INSTITUTE OF MATHEMATICAL ECONOMICS

WORKING PAPERS

No. 235

On the Existence of Equilibrium in Hierarchically Structured Economies

by

Willy Spanjers

October 1994

University of Bielefeld

33501 Bielefeld, Germany

On the Existence of Equilibrium in Hierarchically Structured Economies

Willy Spanjers
Institut für Mathematische Wirtschaftsforschung
Universität Bielefeld
Postfach 100131
33501 Bielefeld
Germany.

First Draft: Oktober 1994

SECOND DRAFT

DECEMBER 1994

Abstract

In this paper the problem of existence of equilibrium in hierarchically structured economies is solved for a broad class of hierarchical structures and institutional characteristics.

A hierarchically structured economy is a pure exchange economy in which the agents are organized as a hierarchical structure which consists of a set of agents, a set of bilateral relationships and a partition of the set of agents in hierarchical levels, such that each level contains exactly one agent.

Each hierarchical relationship is interpreted as a trade relationship in which the dominating agent sets a signal which restricts the set of net-trades the dominated agent can choose from with respect to this trade relationship. The set of signals the dominating agent can choose from and the way these signals restrict the set of net-trades of the dominated agent is described by the institutional characteristic of the trade relationship under consideration.

As equilibrium concept, a suitable adaption the concept of equilibrium in nice plays as introduced in Hellwig and Leininger (1987) is used. It amounts to describing the result of applying of the technique of backward induction to the closures of the sets of feasable actions of the agents. Thus, subgame perfect "almost equilibrium" plays in the corresponding multi-stage game are considered.

1 Introduction

The problem of allocating scarce resources whilst facing unlimited needs is the basic problem of economics.

In 1776, Adam Smith suggested that for certain goods competitive markets allocate scarce resources to the well-being of all, an "invisible hand" coordinating the ruthless persuit of self interest by the participants in the market.

This notion of Smith's has been translated into formal models of competitive markets in, e.g., Walras (1874) and Debreu (1959). They model competitive markets as markets in which all trade takes place at a single vector of prices which holds for all market participants, and which is assumed by each of them not to be influenced by his trades on the market. As stated in the First Theorem of Welfare Economics, in economies with a complete system of interconnected competitive markets, every equilibrium allocation is Pareto efficient. Thus, in the context of these models, Pareto efficiency can be interpreted as describing the notion of "the well-being of all".

Having established this result and recognizing its limitations, a natural next question is whether one can think of more general organizational forms, not necessarily being markets, that lead to desired allocations, not necessarily according to the criterion of Pareto efficiency, for a wide class of different types of agents that may possibly be active in the economy.

In the first half of this century this question was the theme of the socialism debate between, a.o., Lange, v. Mises and v. Hayek. The debate arose over the question whether or not socialist economies are capable of realizing market allocations.

The formal counterpart of this discussion took place over the last two decades, focussing on the question if and how organizational forms, called mechanisms, can be found that realize arbitrary goals for a large class of different environments they may have to cope with. In answering this question, a complicating factor is that agents typically have private information concerning their own characteristics, e.g., concerning their initial endowments and preferences. Such agents may be tempted not to disclose this private information truthfully, hoping to benefit from misrepresenting their characteristics. Therefore, a mechanism must both persuade agents to, directly or indirectly, reveal the relevant information truthfully, and coordinate the actions of the agents as to realize given goals. Unfortunately, the results obtained in trying to answer this question are typically impossibility results, stating that it is impossible to find satisfactory organizational forms that perform those tasks. For a survey we refer to Groves and Ledyard (1987). Lately, emphasis has increasingly shifted to interpreting mechanisms as describing (hypothetical) institutional and organizational structures. Institutions and organizations that have been observed in past and present societies have been formally described as mechanisms, possibly with the hope to find a new angle at the problems described above.

One may want to interpret mechanisms as describing (the results of) the institutional structure of a society as a whole, this being in line with the spirit of the models of Walras and Debreu, and with the socialism debate. In this interpretation, a mechanism describes, implicitly or explicitly, laws, habits, norms of behaviour and the like, as present in a (hypothetical) society. Thus, a change in some of the laws of a society typically results in a different mechanism describing its institutional structure. Having this in mind and being aware of the inertia of habits and norms, it seems impossible to, in the short run, make substantial changes in the institutional structure of a society as to change its functioning from realizing one mechanism to realizing a specific different mechanism. Rather, except for some revolutions, changes in the institutional structure of a society are gradual changes, trying to repair some unwanted effects occuring in some part of the society. Unfortunately, in the present type of formal models of mechanisms it is next to impossible to describe such phenomena, the concepts only allow for "changes" from one mechanism to another as if a revolution would occur.

The aim of this research is the development of formal models that describe the institutional structure of a society consisting of a number of "partial" mechanisms. Partial mechanisms are to describe separate parts of the organizational form that function, in their formal rules, independent from each other. Agents, however, will typically participate in a number of partial mechanisms. Each such agent will coordinate his actions with respect to the different partial mechanisms to his best of interests, given his anticipations regarding the results of his actions on the outcomes of these partial mechanisms. Thus, the anticipations of the agents determine the way the partial mechanisms interact through the behaviour of the agents. Consequently, the anticipations of the agents, which in our models are determined endogenously, play a crucial role in "aggregating" the partial mechanisms to a mechanism describing the functioning of the society as a whole.

In our research, we focus on models in which any partial mechanism contains exactly two participating agents. Furthermore, for technical reasons, our models are constructed as to ensure that in any such partial mechanism, one agent dominates the other agent. Thus, a structure of bilateral asymmetric trade relationships describes the set of institutions in the economy. However, to give economic content to the model, we have to describe for each of the bilateral trade relationships, the rules of trade that apply to this particular trade relationship. This is achieved by endowing every asymmetric trade relationship with an institutional characteristic. An institutional characteristic is a correspondence $\mathcal{T} : X \rightrightarrows Y$ from a space of signals in a space of net trades, such that for each signal $s \in X$ chosen by the dominating agent in the trade relationship, the set of net trades from which the dominated agent can choose, with respect to this trade relationship, is restricted to the set $\mathcal{T}(s)$.

In previous work, we have analyzed models with particular structures of trade relationships and particular institutional characteristics, and, amongst others, considered the question of existence of equilibrium. In Spanjers (1992, Chap. 6) the setting of a pure exchange economy with a finite number of agents and commodities is considered. In this context, an example is given of a model of successive monopolies in which no equilibrium exists.

One way to circumvent this problem is followed in Spanjers (1994a,c) and relies on what we call arbitrage. The main idea is to consider the kind of situation in which some agent acts as a price taker on trade relationships with two different agents. Now, if these price setting agents set different vectors of relative prices (normed to the corresponding unit simplex), then the price taking agent can achieve an arbitrary high income in buying sufficiently large amounts of relatively cheap commodities from one of those price setting agents and selling them to the other. If there are sufficient potential possibilities for arbitrage in the economy, then in equilibrium uniform prices will prevail in (parts of) the economy. Using this property, the existence of equilibrium in the economy is proven with the help of a theorem on the existence of Walrasian equilibrium in pure exchange economies. As a side effect, it is shown that the Walrasian auctioneer can be replaced by a (number of) price setting consumer(s) with zero initial endowments.

Another way to circumvent the problem of the existence of equilibrium has been followed in Spanjers (1992, Chap. 8) and Spanjers (1994b). Attention is restricted to hierarchical trees, this bringing us back to a model with a structure as in the case of successive monopolies. Now, the institutional characteristic of bid and ask prices is considered. This allows the dominating agent to set different vectors of prices for buying and selling, thus enabling him to enforce zero trade for the trade relationship under consideration. Using this property, theorems on the existence of equilibrium are proven. The nature of the proofs makes it clear that, along these lines, there is no hope to obtain results on the existence of equilibrium for a large class of structures of trade relationships and institutional characteristics. This, however, would be necessary to live up to the goal of formulating a useful model with a multitude of different interacting bilateral asymmetric partial mechanisms.

In the present paper we introduce a model of a hierarchically structured economy that does allow for the existence of equilibrium for a large class of hierarchical structures and a large class of institutional characteristics of bilateral asymmetric trade relationships. The sacrifice we have to make in order to obtain this result is to no longer consider (subgame perfect) equilbrium in very nice plays but to consider a suitable modification of the concept of equilibrium in nice plays, as introduced in Hellwig and Leininger (1987). It should be noted that equilibrium in nice plays is not a refinement of subgame perfect equilibrium.

In Section 2 we introduce the model of a hierarchically structured economy. The main theorem on the existence of equilibrium is stated and proven in Section 3. In Section 4 a number of particular institutional characteristics is discussed. In Section 5, the main theorem on existence is applied to hierarchical trees. Finally, in Section 6, some concluding remarks are made and topics of further reseach are suggested.

2 The Model

In this section we define a hierarchically structured economy. We describe such an economy by its hierarchical structure, by its agents and their individual characteristics, and by the institutional characteristics of the trade relationships in the economy. The hierarchical structure is described by a relationship structure that describes between which of the agents in the economy trade relationships exist, and by a hierarchy that partitions the set of agents in hierarchical levels, such that each hierarchical level contains exactly one agent. The hierarchy describes which agents dominate which other agents and thus directs the trade relationships. Furthermore, the hierarchy determines what information the individual agents have about the economy. The agents in the economy are described by their individual characteristics. Since we analyze a pure exchange economy, we describe each agent by his utility function and his initial endowments. Finally, every trade relationship in the economy is described by its institutional characteristic. The institutional characteristic of a trade relationship describes how the signals the dominating agent chooses with respect to this trade relationship restrict the set of net-trades the dominated agent can choose from with respect to the trade relationship under consideration. Thus, the institutional characteristic of a trade relationship describes the rules of trade that apply for it. Finally, we derive the anticipations of the agents concerning the consequences of their actions and define equilibrium.

We start by introducing the concepts we need to describe the hierachical structure of the economy. First, however, we introduce some terminology concerning graph theory. In this paper we restrict ourselves to graphs in which any two vertices have at most one direct connection.

A **(Simple) Undirected Graph** is a pair (A, R) consisting of a finite set of vertices A and a set of distinct edges $R \subset \{\{i,j\} \subset A \mid i \neq j\}$. A **Path** $\gamma(a,b)$ from a to b in a undirected graph (A, R) is a non empty sequence of edges $(\{c_0, c_1\}, \{c_1, c_2\}, \ldots, \{c_{n-1}, c_n\})$ with $c_0 = a$ and $c_n = b$, such that $\forall\, i,j \in \{0, 1, \ldots, n\},\ i \neq j:\ c_i \neq c_j$, with the possible exception that $c_0 = c_n$. A undirected graph (A, R) is **Connected** if $\forall\, a, b \in A,\ a \neq b$ there exists a path $\gamma(a,b)$ from a to b in (A, R).

Definition 2.1 *A* **Relationship Structure** *is a connected, (simple) undirected graph $\mathcal{G} := (A, R)$, where A is the set of vertices and R the set of edges.*

Thus, a relationship structure describes between which agents a bilateral trade relationship exists. A hierarchy of a set of agents gives an ordered partition of the set of agents into hierarchical levels, such that each hierarchical level contains exactly one agent. If one wants to interpret the economy as a multi-stage game, then the hierarchy describes the order in which the agents move. It is assumed that each agent has sufficient information about the agents of lower hierarchical levels for us to be able to apply the technique of backward induction. If one applies a model with a similar structure as the one in this paper to interconnected moral hazard problems, then the interpretation of the term "hierarchy" comes closer to that in every day life.

Definition 2.2 *Let A be a set of agents. An ordered partition $\xi := (S_1, \ldots, S_k)$ of the set A is a* **Hierarchy** *of A if for each $a \in \{1, \ldots, k\}$ it holds that $\#S_a = 1$.*

Let (A, ξ) be a pair consisting of a set of agents and one of its hierarchies. For each $i, j \in A$ we use $i \succ_\xi j$ to denote that $i \in S_a$ and $j \in S_b$ with $a < b$, which is interpreted as stating that i is of a higher hierarchical level then j. For each $i \in A$ we denote $L_i := \{h \in A \mid \{h, i\} \in R \ \ and \ \ h \succ_\xi i\}$ which interpreted as the set of **(Direct) Leaders** of i. Similarly, $F_i := \{j \in A \mid i \in L_j\}$ denotes the set of his **(Direct) Followers**.

Definition 2.3 *Let (A, R) be a relationship structure and ξ a hierarchy of A. The tuple $((A, R), \xi)$ is a* **Hierarchical Tree** *if $\#A = \#R + 1$ and $\forall\, i \in A: \ \#L_i \leq 1$.*

Thus, a hierarchical tree is a tuple consisting of a relationship structure and a hierarchy of the set of agents, such that the following holds. First, the relationship structure is a connected undirected simple graph with a tree structure. Second, every agent has at most one direct leader within the hierarchical structure induced by (A, R) and ξ. The relationship structure of a hierarchical tree has the minimal number of relationships with respect to its connectedness.

If we consider a relationship structure (A, R) and a hierarchy ξ of A, then we can direct the relationships in R through the hierarchy. Now, for each such directed trade relationship we describe the rules of trade that apply. It is only after these rules of trade for the trade relationships are specified, that these trade relationships get economic meaning. [1]

Definition 2.4 *Let (A, R) be a relationship structure and ξ a hierarchy of A. Let $r := \{i, j\} \in R$ and $i \succ_\xi j$. The* **Institutional Characteristic** *of $r \in R$ is the correspondence $\mathcal{T}_r : X_r \rightrightarrows Y_r$. We also write $\mathcal{T}_{ij} := \mathcal{T}_r$.*

The institutional characteristic $\mathcal{T}_r$ of a relationship $r := \{i, j\} \in R$ with $i \succ_\xi j$ is interpreted as specifiying for each signal $s_r \in X_r$, chosen by agent i, the set $\mathcal{T}_r(s_r) \subset Y_r$ of actions agent j can choose from with respect to the relationship r. In Section 4 some examples of different institutional characteristics are discussed.

Definition 2.5 *A* **Hierarchically Structured Economy** *with l commodities is a tuple* $\mathbf{E} = ((A, R), \xi, \{U_i, \omega_i\}_{i \in A}, \{\mathcal{T}_r\}_{r \in R})$ *where:*

[1] Note that in, e.g., North (1990) the opposite position is taken and institutions are interpreted as restricting the possibilities to trade and cooperate. Thus, an institution free environment is considered to be the ideal case, since in such an environment every form of trade or cooperation is feasible.

1. (A, R) *is a relationship structure.*

2. ξ *is a hierarchy of* A.

3. $U_i : \mathbb{R}^l_+ \to \mathbb{R}$ *is the utility function of agent* $i \in A$.

4. $\omega_i \in \mathbb{R}^l_+$ *is the vector of initial endowments of agent* $i \in A$.

5. $\mathcal{T}_r : X_r \rightrightarrows Y_r$ *is the institutional characteristic of the relationship* $r \in R$, *with* $Y_r \subset \mathbb{R}^l$.

We make the following assumption with respect to the individual characteristics of the agent in A throughout the paper.

Assumption 2.6 *Let* $i \in A$. *The function* $U_i : \mathbb{R}^l_+ \to \mathbb{R}$ *is continuous, strictly increasing and strictly quasi-concave.*

We use $L := \{1 \ldots, l\}$ to denote the set of commodities in the economy and with $X_i := \prod_{h \in L_i} Y_{ih} \times \prod_{j \in F_i} X_{ij}$ we denote the set of actions agent $i \in A$ can potentially choose from. We denote $X := \prod_{i \in A} X_i$. The tuple of actions chosen by the agents and the corresponding consumption bundles are described by the trade-signal-allocation tuple.

Definition 2.7 *A* **Trade-Signal-Allocation Tuple** *in the hierarchically structured economy* $\mathbf{E}$ *is a tuple* $(d, s, x) \in X \times \mathbb{R}^{A \times l}_+$ *where:*

1. $d_{ji} \in Y_{ji}$ *is the vector of net-trades over the trade relationship* $\{i, j\} \in R$ *with* $i \succ_\xi j$. *We denote* $d_i := (d_{ih})_{h \in L_i}$.

2. $s_{ij} \in X_{ij}$ *is the vector of signals send on the trade relationship* $\{i, j\} \in R$ *with* $i \succ_\xi j$. *We denote* $s_i := (s_{ij})_{j \in F_i}$.

3. $x_i \in \mathbb{R}^l_+$ *is the consumption bundle of agent* $i \in A$.

For each $i \in A$ we denote:

$\mathcal{L}(i) := \{h \in A \mid h \succ_\xi i\}$.

$\mathcal{S}(i) := \{j \in A \mid i \succ_\xi j\}$.

$\mathcal{S}^+(i) := \{j \in A \mid i \succeq_\xi j\}$.

$Y^{\mathcal{S}(i)} := \prod_{j \in \mathcal{S}(i)} (X_j \times \mathbb{R}^l_+)$.

$Y^{\mathcal{S}^+(i)} := \prod_{j \in \mathcal{S}^+(i)} (X_j \times \mathbb{R}^l_+)$.

$Y^{\mathcal{L}(i)} := \prod_{h \in \mathcal{L}(i)} X_h$.

For agent $i \in A$, the set $\mathcal{L}(i)$ is the set of agents of a higher hierarchical level than agent i. Similarly, the set $\mathcal{S}(i)$ is the set of agent of a lower hierarchical level than agent i. It holds that $\mathcal{S}^+(i) := \mathcal{S}(i) \cup \{i\}$.

The set $Y^{\mathcal{S}(i)}$ is the set of trade-signal-allocation tuples of the agents that are of a lower hierarchical level than agent i. The set $Y^{\mathcal{S}^+(i)}$ is its analogon, when agent the trade-signal-allocation tuple of agent i is included. Finally, the set $Y^{\mathcal{L}(i)}$ is the set of trade-signal tuples, i.e., no consumption bundles of the agents are included, for the agents of a higher hierarchical level than agent i.

For each agent $i \in A$, the anticipated reactions correspondence t_i (if $\mathcal{S}(i) \neq \emptyset$,) the choice correspondence B_i, the optimal actions correspondence Δ_i and the compatible actions correspondence β_i are defined by the following recursive procedure.

We start by the agent of the lowest hierarchical level. For this agent, $m \in S_k$ who does not have any direct followers, we define his choice correspondence B_m, his optimal actions correspondence Δ_m and his compatible actions correspondence β_m. These correspondences are defined through the definitions Def. 2.9, Def. 2.10 and Def. 2.11, respectively.

Assume the recursive procedure has been applied to the agents in the set $\mathcal{S}(i)$ for agent i but not for agent i himself. Let $i \in S_a$. We use $i+1$ to denote the agent $j \in S_{a+1}$. For each $j \in \mathcal{S}(i)$, the correspondences t_j, Δ_j, and β_j are obtained in previous stages of the recursive procedure. Now, the correspondences t_i, B_i, Δ_i and β_i are constructed using the definitions Def. 2.8 through Def. 2.11.

If we apply the procedure to agent $k \in A$ with $\mathcal{L}(k) = \emptyset$, then B_k, Δ_k and β_k are taken to be sets instead of correspondences in the definitions Def. 2.9 to Def. 2.11. The procedure ends after it has been applied to agent $k \in A$ with $\mathcal{L}(k) = \emptyset$.

Definition 2.8 *The* **Anticipated Reactions Correspondence** $t_i : X^{\mathcal{L}(i)} \times X_i \rightrightarrows Y^{\mathcal{S}(i)}$ *such that* $\forall\, ((d_h, s_h)_{h \in \mathcal{L}(i)}, (d_i, s_i)) \in X^{\mathcal{L}(i)} \times X_i$ *we have*

$$\begin{aligned} t_i((d_h, s_h)_{h \in \mathcal{L}(i)}, (d_i, s_i)) \;:=\; & \{(e_j, q_j, y_j)_{j \in \mathcal{S}(i)} \in Y^{\mathcal{S}(i)} \mid \\ & [(e_{i+1}, q_{i+1}, y_{i+1}) \in \Delta_{i+1}((d_h, s_h)_{h \in \mathcal{L}(i)}, (d_i, s_i))] \\ & \textit{and} \;\; [(e_j, q_j, y_j)_{j \in \mathcal{S}(i)} \in \beta_{i+1}((d_h, s_h)_{h \in \mathcal{L}(i)}, (d_i, s_i))]\} \end{aligned}$$

The anticipated reactions correspondence t_i of some agent i denotes the set of reactions of the agents in $\mathcal{S}(i)$ he anticipates given the trade-signal tuples of the agents in $\mathcal{L}(i)$ and a tuple of net-trades and signals he may choose himself.

Next we consider the choice correspondence B_i of agent i. This correspondence describes the actions agent i anticipates to be "almost" feasible, given the anticipated reactions correspondence as defined in the above, and given the trade-signal tuple of the agents in $\mathcal{L}(i)$. Here "almost" feasible means that the actions under consideration can be approached arbitrarily close within the set of feasible actions. Thus, we consider the closure of the set of feasible actions to be the choice set of agent i. The continuity of the utility functions ensures that the optimal "almost feasible" action can be approached arbitrarily close in utility space.

Definition 2.9 *The* **Choice Correspondence** $B_i : X^{\mathcal{L}(i)} \rightrightarrows Y_i$ *of agent* $i \in A$ *is such that* $\forall\,(d_h, s_h)_{h\in\mathcal{L}(i)} \in X^{\mathcal{L}(i)}$:

$$\begin{aligned} B_i((d_h, s_h)_{h\in\mathcal{L}(i)}) := \ \mathrm{cl}\ \{(e_i, q_i, y_i) \in Y_i \mid \ & e_i \in \prod_{g\in L_i} \mathcal{T}_{gi}(s_{gi}), \\ & y_i \le \omega_i + \sum_{g\in L_i} e_{ih} - \sum_{j\in F_i} e_{ji} \\ & \textit{such that if } \mathcal{S}(i) \neq \emptyset : \exists\,(\tilde{e}_j, \tilde{q}_j, \tilde{y}_j)_{j\in\mathcal{S}(i)} \in \\ & t_i((d_h, s_h)_{h\in\mathcal{L}(i)}, (e_i, q_i)) \textit{ with } \ \forall j \in F_i :\ e_{ji} = \tilde{e}_{ji}\}. \end{aligned}$$

The optimal actions correspondence Δ_i of agent i assigns to each tuple of net-trades and signals of the agents in $\mathcal{L}(i)$ the set of optimal tuples of net-trades, signals and consumption bundle of agent i from his corresponding choice set.

Definition 2.10 *The* **Optimal Actions Correspondence** $\Delta_i : X^{\mathcal{L}(i)} \rightrightarrows Y_i$ *of agent* $i \in A$ *is such that* $\forall\,(d_h, s_h)_{h\in\mathcal{L}(i)} \in X^{\mathcal{L}(i)}$:

$$\Delta_i((d_h, s_h)_{h\in\mathcal{L}(i)}) := \mathrm{argmax}_{(e_i,q_i,y_i)\in B_i((d_h,s_h)_{h\in\mathcal{L}(i)})}\ U_i(y_i).$$

The optimal actions from the choice set of agent i may not be (almost) feasble for him when some of his followers take the "wrong" action if they are indifferent between a number of actions. In that case, agent i would face unsolved coordination problems concerning the actions his followers take. We assume this kind of coordination problems not to occur, thus following the lead of Hellwig and Leininger (1987) where equilibrium in very nice plays is considered. In Spanjers (1994b) a simple trade process that relies on the existence of enforcable I-owe-you's is outlined that makes this kind of forward induction plausible. Next, as a matter of notation, we introduce $\tau_i : X^{\mathcal{L}(i)} \rightrightarrows Y^{\mathcal{S}(i)} \times X_i$ such that $\forall\,(d_h, s_h)_{h\in\mathcal{L}(i)} \in X^{\mathcal{L}(i)} : ((e_m, q_m, y_m)_{m\in\mathcal{S}(i)}, (e_i, q_i)) \in \tau_i((d_h, s_h)_{h\in\mathcal{L}(i)})$ if and only if $((e_m, q_m, y_m)_{m\in\mathcal{S}(i)}) \in t_i((d_h, s_h)_{h\in\mathcal{L}(i)}, (e_i, q_i))$.

Definition 2.11 *The* **Compatible Actions Correspondence** $\beta_i : X^{\mathcal{L}(i)} \rightrightarrows Y^{\mathcal{S}^+(i)}$ *of agent* $i \in A$ *is such that* $\forall\,(d_h, s_h)_{h\in\mathcal{L}(i)} \in X^{\mathcal{L}(i)}$:

$$\begin{aligned}\beta_i((d_h, s_h)_{h\in\mathcal{L}(i)}) \;:=\; \{(e_m, q_m, y_m)_{m\in\mathcal{S}^+(i)} \in Y^{\mathcal{S}^+(i)} \mid \\ (e_m, q_m, y_m)_{m\in\mathcal{S}(i)}) \in \mathrm{cl}\,\tau_i((d_h, s_h)_{h\in\mathcal{L}(i)}) \\ \textit{and}\quad y_i \leq \omega_i + \sum_{g\in L_i} e_{ig} - \sum_{j\in F_j} e_{ji}\ \}.\end{aligned}$$

After having defined the choice correspondences of the agents in the economy by the above recursive procedure we define our notion of equilibrium. In the next section we argue that the equilibrium concept we introduce captures, in the context of our models, the essentials of "equilibrium in nice plays", as introduced in Hellwig and Leininger (1987), in the multi-stage game corresponding to our economy. It should be noted, that if the choice correspondences are continuous, as is assumed in Hellwig and Leininger (1987), then our formulation is equivalent to theirs. Here, where the choice correspondences are not continuous, our approach of considering the closures of the feasible actions sets seems to be the natural thing to do, as is argued in Section 3.

Definition 2.12 *Let* **E** *be a hierarchically structured economy. A trade-price-allocation tuple* $(d^*, s^*, x^*) \in X \times \mathbb{R}_+^{A\times l}$ *is an* **Equilibrium** *in* **E** *if there exists a sequence* $(e^t, q^t, y^t)_{t=1}^{\infty} \to (d^*, s^*, x^*)$ *such that* $\forall\, i \in A$:

1. $\forall\, t \in \mathbb{N} : \ (e_i^t, q_i^t, y_i^t) \in \Delta_i((d_h^t, s_h^t)_{h\in\mathcal{L}(i)})$
2. $x_i^* = \omega_i + \sum_{h\in L_i} d_{ih}^* - \sum_{j\in F_i} d_{ji}^*$

For given optimal actions of agent i, one may find that (some of) the agents in $\mathcal{S}(i)$ may have several almost optimal almost feasible (re)actions at their disposal. Typically, only if they make the "right" choices if they are (almost) indifferent, agent i will obtain the result he anticipated. Furthermore, the choice of agent i need not be a best element from the choice set of agent i. It suffices if it is obtained in the limit of a sequence of best elements for a sequence of net-trades and signal of the agents in $\mathcal{L}(i)$ which converges to the equilibrium net-trades and signals. Thus, the agents in $\mathcal{L}(i)$ can choose net-trades and signals sufficiently close to the equilibrium net-trades and signals, such that some corresponding sequence of optimal actions of agent i approaches his equilibrium tuple of net-trades, signals and consumption-bundle (d_i^*, s_i^*, x_i^*) arbitrarily close. In this sense, each agent choose almost feasible and almost optimal actions. From the construction of the correspondences B_i and β_i for each $i \in A$, the following property follows immediately.

Property 2.13 *A tuple $(d^*, s^*, x^*) \in X \times \mathbf{R}_+^{A\times L}$ is an equilibirum in the economy* $\mathbf{E}$ *if and only if the following hold:*

1. $(d_k^*, s_k^*) \in \Delta_k$
2. $(d^*, s^*, x^*) \in \beta_k$

3 The Existence of Equilibrium

In this section we state and prove a theorem on the existence of equilibrium in hierarchically structured economies.

We use backward induction to prove the existence of equilibrium. Proving the existence of an equilibrium through backward induction amounts, in the context of our models, to repeatedly applying the Maximum Theorem. The problem in doing so is that in applying the Maximum Theorem, one starts with a continous constraint correspondence and obtains a upper hemi continuous correspondence of maximizing elements. This correspondence of maximizing elements enters the choice correspondence of the next optimization problem. Now, one may find that this choice correspondence on longer is a continuous correspondence. It may turn out to be a correspondence that fails to be lower hemi continuous. It will still be the case that the correspondence of optimal actions for this second problem has non-empty values, but it may fail to be upper hemi continuous since the Maximum Theorem no longer applies. Next, this corresponcence of optimal actions may enter into the choice set of a third player. As a consequence of the correspondence of optimal reactions not having a closed graph, this choice set may fail to be compact. Thus this third player may fail to have a maximal element in his choice set, and equilibrium may fail to exist.

In the previous section, this kind of problem is overcome by have each agent optimizing over the closure of his set of feasible actions. In our model, under the assumption that the every institutional characteristic is closed valued, it is equivalent to consider the closure of the choice set of the agent or to consider the anticipated reactions correspondences $\hat{\tau}_i$ which are those correspondences the graph of which is the relative closure of the graph of the correspondence t_i with respect to the space of actions of agent i. The latter solution has the advantage that the adaption of the model takes place through the anticipations of the agents, which on their turn influence the choice sets in question. This seems to be more elegant than directly changing the sets of feasible actions, which we modelled as the choice sets of the agents. Indeed, this is the choice made by Hellwig and Leininger (1987) as they introduce nice plays. They argue that:

> Instead of constraining player t's choice by the optimal choice set of player $(t+1)$ we constrain it by the topological closure of this set. This guarantees that the feasibility sets are always closed. However, any play that is added to the original feasibility set by this closure operation in *not* itself a solution to $(t+1)$'s original decision problem and must be rationalized in some other way. This will be done in detail in later sections.
>
> ⋮
>
> In other words, nice plays in subgame e_{t-1} are stable plays in the sense that they can be approximated (in $S_t \times \ldots \times S_T$ and payoff space) by very nice plays (which result from the "pure" backward induction) in slightly perturbed versions of subgame e_{t-1}.

The advantage of the choice we made in this paper is is twofold. Firstly, it allows in Definition 2.12 for a direct formulation of the concept of equilibrium in a form that has similarities with that saying that each agent chooses the best actions from his choice set in such a way that these choices of the agents are, in some sense, compatible. Otherwise, it seems, one can not do better then defining equilibrium to be the tuple $(d^*, s^*, x^*) \in \beta_k$ such that $(s_k^*, x_k^*) \in \Delta_k$. As is seen in Property 2.13, in the formulation chosen in the present paper, this property still characterizes equilibrium.[2] Secondly, the present formulation allows us, in our next theorem on the existence of equilibrium,

[2]It should be noted that in the approach of Hellwig and Leininger (1987) the anticipated reactions correspondence would be different from ours in the previous section. Indeed, in their approach one would consider anticipated reactions correspondences τ_i which are such that their graph is the closure of the graph of the counterparts of the correspondences t_i we found.

not to restrict ourselves to those institutional characteristics for which the images of the signals are closed sets. This seems to be the natural thing to do in this context. It is unclear why an agent i would be happy to consider tuples of his own actions and reactions of his followers that only in the closure of the graph of optimal reactions, given the actions of the agents in $\mathcal{L}(i)$, but would not be happy to consider net-trade in the trade relationships with his direct leaders that are themselves not feasible but can be approached arbitrarily close, given the signals set by the agents in L_i. By the continuity of the utility functions, we have in either case that the utility level the agent is happy to consider as to belong to an optimal choice can be approached arbitrary close in utility space.

The proof of the theorem is along the lines at which Hellwig and Leininger (1987) construct nice plays.

Theorem 3.1 [Existence Theorem]
Let $\mathbf{E}$ *be a hierarchically structured economy for which Assumption 2.6 holds and such that* $\forall\, r := \{i, j\} \in R$:

1. $\exists n \in \mathbb{N} : \; X_r \subset \mathbb{R}^n$.

2. X_r *and* Y_r *are bounded.*

3. $\forall\, s \in X_r : \; 0 \in \mathcal{T}_r(s)$.

4. $\exists\, s \in X_r : \; \mathcal{T}_r(s) = \{0\}$.

Then there exists an equilibrium in $\mathbf{E}$.

In this theorem the first two conditions ensure that the closures of the sets of feasible actions, i.e., the choice sets, of the agents are compact subsets of some $\mathbb{R}^m$. Therefore, they allows us to apply the Maximum Theorem and Weierstrass' theorem. The last two conditions ensure that the choice sets of the agents are non-empty. Thus, the kind of problem with existence of equilbrium we encountered in Spanjers (1992, Example 6.3.1), where the agent of the highest hierarchical level has an empty choice set, is excluded.

As in Section 2, for each agent $i \in A$ such that $\mathcal{S}(i) \neq \emptyset$ with $i \in S_a$ we use $i+1$ to denote the unique agent $j \in S_{a+1}$. Thus, agent $i+1$ is the agent that is of one hierarchical level lower than agent i. For each agent $i \in A$ with $\mathcal{L}(i) \neq \emptyset$ agent $i-1$

is defined to be the agent such that $(i-1)+1=i$.

Proof of Theorem 3.1

We show there exists a tuple $(d^*, s^*, x^*) \in X \times \mathbb{R}^l_+$ such that $(d^*, s^*, x^*) \in \beta_k$ and $(s^*_k, x^*_k) \in \Delta_k$. The proof is structured recursively, starting from the lowest hierarchical level.

Let $j \in A$ with $\mathcal{S}(j) = \emptyset$. Since $F_j = \emptyset$, we have $\forall\, (d_h, s_h)_{h\in\mathcal{L}(j)} \in \prod_{h\in\mathcal{L}(j)} X_h$: $B_j((d_h, s_h)_{h\in\mathcal{L}(j)})$ is bounded. Furthermore, by construction, $B_j((d_h, s_h)_{h\in\mathcal{L}(j)})$ is closed, and therefore, $B_j((d_h, s_h)_{h\in\mathcal{L}(j)})$ is a compact set. By Assumption 3 of the theorem, we have $B_j((d_h, s_h)_{h\in\mathcal{L}(j)}) \neq \emptyset$. Since U_j is a continuous function we have by Weierstrass' theorem that $\Delta_j((d_h, s_h)_{h\in\mathcal{L}(j)})$ is a non-empty compact set, as is $t_{j-1}((d_h, s_h)_{h\in\mathcal{L}(j-1)}, (d_{j-1}, s_{j-1}))$.

Let $i \in A$ be such that the procedure has been applied on the agents in $\mathcal{S}(i) \neq \emptyset$ in previous steps, but not on agent i. From a previous step in the recursive procedure, we have that t_{i+1} is a correspondence with (non-empty) compact values. By Assumption 2 of the theorem, we have that t_{i+1} has a bounded graph. Thus, $\forall\, (d_h, s_h)_{h\in\mathcal{L}(i)} \in X$ we have, by Assumption 2 of the theorem, that $B_i((d_h, s_h)_{h\in\mathcal{L}(i)})$ is a bounded set that is closed by construction, and therefore is compact. By Assumptions 3 and 4 of the theorem we have that $B_i((d_h, s_h)_{h\in\mathcal{L}(i)}) \neq \emptyset$. Since U_i is a continuous function, we have by Weierstrass' theorem that $\Delta_i((d_h, s_h)_{h\in\mathcal{L}(i)})$ is a non-empty compact set. By construction this also holds for $\beta_i((d_h, s_h)_{h\in\mathcal{L}(i)})$. Since A is a finite set, the procedure stops on a finite number of steps after it has been applied to agent k with $L_k = \emptyset$.

From the above procedure we find that Δ_k is a non-empty compact set, as is β_k. Since U_k is a continuous function we find by Weierstrass' theorem that, by construction of β_k, that $\exists\, (d^*, s^*, x^*) \in \beta_k$ such that $(s^*, x^*) \in \Delta_k$. Therefore, an equilibrium in $\mathbf{E}$ exists.

$\mathcal{Q.E.D.}$

4 Some Examples of Institutional Characteristics

In this section we discuss some examples of institutional characteristics. In particular, the institutional characteristics of mono pricing, bid and ask prices, take-it-or-leave-it bids and monopolistic quantity rationing are considered. Finally, the institutional characteristic of non-linear pricing with respect to some set of pricing functions is discussed.

We start with the institutional characteristic of mono pricing.

Definition 4.1 *Let* $\mathbf{E}$ *be a hierarchically structured economy and* $r := \{i, j\} \in R$ *such that* $i \succ_{\xi} j$. *The relationship* r *has the institutional characteristic of* **Mono Pricing** *if its institutional characteristic is the correspondence* $\mathcal{T}^{mon} : X^{mon} \rightrightarrows Y^{mon}$ *where* $X^{mon} := S^{l-1}$ *and* $Y^{mon} := \mathbf{R}^l$ *such that* $\forall\, p \in S^{l-1}$:

$$\mathcal{T}^{mon}(p) := \{d \in \mathbf{R}^l \mid p \cdot d \leq 0\}.$$

If a trade relationship between two agents has the institutional characteristic of mono pricing, then the dominating agent sets a vector of prices for the trade relationship, one for each commodity. Given the vector of prices, the dominated agent decides the amounts of the commodities he wants to buy or sell at these prices. The prices for buying and selling are the same, as, e.g., holds for the price vectors in the general equilibrium model. The dominating agent has the obligation to supply the amounts the dominated agent wants to buy and to accept the amounts the dominated agent wants to sell.

If we want to use the institutional characteristic of mono pricing in our models, we may have a hard time proving the existence of equilibrium. The set Y^{mon} equals $\mathbf{R}^l$, and therefore is not a bounded set. Furthermore, this institutional characteristic does not give the dominating agent the possibility to enforce zero trades, and therefore condition *4* of the existence theorem is violated. This problem, however, does not bite if suitable assumptions of the individual characteristics of the agents are made and the institutional characteristic of mono pricing is only used on trade relationships with agents that have only one direct leader and do not have any direct followers. Then the mean value theorem (or one of its generalizations) ensures that the dominating agent has the possibility to set prices for which zero trade is (one of) the optimal action(s) of the dominated agent.

In Spanjers (1994a,c) it is illustrated that the set of net-trades not being bounded opens the possibility of arbitrage, which is then used to obtain, in equilibrium, uniform prices in the economy, which allows for a proof of a theorem on existence using a standard result on the existence of Walrasian equilibrium in a corresponding pure exchange economy. The counterpart of this theorem within the present model is formulated in Theorem 5.1.
The next institutional characteristic we consider is that of bid and ask prices.

Definition 4.2 *Let* **E** *be a hierarchically structured economy and* $r := \{i, j\} \in R$ *such that* $i \succ_\xi j$*. The relationship* r *has the institutional characteristic of* **Bid and Ask Prices** *if its institutional characteristic is the correspondence* $\mathcal{T}^{bap} : X^{bap} \rightrightarrows Y^{bap}$ *where* $X^{bap} := S^{2l-1}$ *and* $Y^{bap} := \mathbb{R}^l$ *such that* $\forall\,(\underline{p}, \overline{p}) \in S^{2l-1}$:

$$\mathcal{T}^{bap}(\underline{p}, \overline{p}) := \{d \in \mathbb{R}^l \mid \sum_{c \in L} \underline{p}_c \cdot \min\{0, d_c\} + \sum_{c \in L} \overline{p}_c \cdot \max\{0, d_c\} \leq 0\}.$$

If a relationship has the institutional characteristic of bid and ask prices, then, as in the case of mono pricing, the dominating agent acts as a price setter, whereas the dominated agent acts as a price taker with respect to this relationship. The difference is that for the institutional characteristic of bid and ask prices the dominating agent sets two prices for each commodity, one at which he buys and one at which he sells. Thus, he has the possibility to enforce zero trades by setting the price at which he buys for each commodity at zero and setting for each commodity a positive price for selling. In the above definition of the institutional characteristic of bid and ask prices, the set Y^{bap} equals $\mathbb{R}^l$, which is not bounded.
Next we define the institutional characteristic of take-it-or-leave-it bids.

Definition 4.3 *Let* **E** *be a hierarchically structured economy and* $r := \{i, j\} \in R$ *such that* $i \succ_\xi j$*. The relationship* r *has the institutional characteristic of* **Take-it-or-leave-it Bids** *if its institutional characteristic is the correspondence* $\mathcal{T}^{tol} : X^{tol} \rightrightarrows Y^{tol}$ *where* $X^{tol} := Y^{tol} := \mathbb{R}^l$ *such that* $\forall\, t \in \mathbb{R}^l$:

$$\mathcal{T}^{tol}(t) := \{0, t\}.$$

On a trade relationship that has the institutional characteristic of take-it-or-leave-it bids, the dominating agent proposes the dominated agent a bundel of net-trades that he can either accept or reject. If the dominated agent rejects the vector of net-trades, then no trade takes place over this trade relationship.

The following institutional characteristic, that of monopolistic quantity rationing, is introduced in Böhm et al. (1983).

Definition 4.4 *Let* $\mathbf{E}$ *be a hierarchically structured economy and* $r := \{i, j\} \in R$ *such that* $i \succ_\xi j$. *The relationship* r *has the institutional characteristic of* **Monopolistic Quantity Rationing** *if its institutional characteristic is the correspondence* $\mathcal{T}^{mqr} : X^{mqr} \rightrightarrows Y^{mqr}$ *where* $X^{mqr} := S^{l-1} \times \mathbb{R}^l_- \times \mathbb{R}^l_+$ *and* $Y^{mqr} := \mathbb{R}^l$ *such that* $\forall (p, \underline{r}, \overline{r}) \in S^{l-1} \times \mathbb{R}^l_- \times \mathbb{R}^l_+$:

$$\mathcal{T}^{mqr}(p, \underline{r}, \overline{r}) := \{d \in \mathbb{R}^l \mid p \cdot d \leq 0 \quad and \quad \underline{r} \leq d \leq \overline{r}\}.$$

In case of the institutional characteristic of monopolistic quantity rationing, the dominating agent sets vector of prices, as in the case of mono pricing, and vectors of upperbounds on the amounts he buys and sells at these prices. As before, the dominated agent takes these signals as given and decides, whithin these bounds, on the net-trades that take place over this trade relationship.

Clearly, we may also consider "mixtures" of the above institutional characteristics. For instance, we may find that over a particular trade relationship only some of the commodities in the economy can be traded, say, according to mono pricing. Indeed, if we have the institutional characteristic of mono pricing on each trade relationship but only with respect to some strict subset $\tilde{L} \subset L$ of commodities in the economy, then we may, in a context similar to that of Theorem 5.1, end up with a model of a pure exchange economy with incomplete markets.

Another possibility would be that over a given trade relationship some of the commodities in the economy are traded with respect to the trade rules of, say, mono pricing, and others according to that of bid and ask prices. Similarly, one may want to consider an institutional characteristic in which for each commodity one tuple of bid and ask prices holds upto certain amounts of trade, and for higher amounts traded different prices hold.

Finally we introduce the institutional characteristic of non-linear pricing, which contains the institutional characteristics mentioned above as special cases. We consider non-linear prices to be elements of a space of functions from the set of net-trades $\mathbb{R}^l$ to the of prices S^{l-1}.

Definition 4.5 *Let* $\mathbf{E}$ *be a hierarchically structured economy and let* $r := \{i, j\} \in R$ *be such that* $i \succ_\xi j$. *Let* $\emptyset \neq X \subset \{p : \mathbb{R}^l \to S^{l-1}\}$. *The relationship* r *has* *the institutional characteristic of* **(Non-Linear) Pricing** *from* X *if its institutional characteristic can be represented by the correspondence* $\mathcal{T} : X \rightrightarrows \mathbb{R}^l$ *where* $\forall p \in X$:

$$\mathcal{T}(p) := \{d \in \mathbb{R}^l \mid p(d) \cdot d \leq 0\}.$$

The institutional characteristic of non-linear pricing potentially allows the dominating agent to choose, amongst others, each of the institutional characteristics we defined in the above. Indeed, it allows the dominating agent to choose any subset of $\mathbb{R}^l$ for the dominated agent to choose the net-trades from. Clearly, this institutional characteristic typically does not satisfy the conditions of our theorem on the existence of equilibrium.

5 Hierarchical Trees

From the examples in the previous section, it is apparent that for the institutional characteristics we defined here, the theorem on the existence of equilibrium does not readily apply, mainly because for some institutional characteristic $\mathcal{T}_r : X_r \rightrightarrows Y_r$ the set X_r or the set Y_r is not bounded. In order to apply the theorem on existence, we would have to restrict the institutional characteristic to bounded subsets of X_r and Y_r. Typically, different restrictions of these subsets may lead to different equilibrium outcomes for the economy. This presents a new problem, which may severely restrict the scope of applicability of the existence theorem. This time the problem does not as much originate from mathematics, but rather from economics.

The question is to what extend it is possible to find R-tuples of pairs $(X_r^*, Y_r^*)_{r \in R}$ of bounded sets with for each $r \in R$: $X_r^* \subset X_r$ and $Y_r^* \subset Y_r$, such that for each R-tuple of pairs of bounded sets $(\tilde{X}_r, \tilde{Y}_r)_{r \in R}$ such that $X_r \supset \tilde{X}_r \supset X_r^*$ and $Y_r \supset \tilde{Y}_r \supset Y_r^*$ we find the same set of equilibria in the corresponding economics in which the institutional characteristics are restricted to the sets $(\tilde{X}_r, \tilde{Y}_r)_{r \in R}$. Thus, we would be able to restrict attention to "sufficiently large" subsets X_r^* and Y_r^* of the sets X_r and Y_r.

As an example, consider the institutional characteristic of mono pricing, and restrict the set of net-trades that are allowed for to a bounded set that strictly contains the set of net-trades that, in absolute value, do not exeed the total of initial endowments in the economy. Then, in line with Spanjers (1994c), the following theorem holds.

Theorem 5.1 [See also Spanjers (1994c, Theorem 4.2)][3]

Let $\mathbf{E}$ *be a hierarchically structured economy that satisfies Assumption 2.6 with relationship structure* $\mathcal{G} := (A, R)$*. Suppose that for each* $r \in R$ *it holds that* $\forall\, p \in S^{l-1}$: $\mathcal{T}_r(p) = \mathcal{T}^{mon}(p) \cap \{d \in \mathbb{R}^l \mid -\sum_{i\in A}\omega_i \leq d \leq \sum_{i\in A}\omega_i\}$*. Let* $\{G_a := (A_a, R_a)\}_{a\in T}$ *be a family of restricitions of* $\mathcal{G}$*, such that*

1. $\forall\, a, b \in T,\ a \neq b:\ A_a \cap A_b = S_1$.

2. $\cup_{a\in T} A_a = A$.

3. $\cup_{a\in T} R_a = R$.

4. $\forall\, a \in T:\ G_a$ *is 2-connected.*

Suppose that for each $a \in T$ *we have* $\sum_{i\in A_a\setminus S_1}\omega_i \gg 0$*. The tuple* $p^* \in (S^{l-1})^R$ *is a tuple of equilibrium price vectors if and only if it consists of prices that are uniform within every* G_a, $a \in T$*, and these prices are a tuple of differentiated (i.e., thrid degree price discriminated) monopoly prices of agent* $k \in S_1$ *for the set of markets* $(A_a \setminus \{k\})_{a\in T}$.

Thus, the main result of Spanjers (1994c) also holds if the set of net trades allowed for by institutional characteristic of mono pricing is restricted to a sufficiently large bounded set. Some of the results however cannot be obtained in the context of the present paper. The reason for this is that in the model of the present paper we do not allow for more than one agent of the same hierarchical level. The situation of two agents of the same hierarchical level not being able to mutually coordinate their net-trades with the other agents in the economy, which leads to (possibly two-sided)

[3]To see the theorem holds in the form it is presented here, it is important to note that if some agent $i \in A \setminus S_1$ sets a price vector q different from the uniform price p, to obtain the consumption bundle x, then he must anticipate his follower to engage in arbitrage in the maximal amount possible. Therefore the following holds: $\sum_{c\in L} q_c \cdot x_c(q) \leq \sum_{c\in L} q_c \cdot \omega_{ic} - \sum_{c\in L} \mid p_c - q_c \mid \cdot(\sum_{j\in A}\omega_{jc}) \leq \sum_{c\in L} q_c \cdot \omega_i - \sum_{c\in L}(q_c - p_c)\cdot\omega_{ic} \leq \sum_{c\in L} p_c \cdot \omega_{ic}$. Thus, it follows that any consumption bundle the agent can obtain by deviating from setting the uniform price vector p for his followers, is also obtainable for him if he sets the price vector p for his followers. Furthermore, it should be noted that arbitrage over a path in the relationship structure works differently form the way described in Spanjers (1994c), due to the different anticipations used in the paper. Essentially, in the context of the present paper, an agent who wants to benefit from arbitrage quotes the price set by the agent "against" whom he wants to perform arbitrage for the corresponding followers. Now, these followers, although being indifferent, shift the corresponding bundle of net trades such that the agent we started out with improves his consumption bundle. In Spanjers (1994c), each of the agents involved in transferring the "arbitrage" bundle of net-trades benifits form doing so.

rationing, is excluded. As is shown in Spanjers (1994c, Corollary 4.4), if $\mid T \mid= 1$ and $\omega_k = 0$, then Walrasian equilibrium is obtained as a special case of the model under consideration.

Still, the kind of situation where an agent may be dominated by at least two other agents may, in general, make it problematic to restrict the sets of net-trades in the kind of way indicated above. Typically, institutional characteristics may be such that an agent optimally buys very much from one of his leaders and sells most of these trades to the other agent at favorable terms, thus having large amounts of commodities "cycling" through the economy. Institutional characteristics may be formulated in such a way that, e.g., in the unrestricted case no equilibrium exists, wheras in the restricted case equilibrium does exist for every "sufficiently large" bounded subset of the corresponding sets of net-trades. In the case of hierarchical trees, the "cycling" of commodities mentioned above cannot occur. Therefore, in the case of hierarchical trees, we may have some hope that we can, without changing the set of equilibria, restrict attention to sufficienty large bounded sets of signals and net-trades. Precise conditions for which this holds are stated in Theorem 5.3.

Consider an economy that has a hierarchical tree as its hierarchical structure. Consider agents $i \in A \setminus S_1$ and $h \in L_i$ who have a trade relationship with institutional characteristic $\mathcal{T}_r : X_r \rightrightarrows Y_r$. Clearly, for any commodity $c \in L$ it holds that agent i, in equilibrium, cannot buy or sell more of this commodity over this trade relationship then the total amount available for the agents in the economy. Knowing this, we may want to restrict the set of net-trades for the institutional characteristic under consideration to the set

$$\tilde{Y} := \{d \in \mathbb{R}^l \mid -\sum_{i \in A} \omega_i \leq d \leq \sum_{i \in A} \omega_i\}.$$

Things, however, are not this easy. It may be that case that some signal chosen by agent h would lead to an optimal reaction of agent i being such that the corresponding net-trades cannot be delivered by agent h. It may also be the case that the restriction of the set of net-trades to $\tilde{Y}$ would restrict the set of net-trades such that these "prohibitive" net-trades are no longer in the restriction of this set, and knowing this, it becomes advantageous for agent h to set a signal that would otherwise not have been optimal.

Therefore, we need a suitable restriction of the set of net-trades. Consider the correspondence $\hat{B}_r : X_r \times \mathbf{R}^l_+ \rightrightarrows \mathbf{R}^l_+$ such that $\forall (s,z) \in X_r \times \mathbf{R}^l_+$:

$$\hat{B}_r(s,z) := \{x \in \mathbf{R}^l_+ \mid (x-z) \in \mathcal{T}_r(s)\}.$$

Furthermore, define the correspondence $x^*_r : X_r \times \mathbf{R}^l_+ \rightrightarrows \mathbf{R}^l_+$ such that $\forall (s,\omega_i) \in X_r \times \mathbf{R}^l_+$:

$$x^*_r(s,\omega_i) := \mathrm{argmax}_{x_i \in \hat{B}_i(s,\omega_i)} U_i(x_i).$$

Assumption 5.2 *Let* $\mathbf{E}$ *be a hierarchically structured economy. Let* $r := \{i,j\} \in R$ *and let* $\mathcal{T}_r : X_r \rightrightarrows Y_r$ *be its institutional characteristic. Then there exists a bounded set of signals* $\tilde{X}_r \subset X_r$ *and a bounded set of net-trades* $\tilde{Y}_r := \cup_{s\in\tilde{X}_r} \mathcal{T}_r(s) \subset \{y \in \mathbf{R}^l \mid -\sum_{i\in a}\omega_i \leq y \leq \sum_{i\in A}\omega_i\}$ *such that for each* $\tilde{\omega}_i \in \{y \in \mathbf{R}^l_+ \mid U_i(y) \geq U_i(\omega_i)$ and $y \leq \sum_{j\in A}\omega_j\}$ *one of the following holds* $\forall s \in X_r \setminus \tilde{X}_r$:

1. $\exists \tilde{s} \in \tilde{X}_r : x^*_i(\tilde{s},\tilde{\omega}_i) = x^*_i(s,\tilde{\omega}_i)$
2. $x^*_i(s,\tilde{\omega}_i) \not\leq \omega$

If no agent is dominated on more than one of his trade relationships in $\mathbf{E}$ and the above assumption holds for each institutional characteristic $\mathcal{T}_r$ with $r \in R$, then we can without loss of generality restrict ourselves to the economy $\mathbf{E}^*$ which equals $\mathbf{E}$ with the exception that each relationship $r \in R$ has the institutional characteristic $\mathcal{T}^*_r : \tilde{X}_r \rightrightarrows \tilde{Y}_r$ with $\forall s \in \tilde{X}_r : \mathcal{T}^*_r(s) := \mathcal{T}_r(s) \cap \tilde{Y}_r$. This leads to the following theorem of existence of equilibrium in hierarchically structured economies with hierarchical trees.

Theorem 5.3 [Existence Theorem for Hierarchical Trees]
Let $\mathbf{E}$ *be a hierarchically structured economy such that Assumption 2.6 holds. Let* $((A,R),\xi)$ *be the hierarchical tree of* $\mathbf{E}$. *Let for each* $r \in R$ *the institutional characteristic* $\mathcal{T}_r : X_r \rightrightarrows Y_r$ *be such that*

1. $\exists n \in \mathbf{N} : X_r \subset \mathbf{R}^n$.
2. $\forall s \in X_r : 0 \in \mathcal{T}_r(s)$.
3. $\exists s \in X_r : \mathcal{T}_r(s) = \{0\}$.
4. *Assumption 5.2 holds.*

Then there exists an equilbrium in $\mathbf{E}$.

Proof

According to Theorem 3.1 an equilibrium in the restricted economy $\mathbf{E}^*$ exists. Suppose this equilibrium is not an equilibrium in the unrestricted economy $\mathbf{E}$. Then some agent can improve in $\mathbf{E}$ by feasible actions that violate Assumption 5.2. But this implies these actions are not feasible in the unrestricted economy $\mathbf{E}$, which yields a contradiction.

$\mathcal{Q.E.D.}$

In the following corollary, as an example, we apply the above theorem on the existence of equilibruim in hierarchically structured economies that have a hierarchical tree as their hierarchical structure, and that have institutional characteristics from the set of mono pricing, bid and ask pricing, monopolistic quantity rationing and take-it-or-leave-it bids. In a similar way, Theorem 5.3 can be applied to other, larger, sets of institutional characteristics. In particular, this holds for the kind of "mixtures", as discussed in Section 4, of the institutional characteristics mentioned above.

Corollary 5.4

Let $\mathbf{E}$ *be a hierarchically structured economy such that Assumption 2.6 holds and* $\forall\, i \in A:\ \omega_i \gg 0$. *Let* $((A, R), \xi)$ *be the hierarchical tree of* $\mathbf{E}$. *Let* $I := \{\mathcal{T}^{bap}, \mathcal{T}^{tol}, \mathcal{T}^{mqr}\}$. *Let for each* $\{i, j\} \in R$ *with* $F_i \neq \emptyset \neq F_j$ *hold that* $\mathcal{T}_{ij} \in I$ *and for each* $\{i, j\} \in R$ *such that* $F_i = \emptyset$ *or* $F_j = \emptyset$ *we have* $\mathcal{T}_{ij} \in I \cup \{\mathcal{T}^{mon}\}$.
Then an equilibrium in $\mathbf{E}$ *exists.*

Proof

The proof of the corollary consists of three parts. First we show that we can, without loss of generality, consider the economy $\overline{\mathbf{E}}$ instead of the economy $\mathbf{E}$. Then we show that for each of the institutional characteristics in the economy $\overline{\mathbf{E}}$ Assumption 5.2 holds. Finally, Theorem 5.3 is applied to prove the existence of an equilibrium in $\overline{\mathbf{E}}$ and therefore in $\mathbf{E}$.

(i) The economy $\overline{\mathbf{E}}$.

Note that the institutional characteristic of mono pricing only occurs on trade relationships where the dominated agent does not have any direct followers. Therefore, there is, by the mean value theorem, some vector of prices for which an optimal reaction of the dominated agent is a net-trade vector of zero. Furthermore, since Assumption 2.6 holds and for each $i \in A: \ \omega_i \gg 0$, we can restrict attention to price vectors in the interior of the l-dimensional unit simplex S^{l-1}. Therefore we can, without loss of generality, replace the institutional characteristic $\mathcal{T}^{mon}$ by the institutional characteristic $\mathcal{T}^{mon*}: \ \operatorname{int} S^{l-1} \cup \{0\} \rightrightarrows \mathbf{R}^l$ such that for each $p \in \operatorname{int} S^{l-1}: \ \mathcal{T}^{mon*}(p) := \mathcal{T}^{mon}(p)$ and $\mathcal{T}^{mon*}(0) := \{0\}$.

Furthermore, by use of Assumption 2.6 and the property that $\forall i \in A: \ \omega_i \gg 0$, we can, without loss of generality replace the institutional characteristic $\mathcal{T}^{bap}$ by the its restriction to the set of price $\hat{P} := \{(\underline{p}, \overline{p}) \in S^{2l-1} \mid \underline{p} \leq \overline{p} \gg 0\}$, which we denote by $\mathcal{T}^{bap*}$.

The economy $\overline{\mathbf{E}}$ is obtained from the economy $\mathbf{E}$ by replacing the institutional characteristic $\mathcal{T}^{mon}$ in $\mathbf{E}$ by the characteristic $\mathcal{T}^{mon*}$ and the institutional characteristic $\mathcal{T}^{bap}$ by $\mathcal{T}^{bap*}$.

(ii) Assumption 5.2.

In this part of the proof we show that Assumption 5.2 holds and use that for each $i \in A$ we have that $\omega_i \gg 0$.

(iia) Extended Mono Pricing $\mathcal{T}^{mon*}$.

Consider some agent $i \in A$ who is the dominated agent in a trade relationship that has the institutional characteristic $\mathcal{T}^{mon*}$ in the economy $\overline{\mathbf{E}}$. The demand function of this agent corresponding to the correspondence x_r^* on the interior of S^{l-1} is a continuous function of prices that satisfies the usual boundary conditions. Therefore, there exists an $\varepsilon > 0$ such that the set $\hat{X}_r := \{p \in S^{l-1} \mid p \geq \varepsilon \cdot 1_l\}$ satisfies Assumption 5.2.

(iib) Take-it-or-leave-it Bids $\mathcal{T}^{tol}$.

Here the set $\hat{X}_r := \{x \in \mathbf{R}^l \mid \mid x \mid \leq \sum_{i \in A} \omega_i\}$ satisfies Assumption 5.2.

(iic) Monopolistic Quantity Rationing $\mathcal{T}^{mqr}$.

For any trade relationship with the institutional characteristic of monopolistic quantity rationing, the set $\hat{X}_r := \{(p, \underline{r}, \overline{r}) \in S^{l-1} \times \mathbf{R}^l_- \times \mathbf{R}^l_+ \mid -\sum_{i \in A} \omega_i \leq \underline{r} \leq \overline{r} \leq \sum_{i \in A} \omega_i\}$ satisfies Assumption 5.2.

(iid) Bid and Ask Prices $\mathcal{T}^{bap}$.

Let $r \in R$ be a trade relationship with the institutional characteristic of bid and ask prices. First, without loss of generality, we restrict ourselves to the set $\hat{P} := \{(\underline{p}, \overline{p}) \in S^{2l-1} \mid \underline{p} \leq \overline{p} \gg 0\}$, of bid and ask prices where the vector of prices for selling is strictly larger than zero. We consider the correspondence x_r^* as defined above resticted to the set $\hat{P}$. Since Assumption 2.6 holds and $\forall i \in A: \ \omega_i \gg 0$, there exists some $\varepsilon > 0$ such that Assumption 5.2 holds with respect to the bounded set $\tilde{X}_r := \{(\underline{q}, \overline{q}) \in \hat{P} \mid \overline{q} \geq \varepsilon \cdot 1_l\}$.

(iii) Existence of Equilibrium.

Theorem 5.3 applies to $\overline{\mathbf{E}}$ and it follows that an equilibrium in the economy $\overline{\mathbf{E}}$, and therefore in the economy $\mathbf{E}$, exists.

$\mathcal{Q.E.D.}$

6 Concluding Remarks

In this paper the problem of existence of equilibrium in hierarchically structured economies is solved for a broad class of hierarchical structures and institutional characteristics. The existence result is obtained by restricting attention to an equilibrium concept that captures the essential features of equilibrium in nice plays as introduced in Hellwig and Leininger (1987), and considers "almost equilibrium" plays.

The results in this paper open the door to a model in which, for a given hierachy, both the relationship structure and the institutional characteristics of the relationships can be endogenized. Such a model would be the result of a straightforward combination of the ideas of Spanjers (1994d, Section 4.2) and the results of the present paper.

In different contexts, considerations about the trade off between the results a mechanism obtains and the costs of operating it can be found in, e.g., Hurwicz (1972). In Karmann (1981), a model with local competitive markets and individualized transportation costs is considered, which can be interpreted as describing the user costs of operating the local competitive markets. More recently, Gilles, Diamantaras and Ruys (1994) consider the costs of trade infrastructures in large economies. As is noted in Spanjers (1994d), our kind of model can be extended to allow for different institutional characteristics leading to different costs of operating these trade relationships, the costs being paid by the dominated agent in the trade relationship under consideration. Thus, within the context of our model, we would obtain a model of an economy in which the institutional structure is endogenized, taking into account the costs of operating the institutions.

A different road for further research would be to apply the insights obtained in this paper to principal-agent settings. For example, one may want to interpret every asymmetric relationship in a hierarchical structure as representing a principal-agent problem with moral hazard. Once again, one may be interested in the existence of equilibrium and, once this problem is solved, in endogenizing the structure of such relationships for a given hierarchy. This would be a step leading us to a formal model of hierarchical organizations as can be found in firms or bureaucraties.

References

BÖHM, V., E. MASKIN, H. POLEMARCHAKIS AND A. POSTLEWAITE (1983), "Monopolistic Quantity Rationing", *Quarterly Journal of Economics, Vol. 98,* pp. 189-197.

BORDER, K. (1985), *Fixed Point Theorems with Applications to Economics and Game Theory,* Cambridge University Press, Cambridge.

DEBREU, G. (1959), *Theory of Value,* Cowles Foundation, Yale University Press, New Haven.

GILLES, R. (1990), *Core and Equilibria of Socially Structured Economies",* Dissertation, Tilburg University, Tilburg.

GILLES, R., D. DIAMANTARAS AND P. RUYS (1994), "Public Aspects of Trade Infrastructures in Large Economies", *Working Paper E94-01,* Virginia Polytechnical Institute and State University, Blacksburg.

GROVES, T. AND J. LEDYARD (1987), "Incentive Compatibility Since 1972", in *Information, Incentives and Economic Organization,* eds. Groves, Radner and Reiter, University of Minnesota Press, pp. 48-111.

HELLWIG, M., AND W. LEININGER (1987), "On the Existence of Subgame-Perfect Equilibrium in Infinite-Action Games of Perfect Information", *Journal of Economic Theory, Vol. 43,* pp. 55-75.

HURWICZ, L. (1972), "On Informationally Decentralized Systems", in *Decision and Organisation,* eds. Radner and McGuire, North-Holland, pp. 297-336.

KARMANN, A. (1981), *Competitive Equilibria in Spatial Economics,* Anton Hain Verlag, Königstein.

KRELLE, W. (1976), *Preistheorie, 2. Auflage,* Paul Siebeck, Tübingen.

NORTH, D. (1990), *Institutions, Institutional Change and Economic Performance,* Cambridge University Press, Cambridge.

SPANJERS, W. (1992), *Price Setting Behaviour in Hierarchically Structured Economies,* Dissertation, Tilburg University, Tilburg.

SPANJERS, W. (1994a), "Arbitrage and Walrasian Equilibrium Hierarchically Structured Economies", *IMW Working Paper, No. 224,* University of Bielefeld, Bielefeld.

SPANJERS, W. (1994b), "Bid and Ask Prices in Hierarchically Structured Economies with Two Commodities", *IMW Working Paper No. 225,* University of Bielefeld, Bielefeld.

SPANJERS, W. (1994c), "Arbitrage and Monopolistic Market Structures", *IMW Working Paper No. 226,* University of Bielefeld, Bielefeld.

SPANJERS, W. (1994d), "Endogenous Structures of Trade Relationships in Hierarchically Structured Economies with Bid and Ask Prices and Two Commodities", *IMW Working Paper, No. 229,* University of Bielefeld, Bielefeld.

WALRAS, L. (1874), *Eléments d'Economie Politique Pure,* L. Corbaz, Lausanne, English translation of the definitive version by William Jaffé (1954), *Elements of Pure Economics,* Allen and Unwin, London.