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Representation of CII–Games and the Expected Contract Value

by

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Abstract

We consider Cooperative Games with Incomplete Information ("CII–Games"), that is, a version of the NTU–characteristic or coalitional function in the presence of incomplete information about players types. Decisions are parameters entering the utility functions of players, which otherwise depend on randomly chosen types. First of all we study the "canonical choice" of utility functions (e.g., with prevailing linearity in the side payment case). Based on this a "bargaining solution" or "value" is described that results from the expectation of all fair *ex post* (NASH) bargaining solutions of the various realizations of types.

SECTION 1
CII-Games and Fee Games; Motivation and Notations

A *unanimous NTU–Game* (or NASH–bargaining situation) for n players is a pair $(\underline{u},V)$ such that V is a subset of $\mathbb{R}^n$ ("the feasible utility vectors") and $\underline{u} \in V$ (the "status quo" or "threat point").

Within this presentation, we shall always assume that V is a closed, convex, and comprehensive set such that

$$V_+ := \{u \in V \mid u \geq 0\} \tag{1}$$

is compact. Also, we shall only consider $\underline{u} = 0$ and require that 0 is within the interior of V – thus there is $u \in V_+$, $u > 0$.

The elements of V represent collective decisions that are available to the players if they choose to agree upon one of them, that is if they cooperate. In this context it is of course assumed that all players are aware of the consequences of such a decision for each of them; that is, since the elements of V are "utility vectors" it is assumed that it is common knowledge that a decision for $u \in V$ implies the utility u_i for player $i \in I := \{1,...,n\}$.

In this paper we want to consider cooperative games with incomplete information; this theory starts out with the work of HARSANYI–SELTEN [5]. In this context it is assumed that there is a fixed set of decisions available to the players by collective and unanimous agreement, but the consequences or utilities resulting from such a decision may vary and depend in addition on random influence.

Most authors assume that the set of decisions in a cooperative game with incomplete information is finite. However, we shall choose

$$\underline{X} = \{x \in \mathbb{R}^n \mid e\,x \leq 1\} \tag{2}$$

as the set of collective decisions or "parameters"; here $e = (1,...,1) \in \mathbb{R}^n$. If players fail to reach an agreement we want $\underline{x} = 0 \in \underline{X}$ to represent the "status quo" or "threat" parameter. Also $\partial\,\underline{X} = \{x \in \underline{X} \mid e\,x = 1\}$ shall represent the "Pareto efficient" boundary of $\underline{X}$.

$I = \{1,\ldots,n\}$ represents the set of players. For $i \in I$ let T^i denote the possible *types* of i; $T := T^1 \times \ldots \times T^n$ is the set of possible types of all players. The notion of types as used in game theory goes back to HARSANYI [4], other authors prefer to speak about the "state of nature" see e.g. ALLEN [1,2].

The distribution of types is represented by a probability p on T.

Next, assume that for each $i \in I$ a function

$$U^i : T \times \underline{X} \to \mathbb{R}$$

represents player i's utility depending on the actual types and the collective decision agreed upon. Then the six-tupel

$$\Gamma = (I, T, p; \underline{X}, \underline{x}; U) \qquad (3)$$

is a (unanimous) *cooperative game with incomplete information* or for short, a *CII-game.*

There are two stories related to the question as to how this game is being played. The first, naive, or basic interpretation is the obvious one: consider some abstract probability space (Ω, P, F) and let $\tau : \Omega \to T$ be a mapping *with distribution* p.
Nature performs a chance experiment represented by (Ω, P, F), which results in some $\omega \in \Omega$. Each player observes his "true type" $\tau_i(\omega)$ and thereafter the players may agree upon some "collective decision" or parameter $x \in \underline{X}$. If so, each of them receives the utility $U_i^{\tau}(\omega)$ (x). This means that, while bargaining, players have knowledge about their own type and may infer about the types of the other players from the conditional probability resulting from p given that τ_i takes a certain value.

The problem is that in such a situation the utiliti4es of the resulting "bargaining solution" depend on the true types and hence, there is an incentive for players to possibly pretend that they are in a false type, thus increasing their true utility.

This leads to the slightly more refined story of the bayesian incentive compatible mechanism (in the sense of HURWICZ [7]).

According to this story a *Bayesian incentive compatible* or BIC-*mechanism* is a mapping $\mu : T \to \underline{X}$ such that the following inequalities are satisfied.

$$E(U_i^{\tau} \circ \mu^{\tau} \mid \tau_i = t_i) \geq E(U_i^{\tau} \circ \mu^{(\tau_1,\ldots,s_i,\ldots,\tau_n)} \mid \tau_i = t_i) \qquad (i \in \Omega,\ t_i \in T^i,\ s_i \in T^i). \qquad (4)$$

The mechanism is interpreted as an agreement among the players (to be recorded and enforced by some outside agency or court). Accordingly everyone will make an announcement once the types are generally observed and the collective decision will depend on these announcements. A mechanism is BIC if making the true announcements about one's type maximizes each players utilities provided the others stick to this kind of policy. More precisely, in the resulting non-cooperative game induced by the mechanism μ, the strategy to report ones true type is a Nash equilibrium.

For fixed $t \in T$, let

$$V^t := \{U^t(x) \mid x \in \underline{X}\}$$
$$\underline{u}^t := U^t(\underline{x}) \qquad (5)$$

Then, given some suitable conditions to be imposed on $U^{\cdot}(\cdot)$, $(\underline{u}^t, V^t) = (0, V^t)$ is a unanimous NTU-game; clearly it is the one generated by $t \in T$ "with complete information". Thus, $U^t : \underline{X} \to V^t$ can be regarded as a *parametrization*. For various $t \in T$ (i.e. various $\omega \in \Omega$), V^t represents the different consequences resulting from decisions $x \in \underline{X}$.

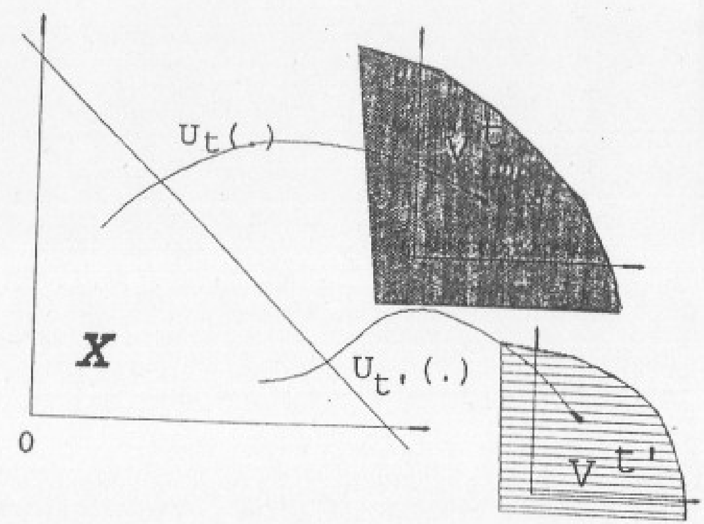

Fig. 1.1.

The appropriate form of the mappings $U^t(\cdot)$ is certainly not without importance. We shall first and for some considerable time be dealing with these mappings. What influence does the parameterization have on possible solution concepts? Can we, at least for some class of games, axiomatically define something like a *canonical representation* or *parametrization* of CII–games?

The first requirement to every $U^t(\cdot)$ is that we want it to be bijective. For, otherwise, the introduction of certain equivalence classes of parameters, which result in equal utility as far as certain players are concerned, would further complicate the situation. Indeed, if our parametrizations are bijective, then we actually attempt the identification of certain points of V^t ($t \in T$), namely those resulting from the same parameter or collective decision $x \in \underline{X}$. Hence a CII induces an *equivalence relation* between various (unanimous) NTU–games, i.e., between the $(0, V^t)$, $t \in T$.

A second requirement we attempt to impose on the parametrizations is monotonicity. It certainly clarifies the situation for the players involved if, when bargaining about some $x \in \underline{X}$, they can behave like when bargaining about utility vectors: by this we mean that player I wants to maximize his coordinate x_i and hence we should require that every coordinate $U_i^t(\cdot)$ of $U^t(\cdot)$ is strictly monotone in x_i.

Furthermore, it seems reasonable to require that players are aware when they are dealing with Pareto efficient utility vectors. Hence it seems reasonable to construct U in such a way that $U^t(x)$ is pareto efficient in V^t whenever $x \in \partial\, \underline{X}$.

The above requirements exhibit the tendency to represent the situation as simple as possible and to include as many of the familiar features of a bargaining situation or NTU–Game as possible in the situation with incomplete information ("entia non sunt multiplicanda praeter necessitatem"). Still, further requirements are certainly necessary (e.g. convexity and continuity).

In order to exhibit a "canonical parametrization" let us start out with the simplest case, that is the side–payment game. As parametrization can be studied without referring to types and the distribution of types, we shall tentatively omit the type vector t and start out with some (unanimous) NTU–game, say $(\underline{u}, V)$ (with $\underline{u} = 0$). The side–payment character is stressed by putting, for $c > 0$ and $e = (1,\ldots,1)$,

$$\begin{aligned} V = V^{e/c} \quad &:= \{u \in \mathbb{R}^n \mid {}^{e}/_{c}\,u \le 1\} \\ &= \{u \in \mathbb{R}^n \mid e\,u \le c\} \end{aligned} \tag{6}$$

Now, as $(0, V^{e/c})$ represents a side–payment situation, we want $U(\cdot)$ to reflect this character as much as possible. This means, that in every coalition $S \subseteq I$ the result of a redistribution of parameter units ("money") does not change total utility. Clearly, linearity is the least we should ask for in this context.

Having this in mind, we come up with

Theorem 1.1 Let $c > 0$ and let

$$U : \underline{X} \to V^{e/c}$$

be a mapping satisfying the following conditions.

1. U is linear
2. U_i is strictly monotone in x_i
3. For any $y \in \mathbb{R}^n$ and any $S \subseteq I$ such that $ey = 0$ and $y_i = 0$ ($i \notin S$) it follows that, for all $x \in \underline{X}$,

$$\sum_{i \in S} U_i(x+y) = \sum_{i \in S} U_i(x). \tag{7}$$

4. U maps $\partial\, \underline{X}$ on $\partial\, V^{e/c}$.

Then there is $C > 0$ and $b^0 \in \mathbb{R}^n$ with $eb^0 < 1$ such that

$$U(x) = C(x - (ex)b^0) \qquad (x \in \underline{X}) \tag{8}$$

Proof:

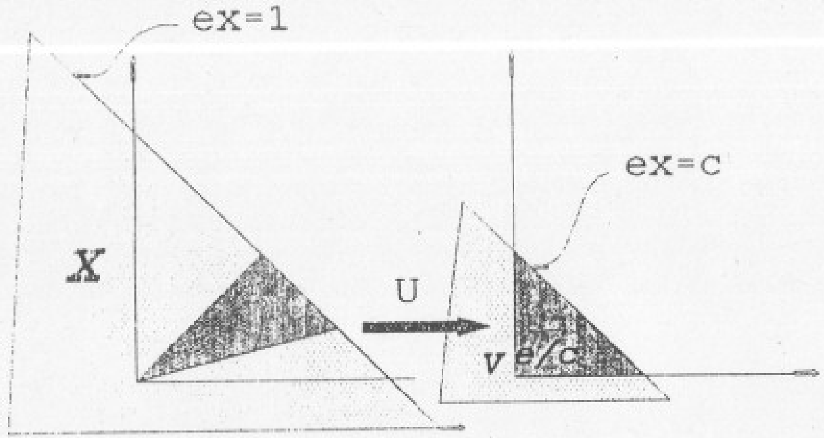

Fig. 1.2.

As U is linear, there is an $n \times n$-matrix A such that

$$U(x) = Ax \qquad (x \in \underline{X}).$$

Consider some $S \subsetneq I$, $S \neq \emptyset$, I.

Clearly, it follows from condition 3. that

$$\sum_{i \in S} A_{i\cdot}\, y = 0$$

for *any* $y \in \mathbb{R}^n_S = \{y \in \mathbb{R}^n \mid y_i = 0 \ (i \notin S),\ ey = 0\}$. Consequently, the S-coordinates of $\sum_{i \in S} A_{i\cdot}$ have to be equal, say

$$\left(\sum_{i \in S} A_{i\cdot}\right)_S = \lambda_S\, e_S \tag{9}$$

for suitable $\lambda_S \in \mathbb{R}$. Replace S by $S + \{i_0\}$ for some $i_0 \notin S$ and subtract equations (9) corresponding to S and $S + \{i_0\}$; it turns out immediately that all $a_{i_0 j}$ $(j \in S)$ must be equal. By varying S we find that all $a_{i_0 j}$ outside the diagonal have to be equal. Thus,

the *rows* $A_{\cdot j}$ of A satisfy

$$A_{\cdot j} = c_j(e^j - b^0)$$

with $C_j \in \mathbb{R}$ and $b^0 \in \mathbb{R}^m$.

Now, condition 2. requires $a_{jj} > 0$, thus $C_j \neq 0$ $(j \in I)$.

Next, by condition 4, the image of e^j has to be Pareto-efficient, that is, we obtain

$$c = e\, A\, e^j = e\, A_{\cdot j} = C_j(1 - eb^0).$$

Hence we conclude that C_j equals $\frac{c}{1 - eb^0}$, i.e.

$$Ax = C(x - (ex)\, b^0,\ C = \frac{c}{1 - eb^0}$$

as $c > 0$, q.e.d.

If $C = 1$ then U clearly is the parametrization used in the context of "fee games" (see [13,14]). That is, we may interpret $\underline{X}$ to be the possible distributions of a unit of money among the players. If players agree on $x \in \underline{X}$ than each one of them has to pay a fee (say to the court or referee) which is proportional to the total amount of money agreed upon. The share of this fee is specific to each player and random in the case of a CII-game (a fee game). In this context, types of players are being distinguished only by the fact that they have to pay different fees according to the chance moves and each of them is aware only of the fee he has to pay for himself.

The constant C amounts to a contraction or extension of the Pareto efficient interval or to a common rescaling of utility for all players; thus C has normalizing character. In the context of Section 2 we shall find that $C = \frac{1}{n}$ is more suitable for our present purpose of defining the canonical representation.

SECTION 2

Canonical Representation of Hyperplane-Games

As previously we write $e = (1,...,1) \in \mathbb{R}^n$, $e^i = (0,...,1,...,0) \in \mathbb{R}^n$ is the i'th unit vector.

Next, if $\mathbb{R} \ni \alpha > 0$ and $x \in \mathbb{R}^n$, we write

$$\frac{1}{\alpha} = (\frac{1}{\alpha_1},..., \frac{1}{\alpha_n})$$

and

$$\alpha \otimes x = (\alpha_1 x_1,..., \alpha_n x_n)$$

(the tensor product – or rather an affine transformation of utility, "a.t.u"); consequently

$$\frac{1}{\alpha} \otimes x = (\frac{1}{\alpha_1} x_1,..., \frac{1}{\alpha_n} x_n) .$$

Note that

$$\alpha (\frac{1}{\alpha} \otimes x) = e\,x = \sum_{i=1}^{n} x_i .$$

Next $\underline{X} = \{x \in \mathbb{R}^n \mid e\,x \leq 1\}$ and $\partial \underline{X} = \{x \in \underline{X} \mid e\,x = 1\}$.

Similarly

$$V^\alpha = \{u \in \mathbb{R}^n \mid \alpha u \leq 1\}$$

and

$$\partial V^\alpha = \{u \in V^\alpha \mid \alpha u = 1\}.$$

For fixed $b^* \in \partial \underline{X}$, define

$$U(\alpha,\cdot) : \underline{X} \to V^\alpha$$
$$U(\alpha,x) = \frac{e\alpha}{\alpha} \otimes x - (ex) \frac{e\alpha-1}{\alpha} \otimes b^* \tag{1}$$

Of course, $U(\alpha,\cdot)$ maps $\partial \underline{X}$ onto ∂V^α (bijectively) because of

$$\begin{aligned} \alpha U(\alpha,x) &= (e\alpha)(ex) - (ex)(e\alpha-1) \underbrace{eb^*}_{1} \\ &= ex \end{aligned} \tag{2}$$

Definition 2.1: $U(\alpha,\cdot)$ is the *canonical parametrization* of V^α.

$U(\alpha,\cdot)$ is linear and "monotone in each coordinate" (cf. SECTION 1). In fact, the connection to "fee-games" as discussed shortly in SECTION 1 (and generally in [ ]) is evident:

Remark 2.2: We may regard V^α as a transformed fee-game as follows:

Write

$$U(\alpha,x) = \frac{e\alpha}{\alpha} \otimes (x - (ex) \frac{e\alpha-1}{e\alpha} b^*) \tag{3}$$

and put

$$C^\alpha := \frac{e\alpha-1}{e\alpha} , \lambda = \frac{\alpha}{e\alpha} , \tag{4}$$

then

$$U(\alpha,x) = \frac{1}{\lambda} \otimes (x - (ex)\, C^\alpha b^*)$$

$$= \frac{1}{\lambda} \otimes (x - (ex)\, b^0 \tag{5}$$

E.g. for $\alpha = t\,(\frac{1}{n},...,\frac{1}{n})$ with $e\alpha = t$ and $\lambda = (\frac{1}{n},...,\frac{1}{n})$, $C^\alpha = \frac{t-1}{t}$, we come up with

$$U(\alpha,x) = \frac{1}{n} (x - (ex))\, b^0 \tag{6}$$

This is the utility as used in fee games – up to the factor $\frac{1}{n}$.
(The factor $\frac{1}{n}$ could be avoided by introducing "$Y^\alpha = \{u \in \mathbb{R}^n \mid \alpha u \leq n\}$", the advantage would be invariance of length measurement as discusses in SECTION 1 – the disadvantage occurs in some notational scramble.)

Remark 2.3: A geometrical interpretation of the mapping $U(\alpha,\cdot)$ is given as follows. Note that

$$U(\alpha,b^*) = \frac{1}{\alpha} \circledast b^* = \hat{b} \tag{7}$$

(the "rescaled fee vector..."). Therefore, if we define

$$a^i = a^{i\alpha} := \frac{1}{e\alpha}\, e^i + \left(1 - \frac{1}{e\alpha}\right) b^* \qquad (i \in \Omega) \tag{8}$$

then

$$U(\alpha,a^i) = \frac{1}{\alpha} \circledast e^i = \frac{e^i}{\alpha^i}.$$

Now, consider the simplex

$$I^\alpha = [a^1,\ldots,a^n] \ . \tag{9}$$

Observe that $U(\alpha,\cdot)$ acts in a way such that a^i is thrown into $\frac{e^i}{\alpha^i}$, b^* is distorted to $\hat{b}$, and the points of I^α are exactly mapped into the Pareto efficient and individually rational utility-vectors of ∂V^α.

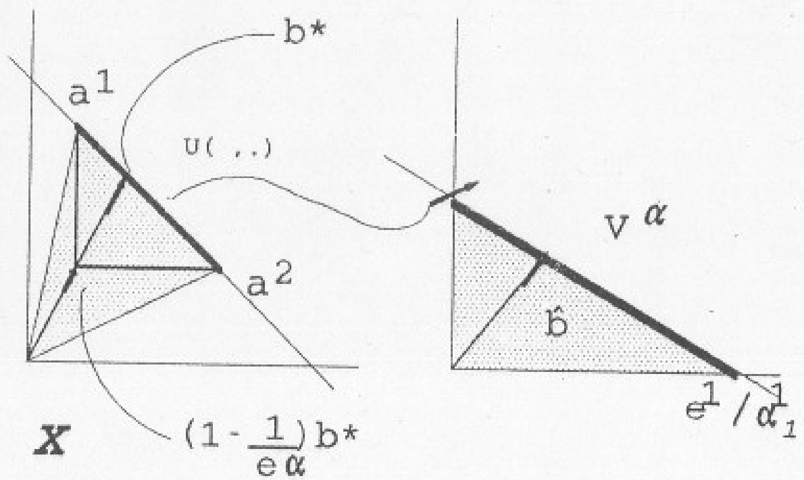

Fig. 2.1.

In particular, if $\alpha = t\,(\frac{1}{n},\ldots,\frac{1}{n})$ (cf. Remark 2.2), then $e\alpha = t$ and thus

$$a^i = \frac{1}{t}\, e^i + \frac{t+1}{t}\, b^*.$$

Thus, distance measurement on $\underline{X}$ is distorted by a factor $\sqrt{n}$.

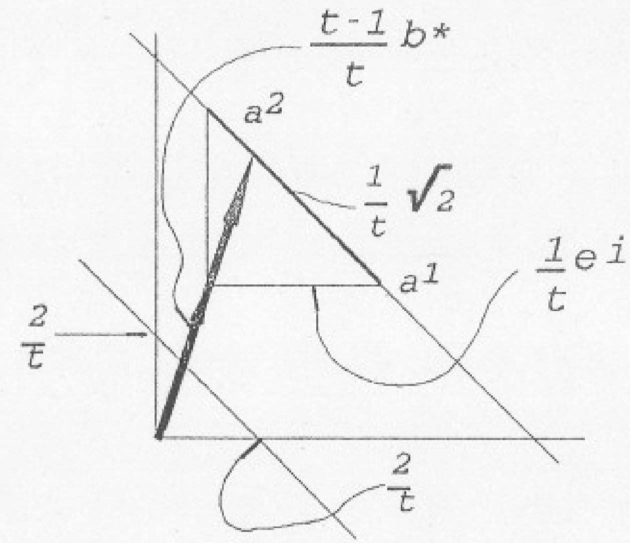

Fig. 2.2.

Lemma 2.4: The inverse mapping to $U(\alpha,\cdot)$ is given by

$$G(\alpha,\cdot) : V^\alpha \to \underline{X}$$

$$G(\alpha,u) = \frac{\alpha}{e\alpha} \circledast u + \frac{e\alpha - 1}{e\alpha}\,(\alpha u)\; b^* \tag{10}$$

Proof: $G(\alpha,\cdot)$ is a linear mapping such that $G(\alpha,0) = 0$ and $G(\alpha, \frac{1}{\alpha_i} e^i) = a^i$; for the role of a^i cf. Remark 2.3 and (8) in particular.

The following monotonicity property of G will be important for our presentation in SECTION 3. In this context, fix $\bar{u} \in \mathbb{R}^n_+$, and let

Then we have

$$\underline{Y}^{\bar u} = \{\alpha \in \mathbb{R}^n \mid \alpha \bar u = 1\} \tag{13}$$

Lemma 2.5: Let $n = 2$. Consider $G_1(\cdot,\bar u) : \underline{Y}^{\bar u} \to \mathbb{R}$. Then $G_1(\cdot,\bar u)$ is strictly monotone in α_1 for all α with $e\,\alpha > 0$. More precisely, if α' is close to α and $\alpha_1 > \alpha_1'$, then

$$G_1(\alpha, \bar u) > G_1(\alpha', \bar u). \tag{14}$$

Proof: Our claim amounts to showing that the directional derivative of $G_1 = G_1(\cdot,\bar u)$

$$(D_{(\frac{1}{\bar u_1}, -\frac{1}{\bar u_2})} G_1)(\alpha) > 0 \tag{15}$$

for $(\frac{1}{\bar u_1}, -\frac{1}{\bar u_2})$ points in direction of increasing α. We shall do this for $\bar u > 0$ only and leave the details for the other cases an an exercise.

To this end, observe that

$$\frac{\partial G_1}{\partial \alpha_1}(\alpha) = \frac{u_1 \alpha_2 + b_1^*}{(e\ \alpha)^2}$$

$$\frac{\partial G_1}{\partial \alpha_2}(\alpha) = \frac{-u_1 \alpha_1 + b_1^*}{(e\ \alpha)^2},$$

such that

$$(D_{(\frac{1}{\bar u_1}, -\frac{1}{\bar u_2})} G_1)(\alpha)$$

$$= \frac{1}{(e\alpha)^2}\left(\alpha_2 + \frac{\bar u_1}{\bar u_2}\alpha_1 + b_1^*\left(\frac{1}{\bar u_1} - \frac{1}{\bar u_2}\right)\right)$$

$$= \frac{1}{(e\alpha)^2}\left(\frac{1}{\bar u_2} + b_1^*\left(\frac{1}{\bar u_1} - \frac{1}{\bar u_2}\right)\right)$$

$$> 0 \quad \text{as } b_1^* \le \frac{1}{\bar u_1}$$

q.e.d.

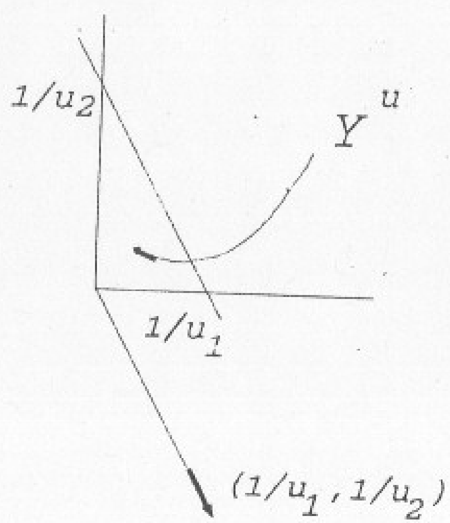

Fig. 2.3

Now we turn shortly to the role of a.t.u.s in connection with the canonical representation.

Definition 2.6: For $\mathbb{R} \ni t > 0$ define

$$S_t : I^{\alpha} \to I^{t\alpha}$$

$$S_t(x) = \frac{1}{t}x + \left(1 - \frac{1}{t}\right) b^* \tag{11}$$

The inverse mapping $T_t : I^{t\alpha} \to I^{\alpha}$ is given by

$$R_t(y) = ty + (1-t)\, b^* = S_t^{-1}(y) = S_{\frac{1}{t}}(y) \tag{12}$$

(or course S_t and $\mathbb{R}_t$ can be defined as mappings $\underline{X} \to \underline{X}$)

Lemma 2.7: For $t > 0$, $\alpha \in \mathbb{R}^n_{++}$, and $x \in \mathbb{R}^n$

$$U(t\alpha,x) = U(\alpha,x) - (ex)\,\frac{t-1}{t}\,\hat b \tag{16}$$

with

$$\hat b = \frac{1}{\alpha} \circledast b^* . \tag{17}$$

Proof: By definition, i.e. (7), $U(\alpha,b^*) = \hat b$, hence

$$U(t\alpha, b^*) = \frac{1}{t\alpha} \circledast b^* = \frac{1}{t}\,\hat b \tag{18}$$

and hence

$$U(t\alpha, b^*) - U(\alpha,b^*) = \left(1 - \frac{1}{t}\right) \hat b. \tag{19}$$

Consequently,

$$U(t\alpha,x) - U(\alpha,x) = -(ex)\left(\frac{te\alpha - 1}{t\alpha} - \frac{e\alpha - 1}{\alpha}\right) \circledast b^*$$

$$= -(ex)\,\frac{t-1}{t}\,\frac{1}{\alpha} \circledast b^* .$$

Lemma 2.8: For $t > 0$ and $\alpha > 0$, $x \in \partial \underline{X}$

1. $\frac{1}{t} V^{\alpha} = V^{t\alpha}$

2. $\frac{1}{t} U(\alpha,x) = U(t\alpha, S_t(x))$

3. Thus, if $u \in \partial V^{\alpha}$ is parametrized by $x \in I^{\alpha}$, then $\frac{1}{t} u \in \frac{1}{t} V^{\alpha} = \partial V^{t\alpha}$ is parametrized by $\frac{1}{t} x + (1 - \frac{1}{t}) b^*$.

Proof: The first statement follows immediately from the definition of V^{α}.

As for the second, we rewrite Lemma 2.7 for $x \in \partial \underline{X}$, we have

$$U(t\alpha,x) - \frac{1}{t} \hat{b} = U(\alpha,x) - \hat{b}; \tag{20}$$

also

$$U(t\alpha, b^*) = \frac{1}{t} \hat{b} = \frac{1}{t} u(\alpha,b^*) . \tag{21}$$

Hence, for $x \in \partial \underline{X}$

$$\begin{aligned} & U(t\alpha, \tfrac{1}{t} x + (1 - \tfrac{1}{t}) b^*) \\ &= \tfrac{1}{t} U(t\alpha, x) + (1 - \tfrac{1}{t}) U(t\alpha, b^*) \qquad (22) \\ &= \tfrac{1}{t} (U(t\alpha, x) - U(t\alpha, b^*)) + U(t\alpha, b^*) \\ &= \tfrac{1}{t} (U(\alpha,x) - U(\alpha, b^*)) + \tfrac{1}{t} U(\alpha, b^*) \\ & \qquad \text{(in view of (20) and (21))} \\ &= \tfrac{1}{t} U(\alpha,x) . \end{aligned}$$

The following sketch relates the a.t.u. induced by $1/t$ and the action of S_t (for $t > 1$).

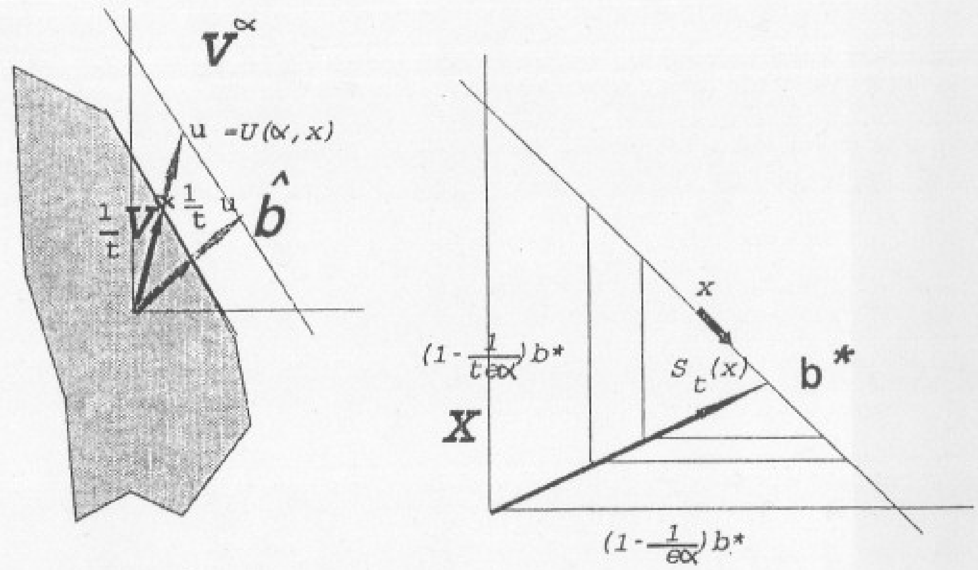

Fig. 2.4.

S_t shrinks the Pareto efficient interval I^{α} to $I^{t\alpha}$.

Theorem 2.9: Let α, β be l.t.u's and let $x \in \partial \underline{X}$. Then

1. $\frac{1}{\beta} \circledast V^{\alpha} = V^{\beta \circledast \alpha}$

2. $\frac{1}{\beta} \circledast U(\alpha,x) = U(\beta \circledast \alpha, S_{\frac{\beta\alpha}{e\alpha}}(x))$

3. That is, if $u \in V^{\alpha}$ is parametrized by α, then $\frac{1}{\beta} \circledast u \in V^{\beta \circledast \alpha}$ is parametrized by $S_{\frac{\beta\alpha}{e\alpha}}(x)$.

Proof: 1st STEP:

If $\beta\alpha = e\alpha$, then

$$\frac{1}{\beta} \circledast U(\alpha,x) = U(\beta \circledast \alpha,\, x) \tag{22}$$

follows immediately from the definition of U by taking into account that $e(\beta \circledast \alpha) = \beta\alpha$. Hence it is seen that in the general case

$$\frac{1}{\beta} \circledast U(\alpha,x) = \frac{e\alpha}{\beta\alpha}\, U(\frac{e\alpha}{\beta\alpha}\, \beta \circledast \alpha,\, x).$$

Taking $t := \frac{\beta\alpha}{e\alpha}$, this means

$$\frac{1}{\beta} \circledast U(\alpha,x) = \frac{1}{t}\, U(\frac{1}{t}\, \beta \circledast \alpha,\, x)$$

which by 2. of Lemma 2.7 is continued by

$$= U(t \cdot \frac{1}{t}\, \beta \circledast \alpha,\, S_t(x))$$

$$= U(\beta \circledast \alpha,\, S_{\frac{\beta\alpha}{e\alpha}}(x))$$

q.e.d.

SECTION 3

Parametrization of the General NTU-Game

Consider an NTU-game (0,V) and some utility vector $\bar{u}$ that is Pareto efficient, i.e., $\bar{u} \in \partial V$.

For the normal vectors $\alpha > 0$ at ∂V in $\bar{u}$ we choose a normalization such that

$$\alpha\bar{u} = 1$$

such that the tangential hyperplane

$$\{u \mid \alpha u = \alpha\bar{u} = 1\} = \partial V^\alpha \tag{1}$$

cuts through axis i in $\frac{1}{\alpha_i} e^i$.

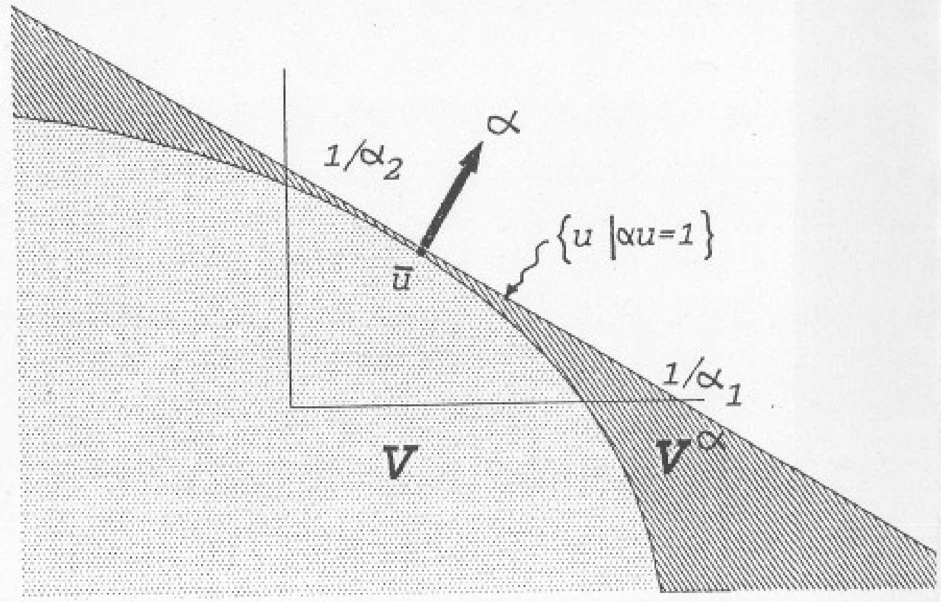

Fig. 3.1.

V^α constitutes the "side payment" alternatives available if, at $\bar{u} \in V$, side payments are permitted "at rate α". Clearly, V^α is "canonically" represented / parametrized by $U(\alpha,\cdot)$.

Denote the normals by

$$N(u) = \{\alpha \mid \alpha \text{ normal in } u,\ \alpha u = 1\} \tag{2}$$

for $u \in \partial V$. Moreover

$$N(V) = \bigcup_{u \in \partial V} N(u) \tag{3}$$

$$= \text{convex hull } (\{N(u) \mid u \in \partial V\}).$$

(Clearly a convex set.) Next, define the (negative) *dual cone of V* to be

$$K(V) = \{u \in \mathbb{R}^n \mid u\,\alpha \leq 0 \qquad (\alpha \in N(V))\}$$

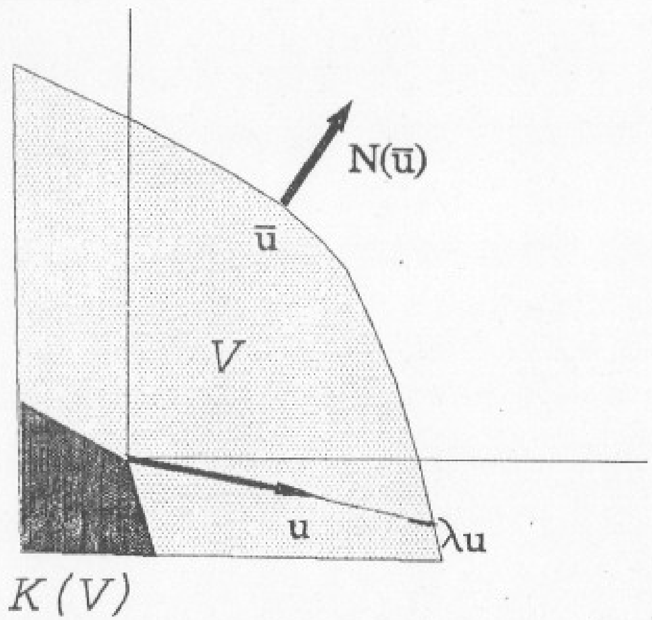

Fig. 3.2.

Note that for any $u \in V$, $u \notin \partial V$, $u \notin K(V)$ we have $\lambda u \in \partial V$ for some $\lambda > 0$.

The dual cone is also defined for $\underline{X}$; clearly $K(\underline{X}) = \{x \in \underline{X} \mid ex \leq 0\}$.

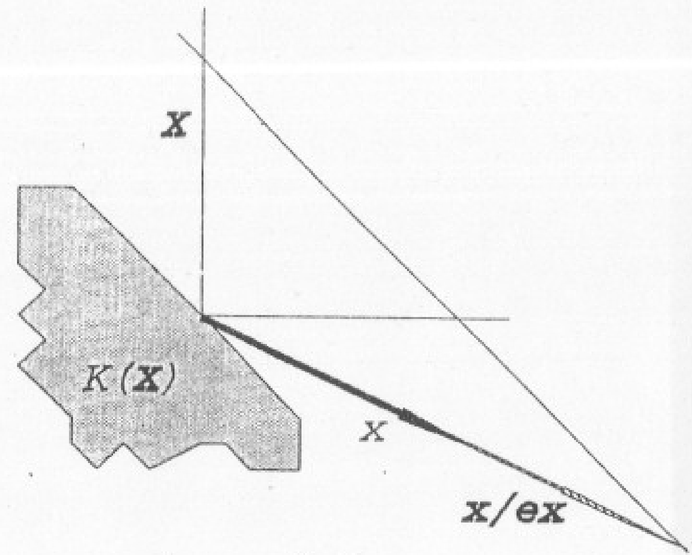

Fig. 3.3.

and, again for $x \in \underline{X}$ and $ex > 0$ it follows that $\frac{x}{ex} \in \partial \underline{X}$.

Our parametrization of V by $\underline{X}$ should desirably transfer halflines $\{\lambda x \mid 0 \leq \lambda \leq \frac{1}{ex}\}$ in halflines $\{\lambda u \mid 0 \leq \lambda \leq \bar{\lambda}\}$ so that players know simultaneously the impact of a proportional reduction of parameter with respect to utility. (At least, this would mean linearity on lines connecting the origin and points in $\partial \underline{X}$.)

Monotonicity is a further requirement, thus player is is interested in increasing x_i – in other words, we want to save as much as possible of the properties of the parametrization $U(\alpha,\cdot)$. And possibly, the α should be the one (anyone) suggested by $u \in \partial V$, i.e., we should consider $\alpha \in N(u)$.

Definition 3.1:

1. Let (0,V) be a standard NTU–game and let

$$A : \partial \underline{X} \to \mathbb{R}^n$$

   be a mapping. Consider the mapping

$$U^A : \{x \in \underline{X} \mid ex > 0\} \to \mathbb{R}^n \tag{4}$$

$$U^A(x) = U(A(\tfrac{x}{ex}), x) \qquad (x \in \underline{X},\ ex > 0)$$

2. Suppose A satisfies the followind conditions

$$\text{A is continuous.} \tag{5}$$

$$A(x) \in N(U^A(x)) \qquad (x \in \partial \underline{X}). \tag{6}$$

$$\text{For all } u \in \partial V,\ A^{-1}(N(u)) \neq \emptyset. \tag{7}$$

Then A is called a (the) (canonical) *normal mapping* of V (or just "A is normal to V"). Any parametrization

$$U : \underline{X} \to \mathbb{R}$$

such that $U = U^A$ on $\{ x \in \underline{X} \mid ex > 0\}$ is called "canonical" as well.

Remark 3.2: Conceivably, the behavior of a mapping U outside the set $\{x \in \underline{X} \mid ex > 0\}$ is of no interest. Once we have established existence and uniqueness of A we shall therefore speak of "the" canonical representation of V as well and mention U^A only.

Indeed, uniqueness of A does not pose a serious problem. For, given $\hat{u} \in \partial V$ with a *unique* $\hat{\alpha} \in N(\hat{u})$, there is a unique $\hat{x} \in \underline{X}$ such that $U(\hat{\alpha}, \hat{x}) = \hat{u}$ (i.e., $\hat{x} = G(\hat{\alpha},\hat{u})$) and clearly A should satisfy $A(\hat{x}) = \hat{\alpha}$. And if $N(\hat{u})$ consists of more then one point, then (again providing the argument for $n = 2$ only) it is an interval. Within this interval we must have

$$G(A(x), \hat{u}) = x$$

i.e.

$$A(x) = G(\cdot, \hat{u})^{-1}(x)$$

provided $G(\cdot,\hat{u})$ is invertible. This property we constitute in passing by pointing out that Lemma 2.5 induces the monotonicity of $G_1(\cdot,\hat{u})$ in the first coordinate.

Perhaps it should be repeated at this instant that $U(\alpha,\cdot)$ is defined with reference to some fixed b^*. Of course the same holds true with respect to the "canonical" representation U^A for some V.

Example 3.3: Let $\alpha' \in \underline{X}$ and $\alpha'' \in \underline{X}$ be as sketched in Fig.3.4.

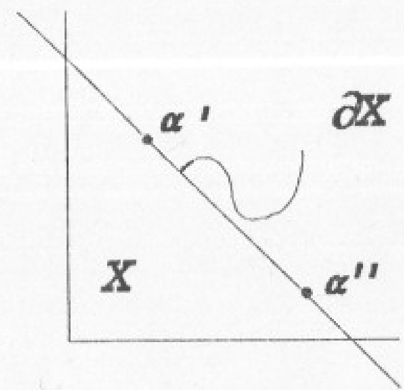

Fig. 3.4.

$\partial \underline{X}$ is decomposed into 3 "intervals" which is in a self explaining notation represented by

$$\partial \underline{X} = (-\infty, \alpha'] \cup [\alpha', \alpha''] \cup [\alpha'', \infty). \tag{8}$$

Define $A : \partial \underline{X} \to \mathbb{R}$ by

$$A(x) = \begin{cases} \alpha' & x' \in (-\infty, \alpha') \\ x & x \in [\alpha', \alpha''] \\ \alpha'' & x'' \in [\alpha'', \infty) \end{cases} \tag{9}$$

Now, as $eA(x) = 1$ $(x \in \partial \underline{X})$, we come up with

$$U^A(x) = \frac{1}{A(x)} \circledast x \qquad x \in \partial \underline{X},$$

i.e.,

$$U^A(x) \begin{cases} \frac{1}{\alpha'} \circledast x & x \in (-\infty, \alpha'] \\ (1,1) & x \in [\alpha', \alpha''] \\ \frac{1}{\alpha''} \circledast x & x \in [\alpha'', \infty) \end{cases} \tag{10}$$

Next, $U(\alpha',\cdot) = \frac{1}{\alpha'} \circledast \cdot$ parametrizes $V^{\alpha'}$ and, in addition $U(\alpha',\alpha') = (1,1)$.

From this it is seen at once that U^A parametrizes $V^{\alpha'} \cap V^{\alpha''}$. Since $\alpha', \alpha'' \in \partial \underline{X}$, it follows that $(1,1) \in \partial V^{\alpha'} \cap \partial V^{\alpha''}$. Thus the parametrization is such that $(-\infty, \alpha']$ is mapped bijectively on $(-\infty, (1,1)]$ and similarly $]\alpha'', \infty)$ on $[(1,1), \infty)$. All $x \in [\alpha', \alpha'']$ are mapped into $(1,1)$ etc.

$A(x)$ yields always the normal in $U(A(x), x) = U^A(x)$ and while x is running through $[\alpha', \alpha'']$, $A(x)$ describes the normal cone at $(1,1)$.

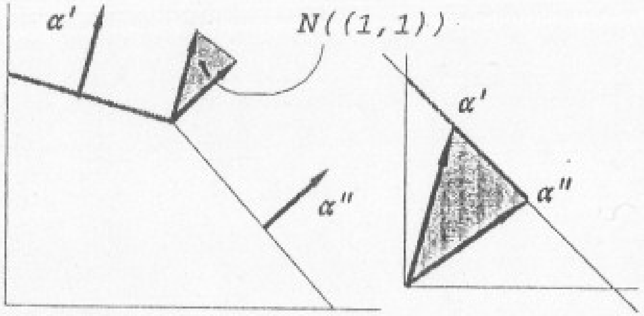

Fig. 3.5.

Note that in this example b^* does not occur as $eA(x) = 1 \quad (x \in \underline{X})$.

Remark 3.4: In view of Theorem 2.9 we may attempt to compute the action of l.t.u's on "normal functions".

If A is normal to V and, for some $\hat{u} \in \partial V$, it so happens that

$$\hat{u} = U(A(\hat{x}), \hat{x})$$

for suitable $\hat{x} \in \partial \underline{X}$, then it follows by virtue of Theorem 2.9 that

$$\frac{1}{\beta} \circledast \hat{u} = U(\beta \circledast A(\hat{x}), S_{\frac{\beta A(\hat{x})}{eA(\hat{x})}}(\hat{x}) . \tag{11}$$

If one can show that

$$x \longrightarrow S_{\frac{\beta A(x)}{eA(x)}}(x) \tag{12}$$

is invertible (we omit this discussion...) with some function Q, say, then clearly (11) reads

$$\frac{1}{\beta} \circledast \hat{u} = U(\beta \circledast A \circ Q(\hat{y}), \hat{y}) ,$$

that is $\beta \circledast A \circ Q$ is normal to $\frac{1}{\beta} \circledast V$.

Example 3.5: (cf. Example 3.3).

For $\alpha', \alpha'' \in \partial \underline{X}$ we know the function $A = A^V$ which is normal to $V = V^{\alpha'} \cap V^{\alpha''}$. This is (cf. (9))

$$A(x) = \begin{cases} \alpha' & x' \in (-\infty, \alpha') \\ x & x \in [\alpha', \alpha''] \\ \alpha'' & x'' \in [\alpha'', \infty) \end{cases} .$$

Now, consider $\frac{1}{\beta} \circledast (V^{\alpha'} \cap V^{\alpha''}) = \frac{1}{\beta} \circledast V$.

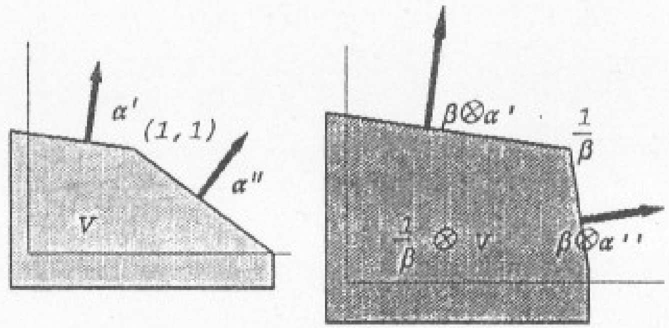

Fig. 3.6.

Now, as $eA(x) = 1$ for $x \in \partial \underline{X}$ we have

$$S_{\frac{\beta A(x)}{eA(x)}}(x) = S_{\beta A(x)}(x),$$

hence, if $\hat{u} = U(A(\hat{x}), \hat{x})$, then

$$\frac{1}{\beta} \circledast \hat{u} = U(\beta \circledast A(\hat{x}), S_{\beta A(\hat{x})}(\hat{x}))$$

$$= U(\beta \circledast A \circ R_{\beta A(\hat{x})}(\hat{y}), \hat{y})$$

with $\hat{y} = S_{\beta A(\hat{x})}(\hat{x})$. Clearly, for $x \in (-\infty, \alpha']$ it follows that $y = S_{\frac{\beta\alpha'}{e\alpha'}}(x) \in (-\infty, S_{\frac{\beta\alpha'}{e\alpha'}}(\alpha')]$.

Thus the canonical normal mapping corresponding to $\frac{1}{\beta} \circledast V$ is given by $A = A^{\frac{1}{\beta} \circledast V}$ via

$$A(y) = \begin{cases} \beta \circledast \alpha & y \in (-\infty,\ S_{\beta\alpha'}(\alpha')] \\ \beta \circledast Q(y) & y \in [S_{\beta\alpha'}(\alpha'),\ S_{\beta\alpha''}(\alpha'')] \\ \beta \circledast \alpha & y \in [S_{\beta\alpha''}(\alpha''),\ \infty) \end{cases} \tag{13}$$

A direct computation is even more revealing since it suggests the unique procedure defining the normal function "at a corner of V".

To this end consider again

$$V = V^{\gamma'} \cap V^{\gamma''}$$

as indicated in Figure 3.7; we assume that $\frac{1}{\beta}$ is the cornerpoint, thus

$$\gamma' \circledast \frac{1}{\beta} = 1 = \gamma'' \circledast \frac{1}{\beta}$$

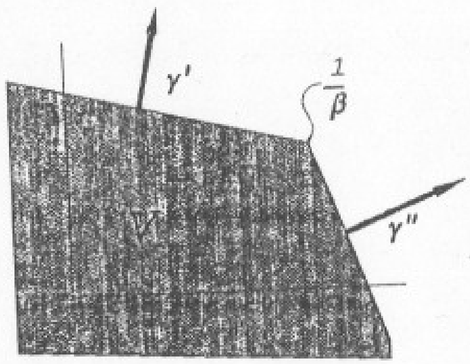

Fig. 3.7.

Clearly, we must have

$$A(x) = \begin{cases} \gamma' & x \in (-\infty,\ G(\gamma', \frac{1}{\beta})] \\ \gamma'' & x \in [G(\gamma'', \frac{1}{\beta}),\ \infty) \end{cases} . \tag{14}$$

In addition we must have

$$U(A(x), x) = \frac{1}{\beta} \quad x \in [G(\gamma', \tfrac{1}{\beta}),\ G(\gamma'', \tfrac{1}{\beta})] \tag{15}$$

Indeed, (15) determines A uniquely within the interval in question for it implies for $A(x) = \alpha$:

$$\frac{e\alpha}{\alpha} \circledast (x - \frac{e\alpha - 1}{e\alpha} b^*) = \frac{1}{\beta},$$

$$\begin{aligned} \alpha &= (e\alpha)\, \beta \circledast (x - \frac{e\alpha-1}{e\alpha} b^*) \\ &= \beta \circledast ((e\alpha)\, x + (1 - e\alpha)\, b^*) \\ &= \beta \circledast R_{t(x)}(x) = A(x); \end{aligned} \tag{16}$$

here $t(x)$ is computed as

$$t(x) = e\alpha = \beta(t(x)\, x + (1 - t(x))\, b^*);$$

i.e.

$$t(x) = \frac{\beta\ b^*}{1 - \beta x + \beta\ b^*} \tag{17}$$

Thus

$$A(x) = \beta \circledast R_{\frac{\beta\ b^*}{1-\beta x+\beta b^*}} \quad x \in [G(\gamma', \tfrac{1}{\beta}),\ G(\gamma'', \tfrac{1}{\beta})] \tag{18}$$

and (14) as well as (18) describe A uniquely.

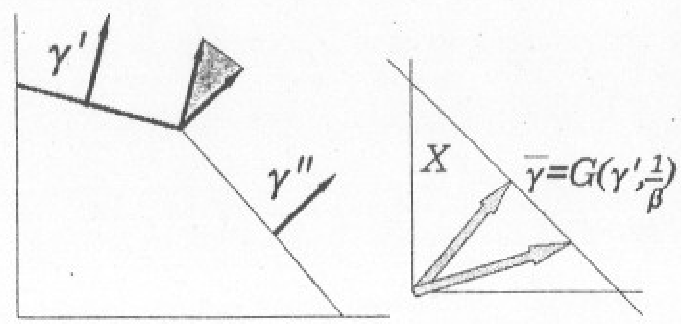

Fig. 3.8.

Remark 3.6: Clearly, the construction for the canonical representation, as indicated by the second part of Example 3.5, makes sense only if we can be sure that

$$G_1(\gamma'', \tfrac{1}{\beta}) > G_1(\gamma', \tfrac{1}{\beta}) \qquad (19)$$

whenever $\gamma''_1 > \gamma'_1$. However, as $\gamma'' \frac{1}{\beta} = \gamma' \frac{1}{\beta} = 1$, this is a consequence of Lemma 2.5, that is, of the monotonicity of $G_1(\cdot, \bar{u})$ in the first coordinate.

Theorem 3.7: For every V the canonical mapping A is uniquely defined.

Proof:

1st STEP: We shall restrict our exposition on the case that V is smooth, i.e., there is a unique supporting hyperplane at any $u \in \partial V$. (For the case of a kink consider Remark 3.2 and Example 3.6.) For any such $u \in \partial V$ denote by $\mathscr{A}(u)$ the unique element in N(u).

Clearly, $G(\mathscr{A}(u), u) \in \underline{X}$ is the element of $\underline{X}$ that should be thrown into $\mathscr{A}(u)$ such that $U(\mathscr{A}(u), G(\mathscr{A}(u), u)) = u$. Therefore, it is conceivably sufficient to show that $G^0(u) := G(\mathscr{A}(u),u)$ induces a bijective mapping G^0.

For indeed, assume that G^0 is bijective and let G^{0-1} denote the inverse function

$$G^{0-1} : \partial \underline{X} \to \partial V.$$

Define

$$A(x) := \mathscr{A} \circ G^{0-1}(x). \qquad (20)$$

Thus, for any $u \in \partial V$ and $x = G^0(u)$, we have to show that

$$U(A(x),x) = u. \qquad (21)$$

However,

$$\begin{aligned} x = G^0(u) &= G(\mathscr{A}(u),u) \\ &= G(\mathscr{A} \circ G^{0-1}(x), u) \\ &= G(A(x), u), \end{aligned}$$

and hence

$$U(A(x),x) = U(A(x), G(A(x),u)) = u; \qquad (22)$$

this concludes our first step, i.e., it is indeed sufficient to show that G^0 is bijective.

2nd STEP: We shall restrict our argument to the case that n = 2 and V is strictly concave, i.e., no line segments appear in ∂ V – otherwise some details will have to be supplied which does not necessarily enlighten the argument.

First of all consider α and α' ($\in \mathbb{R}^2_{++}$) such that

$$\alpha_1 > \alpha'_1.$$

Let $\bar{u} \in \partial V^{\alpha} \cap \partial V^{\alpha'}$, i.e., $\alpha \bar{u} = \alpha' \bar{u} = 1$.

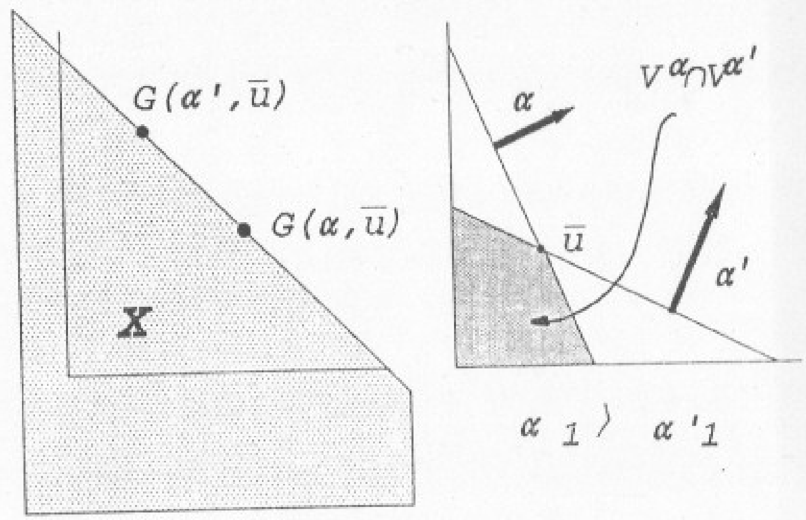

Fig. 3.9.

In view of Lemma 2.5 we know that

$$G_1(\alpha,\bar{u}) > G_1(\alpha',\bar{u}).$$

In other words, given $\bar{u}$, we know that G_1 is strictly monotone in α_1.

Next, consider the situation where α_1' as above, but assume in addition that $\alpha \in N(u)$ and $\alpha' \in N(u')$ with (necessarily) $u_1 > u_1'$. This time, let $\bar{u}$ denote the intersection of ∂V^{α} and $\partial V^{\alpha'}$.

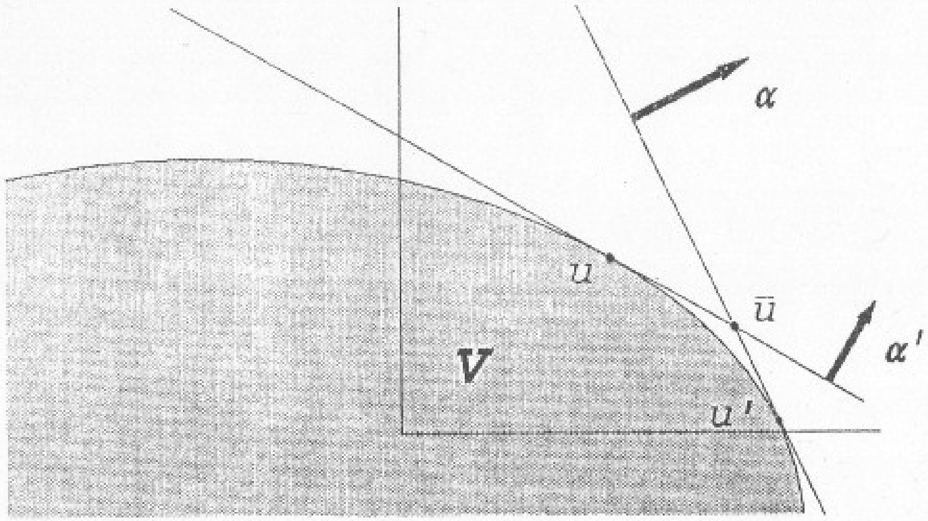

Fig. 3.10.

Clearly,

$$\begin{aligned} G_1(\alpha',u') &< G_1(\alpha',\bar{u}) \\ &< G_1(\alpha,\bar{u}) < G_1(\alpha,u). \end{aligned} \tag{23}$$

This means of course that

$$G(\mathscr{A}(u'),u') \neq G(\mathscr{A}(u),u)$$

for $u \neq u'$, i.e., G^0 is 1–1.

Moreover, as $u_1 \to -\infty$ (this uses the fact that V_* is compact and V is comprehensive!), we observe that $G_1(\mathscr{A}(u),u) \to -\infty$. Similarly, as $u_2 \to \infty$, $G_2(\mathscr{A}(u),u) \to \infty$; thus by continuity we conclude that G^0 indeed mapps ∂ V uniquely *onto* $\partial \underline{X}$, q.e.d.

Remark 3.8: If A is the (canonical) *normal mapping* of some feasible set V which results from a standard NTU–game, then there is a simplex $I^A \subsetneq \underline{X}$ such that U^A maps I^A bijectively on ∂V^+, i.e., on the Pareto efficient and individually rational points of V. If A is constant, hence $V = V^{\alpha}$, then $I^A = I^{\alpha}$ as discussed in Remark 2.3.

Example 3.9: Clearly $A(x) = t\,e$ ($t > 0$, $x \in \partial \underline{X}$) yields a fee game and $A(x) = x$ ($x \in \partial \underline{X}$) yields $V = \{s(1,1) \mid s \leq 1\}$ (not included in our formal framework). This suggests that these two cases in some sense are "extreme".

Consider

$$A(x) = \frac{e}{2} + x \qquad (x \in \partial \underline{X}), \tag{24}$$

here $eA(x) = 2$ and hence for $x \in \partial \underline{X}$

$$U(A(x),x) = \left[\frac{2x_1 - b_1^*}{x_1 + 1/2}, \frac{2x_2 - b_2^*}{x_1 + y1/2}\right] \tag{25}$$

As $u_i = \frac{2x_i - b_i^*}{x_i + 1/2}$ $(i = 1,2)$ we have

$$x_i = \frac{u_i + 2b_i^*}{4 - 2u_i}\,. \tag{26}$$

Thus

$$1 = x_1 + x_2 = \frac{u_1 + 2b_1^*}{4 - 2u_1} + \frac{u_2 + 2b_2^*}{4 - 2u_2} \tag{27}$$

is a description of the Pareto efficient boundary of V. ∂ V is a certain hyperbola asymptotically approaching the lines $u_1 = 2$ and $u_2 = 2$.

If $u_i = 0$ then $x_i = b_i^*/2$ and

$$u_{3-i} = \frac{2}{3 - b_i^*}$$

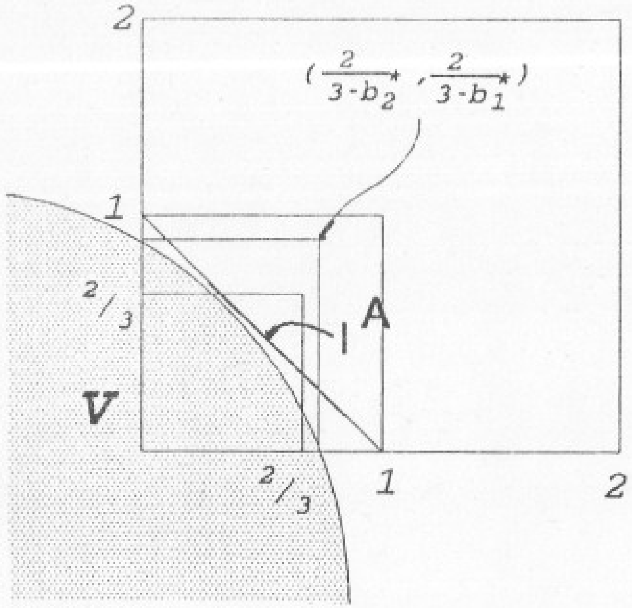

Fig. 3.11.

This way the "interval" $I^A = \left[\left[\frac{b_1^*}{2}, 1 - \frac{b_1^*}{2}\right], \left[1 - \frac{b_2^*}{2}, \frac{b_2^*}{2}\right]\right]$ is uniquely mapped onto the Pareto efficient and individually rational boundary of V.

SECTION 4
The Expected Contract Value

By a "value" or "bargaining solution" we mean a mapping which is defined on CII–games (6–tupels as in SEC.1 (3), that is) and attaining mechanisms, i.e. a mapping

$$\chi : \{\Gamma \mid \Gamma \text{ is a CII–game}\} \to \underline{X}^T \qquad (1)$$

such that

$$\chi(\Gamma) \in \mathscr{M}(\Gamma) = \{\mu \mid \mu \text{ is BIC and "\textit{in mediis}" individually rational}\} \qquad (2)$$

The term BIC has been explained in SEC.1 (see formula (4); as for "i.r. in mediis" ("IR") the term is almost self explaining, of course this means

$$E(U_i^\tau \circ \mu^\tau \mid \tau_i = t_i) \geq 0 \qquad (i \in I, t_i \in T^i). \qquad (3)$$

Note that T is considered fix in (1).

Thus, a value assigns a BIC and IR mechanism to every CII–game. Clearly, the definition of such a value should be governed by a set of axioms. However, in the spirit of the NASH–bargaining solution (NASH [12]), we shall exhibit, as a first approach, a value on "side payment games". It is then a second step to produce an extension on general CII–games using an appropriate version of the IIA–requirement. This extension would then be based on an axiomatic approach.

In the case of complete information, a solution for the side payment case is considered obvious ("equal share of net gains") – or justified by the "obvious axioms" (symmetry, Pareto efficiency, translation covariance). With incomplete information at hand, this is by no means obvious. Let us first discuss the situation within the "side payment" territory.

Definition 4.1: A CII game

$$\Gamma = (I, T; p; \underline{X}, 0; U)$$

is said to be a *fee–game* ("in the wide sense") if, for any $t \in T$, there is $\alpha^t \in \mathbb{R}^n_+$, and $b^{*t} \in \partial \underline{X}$ such that

$$U^t(x) = U(\alpha^t, x) = \frac{e\alpha^t}{\alpha^t} \circledast x - (ex) \frac{e\alpha^t - 1}{\alpha^t} \circledast b^{*t} \qquad (4)$$

holds true.

Remark 4.2: 1. The term *fee–game* is explained at length in ROSENMÜLLER [14]. If we return to Remark 2.2 and formula (5) in SEC.2, then we may write

$$U^t(x) = \frac{1}{\lambda^t} \circledast (x - (ex)\, C^{\alpha^t} b^{*t}) := \frac{1}{\lambda^t} \circledast (x - (ex)\, b^t) \tag{5}$$

with $C^{\alpha^t} = \frac{e\alpha^t - 1}{e\alpha^t}$ and $b^t := C^{\alpha^t} b^{*t}$.

If $\lambda^t := \frac{\alpha^t}{e\alpha^t}$ happens to be $(\frac{1}{n}, \dots, \frac{1}{n}) = \frac{1}{n} e$, then Γ is a *fee–game in the narrow sense.* For, in this case U^t coincides with the version in [14] (up to the factor $1/n$).

2. Clearly, if Γ is a fee game, then $V^t = V^{\alpha^t}$, thus $(0, V^t)$ has side–payment character up to a constant rescaling of utility.

Definition 4.3: Let Γ be a fee–game. For any $t \in T$ define $\bar{u}^t \in V^t$ by

$$\bar{u}^t = \frac{1}{n} \frac{1}{\alpha^t} \tag{6}$$

and $\bar{x}^t \in \underline{X}$ by

$$\begin{aligned} \bar{x}^t &= \frac{1}{n} \sum_{i=1}^{n} a^{i\alpha^t} = \frac{e\alpha^t - 1}{e\alpha^t} b^{*t} + \frac{1}{e\alpha^t} \frac{e}{n} \\ &= C^{\alpha^t} b^{*t} + \frac{1}{e\alpha^t} \frac{e}{n} = b^t + \frac{1}{e\alpha^t} \frac{e}{n}. \end{aligned} \tag{7}$$

See Remark 2.3, formula (8) for the definition of $a^{i\alpha}$, b^t, etc. As we know from SEC.2,

$$U(a^t, \bar{x}^t) = \bar{u}^t \tag{8}$$

is the midpoint of ∂V^{α^t}, thus $\bar{x}^t$ parametrizes the NASH value of $(0, V^t)$ i.e., the *ex post* NTU–game in situation t.

Let us focus on the *ex ante* situation. Note that, for any $x \in \partial \underline{X}$, we have

$$\begin{aligned} E\,U(a^\tau, x) &= E\left(\frac{1}{\lambda^\tau} \circledast (x - C^{\alpha^\tau} b^{*\tau})\right) \\ &=: E\left(\frac{1}{\lambda^\tau} \circledast (x - b^\tau)\right) \\ &= \underbrace{E\left(\frac{1}{\lambda^\tau}\right)}_{\frac{1}{\bar{\lambda}}} \circledast x - E\frac{b^\tau}{\lambda^\tau} \\ &= \frac{1}{\bar{\lambda}} \circledast (x - \underbrace{\bar{\lambda} \circledast E\frac{b^\tau}{\lambda^\tau}}_{\bar{b}}) \\ &= \frac{1}{\bar{\lambda}} \circledast (x - \bar{b}), \end{aligned} \tag{9}$$

where $\lambda^\tau = \frac{\alpha^\tau}{e\alpha^\tau}$, $b^\tau = C^{\alpha^\tau} b^{*\tau}$ in accordance with Remark 2.2.. Therefore

$$V^{\underline{X}} = \{EU(\alpha^\tau, x) \mid x \in \underline{X}\} = V^{\bar{\alpha}} \tag{10}$$

constitutes an NTU–game $(0, V^{\bar{\alpha}})$, here $\bar{\alpha}$ is computed via

$$\bar{\lambda} = \frac{1}{E(\frac{1}{\lambda^\tau})}, \quad \bar{b} = \bar{\lambda} \circledast E\frac{b^\tau}{\lambda^\tau}, \quad \bar{\alpha} = \frac{1}{1 - e\bar{b}} \bar{\lambda} \tag{11}$$

(*mutatis mutandis* for $e\bar{b} = 1$). As (9) provides the parametrization for $V^{\bar{\alpha}}$, we observe that analogously to (7)

$$\bar{x} := \frac{1}{n} \sum_{i=1}^{n} a^{i\bar{\alpha}} = \bar{b} + \frac{1}{e\bar{\alpha}} \frac{e}{n} \tag{12}$$

parametrizes the midpoint of $V^{\bar{\alpha}}$. That is, in a situation *without* information, it seems likely, that players would register $\bar{x}$, thus ensuring the *ex ante* NASH–value. (Any symmetric, efficient, and translation invariant "value" leads to the midpoint of $V^{\bar{\alpha}}$.)

Note that, on the other hand, we have of course that $\bar{u} := E\,u^\tau$ is the midpoint of $V^{\bar{\alpha}}$. Thus, in a world of truth–speaking individuals, players would register the mechanism

$\mu^t = \bar{x}^t$ $(t \in T)$, the *expectation* of which (*ex ante*) is $E(U^{\tau}(\alpha^{\tau},\bar{x}^{\tau})) = \bar{u}$. Thus, agreeing on $\bar{x}$ yields the same *ex ante* expectation as playing the NASH–value *ex post* in the ideal world.

Definition 4.4: If Γ is a fee game, then the *expected contract* is given by

$$
\begin{aligned}
\bar{x} &= \bar{x}(\Gamma) = \frac{1}{n}\sum_{i=1}^{n} a^{t\dot{\alpha}} \\
&= \bar{b} + \frac{1}{e\bar{\alpha}}\frac{e}{n} = C^{\dot{\alpha}}\,\bar{b}^* + \frac{1}{e\bar{a}}\frac{e}{n} \qquad (13)\\
&= \frac{e\bar{\alpha}-1}{e\bar{\alpha}}\,\bar{b}^* + \frac{1}{e\bar{\alpha}}\frac{e}{n},
\end{aligned}
$$

here $\bar{\alpha}$ and $\bar{b}$ are given by (11), $b^* = \dfrac{\bar{b}}{e\bar{b}}$.

It is important to note that $\bar{x}$ and $E\bar{x}^{\tau}$ are different quantities – they coincide, however, in the case of a fee game in the narrow sense. But, of course $EU^{\tau}(\bar{x}) = EU^{\tau}(x^{\tau}) = \bar{u}$ holds always true.

Of course, the constant mechanism $\bar{\mu}$ defined by $\bar{\mu}^t = \bar{x}$ $(t \in T)$ is always BIC. However, it is not necessarily individually rational, i.e., $\bar{\mu} \in \mathscr{M}$ is *not* guaranteed. In [14] it is argued that for fee games (in the narrow sense) a value should *always be equal to* $\bar{\mu}$ *whenever* $\bar{\mu} \in \mathscr{M}$. We shall persue – and extend – this argument in order to justify what we shall consider the "natural" value on fee–games (and the one to be justified axiomatically for the general case later on).

In order to avoid the term "generalized NASH–value" – which it is – we use the term "*expected contract value*" or *EC–value*. Of course, it generalizes the NASH–value – like the ones proposed by HARSANYI–SELTEN [5] and MYERSON [9] (see also WEIDNER [15]).

Within the following, $\nu(0,V)$ denotes the NASH–value of a standard NTU–game $(0,V)$. Also we use

$$V^{\mathscr{M}} := \{E(U^{\tau} \circ \mu^{\tau}) \mid \mu \in \mathscr{M}\} \qquad (14)$$

Definition 4.5: $\chi : \{\Gamma \mid \Gamma \text{ is a CII–game}\} \rightarrow \mathbf{X}^T$ is called a (the) *EC–value* if

$$E(U^{\tau}(\chi^{\tau}(\Gamma)) = \nu(0,V^{\mathscr{M}}) \qquad (15)$$

holds true.

Let us now observe that there is a nice and surveyable class of CII–games allowing for a unique EC–value as characterized by 4.4. This is the case of two players, one of them having full information about his two types (*in mediis*), and the other one resting in one type only. Conveniently we write

$$n = 2\,,\ T = \{\alpha,\beta\} \times \{*\} \qquad (16)$$

$$U^{(\alpha,*)} =: U^{\alpha},\ U^{(\beta,*)} =: U^{\beta} \text{ etc.}$$

and refer to the situation as described by (16) as the case of *Incomplete Information on one side* (the "*II1–CASE* "). For simplicity, we assume always $\frac{b_1^{\beta}}{b_1^{\alpha}} < \frac{\lambda_1^{\beta}}{\lambda_1^{\alpha}} > 1$ in this situation, see (7) and Theorem 3.4 of [14] . This will ensure that β is player 1's "worse situation", as $U_1^{\beta}(x) < U_1^{\alpha}(x)$ whenever $ex > 0$ and $x_1 > 0$.

Theorem 4.5: In the II1–case, the EC–value is uniquely defined.

Proof: As for fee–games in the narrow sense, this is more or less the content of Corollary 3.6 and Lemma 4.2 of [14] . The proof can be extended to fee–games in the wide sense as follows.

Let Γ be a fee–game (in the wide sense). For any system

$$B = (\beta^t)_{t \in T} \qquad \beta^t \in \mathbb{R}^n_{++}\ (t \in T) \qquad (17)$$

define the mapping

$$B : \Gamma \rightarrow \Gamma' = B\Gamma \qquad (18)$$

via the "formal rescaling of utilities", i.e., by fixing the utility functions of Γ' to be

$$U'^t(x) = \frac{1}{\beta^t} \circledast U^t(x) = \frac{1}{\beta^t} \circledast U(\alpha^t, x)$$
$$= U(\beta^t \circledast \alpha^t, S_{\frac{\beta^t \alpha^t}{e\alpha}}(X)) \tag{19}$$

$(t \in T)$. This happens quite in accordance with Theorem 2.9. In particular, we may take

$$\beta^t := \frac{1}{n} \frac{e\alpha^t}{\alpha^t} \qquad (t \in T) \tag{20}$$

such that

$$\beta^t \circledast \alpha^t = \frac{e\alpha}{n} e, \frac{\beta^t \alpha^t}{e\alpha} = 1, \qquad (t \in T) \tag{21}$$

holds true. If so, then

$$U'^t(x) = \frac{1}{\beta^t} \circledast U(\alpha^t, x)$$
$$=: U(\alpha'^t, x) \qquad (t \in T) \tag{22}$$

follows by introducing

$$\alpha'^t := \frac{e\alpha^t}{n} e \qquad (t \in T). \tag{23}$$

In view of Remark 2.2, we may continue by writing

$$U'^t(x) = U(\alpha'^t, x) = \frac{1}{\lambda'^t} \circledast (x - (ex)\, b'^t) \tag{24}$$

such that the quantities involved are given by

$$\lambda'^t := \frac{\alpha'^t}{e\alpha'^t} = \frac{e}{n}, \quad b'^t := C^{\alpha' t} b^{*t}$$
$$C^{\alpha' t} := \frac{e\alpha'^t - 1}{e\alpha'^t} = \frac{e\alpha^t - 1}{e\alpha^t}. \tag{25}$$

Thus,

$$U'^t(x) = \frac{1}{n}(x - (ex)\, b'^t), \tag{26}$$

meaning that Γ' is a fee game in the narrow sense (and up to $\frac{1}{n}$ a fee game in the sense of [14].) On the other hand, (19) says that the utility functions of Γ and Γ' differ by constants only. This leads to the observation that certain properties of mechanisms $\mu \in \mathscr{M}$ are being preserved under the formal rescaling B.

Indeed, as $E(U_1^\tau \circ \mu^\tau \mid \tau_1 = t_1) = U_1'^{t}(\mu'^{t})$ for $t_1 = {}^{\alpha,\beta}$ it is clear that the inequalities (4) of SEC.1 – and hence the BIC–property – are invariant into transformation with B. Similarly, individual rationality (*in mediis*) for player 1 is preserved.

Also, recall that we have assume that $^\beta$ is "player 1's worse situation" i.e., that $\frac{b_1^\beta}{b_1^\alpha} > \frac{\lambda_1^\beta}{\lambda_1^\alpha} > 1$. This is at once transformed into the corresponding property of [14].

Finally, although the I.R. property for player 2 is *not* preserved ($EU_2^\tau \circ \mu^\tau$ is the uninformed players expectation *ex ante* and *in mediis*), the relevant proofs in [14] do only require that a non constant BIC mechanism μ satisfies $U_2^\alpha(\mu^\alpha) > 0$ – and this is preserved by the formal rescaling B.

Thus, Corollary 3.6 and Lemma 4.2 of [14] do also apply in our present II1–case, q.e.d.

The problem with the formal rescaling B is that various types of one player change their utility scale in a different manner. However, this is not compatible with our present philosophy: it is only a player who could apply a change of his utility scale (simultaneously for all his types). This is reflected by the fact that $\mathscr{M}$ will be invariant only under such restricted type of rescaling.

Nevertheless, in the II1–case we may carry over more results from [14], using our above arguments.

Corollary 4.6: The Γ be a fee game (in the wide sense) with II1–property. Also, let $\mu \in \mathscr{M}$ be a Pareto–efficient *ex ante* mechanism. Assume that μ is non–constant, say

$$\mu = (\mu^\alpha, \mu^\beta).$$

Assume, on the other hand, that $\chi(\Gamma)$ is constant, i.e., $\chi^\alpha(\Gamma) = \chi^\beta(\Gamma) = \bar{x}$. Then

$$U_1^\alpha(\mu^\alpha) < U_1^\alpha(\bar{x}),\ U_1^\beta(\mu^\beta) = 0$$
$$E(U_1^\tau \circ \mu^\tau) < E\ U_1^\tau(\bar{x}) \qquad (27)$$

The proof follows from Lemma 4.3 of [14]. Note that our notation for the II1–case ((16) that is) implies that

$$U_1^\alpha(\bar{x}) = E\ (U_1^\tau(\bar{x}) \mid \tau_1 = \alpha) \qquad (28)$$

etc. holds true. (27) implies that player 1's expectation *in mediis* (which is deterministic, as he has full information *in mediis*) is worse at any non constant mechanism – compared to the one at $\bar{x}$ – and the same holds true *ex ante*.

Corollary 4.7: Let Γ be a fee game (in the narrow sense) with II1–property. Then $p^\alpha V^\alpha \subseteq V^{\mathcal{M}}$.

This we prove only for a fee game in the narrow sense. We may directly refer to Theorem 3.4 of [14] (see also Fig.6 of SEC.3 of [14], which characterizes the non constant mechanisms.).

From this it follows that the BIC–mechanisms yielding *ex ante* expectations $\overset{(L)}{u}$, $\overset{(M)}{u}$, and $\overset{(R)}{u}$ are easily identified (see Fig.4.1). We find

$$\overset{(L)}{\mu} = (a^{\alpha,2},0),\quad \overset{(L)}{u} = EU^\tau(\overset{(L)\tau}{\mu}) = p^\alpha(a^{\alpha,2} - b^\alpha) \qquad (29)$$

and

$$\overset{(M)}{\mu} = (a^{\beta 2},a^{\beta 2})$$

with

$$\overset{(M)}{u} = EU^\tau(\overset{(M)\tau}{\mu}) = p^\alpha U^\alpha(a^{\beta 2}) + p^\beta U^\beta(a^{\beta 2}) = a^{\beta 2} - Eb^\tau, \qquad (30)$$

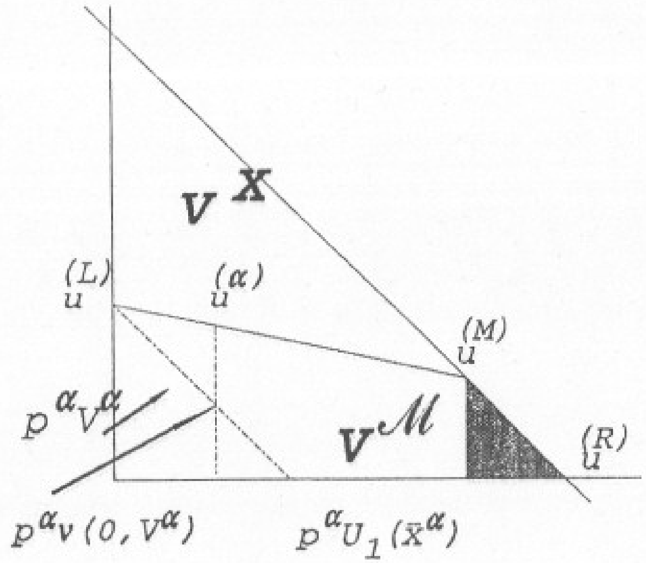

Fig. 4.1

while $u^{(R)}$ results from the fact that $V^{\mathcal{M}} \subseteq V_+^{\underline{X}}$ and $V_+^{\underline{X}}$, has side payment character. Now, as the simplex $p^\alpha V^\alpha$ has vertices $p^\alpha(a^{\alpha,2} - b^\alpha) = u^{(L)}$ and $p^\alpha(a^{\alpha,1} - b^\alpha)$, we observe that the situation has to be as depicted in Fig.4.1, i.e., $p^\alpha V^\alpha \subseteq V^{\mathcal{M}} \subseteq V_+^{\underline{X}}$, q.e.d.

We are now going to collect some arguments in order to support the EC–value in the II1–CASE. Some of this will resemble expositions presented in [14].

Player 2, the uninformed one, will start his analysis by considering (cf. (10))

$$V_+^{\underline{X}} = \{EU^\tau(x) \mid x \in \underline{X}\} \cap \mathbb{R}_+^2.$$

However, in general the elements of $V_{+}^{\underline{X}}$ may not be acceptable for player 1. For, if he observes his type *in mediis*, it might turn out that, for some $x \in \underline{X}$, $EU^{\tau}(x) \in V_{+}^{\underline{X}}$, but $E(U^{\tau}(x) \mid \tau_1 = t_1) < 0$. This is of course what leads the players to discuss mechanisms, i.e., agreements depending on the type player 1 observes and announces.

If so, player 2's interest requires to accept BIC mechanisms only, thus $\mathscr{M}$ emerges as the appropriate class. By Lemma 2.6 of [14], $V^{\mathscr{M}}$ is a convex, compact polyhedron contained in $V_{+}^{\underline{X}}$

$$V^{\mathscr{M}} \subseteq V_{+}^{\underline{X}} \tag{31}$$

From the viewpoint of player 2, the inclusion (31) is not too exciting as it decreases the feasible alternatives. For player 1 this may also be considered a drawback – but for him it constitutes an insurance against finding himself in a non IR situation "in mediis".

Given these general observations, let us now focus to the discussion of a "value" or "bargaining solution".

We are going to distinguish two cases.

1st CASE: Assume that the EC–value is constant. Clearly this means

$$\chi^{\alpha}(\Gamma) = \chi^{\beta}(\Gamma) = \bar{x} = \bar{x}(\Gamma) \tag{32}$$

or, in other words, the constant mechanism $\bar{\mu}$ is IR.

In this case the argument is straight forward as far as the uninformed player (player 2) is concerned. For, in view of (31), he was not very happy anyway with mechanisms $\mu \in \mathscr{M}$. Now, he observes that

$$\nu(0,V^{\mathscr{M}}) = E(U^{\tau}(\chi^{\tau}(\Gamma)) = E(\bar{x}) = \bar{u} = \nu(0,V^{\underline{X}}) \tag{33}$$

holds true. As $V^{\mathscr{M}} \subseteq V_{+}^{\underline{X}}$ player 2, an ardent supporter of the IIA–axiom due to NASH, greatly relieved accepts $\chi(\Gamma)$ as a mechanism that assures what he wanted from the beginning anyway: the NASH–value of $V^{\underline{X}}$.

What could player 1 object to this kind of reasoning? On one hand he could propose another constant mechanism, say $\hat{x}$. There seems to be no sense in his considering none IR (*in mediis*) mechanisms, thus we expect him to choose $\hat{x}$ such that $\hat{u} := EU^{\tau}(\hat{x}) \in V^{\mathscr{M}}$. Since he waves the opportunity to make any use of his *in mediis* information, the argument boils down to proposing some utility $\hat{u} \in V^{\mathscr{M}}$ which differs from the midpoint $\bar{u}$ of $V^{\underline{X}}$ satisfying $\bar{u} \in V^{\mathscr{M}}$. It is difficult to imagine any believer in the NASH–value bringing forward such a proposition.

On the other hand, player 1 could bring up a non constant mechanism, say $\mu = (\mu^{\alpha},\mu^{\beta})$. Since we are still discussing the case in which the constant mechanism $\bar{\mu} = (\bar{x})$ satisfies $\bar{\mu} \in V^{\mathscr{M}}$, we have $U_1^{\beta}(\bar{x}) \geq 0$. Now, (27) shows that ex ante as well as in mediis, player 1 is worse off at μ than at $\chi(\Gamma)$ $(= \bar{x})$ – so why should he forward non constant mechanisms at all?

Combining our argument for the 1st Case, we find it hard to believe that (given the NASH–value as the basis for the discussion in the traditional non random situation) anyone could reject the constant mechanism $\bar{\mu} = \chi(\Gamma) = \bar{x}$, if $\bar{\mu} \in \mathscr{M}$, since $E(U^{\tau}(\chi(\Gamma))) \in V^{\mathscr{M}} \subseteq V^{\underline{X}}$. Of course, this is what is called "the expected contract axiom" in [14].

2nd CASE: Now we are concerned with the case that $\chi(\Gamma)$ is non constant, $\chi = (\chi^{\alpha}, \chi^{\beta})$ (we omit the argument Γ for the moment).

Again we analyze the situation according to whether player 1 argues for alternative mechanisms that are constant or not.

On one hand there could be constant mechanisms that are IR. (in mediis), say $\hat{\mu}$ (= $\hat{x}$). Now, in our present situation, $\bar{\mu}$ (= $\bar{x}$) is not IR. (in mediis). This implies

$$U_1^{\beta}(\bar{x}) < 0 = U_1^{\beta}(\chi^{\beta}) \leq U_1^{\beta}(\hat{x}) \tag{34}$$

(cf. Corollary 4.6 and Formula (27)). (Recall that "$^{\beta}$" is player 1's worse situation!).

As U_1^{β} is strictly monotone in x_1 (see Theorem 1.1!), it follows that

$$\bar{x}_1 < \hat{x}_1 \tag{35}$$

and

$$U_1^{\alpha}(\bar{x}) < U_1^{\alpha}(\hat{x}) \tag{36}$$

as well as

$$EU_1^{\cdot}(x) < EU_1^{\cdot}(\hat{x}). \tag{37}$$

Since $\bar{x} \in \partial\, \underline{X}$ it is seen at once, that the inequalities are the other way around for player 2.

Thus, by proposing $\hat{\mu}$, player 1 is asking not only for more compared to χ but even for more compared to $\bar{x}$. He is asking this by using a constant mechanism, thus without referring to his information *in mediis*, meaning that he argues more or less in view of the *ex ante* situation. Indeed, some direct analysis verifies the situation (*ex ante*) as described in Fig. 4.2: $\bar{u}$ is the midpoint of $V^{\underline{X}}$ (and not available in $V^{\mathscr{M}}$), $E(U^{\cdot}(\chi^{\cdot}))$ is the midpoint of $[u^2, u^1]$ (and hence the NASH-value of $V^{\mathscr{M}}$), and $E(U^{\cdot}(\hat{x})$ is an element of $\partial\, V^{\underline{X}} \cap V^{\mathscr{M}}$.

So the first coordinate of $E(U^{\cdot}(\hat{x}))$ exceeds the expectation of $\bar{x}$ *in mediis* and *ex ante*. As player 1 is essentially arguing from *ex ante*, it seems to be rather impertinent to ask for that much and there is no way to justify such demand by the usual reasons of equity or fairness as presented in the context of the NASH-value.

Finally, let us, on the other hand, deal with the situation that player 1 is proposing a non constant mechanism say $\hat{\mu}$ instead of χ. Because of Corollary 4.6 we have $U_1^{\beta}(\chi^{\beta}) = U_1^{\beta}(\hat{\mu}^{\beta}) = 0$, that is, player 1 always gets 0 in his "worse situation". Now, this could lead him to shift the weight of his argument to situation $^{\alpha}$ ("if $^{\beta}$ occurs I get 0 utility whatsoever, so I want to talk about α; let's treat the *in mediis* situation α only"). Conceivably in this case he could attempt to ask for $\bar{x}^{\alpha}$ and utility $U^{\alpha}(\bar{x}^{\alpha}) = \bar{u}^{\alpha}$ (the midpoint of V^{α}).

This would mean that he is asking for a mechanism $\overset{(\alpha)}{\mu} = (\bar{x}^{\alpha}, \hat{x})$, where $\hat{x}$ is suitably chosen (by Theorem 3.4 and Corollary 3.6 of [14]) yields 0 utility for player 2. That is, the utility is

$$EU_1^{\cdot}(\overset{(\alpha)}{\mu}{}^{\cdot}) = p^{\alpha}\, U_1^{\alpha}(\bar{x}^{\alpha}). \tag{38}$$

Now, let us return to Fig. 4.1. Clearly $p^{\alpha}\, U_1^{\alpha}(\bar{x}^{\alpha})$ is the first coordinate of the NASH-value of $p^{\alpha} V^{\alpha}$. Hence, $\overset{(\alpha)}{u} = E(U^{\cdot}(\overset{(\alpha)}{\mu}{}^{\cdot})$ is obtained as the corresponding point on the Pareto boundary $\partial\, V^{\mathscr{M}}$. And clearly, in this situation, the NASH-value of $V^{\mathscr{M}}$ (i.e., $E(U^{\cdot}(\chi^{\cdot}))$) is to the right of $\overset{(\alpha)}{u}$. (We do *not* refer to (the missing) monotonicity of the NASH-value – the situation is much simpler.)

Thus we conclude: by asking for discussion only *in mediis* and claiming $\nu(0, V^{\alpha})$ in situation $^{\alpha}$, player 1 is worse off compared to χ and again, this is so *ex ante and in mediis* (since the corresponding mechanisms yield always 0 to player 1 in situation $^{\beta}$). So eventually we end up seeing player 1 resolving himself to go back and discuss not "situation $^{\alpha}$ only". His argument will be that since he is so badly off in situation $^{\beta}$, getting 0 at any non constant mechanism, he wants compensation in situation $^{\alpha}$. But this can only mean that he wants to evaluate the situation *ex ante* and ask for fair *ex ante* considerations. In this case there is (for NASH believers) nothing but to stick to just χ, because it yields the NASH–value of $V^{\mathcal{M}}$.

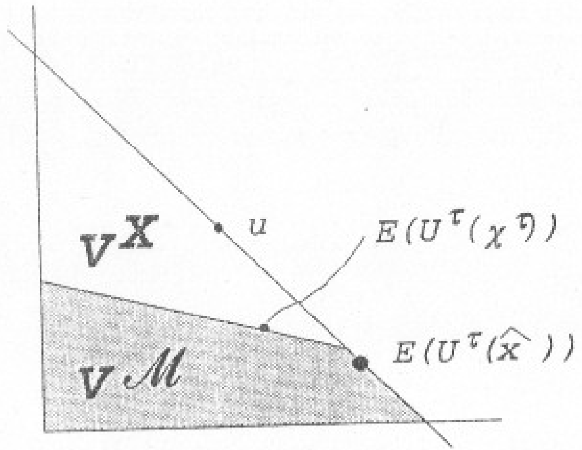

Fig. 4.2

At this stage we shall end our present discussion. We believe to have supported the expected contract value by quite some arguments; hopefully one could consider χ to be the "natural" value for side payment situations (i.e., for fee–games).

The next task will be to establish χ by an axiomatic approach. If this is possible at all, then the canonical representation will play an important role. For, if something like an IIA–axiom will be suitable then the original game and the one used for comparison, ("a fee–game yielding the irrelevant alternatives") would have to be parametrized simultaneously. We feel that the canonical representation is the only "sensible" (that is canonical) way to attempt such a simultaneous representation by reasons that should be clear at this stage of the development.

Literatur

[1] Allen, Beth:
Market games with asymmetric information and nontransferable utility: Representation results and the core.
CARESS Working paper 91–09.
Dep. of Economics. University of Pennsylvania. 38 pp.

[2] Allen, Beth:
Transferable Utility market games with asymmetric information: Representation results and the core.
CARESS Working paper 91–16.
Dep. of Economics. University of Pennsylvania. 37 pp.

[3] D'Aspremont, C. and Gérard–Varet, L.–A.:
Incentives and incomplete information.
Journal of Public Economics 11 (1979), pp. 25 – 45

[4] Harsanyi, J. C.:
Games of incomplete information played by Bayesian players.
Parts I – III. Management Science 14, (1967 – 1968),
pp. 159 – 182, 320 – 334, 486 – 502.

[5] Harsanyi, J.C. and Selten, R.:
A generalized Nash solution for two–person bargaining games with incomplete information.
Management Science 18 (1972), pp. 80 – 106

[6] Holmström, B. and Myerson, R.B.:
Efficient and durable decision rules with incomplete information.
Econometrica 51 (1983), pp. 1799 – 1819

[7] Hurwicz, L.:
On informationally decentralized systems.
Decision and Organisation. (R. Radner and B. McGuire eds.)
North Holland (1972), pp. 297 – 336

[8] Myerson R.B.:
Incentive compatibility and the bargaining problem.
Econometrica 47 (1979), pp. 61 – 73

[9] Myerson, R.B.:
Two–person bargaining problems with incomplete information.
Econometrica 52 (1984), pp 461 – 487

[10] Myerson, R.B.:
Cooperative games with incomplete information.
International Journal of Game Theory 13 (1984),pp.69 – 96

[11] Myerson, R.B. and Satterthwaite, M.A.:
Efficient mechanisms for bilateral trading.
Journal of Economic Theory 29 (1983), pp. 265 – 281

[12] Nash, J. F.:
The bargaining problem.
Econometrica, Vol. 18,(1950), 155 – 162

[13] Rosenmüller, J.:
Remarks on cooperative games with incomplete information.
Working paper 166, Inst. of Math. Econ. (IMW),
University of Bielefeld (1988), 44 p.

[14] Rosenmüller, J:
Fee Games: (N)TU–games with incomplete information.
Rational Interaction, Essays in Honor of John C. Harsanyi.
(Edited by R. Selten), Springer Verlag, Berlin Heidelberg, 1992,
pp.53 – 81, ISBN 3–540–55067–4

[15] Weidner, F:
The generalized Nash bargaining solution and
incentive compatible mechanisms.
International Journal of Game Theory, Vol. 21,
pp. 109 – 121, (1992).

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