INSTITUTE OF MATHEMATICAL ECONOMICS

WORKING PAPERS

No. 271

Foundations of a Theory of Prominence in the Decimal System Part V: Operations on Scales, and Evaluation of Prospects

by

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January 1997

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Abstract

The theory of prominence deals with perception of numbers and creation of numerical responses. An important step was the observation, that the step structure generated by the full steps of the scales $S(100\%, 0)$ with the exactness of prominent numbers do not only serve as patterns that describe typical response behavior in practical situations, but do also define a perception function (by interpolation between the integer values). The obtained step structure permits to substructure large ranges of numerical responses by 'iterated halving or doubling' as going from 100 to 200, 500, 1000, 2000, 5000, etc. in one direction and to smaller values 50, 20, 10, 5, 2, 1, etc. in the other; and – using a smallest unit of perception – also permits to measure the distance from zero. If, for instance, the smallest perceived unit is 10, then the step sequence ..., 100, 50, 20, 10, 0, -10, -20, -50, ... is obtained which permits to evaluate distances of positve and negative numbers. The obtained function has high similarity to the evaluation function for money of the theory of KAHNEMAN-TVERSKY (1992), and permits to consider this theory from new theoretical aspects. The same structure can be applied to the perception of probabilities, where the perception of probabilities follows seps as 0, 5, 10, 20, 50, the perception of counterprobabilities follows the steps as 100, 95, 90, 80, 50. Stitching the scales in 50, one obtains a function which is similar to KAHNEMAN+TVERSKY's π-function for the perception of probabilities. – An important feature of this approach is, that the finest perceived money unit, and finest perceived probability unit are not universal constants, but depend on the specific situation. This phenomenon could be used to detect new, and explain old paradoxa of the perception of prospects (see Part IV). But in order to develop the theory of prominence to a general tool to predict the evaluation of prospects, general rules of task-dependend selection of the smallest perceived money unit, and the smallest perceived utility unit (for probability and counterprobability) were necessary. It took us several years of investigation to clear up the problem and finally create a set of data which was short enough to be answered in reasonable time, and on the other hand large enough to cover the central issues of the problem. The result is a first version of a rule system that describes the selection behavior for smallest perceived money and probability unit. Another subproblem was that had to be solved was the evaluation of losses, which is modelled such that the steps within the range of negative payoffs are simply counted double (factor 2 as a constant of nature). – The result clearly supports the theory of prominence as a general model, and in particular shows that the theory can be applied on the evaluation of prospects. It gives a new access to understand the evaluation of prospects, and permits to reflect the approch of KAHNEMAN+ TVERSKY from a new point of view. But it also raises additional questions, for instance, how prospects with more than two alternatives are evaluated.

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0 Notations

The prominent numbers are $\{a * 10^i : a \in \{1, 2, 5\}, i \,\text{integer}\}$. The spontaneous numbers are $\{a * 10^i : a \in \{1, 1.5, 2, 3, 5, 7, \}, i \,\text{integer}\}$. A presentation of a number is its presentation as a sum of prominent numbers, where each prominent number occurs at most once, and all coefficients are either +1, -1, or 0. The exactness of a presentation is the smallest prominent number with coefficient unequal zero. The exactness of a number $x \neq o$ is the crudest exactness over all presentations of the number. The relative exactness of a number $x \neq 0$ is its exactness devided by $|x|$. The exactness of 0 is ∞, its relative exactness is 1. A number has level of [relative] exactness r, if its [relative] exactness is cruder or equal to r. A set of data has [relative] exactness r, if r is the crudest prominent number such that at least 75% of the data have this [relative] exactness. – A scale $S(r, a)$ is the set of 0 and all numbers numbers with (1) relative exactness $\geq r$, and (2) exactness $\geq a$. Two numbers x, y in $S(r, a)$ are identified when their relative difference $(|y - x|)/max(|x|, |y|)$ is smaller than r. – Examples: The presentation of a number need not be unique, for instance $17 = 10 + 5 + 2 = 20 - 2 - 1$. The exactness of 17 is 2. The exactness of 18 is 2, too. 17 and 18 are identified in $S(5\%, 1)$.

1 Operations on Scales, and Evaluation of Payoffs, Losses, Probabilities, and Prospects

1.1 Anchor Points, and Stitching of Scales

We understand the process of creating a numerical response as a procedure during which proposals for possible responses are offered, and accepted or rejected on stepwise refining levels. From the process model we know that the respective current preliminary result of the process is stepwise refined with decreasing exactness, where the limit is given by the limit of possible (or reasonable) judgement of the decision maker.

Now assume a task, where the decision maker is involved in such a way that he can judge whether he likes or dislikes responses, more precisely, assume that

To make the idea of fineness of judgement more precise, we assume that a decision maker can judge whether a given response is 'essentially different' from another response (and we assume that the fineness of analysis which determines the level to perceive a difference as essential can be fixed during a given task). Then it should be possible by some experimental procedure to ask the decision maker for a finest sequence of numerical responses such that every response is (just) perceived as essentially different from its neighbour.

Under certain conditions the obtained scale is not linear. The typical reason for non-linearity are 'emotional reference points' to which subjects measure the distance, where 'essential differences' are given as sufficiently large relative differences.

Typical emotional reference points are the (respective actual) aspiration levels of decision making processes or negotiation processes. It seems that at a certain point of time and for a certain numeric alternative, only one aspiration level can serve as a reference point (more precisely: one reference point for every attended dimension).

Within a strict mathematical approach we presume that the differences to a reference point (called anchor point) are measured in steps of the perception function. I. e.

> **Definition:** x^0 is called anchor point, if the variable x is replaced by $x - x^0$, and $x - x^0$ is evaluated in steps of a scale $M(0, a) = S(1, a)$. These scales are denoted as $M(0, a, x^0)$ or $S(1, a, x^0)$.

Typical examples for universally used anchor points are the 100% value of the percentage scale, and for part of the subjects also the 50% value. Other examples are prospects as [x(50%),y(50%)] where it can happen that subjects select the higher or the lower of the two alternatives as anchor point when they are asked to determine the money equivalent.

Another aspect of anchor points is that they can serve to create linear scales in a neighbourhood of a the reference point. An example: the scale $M(1, 10)$ applied to the anchor point 100 creates the half steps ..., 50, 70, 80, 90, 100, 110, 120, 130, 150, ..., i. e. a range of linear perception in steps of 10 between 70 and 130. Accordingly, it is theoretically possible that a variable locally (over an extended range) permits the impression of linear evaluation, while the global perception is logarithmic.

As mentioned above, it is possible that in different ranges of the space of alternatives different anchor points are selected to evaluate the differences of alternatives. An example: many subjects have 0% and 100% as anchor points for their perception of probabilities. Assuming that the absolute exactness of perception in the anchor points is 5% (on the full step level), we obtain the two pieces 0, 5, 10, 20, 50, 100 (for the probabilities with anchor point 0), and 0, 50, 80, 90, 95, 100 (for the counterprobabilities with anchor point 100). Assuming that 50% is the break even point, we obtain 0, 5, 10, 20, 50, 80, 90, 95, 100. 50% is denoted as stitching point of the pieces of the new scale. Generally we use the

> **Notation:** Scales can consist of subscales where every subscale has a different anchor point. Two scales $S(r,a,x^0)$ and $S(s,b,y^0)$ can be stitched in two ways: either there is one point z^0 which belongs to both scales, such that $S(r,a,x^0) \leq z^0$ is taken from $S(r,a,x^0)$ for the range below z^0, and $S(s,b,y^0) \geq z^0$ is taken from the scale $S(r,a,y^0)$ for values above z^0. In this case the stitching condition is that the distance of the two maximal elements of $S(r,a,x^0) \leq z^0$ equals the distance of the two minimal elements of $S(b,y^0) \leq z^0$.
> or there are two points $v^0 < w^0$ such that v^0, w^0 are the two maximal elements of a scale $S(r,a,x^0) \leq w^0$, and the two minimal elements of $S(s,b,y^0) \geq v^0$.

The condition ensures that in both cases the lengths of steps of the stitched scales are adjusted in a way which is similar to differentiability.

Notice that in a neighbourhood of the stitching point 50% the scale can be characterized as $M(0,20)$ with the steps (0), 20=30, 50, 70=80, (100) (it is irrelevant if 0% or 100% is selected as anchor point). The $M(1,5)$ scales with anchor points in 0% and 100%, namely 0, 2, 5, 7, 10, 15, 20, 30, 50, 70, ... and ..., 30, 50, 70, 80, 85, 90, 93, 95, 98, 100 have the three points 30, 50, 70 in common. This permits to identify the steps 30-50 and 50-70 as being evaluated in the same way, namely as half steps, by both scales.

It may be remarked that some subjects use more than two anchor points for the probability scale, namely not only 0 and 100, but also 50. Höfelmeier (1996) observed that part of his subjects used the scale 0, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 100, which is stitched from the parts $\{0,5,10,20,30\}$, which is part of $S(26\%,5,0)$, and $\{20,30,50,60,70\}$, which is part of $S(26\%,5,100)$. Here we have case b) with two pairs of stitching points.

1.2 Evaluation of 50%-50% Lotteries

Empirical ananlysis of the evaluation of payoffs is not obvious. A natural tool for the evaluation are lotteries, especially 50%-50% lotteries. A reasonable assumption is that the value of the money equivalent of a 50%-50% lottery is 'in the middle' of the money equivalents of the money alternatives of the lottery. Our experimental results confirmed this. However it is not obvious:

Some authors suggest that the money equivalent of a lottery $m[x(50\%), y(50\%)]$ (with $x < y$) is such that the weighted sums of the money-equivalents of the distances to the respective alternatives are equal: $w_1 * m(x) = w_2 * m(y)$, i. e. $m(x)/m(y) = w_1/w_2$. Our results do not confirm such an approach.

KAHNEMAN+TVERSKY (1992) present a π-function, where the value of 50% is about .40, i. e. $v[x(50\%), y(50\%)]$ (with $x < y$) is evaluated as $\pi(50\%)*v(x)+(1-\pi(50\%))*v(y) = .40*v(x)+.60*v(y)$. The main difference between the approach of KAHNEMAN+ TVERSKY, and our approach is, that they model a money equivalent function which maps into the money space:

$me[x(p) + y(q)] = v(x) * \pi(p) + v(y) * \pi(q)$ (measured in money units) while we model perception in the perception space, i. e.

$per[x(p) + y(p)] = per(x) * \pi(p) + per(y) * \pi(q)$ (measured in the perception space)

the corresponding money equivalent is in our approach obtained by mapping the result from the perception space into the money space what is done by the inverse of the perception function:

$$me[x(p), y(q)] = per^{(-1)}(per[x(p) + y(q)])$$

Accordingly, KAHNEMAN+TVERSKY obtain a π-function which at the same time serves to arrange the remapping from the money space into the perception space. In particular, the pi function has to arrange the curvature of the perception function within the range of positive numbers, which forces $\pi(50\%)$ to be below .50. At the same time, the π-function has to arrange the curvature of the negative numbers, and in case that the curvature is different here from the curvature in the range of positive numbers, one needs a different π-function, as KAHNEMAN+TVERSKY indeed do. However, the big problem arises in the range of around the zero-point, where perception is about linear. Since the π-function takes the task to arrange curvature, it enforces curvature, where the data do not want it. How is this done? The trick is that the kink of the evaluation function for money in the zero point can be used to produce the necessary countereffect, which, however, cannot work concincingly.

In fact, there is something qualitatively different going on in the perception of negative numbers. This is modelled by KAHNEMAN+TVERSKY by the kink. However it would be more reasonable to use a different pi-function as a tool for the analysis, which has not the task to arrange the curvature of the money-function.

(It may also be remarked that the evbaluation of 50% as different from .5 enforces a rule of ordering the money alternatives of a prospect. This has to be in KAHNEMAN+TVERSKY's approach to put the number with smaller absolute value first, to obtain the curvature.)

1.3 Evaluation of Losses

There are clear empirical signs that show, that payoffs referring to losses are perceived in a different way than positive payoffs. The literature suggests different ways to classify and evaluate unliked payoffs involved of a prospect.

Approach A assumes that if one evaluates a prospect $[x(p), y(q)]$ with $0 < x < y$ by its money equivalent z, the possible future results of the lottery x, y are perceived as losses or profits with respect to z. In these approaches, x is assumed to be perceived as a loss (compared to z), y as a profit. (This models the situation where the decider thinks about replacing the money equivalent z by the lottery, which is so to say the control phase after having made the decision to replace the lottery by its money equivalent.) It seems reasonable to assume that losses are evaluated higher than profits, i. e. that the money equivalent will be selected such that the distance $z - x$ is smaller than $y - z$. (Compare the introduction of weight factors w1,w2 in the preceding section.)

Approach B assumes that – having the perception function for money as a refineable step function – steps in the range of negative payoffs are evaluated differently from steps in the range of positive payoffs. Where our experimental results support the rule:

> **Negative Payoffs Perception Rule**
> steps within the range of payoffs are perceived as in the range of positive payoffs, steps within the range of negative payoffs are counted double

In a model of linear perception, Approach B can model the ideas of Approach A, if one assumes that the subjects use z as an anchor point. But empirical evidence shows that this is usually not done, and that perception is not linear. Our empirical results support Approach B.

A Remark Concerning the Curvature of the Perception Function in the Range of Negative Payoffs

It should be remarked that it is possible to ask questions concerning prospects with negative payoffs $[x(p), y(q)]$ with $x < y < 0$ in a way that the responses are nearer to x than to y (what contradicts logarithmic perception of negative payoffs). Some of such situations indicate that subjects can use the lower payoff as an anchor point. Other situations support that subjects on 'mirror' the behavior with positive numbers by $per[x(p), y(q)] = -per[-x(p), -y(q)]$. Other situations show a compromise between these attidtudes on an individual level. We model the situation by the mirror approach, and observe that in certain situations the selection of anchor points can leed to different results.

1.4 Selection of Relative Exactness

To apply the theory of prominence as a predictive tool needs the prediction of relative exactness only in situations where the step structure of responses and the step strucure of perceived alternatives is important. One such field is price setting in resale markets, price perception by consumers, and perception of offers in bi- and multilateral negotiations on numerical payoffs.

These tasks are considered Part II of the Foundations of a Theory of Prominence. Central instrument of Part II is the Exactness Selection Rule.

To present this rule needs the introduction of the term 'range of reasonable alternatives':

How do situations, where numerical responses are given, do typically look like? One case – which we do not want to consider here – is that we memorize certain numbers, for instance the number π, in order to perform certain calculations. In this case the information is stored in a digital precise way, and picked up later. A second case – which we also do not consider here – is that a person tries to hit a correct value by his estimation, where the best response is selected from a set of responses, and rewarded. In such a case it makes sense to give a very precise precise answer for strategic reasons. A third case follows the principle to select a number which appears to be as representative as possible for a given set of numbers. If you select a number between 0 and 100 to demonstrate the separation into prime factors, you will probably not select 50, but some number which appears to serve as a typical representative.

We try to model the numerical response that is given if a subject is asked for a numerical response as in 'how many inhabitants has Cairo'. The typical answer is not the description of a distribution, or a response of several numbers or a lower and upper bound in order to describe a range of numbers. The typical response is to give one number. (It seems that this an attitude which helps to perform rough numerical calculations, as needed for decision processing.) The 'crudeness' of the response informs about the sureness of the response. For instance the response '6 Millions' informs that the judgement has a fineness of at least 1 Million. (In this context it may be remarked that subjects sometimes seem to avoid crude responses as 5 Millions, when they have a very precise judgement as 'the number should be between 4.5 and 5.5 Millions'. In this case they rather decide to respond 4.5 or 5.5 Millions instead of 5 Millions in order to inform better about the preciseness of their knowledge.)

Experimentally we ask the subjects to 'give that response with which you feel most content'. The exactness of the response then depends on the 'range of reasonable alternatives' which a subject (or a decision maker) does not immediately rejected as wrong answers. Practitioners use the terms 'worst case' and 'best case' for the lower and upper bound of this range. (Own empirical investigations suggest that the range of reasonable alternatives of a given distributution is the range obtained by excluding the 10% most unlikely alternatives on both ends of the distributions. But it may be also possible that decision makers adjust the percentage of excluded tails according to their success in previous decisions; they take more extreme alternatives into account when they excluded them in previous unsuccessful decisions.) Correspondingly we get the

Notation: The range of reasonable alternatives is the range of reasonable numerical responses which are not obviously excluded. – When a density function is given, subjects seem to exclude the 10% most unlikely cases at both ends of the distribution.

Based on this notation the following rule can be formulated

Relative Exactness Selection Rule
The relative exactness of a numerical response is selected such that there are between 3 and 5 numbers with this or a cruder exactness in the range of reasonable alternatives.

This rule informs about the preciseness reached in the anaylysis. It says that finer numbers are not considered. The rough idea behind the condition is that at least 3 neighboured numbers (of a given step structure) with sufficiently high intensity of response are needed to make sure to be sufficiently near to the maximum; and more than 5 numbers seem to exceed the short term memory.

The rule has been supported by several empirical investigations.

1.5 Selection of Absolute Exactness

Knowing the general shape of the perception function, absolute exactness is the crucial variable that determines the evaluation of payoffs. The theory of prominence as a tool for prediction needs a prediction of the absolute exactness of perception.

There are several numerical decision situations where the absolute exactness is irrelevant, for instance the (theoretical) evaluation of a prospect $[50(50\%), 100(50\%)]$ is independent from the absolute exactness. (By theory of prominence it is always 70.)

An approach which suggests itself is the extension of the 3-5 alternatives rule to cases where one of the alternatives is nearer to the zero-point, or where alternatives at both sides of the zero point are possible. However then two parameters (absolute and relative exactness) would have to be selected by one condition. – Accordingly, the system is free to get a special condition for absolute exactness.

The first impression from empirical data is that absolute exactness is usually not finer than 3 full steps below the maximum of the absolute values of the alternatives. In most cases it is between 2 and 3 full steps, and there are some situations with linear perception (i. e. absolute exactness 1 full step below the maximum of the absolute values).

To solve the problem of an absolute exactness selection rule we investigated for several years. We now have first results but are not sure that the final formula is found. Accordingly the next paragraphs may serve as an outline of the theory which may need modification in one or another detail.

As the prospect theory of KAHNEMAN+TVERSKY the new theory needs the parallel solution of the perception of numerical payoffs and probabilities. Different from them, we assume that general numerical perception follows the perception functions as predicted by the theory of prominence, and after asuming that negative payoffs are evaluated as described above, the only parameters that can be adjusted (and have to be determined) are the absolute exactness of the money and the probability scale. As could be shown in Part IV that the absolute prominence is not a universal constant, and this fact served to find new, and explain well known paradoxa of prospect perception.

The reader cannot expect a clean mathematical rule, but rather a decision-tree of boundedly rational behavior showing how subjects solve conflicting criteria as a function of given parameters.

1.6 Evaluation of Prospects

A prospect $[x_i(p_i) : i = 1, \ldots, n]$ is a lottery in which exactly one of the monetary payoffs $x_1, \ldots, x_n$ happens, and the probabilities of $x_1, \ldots, x_n$ are $p_1, \ldots, p_n$.
The evaluation of such prospects is as in the approach of KAHNEMAN+TVERSKY. There is an evaluation function for monetary payoffs, v, an evaluation function for probabilities, p_i, and the two components are as usual multiplied and added over all events:

Evaluation of Prospects $[x_i(p_i) : i = 1, \ldots, n]$:

1. evaluation of payoffs via $per(x)$: one anchor point (0 DM), analysis on full step level (perception of halves and quarters is possible), absolute exactness given by rules of next section
2. evaluation of probabilities via $\pi(p)$: two anchor points (0%,100%), parts stitched in 50%, analysis on full step level (perception of halves and quarters is possible), absolute exactness identical in both anchor points (necessarily?), absolute exactness given by rules of next section
3. evaluation of prospect:
   $per[x_i(p_i) : i = 1, \ldots, n] := \sum((per(x_i) - per(x_i - 1)) * \pi(P_i) : i = 1, .., n)$
   after ordering $x_1, \ldots, x_n$ decreasingly, $P_i := \sum(p(j) : j = 1, \ldots, i)$
4. money equivalent: me[prospect] $:= per^{(-1)}(per$[prospect]$)$

From the theoretical point of view it should not matter whether the evaluation in step 3. is presented as $\sum((per(x_i) - per(x_i - 1)) * \pi(P_i) : i = 1, \ldots, n)$ or as $\sum(per(x_i) * (\pi(P_i) - \pi(P_{(i-1)})) : i = 1, \ldots, n)$. Behaviorally this might matter. Since we are not sure, how the aggregated probabilities are computed. Experiments show that subjects percieve 10% as two of the steps 0, 5, 10, 20, 50, 80, 90, 95, 100, i. e. 2 of 8 steps, and perceive 20% as 2 of the steps 0, 10, 20, 50, 80, 90, 100, i. e. 2 of 6 steps. But what happens, if both probabilities are involved in one task? Will subjects then evaluate 20% as 3 of the steps 0, 5, 10, 20, 50, 80, 90, 95, 100, i. e. as 3 of 8 steps? In the analysis of the choice of prospects (see Part IV, Section 3) a similar effect happens: subjects select the finer of two exactnesses when two prospects are considered at the same time. The approach above permits that the probabilities are perceived separately with possibly different smallest probability units. – The alternative to use differences of probabilities (instead of differences of payoffs) only makes sense, when the probabilities are evaluated on a common scale, i. e. with a common finest probability unit for all probability evaluations. (We cannot clarify this point at the present state of our experimental investigations.)

An example illustrating such an evaluation is given in Figure E. In this figure we presume universal finest perceived money unit (5 TDM) and a universal finest perceived probability units (5%) for the complete evaluation.

`Figure E: Cumulative Evaluation of` [50(5%), 30(75%), 10(15%), 1(5%)] `1)`

```
 [TDM]
 100  +--------+--------------------------------------+--------+
      |////////|                    .                 |////////|
  70  +///////                      .                  ///////|
      |////////|                    .                 |////////|
  50  +///////o---o                 .                  ///////|    1*1/8= 1/8
      |////////|  |                 .                 |////////|
  30  +///////    o---+---+---+---o                    ///////|    3*5/8=15/8
      |////////|                  . |                 |////////|
  20  +///////                      . |                ///////|
      |//////me - - - - - - - - - - | - - - - - me//////|
  15  +///////                      . |                ///////|
      |////////|                  . |                 |////////|
  10  +///////                      . o---+---o        ///////|    4*7/8=28/8
      |////////|                  .         |         |////////|
   7  +///////                      .         |        ///////|
      |////////|                  .         |         |////////|
   5  +///////                      .         |        ///////|
      |////////|                  .         |         |////////|
   3  +///////                      .         |        ///////|
      |////////|                  .         |         |////////|
   2  +///////+ - - - - - - - - - - - - - - x - x///////|    0*1/8= 0/8
      |////////////////////////////////////|///|////////|    -----------
   1  +////////////////////////////////////o---o///////|    per=   44/8
      |/////////////////////////////////////////////////|    me= 17 TDM
   0  +---+---+---+---+---+---+---+---+---+---+---+---+
      0   1   2   5  10  20  50  80  90  95  98  99  100    [\%]
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1) the hatched areas are excluded by smallest absolute units of probability (5%), counterprobability (5%), and money (5TDM) ('hick zero lines')

2) the line o--o--o--o gives the initial problem, me---me the money equivalent

Remark: The same procedure can be used to determine a probability equivalent, i. e. a lottery $[100(p)]$ with identical value. For the example of Figure E the result is given by the dotted vertical line at $p = 60\%$.

2 Rules for the Finest Perceived Absolute Money and Probability Units - Results from an Experiment

Our aim is to develop universal rules for the finest perceived money unit (FPMU) and the finest perceived probability unit (FPPU). From our experimental results we know, that these units are not generally universal, but depend on the special problem, here on the given prospect. However, we think that a general rule governs the FPMU-selection function and the FPPU-selection function. The following experiment is our first attempt to develop this universal rule. It has been prepared by several pre-experiments.

The Experiment

In the following we analyze the answers of a questionnaire presented to 20 20 students of business administration and mathematical economics answered a questionnaire. The questionnaire was presented in the beginning of the experimental part of a project seminar, in which the students performed experiments for about 20 hours on different days, thereafter jointly analyzed and tried to model their behavior for another 20 hours within the seminar, and finally had to write seminar papers on self selected topics concerning the results. Thereby subjects were highly motivated to inform about their money equivalents correctly and carefully. The questionnaire session took about 4 hours.

The subjects were asked for their money equivalents of several lotteries. The lotteries were selected with the intention to cover an extended range of possible prospects. Since the theory of prominence is presently not developed in a way to predict how subjects perceive numerical stimuli that are finer than spontaneous numbers (how do they round, truncate), we decided to use only prominent numbers as numerical inputs of the prospects.

The question for the money equivalent was separated into two questions:

Assume in front of you on a table there is the given prospect $[x(p), y(q)]$ on one side, and an amount z of money on the other. You may be able say whether you prefer the the money amount or the lottery for every money amount z. Please give

case (a): the highest money amount at which you prefer the lottery
case (b): the lowest money amount at which you prefer the money

In other words, there may be a range of alternatives which are such that you cannot decide. Please inform us about the upper or lower bound of this range.

The obtained answers in case (a) and (b) have the property: buying price < case (a) < money-equivalent < case (b) < selling price. In prestudies we got the impression that the buying condition is much more influenced by secondary arguments as intended profit than the selling condition, and that condition (b) reflects the 'true' money equivalent better than condition (a). Since we had to restrict the set of questions, we decided to ask only case (b) for all prospects, and case (a) only for certain selected prospects.

(The reader may ask, why we did not apply the MARSCHAK-procedure, i. e. picked up the data as bids for a lottery. There are two problems with this procedure: 1. the lotteries here involve losses, and 2. we are interested in selection behavior for amounts that are really substantial, i. e. for students above DM 1000 (roghly 650 Dollars), since we had observed linear perception for too low amounts in pre-experiments. The only experimental condition we could afford, which fulfilled conditions 1. and 2., was to condition the verification of the decision on the fact that a lotto coupon won sufficiently much to give the subject the necessary initial endowment to cover potential losses. In fact, we started the session with this condition, but found that the induced behavior with the condition that you loose money that you (just) received, is different from loosing 'own money', and therefore returned to hypothetical questions (as they have also been used by KAHNEMAN+TVERSKY in their study). It was made clear that the subjects should adjust to the situation where the losses had to be paid by themselves, not by some rich aunt or grandmother (who sometimes helps in close financial situations). In order to make subjects aware of the situation they were motivated to discuss for some time, how they would cover possible losses of different amounts.

The absolute amounts of the nonzero payoffs involved in the prospects ranged between 500 and 10000 DM. It may be remarked that we observed a behavior which is nearer to evaluation according to expected payoffs, when the money amounts are less substantial. We distinguish the following ranges of payoffs:

(1) for $|x|, |y|$ in $\{0\} \cup [1000DM, 10000DM]$ the theory is developed below
(2) for $|x|, |y|$ in $\{0\} \cup [100DM, 1000DM]$ first steps into the direction of linear perception of money can be observed (see the data below)
(3) for $|x|, |y|$ in $\{0\} \cup [10DM, 100DM]$ the perception of payoffs seems to be essentially more linear (results from pre-experiments, for instance subjects evaluate the lottery $[20DM(50\%), -20DM(50\%)]$ near to 0)

Accordingly we separate questions into those which belong to type (1) and (2).

The analysis involves 19 subjects (one of the 20 did not answer the questions correctly), 7 probability conditions

$$(p, q) = (99\%, 1\%), (90\%, 10\%), (80\%, 20\%), (50\%, 50\%), (20\%, 80\%), (10\%, 90\%), (1\%, 99\%)$$

crossed with 14 money conditions of type (1)

$$(x, y) = (10, 5), (10, 1), (10, .5), (10, 0), (10, -.5), (10, -1), (10, -5), (10, -10), (1, -5), (1, -10), (0, -10), (-.5, -10), (-1, -10), (-5, -10)$$

and 6 money conditions of type (2) (where cases with numbers below 5 are omitted):

$$(x, y) = (10, 5), (10, 0), (10, -5), (10, -10), (0, -10), (-5, -10).$$

The subjects gave the case (b) answers for all questions, and the case (a) answers for $(x, y) = (10, 5), (10, 0)$,
$(10, -10), (0, -10), (-5, -10)$.

Procedure

The difficulty of the analysis was that we had to solve several problems at once:

1. evaluation of negative payoffs
2. possibly different evaluation of amounts below and above money equivalent
3. rule for the selection of FPMU
4. rule for the selection of FPPU
5. question if $\pi(1-p) = 1 - \pi(p)$ should be posed as a condition

Problems 1. and 2. were solved by other investigations (see for instance the experiments in Part IV). Ad 1.: payoffs in the range of negative numbers are simply counted double. Ad 2.: except from 'ad 1.' there are no different evaluations of payoffs below or above the money equivalent.

First part of the Analysis were the prospects with $(p, q) = (50\%, 50\%)$. For these prospects we were quite sure in advance that the p_i-value should be at (about) 1/2 (compare the results in Section V). This value of p_i enabled to make first conjectures concerning the perception function.

A very helpful tool were the four questions with the money amounts (10,5), (-10,-5). For these money-data the step structure of perception is independent from the FPMU, since there is no prominent number between the alternatives. This gave us for every case of probabilities (p, q) a set of 4 prospects which could be used to get a (first) estimate of the evaluation of the probabilities (p, q). In fact, these estimates fitted to the other data.

Thereafter we had to find for every specific prospect that pair of FPMU and FPPU which explained the data best. When several pairs were nearly optimal, we selected that, where the FPPU was the same as obtained above. This gave us FPMU-values for all prospects (see Table 5.1). The table suggests that usually probabilities and counterprobabilities are perceived in the sameway. But this does not hold for $(80\%, 20\%), (20\%, 80\%)$. Accordingly we considered the p_i-values for the cases $(p, q) = (99\%, 1\%), (90\%, 10\%), (50\%, 50\%), (10\%, 90\%), (1\%, 99\%)$ as given, and permitted the two observed alternatives $p_i(20\%) = 1/4$, $p_i(20\%) = 2/8$ within the next part of the investigation process, in order to decide between them later.

Next task was, to investigate whether the obtained FPMU data could be explained by a general behavioral model. How would this model look like? Which noise in the data should we expect? The obtained model of FPMU-selection is given below. It has (for non-50%-50% prospects) 4 parameters to explain a data set of 12*4= 48 data (after extracting those cases where the FPMU does not effect the choice, and not considering the cases $(p, q) = (80\%, 20\%), (20\%, 80\%)$). (The obtained 'formula' deviates from the data in 3 of the 48 entries.) The FPMU-values are presented in Table 5.3.1. The FPMU-values for $(p, q) = (80\%, 20\%), (20\%, 80\%)$ are entered identical to those of the respective cases $(p, q) = (90\%, 10\%), (10\%, 90\%)$. (In one case, (-.5,-10) with (80%,20%) the data fit essentially better, if one replaces 10% by 20%.)

Having obtained the rules of FPMU-selection, and fixed the FPMU-values for the prospects, we can check for every prospect, inhowfar the obtained 19 responses of the subjects support the theory. One criterion is the p_i-value that explains the median of the responses best. (These medians are given as first entries in the brackets of Table 5.3.1.) Another criterion is, 'how far away' the result is from the median. This is measured by the (smallest) number of players who has to change their opinion to obtain the theoretical result as the median, where the theoretical predictions are clear for the all cases, except for the probabilities $(p, q) = (80\%, 20\%), (20\%; 80\%)$, since there are two conflicting predictions (here we made the computations for both cases). (Notice that zeros are replaced by a dot, the insert 'r' refers to cases where subjects replaced the theoretical response by a more prominent number ('rounding'), since it seems reasonable to admit rounding as a behavioral attitude, the necessary changes of position which can be explained by taking the rounding process backword, are not counted. If this rule is applied this is denoted by the insert 'r' before the distance.) The obtained data strongly sipport the developed there are only small deviations (usually below 2) remaining, the median deviation is 0 in every column.

At this point of analysis it is also possible to extend the FPPU-rule to the cases $(p, q) = (80\%, 20\%), (20\%, 80\%)$ (see there), the data clearly suggest the behavior given in the rule.

The described process of data analysis shows that the given data do not permit an essentially different interpretation.

For the prospects of type (2) (smaller payoffs), the best predictors are not all identical to those of type (1). The corresponding values are given in Table 5.3.2, the differences can be seen in Table 5.4.

Results

The following conditions hold for prospects $[x(p), y(q)]$ with substantial payoffs x, y. (Our data suggest that – for students – substantial payoffs are in the range between DM 1000, and DM 10000.)

> **Selection of Finest Perceived Money Unit (FPMU)**
> **('FPMU Selection Rule')**
> Consider $[x(p), y(q)]$. Let MAX the smallest prominent number $\geq \max(|x|, |y|)$. FPMU is a prominent number. It is as fine as possible, subject to the following conditions (which have to be fulfilled in hyrarchical order)
>
> Linear Perception

(X) ('dominant negative value') FPMU is 1 step below MAX, if $y < 0$ and $|y| \geq |x|$ and $q \geq 80\%$

Normal Case

(A) ('upper boundary') FPMU $\leq$ two steps below MAX (follows from (X))

(B) ('necessity') FPMU $> \min(|z| : z \in [x, y])$

(C) ('lower boundary') FPMU $\geq MAX/10$

(D) ('five steps') there are at most 5 full steps cruder than FPMU in $[x, y]$

50%-50%-Lotteries (separate case)

(E) ('normal 50%-50% lottery') FPMU is 2 steps below MAX, except the cases

(F) ('large negative value') perception is linear, if $x < 0 < y$ and $|x|/|y| \geq 4$

(G) ('small negative value') FPMU is 3 steps below MAX, if $x < 0 < y$ and $|y|/|x| \geq 4$

An interesting result is, that the behavior is for 50%-50% prospects different from the other prospects.

(X) gives the condition under which linear perception takes place. This is the case when a negative number with sufficiently large absolute value ($|y| > |x|$) has sufficiently large probability.

For non-50%-50% prospects the behavior can be characterized by 4 rules the 'upper boundary condition' follows from (X). 'Necessity' says that the fineness of the FPMU should be induced by the problem, the FPMU/2 (which is perceived on the level of spontaneous numbers) should not be finer than the smallest number involved in the prospect. 'Lower boundary' restricts the fineness byat most one step below the ordinary case, i. e. two steps below the linear case. the 'five steps condition' picks up the condition of the Relative Exactness Selection Rule.

The predicted values are entered in Table 5.3.1 and permit to conclude on those p_i-values that would have explained the respective median evaluation best. These values are denoted as 'observed p_i-values'. They permit to identify the rule system of the finest perceived probability unit:

Selection of Finest Perceived Probability Unit (FPPU) ('FPPU Selection Rule')

Normal Case: FPPU is largest prominent number strictly below $MIN := \min(p, q)$

Exception: FPPU $= MIN$, if $MIN \leq 1\%$, or ($p = 20\%$ and $x \geq$ and $|x|/|y| \geq 1/10$)

These rules are comparatively simple. There is a normal case, in which the largest prominent number below both probabilities of the given prospect is selected. If at least one of these probabilities is 'very fine' ($\leq 1\%$), then FPPU is by one step cruder. Moreover, the

FPMU is one step cruder when $p = 20\%$ the money payoff obtained with 20% probability is sufficiently large compared to the other money payoff:

The following Table 5.1 shows the stepstructure induced by the prediction, the corresponding p_i-values, their distances to the respective nearest values 0 or 1, and the corresponding empirical data (medians of that pi-values that would explain the respective obtained data best) for lotteries with payoffs in the range DM 1000-10000 / range DM 100-1000.

Table 5.1: Observed Finest Perceived Probability Unit (FPPU)

(p,q) 1)	full steps of probability	FPPU	pi(p)	distance 2)	median 3)
(99%, 1%)	0-1-2-5-10-20-50-80-90-95-98-99-100	MIN	11/12=.917	1/12=.083	.08/.06
(90%,10%)	0-----5-10-20-50-80-90-95-------100	MIN/2	6/8=.750	2/8=.250	.25/.20
(80%,20%)	0-------10-20-50-80-90----------100	MIN/2	4/6=.667	2/6=.333	.33/.30
(50%,50%)	0----------20-50-80-------------100	MIN/2	2/4=.500	2/4=.500	.50/.56
(20%,80%)	0----------20-50-80-------------100	MIN	1/4=.250	1/4=.250	.25/.34
(10%,90%)	0-----5-10-20-50-80-90-5--------100	MIN/2	2/8=.250	2/8=.250	.24/.25
(1%,99%)	0-1-2-5-10-20-50-80-90-95-98-99-100	MIN	1/12=.083	1/12=.083	.07/.09

1) with p>q, except for (p,q)=(0,-10),(-.5,-10)
2) distance of pi(p) from respective next value 0 or 1
3) median over all conditions for lotteries of type [x*10000(p), y*10000(q)] / [x*1000(p), y*1000(q)] (payoffs in DM)

The next table gives the respective 'observed p_i-values' (that explain the obtained median) for all performed prospects. (These are the first two digits of the entries of the bracket. The other entries in the bracket refer to the distance of the median from the prediction measured by '# of deviating subjects'.)

Table 5.2 informs about the frequencies of different 'observed p_i-values' where every condition (p, q) gives one entry (by the median of the responses). This permits to get an impression of the exactness, with which the predictions are met by median behavior. It also partially answers the question, inhowfar the MIN-rule or the $MIN/2$-rule can be clearly separated from one another.

Table 5.2: p_i-Values of $\min(p, q)$ for the Lotteries $[x(p), y(q)]$ (# of Cases)

```
                  pi-values.......................................                     lt-in-gt
   ( p,q )   mult .0      .10     .20     .30     .40     .50     .60 median [MIN/2,MIN] [.,.] 2)
-------------------+---+---+---+---+---+---+---+---+---+---+---+---+-------------------------------

                           m
(99%, 1%)  1000    .   .41331 1. 1 .   .   .   .   .   .   .   .   .   .08 \
-------------------+---+->p<--p+---+---+---+---+---+---+---+---+---+------ .08  >.08< .17   (8)10-2
            100    .   .3  1  1. 1 .   .   .   .   .   .   .   .   .   .08 /                  3)
                         mm                                  *)

                                       m
(90%,10%)  1000    .   .   .   .   2   7 112   .1  .   .   .   .   .   .25 \
-------------------+---+---+---+-p-+-->p<--+---+---+---+---+---+---+------ .25   .17 >.25<  0-15-5
            100    .   .   .   . 12.1  2   .   .   .   .   .   .   .   .20 /
                                   m

                                              m
(80%,20%)  1000    .   .   .   .   .   .  212132 2 1   .   .   .   .   .33 \
-------------------+---+---+---+---+---p---+->p<---+---+---+---+---+------ .32   .25 >.33<  0-15-5
            100    .   .   .   .   .   21 1.1 1.   .   .   .   .   .   .27 /
                                         m
                                                            m
```

```
(50%,50%)  1000  .    .    .    .    .    .    .    .    . 1 . 1 9  11   1   .50 \
-----------------+----+----+----+----+----+----+----+----+-->p<--+----+------ .50  >.50  .50<   2-10-8
            100  .    .    .    .    .    ..   .    .    .    1  1.211.    .56 /
                                                                  m

                                      m
(20%,80%)  1000  .    .    .    . 1 .1  7 13.   .1  .    .    .    .    .   .25 \
-----------------+----+----+----+----+-->p<--+--p+----+----+----+----+------ .25  >.25< .33    3-14-3
            100  .   ..    .    .   .1  11   .1   . 2 .    .    .    .    .   .28 /
                                         m

                                  m
(10%,90%)  1000  .    .    .  1. 21221 5    .    .    .    .    .    .    .   .21 \
-----------------+----+----+----+-p-+-->p<--+----+----+----+----+----+------ .25   .17 >.25<   1-17-2
            100  .    .    .    :    .  4   1.   . 1 .    .    .    .    .   .25 /
                                        m

                      mm
( 1%,99%)  1000  .   313312 1 .    .    .    .    .    .    .    .    .    .07 \
-----------------+----+->p<--p+----+----+----+----+----+----+----+----+------ .07  >.08< .17   (9)-7-0
                 .    .211. 2 .    .    .    .    .    .    .    .    .    .08 /              3)
                       mm
--------------------------------------------------------------------------------------------------
*)  the prognostified pi-values MIN/2,MIN are marked by 'p', the respective medians by 'm'
2)  # of cases below, within, above the range [MIN/2,MIN]
3)  the given step structure of responses does not permit to distinguish between .06 and .08
```

For type (2) lotteries perception is basically similar, although it tends somewhat more into the direction of linearity. (See Table 5.3.2). The differences of the two money-conditions for the FPMU are shown in Table 5.4.

Table 5.3.2: Finest Perceived Money Unit as Proportion of $MAX := \max(|x|, |y|)$ for Different Lotteries $[x(p), y(q)]$, and Corresponding Predictions of p_i-Values (using the respective FPMU, for which the deviation from the p_i-values above is minimal)

case 2: lotteries of type [x*100(p), y*100(q)] (payoffs in DM)

(p,q):	(99%, 1%)	(90%,10%)	(80%,20%)	(50%,50%)	(20%,80%)	(10%,90%)	(1%,99%)
distance to:	.08	.25	.25;.33	.50	.25/.33	.25	.08
(x, y)							
(10, 5)	*) (06..)	*) (25..)	*) (31+1;..)	*) (56+1)	*) (31+1 ..)	*) (37+2)	*) (12..)
(10, 1)							
(10, .5)							
(10, 0)	lin (06..)	20% (25..)	20% (33+2;..)	20% (58+2)	lin (37+2 ..)	lin (25..)	lin (06r.)
(10,-.5)							
(10, -1)							
(10, -5)	20% (14+3)	20% (21-3)	20% (29+1;r.)	20% (57r.)	20% (21r. r.)	20% (29..)	20% (07..)
(10,-10)	20% (17r1)	20% (19r1)	lin (29+6;-2)	20% (56r.)	lin (26+1 r.)	lin (25..)	lin (08..)
(1, -5)							
(1,-10)							
(0,-10)	20% (06..)	10% (25..)	lin (25..;r.)	20% (54r.)	lin (38+8 ..)	lin (17-3)	lin (10..)
(-.5,-10)							
(-1,-10)							
(-5,-10)	*) (06..)	*) (19-1)	*) (25..;r.)	*) (50..)	*) (37+4 ..)	*) (25..)	*) (12+1)
median 2)	(06..)	(20-.)	(30+1;r.)	(56+.)	(34+1 ..)	(25..)	(09..)

footnotes as in the following table

Table 5.3.1: Finest Perceived Money Unit as Proportion of $MAX := \max(|x|,|y|)$ for Different Lotteries [x(p),y(q)] 1) and Corresponding Predictions of p_i-Values - Assuming the FPMU-Selection Rule as given

case 1: lotteries of type [x*1000(p), y*1000(q)] (payoffs in DM)

(p,q): / distance to:	(99%, 1%) .08	(90%,10%) .25	(80%,20%) .25;.33	(50%,50%) .50	(20%,80%) .25;.33	(10%,90%) .25	(1%,99%) .08	SPU /MAX	full.steps in [x,y]	(*)	applied conditions norm	line	50%
(x, y)	absolute value of positive payoff is higher												
(10, 5)	*) (06..)	*) (20-1)	*) (37+4;..)	*) (50..)	*) (25..;r.)	*) (25..)	*) (12..)	*)	5X	(1)	(A)	-	E
(10, 1)	20%'(08..)	20% (20r.)	20% (35+4;..)	20% (50..)	20% (25..;r.)	20% (20r.)	20% (10..)	20%	-25X	(2.5)	B	-	E
(10, .5)	10% (07..)	10% (25..)	10% (29r.;r.)	20% (50..)	10% (28r.;r.)	10% (21..)	10% (04..)	10%	-125X	(3.5)	B(C)	-	E
(10, 0)	10% (06..)	10% (25..)	10% (31+4;..)	10% (50..)	10% (28+1;r.)	10% (25..)	10% (04-1)	10%	0125X	(4)	C	-	E
(10,-.5)	10% (10..)	10% (30+1)	10% (30+2;..)	10% (50..)	10% (25..;-3)	10% (20r.)	10% (05r.)	10%	-0125X	(4.5)	C(D)	-	F
(10, -1)	10% (08..)	10% (25..)	10% (33+4;..)	10% (42-2)	10% (25..;-4)	10% (25..)	10% (04-2)	10%	10125X	(5)	C(D)	-	F
(10, -5)	20%'(14+3)	20% (36+2)	20% (34+5;..)	20% (50..)	20% (21r.;r.)	20% (17-1)	20% (07..)	20%	52025X	(5)	D	-	E
(x, y)	absolute value of negative payoff is higher												
(10,-10)	20%'(17r1)	20% (28r.)	20% (35+6;+2)	20% (50..)	lin (25..;-3)	lin (17-3)	lin (08..)	20%	X52025X	(6)	A	X	E
(1, -5)	20%'(11..)	20% (25..)	20% (29..;..)	lin (47r.)	lin (17-1;-7)	lin (22..)	lin (10..)	20%	52101	(4)	C(D)	X	G
(1,-10)	10%˜(10..)	10%˜(25..)	10% (33+9;..)	lin (53..)	lin (28..;r.)	lin (18-2)	lin (06..)	10%	X52101	(5)	C(D)	X	G
(0,-10)	10% (06..)	10% (25..)	10% (25..;-2)	20% (50..)	lin (37+5;..)	lin (25..)	lin (06..)	10%	X5210	(4)	C	X	E
(-.5,-10)	10% (07..)	10% (14r.)	20%m(27..;r.)	20% (55+2)	lin (33+3;..)	lin (27..)	lin (08..)	10%	X521-	(3.5)	B(C)	X	E
(-1,-10)	20%'(10..)	20% (30r.)	20% (40+5;r1)	20% (60r.)	lin (36+2;..)	lin (21..)	lin (07..)	20%	X52-	(2.5)	B	X	E
(-5,-10)	*) (06..)	*) (25..)	*) (31+1;..)	*) (50..)	*) (25..;r.)	*) (25..)	*) (06..)	*)	X5	(1)	(A)	(Y)	E
median 2)	(08..)	(25..)	(33+1;..)	(50..)	(25..;r.)	(21..)	(07..)						

1) entries - as 20% (08..) - are read as follows: '10%' means that FPMU=MAX*10%; '08'= pi(p); the following two symbols refer to the number of data that have to be changed to obtain the ideal pi-value (see head of column) as the median ('..' := 0, 4 places - as in 20% (35+4..) - refer to two ideal answers (as .25 and .33 for (p,q)=(80%,20%)) 'r' denotes cases where rounding is retransformed (and not counted as modification)

2) median after interchanging the cases (p,q) and (q,p) for (x,y)= (0,-10),(-.5,-10)

*) every prominence explains the data

vv marks column with ideal pi-value predicted via FPPU rule

˜ steps in range of negative payoffs not evaluated double

' data explained better if 10% instead of 20%

Table 5.4: Modification of Finest Perceived Money Unit by Reduction of Payoffs by Factor 10 1)

p=	case 1 [x*1000(p),y*1000(1-p)] 99% 90% 80% 50% 20% 10% 01%	case 2 [x*100(p),y*100(1-p)] 99% 90% 80% 50% 20% 10% 01%
(10, 5)	*)--*)--*)---*)---*)--*)--*)	*)--*)--*) *) *)--*)--*)
(10, 1)	20%'20%-20%--20%--20%-20%-20%	
(10, .5)	10%-10%-10% 20% 10%-10%-10%	
(10, 0)	10%-10%-10% 20% 10%-10%-10%	lin 20%-20%--20% lin-lin-lin ^^^ ^^^ ^^^ ^^^ ^^^ ^^^
(10,-.5)	10%-10%-10% 10%--10%-10%-10%	
(10, -1)	10%-10%-10%--10%--10%-10%-10%	
(10, -5)	20%'20%-20%--20%--20%-20%-20%	20%'20%-20%--20%--20%-20%-20%
(10,-10)	20%'20%-20% 20% lin-lin-lin	20%'20% lin 20% lin-lin-lin ^^^
(1, -5)	20%'20%-20% lin--lin-lin-lin	
(1,-10)	10%-10%-10% lin--lin-lin-lin	
(0,-10)	10%-10%-10% 20% lin-lin-lin	20% 10% 20%--20% lin-lin-lin ^^^ ^^^
(-.5,-10)	10%-10%(20%)-20% lin-lin-lin	
(-1,-10)	20%'20%-20%--20% lin-lin-lin	
(-5,-10)	*)--*)--*)---*)---*)--*)--*)	*)--*)--*)---*)---*)--*)--*)

*) every exactness explains the data
1) ^^^ marks cases where differences occured
20%' = data are explained better by 10% than by 20%

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