is that the principals offer one of the contracts which they proposed for each action if the action was observable. The principals selected the contract which gave them higher payoffs and the agents selected the intended action. The incentive compatibility constraint is replaced by fairness considerations. The action is determined by the proposal of the principal which is considered to be fair or not for a certain action.

The experimental results of the game in which a principal faces an agent who can be of two types (a good and a bad one) of which the correct type is only known to the agent can also be explained by means of the ultimatum game with prospects as payoffs. The agent who is the responder in the ultimatum game determines the minimal level of acceptance. If two types of agents are possible the agent of the good type rejects offers which are below the proposal he accepts if all agents were of his type. This gives the lowest proposal the principal can make if he maximizes his payoff, because rejection is more costly than higher payoffs for the bad type of agent. In the experiment the principals proposed the contract for the good one.

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