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## Foundations of a Theory of Prominence in the Decimal System Part II: Exactness Selection Rule, and Confirming Results

by

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### Abstract

The information that is used create a numerical response is typically diffuse, and cannot be described by a distribution. A criterion to describe the information is its range of reasonable alternatives, corresponding to the worst case - best case analysis of practitioners in decision situations where distributions are missing. Empirical data show, that numerical responses in such situations follow a rule that gives conditions for the exactness of the response. The rule says that the exactness is selected such that there are"between 3 and 5 alternatives on this or a cruder level of exactness in the range of reasonable alternatives. This rule permits to predict the exactness of responses, but also permits to deduce on the exactness of information. Once known, it is a powerful tool to inform about information and motives of  $subjects$  from their numerical responses.  $-$  The paper introduces the rule, and gives some empirical examples that support the theory. These examples concern retail price setting of firms, subjects' estimates of numbers of inhabitants of towns, apd a bearing experiment in which different degrees of diffuseness are simulated.

### Contents



## **o Notations**

The prominent numbers are  $\{a * 10^i : a \in \{1, 2, 5\}, i \text{ integer}\}.$  The spontaneous numbers are  $\{a * 10^i : a \in \{1, 1.5, 2, 3, 5, 7, \}, i \text{ integer}\}.$  A presentation of a number is its presentation as a sum of prominent numbers, where each prominent number occurs at most once, and all coefficients are either  $+1$ ,  $-1$ , or 0. The exactness of a presentation is the smallest prominent number with coefficient unequal zero. The exactness of a number  $x \neq 0$  is the erudest exaetness over all presentations of the number. The relative exaetness of a number  $x \neq 0$  is its exactness devided by |x|. The exactness of 0 is  $\infty$ , its relative exactness is 1. A number has level of [relative] exactness,  $r$ , if its [relative] exactness is cruder or equal to  $r$ . A set of data has [relative] exactness  $r$ , if  $r$  is the crudest prominent number such that at least 75% of the data have this [relative] exactness.  $- A$  scale  $S(r, a)$  is the set of 0 and all numbers numbers with (1) relative exactness  $\geq r$ , and (2) exactness  $\geq a$ . Two numbers  $x, y$  in  $S(r, a)$  are identified when their relative difference  $(|y - x|)/max(|x|, |y|)$ is smaller than r. - Examples: The presentation of a number need not be unique, for instance  $17 = 10 + 5 + 2 = 20 - 2 - 1$ . The exactness of 17 is 2. The exactness of 18 is 2, too. 17 and 18 are identified in *S(5%,* 1).

### **1 Exactness Selection Rule**

The level of exactness of a numerical response depends on the exactness of the stimulus. A crude but - as it seems - behaviorally meaningful information to characterize the exactness of a signal is the 'range of reasonable alternatives', i. e. the range between the lowest and highest 'reasonable response'.

Exactness Selection Rule: The relative and absolute exactness  $r, a$  of a numerical response are selected such that there are at least 3, and at most 5 steps of the scale  $S(r, a)$  in the range of reasonable alternatives.

In empirical and experimental situations, where distributions could be measured, the exactness selection rule could be confirmed under the assumption that the range of reasonable alternatives is the range between the end points of the 10% tales on both sides. I. e. the range of reasonable alternatives carries at least 80% of the mass of the distribution. (See Sections 3.2-3.4 below.)

The rule does not give a unique prescription since two parameters  $r, a$  are adjusted to one variable 'extension of range of reasonable alternatives'. This problem disappears for variables which are sufficiently far awy from the zero point, as most measurements of length, height, strength, or decimal measurement of time. In these cases it is reasonable to set  $a := 0$ , so that only r has to be determined. In the other cases one needs at least two decisions to determine r and *s* (How this can be done for arbitrary prospects is shown in part V of the Foundation.

As an instrument of data-analysis the exactness selection rule can also be used to conclude from the exactness of a response (or a set of responses) on the corresponding range of reasonable alternatives.

The criterion 'range of reasonable alternatives', given as the range between 'worst case' and 'best case', seems to fit to the aspiration structure of decision processing of individuals and groups.<sup>1</sup>

The exactness selection rule as introduced above refers to variables which are perceived in a logarithmic way. However, there seem to be also situations where differences of numbers are perceived linearly. (It seems, for instance, that payoffs in characteristic function games can have this character.) In such a situation, there is only one exactness, *a*, which describes the steps of perception. The Scale is  $S(a) := \{i \text{ times } a : i \text{ integer}\}$ , and the exactness selection rule modifies to: '

Linear Version of the Exactness Selection Rule: The absolute exactness *a* of a numerical response is selected such that there are at least 3 and at most 5 steps of the scale  $S(a)$  in the range of reasonable alternatives.

(Note that specificly for  $a = 20$  the scale  $S(a)$  has the steps ..., 0, 20=30, 50, 70=80. 100, 120=130, etc., where steps as '70=80' may be denoted as '70', '80' or '75'.)

Remark: For us it *is* an open problem, *if* there are really situations where subjects feel genuinely linearly. Our impression is rather, that by setting anchor points (see Part V), subject can create linear pieces with given fineness around arbitrary numbers. An interesting point *is* that these linear pieces- as for instance the linear piece of the scale S(26%,*10)* of spontaneous numbers, namely -30, -20, -10, 0,  $+10$ ,  $+20$ ,  $+30$ , typically have a length of at least 5 elements, so that the exactness selection rule selects a completely linear piece around the anthor point. Anyway, the ranges between prominent numbers seem to be typically subdivided in a linear way, as for instance 10,  $12=13$ ,  $15$ ,  $17=18$ ,  $20$ ; (as part of

 $1$ I remember the report of a dealer on financial markets whose group uses worst case - best case analysis as a key instrument of their decisions.

 $S(.11,0)$ , or 10, 11, 12, ..., 19, 20 (as part of  $S(.05,0)$ .

The exactness selection rule could be verified by several experiments and sets of empirical data, which are presented in the following.

#### **Exactness of Price Setting**  $\overline{2}$



Figure 2.1: Range of Reasonable Alternatives versus Level of Exactness<sup>2</sup> of Retail Pricing for Branded and Unbranded Articles of the Food Sector

Empirical Data: Retail prices of 27 branded and 3 nonbranded articles of the foodsector have been picked up in 35 different shops (size between 200 and 3000  $m^2$ ). For every article the the range of reasonable alternatives was defined by the cuts of the 10% tales of the distribution of observed prices of this article. - The exactness of the set of prices of every article was obtained by a weighted median analysis.<sup>3</sup>

<sup>&</sup>lt;sup>2</sup>In the figure 'level of exactness' is denoted as 'prominence'.

<sup>&</sup>lt;sup>3</sup>Assume a single peaked distribution  $f: [0, 100] \longrightarrow R$  is measured with a tool which can only perceive integer values. Assume the perception  $p: R \longrightarrow Z$  always selects the respective nearest integer value. -Let the result of an empirical investigation be given as frequencies  $F$  on  $Z$ . Let  $M$  the traditional median

**Prediction:** Assuming equal distances between any two numbers selected by the exactness selection rule, the prediction is that the length of the range of reasonable alternatives should be at least 2 times as large as the distance  $\Delta$  of any two prices (case:  $l = p_1 - p_2 - p_3 = u$ , where *l, u* are lower and upper boundary of the range of reasonable alternatives,  $p_1$ , ... are the price steps within the range), and at most 5 times as large (case:  $l - p_1 - p_2 - p_3 - p_4 - p_5 - u$  when l and u are between integer steps, assuming that land *u* are at half steps).

Result: The obtained data are presented in Figure 2.1. For all branded articles the length of the range of reasonable alternatives was between 2 and 5 times as large as the amount of the exactness. Only nonbranded articles (as sliced bread, or apples) did not meet this condition. It may be suggested that for nonbranded articles as apples one item combines different qualities, so that a wider distribution of prices is reasonable. The result strongly confirmes the exactness selection rule.

# **3 Exactness ofResponses Concerning the N umber of Inhabitants of a Town**

Experiment: In part 1 of the questionnaire 19 subjects were asked for the number of inhabitants of 7 towns. - In part 2 the subjects were asked to describe their individual distributions concerning the probabilities of different numbers of inhabitants by the following instruction:

"Consider a town, for instance New York. May be that you have some idea of the number of inhabitants, and may be that.you evaluate certain numbers of inhabitants as more likely than others. To describe the corresponding distribution, select different numbers  $x_i$  of inhabitants as possible estimates (for New York you may for instance select the  $x_i$  in steps of 1 Million, 500.000, or in finer steps), and give for each number  $x_i$  a response  $r_i$  that characterizes the intensity of your feeling that  $x_i$  is the right number of inhabitants. (For some of you it may be helpful to create the response as the subjective probability that  $$ among the selected numbers  $-x_i$  is nearest to the true value.) The sum of the responses should add up to about 100, but do not worry, if it does not: we rather prefer to get your spontaneous responses than the correct sum. To compare the individual responses we will anyway normalize the sums of responses to 100 (by multiplication with a factor)."

Predictions: For every subject and every town the following data were considered:

- 1. exactness  $\Delta$  of the response (number of inhabitants),
- 2. exactness  $\Delta'$  of the set of selected numbers  $x_i$  of inhabitants,

of *F* on *Z*,  $F(\leq M)$  the sum of frequencies of values at or below *M*, and *FF* the total frequency. Then the weighted median is defined as  $M + (FF/2 - F( $M$ ))/(F(M)/2 + F(N)/2)$ , where  $N = M - 1$ if  $F(*M*) > F(>M)$ ,  $N = M + 1$  otherwise. - (The concept interpolates between the integer values according to the observed frequencies.)

- 3. the width  $\beta$  of the 80%-interval of reasonable alternatives,
- 4. the number *n* of self-selected numbers  $x_i$  within the 80%-interval of reasonable alternatives.

The relative exactness selection rule predicts:  $\Delta/\beta \leq 5$ ,  $\Delta'/\beta \leq 5$  (or  $\leq 6$ ), and  $n \leq 5$ (or  $\leq 6$ ) (where the last two conditions are nearly equivalent, since the subjects usually subdivided into intervals of equal length). - Concerning the crudeness of response compared to the crudeness of the scale it may be expected that the response should not be more than one step finer than that of the scale.

**Results:** A first result was that the given response was in most cases different from the maximum of the given distribution, there were several cases in which the response was not even inside the area of the 80% most reasonable alternatives. This suggests that there is no fixed distribution which can be recalled from the memory, but that the distribution was constructed on request, sometimes even from information different from that used for the first decision.

Figure 3.1 gives the graphical presentation of a typical response of one subject for one town.



Figure 3.1: Graphical Representation of the Responses of a Subject for a Town (Example)

Table 3.2 shows the frequencies of different numbers of alternatives in the self-created scale. Compared to the prediction there are 13 of 133 cases, where the scale is too crude,

and 2 (or at most 8) of 133 cases, where the scale is too fine. This is significantly different from chance on the 2% level (binomial test, two-sided).







## 4 **A Bearing-Experiment**

Experiment: A scale (from 100 to 300) was given on top of a screen by a horizontal line on which multiples of ten were marked by small vertical marks. The corresponding values (100, 110, 120, ..., 300) were displayed right above the marks. This scale was fixed throughout the experiment. In a given distance from the scale there was another horizontal line with only one mark. The distance of the horizontal line from the scale was modified in 5 steps. For every distance the position of the mark was varied. 22 subjects had the task to identify the position of the mark on the scale by bearing vertically upward, and to answer which point on the scale they obtained. They were asked to "give that response with which they were most content". The distances of the horizontal line from the scale were  $320, 160, 80, 40, 20$  pixels. The position of the mark was varied in a way that the last digit of the number had each of the values 0, 1, 2, ..., 9 with equal frequency, moreover none of the values appeared a second time before all other values had appeared. The first two digits of the position of the mark (for instance the 13 of a position 137) were selected in a way that the position jumped for essential distances from one question to the next. - The question of the experiment was, how the frequencies of the last digits of the responses depended on the distance of the mark from the scale. A priori, every last digit had the same chance, the last digit of ihe position of the mark had the numbers  $0, 1, 2, \ldots, 9$  all with the same frequencies.

Prediction: The bearing process creates an unprecise image of the given signal, which becomes increasingly unprecise with increasing distance. Accordingly, the exactness of response should become finer when the distance is reduced. - Since the nonrectangular part of the bearing follows the principle of similar triangles, the range of reasonable alternatives is for every single response proportional to the distance of the mark from the scale. Accordingly, the absolute exactness of the responses should increase inversely proportional with the distance. In particular, the responses should be multiples of ten for sufficiently large distance, and be near to identity when the distance is low (notice that even then when the mark is on the scale there remains a judgement to decide at which position the mark is, since only multiples of ten are precisely given by the scale).



Figure 4.1: Screen of the Bearing Experiment

Concerning the order, in which numbers should appear with decreasing distance, the first numbers should be the multiples of ten (ending with 0), at smaller distances, numbers ending with 5 should appear. At even smaller distances, in addition 2 and 8 should be answered as last digits (since they have exactness 2; 2 and 8 are more easily constructed compared to 3 and 7, since the given scale has marks at  $10 = 0$ , so that  $2 = 0 + 2$  and  $8 = 10 - 2$  can be identified easier than  $7 = 5 + 2$  and  $3 = 5 - 2$ , since these umbers need the additional construction of the 5 which is not marked on the scale) fhereafter the respective most prominent numbers in the respective ranges, namely 1,9 and 3,7 should appear, and finally 4 and 6.  $-$  The distances at which the respective numbers appear, should be related to the extension of the 80%-interval, which should be a linear function of the distance. The relation should accord to the prediction of the Exactness Selection Rule.

| distance |  |                 |                         | predicted |                |  |         |   |  |     |  |  |  |   |   |                                  | extension of predicted observed |    |  |  |  |  |
|----------|--|-----------------|-------------------------|-----------|----------------|--|---------|---|--|-----|--|--|--|---|---|----------------------------------|---------------------------------|----|--|--|--|--|
| (pixels) |  |                 | responses (last digits) |           |                |  |         |   |  |     |  |  |  |   |   | 80%-interval exactness exactness |                                 |    |  |  |  |  |
|          |  |                 |                         |           |                |  |         |   |  |     |  |  |  |   |   |                                  |                                 |    |  |  |  |  |
|          |  |                 | 001                     |           | $\overline{2}$ |  | 3 4 5 6 |   |  |     |  |  |  |   | 8 | 90                               |                                 |    |  |  |  |  |
|          |  | 20 <sub>0</sub> |                         |           | 2              |  | 3       |   |  | 456 |  |  |  |   | 8 | 90                               |                                 |    |  |  |  |  |
|          |  | 40 0 1          |                         |           | 2              |  |         | 3 |  | 5   |  |  |  |   | 8 | 90                               |                                 |    |  |  |  |  |
|          |  | 80 0            |                         |           |                |  |         |   |  | 5   |  |  |  | 8 |   |                                  |                                 |    |  |  |  |  |
|          |  | 160 0           |                         |           |                |  |         |   |  | 5   |  |  |  | 8 |   |                                  |                                 |    |  |  |  |  |
|          |  | 320 0           |                         |           |                |  |         |   |  |     |  |  |  |   |   |                                  |                                 | 14 |  |  |  |  |

Table 4.2: Predictions of the Bearing Experiment

r

Results: Figure 4.3 shows the extensions of the 80%-intervals (respective medians over all subjects) as a function of the distance of the mark from the scale (in pixels). The corresponding length's at different distances, and the respective predictions for the exactnesses are given in Table 4.2. The observed exactnesses hit these predictions in all cases. - Figure 4.4 shows the frequencies of the last digits of the selected responses. These frequencies show a general pattern, as predicted by the theory, but a detailed analysis shows, that the subjects do not always follow the prediction. There are subjects who select 9 more easily than 8, or 6 more easily than 7. It seems that some subjects also show other motives and preferences for numbers than those induced by the perception according to the theory of prominence. At distance 0 the numbers  $0, 1, 2, \ldots, 9$  were responded as last digits with roughly the same frequencies.



Figure 4.3: Extension of the 80%-Interval as a Function of the Distance



Figure 4.4: Frequencies of Selected Numbers for Different Distances of the Mark from the Scale

Remark: Not only the described experiment was performed, but also a modified experiment performed with the same subjects), in which the marks of the scale were at the multiples of 5 instead of the multiples of ten. Under this condition, the sequence of appearance of numbers with decreasing distance between mark and scale changed: First appeared numbers that ended with 5, then 0, thereafter 3, 7, then 2, 8, 4, 6 and at last 1, 9 appeared as responses. (For details see VOGT+ALBERS 1992.)

The result confirms the procedure described in Part III, Section 1.2.

# **5 Comments Concerninga Micro-Justification ofthe Exactness Selection Rule**

An interesting question is, why people decide according to the exactness selection rule. Our suggestions go into the following directions:

- 1. Assurne the intensity of a numerical one-dimensional stimulus is given by anormal distribution. Assume that a decision maker can observe the distribution with a coarseness or graneness of her choice, for a given coarseness she selects a scale of possible equidistant responses. Assurne that for a given scale there is an (automatically and unconsciously working) mechanism that assigns to every number  $x_i$  of the scale that part of the mass of the distribution which belongs to possible real-valued responses that are nearer to  $x_i$  than to the neighbours of  $x_i$  on the scale. - Let  $x_{(i-1)}, x_i, x_{(i+1)}$  be 3 neighboured numbers of the scale. For 'sufficiently normal' distributions, each of the 3 alternatives  $x_{(i-1)}, x_i, x_{(i+1)}$  is 'quite near' to the maximum, if each of the 3 numbers carries at least (about) 20% of the mass of the distribution. ('Quite near' is meant with respect to the distribution, not with respect to the scale. For instance, for a normal distribution the criterion ensures that the distance of each of the 3 numbers from the maximum is less than  $\ldots$   $\sigma$ .) For several reasons, for instance that distributions need not be symmetric, it may be reasonable not always to select the middle response  $x_i$ , and instead leave the final decision, which of the 3 alternatives to select, to other criteria. .
- 2. A crucial bottle-neck of decision processes is the short-term memory. This addresses the fact that without of further identification and individualisation of the alternatives a brain can distinguish only up to 5 (at most 7) alternatives. (Example: The experimenter gives arbitrary integers between 10 and 100 one after the other. The subject has to repeat the whole sequence after every new number. Usually the first mistake occurs right after the fifth number. - Explanation: one place of some type of short term memory carries the information of one of these numbers. As soon as the number of spaces used is latger than 5, the restriction of the short term memory applies. $4 -$ Assuming that the subconscious decision process selecting the numerical response uses the short term memory (and needs certain information of all alternatives to be present at the same time, so that the formation of chunks cannot help), then it seems reasonable that - by restriction of the exactness of the analysis - the

<sup>4</sup>Note that experienced subjects can build 'chunks', i. e. combine several individual pieces of information to one complex, which helps to memorize longer sequences. But chunks cannot be used as individual pieces of information, they have to be "unpacked", when their information has to be used. Accordingly the aggregation to chunks cannot be used in the type of decision processing considered here.)

number of reasonable alternatives is reduced to not more than 5. A careful decision maker will (unconsciously) select the degree of exactness as fine as possible under this constraint.

By our empirical data we can presently not decide, which of the two principles "finest exactness, but not more than 5 reasonable alternatives", or "crudest scale, but at least 3 reasonable alternatives that carry at least 20% of the mass of the distribution" describes subjects' behavior more adequately.

In this context it may be remarked that the graneness of response is not only given by the task, but also by the way of presention: if, for instance, the bearing experiment is performed with two persohs doing their individual experiments in the same room, the exactness of the responses is slightly, but significantly finer than if the subjects perform their tasks at different times. Moreover, the task seems not"to be independent from the way, how it is presented: we had the impression that the instruction "give that answer with which you are most content" fits best to the idea of individual decision processing. Less precise descriptions of the task seem to generate in some subjects the effort to hit the correct number. Accordingly, we preferred in other experiments such questions which had no correct answers.

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