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## A General Strategy Proof Fair Allocation Mechanism

by

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Abstract: This paper studies a general problem of efficiently and fairly allocating n indivisible objects like jobs or houses with a certain amount of money to n persons with a requirement that each person be assigned with one object. The precise preferences of individuals over both the objects and money are unknown and manipulable but assumed to follow some general patterns to ensure the existence of a fair allocation. A mechanism is developed that elicits honest preferences over both the objects and money, and that assigns the objects with some money to individuals efficiently and fairly.

Keywords: Indivisibility, money, fairness, efficiency, nonmanipulability.

JEL classification: D61, D63, D71, C6, C62, C68.

## 1 Introduction

The fair allocation existence problem of indivisible objects with money has been studied previously by Svensson (1983), Maskin (1987), Alkan, Demange and Gale (1991), Tadenuma and Thomson (1991), Su (1999), Sun and Yang (2001), and Yang (2001). This paper studies a general problem of efficiently and fairly allocating n indivisible objects like jobs, duties or houses with a certain amount of money to n persons. Each object has a maximum compensation limit. It is required that each person be assigned with one object even if it may be unprofitable to him. The central issue of the problem is that agents may behave strategically rather than truthfully in reporting their preferences. A mechanism is developed that elicits honest preferences over both the objects and money, and that assigns the objects with some money to individuals efficiently and fairly. The mechanism

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is called the optimal fair allocation mechanism and always selects an optimal fair allocation compatible with the maximum compensation limits. The current work is closely related to Groves (1973) and Leonard (1983) which in turn relate to the Vickrey auction. Both authors have developed strategy proof mechanisms that are applied to the quasi-linear utility environment, i.e., all agents have quasi-linear utilities in money. Our work contributes to the literature in two aspects: First, in the current model agents are allowed to manipulate the preferences over both the objects and money, whereas in the existing models agents are allowed to manipulate the preferences over the objects only. Second, the existing models can only apply to the quasi-linear utility environment, whereas the current model can apply to "almost all" possible utility environments. Our results are somehow surprisingly general and robust in the sense that as long as agents behave normally and rationally, and as long as there exists a fair allocation, the optimal fair allocation mechanism will always select an optimal fair allocation and simultaneously achieve efficiency, fairness and nonmanipulability.

This paper is organised as follows. Section 2 sets up the model and presents basic concepts and Section 3 demonstrates the main result.

### 2 The Model

First, we introduce some notation. The set  $I_k$  denotes the set of the first k positive integers. The set  $\mathbb{R}^n$  denotes the n-dimensional Euclidean space. For any two vectors x and  $y \in \mathbb{R}^n$ ,  $x \geq y$  means  $x(i) \geq y(i)$  for all i; x > y means  $x \geq y$  and x(i) > y(i) for some i; and  $x \gg y$  means x(i) > y(i) for all i. The notation  $\sharp A$  denotes the cardinality of a finite set A.

The fair allocation model consists of n agents, n indivisible objects and a certain amount of money. The sets of agents and objects will be denoted by  $I_n$  and N with  $N = I_n$ , respectively. Each object  $j \in N$  has an upper bound compensation limit c(j) units of money. It is required that each agent be assigned with exactly one object even if it may be unprofitable to him. A situation is unprofitable to an agent if what the agent is assigned with is worse than the situation in which he does not participate. This will be called a nonmarket situation. The upper bound compensation limits for the objects are not unusual in many circumstances. For example, in the job assignment problem, since jobs or duties

are in general not identical, a manager could first make an assessment v(j) over every job j and then sets a maximum compensation c(j) for every job j so that the value v(j)+c(j) is the same for every job. The preference of each agent i over the objects and money is represented by a utility function  $u_i: N \times \mathbb{R} \mapsto \mathbb{R}$ . Although agents may have incentive to misreport their utilities, it is reasonable to assume that  $u_i(j,m)$  is a strictly increasing and continuous function in money m for each object  $j \in N$ . By this mild assumption, we mean that agents behave normally and rationally. Clearly,  $u_i(j,m)$  is quite general and covers the quasi-linear utilities in money  $(u_i(j,m)=v(i,j)+m)$  as a particular case. Our goal is to design a mechanism that makes it a dominant strategy for every agent to reveal his true preference over both the objects and money, and that efficiently and fairly allocates the objects among the agents with a compensation scheme compatible with the compensations c(j). This problem differs from the classical fair allocation problem in that the latter has no compensation limit for each object but has a fixed amount M of money that must be completely allocated with the objects among the agents.

Let  $C = (c(1), \dots, c(n))$  be the vector of maximum compensations which will be fixed throughout the paper. An allocation  $(\pi, x)$  consists of a permutation  $\pi$  of the n objects and a compensation scheme  $x : N \mapsto \mathbb{R}$ . At the allocation  $(\pi, x)$ , agent i gets object  $\pi(i)$ and  $x(\pi(i))$  units of money. In case  $x(\pi(i))$  is negative, this means agent i will pay the amount  $|x(\pi(i))|$ . An allocation  $(\pi, x)$  is fair if for every agent  $i \in I_n$ 

$$u_i(\pi(i), x(\pi(i))) \ge u_i(\pi(j), x(\pi(j))), \ \forall j \in I_n.$$

A fair allocation  $(\pi, x)$  is compatible with the vector C if  $x(j) \leq c(j)$  for every  $j \in N$ . A fair allocation is efficient if there exists no other allocation  $(\rho, y)$  such that  $u_i(\rho(i), y(\rho(i))) \geq u_i(\pi(i), x(\pi(i)))$  for every  $i \in I_n$ , and  $u_j(\rho(j), y(\rho(j))) > u_j(\pi(j), x(\pi(j)))$  for some  $j \in I_n$  and  $\sum_{j=1}^n x(j) = \sum_{j=1}^n y(j)$ . A compatible fair allocation  $(\pi, x)$  is optimal if for every compatible fair allocation  $(\rho, y)$  it holds that  $x \geq y$ .

A mechanism is a rule that specifies an allocation for each profile  $(u_1, \dots, u_n)$  of utility functions. A mechanism that selects a fair allocation is strategy proof if no agent i can make himself strictly better off by misreporting his utilities  $u_i(j, m)$  over the objects j and money m, while all other agents k reveal their true utilities  $u_k(j, m)$ . The mechanism that always selects an optimal fair allocation is called the optimal fair allocation mechanism. Our first result is a nonmanipulable impossibility theorem for the classical fair allocation

problem.

Lemma 2.1 The classical fair allocation problem is manipulable.

Proof: We illustrate this by an example to show that it is impossible to achieve nonmanipulability. There are two agents i = 1, 2, and two objects j = 1, 2, and M = 10. Agents have quasi-linear utilities in money, precisely,  $u_i(j,m) = v(i,j) + m$  with v(1,1) = 10, v(1,2) = 0, v(2,1) = v(2,2) = 10. The set of fair allocations can be written as

$$S = \{(\pi, x) \mid \pi(1) = 1, \pi(2) = 2, 0 \le x_1 \le x_2, x_1 + x_2 = 10\}.$$

Let  $(\pi, x^*)$  be any element in S. In case  $x_2^* < 10$ , agent 2 can make himself strictly better off by misreporting:  $v'(2,1) = 15 + 0.5x_2^*$ , v'(2,2) = 10. In case  $x_1^* < 5$ , agent 1 can make himself strictly better off by misreporting: v'(1,1) = 10,  $v'(1,2) = 7.5 + 0.5x_1^*$ .

The precise preferences of individuals over both the objects and money are unknown and manipulable but assumed to follow the following general patterns which are required to ensure the existence of a compatible fair allocation.

Assumption 2.2 For any  $m \in \mathbb{R}$ , any  $i \in I_n$  and any  $j, j' \in N$ , there exists a number L > 0 such that  $u_i(j, m) > u_i(j', -L)$ .

Clearly, quasi-linear utility functions satisfy the condition. It might be worth mentioning that all the results in the paper hold true as long as agents behave normally and rationally, and as long as there exists a fair allocation compatible with C. Assumption 2.2 is just one of the most general existence conditions and used here as an illustration.

Lemma 2.3 Under Assumption 2.2, there exists a compatible fair allocation.

Proof: By assumption there exist  $L^{1*}>0$  and  $L^{2*}>0$  such that  $u_i(j,c(j))>u_i(k,-L^{1*}),$   $u_i(j,0)>u_i(k,-L^{2*})$  for all  $i\in I_n,\, j,k\in N$ . Let  $L^*=\max\{L^{1*},L^{2*}\}$  and let  $M^*=-nL^*$ . Let  $Y=-(n+1)L^*$ . We will prove that for any  $i\in I_n$  and for any  $x\in \mathbb{R}^n$  with  $\sum_{j\in N}x(j)=M^*$ , if  $x(j)\leq Y$ , then

$$u_i(j, x(j)) < \max_{h \in N} u_i(h, x(h)).$$

Since  $\sum_{j=1}^{n} x(j) = M^*$  and  $x(j) \leq Y$ , then there exists some  $k \in N$  such that x(k) > 0. Thus,

$$u_i(j, x(j)) \le u_i(j, -L^*) < u_i(k, 0) < u_i(k, x(k)) \le \max_{h \in N} u_i(h, x(h)).$$

By Theorem 3.1 in Yang (2001) or Sun and Yang (2001) there exists a fair allocation  $(\pi, x^*)$  with  $\sum_{j \in N} x^*(j) = M^*$ . Clearly, there is  $x^*(h) \leq -L^{1*}$ . If  $x \not\leq C$ , there is  $x^*(l) \geq c(l)$ . So, nobody likes to have the bundle  $(h, x^*(h))$ , yielding a contradiction.

Maskin (1987) and Svensson (1983) have shown that every fair allocation is efficient. The following perturbation theorem is due to Alkan et al. (1991).

**Lemma 2.4** If  $(\pi, x)$  is a fair allocation, then for  $\varepsilon > 0$  there exists another fair allocation  $(\rho, y)$  such that  $y \gg x$  with  $y(j) - x(j) < \varepsilon$  for all  $j \in N$ .

Lemma 2.4 implies that if  $(\pi, x)$  is an optimal fair allocation, then some component x(j) of x must be equal to c(j) of the vector C.

**Theorem 2.5** There always exists an optimal fair allocation  $(\pi, x^*)$ . The vector  $x^*$  is unique and moreover one of its components  $x^*(j)$  must be equal to c(j) of the vector C.

Proof: By Lemma 2.3, there exists a fair allocation compatible with the vector C. Suppose to the contrary that there exists no optimal fair allocation. Then there would exist two compatible fair allocations  $(\pi, x)$  and  $(\rho, y)$  with  $x \not\geq y$  and  $y \not\geq x$  so that there exists no other compatible fair allocation  $(\tau, z)$  with z > x. By hypothesis, we know that  $x \leq C$ ,  $y \leq C$ , and  $z \leq C$ . Define three sets A, B and D of objects by  $A = \{j \mid j \in N \text{ with } x(j) < y(j)\}$ ,  $B = \{j \mid j \in N \text{ with } x(j) > y(j)\}$ , and  $D = \{j \mid j \in N \text{ with } x(j) = y(j)\}$ . Then we have that  $A \neq \emptyset$ ,  $B \neq \emptyset$ ,  $A \cap B = \emptyset$  and  $A \cup B \cup D = N$ . For every agent i with  $\pi(i) \in A$ , we have

$$u_i(\rho(i), x(\rho(i))) \le u_i(\pi(i), x(\pi(i))) < u_i(\pi(i), y(\pi(i))) \le u_i(\rho(i), y(\rho(i))).$$

This implies that  $y(\rho(i)) > x(\rho(i))$ , i.e.,  $\rho(i) \in A$ . Let  $\Phi = \{i \mid i \in I_n \text{ with } \pi(i) \in A\}$  and  $\Psi = \{i \mid i \in I_n \text{ with } \rho(i) \in A\}$ . Then we have  $\Phi \subseteq \Psi$ . Moreover it follows from  $\sharp \Phi = \sharp \Psi = \sharp A$  that  $\Phi = \Psi$ . That is,  $\pi(i) \in A$  if and only if  $\rho(i) \in A$ . Similarly, we can show that  $\pi(i) \in B$  if and only if  $\rho(i) \in B$ .

In summary, we have that for every agent  $i \in I_n$ ,

$$\pi(i) \in A \iff \rho(i) \in A; \pi(i) \in B \iff \rho(i) \in B; \pi(i) \in D \iff \rho(i) \in D.$$

Thus we can define a new permutation  $\tau$  by:

$$\tau(i) = \begin{cases} \rho(i) & \text{when } \pi(i) \in A, \\ \pi(i) & \text{when } \pi(i) \in B \cup D. \end{cases}$$

Furthermore, we define

$$z = x \vee y = \{z \in \mathbb{R}^n \mid z(j) = \max\{x(j), y(j)\}, \forall j \in N\}$$

It is clear that  $x < z \le C$ . Now we will prove that  $(\tau, z)$  is in fact a fair allocation.

For every agent i with  $\pi(i) \in A$  and every object  $j \in A$ , we have that

$$u_i(\tau(i),z(\tau(i)))=u_i(\rho(i),y(\rho(i)))\geq u_i(j,y(j))=u_i(j,z(j)).$$

For every agent i with  $\pi(i) \in A$  and every object  $j \in B \cup D$ , we have that

$$u_i(\tau(i), z(\tau(i))) = u_i(\rho(i), y(\rho(i))) \ge u_i(\pi(i), y(\pi(i)))$$

$$> u_i(\pi(i), x(\pi(i))) \ge u_i(j, x(j)) = u_i(j, z(j)).$$

For every i with  $\pi(i) \in B \cup D$  and every object  $j \in A$ , we have that

$$u_i(\tau(i), z(\tau(i))) = u_i(\pi(i), x(\pi(i))) \ge u_i(\rho(i), x(\rho(i)))$$

$$\geq u_i(\rho(i), y(\rho(i))) \geq u_i(j, y(j)) = u_i(j, z(j)).$$

For every agent i with  $\pi(i) \in B \cup D$  and every object  $j \in B \cup D$ , we have that

$$u_i(\tau(i), z(\tau(i))) = u_i(\pi(i), x(\pi(i))) \ge u_i(j, x(j)) = u_i(j, z(j)).$$

So,  $(\tau, z)$  is a fair allocation, yielding a contradiction to the assumption that there exists no other fair allocation  $(\tau, z)$  with z > x.

### 3 The Main Result

Now we are ready to present and demonstrate the principal result of this paper.

**Theorem 3.1** The optimal fair allocation mechanism achieves simultaneously efficiency, fairness and nonmanipulability.

Proof: Efficiency and fairness are obvious. We will show that the mechanism is strategy proof. Let  $(\pi, x)$  be an optimal fair allocation. We can always relabel the objects so that  $\pi(i) = i$  for all  $i \in I_n$ . Suppose to the contrary that there exists some agent, say, agent 1, who can make himself strictly better off by misreporting his utility function  $u_1(j, m)$ . Let

his misreported utility function be  $\bar{u}_1(j,m)$ . Now we construct a new model in which agent 1 has the misreported utility function  $\bar{u}_1(j,m)$  and all other agents have the same utility functions as before. Then with respect to the new model, there exists a fair allocation  $(\rho, y)$  compatible with C such that  $u_1(\rho(1), y(\rho(1))) > u_1(\pi(1), x(\pi(1)))$ . Now define

$$\rho^{(0)}(1) = \pi(1) = 1, \ \rho^{(1)}(1) = \rho(1), \ \cdots, \ \rho^{(k)}(1) = \rho(\rho^{(k-1)}(1)), \ \cdots$$

Then there exists a smallest integer  $k^*(\geq 1)$  such that  $\rho^{(k^*)}(1) \in \{\rho^{(0)}(1), \dots, \rho^{(k^*-1)}(1)\}$ . Let  $S = \{\rho^{(0)}(1), \dots, \rho^{(k^*-1)}(1)\}$ . Now we will show that the set S has the following two properties:

Property 1: The set S is a closed circle, i.e.,  $\rho^{(k^*)}(1) = \rho^{(0)}(1) = 1$ .

Suppose to the contrary that  $\rho^{(k^*)}(1) = \rho^{(k)}(1)$  for some  $1 \le k \le k^* - 1$ . Then it follows from

$$\rho(\rho^{(k^*-1)}(1)) = \rho^{(k^*)}(1) = \rho^{(k)}(1) = \rho(\rho^{(k-1)}(1))$$

that  $\rho^{(k^*-1)}(1) = \rho^{(k-1)}(1) \in \{ \rho^{(0)}(1), \cdots, \rho^{(k^*-2)}(1) \}$ , yielding a contradiction to the assumption that  $k^*$  is the smallest integer.

Property 2: y(j) > x(j) for every object  $j \in S$ .

Recall that  $u_1(\rho(1), y(\rho(1))) > u_1(\pi(1), x(\pi(1)))$ . We have that

$$u_1(\rho(1), y(\rho(1))) > u_1(\pi(1), x(\pi(1))) \ge u_1(\rho(1), x(\rho(1))).$$

This implies that  $y(\rho(1)) > x(\rho(1))$ . If  $k^* = 1$ , i.e.,  $\rho(1) = 1$ , we have proved Property 2. Otherwise, suppose that  $y(\rho^{(k)}(1)) > x(\rho^{(k)}(1))$  holds for some  $k = 1, \dots, k^* - 1$ . Notice that  $\rho^{(k)}(1) \neq 1$ . Then, it follows from

$$\begin{split} u_{\rho^{(k)}(1)}(\rho^{(k+1)}(1),y(\rho^{(k+1)}(1))) &\geq u_{\rho^{(k)}(1)}(\rho^{(k)}(1),y(\rho^{(k)}(1))) \\ &> \ u_{\rho^{(k)}(1)}(\rho^{(k)}(1),x(\rho^{(k)}(1))) \geq u_{\rho^{(k)}(1)}(\rho^{(k+1)}(1),x(\rho^{(k+1)}(1))) \end{split}$$

that  $y(\rho^{(k+1)}(1)) > x(\rho^{(k+1)}(1))$ . So, by induction we have that  $y(\rho^{(k)}(1)) > x(\rho^{(k)}(1))$  for  $k = 1, \dots, k^*$ . In particular, notice that

$$y(1) = y(\rho^{(0)}(1)) = y(\rho^{(k^*)}(1)) > x(\rho^{(k^*)}(1)) = x(\rho^{(0)}(1)) = x(1).$$

Consequently, we have shown Property 2.

Define three sets A, B and D of objects by  $A = \{j \mid j \in N \text{ with } x(j) < y(j)\}$ ,  $B = \{j \mid j \in N \text{ with } x(j) > y(j)\}$ , and  $D = \{j \mid j \in N \text{ with } x(j) = y(j)\}$ . Property 2

implies that  $A \supseteq S \neq \emptyset$ . Note that  $B \cup D \neq \emptyset$ . If not, we have  $x \ll y \leq C$ , i.e.,  $x \ll C$ . But that is impossible, because  $(\pi, x)$  is an optimal fair allocation and so  $x \not\ll C$ . We see that for agent 1 both  $\pi(1) = 1$  and  $\rho(1) \in S \subseteq A$ . Moreover, for every agent  $i \neq 1$  with  $\pi(i) \in A$ , we have

$$u_i(\rho(i), x(\rho(i))) \le u_i(\pi(i), x(\pi(i))) < u_i(\pi(i), y(\pi(i))) \le u_i(\rho(i), y(\rho(i))).$$

This implies that  $y(\rho(i)) > x(\rho(i))$ , i.e.,  $\rho(i) \in A$ . As in the proof of Theorem 2.5, we can show that  $\pi(i) \in A$  if and only if  $\rho(i) \in A$ . Similarly, we have that  $\pi(i) \in B$  if and only if  $\rho(i) \in B$ .

In summary, we have that for every agent  $i \in I_n$ ,

$$\pi(i) \in A \iff \rho(i) \in A; \, \pi(i) \in B \iff \rho(i) \in B; \, \pi(i) \in D \iff \rho(i) \in D.$$

Define a set of agents by  $\Phi = \{i \mid i \in I_n \text{ with } \pi(i) \in A\}$ . Then we define a permutation  $\pi_A : \Phi \mapsto A$  and a compensation scheme  $x_A : A \mapsto \mathbb{R}$  by:

$$\pi_A(j) = \pi(j)$$
 for all  $j \in A$ ; and  $x_A(j) = x(j)$  for all  $j \in A$ .

Clearly,  $(\pi_A, x_A)$  is a fair allocation of the objects in A among the agents in  $\Phi$ . Then, by Lemma 2.4, there exists another fair allocation  $(\tau_A, z_A)$  such that  $z_A \gg x_A$  but  $z_A(j) < y(j)$  for all  $j \in A$ . Now let us define a new permutation  $\pi^* : I_n \mapsto N$  and a new compensation scheme  $x^* : N \mapsto \mathbb{R}$  by:

$$\pi^*(i) = \begin{cases} \tau_A(i) & \text{when } \pi(i) \in A, \\ \pi(i) & \text{when } \pi(i) \in B \cup D; \end{cases}$$

$$x^*(j) = \begin{cases} z_A(j) & \text{when } j \in A, \\ x(j) & \text{when } j \in B \cup D; \end{cases}$$

By construction, it is clear that  $x < x^* \le C$ . Now we will prove that  $(\pi^*, x^*)$  is in fact a fair allocation.

For every agent i with  $\pi(i) \in A$  and every object  $j \in A$ , we have that

$$u_i(\pi^*(i), x^*(\pi^*(i))) = u_i(\tau_A(i), z_A(\tau_A(i))) \ge u_i(j, z_A(j)) = u_i(j, x^*(j)).$$

For every agent i with  $\pi(i) \in A$  and every object  $j \in B \cup D$ , we have that

$$u_i(\pi^*(i), x^*(\pi^*(i))) = u_i(\tau_A(i), z_A(\tau_A(i))) \ge u_i(\pi(i), z_A(\pi(i)))$$

$$> u_i(\pi(i), x(\pi(i))) \ge u_i(j, x(j)) = u_i(j, x^*(j)).$$

For every i with  $\pi(i) \in B \cup D$  and every object  $j \in A$ , we have that

$$u_i(\pi^*(i), x^*(\pi^*(i))) = u_i(\pi(i), x(\pi(i))) \ge u_i(\rho(i), x(\rho(i)))$$

$$\geq u_i(\rho(i), y(\rho(i))) \geq u_i(j, y(j)) > u_i(j, z_A(j)) = u_i(j, x^*(j)).$$

For every agent i with  $\pi(i) \in B \cup D$  and every object  $j \in B \cup D$ , we have that

$$u_i(\pi^*(i), x^*(\pi^*(i))) = u_i(\pi(i), x(\pi(i))) \ge u_i(j, x(j)) = u_i(j, x^*(j)).$$

So,  $(\pi^*, x^*)$  is a fair allocation, yielding a contradiction to the assumption that there exists no other fair allocation  $(\tau, z)$  with z > x.

#### REFERENCES

Alkan, A., Demange, G., Gale, D., 1991. Fair allocation of indivisible objects and criteria of justice. *Econometrica* 59, 1023-1039.

Groves, T., 1973. Incentives in teams. Econometrics 41, 617-631.

Leonard, H.B., 1983. Elicitation of honest preferences for the assignment of individuals to positions. Journal of Political Economy 91, 461-479.

Maskin, E., 1987. On the fair allocation of indivisible objects. in: Arrow and the Foundations of the Theory of Economic Policy, MacMillan, London, pp. 341-349.

Su, F.E., 1999. Rental harmony: Sperner's lemma in fair division. American Mathematical Monthly 106, 930-942.

Sun, N., Yang, Z., 2001. On fair allocations and indivisibilities. DP No. 1347, Cowles Foundation, Yale University, New Haven.

Svensson, L., 1983. Large indivisibilities: an analysis with respect to price equilibrium and fairness. *Econometrica* 51, 939-954.

Tadenuma, K., Thomson, W., 1991. No-envy and consistency in economies with indivisibilities. *Econometrica* 59, 1755-1767.

Yang, Z., 2001. An intersection theorem on an unbounded set and its application to the fair allocation problem. Journal of Optimization Theory and Applications 110, 429-443.

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