INSTITUTE OF MATHEMATICAL ECONOMICS

WORKING PAPERS

No. 334

Self-Supporting Liberals and Their Cliques: An Axiomatic Characterization

by

Dinko Dimitrov and Yongsheng Xu

June 2002

University of Bielefeld

33501 Bielefeld, Germany

Self-Supporting Liberals and Their Cliques:
An Axiomatic Characterization[*]

Dinko Dimitrov

Center for Interdisciplinary Research (ZiF), University of Bielefeld

Wellenberg 1, D-33615 Bielefeld, Germany

Email: dinko.dimitrov@uni-bielefeld.de

Yongsheng Xu

Department of Economics, Andrew Young School of Policy Studies

Georgia State University, Atlanta, GA 30303, U.S.A.

Email: yxu3@gsu.edu

June 2002

Abstract

In this paper we axiomatically characterize a recursive procedure for defining a social group. The procedure starts with the set of all individuals who define themselves as members of the social group. This initial set is then expanded by adding individuals who are considered to be appropriate group members by someone in the initial set, and continues inductively until there is no possibility of expansion any more. *Journal of Economic Literature* Classification Numbers: D63, D71.

Key Words: clique, liberalism, procedure, social identity

1

1 Introduction

The problem of group identification serves as a background in many social and economic contexts. For example, when one examines the political principle of self-determination of a newly formed country, one would like to define the extension of a given nationality. Or when a newly arrived person in Atlanta chooses where to live, the person is interested in finding out a residential neighborhood that would suit her: "Are they my kind of people? Do I belong to this neighborhood?" In all those contexts, it is typically assumed that there is a well-defined group of people who share some common values, beliefs, expectations, customs, jargon, or rituals. Consequently, the questions like "how to define a social group" or "who belongs to the social group" arise.

In a recent paper, Kasher and Rubinstein (1997) provide an answer to the above questions from a social choice perspective. They view that each individual of a society has an opinion about every individual, including oneself, whether the latter is a member of a group to be formed. The collective identity of the group to be formed is then determined by aggregating opinions of all the individuals in the society. For this purpose, they provide, among others, an axiomatic characterization of a "liberal" aggregator whereby the members of the group consist of those and only those who each of them views oneself a member of the group.

The purpose of this paper is to extend the "liberal" aggregator characterized by Kasher and Rubinstein (1997) by adding a *procedural* view to the study of the group identification problem. This procedural view allows us to see a collective as "a family of groups, subcollectives, each with its own view of who is a member of the collective, its own sense of tradition and

2

its own underlying conceptual realm, but each bearing some resemblance to the other ones" (Kasher (1993, p. 70)). More specifically, we axiomatically characterize a recursive procedure for determining "who is a member of a social group". The recursive procedure starts with the set of all individuals who define themselves as members of the social group. This initial set is then expanded by adding individuals who are considered to be appropriate group members by someone in the initial set, and continues inductively until there is no possibility of expansion any more.

The rest of the paper is organized as follows. In Section 2, we present the basic notation and definitions. Section 3 introduces the axioms used for characterization. The main result is contained in Section 4, and Section 5 contains some concluding remarks.

2 Basic Notation and Definitions

Let $N = \{1, \ldots, n\}$ denote the set of all individuals in the society. Each individual $i \in N$ forms a set $G_i \subseteq N$ consisting of all society members that in the view of i have the social identity G. For all $i \in N$, when $i \in G_i$, we also say that i considers himself as a G. A *profile* of views is an n–tuple of vectors $(G_1, \ldots, G_n)$ where $G_i \subseteq N$ for all $i \in N$. Let $\mathcal{G}$ be the set of all profiles of views. A *Collective Identity Function* (*CIF*) assigns to each profile $(G_1, \ldots, G_n) \in \mathcal{G}$ a set $G(G_1, \ldots, G_n) \subseteq N$ of socially accepted Gs. For the purpose of simplicity, we will often write G instead of $G(G_1, \ldots, G_n)$.

For any $(G_1, \ldots, G_n) \in \mathcal{G}$, define $L(0)(G_1, \ldots, G_n) = \{i \in N : i \in G_i\}$. Thus, $L(0)(G_1, \ldots, G_n)$ consists of all individuals in the society who consider themselves as Gs. For any $(G_1, \ldots, G_n) \in \mathcal{G}$, with the help of $L(0)(G_1, \ldots, G_n)$, we now define a CIF being *self-supporting-liberals-and-*

their-cliques, to be denoted by $L(G_1, \ldots, G_n)$, as follows: for each positive integer t, let $L(t)(G_1, \ldots, G_n) = L(t-1) \cup \{i \in N : i \in G_k \text{ for some } k \in L(t-1)(G_1, \ldots, G_n)\}$; and if for some $t \geq 0$, $L(t)(G_1, \ldots, G_n) = L(t+1)(G_1, \ldots, G_n)$, then $L(G_1, \ldots, G_n) = L(t)(G_1, \ldots, G_n)$.

To illustrate the above procedure of defining a CIF, consider the following example. Let $N = \{1, 2, 3\}$ and consider the profile $G_1 = \{1, 2\}, G_2 = \{3\}$ and $G_3 = \emptyset$. Then, for this profile, $L(0) = \{1\}, L(1) = L(0) \cup \{2\} = \{1\} \cup \{2\}$, $L(2) = L(1) \cup \{3\} = \{1, 2, 3\}, L(3) = L(2)$. Therefore, for the given profile of views, we have $L = \{1, 2, 3\}$.

The CIF L defined above is discussed in Kasher and Rubinstein (1997). It starts with $L(0)$ which consists of all members of the society who view themselves as Gs. Thus, the set $L(0)$ reflects a weak notion of *self determination*: if one considers oneself a member of G, then one should be a member of G collectively. In the liberal tradition, those individuals may be called *self-supporting liberals*[1]. The collective identity function L now expands the set $L(0)$ by the following procedure. If, according to some individual $i \in L(0)$, an individual $k \in N$ is viewed as a G, then k should be a G collectively. By adding all such ks to $L(0)$, we obtain the set $L(1)$. We then repeat the above process with $L(1)$ by adding those individuals who are considered as Gs by some individuals in $L(1)$ to $L(1)$ to obtain $L(2)$. Since n is finite, at certain step t, we must have $L(t) = L(t+1)$: the set $L(t)$ can no longer be expanded. The intuition behind each step of the expansion is in line with Kasher's argument (1993): every socially accepted G as being newly added brings a possibly unique new view of being a G collectively with him, and a collective identity function is supposed to aggregate those views and must

---

[1] In Kasher and Rubinstein (1997), the individuals in $L(0)$ are called *liberals*.

pay attention to this new individual's G—concept in order to cover the whole diversity of views in the society about the question "what does it mean to be a G".

3 Axioms

In order to present our axiomatic characterization of the CIF L, we introduce the following axioms for a CIF to satisfy.

A CIF satisfies

Consensus (C) iff, for all $(G_1, \ldots, G_n) \in \mathcal{G}$, $[j \in G_i$ for all $i \in N] \Rightarrow j \in G(G_1, \ldots, G_n)$, and $[j \notin G_i$ for all $i \in N] \Rightarrow j \notin G(G_1, \ldots, G_n)$.

Symmetry (SYM) iff, for all $(G_1, \ldots, G_n) \in \mathcal{G}$, for all $j, k \in N$, if (i) $G_j - \{j, k\} = G_k - \{j, k\}$; (ii) $\forall i \in N - \{j, k\}$, $j \in G_i$ iff $k \in G_i$; (iii) $j \in G_j$ iff $k \in G_k$; (iv) $j \in G_k$ iff $k \in G_j$, then $j \in G(G_1, \ldots, G_n) \Leftrightarrow k \in G(G_1, \ldots, G_n)$.

Independence (I) iff, for all profiles $(G_1, \ldots, G_n)$ and $(G_1', \ldots, G_n')$ in $\mathcal{G}$, and all $i \in N$, if [for every $k \neq i$, $k \in G(G_1, \cdots, G_n)$ if and only if $k \in G(G_1', \ldots, G_n')$], and [for all $k \in N$, $i \in G_k$ if and only if $i \in G_k'$], then $i \in G(G_1, \ldots, G_n) \Leftrightarrow i \in G(G_1', \ldots, G_n')$.

Weak monotonicity (WM) iff, for all $(G_1, \ldots, G_n), (G_1', \ldots, G_n') \in \mathcal{G}$ and all $i, k \in N$, if $[i \notin G(G_1, \ldots, G_n)]$ and $[(G_1', \ldots, G_n')$ is a profile identical to $(G_1, \ldots, G_n)$ except that $i \in G_k$ and $i \notin G_k']$, then $i \notin G(G_1', \ldots, G_n')$.

Equal treatment of insiders' views (ETIV) iff, for all $(G_1, \ldots, G_n)$, $(G_1', \ldots, G_n') \in \mathcal{G}$, and all $i, k, m \in N$, if $[G_h = G_h'$ for all $h \in N -$

5

$\{i,k\}]$, $[m \in G_k,\ m \notin G_i]$, $[G'_k = G_k - \{m\}]$, and $[G'_i = G_i \cup \{m\}]$, then $[k \in G(G_1,\dots,G_n)\ \&\ i \in G(G'_1,\dots,G'_n)] \Rightarrow [m \in G(G_1,\dots,G_n)$ iff $m \in G(G'_1,\dots,G'_n)]$.

Irrelevance of an outsider's view (IOV) iff, for all $i,k \in N$ with $i \neq k$ and all $(G_1,\dots,G_n), (G'_1,\dots,G'_n) \in \mathcal{G}$, if $[G_h = G'_h$ for all $h \in N - \{i\}]$ and $[G'_i = G_i - \{k\}]$, then $[i \notin G(G_1,\dots,G_n)] \Rightarrow [k \in G(G_1,\dots,G_n) \Leftrightarrow k \in G(G'_1,\dots,G'_n)]$.

The first three axioms are introduced and discussed in Kasher and Rubinstein (1997). Weak monotonicity is a weaker version of the monotonicity condition introduced in Kasher and Rubinstein (1997). (WM) requires that if an individual i is not collectively recognized as a G and someone changes his view about i from being a G to being a non$-G$, then i can not be a G after this change.

(ETIV) requires that if two individuals i, k in the society have opposite views about a society member m with $m \in G_k$ and $m \notin G_i$ in a given profile, and if they switch their views concerning m in a new profile and nothing else has changed, then, when k is a G collectively in the original profile and i is a G collectively in the new profile, it must be true that m is a G collectively in the original profile if and only if m is a G collectively in the new profile. This axiom essentially requires that a CIF should treat the views of all the members who are considered to be Gs collectively equally.

Finally, (IOV) stipulates that if someone is collectively defined as a non$-G$, then this person's view about any society member is not relevant in deciding the collective identity of G. The axiom is in the spirit of the exclusive self-determination axiom introduced in Samet and Schmeidler (forthcoming). It should be noted that our axiom is much weaker than the exclusive self-

6

determination axiom used in that paper.

4 The Result

In this section, we give an axiomatic characterization of a CIF being L defined in Section 2.

Theorem 1 *A CIF G satisfies the axioms (C), (SYM), (WM), (I), (ETIV), and (IOV) if and only if $G = L$.*

Proof. It can be verified that the CIF L satisfies the axioms (C), (SYM), (WM), (I), (ETIV) and (IOV). We now show that if a CIF G satisfies these six axioms, we must have $G = L$. Let a CIF G satisfy the six axioms. For all $(G_1, \ldots, G_n) \in \mathcal{G}$, let $G = G(G_1, \ldots, G_n)$ and $L = L(G_1, \ldots, G_n)$ for short. We have to show that, for all $(G_1, \ldots, G_n) \in \mathcal{G}$,

(i) $[i \in G_i$ for some $i \in N] \Rightarrow [i \in G]$;

(ii) $[i \in G$ and $k \in G_i$ for any $k \in N] \Rightarrow [k \in G]$;

(iii) $[i \in L$ and $k \in G_i$ for any $k \in N] \Rightarrow [k \in G]$;

(iv) $G = L$.

(i) The proof of this part is very close to the proof of Theorem 1(a) in Kasher and Rubinstein (1997). Assume that there is a profile $(G_1, \ldots, G_n)$ in which $i \in G_i$ but $i \notin G$. By applying (WM) several times, we arrive at a profile $(G_1', \ldots, G_n')$ that is identical to $(G_1, \ldots, G_n)$ with the possible exception that for all $k \neq i$, $i \notin G_k'$ so that $i \notin G' = G(G_1', \ldots, G_n')$. Let $(G_1'', \ldots, G_n'')$ be the profile where $G_j'' = \{j\}$ for all $j \in G' \cup \{i\}$ and $G_j'' = \emptyset$ for any other j. By (C), $G'' = G(G_1'', \ldots, G_n'')$ does not contain any of the members of $N - G' - \{i\}$. By (SYM), the CIF classifies all members of

$G' \cup \{i\}$ identically. Therefore, there are two possibility: (1) $G''' = \emptyset$; or (2) $G''' = G' \cup \{i\}$. Suppose $G''' = \emptyset$. Let $N'' = \{i \in N : G_i'' = \{i\}\}$. Consider the profile $G_i''' = N''$ for all $i \in N$. Since $G''' = \emptyset$, by the repeated use of (IOV), we obtain that $G'''' = \emptyset$. On the other hand, by (C), $G'''' = N''$, a contradiction. Therefore, it is impossible that $G''' = \emptyset$. Thus, $G''' = G' \cup \{i\}$. Finally, we get a contradiction to (I) because G' and G''' are identical with the exception of member i, and member i is treated equally by all members of N; nevertheless, $i \in G'''$ and $i \notin G'$.

(ii) Assume that there is a profile $(G_1, \dots, G_n)$ where $i \in G$, $k \in G_i$. Note first that if $k \in G_k$, then, according to (i), $k \in G$. For the case where $k \notin G_k$, let $(G_1', \dots, G_n')$ be a profile that is identical to $(G_1, \dots, G_n)$ with the possible exception that $G_i' = G_i - \{k\}$, $G_k' = G_k \cup \{k\}$. Since $k \in G_k'$, from (i), it follows that $k \in G' = G(G_1', \dots, G_n')$. Therefore, $k \in G$ follows from (ETIV).

(iii) This assertion follows from (i) and (ii) directly.

(iv) Let the profile $(G_1, \cdots, G_n) \in \mathcal{G}$ be given. We need to show that $G(G_1, \dots, G_n) = L \,(= L(G_1, \dots, G_n))$. Consider first the profile $(G_1', \dots, G_n')$ such that, for all $i \in L, G_i' = G_i$ and for all $i \in N - L, G_i' = \emptyset$. It is clear that $L(G_1', \dots, G_n') = L$. By (C), $G' \subset L$, and from (iii), $L \subset G'$. Therefore, $G' = L$. Now, starting with the profile $(G_1', \dots, G_n')$ and by appropriately changing G_i' for all $i \in N - L$, we can obtain $(G_1, \dots, G_n)$. Noting that, at each stage of the change, we can invoke (IOV). Then, by the repeated use of (IOV), we obtain $G = G' = L$. $\blacksquare$

5 Concluding Remarks

In this paper, we have axiomatically characterized the procedure that defines the collective identify function L in the framework proposed by Kasher and Rubinstein (1997). Though it is not our intention to advocate the CIF L, we note some interesting features of L. It includes all those individuals $L(0)$ who consider themselves as members of a social group as well as all other individuals who are considered as members of the social group by some member of the social group. In other words, L consists of all those individuals who are self-supporting liberals (self-claimed members of a social group) and their cliques (who are viewed as members of the social group by some member of the social group). Therefore, L reflects a liberal view[2] of collective identity.

References

[1] A. Kasher, Jewish Collective Identity, *in* "Jewish Identity" (D. T. Goldberg and M. Krausz, Eds.), pp. 56-78, Temple University Press, Philadelphia, 1993.

[2] A. Kasher and A. Rubinstein, On the Question "Who is a J?": A Social Choice Approach, *Logique & Analyse* **160** (1997), 385-395.

---

[2] See, for example, Kasher and Rubinstein (1997), and Samet and Schmeidler (forthcoming). If the determination of the membership of a social group is a personal matter, there is indeed some reason to call individuals in $L(0)$ as (self-supporting) liberals (see Sen (1970)).

[3] D. Samet and D. Schmeidler, Between Liberalism and Democracy, *J. Econ. Theory*, forthcoming.

[4] A. K. Sen, The Impossibility of a Paretian Liberal, *J. Political Econ.* **78** (1970), 152-157.