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Supporting Cooperative Multi-Issue Negotiations

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Abstract

In this paper we characterize an intuitive method for implementing cooperative bargaining solutions in multi-issue negotiations. We illustrate our approach with an example of a marketing negotiation where parties' balance of power is determined by a reference point within the bargaining set. The procedure is a generalization of the fair-division algorithm 'Adjusted Winner,' introduced by BRAMS AND TAYLOR (1996). The technique consists of two fundamental steps: Negotiators first focus on joint utility maximization. In a second step they establish the desired balance of power through efficient transfers. Due to its tractability and the fact that it does not require computer support, the algorithm provides an argumentative basis for the complete negotiation process.

Keywords: Bargaining, Reference points, Adjusted Winner

1. Introduction

Whenever two parties negotiate over multiple issues and value these differently, they can trade their individual power to control the issues and, thereby, increase both players' shares of the pie. However, the multiplicity of issues is also what gives rise to the complexity of negotiations, and what makes it so difficult to achieve efficient outcomes, or perhaps even to realize mutual benefits. As a consequence, there is an increasing interest in the development of negotiation support systems in order to provide individuals or groups with a set of tools that helps them to structure a problem and find a cooperative solution.

Cooperative solution concepts, e.g. the axiomatic characterizations offered by mathematical bargaining theory, provide insight into the properties of different solutions. A computer program which merely calculates bargaining solutions is not always very useful, though, because players want to understand the solution as well the process that leads them there. In addition, it is important for them to actively take part in the process. This could explain why SPECTOR (1997) finds empirical evidence that quantitative analytical support to negotiations is not very popular in actual bargaining situations. What is needed, in his view, is a procedural assistance that guides negotiators to efficient outcomes.[1] From the negotiation analytic perspective outlined by SEBENIUS (1992), a prescriptive approach to cooperative negotiation is required. Therefore, procedural models of cooperative bargaining can be helpful for understanding the actual implementation of cooperative solutions.

GUPTA (1989) develops a model of integrative bargaining, which describes negotiation as a process where parties trade their authority over the negotiated issues. He shows that if parties maintain a constant balance of power throughout the nego-

[1]Recent support systems are apparently more successful in this respect. RANGASWAMY AND SHELL (1997), for example, report that negotiators using their electronic bargaining table, *Negotiation Assistant*, were able to achieve Pareto improvements in post-settlement agreements. Indeed, the power of a program such as *Negotiation Assistant* is not the calculation of efficient outcomes, but rather the argumentative support and the visual aid that it offers negotiators during the bargaining process.

tiation, they are implicitly committing themselves to a system of fixed issue prices. With given prices, their joint optimization can be decomposed into two individual problems that can be solved separately. The two solutions are jointly feasible, and they are shown to coincide with the outcome induced by the axiomatic solution of GUPTA AND LIVNE (1988), which is an extension of the solution characterized by KALAI AND SMORODINSKY (1975).

In this paper we formulate a simpler method for implementing cooperative bargaining solutions in multi-issue negotiations. The procedure is a generalization of the fair-division algorithm 'Adjusted Winner,' introduced by BRAMS AND TAYLOR (1996), for players with linear, additively separable preferences. The generalization is crucial when players have non-linear preferences over issues, since a procedure based on linear preferences is then likely to produce an inefficient outcome. In order to illustrate the potential of this approach, we study GUPTA's (1989) example of a negotiation between a manufacturer and a retailer over six different marketing plans for each of six different products. We choose this existing negotiation problem (rather than constructing a new one that conveniently suits our purpose) because of its features that are common in marketing research and its complexity, which provides room for analytical support.

Adjusted Winner is composed of simple steps that are characteristic for a cooperative negotiation process. Instead of keeping the balance of power fixed throughout the negotiation, the algorithm first distributes power efficiently. This introduces an additional cooperative element into the process, because players maximize their joint utility without being concerned about the distribution of gains. In the second step the balance of power is reestablished. This requires a redistribution of gains. As we will see, it is easy to reallocate issues efficiently, when starting with an efficient agreement. Since the definition of balanced power depends on the solution concept, a variety of alternative solutions can be implemented.[2]

In section 2, we describe the structure of the multi-issue negotiation problem

[2] RAITH AND WELZEL (1998) show that Adjusted Winner implements the Kalai-Smorodinsky solution, and that it is also easily adapted to the Nash bargaining solution.

analyzed by GUPTA (1989). In section 3, we then formulate the issue-trading process for an implicit price system and characterize the Gupta-Livne solution as the solution to the two players' individual optimization problems. We describe Adjusted Winner in section 4 and modify it to implement the Gupta-Livne solution. We consider nonlinear preferences in section 5 and illustrate the inefficiency of procedures based on linear preferences in order to emphasize the potential gains of generalizing Adjusted Winner. We also provide an algorithmic formulation of the procedure, which can be reduced to only two fundamental steps. In section 6, we offer some concluding remarks and a general assessment of our approach.

2. The Structure of a Multi-Issue Negotiation

Consider a negotiation between a manufacturer M (e.g. Proctor & Gamble) and a retailer R (e.g. Safeway) over six different marketing plans (A–F) for each of six different products (1–6). Table 1 shows both negotiators' profits for each of the marketing plans in some given monetary unit (say 1 mill. Euros).[3]

A negotiated outcome consists of one plan (an option) for each of the six products (the issues), implying that there are $6^6 = 46,656$ possible discrete outcomes to this negotiation. The complexity of this multi-issue negotiation is illustrated in Figure 1, where we have plotted the players' total payoffs, π^M and π^R, for each of these outcomes. If a Euro is the players' standard of value, the payoffs can be viewed as a characterization of their individual utilities.

Note that the efficient outcomes lie beyond the region with the highest density of outcomes. One can well imagine that an unstructured negotiation, i.e., when players only thrash around with offers and counter offers, will lead to an inefficient or unfair outcome. Therefore, in order to study models of negotiation processes that implement efficient outcomes, it is important to understand the structure the underlying negotiation problem.

Neither player can receive a total profit of less than 3, e.g., because of preferable

[3] This (hypothetical) multi-issue negotiation problem was analyzed both theoretically as well as experimentally by GUPTA (1989). Table 1 reproduces his Table 3a, p. 798.

ISSUES		OPTIONS A	B	C	D	E	F
1	M	0.50	2.15	3.50	4.16	5.50	6.50
	R	2.50	2.15	1.90	1.72	1.50	0.50
2	M	0.50	1.22	1.70	2.00	2.20	2.50
	R	2.50	2.00	1.70	1.50	1.10	0.50
3	M	2.50	2.20	2.00	1.70	1.22	0.50
	R	0.50	1.10	1.50	1.70	2.00	2.50
4	M	0.50	1.50	1.75	2.00	2.30	2.50
	R	8.50	7.17	5.50	4.50	2.22	0.50
5	M	6.50	5.50	4.16	3.50	2.15	0.50
	R	0.50	1.50	1.72	1.90	2.15	2.50
6	M	0.50	1.90	3.30	4.25	5.50	6.50
	R	4.50	3.90	3.30	3.00	2.50	0.50

Table 1: Payoffs of players M and R from plans A–F for products 1–6

outside alternatives. The maximum gains are thus 24 and 20 for player M and R, respectively. In Figure 1, the characterization of the negotiation problem in terms of gains rather than payoffs would simply shift the point of minimum payoff (3,3) to the origin. By considering each issue separately, the gain for each individual option is obtained by comparing it with the least preferred option of that issue. In our example, the bargaining situation can be interpreted in terms of gains by subtracting the minimum profit of 0.5 from each option of Table 1.

For an alternative characterization of utility, the standard of value could also be the aggregate over all 6 issues. In terms of gains, the complete pie is worth 24 units to player M and 20 units to player R. By using these values to normalize Euro gains, one obtains percentage gains as given in Table 2.[4] The maximum percentage gain that a player can achieve or lose on a specific issue can be seen as his interest in this issue. For each player, the sum of interests is equal to 100.

[4]For example, with plan C for product 2, player M can realize a percentage gain of $\frac{1.70-0.50}{24} \times 100 = 5.00$, and player R gains $\frac{1.70-0.50}{20} \times 100 = 6.00$ percent.

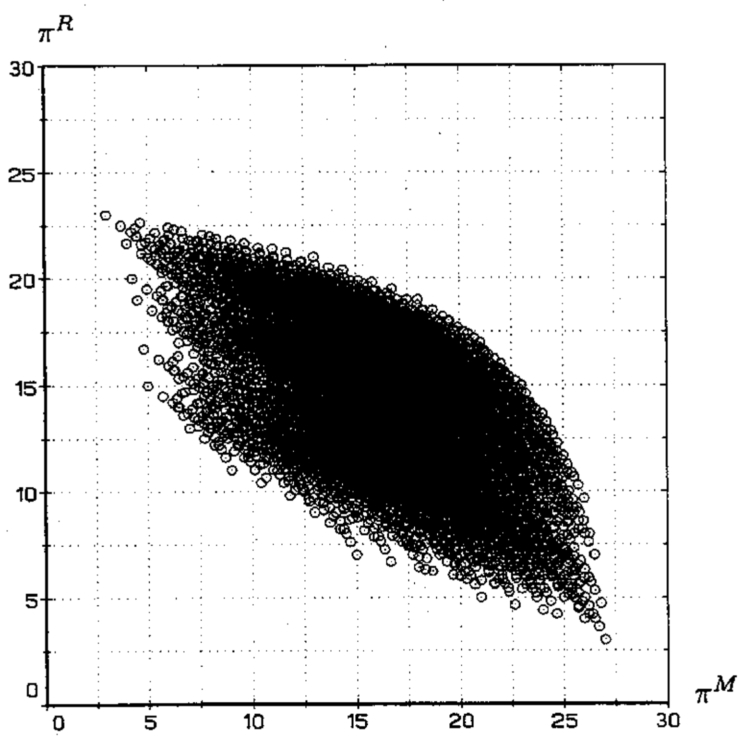

Figure 1: Payoffs for six issues with 6 options each

This scoring method is quite common in negotiation analysis: Players first weight issues by distributing 100 points. They then value the options of an issue by assigning to the best option of an issue the number of points that the issue receives, while the worst option of an issue gets 0 points.[5]

3. The Linear-Programming Problem

GUPTA (1989) develops a model of integrative bargaining where players jointly improve on a given reference point within the bargaining set with the goal of finding an efficient outcome that preserves the initial balance of power. The motivation for a reference point is that players enter the negotiation endowed with a certain amount of 'issue authority,' i.e. their issue-related power to determine which option

[5] KEENEY AND RAIFFA (1991) give an overview of how to structure multi-issue negotiations by using an additive scoring system.

ISSUES		OPTIONS						
		A	B	C	D	E	F	Max
1	M	0.00	6.87	12.50	15.25	$20.8\bar{3}$	25.00	25.00
	R	10.00	8.25	7.00	6.10	5.00	0.00	10.00
2	M	0.00	3.00	5.00	6.25	7.08	$8.3\bar{3}$	$8.3\bar{3}$
	R	10.00	7.50	6.00	5.00	3.00	0.00	10.00
3	M	$8.3\bar{3}$	7.08	6.25	5.00	3.00	0.00	$8.3\bar{3}$
	R	0.00	3.00	5.00	6.00	7.50	10.00	10.00
4	M	0.00	4.17	5.21	6.25	7.50	$8.3\bar{3}$	$8.3\bar{3}$
	R	40.00	33.35	25.00	20.00	8.60	0.00	40.00
5	M	25.00	$20.8\bar{3}$	15.25	12.50	6.87	0.00	25.00
	R	0.00	5.00	6.10	7.00	8.25	10.00	10.00
6	M	0.00	$5.8\bar{3}$	$11.6\bar{6}$	15.63	$20.8\bar{3}$	25.00	25.00
	R	20.00	17.00	14.00	12.50	10.00	0.00	20.00

Table 2: Percentage Gains of players M and R from plans A–F for products 1–6

is implemented. Issue authority may be related to the negotiators themselves (e.g. to their reputation, status, or position) or due to the history of the negotiation. The issue authority combined with players' interests determines their bargaining power. Negotiation then implies that players, driven by their interests, use their power to trade their authority, which generally varies across issues.

GUPTA (1989) assumes that negotiators have linear, additively separable preferences over the negotiated issues. Formally, their preferences over the 6 required divisions can be characterized by utility functions $u^x : [0,1]^6 \to \mathbb{R},\ x = M, R$, where u^x is assumed to be linear on the 6-tuple of divisions.

The linear specification of preferences leads to a simplification of the bargaining problem, because a player's interest in an issue can then be characterized completely by the best option for each issue. This is captured by the interest matrix

$$X \equiv \begin{bmatrix} X_M \\ X_R \end{bmatrix} = \begin{bmatrix} 25.00 & 8.33 & 8.33 & 8.33 & 25.00 & 25.00 \\ 10.00 & 10.00 & 10.00 & 40.00 & 10.00 & 20.00 \end{bmatrix},$$

which follows directly from the far-right column of Table 2.[6] The top row of X describes the preferences of M and the bottom row those of R. The columns of X represent the individual issues.

Players' initial relative authorities over the six issues are assumed to be exogenously given by the matrix

$$C^0 \equiv \begin{bmatrix} C_M^0 \\ C_R^0 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.5 & 0.5 & 1.0 & 0.0 & 0.0 \\ 1.0 & 0.5 & 0.5 & 0.0 & 1.0 & 1.0 \end{bmatrix},$$

where the entries in each column of C^0 sum to one. Here issue authority is "mismatched" in the following sense: A negotiator has no authority over issues that he prefers a lot more than the other player. Players have equal authority, though, over issues that they value similarly (see GUPTA (1989)).

Combining issue authority with interests yields the initial dependence matrix

$$D^0 \equiv \begin{bmatrix} D_{MM}^0 & D_{MR}^0 \\ D_{RM}^0 & D_{RR}^0 \end{bmatrix} \equiv XC^{0'} = \begin{bmatrix} 16.67 & 83.33 \\ 50.00 & 50.00 \end{bmatrix},$$

which shows to what extent players can control their own payoffs and how much their benefits depend on the other player. According to the top row of D^0, player M can secure only 16.67% by exercising issue authority. For the other 83.33% he is dependant on R. The bottom row indicates that R has control over 50% of his interests. The outcome that players can obtain just by exercising their issue authority is taken as the 'reference point' of negotiation; we denote this by $u_r = (u_r^M, u_r^R) \equiv (D_{MM}^0, D_{RR}^0) = (16.67, 50.00)$.

The structure of D^0 suggests that player R has more bargaining power than M. This is manifested in GUPTA's (1989) definition of players' overall power, denoted by $p = (p_M, p_R)$, where

$$p_M \equiv \frac{D_{RM}^0}{D_{MR}^0 + D_{RM}^0} = 0.375 \quad \text{and} \quad p_R \equiv \frac{D_{MR}^0}{D_{MR}^0 + D_{RM}^0} = 0.625 .$$

As a consequence, the balance of power is determined directly by the reference point:

$$\frac{p_M}{p_R} = \frac{D_{RM}^0}{D_{MR}^0} = \frac{100 - D_{RR}^0}{100 - D_{MM}^0} = \frac{100 - u_r^R}{100 - u_r^M} = 0.6 .$$

[6]See also GUPTA (1989): Table 3b, p. 798.

Geometrically, the balance of power is given by the slope of the line connecting the reference point u_r with the 'utopia point' (100,100). This is an attractive feature, since the reference point may be determined by various factors other than issue authority. For a discussion of alternative motivations see GUPTA AND LIVNE (1988).

Negotiation involves the trading of issue authorities in order to obtain an optimal authority structure C^* and, through $D^* = XC^{*\prime}$, the maximum compatible outcomes $u^{M*} = D^*_{MM}$ and $u^{R*} = D^*_{RR}$. Alternatively, we can assume that each player, through his ex-ante issue authority, has a specific percentage of total power, $100p_M$ or $100p_R$, with which he can purchase issue authority at a given system of prices. If we denote the vector of issue prices by v, then the budget constraints for both players are given by $vC' \leq 100p$. The question is, what determines the prices?

In order for the balance of power to remain constant, D_{MM} and D_{RR} must be maximized subject to

$$\frac{p_M}{p_R} = \frac{100 - D_{RR}}{100 - D_{MM}} .$$

This condition implies that $pD^* = 100p$.[7] With $D^* = XC^{*\prime}$ and $vC^{*\prime} = 100p$, the implicit price system is given by $v = pX = (15.63, 9.38, 9.38, 28.13, 15.63, 21.88)$. Hence, the trading prices are the product of bargaining power and interests.

With given prices, the individual linear program of player $i = M, R$ can now be solved separately: $\max_{C_i}\{D_{ii} = X_i C_i' \mid vC_i' \leq 100p_i;\ C_{ij} \leq 1,\ j = 1, \ldots, 6\}$. This yields the optimal outcomes $D^*_{MM} = 57.14$ and $D^*_{RR} = 74.29$. Since

$$\frac{100 - D^*_{RR}}{100 - D^*_{MM}} = \frac{100 - u_r^R}{100 - u_r^M} ,$$

the linear program implements the maximum jointly feasible outcome on the line connecting the reference point with the utopia point. For linear, additively separable preferences, this is the outcome which is also induced by the axiomatic solution of GUPTA AND LIVNE (1988).

In experiments with the negotiation problem of Table 1, GUPTA (1989) finds evidence that negotiators with given reference points try to maintain a constant

[7]In order to see this, note that the condition can be rearranged to $p_M D^*_{11} + p_R D^*_{21} = 100p_M$ or $p_M D^*_{12} + p_R D^*_{22} = 100p_R$.

balance of power during negotiation. As the previous analysis shows, this implies that they are implicitly trading issue authority at fixed prices. Players do, however, have difficulty in reaching an efficient agreement, i.e. the Gupta-Livne solution is not often implemented. This not surprising when one considers the density of agreements in Figure 1 and that a reference point $u_r = (16.67, 50)$ corresponds to a reference payoff of $\pi_r = (7, 13)$. The linear programming method is an analytically straight-forward approach to obtain the solution that players may be aiming for. Unfortunately, its explanatory power depends on players' knowledge of linear programming.

4. Adjusted Winner

If negotiators have linear, additively separable preferences over issues, then negotiation becomes a fair-division problem. Issues can be regarded as divisible goods that are divided between both players. For this type of problem BRAMS AND TAYLOR (1996) introduce an algorithm, which they title 'Adjusted Winner.' The procedure implements an efficient and equitable outcome, where equitability implies that both players achieve the same percentage gains, or, more generally, that players achieve percentage gains according to some given entitlement ratio. The solution, thus, features the same characteristics as the axiomatic solution of KALAI AND SMORODINSKY (1975).[8] Moreover, the algorithm is extremely simple and consists of only two steps: The first step implements an efficient outcome, and the second step adjusts this outcome until it is equitable.

For each issue, step 1 of Adjusted Winner first gives complete issue authority to the player who values this issue most, independent of players' initial authority given by C^0. In GUPTA's (1989) terminology, step 1 ensures that issue authority is completely "matched." If players value an issue equally, then issue authority can be distributed in any form, e.g. by tossing a coin.[9] In our negotiation example (cf. Table

[8] For a more elaborate treatment of this aspect, see RAITH AND WELZEL (1998).

[9] BRAMS AND TAYLOR (1996) propose to first distribute the differently valued issues and then give all equally valued issues to the player with the highest utility in order to preserve his lead.

2), step 1 gives player M complete authority over issues 1, 5, and 6, while player R receives issues 2, 3, and 4. The resulting utilities are $u^M = 75$ and $u^R = 60$.

The intriguing feature of step 1 is that it guarantees efficiency. In order to see this, one must only verify that there is no trade of issue authorities that can benefit both players. If issue authority is distributed according to the rules of step 1 and, for each issue, authority is given to the player who values this issue most, then this distribution also maximizes the sum of players' utilities. Consequently, if one player gains by trading authorities, the other must lose, since trading cannot increase the sum of players utilities. There is also no convex combination of authority distributions that dominates that of step 1, since then at least one distribution must yield a higher sum of players utilities.[10]

Step 2 of Adjusted Winner now shifts issue authority from one player to the other until equity is achieved. Since the outcome of step 1 is already efficient, step 2 is designed to preserve this feature by transferring issue authority at the lowest cost-gain ratio. In our example, authority must be shifted from player M to player R. The only issues to consider are those that have been given to M. Issues 1 and 5 are valued by M at 25 percent and by R at 10 percent. Changing authority would thus imply that M loses 2.5 times as much as R gains. Issue 6 is valued at 25 percent by M and 20 percent by R, implying a cost-gain ratio of only 1.25. Hence, the efficient adjustment shifts the authority over issue 6 first. A complete switch of authority would lead to $u^M = 50$ and $u^R = 80$, which is too strong. Equitability requires a division of issue authority, given by some fraction $\alpha \in [0, 1]$, such that

$$50 + \alpha 25 = 60 + (1 - \alpha)20 \; .$$

The equitable split is given by $\alpha = 2/3$, i.e., M must give up 1/3 of his authority over issue 6. The resulting outcome is $u^{M*} = u^{R*} = 66.67$.

For linear, additively separable preferences, Adjusted Winner implements the axiomatic solution of KALAI AND SMORODINSKY (1975). As GUPTA AND LIVNE (1988) show, their axiomatization is an extension of Kalai-Smorodinsky. The re-

[10] cf. BRAMS AND TAYLOR (1996): Proof of Theorem 4.1.

quired generalization of Adjusted Winner in order to implement the Gupta-Livne solution is straight forward: Only the equitability condition must be modified to acknowledge the balance of power given by the reference point. In particular, the adjustment process must induce an efficient outcome (u^{M*}, u^{R*}) that satisfies the condition

$$\frac{100 - u^{R*}}{100 - u^{M*}} = \frac{100 - u_r^R}{100 - u_r^M} = \frac{p_M}{p_R} .$$

As before, it is issue 6 that must be divided. Denoting by α player M's fraction of issue authority, the above condition becomes

$$\frac{100 - [60 + (1-\alpha)20]}{100 - [50 + \alpha 25]} = 0.6 ,$$

which yields $\alpha = 2/7$. If M passes 5/7 of his authority over issue 6 to player R, then $u^{M*} = 57.14$ and $u^{R*} = 74.29$. This outcome coincides with the solution of the linear programming problem of the previous section.

Although both implementations of the Gupta-Livne solution have similar characteristic features, Adjusted Winner is analytically simpler than the linear-programming problem. The programming method first fixes the balance of power and then moves towards efficiency. With a large number of potential (inefficient) agreements, this can become a difficult task without computer support. With Adjusted Winner, the process is reversed: The first step takes players directly to the efficient boundary by distributing issue authority efficiently. The second step then reestablishes the initial balance of power by adjusting issue authority.

Intuitively, it is the "detour" of letting players change the balance of power which makes Adjusted Winner so easy. This is also the additional cooperative element in the bargaining process: In order to attain efficiency, players first jointly maximize utility and afterwards consider the distribution of gains. The individual maximization underlying the linear-programming method thus has its cost. One must not assume, though, that this approach requires less cooperation than Adjusted Winner. With $v = pX$, trading issue authority at fixed prices requires full commitment to the balance of power and full information of both players' interests.

5. Generalized Adjusted Winner

Both the linear programming approach as well as Adjusted Winner make use of the assumption that players have linear, additively separable preferences over issues and options. This implies that their interests are characterized by the utilities of their best options for each issue. In our example (Table 2), players focus only on combinations of the two extreme marketing plans A and F of each issue. This linearization of the negotiation problem is, indeed, a considerable simplification, because there are then only $2^6 = 64$ discrete outcomes.

However, since the linearization does not acknowledge efficient intermediate options, any solution concept based on this feature will generally lead to an inefficient outcome. This is shown in Figure 2, where we have plotted players' payoffs for the linearized problem together with the outcomes of the linear program and Adjusted Winner, denoted by LP and AW, respectively. As Figure 2 illustrates, the efficiency frontier of the non-linear problem, given by all efficient options of Table 2, lies well beyond that of the linearized problem. As a consequence, the linear program does not implement the Gupta-Livne solution (GL), and Adjusted Winner does not implement the Kalai-Smorodinsky solution (KS) anymore. The efficiency loss is quite significant, considering that 5 utility points are equivalent to 1 mill. Euros for player R and even 1.2 mill. Euros for player M.

In order to take account of all marketing plans, we formally assume that players' preferences over the 6 required divisions are, again, characterized by utility functions $u^x : [0,1]^6 \to \mathbb{R}$, $x = M, R$, with $u^x \equiv u_1^x + u_2^x + \cdots + u_6^x$, but where the subutility functions u_i^x, $i = 1, \ldots, 6$, are now assumed to be piecewise linear and concave.

As long as utility is piecewise linear, i.e. the substitution rates between options are constant, the linear programming problem can be extended in order to acknowledge all efficient options. GUPTA (1989) proposes to fractionate the issues into linear subissues. With 6 options for each of 6 issues, this decomposition produces up to 30 subissues (5 subissues per issue)[11] A linear program with 30 issue prices becomes

[11] The actual number of subissues may be less, since some options can turn out to be inefficient when compared with convex combinations of other options.

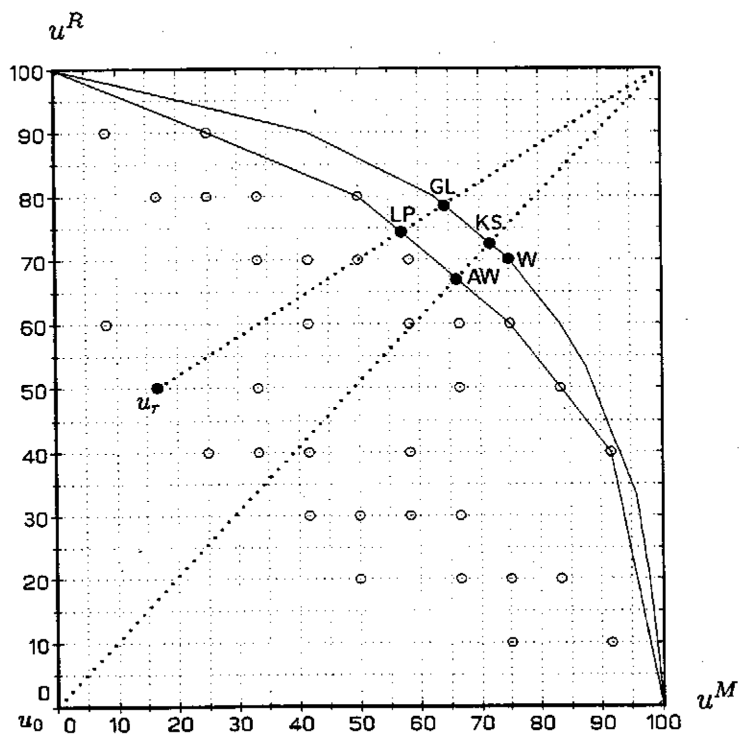

Figure 2: Bargaining solutions for linear and nonlinear preferences over 6 issues

quite difficult to manage without computer support. More problematic, though, is the distribution of issue authority between the artificially constructed new issues. Technically, the authority matrix C must be compatible with the interest matrix X. Otherwise, the balance of power cannot be derived from the authority structure, and then it is not clear how one should motivate the underlying implicit price system. As a consequence, the interpretation of a trading process becomes less intuitive.

The decomposition of issues into subissues also allows the application of Adjusted Winner.[12] However, a decomposition is not necessary, since the procedure can also be adapted to the original, non-decomposed bargaining problem without much additional effort. We demonstrate this by generalizing the algorithm in order to implement the Gupta-Livne solution. The Kalai-Smorodinsky solution follows immediately as a special case, where the origin is the reference point. The whole

[12] A formal proof is given by RAITH AND WELZEL (1998)

procedure still consists of two fundamental steps, which we formulate as a two-step algorithm using our example for illustration.

Step 1: Maximize joint utility without considering the distribution of gains – *For each issue i choose the 'temporary options' iT that maximize the sum of players' utilities, i.e. $iT \in \arg\max_T\{u^M_{iT} + u^R_{iT}\}$. If more than one option satisfies this criterion, then let a referee decide which option to choose from this set. For all temporary options, calculate the sum of each player's utilities, $u^x = \sum_i u^x_{iT}$, $x = M, R$. Determine the 'temporary winner' W and the 'temporary loser' L through the condition*

$$\frac{u^W - u^W_r}{100 - u^W_r} \geq \frac{u^L - u^L_r}{100 - u^L_r} .$$

◇

Consider our example negotiation in Table 2. We denote an agreement by the 6-tuple of chosen options. Step 1 then leads to the temporary agreement [EDCABE], which yields the utilities $u^M = 75$ and $u^R = 70$; in Figure 2 we denote this outcome by W. As before, step 1 induces an efficient agreement, because it maximizes the sum of players' utilities over all issues. With the given reference point of $u_r = (16.67, 50)$, the temporary outcome implies

$$\frac{u^M - u^M_r}{100 - u^M_r} = \frac{58.33}{83.33} > \frac{20}{50} = \frac{u^R - u^R_r}{100 - u^R_r} .$$

According to the above condition, player M is the designated 'temporary winner' W and R the 'temporary loser' L. With the distance between the reference point and the utopia point as a standard of value, player M's percentage gain outweighs that of R, thus disturbing the initial balance of power.

Step 2: Redistribute gains efficiently in order to reestablish the balance of power – *For each issue i and each temporary option iT, consider each efficient alternative option iO_T that gives the temporary loser a higher utility than with the temporary option iT. Calculate the rate of substitution*

$$\mathrm{RS}_{i:TO} := \frac{u^W_{iT} - u^W_{iO_T}}{u^L_{iO_T} - u^L_{iT}} .$$

Select the alternative options iO^*_T that yield the lowest cost-gain ratio, i.e. $iO^*_T \in$ $\arg\min_O \mathrm{RS}_{i:TO}$. If more than one option satisfies this criterion, then let a referee *choose from this set. Now determine the issues i^* with the lowest cost-gain ratio, i.e.* $i^* \in \arg\min_i \mathrm{RS}_{i:TO^*}$. *If there is no unique issue, then let a referee decide. Calculate players' overall utilities $u^W_{O^*}$ and $u^L_{O^*}$ by replacing only option i^*T with its efficient alternative $i^*O^*_T$. If*

$$\frac{u^W_{O^*} - u^W_r}{100 - u^W_r} > \frac{u^L_{O^*} - u^L_r}{100 - u^L_r},$$

then make $i^*O^*_T$ the new temporary option of issue i^* and repeat step 2. Else, calculate the convex combination of options i^*T and $i^*O^*_T$ that establishes the initial balance of power between the winner W and loser L. ◇

It is important to note that the referee decisions in the algorithmic characterization of steps 1 and 2 do not require an actual referee, but can be substituted by a variety of alternatives. These include the toss of a coin or letting the temporary loser or winner decide. If actual issues and options have a meaningful content, practical negotiators would presumably wish to choose the "most practical" alternative.

We illustrate step 2 with the help of our example. For issue 1, the temporary agreement chooses option E. Considering the alternative options D, C, B, and A, we find that $O^*_{1E} = \mathsf{C}$, with $\mathrm{RS}_{1:EC} = 4.165$. For issue 2, we have $\mathrm{RS}_{2:DC} = \mathrm{RS}_{2:DA} = 1.25 < \mathrm{RS}_{2:DB} = 1.30$. Since players are still quite far from the initial balance of power, they would presumably choose the alternative with the highest gain for player R, i.e $O^*_{2D} = \mathsf{A}$. By the same line of reasoning, one obtains $O^*_{3C} = \mathsf{F}$, where $\mathrm{RS}_{3:CF} = \mathrm{RS}_{3:CD} = 1.25$. Issue 4 can be skipped, since player R is already at his maximum here. Issues 5 and 6 both have option D as the efficient alternative, with $\mathrm{RS}_{5:BD} = 4.165$ and $\mathrm{RS}_{6:ED} = 2.08$. The lowest cost-gain ratio over all issues is thus given by issues 2 and 3. If players select $x^* = 2$, this leads to the alternative agreement [EACABE], with total utilities $u^M_{O^*} = 68.75$ and $u^R_{O^*} = 75$. However, with

$$\frac{100 - u^M_{O^*}}{100 - u^M_r} = \frac{52.08}{83.33} > \frac{25}{50} = \frac{u^R_{O^*} - u^R_r}{100 - u^R_r},$$

the adjustment is not sufficient for the required balance of power.

Since issue 2 offers no further possibilities of substitution, negotiators can continue directly with issue 3, without having to calculate further substitution rates. It seems reasonable, though, for them to proceed in smaller steps now, because they are already quite close to the desired solution. They should, therefore, select $O^*_{3C} = \mathsf{D}$, such that M loses only 1.25 and R gains 1.00. The new agreement is [EADABE], and the resulting utilities are $u^M_{O*} = 67.50$ and $u^R_{O*} = 76$. With

$$\frac{u^M_{O*} - u^M_r}{100 - u^M_r} = \frac{50.83}{83.33} = 0.61 > 0.52 = \frac{26}{50}\frac{u^R_{O*} - u^R_r}{100 - u^R_r},$$

the initial balance of power is still not achieved. The next efficient alternative $O^*_{3D} = \mathsf{F}$ leads to the agreement [EAFABE], where $u^M_{O*} = 62.50$ and $u^R_{O*} = 80$. This adjustment is too strong, because

$$\frac{u^M_{O*} - u^M_r}{100 - u^M_r} = \frac{45.83}{83.33} = 0.55 < 0.6 = \frac{30}{50} = \frac{u^R_{O*} - u^R_r}{100 - u^R_r}.$$

The balance of power thus requires a convex combination of options 3D and 3F. Denoting by α the fraction of option 3D, one obtains

$$\frac{[\alpha 67.50 + (1-\alpha)62.50] - 16.67}{100 - 16.67} = \frac{[\alpha 76 + (1-\alpha)80] - 50}{100 - 50},$$

which yields a value of $\alpha = 0.36$. Players' utilities are 'balanced' at $u^{M*} = 64.30$ and $u^{R*} = 78.56$.

The efficient agreement [EA(αD+$(1-\alpha)$F)ABE] implements the Gupta-Livne solution. An important practical feature, which is highlighted by the algorithm underlying Adjusted Winner, is that the agreement requires the convex combination of at most two options of a single issue; the options of all other issues remain undivided.

If a mixture of two specific options is difficult to realize in practice, then the algorithm offers alternatives at each point where a 'referee decision' is required. In our example, negotiators could also consider the equally favorable agreement [E(αA+$(1-\alpha)$C)FABE] as an implementation of the Gupta-Livne solution. Every referee decision thus increases the flexibility of the negotiation process, but it does not create additional conflict, because all alternatives lead to the same payoff.

If any mixture of options is difficult to achieve, Adjusted Winner even offers negotiators 'creative' support. In our example, note that the efficient substitution process bypasses option 3E, since players can achieve a better outcome with a combination of options D and F. Nevertheless, even with the agreement [EAEABE], players can still realize utilities of $u^M = 65.50$ and $u^R = 77.50$, which are quite close to the Gupta-Livne outcome. Hence, negotiators only need to direct their creativity to improve option 3E with a slight bias in favor of player R.

6. Conclusion

As we have shown, the fair-division algorithm Adjusted Winner can be generalized to implement efficient outcomes in complex multi-issue negotiations. In our analysis, we showed how the algorithm implements the Gupta-Livne solution, which is a generalization of the Kalai-Smorodinsky solution with a reference point that is not the origin. The adjustment process along the Pareto frontier can thus be adapted to different solution concepts.

The strongest feature of the algorithm is its simplicity. For a multi-issue negotiation problem, we showed that the implementation of the Gupta-Livne solution involves only a few intuitive steps that require only minor computational effort, without using a computer. In contrast, in order to solve the linear programming problem, negotiators must at least be capable of linear programming (or know someone who is). However, considering the development of modern information technology, the question arises: Do we still need a procedure for which negotiators require at most a pocket calculator in order to implement an efficient outcome?

Apparently, yes: Most solution concepts in bargaining theory have been known for decades, and assessment techniques such as the utility matrix in our example have been in use for over 20 years.[13] Nevertheless, the implementation of cooperative solutions still seems to be a major task for negotiators. The reason is that negotiators in practice require more than just knowledge of an efficient agreement. It must also be supportable by a convincing line of reasoning. Without plausible argumentation, a cooperative solution loses its appeal.

Adjusted Winner in its generalized form is a simple process support tool, which in most cases does not even require computer support. The technique consists of

[13] According to Raiffa (1982), this technique was employed in the Panama-Canal negotiations in 1974.

only two fundamental steps: The first maximizes negotiators' joint utility without considering the distribution of gains. Players focus only on efficiency and they distribute their bargaining power accordingly. The second step achieves the desired distribution of gains through efficient transfers. The options that are changed are plausibly those with the lowest cost-gain ratios. Due to its tractability, the algorithm thus provides an argumentative basis for the complete negotiation process.

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