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Aspirations and Aspiration Adjustment in Location Games

Wulf Albers

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H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

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by

Wulf Albers, Bielefeld

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Summary:

This paper gives a first overview of some central ideas of aspiration approaches for location games. The theories have been developped observing more than 400 experimental location games, mainly with free face to face communication,

The paper is given in 5 sections:

section 1 introduces the paradigm of location games: location games are a generalization of the situation of n players with ideal points $x_1,...,x_n$ in $\mathbb{R}^n$ having to agree on a solution point $x \in \mathbb{R}^n$ by simple majority rule, where each player tries to obtain a result which is as near as possible to his ideal position. It seems that - for instance by using factor analysis methods - the paradigm can be applied to a wide class of political decision problems.

In section 2 aspirations and the aspiration equilibrium are introduced as a rational solution concept: Aspirations are assumed to develop parallel to the bargaining process; they are supposed to be such that within a coalition a player only agrees to an alternative if its utility fulfills his aspiration. In the aspiration approach the aspirations of all players are considered simultaneously. Equilibrium conditions for such aspiration profiles $(a_1,...,a_n)$ are introduced. The result can be interpreted as an extension of the quota concept or of the generalized quota concept (ALBERS, 1974) to location games.

In section 3 aspiration adjustment processes are modeled from a rational point of view. Aspiration adjustment paths are introduced as limits of stepwise aspiration adjustment. The end points of maximal paths have in some respect the character of ϵ-equilibria. Related to the observed behavior a path section rule is given which - under reasonable conditions - seems to reduce the number of paths in a way that all paths have the same end points. So - applying some principles of observed behavior - the rational theory could be refined in a way that a unique aspiration profile can be predicted.

However, these predictions essentially differ from observed bargaining results. The reason for that seems to be the difference between bargaining processes and aspiration adjustment processes. Aspiration adjustment processes as modeled here, are based on the assumption that each player maximizes his aspiration. In bargaining processes this aim is confounded with a necessity to stop the bargaining process in a point, when oneself is in the formed coalition. From this point of view it can become reasonable to reduce ones demands essentially below one's adequate aspiration.

Section 4 gives a more behavioral approach. It is assumed that players can deviate from their aspiration as long as the condition "a stronger player should not get more than a weaker player" is fulfilled (where the strength is given by the theoretical value of the aspiration of section 3). This principle selects a certain set of alternatives for each coalition. - In addition, two conditions are introduced, namely (a), that within a coalition a player i has to justify high outcomes by outcomes in other coalitions, including i, with at least the same utility to him, and (b) that the others have to justify low outcomes of i by outcomes with at most the same utility for i in other coalitions, including i. Applying this principle repeatedly, one obtains for each coalition a set of alternatives as predicted outcomes. - This approach can be interpreted as a consequent extension of the equal share analysis of SELTEN. - Presently, the predictions of section 4 seem to give best descriptive solution concepts for location games.

Section 5 introduces the <u>formation of blocs.</u> Blocs are not-winning coalitions of similar players, who bargain with one vote and with a joint utility function (which is obtained from the utility functions of the bloc members by a rule of fairness). Blocs are only formed if thereby the aspirations of all players of the bloc increase. So the formation of blocs does not make sense by its immediate outcome, but by the related transformation of the game.

1 Basic Tools

1.1 The Space of Alternatives

The task of economic, social, or political decision making is to select one out of many alternatives. Here the space of alternatives is denoted by X. It is modeled as an m-dimensional Euklidean space. The coordinates of the space can be for instance amounts of different budgets or outlays within parts of budgets, depending on the degree of aggregation of the decision or the analysis.

The coordinates can be also obtained by factor analysis. Empirical experiences with factor analysis indicate that for most applied problems the extension to a space with more than 6 or 7 dimensions does not give significant additional insights. In fact, in many cases only the 3 or 4 most significant factors do really have explanatory character.

In the examples below the space is restricted to two dimensions. Then the situation can be presented by drawing.

1.2 Utility Functions

It is assumed that the preferences of the players can be modeled as quasiconcave utility functions, i.e. the utility functions induce iso--utility regions which are borders of convex sets (compare figure 1).

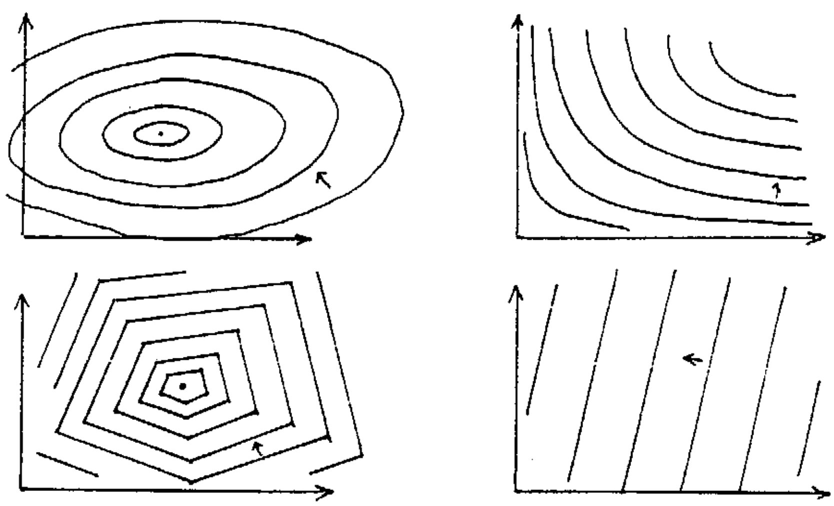

figure 1: examples of utility functions illustrated by the corresponding iso-utility regions

1.3 Pareto-optimality

In n-person decision making it is rational to select Pareto-optimal alternatives:

> **DEFINITION:** An alternative $x \in X$ is Pareto-optimal, if there is no other alternative $y \in X$ which is strictly preferred to x by all decision makers (i.e. $u_i(y) > u_i(x)$ for no $y \in X$, $i = 1,2,\ldots,n$).

By this condition the set of reasonable outcomes is essentially restricted as the examples of figure 2 show:

(a) shows the Pareto-line between two individuals connecting the ideal points of the players.

(b) shows the triangle of ideal points for three players. The boundary of the triangle consists of the Pareto-lines corresponding to the 2-person subcoalitions.

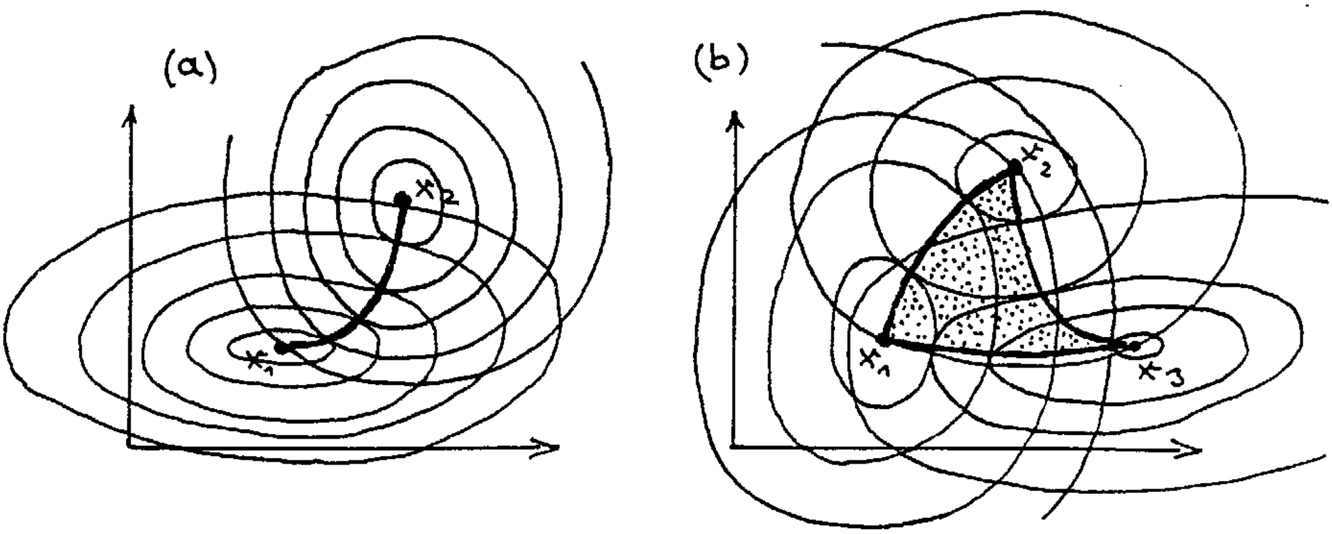

figure 2: Examples of sets of Pareto-optimal points.

Generally the following theorem holds

THEOREM: Given a space of alterantives $X = \mathbb{R}^m$.

(1) If the utility functions of the players are continuous (not necessarily quasiconcave) then for each coalition $S \subseteq N$ the mapping from the set U of possible utility functions to the corresponding sets of Pareto-optimal points of S is continuous (using the Hausdorff topology).

(2) If the utility functions of the players are quasiconcave, then for all utility levels $u^* = (u_1^*, \ldots, u_n^*) \in \mathbb{R}^n$ for each coalition $S \subseteq N$ the corresponding sets

$$X(S,u) := \{x \in X \mid u_i(x) \geq u_i^* \text{ for all } i \in S\}$$

are convex.

(3) If the utility functions of the players are strictly quasiconcave and their maxima are obtained on X, then there is a continuous mapping of an $(|N|-1)$-dimensional simplex[1] (with vertices $s_1, \ldots, s_n$) to X such that:

a) the set of Pareto-optimal points of N is the image of the simplex,

b) for each subcoalition S of N the set of Pareto-optimal points is the image of the facet spanned by the vertices $(s_i | i \in S)$.

c) for each player $i \in N$ the ideal point x_i (which maximizes his utility) is the image of s_i,

d) for each 2-person coalition $\{i,j\}$ the set of Pareto-optimal points is a path connecting their ideal points x_i, x_j. (Note that a), c), d) follow from b).)

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[1] $|S|$ denotes the number of players in S

Difficulties which may arise when the utility functions are only continuous and not quasiconcave are shown by the example of figure 3: The Pareto-sets of two person coalitions need no more define paths between the ideal points of the players.

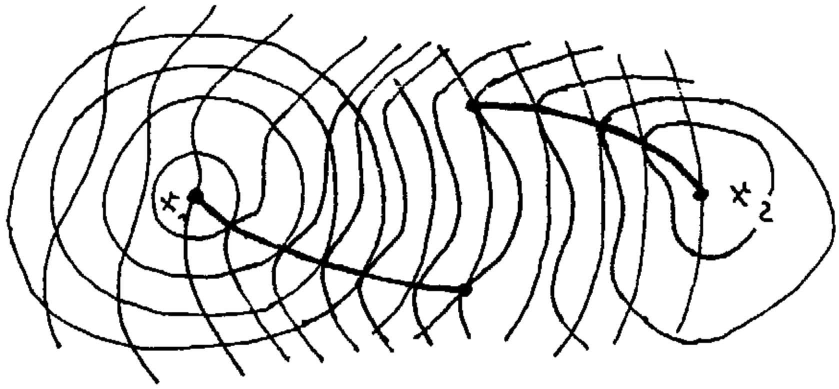

figure 3: example showing that for utility functions which are not quasiconcave the set of Pareto-optimal points of two players needs not define a path between their ideal positions x_1, x_2.

The example of figure 4 shows that the set of Pareto-optimal points of 3 players does not generally need to be isomorphic to a two dimensional simplex.

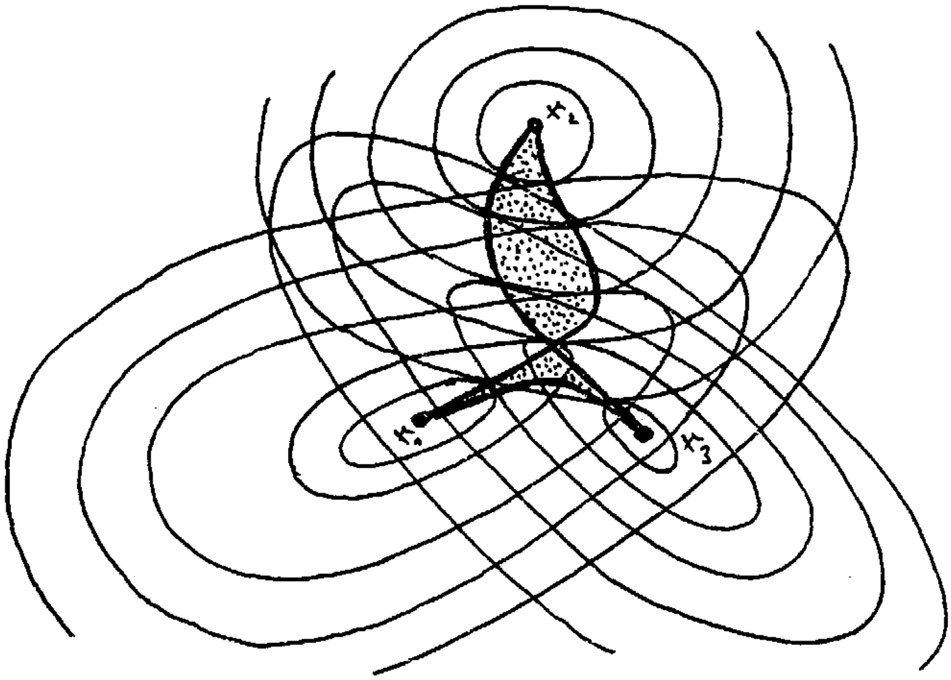

figure 4: example showing that even for strictly quasiconcave utility functions the set of Pareto-optimal points of 3 players does not need to be isomorphic to a triangle.

The example of figure 3 indicates that the quasiconcavity of the utility functions avoids dicontinuities and thereby simplifies the bargaining problem.

1.5 Location Games

DEFINITION: A location game $\Gamma = (N,X,u,W)$ is given by

- a set of n players, $N = \{1,2,\ldots,n\}$ [1]
- a space X of alternatives
- utility functions $u_i : X \longrightarrow \mathbb{R}$ of the players on X
- a set W of coalitions (i.e. of subsets of N), called winning coalitions

The idea of the game is that the players of a winning coalition can determine an alternative $x \in X$ as the outcome of the game. So that the problem of the location game is, which winning coalition is formed, and which alternative is selected by the coalition.

It is assumed that the space X of alternatives is an m-dimensional Euclidean space $\mathbb{R}^m$, that the utility functions of the players are quasiconcave, and that there are no two winning coalitions with empty intersection.

The examples given in this paper and the performed experiments only involve 3-, 4-, and 5-person games with $X = \mathbb{R}^2$. The utility functions are given via ideal points $x_1,\ldots,x_n$ (or via ideal lines $l_1,\ldots,l_n$) by the respective (negative) Euclidean distances from the ideal points (or lines). The winning coalitions are given by simple majority rule.

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[1] The subsets of N are called coalitions.

In this framework it is the aim of a player to arrange a coalition and thereby verify an alternative x which is as near as possible to his ideal position x_i (or his ideal line l_i). Of course, the interests of the players are usually contrary, so that it is a matter of bargaining which coalition is formed, and which alternative is selected.

1.6 Interpersonal Comparison of Utility

In section 4 we assume that there is a strength ordering on the players with the consequence that - if possible - a stronger player should not get less than a weaker player. - This implies a common agreement upon the interpersonal comparison of outcomes, or, more precisely, a common agreement on scales $\bar{u}_1,\dots,\bar{u}_n$ by which the outcomes of the players $1,\dots,n$ can be measured where each scale $\bar{u}_i$ is a mapping from X to $\mathbb{R}$. These scales have the character of utility functions, and it seems reasonable to assume that they can be obtained from the individual utility functions u_i of the players by stricly monotonic transformations.

Under this assumption we can replace the individual utility functions $u_1,\dots,u_n$ by the scales $\bar{u}_1,\dots,\bar{u}_n$ as long as the analysis only refers to the ordinal character of the utility functions. Moreover, $\bar{u}_1, \quad ,\bar{u}_n$ give the additional property that interpersonal comparisons of outcomes are possible.

In this paper the results of section 3 make sense for utility functions of both types u_i and $\bar{u}_i$. However, from a behavioristic point of view, the path selection rule in section 3.5 implicitly requires the interpersonal comparison, since otherwise a behavioral selection of "most symmetric aspiration profiles" is difficult to motivate. - Section 4 should be based on functions of type $\bar{u}_i$.

In our experiments the monetary incentives $\tilde{u}_i(x) := c_i - \alpha |x-x_i|$ were multiples of the (negative) Euclidean distances of X from the respective ideal positions $x^i \in \mathbb{R}^n$, added by constants c_i which were different for different players and not known in advance. (The constants depended on the success of other players in the same position as player i.) In this setup the distances from the ideal positions suggest themselves as evaluation functions $\bar{u}_i$. It seems reasonable to assume that also in other situations the Euclidean distances from the ideal positions can be spontaneously selected as scales to perform the comparison of outcomes.

This means that the outcomes of the players are implicitly compared with their <u>maximal possible</u> outcomes. It is interesting to remark that in characteristic function games (with payoff 0 in all one-player coalitions) the players directly compare their numerical outcomes. So in this situation each player compares his outcome with the <u>worst</u> outcome he can get. - So in both cases canonical reference points have been selected to obtain interpersonal comparability, however, in fully different ways.

2. Aspirations

2.1 Aspiration Profiles

In this approach aspirations are modeled as minimal demands of utility, so that a player i with an aspiration a_i will agree to an alternative x $\in$ X, only if $u_i(x) \geq a_i$.

It is assumed that in each state of the bargaining process each player has an aspiration a_i which may be adjusted at the next stage. The aspirations of the players define an aspiration profile $a = (a_1,\ldots,a_n) \in \mathbb{R}^n$.

> **NOTATIONS:** For an aspiration profile $a \in \mathbb{R}^n$ let
> $X(S,a) := \{x \in X \mid x \text{ Pareto-optimal for } S \text{ and } u_i(x) \geq a_i(x) \text{ for all } i \in S\}$
> $coa\ (a) := \{S \in W \mid X(S,a) \neq 0\}$
> $coa_i(a) := \{S \in W \mid X(S,a) \neq 0,\ i \in S\}$

X(S,a) is the set of those Pareto-optimal alternatives of S, which fulfill the aspirations of all players of S. coa(a) are those winning coalitions, which can fulfill the aspirations of all of their members by an adequate alternative. So the coalitions of coa(a) may be denoted as "feasible coalitions". $coa_i(a)$ are the feasible coalitions containing player i. Correspondingly we introduce feasibility of players:

> **NOTATION:** A player is called <u>feasible</u> (with respect to a) if $coa_i(a) \neq \phi$.

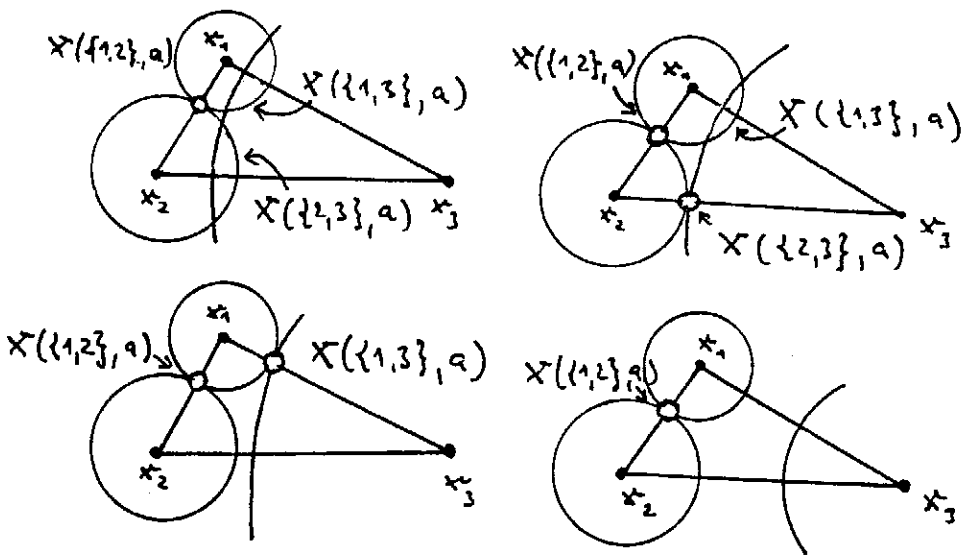

figure 5: examples of feasible points X(S,a) and feasible coalitions for different aspirations in a 3-person location game.

2.2 Aspiration Adjustment Process and Bargaining Process

Analysing people who are bargaining in a location game, one has to distinguish the development of aspirations in an <u>aspiration adjustment process</u> (which can be modeled by the aspiration profiles a^t at different points of time $t \in T$), and the <u>bargaining process</u> which may be given by a sequence of proposals (at different points of time) and by the Information which player agreed to which proposal (at which point of time), where a <u>proposal</u> (x,S) is a pair $x \in X$, $S \in W$. [3]

The problem of empirical observation is that the bargaining process can be observed directly, while the aspiration adjustment process can only be observed indirectly by its influences on the bargaining process.

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[3] The idea behind a proposal (x,S) is that the players of S might, should (or already have) agreed to the alternative x. If the players of S have agreed to x then (x,S) can get the character of an interim agreement or a final agreement.

Relations between these two processes are given by assumptions as

(1) a player i will only agree to a proposal (x,S) if it fulfills his aspiration (i.e., if $u_i(x) \geq a_i$)

(2) a player $i \in S$ who agrees to a proposal (x,S) thereby indicates that his aspiration a_i is not below $u_i(x)$ [1]

(3) A player who actively changes from (x,S) to (y,T) (with $i \in T \cap S$) has an an aspiration $a_i > u_i(x)$.

In ALBERS (1986) the relations of aspiration adjustment process and bargaining process are worked out in detail for Apex Games. In that paper the bargaining process is analysed and the aspiration adjustment process behind the bargaining process is modeled implicitly. - Here the aspiration adjustment process is modeled directly, and the bargaining process is not modeled. (It should be remarked that the approach here can be easily transferred to one-step characteristic function games.)

2.3 Dependence of Players

Let $a = (a_1, a_2, \ldots a_n)$ be an aspiration profile. Then we say that a player i depends on player j if every feasible coalition of i contains player j, while player j has a feasible coalition without player i:

DEFINITION: i depends on j if $coa_i(a) \subsetneq coa_j(a)$.

In such a situation it can be reasonable that player j asks player i to reduce his aspiration and j himself increases his aspiration in a way that afterwards still $coa_i(a) \subseteq coa_j(a)$. In fact, if i gets less than his aspiration in an alternative coalition of player j without player i, then player j can even force i to reduce his aspiration by threatening him otherwise to form a coalition without i.

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[1] follows from (1).

If, on the other hand, for all coalitions in $coa_j(a) \setminus coa_i(a)$, the corresponding proposals (x,S) fulfill the aspirations of player i, then the proposals (x, (S ∪ {j}) ∪ {i}) are also feasible under simple majority rule, and $coa_i(a)$ cannot be a subset of $coa_j(a)$. So, under simple majority rule, the argument that a dependent player can be forced to reduce his aspiration always holds.

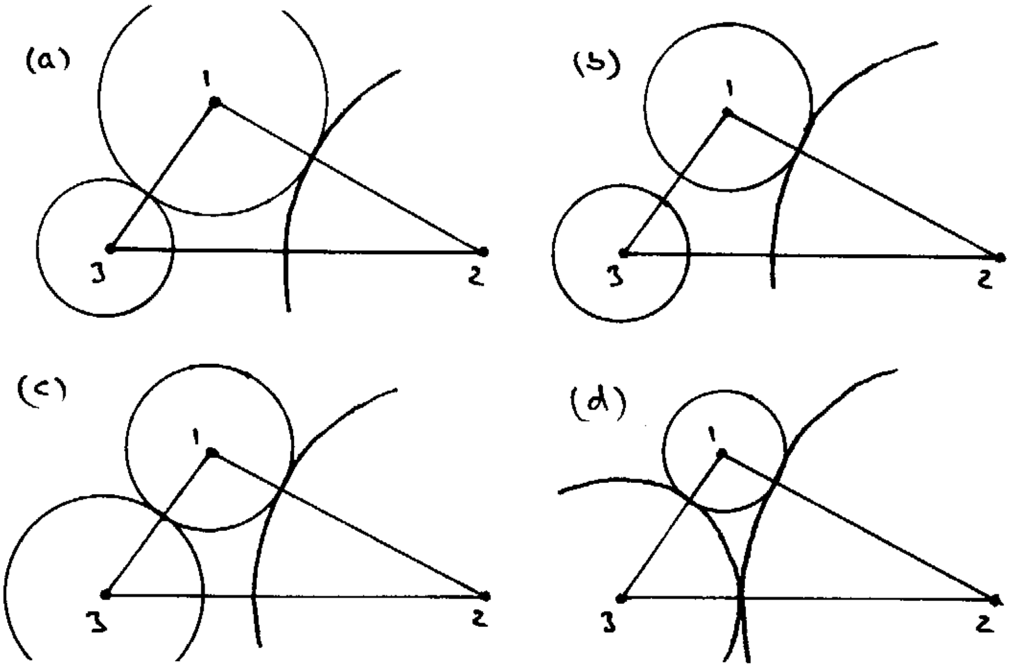

figure 6: aspiration adjustment in a 3-person game

The definition of dependence may be explained by the example of figure 6: consider the situation with aspirations as given in (a). Here player 2 and 3 depend on 1. Player 1 can force player 2 to reduce his aspiration and 1 can increase his aspiration for the same amount (see (b)). Now player 3 has to reduce his aspiration if he wants to find a coalition partner (see c)). Steps (b) and (c) can be repeated unless players 2 and 3 form an alternative coalition and thereby loose their dependence on 1. The corresponding result, where all players are independent, is given in (d)).

In the following we shall use the

NOTATION: $dep_i(a) := \{j \in N \mid i \text{ depends on } j \text{ (with respect to a)}\}$

2.4 The Aspiration Equilibrium

Figure 6 (d) gives a very acceptable solution for this specific game. In order to generalize this to arbitrary location games we give three properties which are met by the example:

- each player is feasible, i.e.
  (A1) $coa_i(a) \neq \phi$ for all $i \in N$

- if a set of players, $S \subseteq N$, increases their aspirations, then at least one of them becomes infeasible or dependent, i.e.
  ($\bar{A}\bar{2}$) $\bar{a}_i > a_i$ (all $i \in S$), $\bar{a}_k = a_k$ (all $k \in N \setminus S$),
  $\Longrightarrow$ either $coa(\bar{a}) = \phi$, or there is $i \in S$, $j \in N$ such that $coa_i(\bar{a}) \subsetneq coa_j(\bar{a})$.

- no player depends on another, i.e.
  (A3) $coa_i(a) \subsetneq coa_j(a)$ for no pair $i, j \in N$

Property (A1) is obvious; (A3) has been discussed above. ($\bar{A}\bar{2}$) can be explained by figure 6, which shows situations where players can increase their aspirations (see the dotted lines) and can afterwards still verify their aspirations without of becoming dependent.

Using these axioms we define

<u>DEFINITION:</u> The <u>aspiration equilibrium</u> is the set of aspiration profiles which meet (A1), ($\bar{A}\bar{2}$), and (A3).

In the following we shall replace $(\bar{\bar{A2}})$ by the condition

- if a set of players, $S \subseteq N$, increases its aspirations, then at least one of them becomes dependent from an additional player, i.e.

(A2) $\bar{a}_i > a_i$ (all $i \in S$), $\bar{a}_k = a_k$ (all $k \in N \setminus S$)
$\Longrightarrow$ either $coa(\bar{a}) = \phi$, or there is $i \in S$, such that $dep_i(\bar{a}) \supsetneq dep_i(a)$ (where
$dep_i(a) := \{j \in N \mid coa_i(a) \neq coa_j(a)\}$ is the set of those players, on whom i depends).

It is easy to prove that condition (A2) can replace $(\bar{\bar{A2}})$:

REMARK: The aspiration equilibrium is the set of aspiration profiles which meet (A1), (A2), (A3).

The advantage of formulation (A2) is, however, that it makes sense to apply it, even if (A3) does not hold. Since (A2) is a condition which reduces the slack (compare the examples of figure (b)), condition (A2) will enable us to consider apsiration profiles which meet (A1) and (A2), i.e. aspiration profiles with no slack (or low slack), and we can define a movement of such profiles in a way that in its end point condition (A3) is more or less fulfilled.

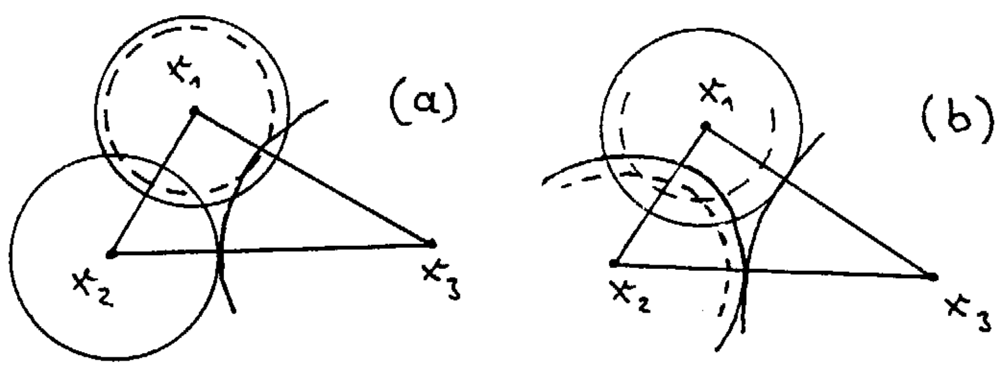

figure 7: aspirations in a 3-person game with ideal points which do not meet condition (A2).

According to conditions (A1) - (A3) the following behavioral rules can be introduced:

(i) a player reduces a_i if $coa_i(a) = 0$

(ii) a set of players, $S \subseteq N$, increases their aspiration a_i $(i \in S)$, if thereby $dep_i(a)$ does not increase for all $i \in S$.

(iii) a player reduces a_i if he depends on another player.

and we get the

LEMMA: An aspiration profile is an aspiration equilibrium iff it is stable with respect to (i) - (iii).

Figure 8 gives some examples of aspiration equilibria in different location games:

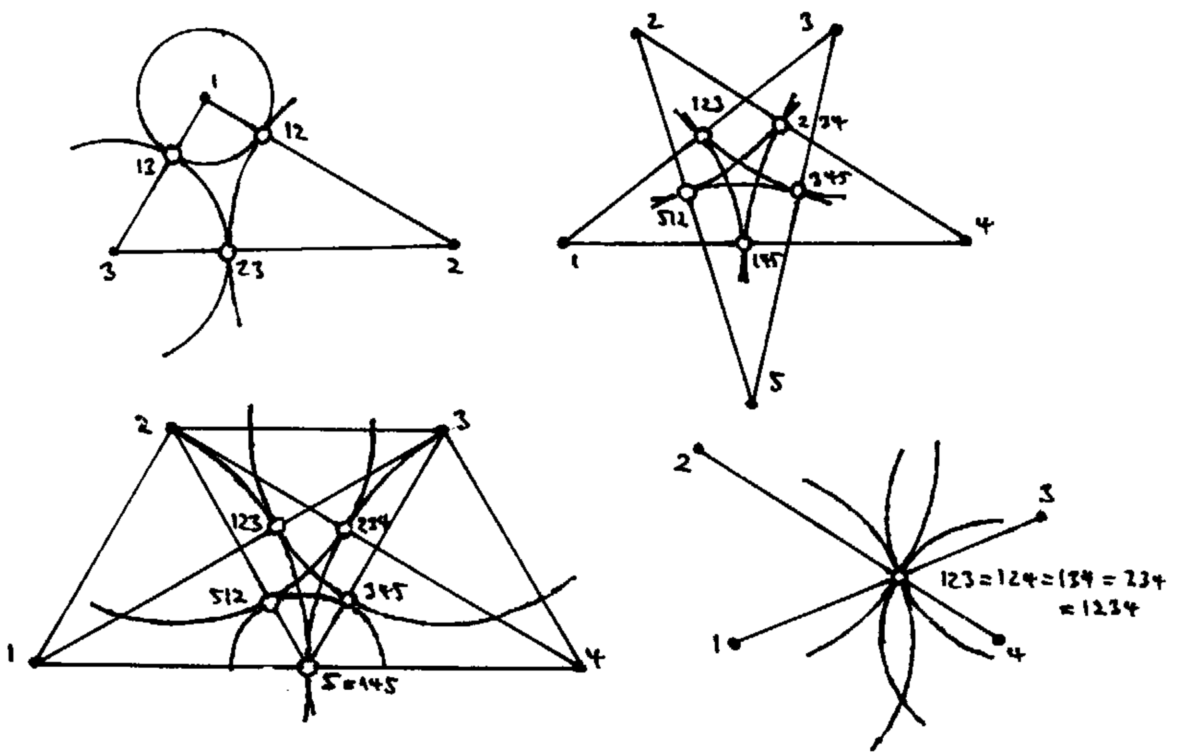

figure 8: aspiration equilibria of different location games

2.5 Predictions Related to the Aspiration Equilibrium

The prediction of the aspiration equilibrium theory is that the aspiration adjustment process stops in an aspiration equilibrium profile a and that the corresponding bargaining results are all proposals (x,S) with $S \in coa(a)$ and $x \in X(S,a)$.

Example: For the 3-person game with ideal points of figure 9 there is only one aspiration equilibrium. The corresponding feasible coalitions are $\langle 1,2 \rangle$, $\langle 1,3 \rangle$, and $\langle 2,3 \rangle$; and the corresponding alternatives are $x_{1,2}$, and $x_{1,3}$, and $x_{2,3}$, respectively.

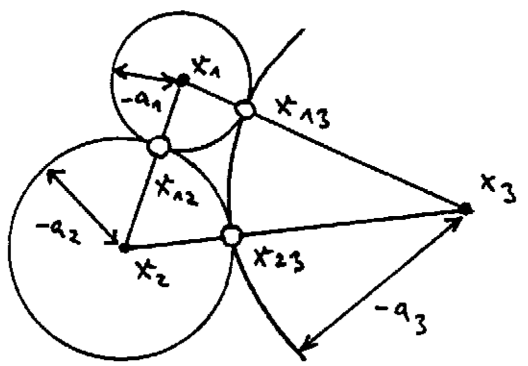

figure 9: predicted results of a 3-person game.

2.6 Modification of the Aspiration Equilibrium

The definition of section 2.3 is a first approach to a reasonable definition of an aspiration equilibrium. One problem of the definition may be illustrated by an example (see figure 10).

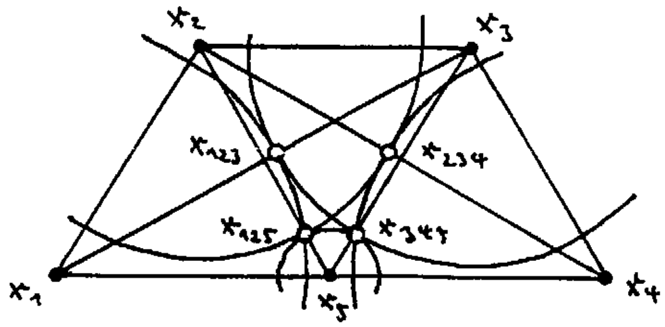

figure 10: example motivating a modification of the aspiration equilibrium definition.

In this example, for the given aspirations, player 1 depends on player 2 (and symmetrically player 4 depends on player 3). However, if player 1 reduces his aspiration, player 2 cannot in return increase his aspiration (neither in coalition ⟨1,2,5⟩ nor in ⟨1,2,3⟩). From this point of view there is no motivation for player 2 to press player 1 to reduce his aspiration, since he cannot take an (immediate) advantage from that.

This consideration motivates to modify condition ($\bar{\bar{A}}2$) in the following way:

($\bar{\bar{A}}2'$) $coa_i(a) \subsetneq coa_j(a)$ only if j becomes infeasible or dependent for all aspiration profiles $\bar{a}$ with $\bar{a}_i < a_i$, $\bar{a}_j > a_j$ and $\bar{a}_k = a_k$ for all $k \neq i$, $k \neq j$

This means that it is impossible that player i reduces his aspiration and player j increases his in a way which permits a feasible coalition to j.

This modification, so to speak, restricts the pressure of player j on player i to such cases, where something similar as side payments from i to j are possible. - In this context it should be remarked that in a similar way as here aspiration equilibria can be defined for characteristic function games (where side payments are always possible). For these games, however, conditions ($\bar{\bar{A}}2$) and ($\bar{\bar{A}}2'$) are equivalent. It must, however, be remarked that the new definition does not solve all problems. In fact, it does not even solve all problems imposed by the example.

Assume players {3,4,5} form a preliminary coalition and agree upon the point x_{345}. What will players 1,2 do? They will propose a coalition {1,2,3} with an outcome $\bar{x}_{123}$ which is nearer to the ideal position of player 3 than x_{123} (and therefore also nearer than x_{345}). Player 3 can of course accept this offer, since the new proposal cannot be dominated by any coalition which does not include player 3 (except the case that one of the players essentially reduces his aspirations). So, implicitly, player 3 can force player 1 (or 1 and 5) to reduce their aspirations.

The dependence of player 1 from player 2 therefore works in disfavor of player 1, because player 2 clearly prefers x_{123} to x_{125}, so that, under certain pre-histories of the bargaining process, player 1 cannot use x_{125} as a counter-argument. Thereby he becomes dependent (from player 3).

The threat described here works different from that given when we introduced the definition of dependence. There we argued that if i depends on j then j can threaten player i to reduce his aspiration and, otherwise form a coalition with somebody else. Here he threatens player i to form a specific coalition with player j. Restricting player i to this alternative, he may become dependent on somebody else, and thereby he can be forced to reduce his aspiration.

This argument again results with the conclusion that the dependent player must reduce his aspiration. However, we are not sure, if in every situation where one player depends on another one, there are forces working in such a way that the dependend player has to reduce his demands.

3 Modeling the Aspiration Adjustment Process

The aim of section 3 is to model the aspiration adjustment process in a normative way with continuous time as an aspiration adjustment path. Such a path is defined as a limit of aspiration adjustment sequences with discrete points of time. Maximal paths are selected, paths are normalized by assuming "a constant speed of change", and - by applying the observations of experiments - a path selection condition is introduced.

3.1 Aspiration Adjustment Chains

DEFINITION: An aspiration adjustment chain is a sequence $\bar{a} = (a^1, a^2, \dots,)$ of aspiration profiles such that for all $r \in \mathbb{N}$, $i \in \mathbb{N}$ one of the following conditions holds:

(1) $a_1^{r+1} < a_i^r$ and $coa_i(a^r) = \phi$

(2) $a_i^{r+1} > a_i^r$ and ex. $S \in coa_i(a^r) \cap coa_i(a^{r+1})$ and $dep_i(a^{r+1}) \subseteq dep_i(a^r)$

(3) $a_i^{r+1} < a_i^r$ and i depends on some player j in $coa(a^r)$, and there is no player k who depends on i $coa(a^r)$

(4) $a_i^{r+1} = a_i^r$.

(*) Moreover, it is assumed that for given r

case (2) is not applied, if there are $i \in N$, $\bar{a}^{r+1} \in \mathbb{R}^N$ fulfilling condition (1),

case (3) is not applied, if there are $i \in N$, $\bar{a}^{r+1} \in \mathbb{R}^N$ fulfilling condition (1) or (2).

Explanation: conditions (1) - (3) refer to the corresponding conditions of the aspiration equilibrium and to conditions (i) - (iii). These conditions are ordered by (*) in a hierarchical way, i.e. (1) is applied before (2) and (2) is applied before (3). Examples are given in figure 11 and figure 12.

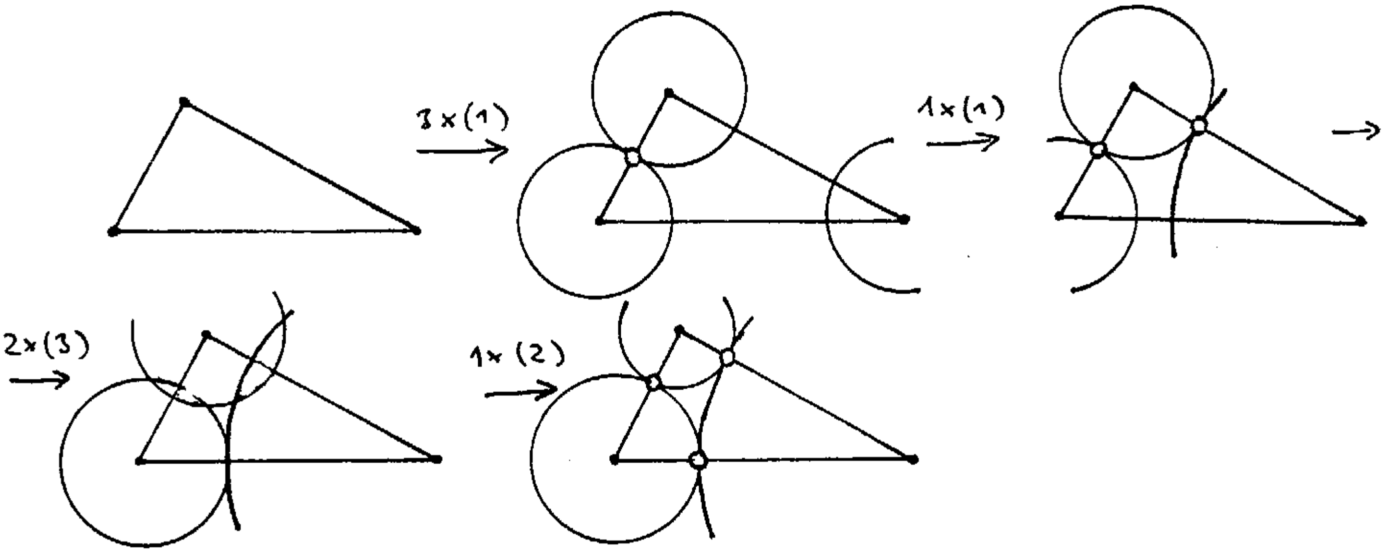

figure 11: an aspiration adjustment sequence in a 3-person game

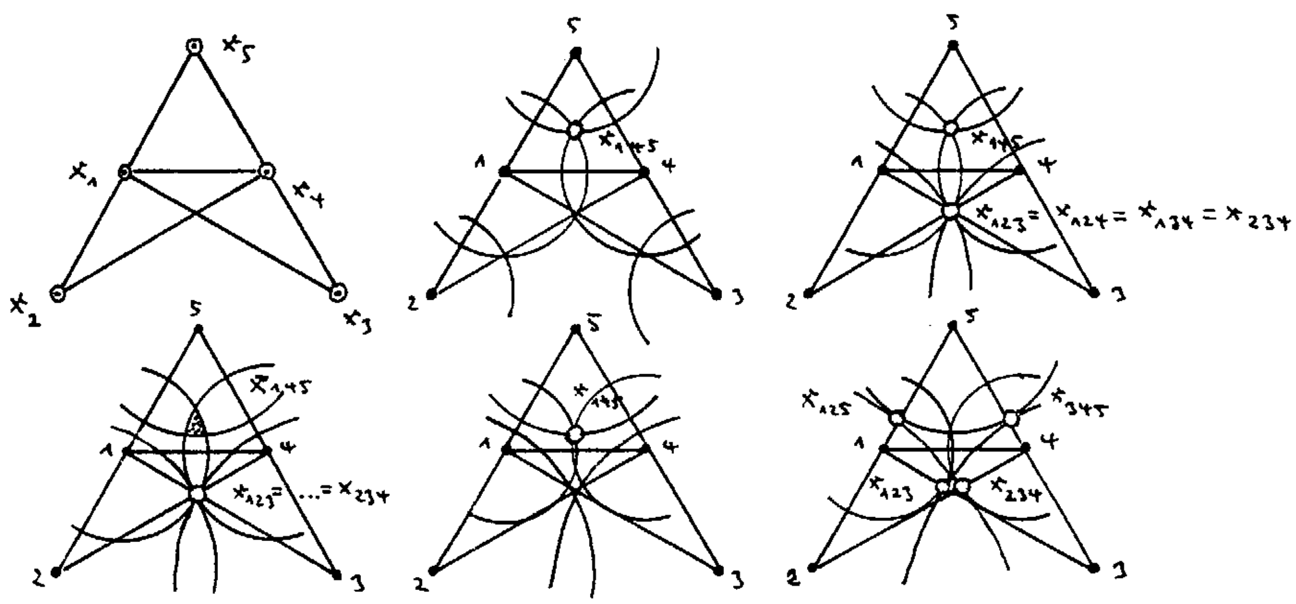

figure 12: an aspiration adjustment sequence in a 5-person game

In these examples the single steps of aspiration adjustment have been performed in a special way, namely such that players in symmetric positions have been treated equally, and that all players who reduced their aspirations from one step to the next reduced them for the same amount.

Of course, it would have been possible to subdivide the processes into finer processes with finer steps of aspiration adjustment. In the limit, paths of aspiration adjustment are obtained. The definition of paths requires to define the fineness of aspiration adjustment sequences:

DEFINITION: An aspiration adjustment sequence $\bar{a}$ has fineness ϵ ($\epsilon \in \mathbb{R}$, $\epsilon > 0$), if $|a_i^{r+1} - a_i^r| < \epsilon$ for all $i \in N$, $r \in \mathbb{N}$.

3.2 Aspiration Adjustment Paths

Let $T = [0,t]$ or $[0,t)$ (i.e. the set of real numbers between 0 and t including 0, but not necessarily including t).

DEFINITION: $\alpha: T \longrightarrow \mathbb{R}^N$ is an aspiration adjustment path, if

(1) there is a sequence of aspiration adjustment sequences ${}^1\bar{a}, {}^2\bar{a}, {}^3\bar{a}, \ldots$, and

(2) there are mappings ${}^i\tau : \mathbb{N} \longrightarrow T$ ($i = 1,2,\ldots$) such that

(a) $\{({}^i\tau(r), {}^i\bar{a}^r) \mid r \in \mathbb{N}\}$ converges to $\{(t,\alpha(t)) \mid t \in T\}$ (Hausdorff topology)

(b) the fineness of the sequences ${}^i\bar{a}$ converges to 0

(c) $\max_{r \in \mathbb{N}} ({}^i\tau(r+1) - {}^i\tau(r))$ converges to 0.

From this definition follows

LEMMA: Every aspiration adjustment path $\alpha: T \longrightarrow \mathbb{R}^N$ is continuous.

For the examples of figures 11 and 12 aspiration adjustment paths can be obtained by refining the drawn process: Then the figure just shows the vertices of the path, and the path is obtainined by connecting these vertices by straight lines of aspiration adjustment in $\mathbb{R}^N$.

3.3 Maximal Aspiration Adjustment Paths

DEFINITION: An aspiration adjustment path α: $T \longrightarrow \mathbb{R}^N$ is maximal if there is no aspiration adjustment path β: $U \longrightarrow \mathbb{R}^N$ such that $\alpha(0) = \beta(0)$ and $\alpha(T) \subsetneq \beta(U)$.

This maximality condition refers to the tails of the paths:

LEMMA: For each aspiration path $\alpha : T \longrightarrow \mathbb{R}^N$ there is a maximal aspiration path $\beta : U \longrightarrow \mathbb{R}^N$ with the same initial point (i.e. $\alpha(0) = \beta(0)$) which extends α (i.e. $\alpha(T) \subseteq \beta(U)$.

Maximal aspiration adjustment paths can lead to aspiration equilibria, and, obviously, each aspiration equilibrium can be presented as the end point of a maximal aspiration adjustment path.

An aspiration adjustment path can be interpreted as a permanent effort to fulfill conditions (A1) - (A3) of the aspiration equilibrium. From the hierarchy of the aspiration modification conditions for aspiration adjustment chains follows that aspiration adjustment chains permanently have to reach aspirations which meet (A1) and (A2), before a new effort can be made to fulfill (A3). From this follows

THEOREM: For each aspiration adjustment path $\alpha : T \longrightarrow \mathbb{R}$ there are points $t^1 < t^2$ in T such that

(1) $\alpha(t)$ meets (A1) iff $t \geq t^1$

(2) $\alpha(t)$ meets (A2) iff $t \geq t^2$.

Of course, the only point t^3 for which $\alpha(t^3)$ meets (A3) can be the end point of the interval T. However, it need not be that the path really meets an aspiration equilibrium. To characterize properties of the end point we introduce

DEFINITION: An aspiration profile $a \in \mathbb{R}^N$ is an <u>aspiration ϵ-equilibrium</u> if

(1) a meets (A1) and (A2)

(2) For no pair $i,j \in N$ with $coa_i(a) \subsetneq coa_j(a)$ there is an aspiration profile $\bar{a}$ such that

$\bar{a}_i < a_i - \epsilon$, $\bar{a}_j > a_j + \epsilon$, $\bar{a}_k = a_k$ for $k \neq i,j$,

and $coa_i(\bar{a}) \subsetneq coa_j(\bar{a})$.

So an aspiration ϵ-equilibrium can be interpreted as a point in which condition (A3) is insofar fulfilled that the dependence of player i from j cannot justify a change of the aspirations for more than ϵ.

Now we can formulate

THEOREM: The end point $\alpha(t^*)$ of a maximal aspiration adjustment path is a limit point of aspiration ϵ-equilibria (i.e. for each $\epsilon > 0$ there is an aspiration ϵ-equilibrium a^ϵ such that $|a^\epsilon - \alpha(t^*)| < \epsilon$).

I strongly assume that the following suggestion is true:

SUGGESTION: An aspiration adjustment path has no cycles.

From this follows easily

SUGGESTION: Every maximal aspiration adjustment path has an end point.

3.4 The Speed Normalization

The following remark says that a monotonic transformation of the time scale T of a path defines a new path:

REMARK: If $\alpha : T \longrightarrow \mathbb{R}^N$ is an aspiration adjustment path and $f: \mathbb{R} \longrightarrow \mathbb{R}$ is a continuous strictly monotonic increasing function with $f(0) = 0$, then $\gamma := \alpha \circ f^{-1} : f(T) \longrightarrow \mathbb{R}^N$ is an aspiration adjustment path.

The only difference of the two paths α and β are the "speeds" $\frac{d\alpha}{dt}$ and $\frac{d\gamma}{dt}$ in different points of the path. Since we are not interested in the speed of the aspiration adjustment, we normalize it:

SPEED NORMALIZATION: In the following we restrict our considerations to aspiration adjustments paths for which all $t \in T$:

$$\Sigma_{\{i \in N | d\alpha_i(t)/dt < 0\}} \left| \frac{d\ \alpha_i(t)}{dt} \right| = 1.$$

3.5 Path Selection Rule and Open Start Condition

Experimental observations indicate that subjects select paths which fulfill a certain "symmetry condition". To define this we introduce the

NOTATION: For each aspiration profile $a \in \mathbb{R}^N$ let lex(a) be the vector obtained from a by reordering the components of a increasingly.

Now we can formulate the

PATH SELECTION RULE (PSR): Let $\alpha : T \longrightarrow \mathbb{R}^N$, $\beta : U \longrightarrow \mathbb{R}^N$ be two aspiration paths, and let $t^* \in T$ be maximal subject to $\alpha(t) = \beta(t)$ for all $t < t^*$.
Then α is preferred to β, if there is an $\epsilon^* > 0$ so that for all ϵ with $0 < \epsilon < \epsilon^*$ lex $\alpha(t^* + \epsilon)$ is lexicographically greater than lex $\beta(t^*)$.

In addition, we define the

OPEN START CONDITION (OSC): A path $\alpha : T \longrightarrow \mathbb{R}^N$ which meets the path selection rule, meets the open start condition if $\beta(U) \supseteq \alpha(U)$ for every path β: $U \longrightarrow \mathbb{R}^N$ with $\beta_i(0) \leq \alpha_i(0)$ for all $i \in N$
(i.e. a path which meets the path selection rule meets the free start condition, if it can be continued at the starting side to arbitrary aspiration profiles below the initial aspiration profile $\alpha(0)$ of the path).

Now we can give two aspiration profiles which are on all "sufficiently large" aspiration adjustment paths which meet the path selection rule and the free entry condition, namely

NOTATION: For each location game Γ let

$a^1(\Gamma)$ the unique aspiration profile which meets (A1) and for which lex(a) is lexicographically maximal

$a^2(\Gamma)$ the unique aspiration profile which meets (A2) and for which lex(a) is lexicographically maximal.

These points have the following properties:

THEOREM: For each aspiration adjustment path $\alpha : T \longrightarrow \mathbb{R}^N$ which meets PSR and OSC

either (1) there is a value $t^1 \in T$ such that (a) $a^1(r) = \alpha(t^1)$, and (b) $[\alpha(t)$ meets $(A1)] \Longleftrightarrow t \geq t^1$

or (2) there is an aspiration adjustment path $\beta: U \longrightarrow \mathbb{R}^N$ which meets: (a) the path selection rule, (b) the open start condition, (c) condition (1) above, and (d).$\alpha(T) \subseteq \beta(U)$.

THEOREM: For each aspiration adjustment path $\alpha : T \longrightarrow \mathbb{R}^N$ which meets PSR and OSC either (1) there is a value $t^2 \in T$ such that (a) $a^2(r) = \alpha(t^2)$, and (b) $[\alpha(t)$ meets (A1) and $(A2)] \Longleftrightarrow t \geq t^2$ or (2) there is an aspiration adjustment path $\beta: U \longrightarrow \mathbb{R}$ which meets: (a) the path selection rule, (b) the open start condition, c) condition (1), and (d) $\alpha(T) \subseteq \beta(U)$.

COROLLARY: Each maximal aspiration adjustment path which meets PSR and OSC, and which contains $a^1(r)$ also contains $a^2(r)$.

It is suggested, but not proven, that by maximal aspiration adjustment paths also a third point can be characterized:

SUGGESTION: Each maximal aspiration adjustment path which meets ASR and OSR ends with the same aspiration profile, called $a^*(r)$.

The idea is that by the (suggested) theorem a unique solution profile can be assigned to each location game. But even if the theorem is wrong, the set of possible end points of aspiration adjustment paths which meet the aspiration adjustment rule and the free start condition essentially restricts the set of predicted results of the aspiration adjustment process.

<u>Examples:</u> The aspirations shown in figures 11 and 12 give the vertices of the respective maximal aspiration adjustment paths which meet ASR and OSC. In both examples the end points $a^*(\Gamma)$ are unique. Both paths are started with aspirations given by the ideal positions of the players. For both games $A^1(\Gamma)$ is given by figure (b) and $A^2(\Gamma)$ by figure (c).

3.6 Strategic Behavior

The preceding section describes in which way aspiration adjustment processes develop. It seems reasonable that rational players can foresee the further development of the process and the question arises, if this might cause them to change their behavior.

For instance, if a player i depends on another player j, then player j can refuse to ask player i to reduce his aspiration and to increase his own aspiration. Thereby the aspiration adjustment process can stop at an early point of the aspiration adjustment procedure.

In fact, in many experiments the aspiration adjustment process was already stopped between the points $a^1(\Gamma)$ and $a^2(\Gamma)$. The consequence of this is that many in 3-person games only the coalition $\{1,2\}$ is formed.

The main difference between the aspiration adjustment process and the bargaining process is, that the aspiration adjustment process is modeled in a way which assumes that it is in the interest of every player to maximize his aspiration value. However, the aspiration has to be verified as an outcome! And since usually a player is not contained in all feasible coalitions, he cannot be sure to verify his aspiration.

From this point of view it may even be reasonable to demand essentially less than one's adequate aspiration. It might perhaps even happen that all players are willing to reduce their aspirations below the aspirations reached in the aspiration adjustment path. But there is a border to such aspiration reductions. Aspirations are not an the free disposal of the players, they must be regarded by the others as adequate demands. The question arises, which deviations of aspirations from those of the aspiration adjustment process are accepted by the players. Experimental results suggest that players accept deviations from reasonable aspiration profiles as long as players with higher aspiration values get higher outcomes than players with lower aspiration values. These conditions reduce the purpose of the aspiration adjustment process to finding an order of strength on the set of players, with the implication that - if possible - a stronger player should get more than a weaker player. This idea will be modeled in section 4.

3.7 Relation to the Competitive Solution

Although defined in a very different way, the concept here is closely related to the "competitive solution" of McKELVEY, ORDESHOOK, and WINER (1978), the following definition is given according to LAING, OLMSTED (1978):

DEFINITION: A set C of proposals is called a competitive solution if

(1) (internal stability) $(x,S),\ (y,T) \in C$
$\Longrightarrow$ (y,T) does not dominate (x,S)

(2) (external stability) for each $(x,S) \notin C$ which dominates a proposal in C there is a proposal $(y,T) \in C$ such that (y,T) dominates (x,S)

where: (y,T) dominates (x,S) iff $u_i(y) > u_i(x)$ for all $i \in T$

For many games the competitive solutions are given by the sets $\cup\{X(a,S) \mid S \subseteq N\} = \cup\{X(a,S) \mid S \in coa(a)\}$ of the aspiration equilibria a.

Figure 13 (a) gives an example of a location game which has a unique aspiration equilibrium (given by the radii drawn in the figure). But the sets related to the aspiration equilibrium do not form a competitive solution, since $(x_{235}, \langle 2,3,5\rangle)$ is dominated by $(x_{145}, \langle 1,4,5\rangle)$. However, $\langle(x_{123}, \langle 1,2,3\rangle), (x_{,145}, \langle 1,4,5\rangle), (x_{234}, \langle 2,3,4\rangle)\rangle$ form a competitive equilibrium

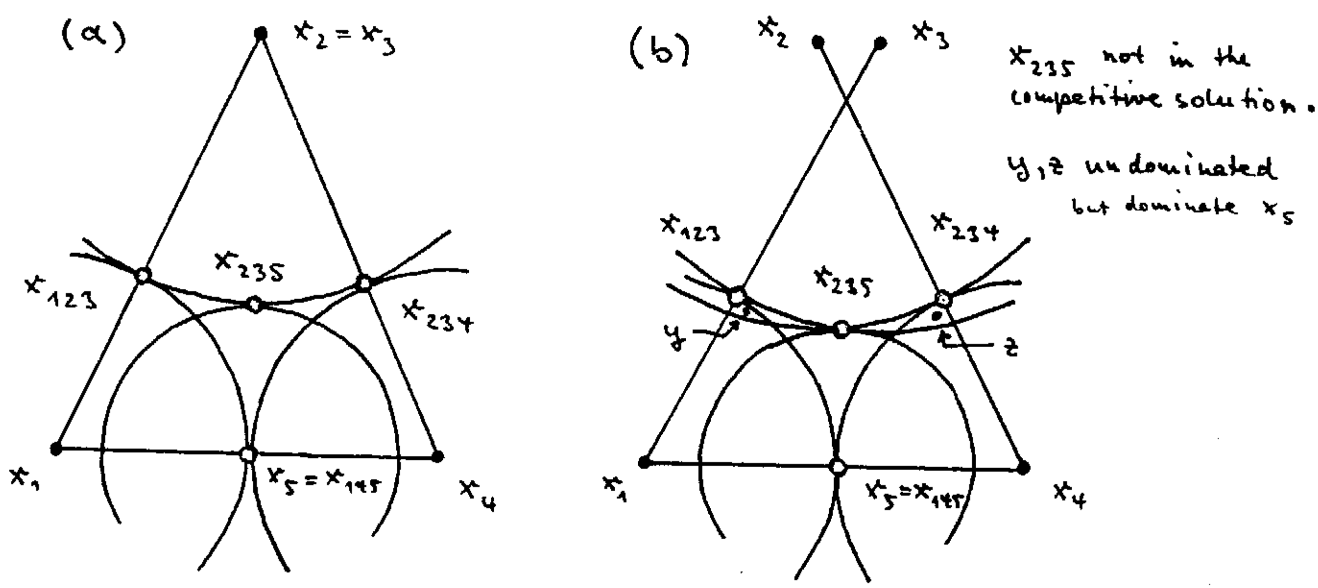

figure 13: two examples of location games and corresponding aspiration equilibria given by the drawn radii. - game (b) has no competitive solution.

The situation in this game can be characterized as one, where player 5 has the only function to support the proposal $\langle x_{145}, \langle 1,4,5 \rangle\rangle$ if coalition $\langle 1.4 \rangle$ is formed. This consideration reduces the essential part of the game to the active interest groups "1", "4", and "2,3", where the coalitions "1,2,3", "4,2,3", and "1,4" are possible, of which "1,4" is necessarily supported by player 5

A slight modification of the example (see figure 13 (b)), generates the game for which for each aspiration equilibrium the corresponding set of proposals neither fulfills the internal nor the external stability condition. In this example omitting the dominated proposal $(x_{235}, \langle 2,3,5 \rangle)$ still leaves the external stability violated. I strongly suggest that this game <u>has no competitive solution</u>. The role of player 5 is similar to that in example (a); however, the idea of the aspiration equilibrium can no more be presented in the framework of the competitive solution approach.

4 Order of Strength and Generalized Equal Share Analysis

4.1 The Order of Strength

The following concept is based on a strength ordering on the set of players:

> **<u>DEFINITION:</u>** $\succ$ is a <u>strength ordering</u> on N, if it is
> transitive (i.e. $i \succ j,\ j \succ k \Rightarrow i \succ k$)
> reflexive (i.e. $i \succ i$ for all $i \in N$)
> complete (i.e. $i \succ j$ or $j \succ i$ for all $i,j \in N$).

Within this section it is not important, where the strength ordering comes from. Observing experimental games it seems that it is one of the subjects' questions to the game to find out their bargaining strengths, i.e. to find out who is stronger, and who is weaker than oneself. It seems that the answer is obtained during the bargaining process, sometimes by hypothetical bargaining.

According to the aspiration approach, it may be suggested that the strength ordering is induced by the aspiration values of the unique joint final aspiration profile of all maximal aspiration adjustment paths which follows ASR and OSC. And, in fact, this seems to be the most reasonable candidate to induce a strength ordering.

But it may also happen that players follow the aspiration adjustment path only in the beginning of their considerations and then switch to another criterion which explains the strength ordering obtained at that state. ALBERS and BRUNWINKEL (1987) consider such a criterion (it says that a player i is stronger than another one, if there are more players, whose ideal positions are nearer to the ideal position of i than players, whose ideal positions are nearer to that of the other one). Another criterion may be the distance from the gravicenter of the set of ideal points (a player is stronger, if he is nearer to the gravicenter).

However, presently it seems to the author that these alternatives are only pseudo-criteria which are used to confirm the players' feeling of strength which they developped during the bargaining process.

4.2 Predictions Related to the Order of Strength

The idea related to the strength ordering is that within a coalition a stronger player must not agree to get "less" than a weaker player. This is made precise by the following definition of dominance:

> **DEFINITION:** Let $\succ$ be a strength ordering on N, S a winning coalition, and x,y two alternatives.
> Then <u>y dominates x with respect to S and $\succ$</u>, if for each $i \in S$ one of the following conditions holds true:
> (1) $u_i(y) > u_i(x)$
> (2) $u_i(y) \geq u_i(x)$ and there is a player $j \succ i$ with $u_j(x) < u_i(x)$ and $u_j(y) > u_j(x)$.

It is reasonable that a player will agree to a change from x to y if his utility increases (condition (1)). Moreover, we assume that a player can be forced to accept a point y with a lower utility than x if there is a player j who is not weaker than i ($j \succ i$) so that j has the right not to accept the alternative x (in which he gets less than j), and to suggest an alternative proposal instead, which increases his utility ($u_j(y) > u_j(x)$).

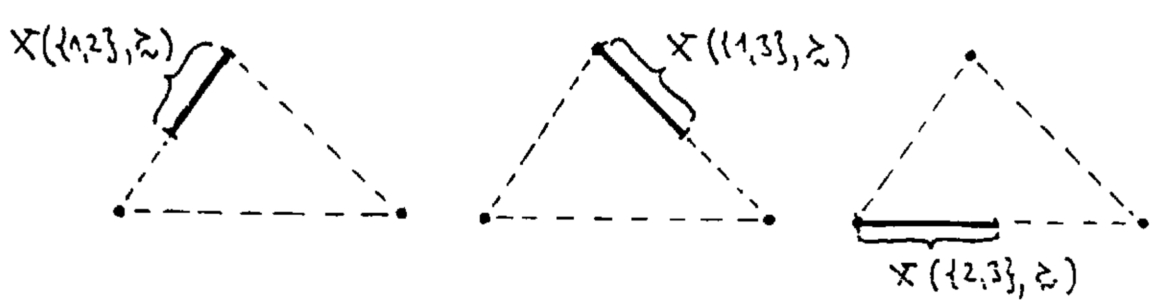

figure 14: The sets $X(S,\succ)$ of the two person coalitions in a three-person game with $1\succ 2\succ 3$

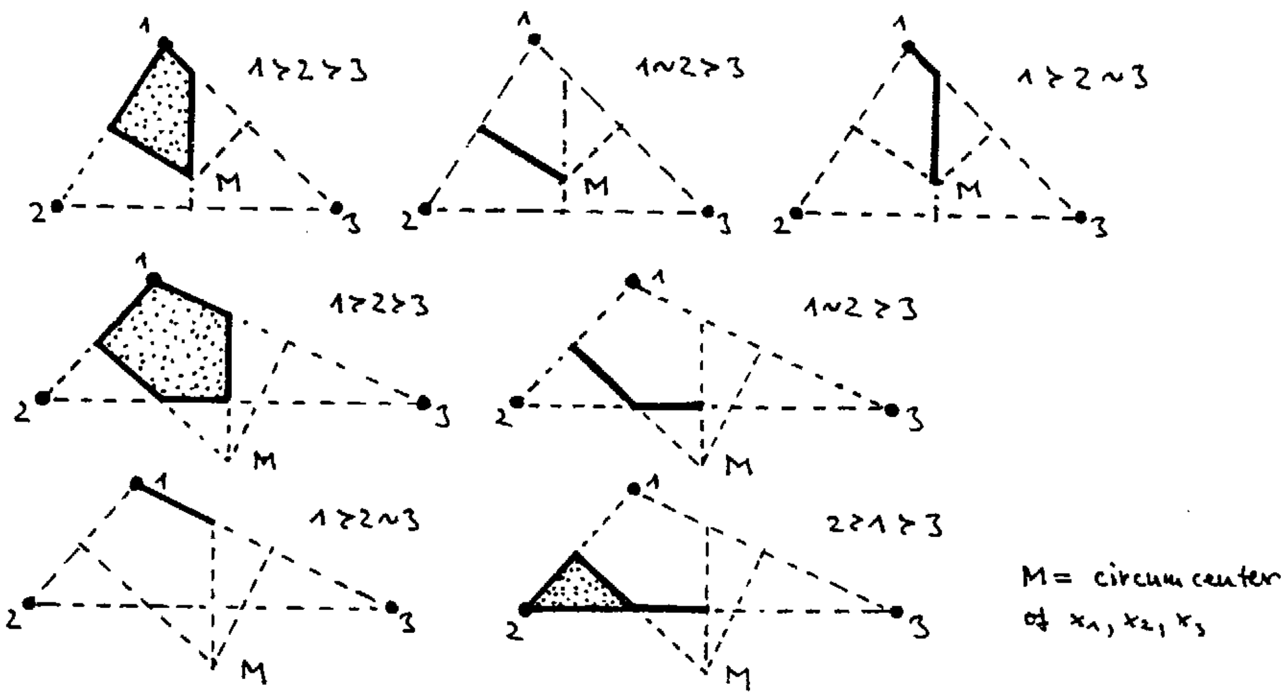

figure 15: examples of sets X(S,≻) of three-person coalitions with different orders of strength.

The arguments which are stable with respect to this dominance for a given coalition S are

> **DEFINITION:** If S is a winning coalition then <u>X(S,≻)</u> is the set of all alternatives which are not dominated with respect to S and ≻. If S is not winning then X(S,≻) is defined to be empty.-
> X(S,≻) is denoted as the <u>set of stable alternatives of S with respect to ≻</u>.

From the definition of dominance follows immediately

> **REMARK:** All alternatives of X(S,≻) are Pareto-optimal (for all $S \subseteq N$ and all strength orderings on N).

Examples for sets X(S,≻) are given in the figures 13 and 14. The examples show that it can happen that a player gets more than another player, although he is not stronger than the other one.

4.3 The Adjustment Process of Solution Sets X(S), $S \subseteq N$

It seems reasonable to assume that the solution sets $X(S,\succ)$ induce strategical considerations of the players by arguing with possible outcomes in other coalitions. Specifically, if a coalition S is formed, a player i can argue that he should not get less than in his worst alternative proposal which does not involve the other players of S, and the partners of i can argue, that he should not get more than his maximal outcome in an alternative coalition, which does not involve them. The argument of player i will be extended to arguments of subsets of S. Moreover, the alternatives involved in the arguments of a player i must be restricted to such proposals (y,T), which involve player i as as essential decision maker, i.e. which are not Pareto-optimal for $T\backslash\{i\}$.

Applying this idea induces a shrinking of the solution sets $X(S,\succ)$ to $X'(S,\succ)$. Then the procedure can be applied to the new sets, etc, so that a set of adjustment processes is obtained. To formulate this procedure for an arbitrary step, it is defined for sets $X^r(S)$, $S \subseteq N$:

<u>SET ADJUSTMENT PROCEDURE:</u> Let $X^r(S)$ be given for all $S \subseteq N$. Then $x \in X^{r+1}(S)$ if

(1) $x \in X^r(S)$

(2) For no $I \subseteq S$ there is $T \subseteq N$ (with $T \cap S = I$) and $y \in X^r(T)$ (y not Pareto-optimal for $T\backslash I$) such that $u_i(x) < u_i(y)$ for all $i \in I$

(3) For all $i \in S$ <u>either</u> x is Pareto-optimal for $N\backslash\{i\}$ <u>or</u> there is $T \subseteq N$ (with $T \cap S = \{i\}$) and $y \in X(T)$ y not Pareto-optimal for $T\backslash\{i\}$) such that $u_i(x) \leq u_i(y)$.

Here (2) can be interpreted in a sense that so subcoalition must accept a proposal which is worse than its worst alternative in another coalition, (3) says that no player should get more than he gets in his best alternative in another coalition.

It must be remarked that from experimental observations we are not sure whether to include condition (3) in the reduction procedure or not. And, in fact, there are good reasons not to apply condition (3) if one assumes that players mainly examine if a proposal gives high enough amounts to the others, since danger comes from players who get too low amounts and therefore change the coalition. However, giving somebody else more than he should get is the basic intention of the definition of the sets Y(S,≻), since by giving high outcomes to the others, one can stabilize a results in favor of oneself.

REMARK and DEFINITION: Applying the procedure of the preceding definition repeatedly, one obtains for each coalition $S \subseteq N$ a sequence $X^1(S)$, $X^2(S)$,... which converges to a set $X^\infty(S) := \bigcap_{r \in N} X^r(S)$.

Figure 16 gives an example showing the devolopment of the sets $X^r(S,\succ)$.

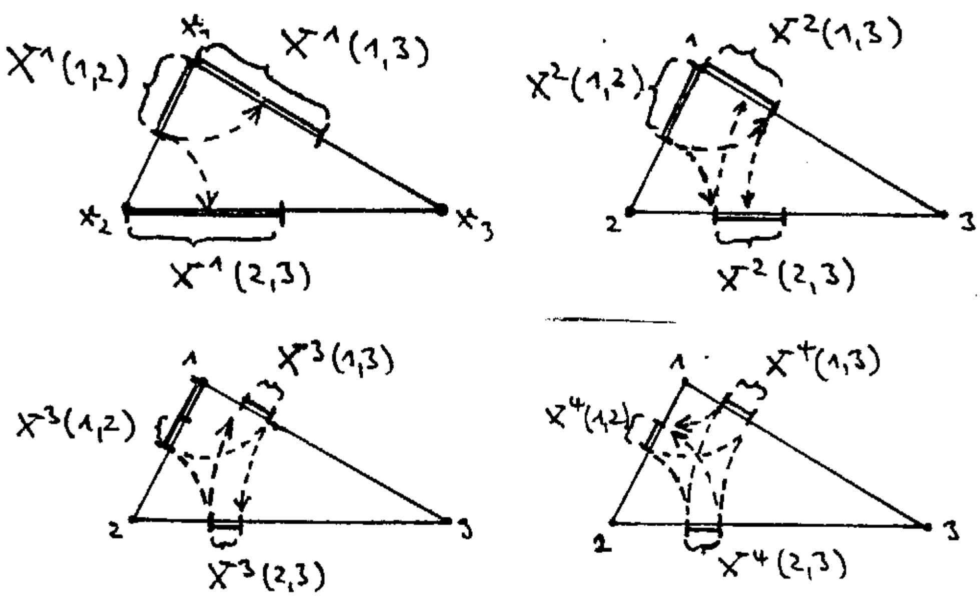

figure 16: an example explaning the set adjustment process (location game with ideal points and strength 1≻2≻3, according to $a^*(\Gamma)$)

4.4 The Set Adjustment Process Starting With $X(S,\succ)$, $S \subseteq N$

Now the procedure is applied to the $X(S,\succ)$ of the preceding section. It is assumed that the strength ordering is obtained by an aspiration profile, which is the end point of a maximal aspiration adjustment path:

> <u>**THEOREM:**</u> Let a be the end point of a maximal aspiration adjustment path and let a $\succ^{a}$ the strength ordering induced by a (i.e. $i \succ j \Leftrightarrow a_i \geq a_j$), and let $X^{1}(S) := X(S,\succ)$ for all $S \subseteq N$. Then for all $S \subseteq N$
>
> (1) $X(a,S) \cap X(S,\succ) \subseteq X^{r}(S,\succ)$ for all $r \in \mathbb{N}$ and therefore
>
> (2) $X(a,S) \cap X(S,\succ) \subseteq X^{\infty}(S,\succ)$
>
> (3) $X^{\infty}(S,\succ) \neq 0$ for all $S \in coa(a)$.

Generally one can say that for an end point of a maximal aspiration adjustment path the definitions of this section have the following character: the solution sets $X(S,\succ^{a})$ are by and large essential extensions of the sets $X(a,S)$ and the set adjustment procedure reduces these sets into the direction of the sets $X(a,S)$.

Again, it should be remarked, that it is not sure whether one should exclude condition (3) of the set adjustment procedure. If it is excluded, then the obtained sets are larger and the theorem holds as well. Specifically, - and this was the aim of section 4 -, this theorem can be applied to the possibly unique end point a^{*} of the maximal aspiration adjustment path, which meets PSR and OSC. In this case the aspiration adjustment procedure extends the predictes areas $X(S,a)$ to $X^{\cdot\cdot}(S,\succ^{*})$.

Accordingly, we obtain the

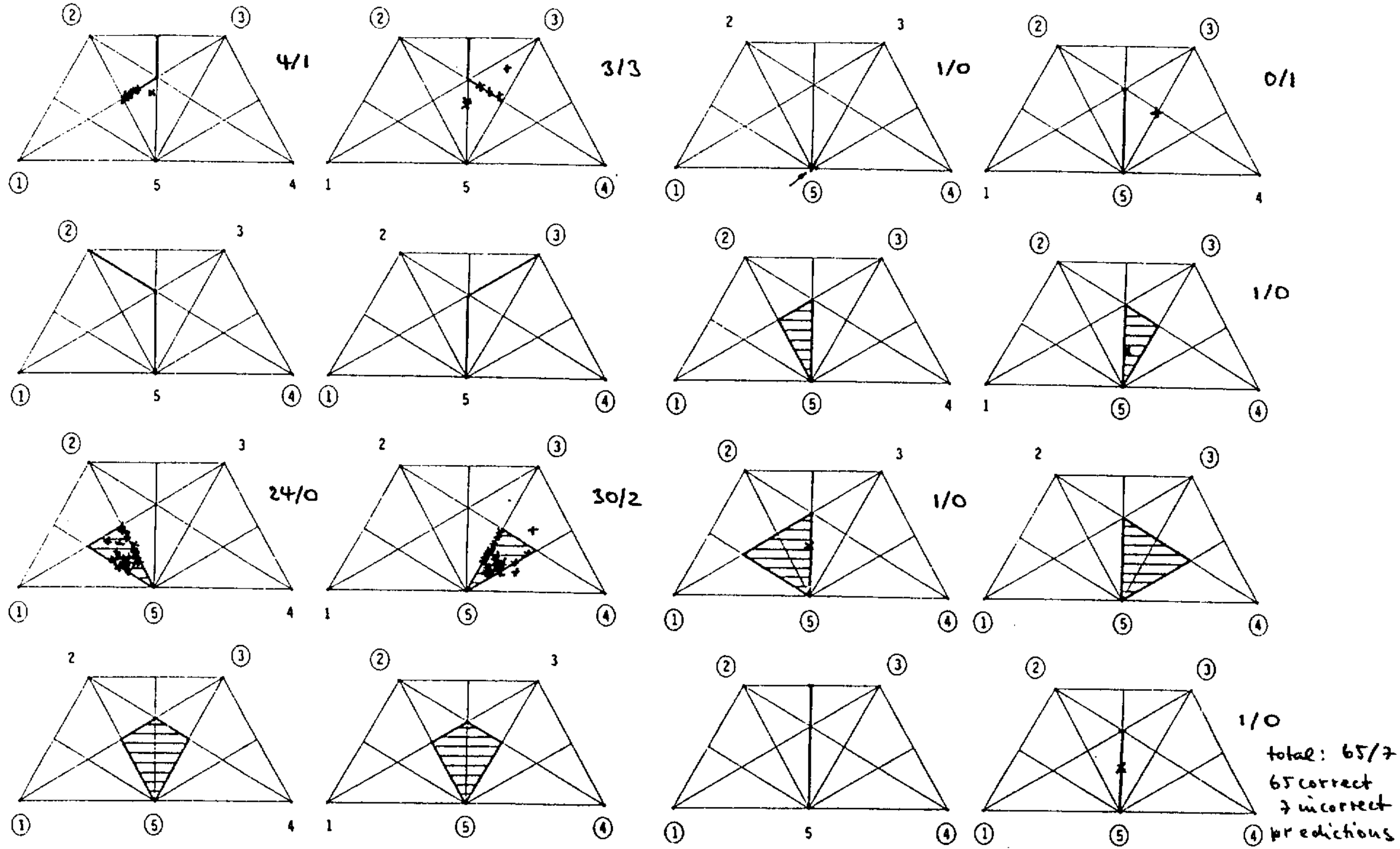

figure 17: solution sets $X^1(S,\zeta^{a*})$ and experimental results of a location game with ideal points. players within a coalition are marked by a circle. - the numbers x/y refer to correct/incorrect predictions (including ε-neighborhoods).

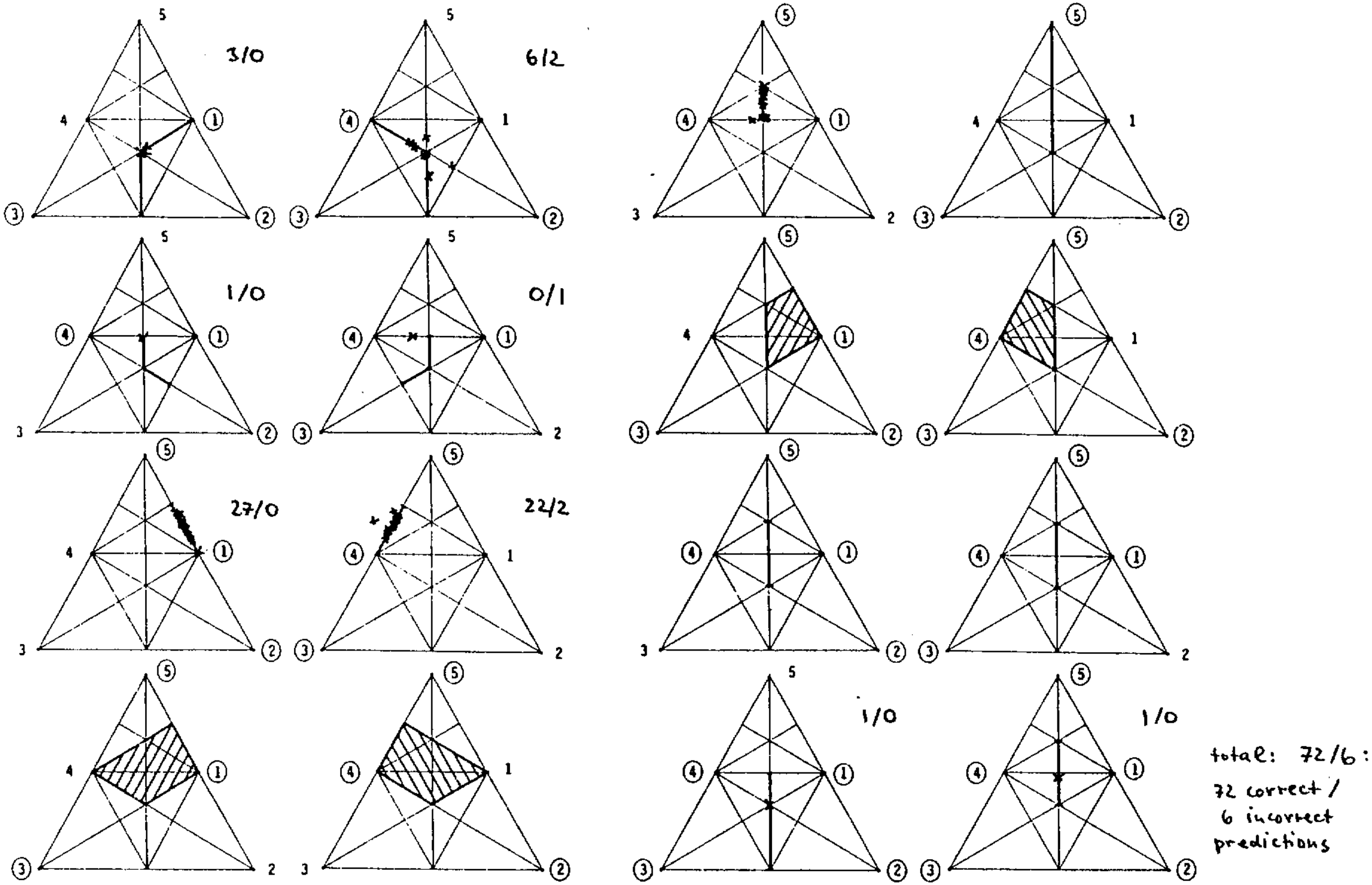

figure 18: solution sets $X^1(S,\succsim^{q*})$ and experimental results of a location game with ideal points. players within a coalition are marked by a circle. - the numbers x/y refer to correct/incorrect predictions (including ε-neighborhoods).

PREDICTION: For each coalition $S \subseteq N$ the predicted outcomes are $X^{\dagger}(S,\succ^a)$, where $\succ^a$ is the strength ordering induced by that aspiration profile a which is obtained as the unique joint end point of all maximal aspiration paths which meet PSR and OSC.[5]

It must be remarked that from the present state of experimental observation it cannot be definitely decided if the sets $X^1(S,\succ^a)$, $X^2(S,\succ^a)$ or $X^{\infty}(S,\succ^a)$ are the best predictors of experimental outcomes. This may also depend from the question to what extend social phenomena can influence the result and thereby from the experimental presentation of the game.

The examples of figures 13 and 14 show that this extension of the predicted areas is essential and necessary to explain experimental results. Moreover, the pure aspiration adjustment path concept (with maximality, PSR, and OSC) leads to point predictions, which are usually not met by experimental results. Overall the procedures of section 4 extend the predicted regions in a way which fits with experimental results quite well.

4.5 Relations to Equal Share Analysis and Equal Division Bounds

The procedure described here is related to the equal share analysis (SELTEN, 1968, 1972) and the equal division bounds concept (SELTEN, 1982, 1985). Both concepts have been developped for characteristic function games. The latter concept is only defined for 3-person games, However, SELTEN only considers sets similar to $X^2(S,\succ^a)$ (in the equal share analysis) and similar to $A^3(S,\succ^a)$ (in the equal division bounds concept).

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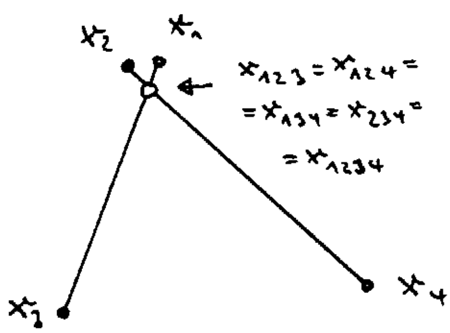
[5] In the case that our suggestion above is wrong and there is more than one possible end point for these paths, all of these paths have to be considered and the predicted area is obtained as a union of the corresponding solution areas.

5 Formation of Blocs

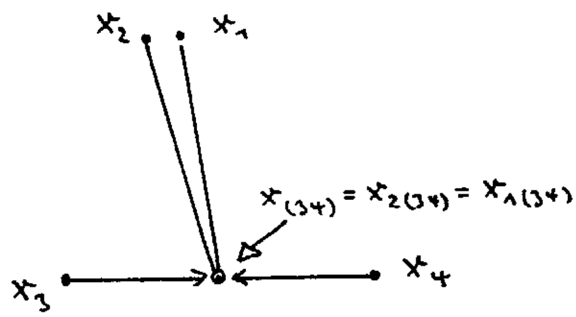
ALBERS (1978) described phenomena in characteristic function games which can also be detected in location games, namely the formation of blocs. We introduce this idea by an example (see figure 17)

(a)



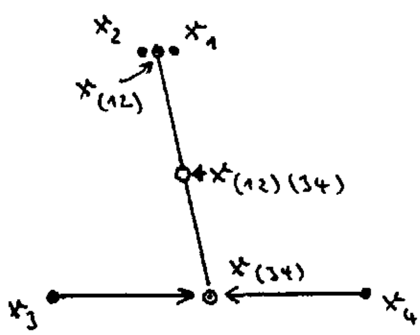
result if all players behave as individuals

(b)



result if players 3,4 form a bloc

(c)



result if ⟨1,2⟩ and ⟨3,4⟩ form a bloc, respectively

figure 17: example describing the formation of blocs in a 4-person location game with ideal points.

In the original game the aspiration equilibrium is given by the intersection point of the diagonals $\overline{24}$ and $\overline{13}$ all aspirations are fulfilled within points (the point is the core point of the game) (see figure (a)).

Now, players 3 and 4 might have the idea to form a subcoalition. Although this does not not give them an additional outcome immediately (since {3,4} is not winning), this improves their bargaining situation, if they form then on replace their different ideal points x_3 and x_4 by a joint ideal point, for instance the mid point x_{34} of x_3 and x_4, and if they from then on try to verify a result which is as near as possible to x_{34}. In this case the result of the modified game is x_{34} (compare (b)).

Of course, players 1 and 2 will answer by forming a subcoalition {1,2} with a joint proposal x_{12} which may be the mid point of x_1 and x_2. Then the result of the aspiration adjustment process will be the mid point of x_{12} and x_{34} (see figure (c)). This result shows that the formation of the subcoalition {3,4} overall improves the result of players 3 and 4.

Generally we define

> **DEFINITION:** A bloc is the formation of a non winning coalition S which replaces the utility function of its members by a joint utility function and from then on behaves as one player (with the aggregated number of votes).

Here we only consider the case that the joint utility function is obtained by the individual utility functions by selecting a new ideal point.

Presently we cannot generally say by which principle this new ideal point is selected. One might think of the center of the smallest circle containing all positions of the bloc members, or of the gravicenter of their ideal points, or of a proportional reduction (or an equal amount reduction) of the aspirations of the original game until a point in the

Pareto surface of the bloc-coalition is obtained. Presently it seems reasonable to predict the convex closure of these alternatives as reasonable agreement points of the bloc players for a joint ideal position.

The question arises, under which circumstances players will form a bloc. One point is, that the conditions of communication must permit to agree on a joint utility function. The central idea, however, is the

> **BLOC FORMING PRINCIPLE:** A bloc $S \subseteq N$ is formed if thereby the aspirations given by the end points $a^{*}(r)$ of the aspiration paths increase for all players of the bloc.

In our example this principle leads to bloc $\langle 3,4 \rangle$ in a first step and to bloc $\langle 1,2 \rangle$ in a second step.

However, we also observed formations of blocs which did not increase the aspirations of its members. These did not refer to the aspirations or the aspiration adjustment process, but to specific proposals of the bargaining process:

If a certain proposal (S,x) is regarded as the final state of the bargaining process by all members of S, then the players outside S will definitely not consider their aspirations as possible outcomes but (usually) less than that. It can then be that these players (or a part of them) by forming a bloc (i.e. a subcoalition with a new joint utility function) changes the game in a way that the new aspiration values permit new coalitions including the bloc, and that all bloc members afterwards have adequate aspirations which are higher than their outcomes in (S,x).

The central question of such a situation is, however, if the bloc will hold afterwards, or if this coalition is only used as a tool to make the bargaining process not stop in a point which is unfavorable for the players of the bloc.

ALBERS (1978) could show that there are situations, where blocs do hold, even if breaking the bloc would increase the aspirations of all of its members (however, these aspirations could not be verified in a coalition including the whole bloc). It seems that the reason that such blocs do not break, can be modeled by loyalty potentials (see ALBERS, 1986), which are built up among "similar" players in "similar" positions, and which influence the decision behavior in a similar way as additional outcomes of the bloc players. The corresponding examples are characteristic function games.

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