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Kenneth Arrow's Contribution to Economic Science.*

Kenneth Arrow has contributed by his research, his teaching and his stimulating personal influence on other economists as much to the advancement of economic theory as any other living economist has. His research will be discussed below in more detail. Arrow's contributions have been of a decisive character in several fields of economics. But, I believe, economic research owes Arrow much more than what has become visible in his numerous scientific publications. In working closely with a large number of economists, who have become leading contributors to the growth of knowledge, he has greatly stimulated them and indirectly through them again others in their work. A visible sign of this long series of exceptionally successful collaborative efforts is the large portion of papers which Arrow has written with others.

Most of Arrow's work addresses itself to the fundamental problems of economic theory, such as general competitive equilibrium and welfare economics in the broad sense of this term. The historical trends and events of the last decade in East and West alike must have destroyed the belief of a superficial pragmatism that we need not burden ourselves with fundamental theoretical problems. The social relevance of theoretical insight becomes increasingly appreciated. Why was Arrow's contribution to economic theory so important? In my opinion, because in several fields of research he was able to develop clear and convincing concepts and models so that unambiguous answers could be found by mathematical analysis, where before the problems had not been well defined or perhaps had not even been conceived as problems.

* An earlier version of this evaluation of Arrow's scientific work was written for the Committee on Economic Science of the Royal Academy of Sciences in Stockholm. This paper will be published in the Swedish Journal of Economics together with the Official Announcement of the decision of the Academy to award the Alfred Nobel Memorial Prize in Economic Science for 1972 to Kenneth Arrow and John R. Hicks. In writing this article I have greatly benefited from discussions with Felix FitzRoy (Heidelberg) and Werner Hildenbrand (Bonn).

Conspicuous examples are Arrow's "Social Choice and Individual Values" [A1], and his contribution to Welfare economics in the second Berkeley Symposium on Mathematical Statistics and Probability [A2], his publications on the economics of uncertainty (e.g. [A3]) or on the "Economic Implications of Learning by Doing" [A4]. Only after Arrow had given the basic formulations of the models, were other economists (or he himself) able to prove unambiguous theorems.

For the purpose of their evaluation I have divided Arrow's more than 100 scientific publications into eight groups:

- 1) General Equilibrium and Efficiency Theory
 - a) Existence Theorems
 - b) Stability
 - c) Optimality and Decentralization
- 2) Economics of Uncertainty
- 3) Growth, Productivity Theory and Technical Progress
- 4) Intertemporal Decision Theory and Optimal Growth
- 5) Mathematical Programming and Statistical Decision Theory
- 6) Social Choice and Individual Values (Foundations on Welfare Economics)
- 7) Surveys
- 8) Other Scientific Publications

1) General Equilibrium Theory and Efficiency Theory

a) Existence of Economic Equilibrium.

The by now canonical model of General Equilibrium is, what is called the Arrow-Debreu model. It was first presented in a paper by Arrow and Debreu which appeared in 1954 in *Econometrica* [A5]. It was later modified, refined and improved by several authors until it received its definite form in Debreu's *Theory of Value* (1959) [B1]. Ever since it has been the starting point for theoretical investigations in general equilibrium theory, welfare economics, the economics of uncertainty, the theory of money and other fields of theoretical research. Its direct and indirect influence on the modes of thinking in economic theory cannot easily be overestimated. In making available a firm and secure basis for any kind of general equilibrium analysis it was an important prerequisite for the general trend in the profession away from the weaknesses of partial equilibrium analysis toward a comprehensive inclusion of all the

relevant variables in any given context.

General equilibrium theory, as we understand it, started with Walras [B2]. But the mathematical problems of existence of a nonnegative solution for the system of relations describing a model of general interdependence in a market economy was not really attacked successfully until Abraham Wald wrote his famous but extremely complex and difficult papers on the subject in the thirties [B3, B4, B5]. The development of the subject was then substantially assisted by the use of fixed point theorems, saddle point theorems and theorems on convex sets which were used or indeed proved in the theory of games and mathematical programming (by von Neumann, Kakutani, Dantzig, Kuhn, Tucker, Nash, and others). Using these important tools Arrow and Debreu and independently McKenzie presented more general and much simpler existence theorems for general equilibrium models in 1954. It would lead too far to discuss the assumptions made in these models in detail. What is important for the great influence of the Arrow-Debreu model is its abstractness. In replacing differentiability assumptions and first order and second order conditions for derivatives by convexity assumptions, generality as well as simplicity could be gained. The abstract definition of a commodity and of production stressed the parallels between atemporal and intertemporal as well as between deterministic and probabilistic allocation problems. The theory of inequalities and mathematical instruments from the theory of convex sets and correspondences made it possible to consider non-interior solutions - any realistic solution is non-interior- and surfaces and curves lacking smoothness and strict convexity. It appears perhaps as a paradox result, but upon reflexion it can be clearly understood, that the use of mathematically powerful and abstract methods reduces the set of assumptions in the model more and more to those which have a definite economic meaning and eliminates the need for assumptions which are only of a technical nature. This property of abstract theory also explains the great help which heuristic reasoning in economics gets from these highly mathematical methods.

Arrow and Debreu proved the existence of general equilibrium basically under the assumptions that (I) production sets of firms

are well defined (implying a certain property of complete information), closed and convex (excluding indivisibilities or increasing returns to scale), do not contain positive vectors (implying that there are no outputs without inputs), and are not reversible (you cannot reproduce the inputs by means of the outputs of these inputs); (II) that consumption sets are bounded from below (meaning that there is a limit for the supply of services by any household), that preferences are well defined (implying the ability of the household to know exactly what it wants), that the (ordinal) utility function is continuous (small changes in consumption cannot cause large changes in the level of satisfaction) and quasiconcave (if there is a price vector such that demand is higher than a given quantity x_1 and another price vector such that it is lower than x_1 then there is a price vector such that demand is equal to x_1), (III) every consumer has sufficient initial endowments as to be able to supply each commodity in a positive amount. While III is clearly unrealistic and indeed has been modified by the authors in the same paper and later has been weakened further, the set of assumptions under which they were able to derive equilibrium was a great step forward towards a rigorous, general and relevant theory of equilibrium. Some of the assumptions have been made superfluous or weakened since. Others retain their force, such as the completeness of information and the convexity of the production sets.

A year ago the long expected book of Arrow and Hahn on "General Competitive Analysis" has appeared A6 . It is not only an impressive account of the "nonmeasure theoretic" part of General Equilibrium Theory. It contains quite a few new results or new and simpler proofs for known results. Among the new results I should like to mention those which are contained in the chapter "General Equilibrium under Alternative Assumptions" where the existence of equilibrium is shown for a model with externalities, an intertemporal model, which lacks future markets, and for a model of monopolistic competition. All three generalisations are analytically not trivial and, of course, steps in important directions. They indicate the potential of general equilibrium analysis for models which come

closer to reality than the canonical Arrow-Debreu model does.

b) Stability of Economic Equilibrium.

In a series of about a dozen papers, mostly written with coauthors, Arrow and his collaborators present results concerning stability of general economic equilibria. It was again Walras who discussed this subject first in greater detail. His tâtonnement process remained the central object of investigation concerning stability. In its modern form this process can be described by the differential equations.

$$\dot{p}_k = a_k x_k(p) \quad \text{or, in general, } \dot{p}_k = f_k(x_k(p))$$

where x_k is excess demand on the k -th market, p_k the price vector, \dot{p}_k is the instantaneous rate of change of the k -th price, a_k is a positive constant and f_k is a sign preserving continuous function. Building on the work of Hicks, Hotelling, Samuelson, Allen, Mosak and Metzler, Arrow and his different coauthors were able to establish a rather coherent body of knowledge concerning sufficient conditions for the stability of tâtonnement processes. In their paper of 1959 Arrow, Hurwicz and Block [A6] presented the following important results: Stability of the tâtonnement process is guaranteed, if all commodities are gross substitutes or if the market satisfies the weak axiom of revealed preference or if the matrix of partial derivatives of the excess demand function has a dominant diagonal. Stability also prevails under these conditions, if one of the commodities is chosen as a numéraire i.e. its price remains constant. The paper proves global stability and uses methods of proof more related to Liapunov's method than to the classical stability analysis working with matrix theory as it prevailed in the work of the predecessors who mainly were able to prove local results. The results of this paper have remained the basic results concerning stability of tâtonnement processes. Just as the Arrow-Debreu model is the starting point for investigations concerning existence, the Arrow-Hurwicz-Block paper is until today the starting point concerning stability of multi-market equilibria. Arrow and Hurwicz have generalized some of the results in a series of papers, particularly by analyzing the case of weak gross substitutability and the case of multiple equilibria as well as non linear adaptation processes [A8,A9]. Theorems concerning local stability were generalized

earlier by Arrow and Nerlove [A10] to the case of adaptive expectations in which expected prices influence present demand behaviour and price expectations were formed by basically averaging price observations of the past. Enthoven and Arrow [A11] showed that extrapolative expectations would imply the same stability behaviour as the model with static expectations as long as the extrapolations are sufficiently cautious. Arrow generalized the model to the case of a set of markets in which demand exhibits rising trends [A12].

In his paper "Toward a Theory of Price Adjustment" [A13] Arrow argues that traditional theory does not really have a disequilibrium price theory, since, as he shows, the theory of perfect competition is only valid in equilibrium situations, whereas in disequilibrium situations any exchange contract must be influenced by monopoly situations, transitory as they may be. Although the paper does not really develop this view into a more general theory and although unfortunately the profession has not supplied such a theory since (only a few papers have appeared recently), Arrow's ideas did not go unnoticed. Thus for example Leijonhufvud in his important book on Keynes [B6] makes the interesting and convincing point that Keynes' theory of equilibrium or disequilibrium may become better understood, if one would extrapolate the ideas which Arrow had proposed in his paper.

c) Optimality and Decentralization

The modern version of Adam Smith's "invisible hand" is the theorem on Pareto-Optimality of a multimarket equilibrium. This theorem traditionally had been proved by marginalist methods (see for example Lange, [B7]). These methods not only presupposed differentiability of utility and production functions. They also had to assume that the market equilibrium under consideration was an interior solution in all quantity variables, an utterly unrealistic assumption. In his paper "An Extension of the Basic Theorems of Classical Welfare Economics" [A2] Arrow succeeds in generalizing these welfare-theoretic propositions. His mathematical tool is the theory of convex sets and the paper is written in the spirit of the theory of mathematical programming as it had been developed in the preceding years. He not only proves under fairly

general conditions that the absence of external economies and diseconomies implies the Pareto-Optimality of a market equilibrium. He proves the converse and deeper theorem that any Pareto-Optimal allocation can be obtained by market forces, given the choice of appropriate distribution of initial resources. Any kind of corner solutions now causes almost no problems in the method of proof. Differentiability no longer has to be assumed. The results of this paper were later further improved and are now one of the cornerstones of modern equilibrium theory and welfare economics. But apart from its specific welfare-theoretic results, this paper, as it were, set the stage for the development of modern general equilibrium theory by formulating the abstract model which later on was so successfully used to prove the theorems of this theory. This is the reason why I consider it to be one of the most influential papers written in mathematical economics after the war.

In a long paper Arrow and Hurwicz [A14] investigate the problem of decentralization in a model, in which a central authority tries to maximize a utility function under constraints by means of a large number of activities. Let the utility function and the constraints be concave. By using the Kuhn-Tucker theorem the authors show the existence of a price system such that the maximization can be achieved with decentralized decision procedures: every manager of an activity simply has to maximize the profits of his activity. The authors then consider gradient methods for finding the optimal allocation and the optimal prices. By means of a theorem of the authors on concave programming they are able to show that the gradient process converges to the optimum. The gradient process is of a generalized tâtonnement type so that a link is established between the decentralisation and the stability problem. While these results are global results similar local results can be obtained if one relaxes the non-increasing returns to scale assumption. The mathematical method of "concavication", due to the authors, allows to reduce the informational needs for the optimization process substantially even under conditions of increasing returns to scale. An interesting "imperfect competition" interpretation is given.

This paper gives for the first time a firm mathematical foundation to the well known Taylor-Lange-Lerner theory of market socialism or to the theory of internal decentralization of decisions in large enterprises. Both fields of application have become more and more important in recent years. Thus for example the work of the Hungarian economist Kornai [B8] and of others in Eastern Europe on the theoretical level and certain reforms in the planning process in these countries give an indication of the importance of this area of research to which the work of Arrow and Hurwicz was a major contribution.

2. Economics of Uncertainty.

The economics of uncertainty have become a major field of research in economic theory and applied economics. Arrows contributions have stimulated this field greatly. In his paper "Le rôle des valeurs boursières pour la repartition la meilleure des risques" (1953) [A3] develops in a cohesive manner the general equilibrium theory of choice under uncertainty. Adopting the "state of the world" model which later became the standard model in the economics of uncertainty, he incorporated uncertainty into a model of general equilibrium by introducing the concept of a contingent commodity. While usually a commodity is defined by its physical qualities, its location and its moment of delivery, contingent commodities in addition are characterized by a single state of the world such that they will be delivered only, if this state of the world occurs. This concept of contracts in contingent commodities has real world counterparts such as insurance contracts, lottery tickets etc. Arrow shows that the usual properties of general equilibrium models generalize to such a world of uncertainty. But if the possible states of the world are numerous so will be the markets on which transactions have to take place. If there are S states of the world and C different commodities (of the noncontingent type) then C times S markets for the same number of contingent commodities have to exist. Arrow now argues that it is possible to reduce the number of markets necessary to C spot markets of (noncontingent) commodities and S securities markets in which S securities are traded each promising to pay one dollar contingent on the event that state s occurs.

This interesting argument is certainly correct and sheds light on the question of the function of securities markets. But I do not think it is quite correct to maintain, as Arrow does, that these C + S markets are able to accomplish exactly the same Pareto-optimal allocation of resources as the S times C markets. For this to be the case the consumers would have to know all prices of all commodities prevailing in all states of the world, before they can make the correct purchasing decisions on the securities markets. But they do not know these prices unless all these C times S markets really exist. This criticism may not be very relevant, since subjective price uncertainty conditional on any given state of the world may be much lower than unconditional price uncertainty. Thus the S securities markets may be able to reallocate the bulk of the risk in the economy optimally and with low transaction costs.

In the three Yrjö Jahnsson lectures, delivered in 1963 in Helsinki (published 1965) [A15] and published in revised form in Arrow's "Essays in the Theory of Risk Bearing" (1971) [A16], he surveys important aspects of the economics of uncertainty. But in doing so he does much more, since it was probably for the first time that the professional reader was able to see the connections, possibilities and limits of the different propositions which constitute the field of the economics of uncertainty. The first of these lectures deals with the axiomatic foundations of behaviour under conditions of uncertainty. Arrow presents and motivates a set of axioms leading to the expected utility theorem by way of a new and heuristically appealing proof. He also derives the concept of personal (subjective) probabilities in the tradition of Ramsey, DeFinetti and Savage with a new and interesting proof. In the second lecture he discusses the concept of risk aversion as it was introduced by von Neumann and Morgenstern and analysed by Friedman and Savage, Pratt and others. He especially concentrates on the question of increasing or decreasing relative risk aversion and relates it to a thorough investigation of the St. Petersburg Paradox, showing that avoidance of paradoxes of this type imply the boundedness of the utility function which in turn implies a tendency for increasing relative risk aversion as wealth or income rises. This analysis has impor-

tant implications for the theory of money and portfolio selection. In his third lecture Arrow looks at the implications of risk aversion on the institutions of economic life. He gives an illuminating discussion of such risk-shifting and risk-spreading institutions as insurance and common stocks and their limitations due to "moral hazard."

These questions are again taken up in the stimulating analysis of the market for medical care [A16] as well as in his lecture "Control in Large Organizations" [A16]. The former paper is particularly interesting, since here we have one of the few cases, where an economist analyses and explains behaviour which contradicts the profit maximizing assumptions economists usually make. Arrow argues cogently that medical doctors and hospitals by and large do not maximize profits and explains the risk related function of medical ethics which among other things prevent such profit maximizing behaviour. Most economists are aware that the assumption of the "economic man" has clear limitations of applicability. While it seems to be fruitless to continue a quarrel about the exact range of applicability of this assumption, it seems to be promising to try to explain different behavioural patterns, in the way Arrow does, by means of a comprehensive theory - in this case the economic theory of uncertainty. The problem of control in large organizations is closely related to the problems of information and hence of uncertainty. Arrow surveys these questions from the point of view of the general theory of uncertainty. As one example of such an analysis we may mention the conflict of interest between share-holders and managers of firms with respect to risky investments. Given that the risks involved are largely independent of other risks in the economy, share-holders are interested that the company take any risks with positive expected monetary value, since they can diversify their portfolio, whereas managers do not have such possibilities to shift and spread the risks, which is due to the problem of "moral hazard" or incentives. This analysis sheds light, for example, on the limited applicability of the Modigliani-Miller theorem on the corporate finance structure.

In the paper with R.C. Lind [A16] on Uncertainty and the Evaluation of Public Investment Decisions the authors prove the follow-

ing: If the risks of a public investment project are independent of other risks in the economy then the expected discounted monetary value of the project is the correct decision criterion.

Arrow and Lind argue that there is no unjustified bias towards public investment projects, if one adopts this criterion, even if private investors will demand a higher expected rate of return than corresponds to the rate of interest, due to risk aversion. This is so since the public has a greater riskshifting capability than the private investor, which explains the difference of the critical expected rates of return.

3. Theory of Production and Technical Progress

There exist three well known papers of Arrow which have had great influence on the thinking of the profession in this area. The first was written jointly with Chenery, Minhas and Solow [A17]; it made available and popular among theoretical and empirical economists the constant elasticity of substitution production function. For the first time there was a workable instrument to measure the elasticity of substitution between capital and labour. For the question of the practical relevance of traditional marginal productivity theory of income distribution nothing was more important than to be able to isolate the elasticity of substitution between capital and labour empirically. While it so far has not been worked out properly, I believe it can be shown that marginal productivity theory only stands a chance to be relevant for the question of income distribution between capital and labour, if the elasticity of substitution between these two factors of production is sufficiently large. The CES production function has been used extensively in the last ten years by empirical economists to measure the elasticity of substitution in many countries and industries of the western world. But also for the purposes of theoretical and empirical studies of economic growth this instrument has become indispensable. Thus the paper of Arrow, Chenery, Minhas and Solow is one of the most frequently quoted papers in the profession.

In his paper "Economic Welfare and the Allocation of Resources for Invention" [A18], Arrow gives a theoretical analysis of the

problems which a market economy faces with respect to the production of technical progress. This analysis is partly related to his analysis of the problems of uncertainty as they arise in this context. But Arrow is then mainly concerned with what one calls the public goods properties of inventions which cause difficulties for the market mechanisms to work efficiently. In particular Arrow discusses the welfare theoretical implications of the patent system as it at present exists and its possible alternatives. As in so many areas of economics this paper of Arrow has become a reference point with respect to which numerous economists have oriented their own research.

The third well known contribution of Arrow in this field is his beautiful paper on "The Economic Implications of Learning by Doing" [A4]. Here he stresses the point that in the process of production society also learns how to produce more efficiently. This social learning process, in so far as it cannot be appropriated by single individuals or firms (which also would be inefficient) is left without economic compensation for those who cause it to take place. This then is a case of market failure which calls for correction by government intervention. The model, which Arrow uses to make precise his point, is surprisingly simple in its mathematical argument which may be one of the reasons why it has become and remained so well known and important in the literature on growth and technical progress.

4. Intertemporal Decision Theory and Optimal Growth

One major field of interest for Arrow has always been the problems of optimization in the context of intertemporal decisions. In the fifties it was mainly optimal inventory theory, in the sixties and since it was the problem of optimal accumulation.

In 1951 Arrow, Harris and J. Marshak [A19] published a paper on Optimal Inventory Policy which was to become one of the classic references in the literature on optimal inventory problems. The authors consider different models with stochastic demand and in particular they discuss, I believe for the first time, the (S,s) -policies which since have remained

a central topic in optimal inventory theory and its applications. This paper also for the first time (apart from Massé's work which only later became known) makes use of the recursive nature of intertemporal decision problems. In this respect the paper can also be considered a forerunner to the theory of dynamic programming as it was then developed by Bellman.

In 1958 Arrow together with Karlin and Scarf edited a volume on the mathematical theory of inventory and production [A20] in which he contributed several specific papers (with Karlin and Beckmann as co-authors) on the deterministic models of optimal production scheduling and related problems. He also is co-author (with Karlin and Scarf) of a survey on inventory problems and has written a paper on the historical background of inventory problems with a strong emphasis on the analogies to monetary theory which is recommended to everybody interested in monetary theory.

Since then Arrow has written numerous articles on the problems of optimal accumulation. There are several papers on the problems of optimal capital investment for private firms, of which the first is one by Arrow, Karlin and Beckmann in the volume on inventory theory mentioned above. These papers culminate in Arrow's paper in the Hicks-Festschrift [A21] in which he considers the problem of irreversible capital investment by means of Pontryagin's maximum principle. He thereby succeeds in obtaining a relatively easy way to solve such problems.

A number of papers are concerned with criteria for public investment projects. The central and convincing argument is that the appropriate choices concerning public investment have to be made by means of an explicit optimization model. Such models are treated extensively in the book by Arrow and Kurz on "Public Investment, the Rate of Return and Optimal Fiscal Policy" which appeared in 1970 [A22]. Using extensively Pontryagin's maximum principle, the authors treat different models, in which production depends on the supply of a public good, which is called public capital. They put particular

emphasis on the problem of controllability of an optimal policy with limited instruments such as certain taxes and public debt under alternative assumptions concerning individual savings behaviour. The book is an important step towards closer interaction between the theory of public finance and the modern theory of intertemporal decision making.

5. Statistical Decision Theory and Mathematical Programming

I do not really feel competent to evaluate Arrow's contribution in statistical decision theory. His contributions to the theory of nonlinear programming are related to Arrow's work on stability and decentralization of economic allocation processes. But they are of independent interest. In a series of papers Arrow and Hurwicz provide important generalization of the Kuhn-Tucker theorem on nonlinear programming [A23]. Arrow and Enthoven generalize the Kuhn-Tucker theorem to quasi concave objective functions which frequently occur in economics. Arrow, mostly in collaboration with Hurwicz but also with Uzawa, has written several papers providing proofs for the convergence of gradient methods in solving nonlinear programming methods. Several of these theorems have been used in the paper mentioned above by Arrow and Hurwicz on decentralization in resource allocation.

These numerous and interesting contributions to the theory of nonlinear programming are also important contributions to economic theory since, as Arrow and his co-authors themselves have proved, they can be used to give rigorous proofs for much more general theorems in equilibrium theory than hitherto were available.

6. Social Choice and Individual Values

The work which made Arrow's name better known than any of his other contributions is his book on Social Choice and Individual Values (first edition 1951, second edition 1963) [A1]. It was by this book that certain deep difficulties in developing satisfactory mechanisms for arriving at social choices became known to economists interested in welfare economics and related questions. The voting paradox, known earlier,

was thereby generalized to a general impossibility theorem about the existence of a function relating individual preferences to social "preferences" or choices, if this function were to fulfill certain conditions. The conditions stipulated by Arrow were

- 1). The function should have a domain containing all logically possible combinations of individual preferences with respect to at least three alternative choices for the society, A, B, C.
- 2). If a certain alternative becomes preferred to some other alternative with certain people while this had not been the case before and if this alternative was socially preferred to the other one before, then it should *ceteris paribus a fortiori* be preferred socially after this change of preferences. (Positive association of social and individual values).
- 3). A change in individual preferences only with respect to third alternatives should have no influence on the social preference ordering with respect to two given alternatives (independence of irrelevant alternatives).
- 4). A given social preference of one alternative over the other cannot prevail for every conceivable combination of individual preference orderings (the social preference is not allowed to be imposed independently of individual preferences).
- 5). There is no dictator dictating his preferences to the society as a whole.

Arrow proves that there exists no social choice function fulfilling all these conditions although each of them appears to be a reasonable requirement for such a social choice function. Since the existence of social choice functions seemed to be a necessary requirement for any kind of rational social decision making, his theorem was a great challenge to those who were interested in the foundations of welfare economics and, indeed, political theory.

It was particularly the third axiom (independence of irrelevant alternatives) which was criticized and scrutinized, since it seemed to exclude the introduction of the criterion of intensity of preference: If 51 people just barely prefer A to B and 49 are willing to pay a high price to have B instead of A, then this appears to be different from a situation in which the same 51 people intensely prefer A to B and the 49 prefer B to A, but very mildly so. Indeed, it is reasonable to expect that a sufficiently coherent democratic society will in the first case choose B and will in the second case choose A, and many would not object to this difference of outcomes. Yet Arrow's condition 3 implies that the decision has to be the same in both cases. Murakami [B9], Sen [B10] and others have pointed out that condition 3 really contains two conditions: one which says that the ranking of third alternatives as such should not have an influence on the social ranking of any given two alternatives; and another one which says that the social ranking of two alternatives should only depend on the ordinal individual ranking of the alternatives. It is this second condition which causes doubt: in so far as the "intensity" of preferences between two alternatives is measured by their relative position with respect to other alternatives, the third condition of Arrow may be too strong. The appropriate formulation of axioms for social welfare functions replacing Arrow's condition 3 is still a debated subject. Since Arrow's axiom is inconsistent with traditional utilitarianism and the "new welfare economics", the central place of this debate in the field of normative economics is obvious.

Another attempt to bypass the impossibility theorem is to try to reduce the scope of the social welfare function, which means not to accept Arrow's condition 1. Tullock argues in his book [B11] that it is empirically irrelevant to try to find a social welfare function covering all logically possible preference relations of individuals, since the variety of preferences of reasonable voters is much smaller; and he tries to convince the reader that, whenever the number of (reasonable) voters is large, single majority voting will work satisfactorily. However such a basic problem as income distribution cannot be solved by majority voting, as Arrow notes in his review of

Tullock's book [A24] .

The discussion on these problems seems even to be increasing in recent years. So it should be expected that other interesting results will come forward soon. The excellent introduction and survey of the state of the art on social welfare functions by Sen [B10] shows clearly how much this field and hence the foundations of welfare economics as well as of political theory owe to Arrow's pioneering work.

7. Surveys

Ever since Arrow has been active in economic research and teaching he has provided the profession and graduate students with excellent surveys of and introductions into relevant fields of economics and related sciences. I should particularly like to mention: Mathematical Models in the Social Sciences (1951) [A25], Alternative Approaches to the Theory of Choice in Risk-Taking Situations (1951) [A26] , his introduction into the books "Studies in Linear and Non-linear Programming" (1958 [A27] , and "Studies in the Mathematical Theory of Inventory and Production" (1958) [A20] , then "The Measurement of Price Changes" (1958) [A28] , his Finland lectures on the Economics of Uncertainty (1964 [A15] , the article "Economic Equilibrium" in the International Encyclopedia of The Social Sciences (1968 [A29] , "Applications of Control Theory to Economic Growth" (1968) [A30] and "The Organization of Economic Activity: Issues Pertinent to the Choice of Market Versus Nonmarket Allocation" (1969) [A31] . Their lucidity and comprehensiveness is outstanding in our profession. They indirectly make Arrow one of the most influential teachers of economics. And they show how much their author has the virtue of a balanced judgement. After having reread some of them and after having read others for the first time on the occasion of writing this paper I would very much recommend to the author to collect them and publish them together so that they will be easily accessible to the profession. Even though some of them may no longer reflect the present state of the field, they are still an optimal way to get acquainted with the state of a subject at the time when they were written. Since it is com-

paratively recent and therefore reflects the present state of knowledge adequately I want especially to mention the survey of the issues "pertinent to the choice of market versus nonmarket allocation." The theory of general equilibrium, externalities, uncertainty, public goods, indivisibilities, games etc. have been surveyed here under the point of view of the virtues and limitations of the market mechanism by someone who has done research in almost any of the special subjects relevant for this topic. I find it a brilliant and surprisingly short account of this core of economic theory.

8. Other Publications

About one fifth of Arrow's scientific publications are not covered by the seven titles which I have introduced to classify his contributions. Among them are several empirical studies, lectures and papers on problems of management science, economic development and the urban crisis, and comprehensive evaluations of the scientific work of fellow economists, among them Ragnar Frisch and Paul Samuelson [A31, A32]. The broad scope and the analytic depth of Arrow's scientific work is quite outstanding in economic science.

A. List of those References of Arrow's Publications
which are quoted in the text

- (A1) Social Choice and Individual values; New York 1951
- (A2) An Extension of the Basic Theorems of Classical Welfare Economics; in: J.Neyman (ed.), Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. Berkeley and Los Angeles, 1951
- (A3) Le rôle des valeurs boursières pour la répartition la meilleure des risques; Econometrie, Colloques Internationaux du Centre National de la Recherche Scientific, Vol. XI, Paris 1953
- (A4) The Economic Implications of Learning by Doing, Rev. of Ec. Studies, 29, 1962
- (A5) (with G.Debreu), Existence of Equilibrium for a Competitive Economy, Econometrica, 22, 1954
- (A6) (with F.H. Hahn), General Competitive Analysis, San Francisco, Edinburgh 1971
- (A7) (with H.D. Block and L.Hurwicz), On the Stability of the Competitive Equilibrium, II, Econometrica 27, 1959
- (A8) (with Hurwicz), Competitive Stability under Weak Gross Substitutability, The Euclidean Distance Approach, Int. Ec. Rev.1, 1960
- (A9) (with Hurwicz), Some Remarks on the Equilibria of Economic Systems, Econometrica, 28, 1960
- (A10) (with M.Nerlove), A Note on Expectations and Stability, Econometrica 26, 1958
- (A11) (with A.C.Enthoven), A Theorem on Expectations and the Stability of Equilibrium, Econometrica 24, 1956
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