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Arbitrage and Walrasian Equilibrium in Hierarchically Structured Economies

by

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Abstract

In this paper we present a model of a pure exchange economy in which markets with uniform prices result from suitalbe exogenously given structures of asymmetric bilateral trade relationships. These trade relationships are interpreted as price setting relationships, so for any price taking agent in a trade relationship we have another agent setting a vector of prices. We assume each agent can only observe those agents with respect to whom he behaves as a price setter. The agents form their anticipations about the consequences of their actions on the basis of their limited information about (the state of) the economy.

We give a theorem on the existence of equilibrium which states that if the structure of trade relationships allows for sufficient potential possibilities for arbitrage, then an equilibrium exists, and any such equilibirum has uniform prices, i.e., on every trade relationship the same vector of prices is set. Finally, we state a theorem, with additional restrictions, on equivalence of equilibria in our economy with Walrasian equilibria.

1 Introduction

The general equilibrium model as formulated by Debreu (1959) is one of the fundamental models in economics. It is a model in which decentralized selfish decision making leads to equilibrium outcomes that are efficient for the economy as a whole, as is stated in the First Theorem of Welfare Economics. Unfortunately, the model has some weaknesses, two of which are mentioned here. Firstly, all agents in the economy are assumed to act as price takers, without any agent setting the prices. Secondly, it is assumed that each agent can trade with every other agent in the economy only through "the market", so a very particular trade or communication structure of the economy is assumed. In this paper we discuss a model that tackles both problems.

The problem of price setting in general equilibrium models has occupied economists for some time and it continues to do so. We refer to Negishi (1961) who first analyzed general equilibrium models with monopolistic competition.

An important line of research investigates dynamic models in which agents meet pairwise and then trade according to specified trade rules. The agents typically meet often, either in a prespecified order or at random. This line of research is initiated by Feldman (1973). Recently, in McAffee (1993), a model of a large market is analyzed where sellers to some extend determine the mechanism, and therefore to some extend the rules of trade, on the trade relationships. In these models, however, there is no fixed and given structure of trade relationships that may be interpreted as reflecting the institutional structure of real life economies, which seems to be characterized by a rather high degree of inertia.

Economic models with a fixed restrictive structure of exchange institutions can be found in the fields of industrial organization and spatial economics. For models of successive monopolies we refer to Krelle (1976). In Vind (1983) an example of a model of an economy with a fixed structure of exchange institutions is given. The first part of Karmann (1981) gives a spatial general equilibrium model with transportation technologies. These transportation technologies can be interpreted as describing the user's costs of exchange institutions that take the form of competitive markets. For more recent research on economic models with restrictive fixed communication structures we refer to Gilles (1990) and Spanjers (1992).

In the present paper we consider models with a fixed and given structure of asymmetric bilateral trade relationships. In order to give the model some contents, we have to specify the rules for the use of the trade relationships. In particular, we assume that on every trade relationship one agent acts as a price setter and the other acts as a price taker with respect to this trade relationship. We focus on structures of trade relationships in which arbitrage may potentially occur. In the context of our model, arbitrage means that some agent buys some commodities on one trade relationship at a relatively low price, and sells them on another trade relationship where they have relatively high price, thus being in a position to make arbitrary high "profits".

The paper is organized as follows. In Section 2 we introduce a model of hierarchically structured economies. In Section 3 we state a restrictive theorem on the existence of equilibrium in such economies. The theorem states that if the structure of trade relationships allows for sufficient potential possibilities for arbitrage, then an equilibrium exists. We state an even more restrictive theorem on the equivalence of equilibrium in hierarchically structured economies with Walrasian equilibrium. Finally, in Section 4, some concluding remarks are made.

2 The Model

In this section we define a hierarchically structured economy. We describe such an economy by its hierarchical structure, by the individual characteristics of the agents and by the institutional characteristics of the trade relationships in the economy. The hierarchical structure describes between which pairs of agents a trade relationship exists and which of the agents in such a relationship dominates the other. Each agent is described by his individual characteristics, being his utility function and his initial endowments. Additionally, we describe the rules of trade of each trade relationship by its institutional characteristic. We describe the information the agents have about of the economy and use this information to derive their anticipations about the consequences of their actions for the behaviour of (some of) the other agents.

The hierarchical structure of an economy is described by a directed graph which is called the hierarchical graph. A simple directed graph is a pair (A, D) consisting of a finite non-empty set of vertices A and a set of arcs $D \subset \{(i,j) \in A \times A \mid i \neq j\}$. A directed graph is weakly connected if there do not exist two simple directed graphs (A_1, D_1) and (A_2, D_2) such that $A = A_1 \cup A_2$ and $D = D_1 \cup D_2$. A **Hierarchical Graph** (A, D) is a weakly connected simple directed graph, where A the set of vertices and D is the set of arcs.

For each agent $i \in A$ we define $F_i := \{j \in A \mid (i,j) \in D\}$ to be the set of (direct) followers of agent i in the hierarchical graph (A, D). We define $L_i := \{h \in A \mid (h,i) \in D\}$, as the set of (direct) leaders of agent i. We define $S_1 := \{a \in A \mid L_a = \emptyset\}$ to be the set of agents that do not have a leader.

The rules of trade over an asymmetric trade relationship are described by its institutional characteristic. The **Institutional Characteristic** of a trade relationship $w = (i,j) \in D$ is a correspondence $\mathcal{T}_w : \tilde{X}_w \rightrightarrows \mathbb{R}^l$ such that for each signal $s \in \tilde{X}_w$ chosen by the dominating agent, i, the dominated agent, j, can choose for the trade relationship w any vector of net trades in the set $\mathcal{T}_w(s) \subset \mathbb{R}^l$. Here, we restrict ourselves to the institutional characteristic of mono pricing. Let $S^{l-1} := \{x \in \mathbb{R}^l_+ \mid \sum_{c=1}^{l} x_c = 1\}$ be the $(l-1)$-dimensional unit simplex. The institutional characteristic of **Mono Pricing** is the correspondence $\mathcal{T}^{\text{mon}} : S^{l-1} \rightrightarrows \mathbb{R}^l$ such that for each $p \in S^{l-1}$ we have $\mathcal{T}^{\text{mon}}(p) := \{d \in \mathbb{R}^l \mid p \cdot d \leq 0\}$. Thus, for the institutional characteristic of mono pricing, the leader in a trade relationship sets the

prices for the net trade within that relationship. The prices for buying and selling are the same. The follower determines the amounts that are traded and the leader has the obligation to buy or sell whatever amount the follower decides to trade at the given prices.

A **Hierarchically Structured Economy** is a tuple $\mathbf{E} := ((A, D), \{U_i, \omega_i\}_{i \in A}, \{\mathcal{T}_w\}_{w \in D})$. Here (A, D) is a hierarchical graph where A is the set of agents in the economy and D is the set of asymmetric trade relationships. The tuple $\{U_i, \omega_i\}_{i \in A}$ describes the individual characteristics of the agents. Each agent $i \in A$ is described by a strictly monotonic, continuous and strictly quasi concave utility function $U_i : \mathbb{R}^l_+ \to \mathbb{R}$, and initial endowments $\omega_i \in \mathbb{R}^l_+$. We assume $\sum_{i \in A} \omega_i \gg 0$. We use $L := \{1, \ldots, l\}$ to denote the set of commodities in $\mathbf{E}$. Finally, $\{\mathcal{T}_w\}_{w \in D}$, describes the institutional characteristics of the trade relationships in D. We assume that every trade relationship has the institutional characteristic of mono pricing, i.e., $\forall w \in D : \mathcal{T}_w = \mathcal{T}^{mon}$.

We define $X_i := \mathbb{R}^{L \times L_i} \times (S^{l-1})^{F_i}$ to denote the space of net trades and prices agent i can choose from. A **Trade-Price-Allocation System** in the hierarchically structured economy $\mathbf{E}$ is a tuple $(d, p, x) \in X \times \mathbb{R}^{L \times A}_+ := \mathbb{R}^{L \times D} \times (S^{l-1})^D \times \mathbb{R}^{L \times A}_+$. The vector $d_{ji} \in \mathbb{R}^l$ denotes the net trades over the relationship $(i,j) \in D$. We define $d_j := (d_{jk})_{k \in L_j}$. The vector $p_{ij} \in S^{l-1}$ is the price vector on trade relationship $(i,j) \in D$. We define $p_i := (p_{ij})_{j \in F_i}$. Finally, $x_i \in \mathbb{R}^l_+$ to is the consumption bundle of agent $i \in A$.

The prices an agent sets on a trade relationship he dominates depend, amongst others, on what he anticipates to be the consequences of setting these prices. We assume an agent, i, knows, given the state of the economy as described by the trade-price tuple (d, p), his individual characteristics, the utility function of his followers, their initial endowments, the aggregate of the net trades between each $j \in F_i$ of them and his direct followers in F_j in a given state of the economy, and the prices other direct leaders in L_j set for agent j. We assume agent i forms his anticipations about the net trades that result from a change in the prices he sets by solving the optimization problem of each of his followers, say $j \in F_i$, assuming the prices set by the other leaders of agent j and the net trades between agent j and his followers in F_j do not change. Furthermore, we assume that if agent i does not change the prices he sets for agent j, then he anticipates the net trade between him and agent j not to change. The resulting anticipations of agent i about the consequences of a change in the prices he sets for the behaviour of $j \in F_i$ are described by the **Anticipated Net Trade Correspondence** $t_{ij} : X \times X_i \rightrightarrows \mathbb{R}^l$.

If for a follower $j \in F_i$ of some agent i it holds that $L_j = \{i\}$, then the anticipated net trade correspondence of agent i for agent j can be represented by a continuous function. [1] If agent j has more than one leader we lose this property.

In the case some agent j has, say, two leaders who set the same prices on their

[1] See Spanjers (1992, Chap. 5)

trade relationships with agent j, only the sum of the net-trades with those leaders matters for him. This results in anticipated net trade correspondences for the leaders of agent j which have a hyperplane in $\mathbf{R}^l$ as their values in the case the prices for trade with agent j are the same.

If the, say two, leaders of agent j set different prices in their respective trade relationships with agent j, then agent j may perform arbitrage in buying some commodity on the trade relationship where it is relatively cheap and selling it on the other trade relationship where it is relatively expensive. Since there are no upperbounds on the amounts of commodities agent j can buy or sell on these trade relationships, he can achieve an arbitrary high income and can affort any consumption bundle $x_j \in \mathbf{R}^l_+$. Therefore, the optimization problem of agent j would have no solution, since agent j will generate net trades which are infinitly large in absolute values. Similarly, if an agent i considers changing the prices he sets for some agent $j \in F_i$ such that such a situation would occur, then the optimization problem that defines the anticipated net trade correspondence t_{ij} of agent i for agent j would have no solution. To circumvent the problems this would cause for our models, we assume the net trades in these cases to be anticipated by agent i to be "sufficiently large" instead of infinitly large in absolute value. We say anticipated net trades are **Sufficiently Large** if for agent $i \in L_j$ who provokes them by setting a vector of prices different from that set by another leader in L_j, as described by the trade-price tuple (d,p), one of the following holds. Either the anticipated net trades, or any other net trades that would make agent j still better off, are large enough to make sure agent i can not deliver. Or they are large enough to ensure that agent i can take actions, given these anticipated net trades, that he anticipates to make him better off than he was before he induced these net trades, and any higher utility level for agent j can be attained by agent j by net trades that enable agent i to choose his other actions such that he anticipates to be better off still. The latter situation may arise if agent i anticipates to be able to transfer the anticipated net trades with agent j to one of his leaders or another of his followers at profitable prices. Such a leader $h \in L_i$ may or may not be able absorb the net trades agent i plans on their trade relationship.

Because of the difficulties mentioned above we can only successfully analyze hierarchically structured economies if we make severe restrictions on the hierarchical graphs we allow for. Loosely speaking, we only consider hierarchical grahps in which there are sufficient potential possibilities for arbitrage. In economies with this kind of hierarchical graphs, the only information the agents use about the individual characteristics of their followers it that their utility functions are strictly monotonic.

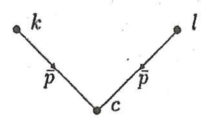
The set of actions an agent anticipates to be feasible for him is called his choice set. An arbitrary agent i anticipates a tuple of actions $(e_i, q_i, x_i) \in X_i \times \mathbf{R}^l_+$ to be feasible given the state (d,p), if they are compatible with some system of net trades $(e_{ji})_{j \in F_i}$ with his followers where for each $j \in F_i$ it holds that $e_{ji} \in t_{ij}((d,p),(e_i,q_i))$. This implies we assume the agents to be "optimistic" about the consequences of their actions. The choice set of agent i depends on the state of the economy as described

by the current trade-price system and, of course, on the anticipated net trade correspondences of agent i with respect to his followers. The **Choice Correspondence** $B_i : X \rightrightarrows X_i \times \mathbf{R}^{L \times A}$ of agent i gives the choice set $B_i(d,p)$ of agent i as a function of the ruling trade-price system (d,p).

Each agent is assumed to choose his actions as to maximize his utility over his choice set. An equilibrium in a hierarchically structured economy $\mathbf{E}$ is a state that is feasible and is such that no agent anticipates to possibly be better off if he chooses some (different) actions in his choice set. Formally, a trade-price-allocation system $(d^*, p^*, x^*) \in X \times \mathbf{R}^{L \times A}_+$ is an **Equilibrium** in the economy $\mathbf{E}$ if $\forall i \in A$ it holds that $(d^*_i, p^*_i, x^*_i) \in B_i(d^*, p^*)$ and $\nexists (e_i, q_i, y_i) \in B_i(d^*, p^*)$ such that $U_i(y_i) > U_i(x^*_i)$. An equilibrium in which the same vector of prices is quoted on each trade relationship is called a uniform price equilibrium.

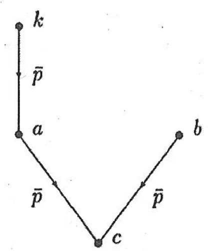
3 Some Results

As indicated above, we focus on economies with hierarchical graphs that allow for sufficient potential possibilities for arbitrage. Before we give a theorem on the existence of equilibrium and a theorem on equivalence of equilbrium with Walrasian equilibrium we discuss three lemmas.



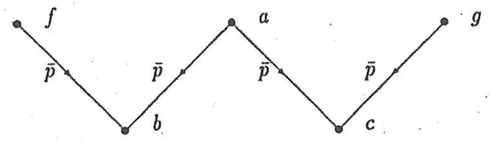
Lemma 3.1 *Suppose (d^*, p^*, x^*) is an equilibrium in the hierarchically structured economy* $\mathbf{E}$. *Let $c \in A : |L_c| \geq 2$. Then $\forall k, l \in L_c : p^*_{kc} = p^*_{lc}$.*

Lemma 3.1 states that if two agents have the same follower they, in equilibrium, set the same prices for this follower. The intuition is that if they do not, their common follower could improve his allocation by performing arbitrage between these two leaders and (d^*, p^*, x^*) would not be an equilibrium.



Lemma 3.2 *Suppose (d^*, p^*, x^*) is an equilibrium in the hierarchically structured economy* **E**. *Let $a \in A$ and $k \in L_a$. Let $c \in F_a$ and $b \in L_c$, with $b \neq a$. Then $p^*_{ka} = p^*_{ac} = p^*_{bc}$.*

From Lemma 3.1 it follows that, in equilibrium, we must have that $p^*_{ac} = p^*_{bc}$. Now suppose that $p^*_{ka} \neq p^*_{bc}$. Agent a takes the vector of prices p^*_{bc} as given. He can choose prices p_{ac} which are "inbetween" the prices p^*_{ka} and p^*_{bc}.[2] In choosing these prices agent a anticipates to induce arbitrage by agent c. By the choice p_{ac} this arbitrage by agent c also benefits agent a, since agent a anticipates to be able to transfer the net trades with agent c to agent k at the (for agent a profitable) vector of prices p^*_{ka}. This implies that agent a has actions in this choice set which he anticipates to make him better off. This contradicts (d^*, p^*, x^*) being an equilibrium.



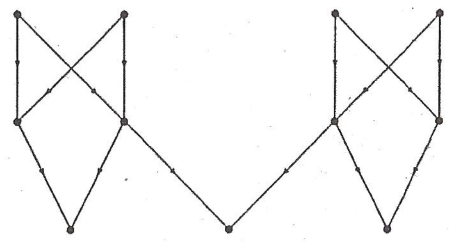
Lemma 3.3 *Suppose (d^*, p^*, x^*) is an equilibrium in the hierarchically structured economy* **E**. *Let $a \in A$, let $b, c \in F_a$ with $b \neq c$. Suppose $a \neq f \in L_b$ and $a \neq g \in L_c$. Then $p^*_{fb} = p^*_{ab} = p^*_{ac} = p^*_{gc}$.*

By Lemma 3.1 we have that $p^*_{fb} = p^*_{ab}$ and $p^*_{ac} = p^*_{gc}$. Suppose that $p^*_{fb} \neq p^*_{gc}$. Now there exist prices p_{ab} "inbetween" p^*_{fb} and p^*_{gc}, and prices p_{ac} "inbetween" p_{ab} and p^*_{gc}. If agent a sets these prices, he anticipates arbitrage by the agents b and c and he anticipates this arbitrage to be profitable for him, because he is "optimistic" and anticipates the net trades on the two trade relationships to be such that they "cancell out" and leave him precisely with the consumption bundle he likes, as if they were coordinated by him. Thus, setting the prices p_{ab} and p_{ac} is anticipated by agent a to be feasible and to make him better off then he is under (d^*, p^*, x^*). This contradicts (d^*, p^*, x^*) being an equilibrium.

Lemma 3.1, Lemma 3.2 and Lemma 3.3 enable us to prove a theorem on the existence of equilibrium in hierarchically structured economies. The intuition behind the existence theorem is that if there are sufficient potential possibilities for arbitrage in the economy and if there is a uniform price such that the total net trades with the agents from $A \setminus S_1$ can be supplied (absorbed) by the agents in S_1, then this uniform price is an equilibrium price. No agent wants to deviate unilaterally from the given

[2] Let $q, \bar{q}, \hat{q} \in S^{l-1}$. We say q is "inbetween" $\bar{q}$ and $\hat{q}$ if $\bar{q} \neq q \neq \hat{q}$ and $\forall c \in L$: $\min\{\bar{q}_c, \hat{q}_c\} \leq q_c \leq \max\{\bar{q}_c, \hat{q}_c\}$.

uniform price since he anticipates such a deviation to result in arbitrage which is disadvantageous for him.



Theorem 3.4 [Existence Theorem]
Let **E** *be a hierarchically structured economy that has (A, D) as its hierarchical graph. Suppose $\forall c \in A \setminus S_1 : |L_c| \geq 2$. Then there exists an equilibrium in* **E**. *Furthermore every equilibrium in* **E** *is a uniform price equilibrium.*

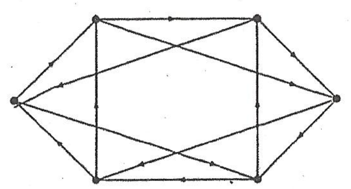
Let **E** be an economy as in Theorem 3.4. Let $\mathbf{E}^W := \{U_i, \omega_i\}_{i \in A}$ be the pure exchange economy with the same agents as the economy **E**. Let (p^W, x^W) be a Walrasian equilibrium in $\mathbf{E}^W$. Consider the trade-price-allocation system (d^*, p^*, x^*), where $p^* := (p^W)_{w \in D}$, $x^* := x^W$, and $d^* := (d^*_w)_{w \in D}$ are net trades with $\forall w \in D : d_w \in T^{\text{mon}}(p^W)$ such that x^* results. Now (d^*, p^*, x^*) is an equilibrium in **E** since it is feasible, x^*_i is optimal for each agent i given the price system p^*, and no agent wants to unilaterally deviate by setting a price $p \neq p^W$ on a trade relationship he dominates, because he anticipates such a deviation to result in arbitrage that is unfavorable for him.

On the other hand, if some trade-price-allocation system (d^*, p^*, x^*) is an equilibrium in an economy **E** which satisfies the conditions of Theorem 3.4, then it follows from applying Lemma 3.1, Lemma 3.2 and Lemma 3.3 that this is an equilibrium with uniform prices.

Theorem 3.4 shows that even if the individual agents in the economy know almost nothing about the economy they participate in, an equilbrium may exist. The theorem, of course, does not give any indication as to how this equilibrium is reached. To attain an equilibrium, actions between agents who may not even know about each others existence have to be coordinated, one way or another.

Since every Pareto efficient allocation in **E** can be supported by a Walrasian equilibrium in $\mathbf{E}^W$, and every Walrasian equilibrium allocation in $\mathbf{E}^W$ can be supported by an equilibrium in **E**, the counterpart the Second Theorem of Welfare Economics holds for economies satisfying the conditions of Theorem 3.4. However, not every equilibrium in such an economy need to be Pareto efficient. For instance, if there are at least two agents in S_1 who have initial endowments that are strictly greater then zero, then there exist uniform price equilibria with a Walrasian equilibrium price in which all agents except the agents in S_1 end up with the corresponding consumption bundle. This may happen because there is no way for the agents in S_1 to coordinate their net trades with their followers in such a way that they also end up with their consumption bundles corresponding to the Walrasian equilibirum. This is example indicates there may be a continuum of equilibrium allocations. Indeed, there will even be a continuum of uniform equilibrium price systems in this economy.

To end this section we give a theorem on equivalence of equilibrium in hierarchically structured economies with Walrasian equilibrium. It states that if the agents of S_1 have zero initial endowments in an economy with sufficient potential possibilities for arbitrage, then every equilibrium in the hierarchically structured economy $\mathbb{E}$ corresponds to Walrasian equilibrium in the corresponding pure exchange economy $\mathbb{E}^W$ and vice versa. Note that this in particular is the case if $S_1 = \emptyset$. The intuition behind this result is that under this additional condition every agent that "matters" for the economy (i.e., has non-zero initial endowments) acts as a prices taker, which essentially makes the economy $\mathbb{E}$ equivalent to the pure exchange economy $\mathbb{E}^W$.



Theorem 3.5 [Walrasian Equivalence]
Let $\mathbb{E}$ be as in Theorem 3.4. Assume additionally that $\forall a \in S_1 : \omega_a = 0$. Then $p^* = (\overline{p})_{w \in D}$ is a uniform equilibrium price for $\mathbb{E}$ if and only if $\overline{p}$ it is a Walrasian equilibrium price in $\mathbb{E}^W$. Furthermore, the equilibrium allocation in $\mathbb{E}$ for p^* is the Walrasian allocation for $\overline{p}$ in $\mathbb{E}^W$ and vice versa.

4 Concluding Remarks

In this paper a model of hierarchically structured economies is introduced that allows for price setting agents within the framework of a fixed structure of asymmetric bilateral trade relationships. The primitives of the model are a set of agents with their individual characteristics and a set of hierarchical trade relationships between agents with their institutional characteristics.

Each trade relationship is assumed to have the institutional characteristic of mono pricing. Thus, the leader in a trade relationship acts as a price setter and the follower

as a price taker with respect to this relationship. Furthermore, it is assumed that agents only have limited information about the economy.

The existence of equilibrium can be proven if the structure of trade relationships in the economy allows for "sufficient" potential possibilities for arbitrage. If, additionally, the agents that are not dominated in any of their trade relationships have zero initial endowments, then we find that every equilibrium in the hierarchically structured economy corresponds to a Walrasian equilibrium in the pure exchange economy with the same set of agents and vice versa.

The model of a hierarchically structured economy is a model in which a number of exchange institutions, i.e., the asymmetric bilateral trade relationships with their institutional characteristics, occur and interact through the actions and anticipations of the agents. Indeed, the model discussed in this paper is part of an attempt to find a rather general model that allows for the existence of equilibria for a broad class of exchange institutions, interpreted as trade relationships with different institutional characteristics.

One of the most intriguing questions raised by this kind of models is: "What determines the structure of trade relationships and their institutional characteristics?" To answer this question, further research into models of economies with different hierarchical structures, different institutional characteristics and different assumptions about the information structure is necessary. Only after having gained an increased understanding of this kind of models, one might hope to succesfully make both the hierarchical structure and the institutional characteristics of the trade relationships an endogenous part of the model.

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