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Excessive and Under–Investment: On the Incentives to Adopt New Technologies under Pigouvian Taxes and Tradeable Permits

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Abstract

The paper investigates the firms' incentives to adopt a new cleaner technology under pollution control policies like effluent taxes and tradeable permits. We consider a competitive industry where firms produce a homogeneous marketable commodity and generate pollution. Firms have convex variable costs and positive fixed costs. There are two types of technologies, conventional and innovative ones, where the latter generates less pollution but incurs higher private costs. The number of firms of each type is determined endogenously by zero profits.

We find that taxes almost always induce complete innovation or no innovation at all. Permits, on the other hand, allow for partial innovation. Considering the socially optimal degree of innovation we find that no innovation should occur if the social damage function is sufficiently flat, complete innovation is optimal if the damage function is sufficiently steep. For intermediately steep damage functions partial innovation is optimal. Under the hypothesis that there is optimal regulation before innovation by taxes or permits, taxes lead to excess innovation whereas permits cause too little innovation, whenever partial innovation is optimal. If complete innovation is optimal, the pattern is reversed: too many firms with the new technology enter the market under permits, too little a number of firms under taxes.

JEL: H23, L51

Keywords: Emission taxes, tradeable permits, innovation, adoption of new technology, over–investment, under–investment, policy adjustment.

1 Introduction

Environmental policies based on prices and decentralized decision making, like emission taxes or tradeable permits, have been proved as powerful and efficient instruments of pollution control. It is well known that these policy tools are equivalent under sufficient information and perfect competition. This equivalent result has been derived, for the most part, under static conditions, i.e. the firms' technologies are assumed to be exogenously given. But as KNEESE & C. SCHULTZE [1975, p. 38] have stressed "...Over the long haul, perhaps the most important single criterion on which to judge environmental policies is the extent to which they spur new technology toward the efficient conservation of environmental quality."[1]

More recently DOWNING & WHITE [1989], MILLIMAN & PRINCE [1989], and MALUEG [1989] have examined the firms' incentives to adopt new abatement technologies. In particular, MILLIMAN & PRINCE provide a detailed comparison of those incentives under different policies as command and control, effluent taxes, auctioned permits, free permits, and subsidies on abatement. Under different scenarios they rank those policies with respect to the firms' incentives to adopt less polluting technologies In most cases, auctioned permits turn out to be the winner. As fashionable in the environmental economics literature, however, those authors pursue "partial partial" analysis, i.e. they focus solely on the pollution sector and do not pay attention to the output market.[2]

This paper also focuses on the impact of taxes and auctioned permits to promote the adoption of new technologies. Similar to SPULBER [1985] we set up a model which pays explicit attention to the output market. In contrast to SPULBER who claims "long run optimality" and symmetry of taxes and permits, however, we show that in the very long run, when new technologies become available, static tax and permit policies are neither symmetric, nor optimal. We also demonstrate that unique policy rankings as stated by MILLIMAN & PRINCE cannot be obtained any longer in a model which takes into account the feedback on the output market. The framework of the model is fairly general. Firms incur convex variable costs and positive fixed cost. There are two types of technologies, a "conventional" and an "innovative" one. The innovative technology incurs higher fixed costs than the conventional one (due to installation and maintenance of abatement equipment), but leads to lower marginal abatement costs.

The paper contains tree major parts. First (Section 4) we study first best allocations if both types of technologies are available. We find that the new technology should

[1] See also ORR [1978].

[2] The authors mention, however, that taking into account the output market could change the results.

not be introduced if the social damage function relatively flat. *Complete innovation* is optimal if the damage from pollution is relatively severe, i.e. if the social damage function is relatively steep. For intermediately steep damage functions, however, *partial innovation* is optimal, i.e. *both* types of technologies should be employed.

Then, in Section 5, we consider competitive market equilibria if firms are regulated by any emission tax or any number of permits. Since the number of firms are determined endogenously by zero profits, we are in particular interested how many firms will run the conventional technology and how many will invest to buy the new technology under either of the two policies. It turns out that for almost all tax levels only conventional firms *or* only innovators can stay in the market at the same time. For exactly one tax level, however, there is a whole set of competitive equilibria where the number of conventional firms and the number of innovators is not uniquely determined. Under permits, in contrast, there is always a unique competitive equilibrium, and both types of firms can stay together in the market for a whole interval of different quotas of permits. So whilst a small tax raise may cause an industrial revolution, a small variation of the number of permits causes a continuous variation of all involved variables, in particular of industry output, pollution, and the numbers of firms.

These results have interesting consequences for the third part (section 6) where we assume that initially all the firms are alike and are due to optimal regulation by either taxes or auctioned permits, and then a new technology becomes available. The regulator, however, cannot anticipate this, and the institutional framework does not allow for fast and appropriate policy adjustment (which is not necessarily a bad assumption). It turns out that for a considerable range of parameters for which partial innovation is optimal, taxes lead to *excessive adoption* whereas permits cause *too little adoption* of the new technology. Even more striking, welfare may decrease through innovation under taxes whereas this can never happen under permits! These results apparently provide strong arguments in favor of auctioned permits instead of emission taxes, at least if the regulator expects *partial* innovation to be optimal. If *complete* innovation is socially optimal, however, the pattern is reversed for a considerable range of parameters. There is *excess entry* of innovators under permits, and *too little entry* of innovators under taxes. So in contrast to the findings of MILLIMAN & PRINCE [1989], none of these policies is strictly superior in general. These results may call for a more dynamic system of taxes and permits whose design and analysis, however, is beyond the scope of this paper.

Very recently LAFFONT and TIROLE [1994a,b] analyzed permit markets where firms can bypass the cost of buying permits by investing into emission free technology [1994a] or by engaging in R&D in order to develop a pollution free technology [1994b] (in contrast, the model considered here does not assume that production free pollution is possible). In their first model the authors claim that a stand alone spot market for permits always leads to over-investment[3] whereas in the second model permit markets always lead to under-investment.[4] But note that also LAFFONT and TIROLE do not pay explicit attention to the output market.

The next section contains basic assumptions. In Section 3 we briefly summarize some crucial results for the symmetric case, i.e. for regulation before innovation. As mentioned, Section 4 investigates the social optimum if the new technology is available, in Section 5 we examine the free entry equilibria under any tax or permit policy, whilst in Section 6 we consider the welfare consequences of innovation under the hypothesis that the conventional firms are regulated optimally. Section 7 investigates optimal agency response, and the final section concludes. Technical proofs are relegated to the appendix.

2 The Model

Throughout this paper we consider a partial model where an endogenous number of n firms causes pollution while producing a homogenous consumption good. Let q_i and e_i, $i = 1, \ldots, n$, denote firm i's output and emission level, respectively, Industry output is written as $Q := \sum_{i=1}^{n} q_i$, total emissions as $E := \sum_{i=1}^{n} e_i$. Welfare, as typical for partial models, is the sum of consumers' surplus, minus the damage from pollution, minus production costs:

$$W(q_1, \ldots, q_n, e_1, \ldots, e_n; s) := \int_0^Q P(z)dz - S(E, s) - \sum_{i=1}^{n} C^i(q_i, e_i) \tag{2.1}$$

where $P(\cdot)$ is the inverse demand for the consumption good, $S(\cdot, \cdot)$ is the social damage function depending on a damage parameter s, and $C^i(\cdot, \cdot)$ is firm i's cost function. We make the following assumptions:

Assumption 1 *Inverse demand P is a downward sloping function of aggregate output only, it has a finite choke-off price $\overline{p} := P(0) := \min\{p|D(p) := P^{-1}(p) = 0\}$, and satisfies $P''(Q) < -2P'(Q)/Q$ for all $Q > 0$, i.e., $|P''|$ is sufficiently bounded.*

The upper bound for P'' is sufficient to guarantee the second order conditions.

[3][1994a], p.2: "Stand alone spot markets (in which the government sets at the beginning of each period the number of permits for that period) create excessive incentives for investment. ... This incentive can be reduced by the introduction of a futures market."

[4][1994b], p.2: " ...while spot markets destroy incentives for innovation, futures markets bring limited improvement." Unfortunately the authors do not discuss the reason for these contrary results.

Assumption 2 *o) S is at least twice continuously differentiable w.r. to E and s; in $(0,0)$ the right sided partial derivatives exist. i) $S(0,s)=0$ $\forall s \geq 0$. ii) $S(E,0)=0$ $\forall E \geq 0$. iii) $S_1(E,s)>0$ $\forall s>0$ $\forall$ $E>0$. iv) $S_1(0,s)=0$ $\forall s \geq 0$. v) $S_{11}(E,s) \geq 0$ $\forall s>0$ and strictly greater for $E>0$. vi) $S_{12}(E,s)>0$ $\forall E>0, s>0$.*

So, S is increasing and convex in E. Marginal damage increases in s. The idea of the damage function is that it represents the disutility that consumers suffer from pollution, plus the economic damage that other industries incur from the pollution in this industry. The damage parameter s is an exogenous parameter of the model and can be interpreted as an indicator of how hazardous the pollutant is. It also determines the slope of the social marginal rate of substitution between consumption and pollution (or abatement). Parameterizing S via s allows us to completely characterize the social optimum and also regulatory policies as a function of the damage function's steepness. Note that the steepness of the damage function matters also in related models, for example for the choice between price versus quantity regulation in WEITZMAN'S [1974] seminal paper on regulation under imperfect information.[5]

The firms' **technologies** are given by their reduced cost functions $C^i:(q_i,e_i)\rightarrow C^i(q_i,e_i)$, i.e. firm i's cost depend on firm i's output q_i and emissions e_i, and slit up into a fixed cost $F>0$ and the variable cost v, i.e.

$$C(q,e)=\begin{cases} 0 & \text{if } (q,e)=(0,0) \ , \\ F+v(q,e) & \text{else} \ . \end{cases} \tag{2.2}$$

Since the derivatives of v and C coincide, we write all assumptions about v in terms of the total cost function C. Assuming sufficient smoothness of C we define

$$Y_{MAC}(C)\equiv\left\{(q,e)>0 \;\middle|\; \frac{C(q,e)}{\langle\nabla C(q,e),(q,e)\rangle}=1\right\} \tag{2.3}$$

as the set of all quantities and emission levels (q,e) for which the degree of scale economics equals 1,[6] or for which we have (generalized) *minimized average costs* (hence the index "MAC"). Further let

$$\tilde{Y}_{MAC}(C)\equiv\left\{\left(\frac{q}{C(q,e)},\frac{e}{C(q,e)}\right) \text{ with } (q,e)\in Y_{MAC}(C)\right\} \tag{2.4}$$

be the set of all quantities and emission levels (q,e) for which the degree of scale economics equals 1, normalized by corresponding cost $C(q,e)$.

Assumption 3 *For all $i=1,\ldots,n$ the firms' cost functions $C^i:\mathbb{R}^2_+\rightarrow\mathbb{R}$ are twice continuously differentiable and satisfy (we omit the superscript i):*

[5] See also ADAR and GRIFFIN [1976], FISHELSON [1976], and BAUMOL/OATES [1988].

[6] The concept of scale economics in a multi product technology is taken from BAUMOL et. al. [1982]. The degree of scale economics is defined by $S(q,e)=C(q,e)/\langle\nabla C(q,e),(q,e)\rangle$.

i) $C_1 > 0$, $C_{11} > 0$, $C_{22} > 0$, $C_{12} < 0$.

ii) *For all q there is $e(q)$ such that $C_2(q, e(q)) = 0$, and $C_2(q, e) < 0$ if $e < e(q)$, and $C_2(q, e) \geq 0$ if $e > e(q)$.*

iii)
$$C_{11}C_{22} - [C_{12}]^2 > 0 \ . \tag{2.5}$$

iv) *For all (q, e) in an open stripe containing $Y_{MAC}(C)$ we have*

$$\langle \nabla C_1, (q, e) \rangle = C_{11}q + C_{12}e \ > \ 0 \tag{2.6}$$
$$\langle \nabla C_2, (q, e) \rangle = C_{12}q + C_{22}e \ > \ 0 \tag{2.7}$$

iv) implies that $\tilde{Y}_{MAC}(C)$ is a one to one relation, i.e. there is a range D and a function $h : D \to \mathbb{R}$ such that $\forall (y_1, y_2) \in \tilde{Y}_{MAC}(C)$ we have $y_2 \in D$ and $y_1 = h(y_2)$.

v) *Moreover h is concave.*[7] [8]

The assumption implies that the *variable* cost function is convex. In particular we have increasing marginal costs for fixed emission levels, abatement costs are convex for each fixed output, output and emissions are cost complements ($C_{12} < 0$), and each output level has a cost minimizing emission level which the firms would choose in the absence of regulation. In that case we could define a further reduced cost function by $\tilde{C}(q) := C(q, e(q))$, and define $\overline{q}$ by

$$\frac{\tilde{C}(\overline{q})}{\overline{q}} = \tilde{C}'(\overline{q}) \tag{2.8}$$

as the output level which minimizes the average cost in the absence of regulation. By Assumption 3 such an $\overline{q}$ exists. Finally we require a joint condition on cost and demand functions:

Assumption 4 i) *(Existence of a market in the absence of regulation) $\widetilde{C^i}'(\overline{q}) < \overline{p}$.*

ii) *(Impossibility of emission-free production) For all $q > 0$ there is e^* such that for all $e < e^*$ we have $C(q, e)/q > \overline{p}$.*

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[7] One can show that h is concave if the third derivatives of the cost function are sufficiently bounded. To write down explicit conditions, however, is tedious and does not yield further insight.

[8] A cost function with the properties of Assumption 3 can be derived from a Cobb–Douglas production function where one input is energy, which has a non-zero factor price, and pollution is proportional to the use of energy. A Cobb–Douglas function where one input is pollution does not satisfy ii) since in that case $e(q)$ would be infinite which is certainly not quite realistic. (I am grateful to CEES WITHAGEN for asking about underlying production functions.)

i) says that in the absence of regulation there exists a market for the commodity. By continuity this implies that there is also a market under moderate regulation, i.e., if the government sets a lax emission standard $\overline{\epsilon}$ slightly smaller than $\epsilon(\overline{q})$ (or charges a sufficiently low emission tax). ii) says that if for any fixed output $q > 0$ the firms' emission level is sufficiently low, the minimal average cost exceeds the choke–off price, inducing the firm to close down. ii) implies that production is not possible without any pollution, which is certainly realistic. It also excludes corner solutions (i.e. $q_i > 0$ but $e_i = 0$ is impossible[9]).

Two Different Types of Firms: Assume now that there are two types of technologies represented by their cost functions, a conventional one, denoted by C^0, and an innovative one, denoted by C^I. The corresponding quantities and emission levels are denoted by q_0, q_I, e_0, e_I. The innovator's fixed cost F^I include fixed costs for buying and installing the new technology, i.e. possible switching costs, such that $F^I > F^0$. Since we allow for free entry, up to the fixed costs, we assume for simplicity that new entry and switching technology causes the same fixed cost.

Now we make a joint assumption on the two technologies. We assume that the conventional technology provides a cost advantage in the absence of regulation. If, on the other hand, a sufficiently small emission level is required, the new technology has lower average costs. For simplicity we assume that there is no back switching of the cost advantages. These properties are depicted in Figure 1 and can be formalized as follows:

Assumption 5 *The two cost functions C^0, and C^I satisfy the following conditions:*

i)
$$\frac{\overline{q}_0}{\tilde{C}^0(\overline{q}_0)} > \frac{\overline{q}_I}{\tilde{C}^I(\overline{q}_I)} \ . \tag{2.9}$$

ii) *There are $(\hat{q}_0, \hat{e}_0) \in Y_{MAC}(C^0)$, $(\hat{q}_I, \hat{e}_I) \in Y_{MAC}(C^I)$, and $\lambda > 0$ such that*

$$\nabla C^0(\hat{q}_0, \hat{\epsilon}_0) = \lambda \nabla C^I(\hat{q}_I, \hat{e}_I) \ , \tag{2.10}$$

$$\frac{\hat{e}_0}{C^0(\hat{q}_0, \hat{\epsilon}_0)} < \frac{\hat{e}_I}{C^I(\hat{q}_I, \hat{e}_I)} \ , \tag{2.11}$$

and

$$\frac{\frac{q_0}{C^0(\hat{q}_0, e_0)} - \frac{\hat{q}_I}{C^I(\hat{q}_I, \hat{e}_I)}}{\frac{\hat{e}_0}{C^0(\hat{q}_0, \hat{e}_0)} - \frac{e_2}{C^I(\hat{q}_I, \hat{e}_I)}} < -\frac{C^i_2(\hat{q}_i, \hat{e}_i)}{C^i_1(\hat{q}_i, e_i)} \qquad i = 0, I. \tag{2.12}$$

iii) *The functions h_0 and h_I intersect only once.*

[9] This implies that complete bypass as assumed in LAFFONT and TIROLE [1994a,b] and [1990] is not possible, which is certainly realistic to assume.

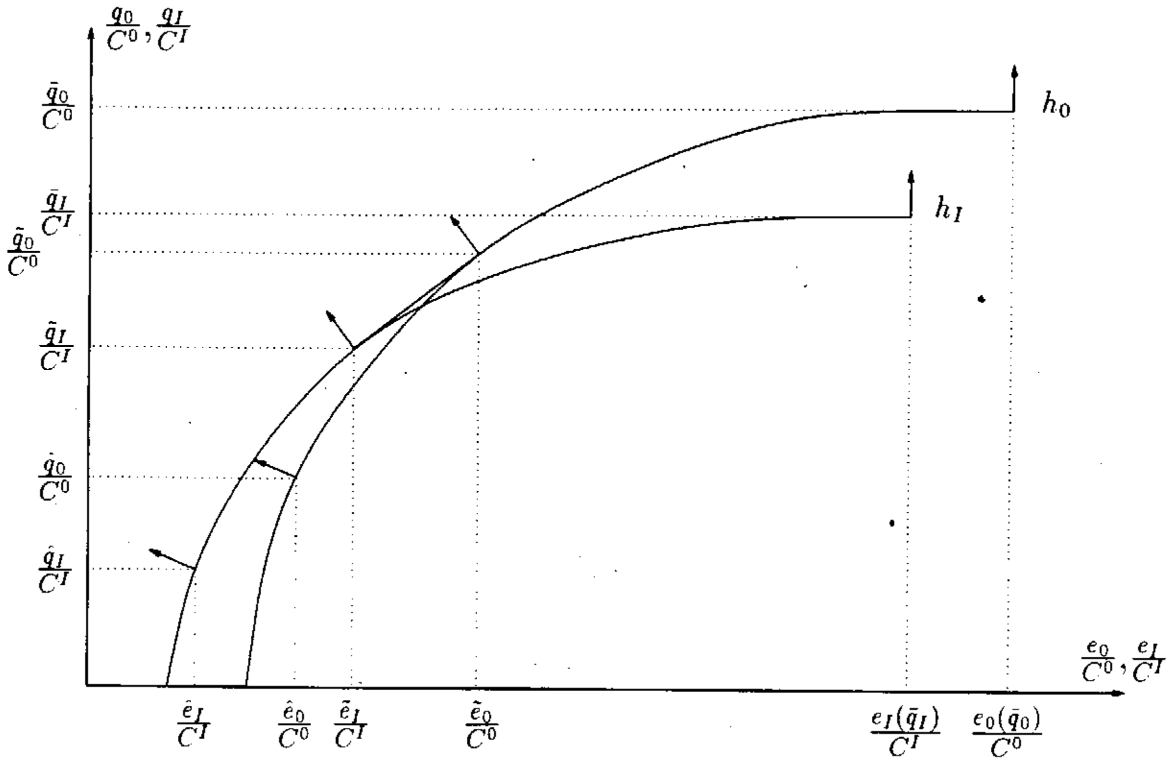

Figure 1: *The sets $\widetilde{Y}_{MAC}(C^0)$ and $\widetilde{Y}_{MAC}(C^I)$ represented by the functions h_0 and h_I. In order to not burst the labels of the points on the axes we wrote for short $\frac{q_i}{C^i}$ instead of $\frac{q_i}{C^i(q_i,e_i)}$. The arrows give the directions of the gradients of the cost function along the h_i-curves. Note that they are perpendicular to those curves.*

Here h_0 and h_I are the corresponding functions from Assumption 3.iv with respect to the cost function C^0 and C^I, respectively. The assumption looks awkward, however, the idea is actually very simple, and it is illustrated in Figure 1. i) says that in the absence of regulation, i.e. if the firms pick ϵ such that $C_2(q, \epsilon) = 0$, the conventional firms of type 0 have the lower average cost than the innovators, who face the higher fixed costs due to installation of new technology. Or equivalently, the output/cost ratio is larger for the conventional firms. Note that the point $\left(\frac{\overline{q}_0}{C^0(\overline{q}_0, e_0(\overline{q}_0))}, \frac{e_0(\overline{q}_0)}{C^0(\overline{q}_0, e_0(\overline{q}_0))}\right)$ lies above the h_I-curve.

ii) says that there is a region of output and emission levels where the innovative firms of type I have a cost advantage. This means that if we have two production plans for which the gradients of marginal cost of both firms point in the same direction, then the point $\left(\frac{\hat{q}_0}{C^0(\hat{q}_0, \hat{e}_0)} \frac{\hat{e}_0}{C^0(\hat{q}_0, \hat{e}_0)}\right)$ lies below the h_I-curve. Note that we could have stated i) and ii) symmetrically, and indeed they almost are. For if we substitute $\left(\frac{\hat{q}_i}{C^i(\hat{q}_i, \hat{e}_i)}, \frac{\hat{e}_i}{C^i(\hat{q}_i, \hat{e}_i)}\right)$ by $\left(\frac{\overline{q}_i}{C^i(\overline{q}_i, e_i(\overline{q}_i))}, \frac{e_i(\overline{q}_i)}{C^i(\overline{q}_i, e_i(\overline{q}_i))}\right)$ for $i = 0, I$, then (2.9) together with $C_2^i(\overline{q}_i, e_i(\overline{q}_i)) = 0$ imply both, (2.10) and the reversed inequality (2.12).

Note further that if (2.11) holds, (2.12) is equivalent to

$$\frac{C_1^I(\hat{q}_I, \hat{e}_I)\hat{q}_0 + C_2^I(\hat{q}_I, \hat{e}_I)\hat{e}_0}{C^0(\hat{q}_0, \hat{e}_0)} < 1 < \frac{C_1^0(\hat{q}_0, \hat{e}_0)\hat{q}_I + C_2^0(\hat{q}_0, \hat{e}_0)\hat{e}_I}{C^I(\hat{q}_I, \hat{e}_I)} . \tag{2.13}$$

This relation will be needed in the proof of Proposition 1, below. iii) is made for convenience and guarantees that the cost advantages do not switch back. Note that i) and ii) imply that h_0 and h_I intersect at least once.

3 Social Optimum and its Implementation under Symmetry

In this section we assume that only conventional firms are around. It is well known (see e.g. SPULBER [1985]) that in this case the socially optimal quantity q^* and the emission ϵ^*, produced by a single firms, as well as the socially optimal number of firms n^* satisfy the following first order conditions

$$P(n^* q^*) = C_1(q^*, \epsilon^*) \tag{3.1}$$

$$S_1(n^* \epsilon^*, s) = -C_2(q^*, \epsilon^*) \tag{3.2}$$

$$0 = P(n^* q^*) q^* - S_1(n^* e^*, s) e^* - C(q^*, e^*) \tag{3.3}$$

i.e. the (output) price equals marginal cost of each firm, social damage equals marginal abatement cost, and firms make a zero profit if pollution is priced at marginal damage. We handle the number of firms as a continuous variable in this model, assuming that the number if firms is large.

Denote in the following by $Q_0^* = n^* q^*$, and $E_0^* = n^* e^*$, the optimal aggregate output and emissions levels, respectively, if only the conventional technologies are available. If we want to emphasize the dependence on the damage parameter s, we write $Q_0^*(s)$ and $E_0^*(s)$, respectively.

It is also well known (see also SPULBER [1985]) that under perfect information (which is assumed throughout) the social optimum can be implemented by a tax $\tau^0(s)$ equal to marginal damage, i.e.:

$$\tau^0(s) = S_1(E_0^*(s), s) \ , \tag{3.4}$$

or by auctioning a number of permits $L(s)$ satisfying[10]

$$L(s) = E_0^*(s) \ . \tag{3.5}$$

The following result shows how output, pollution, the number of firms, and the marginal rate of substitution varies with the damage parameter s.

Proposition 1 *Under Assumptions 1, 2, and 3.i – iii, the system (3.1) – (3.3) has a unique solution consisting of socially optimal (single firms') outputs $q^*(s)$, emissions $e^*(s)$, and an optimal number of firms $n^*(s)$. Aggregate emissions $E_0^*(s)$ and output $Q_0^*(s)$ are decreasing in s, thus $P(Q_0^*(s))$ is increasing in s. Moreover, marginal damage $S_1(E_0^*(s), s)$, as well as social marginal rate of substitution between pollution and consumption denoted by*

$$MRS(s) \equiv \frac{S_1(E_0^*(s), s)}{P(Q_0^*(s))} \tag{3.6}$$

are increasing in s.

If in addition Assumption 3.iv holds, then $n^*(s)$, and $e^*(s)$ are decreasing whereas $q^*(s)$ is increasing in s.

Proof: see the appendix.

Not surprisingly, Proposition 1 as well as (3.4) and (3.5) imply that the optimal Pigouvian tax is increasing, and the optimal number of permits to be issued is decreasing as the damage function becomes steeper. One can show by examples that without Assumption 3.iv) the single quantities and the number of firms may not behave monotonically in s.

Note that due to free entry only auctioned permits lead to a socially optimal number of firms, whereas grandfathering leads to excess entry (see also SPULBER's [1985]).

[10] This is not completely trivial if the number of firms is endogenous. To get the social optimum as a unique market equilibrium one has to show (and indeed can show) that the factor demand for permits is strictly decreasing if the price for permits rises and the output market stays in equilibrium. A proof can be obtained by the author on request.

4 The Social Optimum after Innovation

The main goal of this paper is to investigate the impact of taxes and permits on the market equilibrium if a new technology is available. Since we also want to investigate the efficiency of those tools it is useful to study first how an optimal allocation looks like if both types of technologies, the conventional and the innovative one, are available.

The social planner would maximize welfare over quantities, emissions levels, and numbers of firms with respect to the constraint that all quantities, emissions levels, and numbers of firms are non-negative. Let n_0 and n_I denote the numbers of firms with the conventional cost function C^0 and with the new cost function C^I, respectively. Since the variable cost functions are convex, the conventional firms must have the same production plan (q_0, e_0), and the innovative firms must have the same plan (q_I, e_I). Hence the social planner's program is

$$\begin{aligned} \max_{q_0,e_0,q_I,e_I,n_0,n_I} W(q_0, q_I, e_0, e_I, n_0, n_I; s) \equiv & \\ \max_{q_0,e_0,q_I,e_I,n_0,n_I} \left\{ \int_0^{n_0q_0+n_Iq_I} P(z)dz \right. & - S(n_0e_0 + n_Ie_I, s) - \\ & \left. - n_0C^0(q_0, e_0) - n_IC^I(q_I, e_I) \right\} . \end{aligned} \tag{4.1}$$

Denote by $q_0^*(s), q_I^*(s), e_0^*(s), e_I^*(s), n_0^*(s), n_I^*(s)$ the solution, and by $Q_I^*(s) = n_0^*(s)q_0^*(s)+ n_I^*(s)q_I^*(s)$, and $E_I^*(s) = n_0^*(s)e_0^*(s)+n_I^*(s)e_I^*(s)$ the aggregate output and emission levels, respectively.

Theorem 1 *Let there be two technologies with corresponding cost functions C^0 and C^I, satisfying Assumptions 3 - 5.*

Then there are damage parameters $\underline{s}$, and $\overline{s}$, with $0 < \underline{s} < \overline{s} \leq \infty$ such that the socially optimal solution of the program (4.1) has the following properties:

i) For all $s < \underline{s}$ the new technology should not be employed, i.e. $n_I^*(s) = 0$. All involved quantities satisfy the properties of Proposition 1.

ii) For all $s \in [\underline{s}, \overline{s}]$ we have that

$$\begin{aligned} q_0^*(s) &\equiv \tilde{q}_0, & e_0^*(s) &\equiv \tilde{e}_0, & Q_I^*(s) &\equiv \tilde{Q}, \\ q_I^*(s) &\equiv \tilde{q}_I, & e_I^*(s) &\equiv \tilde{e}_I, & S_1(E_I^*(s), s) &\equiv \tilde{S} \end{aligned}$$

are constant in s. $E_I^*(s)$ is decreasing, $n_0^*(s)$ is decreasing, $n_I^*(s)$ is increasing in s, and these variables satisfy

$$P(\tilde{Q}) = C_1^0(\tilde{q}_0, \tilde{e}_0) = C_1^I(\tilde{q}_I, \tilde{e}_I) \tag{4.2}$$

$$\tilde{S} = S_1(E_I^*(s), s) = C_2^0(\tilde{q}_0, \tilde{e}_0) = C_2^I(\tilde{q}_I, \tilde{e}_I) \tag{4.3}$$

$$0 = P(\tilde{Q}) \cdot \tilde{q}_0 - C^0(\tilde{q}_0, \tilde{e}_0) - \tilde{S} \cdot \tilde{e}_0 \tag{4.4}$$

$$0 = P(\tilde{Q}) \cdot \tilde{q}_I - C^I(\tilde{q}_I, \tilde{e}_I) - \tilde{S} \cdot \tilde{e}_I \tag{4.5}$$

iii) For all $s > \overline{s}$ the conventional technology should not be employed, i.e. $n_0^*(s) = 0$. All involved quantities satisfy the properties of Proposition 1.

iv) There is $\overline{\overline{s}} > \overline{s}$ such that for all $s \in (\underline{s}, \overline{\overline{s}})$ we have more aggregate output, i.e. $Q_I^*(s) > Q_0^*(s)$, and less aggregate pollution, i.e. $E_I^*(s) < E_0^*(s)$.

Proof: See the appendix.

Theorem 1 says the following: If the damage parameter is low, only the conventional type of firms should produce. This is quite intuitive since in the absence of regulation those firms have the lower average cost by Assumption 5.i. If the damage parameter is zero, or close to zero, there is no, or only little need for regulation. But also if emissions are to be reduced only by little, it is still efficient to employ the conventional firms only. If the damage parameter is very high [part iii)], clearly only innovative firms should be active since those have the lower average cost for low emission levels. For intermediate values of damage parameters it is optimal to employ *both* types of technologies. Production is shifted continuously from the conventional to the innovative firms, as s increases. But in that case only the number of firms varies. Each type of firms keeps its efficient production plan $(\tilde{q}_i, \tilde{e}_i)$ for $i = 0, I$. Maybe surprisingly, total output remains constant on the whole interval $[\underline{s}, \overline{s}]$ whereas emissions go down in that interval, marginal damage, however, is also kept constant on $[\underline{s}, \overline{s}]$.

Outside the interval $[\underline{s}, \overline{s}]$, i.e. for $s < \underline{s}$ and $s > \overline{s}$, the properties of all involved variable, $Q_I^*(s)$, $E_I^*(s)$, and so on, are same as for the case where only one type of firm is around. These properties have already been summarized in Proposition 1.

Note that for s sufficiently high, emissions may go up through innovation, i.e. $E_I^*(s) > E_0^*(s)$. We cannot, however, show that emissions go up in general if s is sufficiently high. Due to the fixed costs it may happen that output is too low for a single firm to survive, i.e., the market may break down.[11]

The shape of the aggregate quantity of output is depicted in Figure 2, the shape of aggregate emissions in Figure 3. For marginal social damage and the optimal numbers of firms see Figure 4.

Note that whereas in the one dimensional models of WEITZMAN'S [1974], ADAR and GRIFFIN [1976], FISHELSON [1976], the relative slopes of the damage functions compared to the marginal abatement curve is crucial, in our more general model with two commodities (the consumption good and pollution), the relative slopes of the social

[11]For a polynomial cost function of type $C(q, e) = \frac{1}{2}\left[(\beta q + \alpha - e)^2 + \gamma q^2\right] + F$, one can show for suitable parameters α, β, γ, F, and s sufficiently high that both can happen: $E_I^*(s)$ exceeds $E_0^*(s)$ and the number of firms is greater than 1, but also that $E_I^*(s)$ exceeds $E_0^*(s)$ in a region where the number of firms alls short of 1 (if we treat it as a continuous variable), i.e. there is no market anymore.

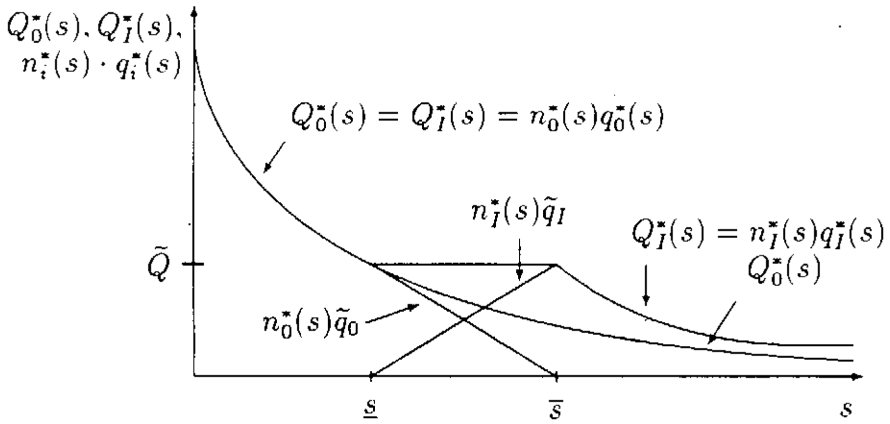

Figure 2: *Optimal aggregate output as a function of the damage parameter s before and after the new technology is available.* $Q_0^*(s)$ *and* $Q_I^*(s)$ *denote optimal aggregate outputs before and after innovation, respectively.* $n_0^*(s)q_0^*(s)$ *and* $n_I^*(s)q_I^*(s)$ *are the optimal aggregate outputs after innovation produced by the conventional industry and the innovators, respectively. Note that* $Q_0^*(s) = Q_I^*(s)$ *for* $s \leq \underline{s}$.

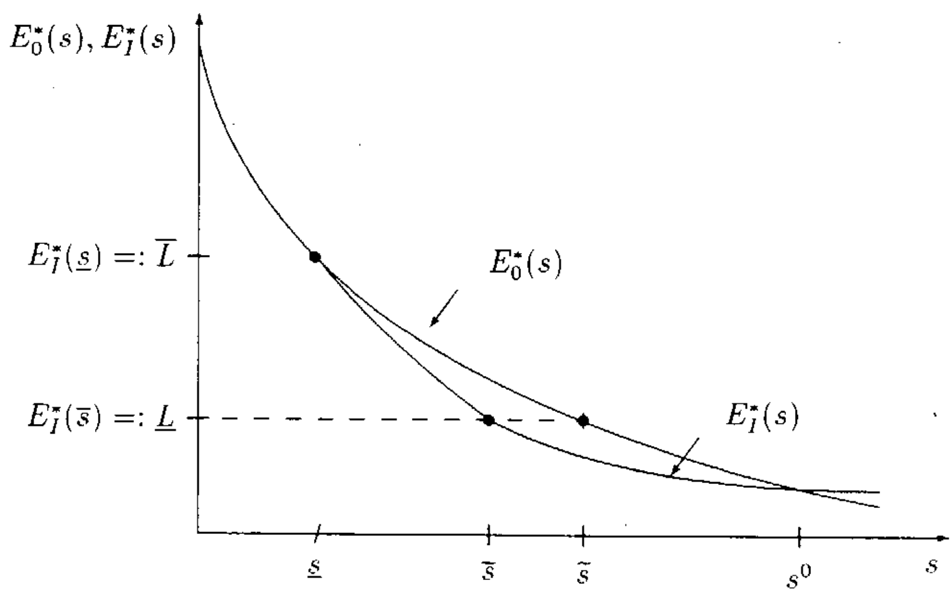

Figure 3: *The optimal aggregate emissions (= optimal permit policy) before and after innovation.*

marginal rate of subsitution between consumption and pollution versus the slope of the marginal rate of transformation between output and abatement is crucial.

5 Free Entry Market Equilibrium under Taxes and Permits with both Types of Technologies

Let us step back for a moment from optimality and let us examine the market equilibrium under any tax or under any number of permits if the two types of firms potentially may enter the market. From "normal" partial analysis with one commodity we know that different types of firms cannot survive under free entry, unless the minimum average costs happen to coincide. *Our* model can be considered as having two commodities, the marketable one and the public bad, i.e. pollution. Hence the situation is similar. However, the minimum average cost depends heavily on the level of regulation, i.e., on the size of the emission tax or the number of permits being issued by the government. Thus we will investigate how a competitive free entry equilibrium looks like if an arbitrary tax is charged on emissions, or an arbitrary number of permits is being issued.

5.1 Free Entry Market Equilibrium under Taxes

Suppose first that any tax τ is given. Let $q_0(\tau)$, $q_I(\tau)$, $e_0(\tau)$, $e_I(\tau)$ denote the individual firms' outputs and emission levels, as a function of the tax τ (these are indeed functions). On the other hand, we cannot write the numbers of firms as a function of the tax rate since an equilibrium will turn out to be not unique in general under taxes. Therefore, also the emission level is not a function of τ. The next proposition characterizes the competitive free entry equilibria for arbitrary taxes.

Recall from the last section that $\tilde{S}$ denotes the marginal social damage in social optimum if both types of firms produce, i.e., for $s \in (\underline{s}, \overline{s})$, and $\tilde{Q}$ is the corresponding aggregate output. Moreover, recall that $(\tilde{q}_i, \tilde{e}_i)$ are the production plans of the two types of firms which are in $Y_{MAC}(C^i)$, and for which the firms marginal production and abatement costs equalize (see Figure 1).

Theorem 2 *Under an emissions tax* τ,

i) there is a unique free entry competitive equilibrium with only the conventional firms being active if $\tau < \tilde{S}$.

ii) There is a unique free entry competitive equilibrium with only the innovative firms being active, or there is no market at all, if $\tau > \tilde{S}$.

iii) For $\tau = \tilde{S}$, there is a whole set of equilibria with $n_0 \in [0, \tilde{Q}/\tilde{q}_0]$ and $n_I = (\tilde{Q} - n_0\tilde{q}_0/\tilde{q}_I)$, and $q_0(\tau) = \tilde{q}_0$, $q_I(\tau) = \tilde{q}_I$, $e_0(\tau) = \tilde{e}_0$, $e_I(\tau) = \tilde{e}_I$.

Aggregate output is independent of (n_0, n_I), whereas aggregate emissions are the higher, the higher n_0.

All involved quantities satisfy the conditions (5.1) - (5.3) below for all the active firms.

So for τ less or greater than $\tilde{S}$ only one type of firms produces under free entry and free choice of technology. For $\tau = \tilde{S}$, on the other hand, the equilibrium is undetermined with repect to the numbers of firms of each type. Whilst aggregate output does not vary with the allocation of production among the two types, pollution does.

Proof: The firms' first order conditions of profit maximization lead to:

$$P(Q(\tau)) = C_1^i(q_i(\tau), e_i(\tau)) \qquad i = 0, I \tag{5.1}$$

$$\tau = -C_2^i(q_i(\tau), e_i(\tau)) \qquad i = 0, I, \tag{5.2}$$

Free entry leads to

$$P(Q(\tau)) - \tau e_i(\tau) - C^i(q_i(\tau), e_i(\tau)) = 0 \qquad i = 0, I\ , \tag{5.3}$$

if both types of firms are active, otherwise only for $i = 0$ or $i = I$. Assume first $\tau = \tilde{S}$. Clearly for such a τ there exists a competitive equilibrium with $q_0 = \tilde{q}_0$, $q_I = \tilde{q}_I$, $e_0 = \tilde{e}_0$, $e_I = \tilde{e}_I$, and numbers n_0, n_I satisfying $n_0\tilde{q}_0 + n_I\tilde{q}_I = \tilde{Q}$, where $\tilde{Q}$ is the socially optimal output if both types are supposed to produce in social optimum, i.e. for the interval $(\underline{s}, \overline{s})$. Note that for this interval the marginal social damage is equal to $\tilde{S}$. Thus all n_0, n_I with $n_0 \in [0, \tilde{Q}/\tilde{q}_0]$ and $n_I = \tilde{Q} - n_0\tilde{q}_0/\tilde{q}_I$ yield a competitive free entry equilibrium. On the other hand, no further equilibrium with $n_0 > \tilde{Q}$ or $n_I > \tilde{Q}/\tilde{q}_I$ can exist.

Suppose now that only one type of firm is active, and consider the ratio between the price for emissions, i.e. the tax, and the output price $P(Q(\tau))$, where $Q(\tau)$ is the aggregate output (which is also uniquely determined if only one type of firms is around and hence it can be written as a function of τ). This rate is the marginal rate of transformation between production and abatement for the firms, hence denoted by

$$MRT(\tau) \equiv \frac{\tau}{P(Q(\tau))}\ . \tag{5.4}$$

and we show that it is increasing in τ. Differentiating with respect to τ yields

$$MRT' = \frac{P - \tau P'Q'}{P^2}\ . \tag{5.5}$$

Now differentiating the system (5.1) - (5.3) for $i = 0$ or $i = I$ w.r. to τ, solving for q_i', e_i', and n_i' and substituting into $Q' = nq' + n'q$ we get after some rearranging the simple expression $Q' = e/(P'q)$. Plugging into (5.5) gives

$$MRT' = \frac{Pq - \tau e}{P^2 q} > 0 \ .$$

The term is positive since the numerator must be positive if (5.3) is satisfied. Assure that by Proposition 1 and Theorem 1 the tax/price–ratio must be equal to $\tilde{R} \equiv \tilde{S}/\tilde{P}$ if only the conventional firms are active and $n_0 = \tilde{Q}/\tilde{q}_0$, or if only the innovative firms are active and $n_I = \tilde{Q}/\tilde{q}_I$. Hence, for $\tau > \tilde{S}$ we have $\tau/P(Q(\tau)) > \tilde{R}$. For $\tau < \tilde{S}$ we get $\tau/P(Q(\tau)) < \tilde{R}$. But since for each active firm profit maximization implies

$$-\frac{C_2^i(q_i, e_i)}{C_1^i(q_i, e_i)} = \frac{\tau}{P(Q(\tau))} \qquad i = I, 0 \ ,$$

and since

$$\frac{-C_2^i(\tilde{q}_i, \tilde{e}_i)}{C_1^i(\tilde{q}_i, \tilde{e}_i)} = \tilde{R} \qquad i = I, 0 \ ,$$

we must have

$$\frac{\frac{q_0}{C^0(q_0,e_0)} - \frac{q_I}{C^I(q_I,e_I)}}{\frac{e_0}{C^0(q_0,e_0)} - \frac{e_I}{C^I(q_I,e_I)}} > \ (=, <) \ \frac{-C_2^i(q_i, e_i)}{C_1^i(q_i, e_i)} \qquad i = 1, 2 \tag{5.6}$$

as $\tau < \tilde{S}$ ($\tau = \tilde{S}$, $\tau > \tilde{S}$) for q_0, q_I, e_0, e_I satisfying (5.1) and (5.2). But this implies that only the conventional firms are active if $\tau < \tilde{S}$, and only the innovative firms are active if $\tau > \tilde{S}$. Q.E.D.

So far we have actually treated both types of firms symmetrically in the theorem, and the notions of "conventional" and "innovative" firms seemed to be somewhat arbitrary. If we take the interpretation seriously and assume that type 0 firms are incumbent, and that after a tax has been imposed, innovators consider to enter (or some incumbents consider to switch technology), the multiplicity of equilibria vanishes for $\tau = \tilde{S}$. For if τ has been set equal to $\tilde{S}$ before the new technology has been available, the number of conventional firms must be $n_0 = \tilde{Q}/\tilde{q}_0$. But this is still an equilibrium after the new technology is available! The innovative firms do not have a comparative advantage to crowd out the conventional firms. Under this story we would have

$$\begin{aligned} n_I(\tau) &= 0 \quad \text{if} \quad \tau \leq \tilde{S} \ , \\ n_0(\tau) &= 0 \quad \text{if} \quad \tau > \tilde{S} \ . \end{aligned}$$

We observe that the market equilibrium may behave very discontinuously as the tax rises. In particular, a small tax increase from, say, $\tau_1 = \tilde{S} - \varepsilon$ to $\tau_2 = \tilde{S} + \varepsilon'$ may lead to an industrial revolution in the long run in the sense that under free entry, or free choice of technology, the conventional technologies will be completely substituted by the new ones. Note that in this case also the pollution level varies discontinuously, making a jump downwards for a small tax increase from $\tau_1 = \tilde{S} - \varepsilon$ to $\tau_2 = \tilde{S} + \varepsilon'$ whereas aggregate output varies continuously.

On the other hand, varying the tax within $(0, \tilde{S})$ or for $\tau > \tilde{S}$ leads to continuous changes of all involved variables. The comparative statics can be read from Proposition 1, i.e. $Q(\tau)$ decreases, $S_1(E(\tau), s)$ and $MRT(\tau)$ increase as τ increases. Under Assumption 3.iv, also $n_i(\tau)$ and $e_i(\tau)$ decrease (for $i = 0, I$) whereas each single firm serves a higher market share, i.e. $q_i(\tau)$ increases.

5.2 Free Entry Market Equilibrium under Permits

Assume now that any number of permits L is given, and define

$$\overline{L} \equiv \frac{\tilde{Q}}{\tilde{q}_0}\tilde{e}_0 \qquad \text{and} \qquad \underline{L} \equiv \frac{\tilde{Q}}{\tilde{q}_I}\tilde{e}_I \, .$$

Note that $\overline{L}$ and $\underline{L}$ are equal to the socially optimal emission levels if $s = \underline{s}$ and $s = \overline{s}$, respectively. But it is possible to define those terms without referring to the damage function. Denote by $q_i(L)$, $e_i(L)$ the quantities and emission levels of firm $i = 0, I$ under a permit regime with L permits. Then we can state the following result:

Proposition 2 *For each number of permits L being issued, there is a unique competitive free entry equilibrium characterized by the following properties.*

i) If $L \geq \overline{L}$, only the conventional firms are in the market, and the market price for permits $\sigma(L)$ does not exceed $\tilde{S}$.

ii) For all $L \in (\underline{L}, \overline{L})$, both types of firms are active, the market price is $\sigma(L) = \tilde{S}$, and the there is a unique allocation of permits between the conventional and innovative firms with

$$n_0 \;=\; (L - \underline{L})\frac{\tilde{q}_I}{\tilde{q}_I\tilde{e}_0 - \tilde{q}_0\tilde{e}_I} \tag{5.7}$$

conventional firms, holding $\tilde{e}_0$ many permits each, and

$$n_I \;=\; (\overline{L} - L)\frac{\tilde{q}_0}{\tilde{q}_I\tilde{e}_0 - \tilde{q}_0\tilde{e}_I} \tag{5.8}$$

innovative firms, holding $\tilde{e}_I$ many permits each.

Moreover, $q_i(L) = \tilde{q}_i$ for $i = 0, I$.

iii) If $L \leq \overline{L}$, only the innovative firms are in the market, and the market price $\sigma(L)$ does not fall short of $\tilde{S}$, or there is no market any more, i.e. if the number of permits is so small that the firms' marginal cost exceeds the choke-off price.

Proof: See the appendix.

So in contrast to the tax regime we get uniqueness of equilibrium under permits. Since by (5.7) and (5.8) n_0 and n_I vary continuously in L even if $L \in (\underline{L}, \overline{L})$ and both types of firms are active, we do not get jumps when the number of permits is reduced or increased. Note that for $L \in (\underline{L}, \overline{L})$ the market price for permits equals $\tilde{S}$, equal to the tax where both types of firms produce. This should not be surprising after all since for σ less than $\tilde{S}$ the innovators could not compete, and for permit prices exceeding $\tilde{S}$ the conventional firms would make losses. Thus the market price alone cannot enforce the unique equilibrium. Market clearing on the market for permits, however, leads to a unique equilibrium. That is, the fixed supply of permits can match demand for permits only for one unique allocation, leading to a unique equilibrium.

6 Free Entry Equilibrium after Innovation under Optimal Regulation before Innovation

So far we have investigated what happens in the market under any tax or any number of permits if a new technology is available. We want to investigate now the impact of those policies when the conventional industry is regulated *optimally*, and the new technology is suddenly available. So the regulator is assumed to neither be able to forcast the new technology, nor to be able to adjust his policy immediatly. In particular we are interested whether taxes or permits are likely to induce excess innovation, or possibly under–innovation. MILLIMAN and PRINCE [1989] have been heavily criticized by MARIN [1989] for assuming optimal regulation. MARIN argues that this is not realistic, and that the levels of taxes or permits were set rather arbitrary. This, of course, is not true, either. Even if policy tools are not set optimally in general, they can often be considered as resulting from a compromise between environmental departments, which prefer high emission taxes, and industrial departments which prefer no or low emission taxes. Certainly lobbying, sometimes even bribing, plays an important role. If we want to investigate, however, whether possibly under– or over– innovation happens to occur, we need a criterion to measure this. Hence, as a benchmark, it is reasonable to look what happens if the conventional industry is regulated optimally.

6.1 Equilibrium under the Original Tax

Before innovation, the optimal emission tax is given by

$$\tau_0(s) = S_1(E_0^*(s), s) \ . \tag{6.1}$$

Now we obtain the following striking result:

Proposition 3 *If the conventional industry is optimally regulated by a Pigouvian tax and the new technology is available to any firm, then*

i) if $s \leq \underline{s}$, no firm with the new technology enters the market,

ii) if $s > \underline{s}$, the conventional industry will be completely eliminated, and only firms with the new technology are in the market.

Proof: Proposition 1 and Theorem 1 imply that under the originally optimal policy. we have $\tau_0(s) < \tilde{S}$ if $s < \underline{s}$, $\tau_0(s) = \tilde{S}$ if $s = \underline{s}$, and $\tau_0(s) > \tilde{S}$ if $s > \underline{s}$. By virtue of Proposition 2 the result follows immediately for $s < \underline{s}$ and $s > \underline{s}$. For $s = \underline{s}$ we employ the fact that the conventional firms are already incumbent, and $(n_0, n_I) = (\tilde{Q}/\tilde{q}_0, 0)$ together with the corresponding output and emission levels is a free entry equilibrium. leaving no place for innovators. Q.E.D.

So the result predicts that after technological change a Pigouvian tax does not allow conventional and new technologies to "live together", once the conventional firms are incumbent. For $s < \underline{s}$, i.e. $\tau_0(s) < \tilde{S}$, the tax is too low as to give the new, less polluting technology a chance. For $s > \underline{s}$, i.e. $\tau_0(s) > \tilde{S}$, the innovators can fully exploit their cost advantage and drive out the conventional firms completely. For $s = \underline{s}$ the tax is $\tau_0(s) = \tilde{S}$, allowing both types of firms to stay in the market. Yet, since the conventional firms are incumbent, there is no way for the innovators to drive them out without making losses. Hence, in the long run, we will observe only one type of firm in the market, only conventional firms or only innovators.

Now for $s < \underline{s}$, we get $\tau_0(s) < \tilde{S}$, and it is even optimal that potential innovators stay out. In that case the new technology can be considered as too costly, or the social damage function as too flat for justifying introduction of the new technology. For $s \in (\underline{s}, \overline{s})$, on the other hand, the original tax always induces complete innovation although only partial innovation is optimal as we know. As a consequence decentralized innovation under taxes may even result in a decrease in welfare compared to the situation before the new technology has been available:

Corollary 1 *For $s > \underline{s}$, but sufficiently close to $\underline{s}$, innovation under the original optimal tax leads to a <u>decrease</u> in welfare.*

Proof: The social optimum requires the number of innovators to be small for s close to $\underline{s}$. But since $\tau_0(s) > \tilde{S}$ for $s > \underline{s}$, we get complete innovation by Prop. 2. Q.E.D.

The intuition is as follows: Since output only changes by little if the tax only slightly exceeds $\tilde{S}$ and since the environment becomes much cleaner through innovation thus rising welfare, other things being equal, the net loss in welfare must result from a dissipation of resources (costs) for the sake of too clean an environment. I.e. too much money will be spent on new abatement equipment.

For $s \geq \overline{s}$, on the other hand, complete innovation is optimal. Since the original tax is higher than socially optimal marginal damage, however, the original tax induces the wrong number, i.e. too little a number of firms to be in the market. This can be considered as under–innovation.

Finally, if the damage parameter is sufficiently high, the optimal emission level after innovation may exceed the optimal emission level before innovation (see Figure 4), and thus optimal marginal damage after innovation may exceed optimal marginal damage before innovation. Hence also the number of firms induced by the original tax policy *exceeds* the optimal number of firms after innovation. By continuity of $\tau_0(s)$ and the socially optimal marginal damage in s, we immediately obtain the following result:

Theorem 3 *Suppose the originally optimal tax $\tau_0(s)$ is still valid and the new technology is available. Then there is an interval (s_a, s_b), with $s_a \in (\underline{s}, \overline{s})$ and $s_b > \overline{s}$, such that:*

i) there is excess innovation for all $s \in (\underline{s}, s_a)$, and possibly for $s > s_b$ if there is still a market for those s.

ii) there is too little innovation, i.e. too little a number of innovative firms for all $s \in (s_a, s_b)$.

iii) For $s = s_a$, $\tau_0(s)$ induces the optimal number of innovators but too little a number (zero) of conventional firms. For $s = s_b$, $\tau_0(s)$ induces the optimal number of innovators.

The result is illustrated in Figure 4.

6.2 Equilibrium under the Original Permit Policy

Let us turn to the permit regime now. The optimal number of permits issued before innovation is given by $L_0(s) = E_0^*(s)$.

Proposition 4 *If the conventional industry is optimally regulated by issuing $L_0(s)$ permits and the new technology is available to any firm, then*

i) for all $s \leq \underline{s}$, no firm with the new technology enters the market (and no incumbent switches technology).

ii) There is $\tilde{s} > \overline{s}$ such that for all $s \in (\underline{s}, \tilde{s})$ there is a unique equilibrium such that both types of firms are active and the price for permits is equal to $\widetilde{S}$. For $L = L_0(s)$ the number of conventional firms, holding $\tilde{e}_0$ many permits each, is given by (5.7), and the number of innovators, holding $\tilde{e}_I$ many permits each, is given by (5.8).

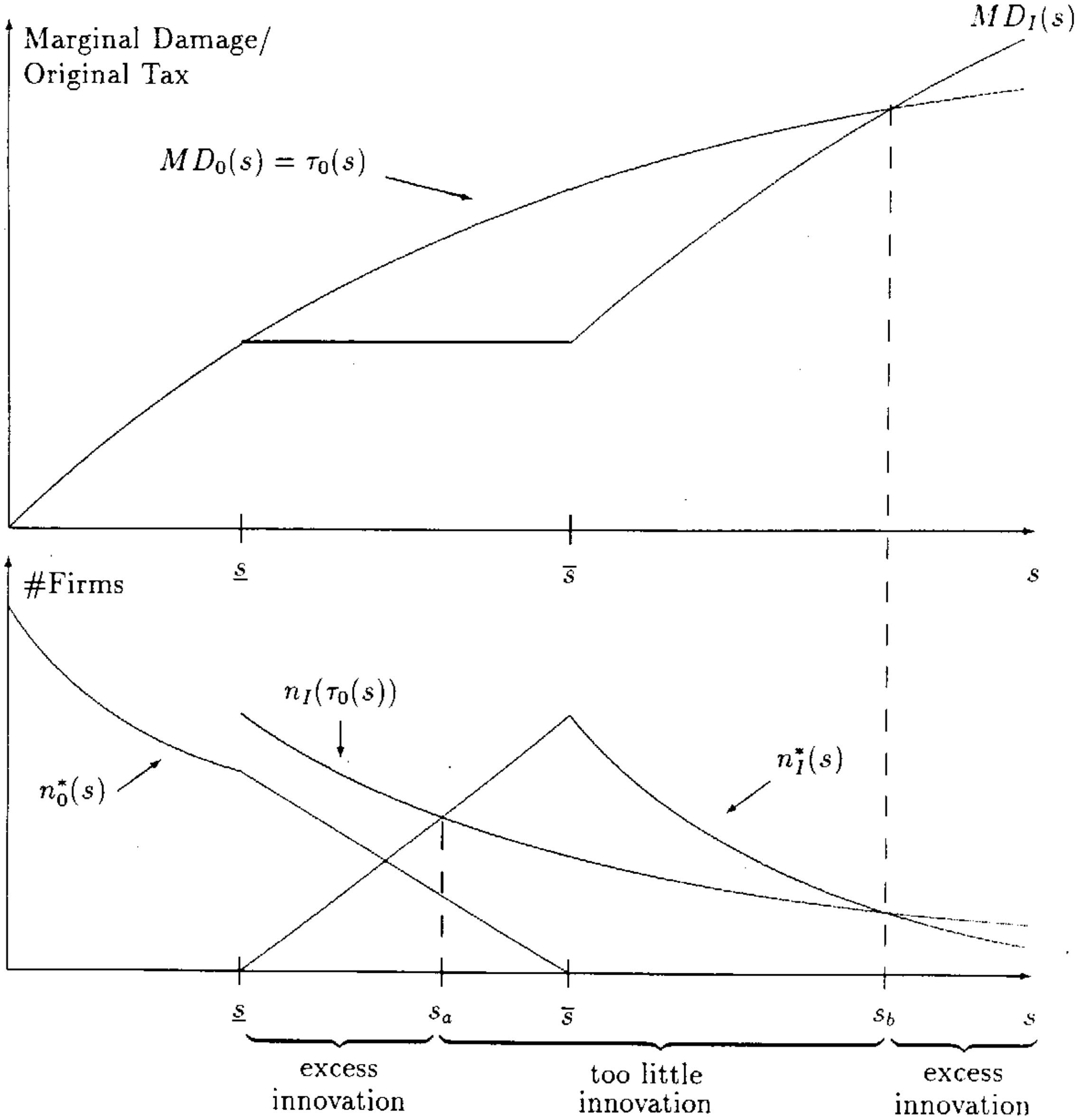

Figure 4: *The upper diagram depicts socially optimal marginal damage after innovation, denoted by MD_I, and the originally optimal tax $\tau_0(s)$ which is equal to optimal marginal damage before innovation, denoted by MD_0. The lower diagram depicts the socially optimal number of firms of type 0, denoted by $n_0^*(s)$, and of type I, denoted by $n_I^*(s)$, and the number of innovators $n_I(\tau_0(s))$ under the original tax $\tau_0(s)$. Note that at point $\underline{s}$ the number $n_I(\tau_0(\underline{s}))$ can also be less than $n_0^*(\underline{s})$.*

iii) For $s \geq \tilde{s}$, the conventional industry will be completely eliminated, and only firms with the new technology are in the market.

Proof: By Theorem 1, $L_0(s)$ exceeds $E_I^*(s)$ for $s \in (\underline{s}, \overline{s})$ and is strictly decreasing. Define $\tilde{s}$ by $L_0(\tilde{s}) = \underline{L}$. Since $L_0(\overline{s}) > E_I^*(\overline{s}) = \underline{L}$, we get $\tilde{s} > \overline{s}$. Thus $L_0(s) \in (\underline{L}, \overline{L})$ for all $s \in (\underline{s}, \tilde{s})$. By Proposition 2 both firms are active for $L \in (\underline{L}, \overline{L})$, leading to numbers of firms given by (5.7) and (5.8). Everything else follows from Proposition 2. Q.E.D.

Since for the whole interval $(\underline{s}, \tilde{s})$ the original number of permits $L_0(s)$ is greater than the optimal number of permits, which equals $E_I^*(s)$ (see Figure 5), we conclude:

Corollary 2 *A permit policy which has been set optimally with respect to an incumbent industry induces <u>too many conventional firms</u> to stay in the market whenever partial innovation is optimal but also a range for parameters $(\overline{s}, \tilde{s})$ for which complete innovation is optimal.*

Corollary 2 implies that too little a number of firms adopts the new technology for s greater but sufficiently close to $\underline{s}$. On the other hand, for s close to $\tilde{s}$ the original number of permits $L_0(s)$ *exceeds* the optimal pollution level after innovation, $E_I^*(s)$. This follows from Theorem 1.iv which tells us that the optimal emission level and thus the number of permits should be reduced after innovation. Hence the price for permits falls short of the optimal marginal damage and too many firms adopt the new technology if s is close to $\tilde{s}$. Finally, if there is a region where $E_I^*(s) > E_0^*(s)$ and the market does not break down, we again have too little innovation for sufficiently large values of s. This gives us a result upside down to Theorem 3:

Theorem 4 *There is an interval (s_c, s_d), with $s_c \in (\overline{s}, \tilde{s})$ and $s_d > \tilde{s}$, such that*

i) there is <u>too little innovation</u> for all $s \in (\underline{s}, s_c)$ and possibly for all $s > s_d$ if there is still a market for those s.

ii) There is <u>excess innovation</u>, i.e. too large a number of innovating firms, for all $s \in (s_c, s_d)$.

iii) For $s = s_c$, $L_0(s)$ induces the optimal number of innovators but too large a number of conventional firms. Only for $s = s_d$, $L_0(s)$ induces the optimal number of innovators and conventional firms ($= 0$).

The result is illustrated in Figure 5. Although also permits – like taxes – may lead to excess innovation as well as to under–innovation, permits have a crucial advantage over taxes:

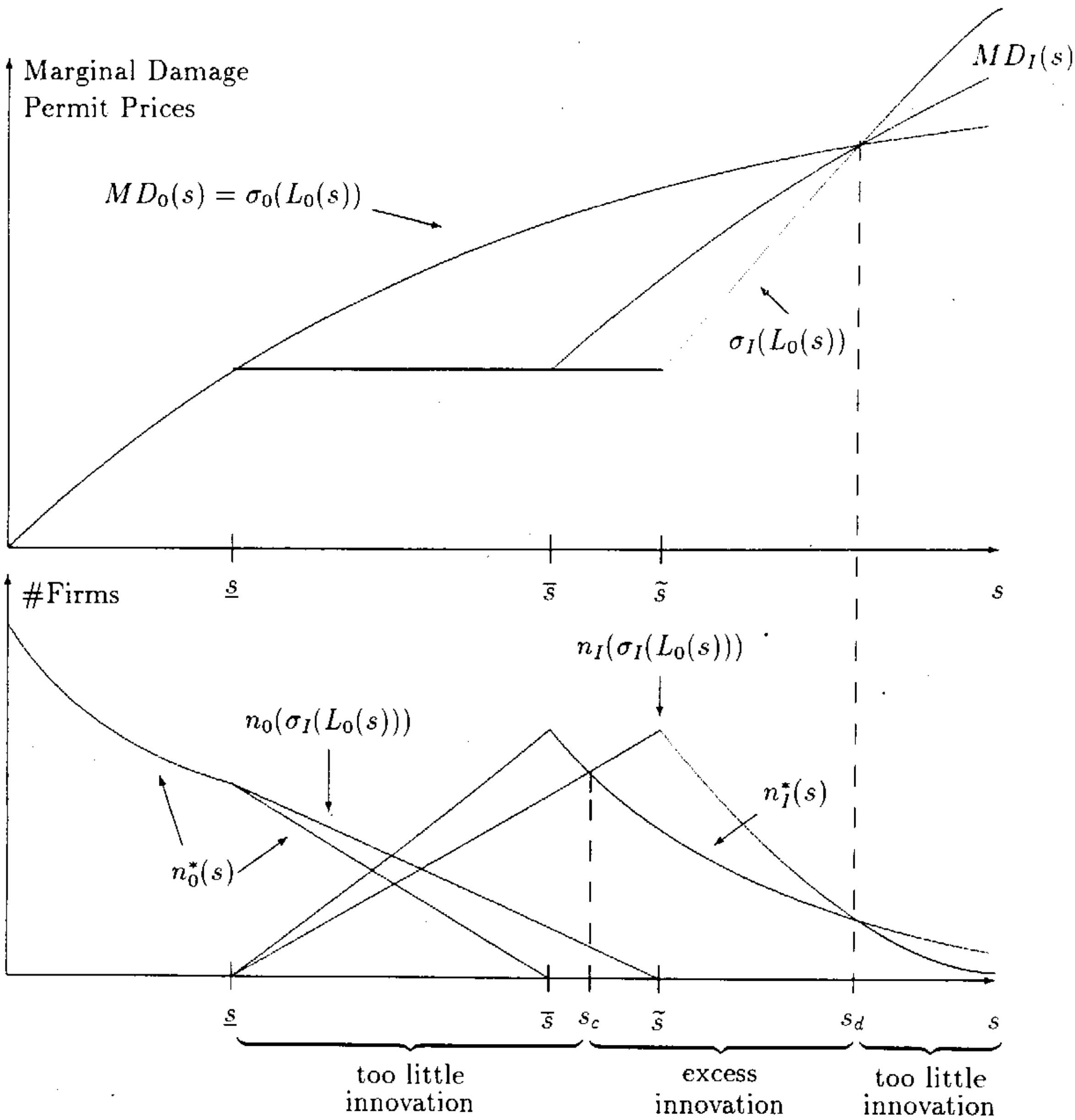

Figure 5: *The upper diagram depicts the socially optimal marginal damage after innovation, denoted by MD_I, the originally permit price before innovation, denoted by $\sigma_0(L_0(s))$ (which is equal to optimal marginal damage before innovation), and the new permit price $\sigma_I(L_0(s))$ after innovation but under the original quota of permits $L_0(s)$. The lower diagram depicts the socially optimal number of firms of type 0, denoted by $n_0^*(s)$, the optimal number of firms of type I, denoted by $n_I^*(s)$, and the number of conventional firms $n_0^*(\sigma_I(L_0(s)))$ and innovators $n_I(\sigma_I(L_0(s)))$, respectively, under the original permit policy $L_0(s)$.*

Corollary 3 *Under permits innovation does never lead to a decrease in welfare.*

Proof: Total emissions are the same before and after innovation. But since the new technology is also employed for $L_0(s) < \overline{L}$, emissions as a scarce resource are used more efficiently, leading to lower abatement costs as before and to more output $\tilde{Q} > Q_0(s)$. Q.E.D.

So the permit regime differs from the emission tax regime in two main aspects. First, the permit regime allows for two different types of firms staying in the market. Second, the introduction of the new technology does not lead to a decrease in welfare.

7 Optimal Adjustment after Innovation

Clearly the original levels of our policy instruments are in general not optimal any longer as soon as a new technology is available. This gives rise to policy adjustment, sometimes called *ratcheting* (c.f. MILLIMAN & PRONCE [1989]). Fortunately, if neither partial nor complete innovation is optimal, i.e. for $s \leq \underline{s}$, there is no reason to do anything since the original levels do not induce technological change. So we can concentrate on the case $s > \underline{s}$. Recall first that by Proposition 1.iv total pollution and marginal damage go down through innovation if s is not too large, that is,

$$E_0(s) > E_I(s) , \tag{7.1}$$

$$S_1(E_0(s), s) > S_1(E_I(s), s) . \tag{7.2}$$

For extremely large damage parameters the pattern may be reversed.

Since the optimal tax is equal to marginal damage, and the optimal number of permits to be issued is equal to the optimal level of total pollution, ratcheting obviously requires to cut taxes, or to reduce the number of permits, respectively, if (7.1) holds. At least for permits we immediately get the following result:

Proposition 5 *After innovation, the optimally adjusted number of permits given by $L_I(s) = E_I(s)$, implements the social optimum.*

There is $\overline{\overline{s}} > \overline{s}$ such that for all $s \in (\underline{s}, \overline{\overline{s}})$ the number of permits has to be reduced, inducing an increase of the price for permits.

For s sufficiently large such that (7.1) is reversed, the number of permits has to be increased, inducing the permit price to fall.

For taxes, unfortunately, but not surprising after all, matters do not look that lucky. For $s \geq \overline{s}$ we are fine since social optimum requires complete innovation, and hence it can obviously be implemented by optimal tax adjustment. For $s \in (\underline{s}, \overline{s})$, on the other hand, adjustment of taxes cannot implement the social optimum, in general, since optimal marginal damage would be equal to $\tilde{S}$ for $s \in (\underline{s}, \overline{s})$. From section 6.1 we know, that there is a multiplicity of free entry equilibria for $\tau = \tilde{S}$. In particular there is one with only conventional firms but also one with only incumbent firms in the market. If the government reduces the tax to $\tau = \tilde{S}$, it is not quite clear what will happen. Since the tax is lower than before, there is room for more firms again. It could happen that this gap may be filled up by owners of the conventional technologies who after having been crowded out by the innovators, get a new chance to revive their dinosaur technologies. But it also may happen that if the reaction lag between innovation and policy adjustment is sufficiently large, the dinosaurs have died out, i.e. capital as been depreciated. In such a case an entrepreneur who reenters the market might immediately start with the new technology.

Hence for s smaller but close to $\overline{s}$ tax adjustment should lead to $\tau = \tilde{S} + \varepsilon$ in order to keep out the dinosaur technology of type 0. For s close to $\underline{s}$ one might suggest to argue similarly and require the tax to be set equal to $\tau = \tilde{S} - \varepsilon$. However, once the new technologies have been installed, certain fixed costs are sunk, and switching back to the conventional technologies could incur new costs to the firms. Hence, it is not likely that trying to get back exclusively to dinosaur technologies does improve welfare. Hence we can only state the following result:

Proposition 6 *i) If $s \in (\underline{s}, \overline{s})$, tax adjustment requires to set the tax equal to $\tilde{S}$ (or $\tilde{S} + \varepsilon$). However, the tax cannot implement the social optimum, in general.*

ii) Tax adjustment can implement the social optimum whenever complete innovation is socially optimal, i.e. for $s > \overline{s}$.

iii) There is an interval $(\overline{s}, \overline{\overline{s}})$, for which tax adjustment results in a tax cut.

iv) For $s > \overline{\overline{s}}$ optimal tax adjustment results in a tax increase.

8 Concluding Remarks

We have demonstrated that in general both types of policies, taxes and permits, may lead to excessive innovative activity but can also induce too little innovation. Taxes cause too much innovation for parameters where partial innovation is optimal but also too little innovation for parameters where complete innovation is optimal. For permits the pattern is reversed. They induce too many conventional firms to stay in the market whenever partial innovation is optimal. However, they lead to excess entry of innovators for parameters where complete innovation is optimal.

Yet, permits have two crucial advantages over taxes. They do never lead to a decrease in welfare whereas taxes sometimes do. Moreover, by adjustment (ratcheting) of the permit policy one can always implement first best ex post, once the new technology is available. In other words, excess innovation under permits — if it occurs — can be considered as less severe than excess innovation under taxes.

However, even the permit system could be improved by reviving a mixed system of permits, taxes, and subsidies, as has been proposed by ROBERTS and SPENCE, or by a system of different permits being proposed by COLLINGE and OATES [1982], or HENRY [1989] (chapter 2). Unfortunately, very little attention as been paid to those systems. The joint idea of all these systems is to make the inelastic supply of permits more elastic by approximating the marginal social damage function by different types of permits. To examine such a system under free entry is beyond the scope of this paper and is left to further research. My conjecture however is that such a system may always lead to first best (or almost first best if the social damage function is approximated by a step function), even if the regulator does not have any prior information about future technologies.

9 Appendix

Proof of Proposition 1: Let $q(s)$, $e(s)$, and $n(s)$ be the socially optimal solution as a function of s (we omit the $*$'s). Differentiating (3.1) - (3.3) w.r. to s and solving for q', e', and n' yields

$$n' = \frac{([C_{12}]^2 - C_{11}C_{22})e + nP'(C_{12}q + C_{22}e)}{D} \tag{9.1}$$

$$q' = -\frac{P'S_{12}q(C_{12}q + C_{22}e)}{D} \tag{9.2}$$

$$e' = \frac{P'S_{12}q(C_{11}q + C_{12}e)}{D} \tag{9.3}$$

where $D = ([C_{12}]^2 - C_{11}C_{22})(P'q^2 - S_{11}e^2) - nP'S_{11}(C_{11}q^2 + 2C_{12}qe + C_{22}e^2) > 0$. Since $Q' = nq' + n'q$, and $E' = ne' + n'e$, we can employ (9.1) - (9.3), and rearrange to obtain:

$$Q' = \frac{[C_{12}]^2 - C_{11}C_{22})S_{12}qe}{D} < 0 \tag{9.4}$$

$$E' = \frac{S_{12}\{([C_{12}]^2 - C_{11}C_{22})e^2 + nP'(C_{11}q^2 + 2C_{12}qe + C_{22}e^2)\}}{D} < 0 . \tag{9.5}$$

Next

$$\frac{d}{ds}S_1(E(s), s) = S_{11}E'(s) + S_{12} = \frac{S_{12}}{D}([C_{12}]^2 - C_{11}C_{22})P'q^2 > 0 . \tag{9.6}$$

Finally,

$$MRS'(s) \equiv \frac{d}{ds}\left(\frac{S_1(E(s), s)}{P(Q(s))}\right) = \frac{P'}{D}S_{12}q([C_{12}]^2 - C_{11}C_{22})(Pq - S_1e) > 0 . \tag{9.7}$$

The last factor is clearly positive by (3.3). One can show that the second order conditions are also satisfies under Assumptions 1 - 3. Q.E.D.

Proof of Theorem 1:

The Lagrange function of the maximization problem is

$$L(\ldots) = W(q_1, q_2, e_1, e_2, n_1, n_2, s) + \lambda_1 q_1 + \lambda_2 q_2 + \mu_1 e_1 + \mu_2 e_2 + \nu_1 n_1 + \nu_2 n_2$$

where λ_1, λ_2, μ_1, μ_2, ν_1, ν_2 are the Kuhn Tucker multipliers of the non-negativity constraints. The first order conditions are

$$\begin{aligned}
P(Q) - C_1^1(q_1, e_1) + \lambda_1 &= 0 & (9.8)\\
P(Q) - C_1^2(q_2, e_2) + \lambda_2 &= 0 & (9.9)\\
-S_1(E, s) - C_2^1(q_1, e_1) + \mu_1 &= 0 & (9.10)\\
-S_1(E, s) - C_2^2(q_2, e_2) + \mu_2 &= 0 & (9.11)\\
P(Q)q_1 - C^1(q_1, e_1) - S_1(E, s)e_1 + \nu_1 &= 0 & (9.12)\\
P(Q)q_2 - C^2(q_2, e_2) - S_1(E, s)e_2 + \nu_2 &= 0 & (9.13)
\end{aligned}$$

Suppose first that there is an interior solution, i.e. all the multipliers are zero. Differentiating (9.12) and (9.13) w.r. to s and employing (9.8) - (9.11) yields:

$$\begin{aligned}
P'(Q)Q'(s)q_1(s) - [S_{11}(E(s), s)E'(s) + S_{12}(E(s), s)]e_1(s) &= 0 & (9.14)\\
P'(Q)Q'(s)q_2(s) - [S_{11}(E(s), s)E'(s) + S_{12}(E(s), s)]e_2(s) &= 0 & (9.15)
\end{aligned}$$

Now suppose $Q'(s) \neq 0$. This implies by (9.12) and (9.13) that $[S_{11}(E(s), s)E'(s) + S_{12}(E(s), s)] \neq 0$ and hence by (9.12) - (9.15) we get $\frac{q_1}{q_2} = \frac{e_1}{e_2} = \frac{C^1(q_1, e_1)}{C^2(q_2, e_2)}$, or

$$\left(\frac{q_1(s)}{C^1(q_1(s), e_1(s))}, \frac{e_1(s)}{C^1(q_1(s), e_1(s))}\right) = \lambda \left(\frac{q_2(s)}{C^2(q_2(s), e_2(s))}, \frac{e_2(s)}{C^2(q_2(s), e_2(s))}\right)$$

for some $\lambda > 0$. But this contradicts Assumption 5, since if the two Y_{MAC}–curves intersect, the gradients cannot have the same direction. Hence, $Q(s) = \tilde{Q}$, and $Q'(s) = 0$. But then (9.14) or (9.15) imply

$$\frac{d}{ds} S_1(E(s), s) = S_{11}(E(s), s)E'(s) + S_{12}(E(s), s) = 0\ . \qquad (9.16)$$

Thus: $E' = -S_{12}/S_{11} < 0$. Differentiating (9.8) - (9.11) w.r. to s, and using (9.16) we get a homogeneous linear system of equations in $q_1'(s)$, $q_2'(s)$, $e_1'(s)$, and $e_2'(s)$. Hence $q_1'(s) = q_2'(s) = e_1'(s) = e_2'(s) = 0$. Next we have $\tilde{Q} = n_1(s)\tilde{q}_1 + n_2(s)\tilde{q}_2$ and $E(s) = n_1(s)\tilde{e}_1 + n_2(s)\tilde{e}_2$ giving $E'(s) = n_1'(s)[\tilde{e}_1 - \tilde{e}_2 \cdot (\tilde{q}_1/\tilde{q}_2)]$. This implies that $n_1(s)$ is decreasing and $n_2(s)$ is increasing, or vice versa. We will exclude below that $n_1(s)$ is increasing.

Now let s be close to zero. Then $S_1(E,s)$ is close to zero. Since $C^1(\tilde{q}_1,\tilde{e}_1) > 0$ and $C^2(\tilde{q}_2,\tilde{e}_2) > 0$, (9.10) – (9.11) cannot be satisfied for $\mu_1 = \mu_2 = 0$, and s sufficiently close to zero. On the other hand, for s close to zero we have

$$P(Q(s)) = C_1^1(q_1(s), e_1(s)) \approx \frac{C^1(q_1(s), e_1(s))}{q_1(s)} \approx \frac{\tilde{C}^1(\overline{q}_1)}{\overline{q}_1}$$

or

$$P(Q(s)) = C_1^2(q_2(s), e_2(s)) \approx \frac{C^2(q_2(s), e_2(s))}{q_2(s)} \approx \frac{\tilde{C}^2(\overline{q}_2)}{\overline{q}_2}.$$

Since we have

$$\frac{\overline{q}_1}{\tilde{C}^1(\overline{q}_1)} > \frac{\overline{q}_2}{\tilde{C}^2(\overline{q}_2)}$$

by Assumption 5 we conclude $\nu_2 > 0$, $\nu_1 = 0$, and hence $n_1 > 0$ and $n_2 = 0$.

Next observe that since $Q_2(s) = \tilde{Q}$, $q_1(s) = \tilde{q}_1$, and $q_2(s) = \tilde{q}_2$ are constant for those s for which there is an interior solution, $n_1(s)$ and $n_2(s)$ must be bounded. Since also $e_1(s) = \tilde{e}_1$, and $e_2(s) = \tilde{e}_2$ are constant and (by Assumption 4) greater than zero, $E(s)$ must be greater than 0 for those s. But this implies that for s sufficiently high $S_1(E(s), s)$ must exceed $-C_2^1(\tilde{q}_1,\tilde{e}_1)$, and $-C_2^2(\tilde{q}_2,\tilde{e}_2)$. Thus (9.10) – (9.11) cannot be satisfied for $\mu_1 = \mu_2 = 0$ and s sufficiently high. This means that only one type of firm can be active, hence $\nu_1 > 0$ or $\nu_2 > 0$. Since by Proposition 1, $MRS(s) \equiv S_1(E(s), s)/P(Q(s))$ is increasing, and since it becomes greater than $\tilde{R} = \tilde{S}/\tilde{P} = -C_2^i(\tilde{q}_i,\tilde{e}_i)/C_1^i(\tilde{q}_i,\tilde{e}_i)$ for $i = 1, 2$, the inequalities (2.13) must hold. Now assume there is a solution with $P(Q) = C_1^2(q_2, e_2)$ and $S_1(E,s) = -C_2^2(q_2, e_2)$. Then (9.12) and (9.13) become

$$C_1^2(q_2, e_2)q_1 - C_2^2(q_2, e_2)e_1 - C^1(q_1, e_1) + \nu_1 = 0 \tag{9.17}$$

$$C_1^2(q_2, e_2)q_2 - C_2^2(q_2, e_2)e_2 - C^2(q_2, e_2) + \nu_2 = 0 \tag{9.18}$$

Implying $\nu_2 = 0$, and $\nu_1 > 0$ by the first inequality of (2.13). Assuming $P(Q) = C_1^1(q_1, e_1)$ and $S_1(E,s) = -C_2^1(q_1, e_1)$ would lead to $\nu_2 < 0$ by the second part of (2.13), which is impossible.

One can show that Assumptions 1, 2, and 3 imply that the objective function of the social planner is strictly concave, thus the solution is unique.

Since all involved functions are continuous, the solution must be continuous. Hence there must exist parameters $\underline{s}$ and $\overline{s}$ such that $n_1(s) > 0$ and $n_2(s) = 0$ for $s < \underline{s}$ and $n_1(s) = 0$ and $n_2(s) > 0$ for $s > \overline{s}$. Q.E.D.

Proof of Proposition 2:

We proceed indirectly again. Since aggregate factor demand for permits is strictly decreasing (see footnote 10) in the price for permits if only one type of firms is around, and the output market clears. This in turn implies that the price for permits is unique and rises if the supply of permits goes down. Hence, if only type 1 firms are around, and $L = \overline{L}$, then $\sigma(\overline{L}) = \tilde{S}$, and if only type 2 firms are around, and $L = \underline{L}$, then also $\sigma(\underline{L}) = \tilde{S}$.

Now suppose $\sigma > \tilde{S}$. Then by the same arguments as for the emission tax (in section 5), only type 2 firms can be active, and $L < \underline{L}$. Similarly, suppose $\sigma < \tilde{S}$. Then only type 1 firms can be active, and $L > \overline{L}$. Now suppose $L \in (\underline{L}, \overline{L})$. Then $\sigma(L) = \tilde{S}$, otherwise L would be smaller than $\underline{L}$ or greater than $\overline{L}$. But if $\sigma(L) = \tilde{S}$, both types of firms can be active with $q_i(L) = \tilde{q}_i$, and $\epsilon_i(L) = \tilde{e}_i$ for $i = 1, 2$. Contrary to the tax solution, however, there are unique numbers of firms $n_1(L)$, and $n_2(L)$, determined by the linearly independent system:

$$\begin{aligned} \tilde{Q} &= n_1(L)\tilde{q}_1 + n_2(L)\tilde{q}_2 \\ L &= n_1(L)\tilde{e}_1 + n_2(L)\tilde{e}_2 \end{aligned}$$

Solving for $n_1(L)$, and $n_2(L)$ yields (5.7) and (5.8). Q.E.D.

References

ADAR, Z. AND J.M. GRIFFIN (1976): "Uncertainty and the Choice of Pollution Control Instruments," *Journal of Environmental Economics and Management*, 3, 178–188.

BAUMOL, W.J, P.C. PANZAR AND R.D. WILLIG (1982): *Contestable Markets and the Theory of Industrial Structure*, Harcourt Brace Javanovich, Inc.

BAUMOL, W.J. AND W.E. OATES (1988): *The Theory of Environmental Policy*, Cambridge, MA: Cambridge University Press.

COLLINGE, R.A. AND W.E. OATES (1982): "Efficiency in Pollution Control in the Short and the Long Run: A System of Rental Emissions Permits," *Canadian Journal of Economics*, 15, 346–54.

DOWNING, P.B. AND L.J. WHITE (1986): "Innovation in Pollution Control," *Journal of Environmental Economics and Management*, 13, 18–29.

FISHELSON, G. (1976) "Emission Control Policies under Uncertainty," *Journal of Environmental Economics and Management*, 3, 189–197.

HENRY, C. (1989): "Microeconomics for Public Policy - Helping the Invisible Hand," *Clarendon Press*, Oxford

KNEESE, A.V., AND C.L. SCHULTZE [1975]: *Pollution, Prices and Public Policy*, Brookings, Washington, D.C.

LAFFONT, J.J. AND J. TIROLE (1990): "Optimal Bypass and Creamskimming," *American Economic Review*, 80, 1042–1061.

LAFFONT, J.J. AND J. TIROLE (1994): "Pollution Permits and Compliance Strategies," Invited conference paper presented at the 50th congress of the IIPF, held at Harvard, August'94

LAFFONT, J.J. and J. TIROLE (1994): "A note on Environmental Innovation," mimeo.

MAGAT, W.A. (1978): "Pollution Control and Technological Advance: A Dynamic Model of the Firm," *Journal of Environmental Economics and Management*, 5, 1–25.

MALUEG, D.A. (1989): "Emission Credit Trading and the Incentive to Adopt New Pollution Abatement Technology," *Journal of Environmental Economics and Management*, 18, 52–57.

MARIN, A. (1989): "Comment: Firms Incentives to Promote Technological Change in Pollution Control, comment" *Journal of Environmental Economics and Management*, 21, 297–300.

MILLIMAN, S.R. AND R. PRINCE (1989): "Firms Incentives to Promote Technological Change in Pollution Control," *Journal of Environmental Economics and Management*, 17, 247–265.

ORR, L. [1976]: "Incentives for Innovation as the Basis for Effluent Charge Strategy," *American Economic Review, Papers and Proceedings*, 66, 441–447.

ROBERTS, M.J. and SPENCE, M. (1976): "Effluent Charges and Licences under Uncertainty", *Journal of Public Economics*, 19, 193–208.

SPULBER, D.F. (1985): "Effluent Regulation and Long–Run Optimality," *Journal of Environmental Economics and Management*, 12, 103 – 116.

WEITZMAN, M.L. (1974): "Prices vs. Quantities," *Review of Economic Studies*, 41, 477 – 491.