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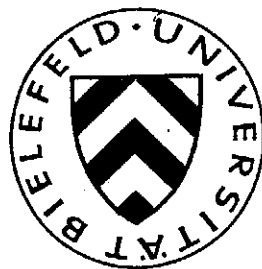
WORKING PAPERS

No. 234

**Green Taxes in Oligopoly Revisited:
Exogenous versus Endogenous Number of Firms**

by

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September 1994

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Green Taxes in Oligopoly Revisited: Exogenous versus Endogenous Number of Firms [*]

Abstract

The paper considers Pigouvian taxes in Cournot oligopoly models with a fixed number as well as with an endogenous number of firms and generalizes existing results from the literature. Whereas with a fixed number of firms the tax always falls short of marginal damage if firms are symmetric, nothing can be said in general if the number of firms is endogenous. For the special case where pollution is determined completely by output, i.e., if there is no further abatement technology, we show that the optimal Pigouvian tax exceeds (falls short of, equals) marginal damage, as if demand is concave (convex, linear).

Keywords: Emission taxes, Cournot oligopoly

1 Introduction

Internalizing externalities by taxes has attracted much attention since PIGOU's [1938] pioneering work. Under perfect competition the optimal tax is equal to marginal damage as is well known. This rule, referred to as the Pigouvian tax rule, has been criticized first by BUCHANAN [1969] who investigates Pigouvian taxes for monopoly and shows that a tax equal to marginal damage fails to be efficient if the market structure is monopolistic. BUCHANAN, however, (ab)uses this result to ride an attack against taxation of pollution in general whenever polluting firms compete imperfectly on the output market.

Since BUCHANAN, other authors have alluded to taxation under imperfect competition. SIEBERT [1975], LEE [1976], BARNETT [1980], and REQUATE [1993a] investigate monopolies. LEVIN [1985] is the first to provide an analysis on emission taxes for Cournot oligopoly. Whereas LEVIN does not focus on *optimal* taxation, EBERT [1992] gives a tax rule for symmetric oligopoly without abatement technology. REQUATE [1993b] investigates asymmetric Cournot duopoly without abatement technologies. All these models show that under imperfect competition the optimal emission tax falls short of marginal damage if firms are not too different. The reason is that the monopolistic as well as the oligopolistic firms create two external diseconomies. Besides polluting they hold down output and thus voluntarily reduce pollution compared to perfect competition. Just recently KATSOULACOS and XEPAPADEAS [1993] considered Pigouvian taxes in an oligopoly where the number of firms is endogenously determined by zero profits. Assuming linear demand and an additively separable cost function they find that the optimal Pigouvian tax exceeds marginal damage in contrast to the findings for an exogenous number of firms.

In this paper I generalize all the Cournot oligopoly models by LEVIN, EBERT, REQUATE [1993b] and KATSOULACOS and XEPAPADEAS. In all of these cases the optimal tax consists of two parts, marginal damage and a strategic term which depends on the slope of inverse demand function, but possibly also on its curvature, i.e., on the second derivative of inverse demand.

In section 3, I investigate oligopoly models with a *fixed* number of firms. Whereas LEVIN, EBERT, and REQUATE [1993b] consider models where pollution is completely determined by output, i.e., where there are no further abatement technologies, I do allow for abatement here. I also give a condition for uniqueness of Nash equilibrium which is not covered by existing results.

In section 4, I consider models where the number of firms is *endogenous*, and I distinguish the cases where the firms have an abatement technology and where pollution is determined completely by output. In particular I show that the result of

KATSOULACOS & XEPAPADEAS [1993] does not hold in general, i.e., the optimal tax does not in general exceed marginal damage if the number of firms is endogenous. In the case of no abatement technology I derive a very neat result which says that the optimal Pigouvian tax exceeds marginal damage if demand in strictly concave, falls short of marginal damage if demand in strictly convex, and equals marginal damage for linear demand. Since theoretically and empirically, demand is more likely to be convex than concave, the optimal Pigouvian tax is likely to fall short of marginal damage even if the number of firms is endogenous. If demand, however, is about to be linear and if there are no severe barriers to entry, a green tax equal to marginal damage is not necessarily a bad rule of thumb even under imperfect competition.

2 Basic Assumptions

Throughout this paper we consider a partial model with one consumption good and one pollutant which is generated by the production process. The consumption good is produced by $n \geq 1$ quantity setting firms which engage in Cournot competition. Let q_i and e_i, $i = 1, \ldots, n$, denote quantity and emission level of firm i, respectively. The society's preferences are represented by an inverse demand function $P(\cdot)$ depending on aggregate output $Q = \sum_{i=1}^{n} q_i$ and by a social damage function $S(\cdot)$ depending on aggregate pollution $E = \sum_{i=1}^{n} e_i$. P is downward sloping, has a finite choke–off price $\bar{p}$, and is not too convex. More precisely, it satisfies [1]

$$\frac{P''(Q)}{P'(Q)} Q > -1 \ . \tag{2.1}$$

The social damage function depends on aggregate emissions only and is increasing and convex, i.e. $S' > 0$, $S'' > 0$.

The firms' technologies are given by their reduced cost functions. We will make alternative assumptions about those. The first type of technology does not allow for abatement, i.e., pollution is determined completely by output. The second type of technology does allow for abatement, i.e., pollution can be substituted by using more or more expensive inputs which in turn incur higher costs.[2]

More formally we assume for the first type of technology without abatement:

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[1]This assumption implies $(P''(Q)/P'(Q))Q > -2$ which implies $P''(x+y)x + 2P'(x+y) < 0$ for all $x, y \geq 0$. The last inequality should be familiar from many standard oligopoly models (see HAHN [1962]). In our analysis, we need a bit stronger an assumption than this, i.e. that the elasticity of the derivative of the inverse demand function is greater than -1 rather than -2.

[2]Some authors, for example BARNETT [1980] and CONRAD [1993], specify certain inputs which can be used to reduce pollution (CONRAD talks of *abatement activity*). We assume that those are already incorporated in the reduced cost function.

Assumption 1 *For all $i = 1, \ldots, n$ the firms' cost function $C^i : \mathbb{R}_+ \to \mathbb{R}_+$ are twice continuously differentiable, split into variable cost v and a fixed cost F (we omit the superscript i):*

$$C(q) = \begin{cases} v(q) + F & \text{if } q > 0, \\ 0 & \text{if } q = 0. \end{cases} \qquad (2.2)$$

where v satisfies

i) $v' >$, $v'' \geq 0$.

ii) *Moreover, there is a continuously differentiable function $f : \mathbb{R}_+ \to \mathbb{R}_+$ with $e = f(q)$ and $f' > 0$.*

iii) *Minimum average costs are smaller than $\overline{p}$.*

For the second type of technology firm i's cost function depends on firm i's output q_i and its emissions e_i. About this type of cost function we assume:

Assumption 2 *For all $i = 1, \ldots, n$ the firms' cost functions $C^i : \mathbb{R}_+^2 \to \mathbb{R}$ are twice continuously differentiable for $(q_i, e_i) >> 0$ and split up into a variable cost $v(\cdot, \cdot)$ and a fixed cost F (we omit the superscript i):*

$$C(q, e) = \begin{cases} v(q, e) + F & \text{if } q > 0, \\ 0 & \text{if } q = 0. \end{cases} \qquad (2.3)$$

where v satisfies

i) $v_1 > 0$, $v_{11} > 0$, $v_{22} > 0$, $v_{12} < 0$,

ii) *for all q there is $e(q)$ such that $v_2(q, e(q)) = 0$, and $v_2(q, e) < 0$ if $e < e(q)$, and $v_2(q, e) \geq 0$ if $e > e(q)$,*

iii) $\qquad v_{11}v_{22} - [v_{12}]^2 > 0 .$ $\qquad (2.4)$

Now let $\widetilde{C}(q) := C(q, e(q))$ be the reduced cost function in the absence of regulation, and let $\overline{q}$ be the optimal scale of a firm w.r. to $\widetilde{C}$, i.e. $\overline{q}$ satisfies:

$$\frac{\widetilde{C}(\overline{q})}{\overline{q}} = \widetilde{C}'(\overline{q}) . \qquad (2.5)$$

(Note that such a $\overline{q}$ exists by the fixed costs and since v is convex.)

We make a joint condition on cost and demand functions:

vi) *(Existence of a market) $\widetilde{C}(\overline{q})/\overline{q} < \overline{p}$.*

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v) *(No emission–free production)* There is e^* such that for all $e < e^*$ we have

$$min_q \left\{ \frac{C(q,e)}{q} \right\} > \bar{p} \ . \tag{2.6}$$

The assumption implies that the *variable* cost function is convex. In particular we have increasing marginal costs for fixed emission levels, abatement costs are convex for each fixed output, output and emissions are complements ($v_{12} \leq 0$), and for each output level there is a cost minimizing emission level which would be chosen by the firms in the absence of regulation.[3] iv) says that in the absence of regulation there exists a market for the commodity. By continuity this implies also that there is a market under moderate regulation, for example if the government sets a lax emission standard $\bar{e}$ slightly smaller than $e(\bar{q})$ (or charges a sufficiently low emission tax). v) says that if the firms' emission levels are sufficiently low, the minimal average cost exceeds the choke–off price, and the market breaks down. This implies that production is not possible totally free of emissions, which is not necessarily a bad assumption. This assumption excludes corner solutions for the e_i.

Now we can define welfare[4]. If Assumption 2 holds, welfare is given by:

$$W(q_1, \ldots, q_n, e_1, \ldots, e_n) := \int_0^Q P(z)dz - S(E) - \sum_{i=1}^n C^i(q_i, e_i) \tag{2.7}$$

Under Assumption 1 the $C^i(q_i, e_i)$ are substituted by $C^i(q_i)$.

3 Emission Taxes in Cournot Oligopoly with an Exogenous Number of Firms

Let τ be a uniform emission tax. Let the firms' technologies satisfy the more general Assumption 2.[5] Then firm i's profit function is given by

$$\Pi^i(q_i, e_i, q_{-i}) = P(Q)q_i - C^i(q_i, e_i) - \tau e_i \ , \tag{3.1}$$

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[3]Such a cost function can be derived from a Cobb Douglas production function where one input, say energy, causes pollution proportional to input, and where energy has a positive market price. (If energy were available for free, $e(q)$ would be infinity).

[4]Note that this kind of objective function is standard in partial analysis and has also been employed by BAUMOL & OATES [1988], BARNETT [1980], SPULBER [1985], EBERT [1992], recently by KATSOULACOS and XEPAPADEAS [1994].

[5]Thus the technologies of the Cournot model presented here are more general than those in the models of LEVIN [1985] and EBERT [1992]. Both authors do not allow for abatement technologies. Rather, pollution is an increasing function of output (a linear function in LEVIN's model). Moreover, LEVIN does not consider *optimal* taxation. EBERT considers only *symmetric* oligopolies.

where $q_{-i} := (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n)$ and $Q := \sum_{i=1}^n q_i$. If we assume the existence of an interior Nash equilibrium $(q_1^*, \ldots, q_n^*, e_1^*, \ldots, e_n^*)$, i.e., $q_i^* > 0$, $e_i^* > 0$, such an equilibrium satisfies for all $i = 1, \ldots, n$:

$$P'(Q^*)q_i^* + P(Q^*) - v_1^i(q_i^*, e_i^*) = 0 \tag{3.2}$$

$$\tau + v_2^i(q_i^*, e_i^*) = 0 \tag{3.3}$$

where again $Q^* = \sum_{i=1}^n q_i^*$. We obtain the following result:

Proposition 1 *If for all i we have*

$$v_{122}^i v_{12}^i \le v_{112}^i v_{22}^i , \tag{3.4}$$

a unique Nash equilibrium exists. If the fixed costs and the tax are sufficiently low, there is an interior Nash equilibrium.

Proof: Without the fixed cost, existence follows from concavity of the firm's profit function. Revenue is concave by condition (2.1). The variable cost function is convex by Assumption 2. (iv) and (v) of Assumption 2 guarantee an interior Nash equilibrium if the tax is not too large. The fixed cost play only a role when the firms decide to produce or not to produce. For uniqueness we need (3.4). This is demonstrated in the appendix.   Q.E.D.

Note that in the previous literature on Pigouvian taxation in oligopoly, e.g., in the work of LEVIN [1985] and EBERT [1992], the cost functions have one variable only. Hence, uniqueness of Nash equilibrium could be established by well known results such as $P'(Q) < v_i''(q_i)$ for all i, Q and all $q_i \le Q$. If $P' < 0$, a sufficient condition for this is that the cost functions are (weakly) convex.[6] In our case the cost function has two arguments. ROSEN [1965] provides conditions (Theorem 2 and 6 in Rosen, 1965) for uniqueness if the strategy space is multidimensional. However, his tractable condition (from his Theorem 6) is quite restrictive and, unfortunately, not satisfied in this model. However, we can state a condition similar to the HAHN'S simple one by defining a reduced cost function in the following way. Set $\tilde{v}(q, \tau) := v(q, e(q, \tau))$, where for any τ, $e(q, \tau)$ is defined as the solution of $\tau = -v_2(q, e)$ in e. By Assumption 2 this solution is unique for all $q > 0$. If the reduced variable cost function is convex, uniqueness of Nash equilibrium is guaranteed. Now, condition (3.4) is sufficient to guarantee that $\tilde{v}(q, \tau)$ is convex.[7] The condition does not look very intuitive at first glance. Note, however, that (3.4) is similar to (2.4). For example, condition (3.4) is satisfied for bi–quadratic cost functions.

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[6]See TIROLE [1988], chapter 5, FRIEDMAN [1983], or also F. HAHN [1962].

[7]Actually condition (3.4) could be weakened to

$$v_{22}[v_{11}v_{22} - v_{12}^2] + v_2[v_{122}v_{12} - v_{112}v_{22}] \ge 0 ,$$

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Given uniqueness, denote the equilibrium (omitting the *-superscripts) for a given tax rate τ by $(q_1(\tau), \ldots, q_n(\tau), e_1(\tau), \ldots, e_n(\tau))$. As usual we write $Q(\tau) := \sum_{i=1}^{n} q_i(\tau)$, and $E(\tau) := \sum_{i=1}^{n} e_i(\tau)$.

Let us now consider the regulator's problem. If he or she can set a tax on emissions only, but cannot regulate output directly,[8] he or she maximizes:

$$W(\tau) := \int_0^{Q(\tau)} P(z)dz - \sum_{i=1}^{n} C^i(q_i(\tau), e_i(\tau)) - S(E(\tau)) . \tag{3.5}$$

Differentiating (3.5) with respect to τ and employing (3.2) and (3.3) one gets

$$\begin{aligned}
\frac{dW}{d\tau} &= \sum_{i=1}^{n} \left[P(Q) - v_1^i(q_i, e_i) \right] q_i' - \sum_{i=1}^{n} v_2^i(q_i, e_i)e_i' - S'(E)\sum_{i=1}^{n} e_i' \\
&= -\sum_{i=1}^{n} P'(Q)q_i q_i' + \sum_{i=1}^{n}[\tau - S'(E)]e_i' .
\end{aligned}$$

Setting $W'(\tau) = 0$ and "solving" for τ (note that the right hand side also depends on τ) yields for $i = 1, \ldots, n$:

$$\tau = S'(E) + \frac{P'(Q) \sum_{i=1}^{n} q_i q_i'}{E'} . \tag{3.6}$$

So the tax is equal to marginal damage plus a strategic term which depends on the signs of q' and E'. However, these terms cannot unambiguously be signed in general. Thus the strategic term can be positive or negative in general.

What we can say, however, is the following:

Proposition 2 *If (2.1) holds, aggregate output is decreasing in τ.*

Proof: Differentiate (3.2) and (3.3) with respect to τ:

$$[P''q_i + P']Q' + [P' - v_{11}^i]q_i' - v_{12}^i e_i' = 0 , \tag{3.7}$$

$$-v_{12}^i q_i' - v_{22}^i e_i' = 1 . \tag{3.8}$$

---

which, however, looks even more awkward whereas condition (3.4) is certainly easier to check. As an alternative we could also simply require

$$P' < \frac{d^2}{(dq)^2}\widetilde{v}(\cdot, \tau)$$

for all $\tau \geq 0$. This, however, is a joint condition on cost and demand, and thus difficult to check in general.

(3.8) is the same as

$$e_i' = -\frac{1 + v_{12}^i q_i'}{v_{22}^i} .$$

(3.9)

Substituting into (3.7) yields

$$[P''q_i + P']Q' + \left[P' - v_{11}^i + \frac{[v_{12}^i]^2}{v_{22}^i}\right] q_i' + \frac{v_{12}^i}{v_{22}^i} = 0 .$$

(3.10)

Solving (3.10) for q_i' yields

$$q_i' = -\frac{v_{22}^i[P''q_i + P']\sum_{j\neq i} q_j' + v_{12}^i}{v_{22}^i(P''q_i + 2P') - A_i} .$$

(3.11)

where $A_i = v_{11}^i v_{22}^i - [v_{12}^i]^2$. Since the sign of the first term of the numerator is ambiguous, we cannot sign q_i', unless we have monopoly. To put it another way we rearrange (3.10) to get

$$q_i' = -\frac{v_{22}^i[P''q_i + P']Q' + v_{12}^i}{v_{22}^i P' - A_i} .$$

(3.12)

By substituting into (3.9) we get

$$e_i' = -\frac{v_{12}^i[P''q_i + P']Q' + v_{11}^i - P'}{v_{22}^i P' - A_i} .$$

(3.13)

Summing over (3.12) one gets

$$Q' = -\sum_{i=1}^n \frac{v_{22}^i[P''q_i + P']Q' + v_{12}^i}{v_{22}^i P' - A_i} .$$

Solving for Q' yields

$$Q' = -\left[\sum_{i=1}^n \frac{v_{12}^i}{v_{22}^i P' - A_i}\right] \cdot \left[1 + \sum_{i=1}^n \frac{v_{22}^i[P''q_i + P']}{v_{22}^i P' - A_i}\right]^{-1} .$$

(3.14)

Studying these terms, we see that Q' is negative by (2.1) and Assumption 2.   Q.E.D.

Note that Proposition 2 generalizes results of LEVIN [1985] and EBERT [1992] who found this under much more restrictive assumptions.

On the other hand, signing E' is not possible in general if firms are asymmetric. For summing over (3.13) yields

$$E' = \sum_{i=1}^n \frac{v_{11}^i - P'}{v_{22} P' - A_i} + Q' \cdot \sum_{i=1}^n \frac{v_{22}^i[P''q_i + P']}{v_{22}^i P' - A_i} .$$

Here the first term is negative, but by (2.1), the second one is positive, so that the sign of the whole expression is undetermined. Yet, a more intelligent analysis possibly could sign E'. However, LEVIN [1985] gives a numerical example where a small tax raise

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from $\tau = 0$ increases pollution in an even simpler framework. He chooses an inverse demand function of the following type: $P(Q) = -a + b(q_1 + q_2) - c(q_1 + q_2)^2$ where a, b, c are positive constants. Moreover he chooses extremely asymmetric firms with different constant marginal costs and pollution proportional to output, i.e. $e_i = d_i q_i$. The example is elaborated in appendix B of his paper. We do not repeat it here.

Since aggregate output unambiguously goes down as the tax goes up, we can conclude that in LEVIN's example a small subsidy on emissions increases welfare. For, output goes up, emissions go down, and costs do not rise. From this we cannot conclude, however, that it is *optimal* to subsidize pollution whenever $E'(0) > 0$. For, the reason that emissions go up while industry output goes down, must be that some firms with high pollution increase output whereas those with low pollution cut down production. If the tax, however, is set sufficiently high, such that each firm's output goes down, then also aggregate emissions have to go down – compared to the laissez faire level. It is not difficult to find examples with sufficiently steep damage functions where a sufficiently high tax improves welfare compared to the laissez faire level, although emissions rise for a small tax increase from zero.

On the other hand, total pollution goes done with rising taxes if firms are symmetric or not too different, as we will see now:

Some Special Cases

A) Symmetric Firms: Assume that all the firms are alike, i.e., $C^i(\cdot, \cdot) = C(\cdot, \cdot)$ for all i. By symmetry, uniqueness of equilibrium requires a symmetric equilibrium, i.e., $q_i = q = Q/n$. In this case (3.14) becomes

$$Q' = -\frac{nC_{12}}{C_{22}[P''Q + (n+1)P'] - A} < 0 . \qquad (3.15)$$

Summing over (3.9) yields

$$
\begin{aligned}
E' &= -\frac{n}{C_{22}} - \frac{C_{12}}{C_{22}}Q' \\
&= -\frac{n}{C_{22}} + \frac{n[C_{12}]^2}{C_{22}[P''Q + (n+1)P'] - A} < 0 ,
\end{aligned}
$$

and hence also $e' = E'/n < 0$.

Moreover, (3.6) reduces to (note $q = Q/n$):

$$\tau = S'(E) + P'(Q)\frac{qQ'}{E'} .$$

The second term is clearly smaller that zero. So under symmetry, the tax falls short of marginal social damage. Let τ^* denote the *optimal* Pigouvian tax for symmetric

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oligopoly, and let τ^c denote the optimal tax under perfect competition. Since the oligopolistic industry output $Q(\tau^*)$ is less than the competitive output under the same tax τ^*, and since $\frac{\partial}{\partial q}e(q,\tau) > 0$, aggregate emissions $E(\tau^*)$ in oligopoly must be less than emissions under perfect competition at the same tax τ^*. Hence, the optimal Pigouvian emission tax τ^* for oligopoly must be lower than the optimal tax τ^c for perfect competition. So all together we obtain:

Corollary 1 *Let Assumption 2, and inequality (2.1) hold, and assume that all the firms are alike. Then:*

 a) *Aggregate output and emissions are decreasing in the emission tax rate. The tax rate is smaller than the marginal damage of pollution.*

 b) *If parallel to taxing emissions, output can be subsidized, the optimal tax/subsidy-system (τ, ζ) implements the social optimum and satisfies:*

$$\tau = S'(E^*) , \tag{3.16}$$
$$\zeta = -P'(Q^*)\frac{Q^*}{n} . \tag{3.17}$$

Part b) is easy to prove and left to the reader.

B) Symmetric firms without abatement technology [EBERT'S model]:

If Assumption 1 holds, a single equation characterizes the Cournot equilibrium:

$$P'(Q)q + P(Q) - v'(q) - \tau f'(q) = 0 .$$

Social optimum requires that the market price equals private plus social marginal cost of production, i.e.,

$$P(Q^*) = v'(q^*) + nf'(q^*)S'(nf(q^*)) .$$

Since $E' = nf'(q)q' = f'(q)Q'$, (3.6) reduces to

$$\tau^* = S'(E^*) + \frac{P'(Q^*)Q^*}{nf'(q^*)} . \tag{3.18}$$

Notice that in this case the optimal emission tax implements the social optimum. It internalizes both, over–pollution, and under–production. If we compare this term with (3.17), we observe that the term $-P'(Q)Q/n$ is equal to the optimal subsidy ζ on output. Hence we can rewrite (3.18) as

$$\tau^* = S'(E^*) - \frac{\zeta}{f'(q^*)} , \tag{3.19}$$

i.e., the tax equals marginal damage minus the optimal subsidy divided by the marginal response of pollution on output, which seems reasonable. The main insights of this subsection can be summarized in

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Corollary 2 *a) [Ebert] If oligopoly is symmetric, and firms do not have an abatement technology, then there is always a Pigouvian tax which implements the social optimum.*

b) In such a case the tax can also be charged on output. The optimal output tax (or subsidy) would be given by

$$\zeta = -P'(Q^*)\frac{Q^*}{n} - f'(q^*)S'(f(Q^*)) \ .$$ (3.20)

The last statement is easily calculated.

Note that if the firms are not symmetric, a tax/subsidy scheme cannot implement first best of course. Individual taxes and subsidies had to be paid in order to achieve first best. This is not possible in many cases, either by information problems or by law. Nevertheless, one could calculate the second best uniform tax/subsidy system. This, however, gives huge formulas and does not yield further insight.

4   Endogenous Number of Firms

So far we have assumed the number of firms to be exogenously given. Let us now consider a situation where firms enter the market as long as they earn a non-negative profit.

Note that under oligopolistic competition with free entry the firms impose three diseconomies to the society. The single firms tend to produce too little and pollute too much, but they enter excessively, and thus they waste resources which could be used to produce other commodities. If the regulator only has the emission tax as a policy instrument, he must take into account that the tax can also serve to mitigate excess entry.

For simplicity we consider only the case of identical firms in this section.[9] Whereas in such a case the tax falls short of marginal damage of pollution as we have seen, now it will depend very much on the curvature of inverse demand, i.e. on P'', and the complementarity between output and pollution, i.e. on v_{12}, whether the tax exceeds or falls short of marginal damage. In the following we will again consider the case where abatement is possible, and then the case without abatement. In particular for the latter one, we will obtain a neat result.

To model entry we can think of a two stage game in which the firms decide to enter the market or to stay out in a first stage, and where they engage in Cournot competition in a second one. If n firms have entered the market in the first stage, then under a given tax the Cournot Nash equilibrium is given by (3.2) and (3.3). Clearly

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[9]One can show that with free entry, i.e. firms earning zero profits, at most two types of different firms can stay in the market. The proof, however, is tedious and goes beyond this paper.

q and e depend on n and τ. In the first stage a number of n firms enters the market such that

$$P(nq(n,\tau)) - C(q(n,\tau), e(n,\tau)) - \tau e(n,\tau) \geq 0 , \qquad (4.1)$$

$$P([n+1]q(n+1,\tau)) - C(q(n+1,\tau), e(n+1,\tau)) - \tau e(n+1,\tau) < 0 . \qquad (4.2)$$

If we assume that the market is relatively large, we can treat n as a continuous variable and simply substitute the two equations (4.1) and (4.2) by a single zero profit condition:

$$P(nq)q - C(q,e) - \tau e = 0 . \qquad (4.3)$$

The system (3.2), (3.3), and (4.3) then describes the reaction of the market on an emission tax τ. Let $q(\tau)$ and $e(\tau)$ denote each firm's quantity and emission level, and $n(\tau)$ the corresponding number of firms in a free entry Nash equilibrium. Given this reaction of the market and assuming that an environmental authority can only regulate emissions but cannot influence output or the number of firms directly, the regulator maximizes

$$W(\tau) := \int_0^{Q(\tau)} P(z)dz - n(\tau) \cdot C(q(\tau), e(\tau)) - S(E(\tau)) \qquad (4.4)$$

where $Q(\tau) = n(\tau)q(\tau)$ and $E(\tau) = n(\tau)e(\tau)$. Differentiating with respect to τ and employing (3.2), (3.3), (4.3), yields:

$$
\begin{aligned}
W' &= P(Q) \cdot [n'q + nq'] - n'C(q,e) - nv_1(q,e)q' - nv_2(q,e)e' - \\
&\quad - S'(E) \cdot [n'e + ne'] \\
&= [P(Q)q - C(q,e) - S'(Q)e]n' + n[P(Q) - v_1(q,e)]q' - n[v_2(q,e) + S'(E)]e' \\
&= [\tau - S'(Q)]en' - nP'(Q)qq' + n[\tau - S'(E)]e' = 0 .
\end{aligned}
$$

Solving for τ yields

$$\tau = S'(E(\tau)) + P'(Q(\tau))\frac{n(\tau)q(\tau)q'(\tau)}{E'(\tau)} . \qquad (4.5)$$

At first glance this relationship between the optimal Pigouvian tax on the one side, and the marginal social damage and the remaining quantities on the other side seems to be the same as (3.6), which yields the optimal tax for the symmetric case when the number of firms is exogenous. However, for n exogenous, we have $Q' = nq'$, whereas here aggregate output changes by $Q' = nq' + n'q$. If E' and q' were negative, the same conclusions as in the model without free entry could be drawn, i.e., the tax would fall short of marginal social damage. To find out the signs of q' and E' we have to differentiate (3.2), (3.3), and (4.3) and solve for q', e', and n'. This yields after some

manipulations:

$$q' = \frac{v_{22}e[P''q + P'] + P'v_{12}q}{D} , \qquad (4.6)$$

$$e' = \frac{P'[q(P''q + 2P' - v_{11})] - v_{12}e(P''q + P')}{D} , \qquad (4.7)$$

$$n' = \frac{(v_{11}v_{22} - [v_{12}]^2)e - nv_{22}e[P''q + P'] - P'[2v_{22}e + (n-1)v_{12}q]}{D} . \qquad (4.8)$$

where the denominator

$$D = P'q[(v_{11}v_{22} - [v_{12}]^2) - v_{22}(P''q + P')]$$

is clearly negative by the assumptions on P and v. The numerators are more difficult to sign. The first term in the numerator of q' is negative by (2.1), whereas the second one is non–negative. But if $v_{12} = 0$ or not too large, the numerator is negative, thus q' will be positive.

We can also rearrange the numerator into $P''v_{22}qe + P'[v_{12}q + v_{22}e]$. Then the numerator is negative if $P'' < 0$, i.e., P is concave, and $v_{12}q + v_{22}e \geq 0$. So also in this case q' is positive. This stands in contrast to the model with an exogenous number of firms where q is decreasing if firms are symmetric.[10]

Little can be said about the signs of e' and n'. But what about aggregate quantities and emissions? Since $Q' = n'q + nq'$ and $E' = n'e + ne'$, we get after some rearranging:

$$Q' = \frac{N(Q')}{D} , \qquad (4.9)$$

$$E' = \frac{N(E')}{Dq} . \qquad (4.10)$$

The numerator of Q', given by

$$N'(Q) = (v_{11}v_{22} - [v_{12}]^2)e - P'(v_{22}e - v_{12}q) ,$$

is clearly positive. Since $D < 0$, aggregate output falls as the emission tax rises.

Aggregate emissions, on the other hand, are difficult to sign again. We get

$$N(E') = (v_{11}v_{22} - [v_{12}]^2)e^2 + nP'q^2[P''q + 2P'] - P'(v_{22}e^2 - v_{12}qe) - $$
$$- nP'[v_{22}e^2 + 2v_{12}qe + v_{11}q^2] + nP''q[v_{11}q^2 + v_{12}qe] .$$

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[10]Note that q' is positive since we treat n as a real number. Actually the single firm's output will change step wise if the number of firms is an integer. For each fixed number of firms the quantity will go down as long as this number of firms can earn a non–negative profit. If the tax rises further, one more firm will have to drop out. Then the remaining firms' output makes a jump upwards.

Inequality (2.4) implies that $v_{22}e^2 + 2v_{12}qe + v_{11}q^2$ nonnegative (the Hessian is positively semi-definite). Hence, all the terms are always positive but the last. The last one vanishes for linear demand, *or* it is non-negative if $v_{12} = 0$ and if inverse demand is non-concave. We see that here we need stronger conditions compared to the model with a fixed number of firms in order to sign aggregate emissions. If we get back now to formula (4.5), the considerations about the signs of q' and E' give rise to the following result.

Proposition 3 *Under symmetric oligopoly with free entry, the optimal emission tax exceeds the marginal social damage if*

i) $v_{12} = 0$, *and, in addition to (2.1), P is non-concave.*

or if

ii) $v_{12}q + v_{22}e \geq 0$, *and P is linear.*

Note that $v_{12}q + v_{22}e \geq 0$ is satisfied if C is bi-quadratic.

Proposition 3 generalizes the main result from KATSOULACOS and XEPAPADEAS [1993] who, for the sake of analytical tractability, assume C to be additively separable, i.e., $C(q,e) = aq + be + F$ with $a, b, F > 0$. Assuming $v_{12} = 0$, however, is certainly not very realistic since pollution and output typically are complements rather than independent. The more general analysis presented here shows which qualifications are important for the result and which can be relaxed.

Note that if the optimal Pigouvian tax exceeds the marginal social damage, loosely speaking, the effect of excess entry dominates the effects of too little production of the individual firm. Thus the regulator chooses a tax in excess of the social marginal damage in order to mitigate excess entry.

The result of Proposition 3 is important in so far as it does not allow to conclude that "imperfect competition in general calls for lower emission taxes than perfect competition". Under free entry it might just be the other way round. This again sheds an interesting light on BUCHANAN's [1969] early campaign against taxation under imperfect competition.

On the other hand, if P is sufficiently convex and v_{12} is sufficiently large, it can happen that q' and E' are negative such that the conclusion of Proposition 3 fails to hold. This can be seen more precisely when the cross cost effect v_{12} is infinity, i.e. if abatement drives up marginal cost prohibitively high. Then we are in the situation of a cost function satisfying Assumption 1:

No Abatement Technology:

Assume now that the firms' cost function satisfies Assumption 1. For simplicity we assume that $e = f(q) = dq$. Then a free entry equilibrium is given by

$$P'(Q)q + P(Q) - v'(q) - \tau d = 0 , \tag{4.11}$$

$$P(Q)q - C(q) - \tau dq = 0 . \tag{4.12}$$

The social planner maximizes welfare w.r. to τ. Similar calculations as in the previous case lead to

$$\tau = S'(E) + \frac{P'Qq'}{E'} . \tag{4.13}$$

Differentiating (4.11) and (4.12) w.r. to τ yields

$$q' = \frac{P''q}{P'[v'' - 2P' - P''q]} , \tag{4.14}$$

$$n' = \frac{v'' - nP''q - 2P'}{P'q[v'' - 2P' - P''q]} . \tag{4.15}$$

By this we get

$$Q' = n'q + nq' = \frac{v'' - 2P'}{P'[v'' - 2P' - P''q]} .$$

By inspection we see that aggregate output, and thus also aggregate emissions are clearly decreasing in the tax rate. Surprisingly, however, individual output *decreases* if inverse demand is strictly *convex*, it *increases* if inverse demand is strictly *concave*, and it is *constant* for *linear* demand.

Note that the number of firms does not necessarily behave monotonically if $n > 2$, and P convex. For concave demand the number of firms is strictly decreasing. Since q' also determines the strategic term in (4.13) we get the following result:

Proposition 4 *In a symmetric oligopoly with an endogenous number of firms the optimal (second best) emission tax is given by*

$$\tau = S'(E) + \frac{P'QP''q}{d[v'' - 2P']} . \tag{4.16}$$

The optimal tax:

   a) <u>exceeds</u> marginal damage if demand is strictly concave,

   b) <u>falls short of</u> marginal damage if demand is strictly convex,

   c) <u>is equal to</u> marginal damage for linear demand.

Of course either case can happen if P' is not monotonic.

We see that, despite imperfect competition, the optimal (second best) emission tax equals marginal damage for linear demand. Thus we get the same result as under perfect competition, which again is interesting in the light of BUCHANAN'S [1969] attack against the Pigouvian tax rule. This result also contrasts from KATSOULACOS and XEPAPADEAS [1993], who find that for linear demand the optimal tax exceeds marginal damage if emissions are not complementary to output.

However, a caveat is necessary. For we can neither conclude that the emission tax implements first best under linear demand, nor can we conclude that the optimal emission tax for endogenous oligopoly is the same as for a competitive market for linear demand. Since the firms price higher than marginal cost, they produce less than under perfect competition. Thus under the same tax they also pollute less than under Cournot competition. Hence, even under linear demand, in oligopoly with an endogenous number of firms the optimal tax has to be set lower than under perfect competition!

5 Conclusions

In this paper we examined Pigouvian taxation of emissions in oligopolies with exogenous and endogenous numbers of firms. We saw that whereas the optimal tax falls short of marginal damage if the number of firms is exogenous, and firms are not too different, little could be said in general if the number of firms is endogenous. The latter case, however, certainly is the more realistic one. Since the tax also serves to mitigate excess entry, it will be higher in general under free entry than under a fixed number of firms. If pollution is proportional to output, it depends crucially on the curvature of the inverse demand function whether the tax is lower, higher or equal to marginal damage. Interestingly, the latter happens to be the case for linear demand. So a rule of thumb "green taxes equal to marginal damage" is not necessarily a bad one if there are no barriers to entry apart from fixed cost and if the demand on the output market does not behave too erratic. To check the latter condition is, of course, an empirical question.

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6 Appendix

Proof of uniqueness of Nash–equilibrium in Proposition 1:

It is sufficient to show that $\tilde{C}(\cdot, \tau)$ is convex in q. Now given q and τ, $e(q, \tau)$ satisfies

$$-C(q, e(q, \tau)) = \tau \ . \tag{6.17}$$

Differentiating with respect to q yields

$$e_1(q, \tau) := \frac{\partial e}{\partial q} = -\frac{C_{12}}{C_{22}} > 0 \ . \tag{6.18}$$

Differentiating again and using (6.18) one gets

$$e_{11}(q, \tau) := \frac{\partial^2 e}{(\partial q)^2} = \frac{-C_{112} C_{22} + C_{122} C_{12}}{[C_{22}]^2} \ . \tag{6.19}$$

Then

$$\frac{d^2 \tilde{C}}{(dq)^2} = C_{11} + 2 C_{12} e_1(q, \tau) + C_{22} [e_{11}(q, \tau)]^2 + C_2 e_{11}(q, \tau) \ .$$

Using (6.18) and (6.19) we get

$$\frac{d^2 \tilde{C}}{(dq)^2} = \frac{1}{C_{22}^2} \left[C_{22} [C_{11} C_{22} - C_{12}^2] + C_2 [C_{122} C_{12} - C_{112} C_{22}] \right] \ . \tag{6.20}$$

Since $e(q, \tau) < e(q, 0)$ for all $\tau > 0$ by Assumption 2, we have $C_2(q, e(q, \tau)) < 0$. This and the fact that $C_{11} C_{22} - C_{12}^2 > 0$, together with condition (3.4) guarantee that $\frac{d^2 \tilde{C}}{(dq)^2} > 0$. Q.E.D.

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