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**Power Measurement as Sensitivity Analysis  
- A Unified Approach**

by

Stefan Napel and Mika Widgrén

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University of Bielefeld

33501 Bielefeld, Germany

# Power Measurement as Sensitivity Analysis – A Unified Approach\*

Stefan Napel  
Institute of SocioEconomics  
University of Hamburg  
Von-Melle-Park 5  
20146 Hamburg  
Germany  
Phone +49-40-42838-4026  
Fax +49-40-42838-3957  
napel@econ.uni-hamburg

Mika Widgrén  
Turku School of Economics  
CEPR and CESifo  
Rehtorinpellonkatu 3  
20500 Turku  
Finland  
Phone +358-2-3383337  
Fax +358-2-3383302  
mika.widgren@tukkk.fi

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UB BIELEFELD  
030/4094127+1



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### Abstract

This paper proposes a unified framework that integrates the traditional index-based approach and the competing non-cooperative approach to power analysis. It rests on a quantifiable notion of ex post power as the (counterfactual) sensitivity of the expected or observed outcome to individual players. Thus, it formalizes players' marginal impact on outcomes in both cooperative and non-cooperative games, for both strategic interaction as well as purely random behavior. By taking expectations with respect to preferences, actions, and procedures one obtains meaningful measures of ex ante power. Established power indices turn out to be special cases.

**Keywords:** Power indices, spatial voting, equilibrium analysis, decision procedures

# 1 Introduction

Scientists who study power in political and economic institutions seem divided into two disjoint methodological camps. The first one uses non-cooperative game theory to analyze the impact of explicit decision procedures and given preferences over a well-defined – usually Euclidean – policy space.<sup>1</sup> The second one stands in the tradition of cooperative game theory with much more abstractly defined voting bodies: The considered agents have no particular preferences and form winning coalitions which implement unspecified policies. Individual chances of being part of and influencing a winning coalition are then measured by a *power index*.<sup>2</sup>

Proponents of either approach have recently intensified their debate in the context of decision-making in the European Union (EU).<sup>3</sup> The non-cooperative camp's verdict is that "power indices exclude variables that ought to be in a political analysis (institutions and strategies) and include variables that ought to be left out (computational formulas and hidden assumptions)" (Garrett and Tsebelis, 1999a, p. 337). The cooperative camp has responded by clarifying the assumptions underlying its power formulas and giving some reasons for *not* making institutions and strategies – corresponding to decision procedures and rational preference-driven agents – more explicit.<sup>4</sup> There also have been some attempts to include actors' preferences in the cooperative approach.<sup>5</sup>

Several authors have concluded that it is time to develop a *unified framework* for mea-

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<sup>1</sup>See e.g. Steunenberg (1994), Tsebelis (1994, 1996), Crombez (1996, 1997), and Moser (1996, 1997).

<sup>2</sup>See e.g. Brams and Affuso (1985a, 1985b), Widgrén (1994), Hosli (1993), Laruelle and Widgrén (1998), Baldwin et al. (2000, 2001), Felsenthal and Machover (2001b), and Leech (2002) for recent applications of traditional power indices. Felsenthal and Machover (1998) and Nurmi (1998) contain a more general discussion regarding index-based analysis of power.

<sup>3</sup>Cf. the contributions to the symposium in *Journal of Theoretical Politics* 11(3), 1999, together with Tsebelis and Garrett (1997), Dowding (2000), Garrett and Tsebelis (2001), and Felsenthal and Machover (2001a).

<sup>4</sup>See, in particular, Holler and Widgrén (1999), Berg and Lane (1999), Felsenthal and Machover (2001a), and Braham and Holler (2002).

<sup>5</sup>See e.g. Straffin (1977, 1988), Widgrén (1995), Kirman and Widgrén (1995), and Hosli (2002).

suring decision power (cf. Steunenberget al., 1999, and Felsenthal and Machover, 2001a).<sup>6</sup> On the one hand, such a framework should allow for predictions and ex post analysis of decisions based on knowledge of procedures and preferences. On the other hand, it must be open to ex ante and even completely a priori<sup>7</sup> analysis of power when detailed information may either not be available or should be ignored for normative reasons. Unfortunately, the first attempt to provide such a framework, by Steunenberget al. (1999), is problematic.<sup>8</sup> It confounds power and the success that may, but need not, result from it. This paper proposes an alternative framework.

In particular, we generalize the concept of a player's *marginal impact* or *marginal contribution* to a collective decision in order to establish a common (ex post) primitive of power for cooperative and non-cooperative analysis. Its evaluation amounts to the comparison of an actual outcome with a counterfactual *shadow outcome* which alternatively could have been brought about by the considered player. That is we look at the *sensitivity* of a given outcome to the considered player's behavior. In our view, sensitivity analysis of outcomes goes a long way towards a reconciliation of equilibrium-based non-cooperative measurement and winning coalition-based traditional power indices. One can transparently measure ex ante power as a player's expected ex post power. This is in line with the probabilistic interpretation of traditional power indices (cf. Owen, 1972, 1995 and Straffin,

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<sup>6</sup>Gul (1989) and Hart and MasColell (1996) already give non-cooperative foundations for the Shapley value and thus indirectly the Shapley-Shubik index.

<sup>7</sup>There has been controversy at several workshops on power analysis about the correct usage of the terms 'a priori' and 'a posteriori'. We use the term 'ex ante' to mean 'before a decision is singled out or taken' and the term 'ex post' to mean 'for a particular expected or observed decision'. We reserve 'a priori' to the complete ignorance about any aspect of decision-making other than – somewhat arbitrarily – voting weights and quota. Note that this definition makes meaningful ex ante or ex post analysis of real institutions for which more information than weights and quota – e.g. the strategic resources awarded to players by a particular agenda-setting or multi-stage voting procedure – is known and relevant necessarily 'a posteriori'.

<sup>8</sup>Partly in response to Widgrén and Napel (2002) and an earlier version of this paper, Steunenberget al. and Schmidtchen have suggested modifications of their framework that promise to significantly improve it at the LSE Workshop on Voting Power Analysis, 2002.

1977, 1978, 1988; Laruelle and Valenciano, 2002, provide an up-to-date discussion and extensions). Expectation is to be taken with respect to an appropriate probability measure on the power-relevant states of the world.

The proposed framework is flexible and allows for different ‘degrees of a priori-ness’, concerning players’ either purely random or preference-based actions as well as details about decision procedures. Only inclusion of the latter allows to capture the power implications of resources other than pure voting weight – players’ procedural and strategic resources. Traditional (ex ante) power indices, such as the Penrose index or Shapley-Shubik index, are obtained as special cases.

The index approach to power analysis has evolved significantly in the last 50 years. It has reached a point where its integration into a framework that also allows for explicit decision procedures and preference-driven strategic behavior seems a natural step. We first give a short overview of the index approach in section 2, which takes up some arguments from the fundamental critique of index-based studies by Garrett and Tsebelis. The creative response by Steunenberget al. to the latter is sketched and briefly discussed in section 3. The main section 4 then lays out our unified framework. Section 5 concludes.

## 2 The Traditional Power Index Approach

The traditional object of studies of decision power has been a *weighted voting game* characterized by a set of players,  $N = \{1, \dots, n\}$ , a voting weight for each player,  $w_i \geq 0$  ( $i \in N$ ), and a minimal quota of weights,  $k > 0$ , that is needed for the passage of a legislative proposal. Subsets of players,  $S \subseteq N$ , are called *coalitions*. If a coalition  $S$  meets the quota, i. e.  $\sum_{i \in S} w_i \geq k$ , it is a *winning coalition*. Formation of a winning coalition is assumed to be desirable to its members. More generally, a winning coalition need not be determined by voting weights. One can conveniently describe an abstract decision body  $v$  by directly stating either the set  $W(v)$  of all its winning coalitions or its subset of *minimal winning coalitions*,  $M(v)$ .<sup>9</sup> The latter contains only those winning coalitions which are turned into

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a losing coalition if one of its members leaves the coalition. An equivalent representation is obtained by taking  $v$  to be a mapping from the set of all possible coalitions,  $\wp(N)$ , to  $\{0, 1\}$ , where  $v(S) = 1$  (0) indicates that  $S$  is winning (losing). Function  $v$  is usually referred to as a *simple game*. The difference  $v(S) - v(S - \{i\})$  is known as player  $i$ 's *marginal contribution* to coalition  $S$ .

The most direct approach to measuring players' power is to state a mapping  $\mu$  – called an index – from the space of simple games to  $\mathbb{R}_+^n$  together with a verbal story of why  $\mu_i(v)$  indicates player  $i$ 's power in the considered class of decision bodies. But despite the plausibility of some 'stories', their verbal form easily disguises incoherence or even inconsistency. The *axiomatic* or *property-based approach*, in contrast, explicitly states a set of mathematical properties  $\{A_1, \dots, A_k\}$  that an index is supposed to have – together with an (ideally unique) index  $\mu$  which actually satisfies them. The requirements  $A_j$  are usually referred to as *axioms*. A prominent example for the axiomatic approach is the Shapley-Shubik index  $\phi$  (cf. Shapley, 1953, and Shapley and Shubik, 1954). Though this may not be immediately obvious, four properties  $A_1$ – $A_4$  imply that  $\phi_i(v)$  must be player  $i$ 's weighted marginal contribution to all coalitions  $S$ , where weights are proportional to the number of player orderings  $(j_1, \dots, i, \dots, j_n)$  such that  $S = \{j_1, \dots, i\}$ .<sup>10</sup> Axioms can give a clear reason of why  $\phi$  and no other mapping is used – in particular, if a convincing story for them is provided. A drawback of the axiomatic approach is, however, that axioms clarify the tool with which one measures,<sup>11</sup> but not what is measured based on which (behavioral

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<sup>9</sup>Typically, one requires that the empty set is losing, the grand coalition  $N$  is winning, and any set containing a winning coalition is also winning.

<sup>10</sup>The Shapley-Shubik index is characterized by the requirements that ( $A_1$ ) a (dummy) player who makes no marginal contribution in  $v$  has index value 0, ( $A_2$ ) that the labelling of the players does not matter, ( $A_3$ ) that players' index values add up to 1, and ( $A_4$ ) that in the composition  $u \vee v$  of two simple games  $u$  and  $v$ , having the union of  $W(u)$  and  $W(v)$  as its set of winning coalitions  $W(u \vee v)$ , each player's power equals the sum of his power in  $u$  and his power in  $v$  minus his power in the game  $u \wedge v$  obtained by intersecting  $W(v)$  and  $W(u)$ .

<sup>11</sup>Even this cannot be taken for granted. Axioms can be too general or mathematically complex to give much insight.

and institutional) assumptions about players and the decision body.

The *probabilistic approach* to the construction of power indices entails explicit assumptions about agents' behavior together with an explicit definition of what is measured. Agent behavior is specified as a probability distribution  $P$  for players' *acceptance rates*, denoting the probabilities of a 'yes'-vote by individual players. A given player's ex ante power is then taken to be his probability of casting a *decisive vote*, i. e. to pass a proposal that would not have passed had he voted 'no' instead of 'yes'. Thus power is inferred from the *hypothetical consequences of an agent's behavior*. The object of analysis is with this approach no longer described only by the set of winning coalitions or  $v$ , but also an explicit model of (average) behavior.<sup>12</sup> For example, the widely applied Penrose index (Penrose, 1946) – also known as the non-normalized Banzhaf index – is based on the distribution assumption that each player independently votes 'yes' with probability  $1/2$  (on an unspecified proposal). The corresponding joint distribution of acceptance rates then defines the index  $\beta$  where  $\beta_i(v)$  turns out to be, again, player  $i$ 's weighted marginal contribution to all coalitions in  $v$ , where weight is this time equal to  $1/2^{n-1}$  for every coalition  $S \subseteq N$ .<sup>13</sup>

Just as the direct approach and the axiomatic approach require stories to justify the index  $\mu$  or  $\{A_1, \dots, A_k\}$ , respectively, the assumption of a particular distribution  $P$  of acceptance rates has to be motivated. This points towards drawbacks of the probabilistic index approach. First, the described behavior is usually not connected to any information on the agents' preferences or decision procedures.<sup>14</sup> Second, decisions by individual players are assumed to be stochastically independent. This will, in practice, only rarely be the case since it is incompatible with negotiated coalition formation and voting based on stable

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<sup>12</sup>Acceptance rates and assumptions on their stochastic relation among different players can be interpreted as an implicit way of taking preferences into account in power index models (see Straffin, 1988, for discussion).

<sup>13</sup>This means that  $\beta_i(v)$  is the ratio of the number of swings that player  $i$  does have to the number of swings that  $i$  could have. One can alternatively derive  $\beta$  from the assumption that players' acceptance rates are independently distributed on  $[0, 1]$  with mean  $1/2$ .

<sup>14</sup>If such information is not available, the principle of insufficient reason seems a valid argument for the assumptions behind the Penrose index.



player preferences. The imposition of stochastic or deterministic restrictions for coalitions containing particular players or sub-coalitions can alleviate some of these shortcomings of traditional indices (see van den Brink, 2001, or Napel and Widgrén, 2001). Still, the application of traditional power indices has been severely criticized. Concerning decision-making in the European Union, Garrett and Tsebelis (1999a, 1999b, and 2001) have taken a particularly critical stance, pointing out indices' ignorance of decision procedures and player strategies.<sup>15</sup>

We agree with Garrett and Tsebelis that institutions and strategies have to be taken into account by political analysis. Nevertheless, both the normative or constitutional *ex ante* (possibly even *a priori*) analysis of political institutions and the positive or practical political analysis of actual and expected decisions are valuable. It is legitimate to ask: Which voting weights in the EU Council of Ministers would be equitable? For an answer, countries' special interests and their potentially unstable preferences in different policy dimensions should not matter. Hence they are best concealed behind a 'veil of ignorance' – as accomplished by the Penrose index. When multi-level decision bodies are designed, the objective of minimizing the probability for the referendum paradox (an upper-level decision taken against a majority at a lower level) provides a similar case for *a priori* analysis. In contrast, evaluation of the medium-run expected influence on EU policy from a particular country's point of view benefits if available preference information is taken into account.<sup>16</sup> Garrett and Tsebelis's goal of "understanding ... policy changes on specific issues ... and negotiations about treaty revisions" (Garrett and Tsebelis, 1999b, p. 332) or, in general, "understanding decision-making in the EU" (Garrett and Tsebelis, 2001, p. 105) seems impossible to achieve by looking *only* at the voting resources of different EU members and

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<sup>15</sup>See the replies of Berg and Lane (1999), Holler and Widgrén (1999), Steunenberg, Schmidtchen, and Koboldt (1999), and Felsenthal and Machover (2001a).

<sup>16</sup>The 'medium run' can, of course, be short-lived: A social democratic Council member can quickly turn into a right-wing conservative through elections. News about the first national or another foreign outbreak of mad-cow disease have quickly changed voters' and a government's views on agricultural, health, or trade policy.

the relevant qualified majority rule. The traditional power index approach is not a suitable framework to discuss these positive questions related to power.

We disagree with Garrett and Tsebelis' (2001) call for "a moratorium on the proliferation of index-based studies" (p. 100). As already pointed out by others, it is a matter of taste whether one deems the pursuit of positive or normative analysis more worthwhile. We believe in both and agree with Garrett and Tsebelis that the strategic implications, which are hard to separate from players' preferences, of particular institutional arrangements in the EU and elsewhere have received too little attention so far. We think it desirable to have a general unified framework which allows for positive and normative analysis, actual political and constitutional investigations.

### 3 The Strategic Power Index of Steunenberget al.

Replying to the critique by Garrett and Tsebelis, Steunenberget al. (1999) have proposed a framework originally believed to reconcile traditional power index analysis and analysis of non-cooperative games, which explicitly describe agents' choices in a political procedure and (their beliefs about) agents' preferences. They consider a spatial voting model with  $n$  players and an  $m$ -dimensional outcome space. In our notation, let  $N = \{1, \dots, n\}$  be the set of players and  $X \subseteq \mathbb{R}^m$  be the outcome or policy space.  $\Gamma$  denotes the procedure or game form describing the decision-making process and  $q \in X$  describes the status quo before the start of decision-making. Players are assumed to have Euclidean preferences with  $\lambda_i \in X$  ( $i \in N$ ) as player  $i$ 's ideal point. A particular combination of all players' ideal points and the status quo point define a 'state of the world'  $\xi$ . Assuming that it exists and is unique, let  $x^*(\xi)$  denote the equilibrium outcome of the game based on  $\Gamma$  and  $\xi$ .<sup>17</sup> Steunenberget al. are aware of Barry's (1980) distinction between 'power' and 'luck' and explicitly strive to isolate "the ability of a player to make a difference in the outcome" (p. 362). They note that "[h]aving a preference that lies close to the equilibrium outcome

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<sup>17</sup>Non-uniqueness may be accounted for by either equilibrium selection or, in ex ante analysis, explicit assumptions about different equilibria's probability.

of a particular game does not necessarily mean that this player is also ‘powerful’” (p. 345). Therefore, they suggest to consider not one particular state of the world  $\xi$  but many.

In particular, one can consider each  $\lambda_i$  and the status quo  $q$  to be realizations of random variables  $\tilde{\lambda}_i$  and  $\tilde{q}$ , respectively. If  $P$  denotes the joint distribution of random vector  $\tilde{\xi} := (\tilde{q}, \tilde{\lambda}_1, \dots, \tilde{\lambda}_n)$  and  $\|\cdot\|$  the Euclidean norm, then

$$\Delta_i^\Gamma := \int \|\tilde{\lambda}_i - x^*(\tilde{\xi})\| dP \quad (1)$$

gives the expected distance between the equilibrium outcome for decision procedure  $\Gamma$  and player  $i$ 's ideal outcome. Steunenberget al. “all other things being equal” consider “a player ... more powerful than another player if the expected distance between the equilibrium outcome and its ideal point is smaller than the expected distance for the other player” (p. 348). In order to obtain not only a ranking of players but a cardinal measure of their power, they proceed by considering a dummy player  $d$  – either already one of the players or added to  $N$  – “whose preferences vary over the same range as the preferences of actual players.” This leads to their definition of the *strategic power index (StPI)* as

$$\Psi_i^\Gamma := \frac{\Delta_d^\Gamma - \Delta_i^\Gamma}{\Delta_d^\Gamma}.$$

The remainder of Steunenberget al.'s paper is then dedicated to the detailed investigation of particular game forms  $\Gamma$  which model the consultation and cooperation procedures of EU decision-making. They derive the subgame perfect equilibrium of the respective policy game for any state of the world  $\xi$ , and aggregate the distance between players' ideal outcome and the equilibrium outcome assuming independent uniform distributions over a one-dimensional state space  $X$  for the ideal points and the status quo. The uniformity assumption is not innocuous (see Garrett and Tsebelis, 2001, p. 101), but Steunenberget al.'s numerical calculations could quite easily be redone with more complex distribution assumptions. The important question is: Does  $\Psi_i^\Gamma$  measure what it is claimed to, i. e. “the ability of a player to make a difference in the outcome”?

The answer is: Only under very special circumstances. In particular, (1) turns out to define  $\Delta_i^\Gamma$  to be player  $i$ 's *expected success*. Just like actual distance measures success (a

function of luck and power), so does average distance measure average success. Unless one regards average success as the defining characteristic of power (which neither Steunenberg et al. nor many others do), taking expectations will only by coincidence achieve what Steunenberg et al. aim at, namely to “level out the effect of ‘luck’ or a particular preference configuration on the outcome of a game” (p. 362). This point is discussed in considerable detail in Napel and Widgrén (2002, pp. 9ff). There, various examples illustrate that the StPI is a good measure of expected success but in general fails to capture power;<sup>18</sup>  $\Psi_i^\Gamma$  may also become negative. Only for particular distribution assumptions is luck ‘levelled out’ by taking averages. Similarly, only under special conditions – which eliminate all strategic aspects from the StPI – does a link between the StPI and Penrose index discovered by Felsenthal and Machover (2001a) exist (see Napel and Widgrén, 2002, pp. 12f).

These points seem to have been taken. In particular, Steunenberg and Schmidtchen<sup>19</sup> have proposed the incorporation of a distinct ‘dummy player’ (meaning a reference player without decision rights) for each individual player. This solves some of the problems discussed in Napel and Widgrén (2002), but not all.

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<sup>18</sup>A non-technical example refers to a group of boys with a leader who makes proposals of what to do in the afternoon (play football, watch a movie, etc.) which have to be accepted by simple majority. Boys’ preferences (mappings from the weather conditions, pocket money, etc.) are assumed to be identically but independently distributed. The agenda setter enjoys smaller average distance to the equilibrium outcome than the others; amongst the latter, expected distance is the same. Then, the little brother of the group’s leader is allowed to participate in the group’s afternoon activity albeit without *any* say in selecting the daily programme. He does not always agree with his elder brother’s most desired outcome, but does so more often than the others (the brothers’ ideal points are positively correlated). Then, mean distance between the group’s equilibrium activity and its new member’s most desired recreation is smaller – and hence his StPI power value is larger – than that of the established members who actually have a vote on the outcome.

<sup>19</sup>Presentation at the above-mentioned LSE Workshop.

## 4 An alternative approach

Steunenberget al.'s original framework is suited to study success both *ex post* and, by taking expectations, *ex ante*. The chief reason why the StPI does not measure power is its reliance on information only about the *outcome* of strategic interaction. Power refers to the ability to make a difference to something (which is implicitly regarded as subjectively valuable to someone); a player's power derives from – and needs to be measured by recurring to – his available strategies in the considered decision procedure or game form. Power means potential and thus refers to consequences of both actual and hypothetical actions.

In our view, the key to isolating a player's power in the context of collective decision-making is his *marginal impact* or his *marginal contribution* to the outcome  $x^*$ . As mentioned in section 2, this concept is well-established in the context of simple games and also general cooperative games, where it measures the implication of some player  $i$  joining a coalition  $S$ . The fundamental idea of comparing a given outcome with one or several other outcomes, taking the considered player  $i$ 's behavior to be variable, amounts to analysis of the *sensitivity of the outcome with respect to player  $i$ 's actions*. This can be generalized to a non-cooperative setting which explicitly describes a decision procedure.

### 4.1 An Example

For illustration, consider the player set  $N \cup \{b\}$ , the rather restricted policy space  $X = \{0, 1\}$  embedded in  $\mathbb{R}$ , and status quo  $\tilde{q} = 0$ . Let the decision procedure  $\Gamma$  be such that, first, bureaucrat  $b$  sets the agenda, i. e. either proposes 1 or ends the game and thereby confirms the status quo. Formally, he chooses an action  $a_b$  from  $A_b = \{1, q\}$ . If a proposal is made, then all players  $i \in N$  simultaneously vote either 'yes', denoted by  $a_i = 1$ , or 'no' ( $a_i = 0$ ). This makes  $A_i = \{0, 1\}$  their respective set of actions. The proposal is accepted if the weighted number of 'yes'-votes meets a fixed quota  $k$ , where the vote by player  $i$  is

weighted by  $w_i \geq 0$ . Otherwise, the status quo prevails. Formally, the function

$$x(a) = x(a_b, a_1, \dots, a_n) = \begin{cases} 1; & a_b = 1 \quad \wedge \quad \sum_{i \in N} a_i w_i \geq k \\ 0; & \text{otherwise} \end{cases}$$

maps each action profile  $a = (a_b, a_1, \dots, a_n)$  to an outcome. Traditional power index analysis for players  $i \in N$  can easily be mimicked with this setting:<sup>20</sup> Take

$$D_i^0(a) := x(a_b, a_1, \dots, a_i, \dots, a_n) - x(a_b, a_1, \dots, 0, \dots, a_n) \quad (2)$$

as player  $i$ 's marginal contribution for action profile  $a$  – corresponding to  $v(S) - v(S \setminus \{i\})$  for coalition  $S = \{j | a_j = 1\}$  – and make probabilistic assumptions over the set of all action profiles. The latter replaces the probability distribution over the set of all coalitions which is usually considered via assumptions on acceptance rates. Assume, for example, that bureaucrat  $b$  always chooses  $a_b = 1$  and let  $P$  denote the joint distribution over action profiles  $a \in \prod_{i \in \{b\} \cup N} A_i$ . Then,

$$\mu_i^\Gamma := \int D_i^0(a) dP(a) \quad (3)$$

corresponds exactly to the traditional probabilistic measures obtained via Owen's multilinear extension (1972, 1995) and Straffin's power polynomial (1977, 1978, 1988). For example,

$$P(a) = \begin{cases} 1/2^{n-1}; & a_b = 1 \\ 0; & a_b = 0, \end{cases}$$

makes  $\mu^\Gamma$  the Penrose index. For

$$P(a) = \begin{cases} \frac{(\sum_{i \in N} a_i - 1)! (n - \sum_{i \in N} a_i)!}{n!}; & a_b = 1 \\ 0; & a_b = 0, \end{cases}$$

it is the Shapley-Shubik index.

Economic and political actions are in most modern theoretical analyses regarded to be the consequence of rational and strategic reasoning based on explicit preferences. Therefore directly considering (probability distributions over) players' action choices without

<sup>20</sup>Here we neglect the agenda setter  $b$  in order to stress the equivalence to traditional indices.

recurring to the underlying preferences is methodologically somewhat unsatisfying.<sup>21</sup> It is usually not difficult to find (probability distributions over) preferences which rationalize given behavior. In above example, one may e. g. assume that players  $i \in N$  have random spatial and procedural preferences with uniformly distributed ideal points  $\tilde{\lambda}_i$  taking values in  $X$  and the procedural component that for given policy outcome  $x \in X$  they prefer to have voted truthfully.<sup>22</sup> Let the bureaucrat have only a procedural preference, namely one for putting up a proposal if and only if it is accepted.

Now consider a particular realization of ideal points  $\lambda = (\lambda_1, \dots, \lambda_n)$ . In the unique equilibrium of this game, first, the bureaucrat chooses  $a_b = 1$ , i. e. he proposes 1, if and only if the set  $Y := \{i | \lambda_i = 1\}$  meets the quota, i. e.  $\sum_{i \in Y} w_i \geq k$ . Second, every voter votes truthfully according to his preference in case that a proposal is made. Hence, the function

$$x^*(\lambda) = x^*(\lambda_1, \dots, \lambda_n) = \begin{cases} 1; & \sum_{i \in N} \lambda_i w_i \geq k \\ 0; & \text{otherwise} \end{cases}$$

maps all preference profiles (as determined by the vector of players' ideal points) to a unique equilibrium outcome. Assumptions about the distribution  $P'$  of random vector  $\tilde{\lambda} = (\tilde{\lambda}_1, \dots, \tilde{\lambda}_n)$  can be stated such that  $P'$  induces the same distribution  $P$  over action profiles  $a$  in equilibrium which has been directly assumed above. For example,  $P'(\lambda_1, \dots, \lambda_n) \equiv 1/2^{n-1}$  implies equilibrium behavior which lets  $\mu_i^\Gamma$  in (3) equal the Penrose index.

## 4.2 Measuring Ex Post Power

We propose to extend above analysis from the simple coalition framework typical of ex ante power measurement and the very basic example voting game just considered to a

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<sup>21</sup>It is a very convenient short-cut, however. Also note that models of boundedly rational agents who do not optimize but apply heuristic rules of thumb receive more and more attention in the game-theoretic literature (see e. g. Samuelson, 1997, Fudenberg and Levine, 1998, and Young, 1998).

<sup>22</sup>See Hansson (1996) on the importance of procedural preferences in the context of collective decision-making. Our procedural preference assumption can e. g. be motivated by regarding each player as the representative of a constituency to which he wants to demonstrate his active pursuit of its interests.

more general setting. Player  $i$ 's marginal contribution is a measure for the sensitivity of the outcome to player  $i$ 's behavior. We consider it the best available indicator of player  $i$ 's potential or ability to make a difference for a given (expected or observed) collective decision, i. e. his ex post power. If this is of normative interest or for the lack of precise data, one can calculate ex ante power based on ex post power as the latter's expected value. This equates ex ante power of player  $i$  with the expected sensitivity of the outcome to  $i$ 's behavior. Expectations can be taken with respect to several different aspects which affect ex post power such as actions, preferences, or the procedure. This allows for the (re-)foundation of ex ante measures (including a priori indices) on a well-specified notion of ex post power.

There are several – to us, at this stage, all promising – directions in which the notion of 'power as marginal impact' or 'power as sensitivity' can be made precise. The uniting theme is the identification of the *potential to influence* an outcome of group decision-making by looking at the *sensitivity* of that outcome with respect to the considered set of agents. Influence can equally refer to the impact of a random 'yes' or 'no'-decision, as assumed by the traditional probabilistic index approach, and to the impact of a strategic 'yes' or 'no'-vote based on explicit preferences.

Crucially, 'impact' is always relative to a what-if scenario or what we would like to call the *shadow outcome*. The shadow outcome is the group's decision which would have resulted if the player  $i$  whose power is under consideration had chosen (were to choose) differently than he actually did (is expected to), e. g. if he had stayed out of coalition  $S$  when he ex post belongs to it, or had ideal point 0 instead of 1. In simple games the difference between shadow outcome and actual outcome, i. e. the sensitivity of the outcome to  $i$ 's behavior for a given action profile or coalition, is either 0 or 1. A richer decision framework allows for more finely graded ex post power. It also requires a choice between several candidates for the shadow outcome and, possibly, the subjective evaluation of differences.

A natural way to proceed is to measure player  $i$ 's ex post power as the difference in outcome for given actions of all players, e. g.  $a^* = (a_b^*, a_1^*, \dots, a_i^*, \dots, a_n^*)$  also denoted by  $(a_i^*, a_{-i}^*)$ , and the case in which – for whatever reasons – player  $i$  chooses a different action,



i. e. for  $(a'_i, a^*_{-i}) = (a^*_b, a^*_1, \dots, a'_i, \dots, a^*_n)$  with  $a^*_i \neq a'_i$ . For *preference-based actions* with a unique equilibrium  $a^*$ , this defines a player's ex post power as the hypothetical impact of a *tremble* in the spirit of Selten's (1975) perfectness concept, i. e. of irrational behavior or imperfect implementation of his preferred action. More generally, a tremble can refer to just any deviation from reference behavior or reference preferences.

If  $A_i$  consists of more than two elements, different degrees of irrationality or – if preferences are left out of the picture – potential deviations from the observed action profile can be considered. In particular, one may confine attention to the impact of a local *action tremble*. If  $A_i = X = \{0, \delta, 2\delta, \dots, 1\}$  for some  $\delta > 0$  that uniformly divides  $[0, 1]$ ,<sup>23</sup> a suitable definition of player  $i$ 's marginal contribution is

$$D_i^{1'}(a^*) := \begin{cases} \frac{x(a^*_i, a^*_{-i}) - x(a^*_i - \delta, a^*_{-i})}{\delta}; & a^*_i - \delta \geq 0 \\ 0; & \text{otherwise.} \end{cases} \quad (4)$$

If  $\delta = 1$ , this corresponds exactly to  $D_i^0(\cdot)$  and the marginal contribution defined in the traditional power index framework (see p. 11). As players' choice set approaches the unit interval, i. e.  $\delta \rightarrow 0$ , one obtains

$$D_i^1(a^*) := \lim_{\delta \rightarrow 0} \frac{x(a^*_i, a^*_{-i}) - x(a^*_i - \delta, a^*_{-i})}{\delta} = \left. \frac{\partial x(a)}{\partial a_i} \right|_{a=a^*} \quad (5)$$

for  $a^*_i \in (0, 1)$ .

Both (4) and (5) measure player  $i$ 's power in a given situation, described by the ex post action vector  $a^*$ , as the (marginal) change of outcome which would be caused by a small (or marginal) change of  $i$ 's action. It is, however, not necessary to take only small trembles into account; one may plausibly use

$$D_i^{1''}(a^*) := \max_{a_i \in X} [x(a^*_i, a^*_{-i}) - x(a_i, a^*_{-i})] \quad (6)$$

to define an alternative measure of ex post power.<sup>24</sup>  $D_i^1$ ,  $D_i^{1'}$ , and  $D_i^{1''}$  are all based on the question:

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<sup>23</sup>This assumes that elements of  $X$  have a cardinal meaning. Clarification of the possibilities for extension of our sensitivity analysis-framework to ordinal or nominal outcome spaces, as studied e. g. by Freixas and Zwicker (2002), is left for future work.

- If a player acted differently, would this alter the outcome of collective decision-making and, if yes, by how much?

Making different assumptions about which possible ‘differences’ in a player’s behavior are relevant, they give a different cardinal answer to this question.

Players’ preferences may enter (4)–(6) to define  $x^*(\lambda_1, \dots, \lambda_n) \equiv x(a^*)$  as the reference point for action trembles, i. e. the point at which the (generalized) derivative of outcome function  $x(\cdot)$  with respect to player  $i$ ’s action is evaluated. If  $x^*(\lambda_1, \dots, \lambda_n)$  is the unique equilibrium outcome, any action deviation resulting in a distinct outcome is irrational. A meaningful alternative to studying the potential damage or good that a player’s irrationality could cause is to instead consider the effect of variations in his preferences while maintaining rationality. This refers to the following two criteria for ex post power as sensitivity for given preferences:

- If a player wanted to, could he alter the outcome of collective decision-making?
- Would the change of outcome in magnitude (and direction) match the considered change in preference?

Precise answers to these questions can be given by replacing  $x(a^*)$  by  $x^*(\lambda_1, \dots, \lambda_n)$  in above definitions. As in the case of hypothetical action changes, one may consider either any conceivable change of preferences relative to some reference point or restrict attention to slight variations. The latter requires some metric on preferences, which is, however, naturally given for Euclidean preferences.

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<sup>24</sup>The total range  $D_i^{\max} := \max_{a_{-i} \in X^{n-1}} [\max_{a_i \in X} x(a_i, a_{-i}) - \min_{a_i \in X} x(a_i, a_{-i})]$  of player  $i$ ’s possible impact on outcome lacks any ex post character. It seems a reasonable a priori measure which requires no distribution assumptions on  $\bar{a}^*$  or  $\bar{\lambda}$ . However, it is a rather coarse concept and would only discriminate between dummy and non-dummy players in the context of simple games.  $D_i^{\max'}(a_{-i}^*) := \max_{a_i \in X} x(a_i, a_{-i}^*) - \min_{a_i \in X} x(a_i, a_{-i}^*)$  holds an interesting intermediate ground. Another promising possibility, suggested to us by Matthew Braham, is considering (the inverse of) the minimal tremble size by which a player  $i$  would affect a given outcome as a measure of his ex post power.

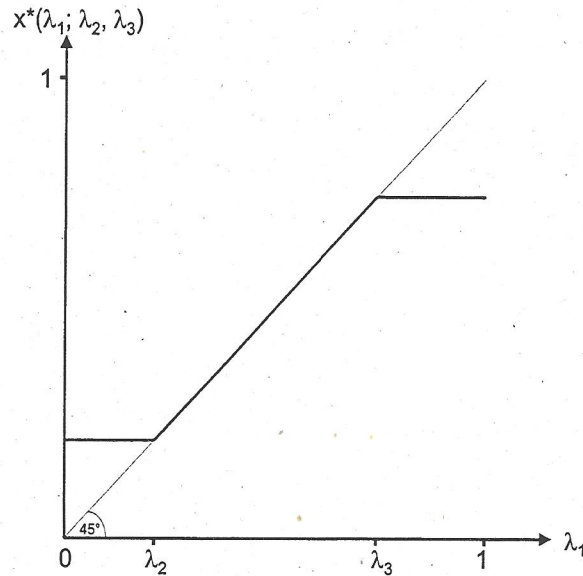


Figure 1: Equilibrium outcome of simple majority voting as  $\lambda_1$  is varied

For illustration, consider players  $N = \{1, 2, 3\}$  with Euclidean preferences on policy space  $X = [0, 1]$ , described by individual ideal points  $\lambda_i \in X$ , and simple majority voting on proposals made by the players. Let  $\lambda_{(j)}$  denote the  $j$ -th smallest of players' ideal points, i. e.  $\lambda_{(1)} \leq \lambda_{(2)} \leq \lambda_{(3)}$ . Depending on the precise assumptions on the order of making proposals and voting, many equilibrium profiles of player strategies exist. However, they yield the median voter's ideal point,  $\lambda_{(2)}$ , as the unique equilibrium outcome  $x^*(\lambda_1, \lambda_2, \lambda_3)$ . One can then investigate player 1's power for given  $\lambda_2$  and  $\lambda_3$ , where without loss of generality we assume  $\lambda_2 \leq \lambda_3$ . The mapping

$$x^*(\lambda_1, \lambda_2, \lambda_3) = \begin{cases} \lambda_2; & \lambda_1 < \lambda_2 \\ \lambda_1; & \lambda_2 \leq \lambda_1 \leq \lambda_3 \\ \lambda_3; & \lambda_3 < \lambda_1 \end{cases}$$

describes the equilibrium outcome (see figure 1), and

$$D_i^2(\lambda) = \frac{\partial x^*(\lambda)}{\partial \lambda_i} = \begin{cases} 0; & \lambda_1 < \lambda_2 \vee \lambda_3 < \lambda_1 \\ 1; & \lambda_2 < \lambda_1 < \lambda_3 \end{cases}$$

is a measure of player 1's power as a function of players' ideal points in  $X$  based on local *preference trembles*. According to  $D_i^2(\cdot)$ , player 1 is powerless (in the sense of not being able to influence collective choice although he would like to do so after a small change of preference) if he is not the median voter. For  $\lambda_1 \in (\lambda_2, \lambda_3)$ , he has maximal power in the sense that any (small) change in individual preference shifts the collective decision by exactly the desired amount.

So far, we have only considered ideal points in one-dimensional policy spaces  $X$ . These are analytically convenient.<sup>25</sup> Both the derivation of ex post power and formation of expectations are more complicated for higher-dimensional spaces. However, this is no obstacle in principle.

To illustrate this, let  $\Lambda = (\lambda_1, \dots, \lambda_n)$  be the collection of  $n$  players' ideal points in  $\mathbb{R}^m$  (an  $m \times n$  matrix having as columns the  $\lambda_i$ -vectors representing individual players' ideal points).<sup>26</sup> In a policy space  $X \subseteq \mathbb{R}^m$ , the opportunities even for only marginal changes of preference are manifold. A given ideal point  $\lambda_i$  can locally be shifted to  $\lambda_i + \alpha$  where  $\alpha$  is an arbitrary vector in  $\mathbb{R}^m$  with small norm. It will depend which tremble directions are particularly meaningful in applications.<sup>27</sup> Multiples of the vector  $(1, 1, \dots, 1) \in \mathbb{R}^m$  seem reasonable if the  $m$  policy dimensions are independent of each other. In any case,

$$D_i^2(\Lambda) := \lim_{t \rightarrow 0} \frac{x^*(\lambda_i + t\alpha, \lambda_{-i}) - x^*(\lambda_i, \lambda_{-i})}{t} = \frac{\partial_\alpha x^*(\lambda_i, \lambda_{-i})}{\partial \lambda_i} \quad (7)$$

defines a suitable measure of player  $i$ 's ex post power provided that above limit exists. This is simply the directional derivative of the equilibrium outcome in direction  $\alpha$ . Alternatively,

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<sup>25</sup>See Cooter (2002) on practical pros and cons of one-dimensional *median democracy* – asking for ‘yes’ or ‘no’-decisions on single issues, as widely used e.g. in Switzerland or California – in comparison with the more dead-lock prone multi-dimensional *bargain democracy*.

<sup>26</sup>At the level of national elections,  $m = 2$  is for most countries a sufficiently ‘high’ dimension (Norman Schofield, personal communication).

<sup>27</sup>For example, one may be interested in the expected effect of a general swing towards economically and/or socially more liberal or conservative positions across parties. One may also consider not a particular direction  $\alpha$  but rather an entire neighborhood of  $\lambda_i$  (e.g. taking the supremum of (7) for all possible directions  $\alpha$ ).

measures for the multidimensional case can be based on the gradient of  $x^*(\lambda_i, \lambda_{-i})$  (holding  $\lambda_{-i}$  constant). In case of ideal points in a discrete policy space, a preference-based measure  $D_i^{2'}(\lambda)$  can be defined by replacing the derivative in (7) with a difference quotient in analogy to (4) and (5). Another candidate for a meaningful ex ante (or even a priori) measure of player 1's power is – in analogy with  $D_i^{1''}(a^*)$  –

$$D_i^{2''}(\lambda) := \max_{\lambda'_i \in X} [x^*(\lambda_i, \lambda_{-i}) - x^*(\lambda'_i, \lambda_{-i})]. \quad (8)$$

This is based on the consideration not only of small preference modifications but also of a complete relocation of the player's ideal outcome (see also fn. 24).

### 4.3 Calculating Ex Ante Power

Mappings  $D_i^1$ ,  $D_i^2$ , and their discrete versions measure ex post power as the difference between distinct shadow outcomes and the expected or observed (equilibrium) collective decision. We do not want to discuss at this point which is the most relevant shadow outcome and hence measure.<sup>28</sup> All of them clearly distinguish power from the luck of a satisfying group decision; they do not require any 'averaging out' of luck. Taking expectations merely serves the purpose of obtaining ex ante conclusions if these are of interest.

Having selected a meaningful measure of ex post power, it is straightforward to define a meaningful ex ante measure. It has to be based on explicit informational assumptions concerning players' preferences or – if one does not want to assume preference-driven behavior – actions. Denoting by  $\tilde{\xi}$  the random state of the world as given either by preferences (and status quo) or players' actions, and by  $P$  its distribution,

$$\mu_i^\Gamma := \int D_i(\tilde{\xi}) dP \quad (9)$$

is the ex ante power index based on ex post measure  $D_i(\cdot)$  and decision procedure or game form  $\Gamma$ .<sup>29</sup>

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<sup>28</sup>Note that  $D_i^{1''}(a^*)$  and  $D_i^{2''}(\lambda)$  produce identical power indications if each action can be a player's most preferred one.

Traditional power indices, such as the Penrose or Shapley-Shubik index, consider the particularly simple decision procedure in which players  $i \in N = \{1, \dots, n\}$  choose an action  $a_i \in A_i = \{0, 1\}$  and the outcome of decision-making,  $x(a)$ , is 1 if set  $Y := \{i | \lambda_i = 1\}$  is a winning coalition, i. e. if  $v(Y) = 1$ , and 0 otherwise. They use  $D_i^0(a)$  (or  $D_i^1(a)$  with  $\delta = 1$ ). The StPI proposed by Steunenberget al., too, is a linear transform of (9), albeit using the unreasonable a posteriori power measure  $D_i(\tilde{\xi}) = \|\tilde{\lambda}_i - x^*(\tilde{\xi})\|$ .

Let us illustrate our sensitivity analysis approach to measuring power more explicitly. As an example assume a simple procedural spatial voting game where a fixed agenda setter makes a ‘take it or leave it’ offer to a group of 5 voters and needs 4 sequentially cast votes to pass it.<sup>30</sup> The policy space is  $X = [0, 1]$ , voters’ ideal points are  $\lambda_1, \dots, \lambda_5$ , that of the agenda setter is  $\sigma$ , and the status quo is 0. The unique equilibrium outcome of this game is

$$x^*(\lambda) = x^*(\lambda_1, \dots, \lambda_5) = \begin{cases} 2\lambda_{(2)} & 0 \leq \lambda_{(2)} < \frac{1}{2}\sigma \\ \sigma & \frac{1}{2}\sigma \leq \lambda_{(2)} \leq 1 \end{cases}$$

where  $\lambda_{(2)}$  is the second-smallest of the  $\lambda_i$ . For the agenda setter  $j$  we get the ex post power

$$D_j^2(\sigma, \lambda) := \begin{cases} 0; & 0 < \lambda_{(2)} < \frac{1}{2}\sigma \\ 1; & \frac{1}{2}\sigma < \lambda_{(2)} < 1 \end{cases}$$

and for voter  $i$

$$D_i^2(\sigma, \lambda) := \begin{cases} 2; & 0 < \lambda_{(2)} < \frac{1}{2}\sigma \wedge \lambda_i = \lambda_{(2)} \\ 0; & \text{otherwise.} \end{cases}$$

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<sup>29</sup>We have omitted a sub- or superscript  $\Gamma$  in the definition of ex post measures for a concise notation. The procedure is, however, the central determinant of outcome functions  $x(\cdot)$  or  $x^*(\cdot)$ . If several different game forms  $\Gamma \in G$  are to be considered ex ante, one has to take expectation over  $\mu_i^\Gamma$  with the appropriate probability measure on  $G$ .

<sup>30</sup>This setting has occasionally been used in the literature as the simplest one allowing to compare the effects of simple majority, qualified majority, and unanimity rules.

Based on this, one can derive ex ante strategic power measures  $\mu_j^\Gamma$  and  $\mu_i^\Gamma$  for the considered decision procedure. Suppose that all ideal points are uniformly distributed on  $[0, 1]$ . We get

$$\mu_j^\Gamma = \int D_j^2(\sigma, \lambda) dP = \int_0^1 \int_0^1 D_j^2(x, y) f_\sigma(x) f_{\lambda_{(2)}}(y) dy dx$$

where  $f_\sigma$  and  $f_{\lambda_{(2)}}$  are the densities of the independent random variables  $\sigma$  and  $\lambda_{(2)}$ , respectively. It follows that

$$\mu_j^\Gamma = \int_0^1 \int_{x/2}^1 1 \cdot 1 \cdot f_{\lambda_{(2)}}(y) dy dx = \int_0^1 \left(1 - F_{\lambda_{(2)}}(x/2)\right) dx = 0.625$$

with

$$F_{\lambda_{(2)}}(x) = \int_0^x 5 \binom{4}{1} s [1-s]^3 ds = 10x^2 - 20x^3 + 15x^4 - 4x^5$$

for  $x \in [0, 1]$  as the cumulative (Beta-)distribution function of  $\lambda_{(2)}$ . Analogously, one can compute

$$\mu_i^\Gamma = \int D_i^2(\sigma, \lambda) dP = 0.15,$$

which is the product of the probability 0.075 of player  $i$  having a swing that matters to the outcome and ex post power  $D_i^2(\sigma, \lambda) = 2$  for these preference configurations. This means that ex ante a shift of the agenda setter's ideal point  $\sigma$  (voter  $i$ 's ideal point  $\lambda_i$ ) by one marginal unit will induce an expected shift of the outcome by 0.625 units (0.15 units). So the agenda setter's leverage and influence on the outcome are ex ante more than four times larger than that of any given voter. One may want to compare agenda setting power to the power of the complete council consisting of all five voters. This can be done by considering self-enforcing agreements among council members before actual voting, hence ascribing the ideal point  $\lambda_{(2)}$  to the council. One obtains

$$\mu_{(2)}^\Gamma = \int D_{(2)}^2(\sigma, \lambda) dP = \int_0^1 \int_0^{x/2} 2 f_\sigma(x) f_{\lambda_{(2)}}(y) dy dx = 0.75,$$

i. e. the council-of-five has ex ante slightly more power in aggregate than the agenda setter.

## 5 Final Remarks

For more complex and more realistic assumptions about preferences and procedures, the proposed two-step sensitivity approach to measurement of power remains valid. The only difference to our simple illustrations is that the calculations for preference-driven strategic behavior – i. e. determination of the equilibrium outcome as a function of parameterized preferences, its derivative with respect to players' preference parameters, and expected values – will be more complicated if e. g. ideal points are not uniformly distributed or for complex bargaining protocols. This is not a big problem if one does not insist on closed analytical solutions, but is primarily concerned with numerical values. In particular, Monte Carlo simulation allows to approximate the required probabilities and expectations with arbitrary precision even of complicated voting bodies.

We have above defined ex post power as the *objective* marginal impact which a player's action or underlying preference has on the outcome of collective decision-making. It is possible to go one step further. Namely, we have in passing hinted at the opportunity to understand – and measure – power as a *subjective* concept.

Consider a multi-dimensional policy space. Let the decision procedure give player  $i$  dictator power in some dimension  $d_i$  and but no say in all other dimensions. The opportunity to define the collective decision in this dimension could be all that player  $i$  cares for. Then, judged in terms of his own preferences he has maximal power. The other players may be completely indifferent towards their joint decision's component in dimension  $d_i$ . Judged in terms of their preferences,  $i$  is a dummy who can never have an impact on their well-being. Alternatively, player  $i$  can hold dictator power on a dimension he does not care about (e. g. to pardon an unknown convict sentenced to death), but which is all-important to some, perhaps not all, other players. Player  $i$  is powerful depending on one's view-point, i. e. preferences.<sup>31</sup>

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<sup>31</sup>This assumes that there is no (perfect) 'market for influence' which would imply that – after all gains from exchange of direct influence are realized – each player  $i$ 's power generically depends on all players' preferences but is the same as judged by any player  $j$  (see Coleman, 1966).



Given the often entirely personal evaluation of power in real life, it seems worthwhile to study the *subjective sensitivity of outcomes* to players' actions or preferences. It is straightforward to replace the derivative of outcome function  $x^*(\cdot)$  in above definitions by the derivative of players' utility of outcome,  $u_i(x^*(\cdot))$ , taken with respect to their own and other players' parameterized actions or preferences. A player's power is then not simply a real number, but a vector of subjective evaluations of it by all players (including himself). The corresponding index function is matrix-valued.

Subjective evaluation of players' power may be meaningless in the context of normative analysis of constitutional designs. However, it seems relevant for positive analysis, and is arguably the most relevant aspect to participants when decision procedures, e.g. in the EU or the WTO, are the object of multilateral negotiations.

Scholars of equilibrium-based ex post analysis and those favoring axiomatic ex ante or a priori analysis will possibly not see an urgent need to merge their fields. However, the suggested unified approach should clarify that they are not as far apart as it may seem. The axiomatic camp has been very little concerned with the notion of ex post power which is implicitly underlying their indices. Its members have leaped from an abstractly defined voting body to individual (ex ante or a priori) power values – without specifying how agents can and do (inter-)act and investigating which ex post power is associated with this. The non-cooperative camp has been interested almost entirely in equilibrium behavior and its consequences for individual success. If the latter's attention is extended from success to power, which in case of subjective evaluations is the derivative of success, and if the former's large jump is decomposed into two smaller steps, the work of both methodological camps turns out to neatly complement each other.

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