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Arbitrage and Monopolistic Market Structures

by

Willy Spanjers

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University of Bielefeld
33501 Bielefeld, Germany

Arbitrage and Monopolistic Market Structures [1] [2]

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Abstract

Usually, markets are modelled as abstract institutions where demand and supply of commodities meet. This way of modelling markets often veils an underlying organizational structure.

In this paper we present a model of a pure exchange economy in which markets result from suitable exogenously given structures of asymmetric bilateral trade relationships. These trade relationships are interpreted as price setting relationships, so for any price taking agent in a trade relationship we have another agent setting a vector of prices. Consequently, we can do without the artifact of an auctioneer.

We explicitly model arbitrage, which enables us to establish uniform prices in (parts of) the economy. We find that, depending on the structure of trade relationships and the initial endowments of particular agents, monopolistic markets with and without price differentiation and the Walrasian market are obtained as special cases.

1 Introduction

The general equilibrium model, as introduced by Walras (1874) and formulated by Arrow and Debreu (1954) and Debreu (1959), is one of the fundamental models in economics. It is a model in which decentralized selfish decision making at equilibrium prices leads to outcomes which are efficient for the economy as a whole, as is stated in the first theorem of welfare economics. Here, we are concerned with two weaknesses of the model. Firstly, all agents in the economy, with exception of an artificial auctioneer who strictly speaking is not an agent, are assumed to act as price takers, without any agent setting the prices. Secondly, it is assumed that each agent can trade with every other agent in the economy through "the market", an abstract institution where supply and demand of commodities meet and trade takes place. Thus, the problem of specifying and analyzing trade relationships and communication structures that give rise to such a market is not addressed.

In dropping the assumption of price taking agents, one encounters the wider problem of integrating imperfect competition in general equilibrium models. In describing the behaviour of economic agents in general equilibrium models with imperfect competition, it seems appropriate to make explicit what each agent anticipates to be the consequences of (a change in) his actions. In Negishi (1961) it is assumed that some firms anticipate the prices for some of their inputs and outputs to be a linear function of the current state of the economy and the change in their demand for inputs and supply of outputs. In Hahn (1978) and Gale (1978), the notion of rational conjectures is introduced which is shown to be too strong to be of any use. In Vind (1983) the concept of conjectures is generalized through the model of equilibrium with coordination. Here agents may have veto power over changes in actions planned by others that would make them worse off. Once again, the existence of equilibrium is a problem. Interestingly, Vind introduces a formal concept of exchange institutions and provides a simple example in which the anticipations of the agents, called expectation functions, are derived from the set of exchange institutions in the economy.

Recently, a lot of attention is paid to models that specify the trade and communication structure of (Walrasian) markets. An important line of research investigates dynamic models in which agents meet pairwise and then trade according to specified trade rules. The agents typically meet often, either in a prespecified order or at random. Eventually, some equilibrium allocation is approached. This line of research is initiated by Feldman (1973) and developed further by, amongst others, Rubinstein and Wollinski (1985). In the context of exchange economies with a continuum of agents we refer to Gale (1986) and, more recently, McLennan and Sonnenschein (1991). In a more restrictive setting, McAffee (1993) allows individual sellers to choose the transaction mechanism they want to use. Thus, what are called the transaction institutions in the market arise endogenously. A different approach to modelling communication in markets is sketched by Kirman (1983), who uses stochastic graph theory in analyzing communication structures.

Economic models with fixed restrictive communication structures have focussed on theories of spatial economics and theories on intermediaries. In the first part of Karmann (1981) a spatial general equilibrium model is given in which transportation technologies play an important role. These transportation technologies can be interpreted as describing the user's costs of exchange institutions that take the form of competitive markets. Models on intermediaries are mostly partial equilibrium models. We refer to Krelle (1976) for models of successive monopolies and vertical intergration. More recently, models of successive monopolies in the context of pure exchange economies have been analyzed in Spanjers (1992). Finally, in Grodal and Vind (1989) competitive markets are modelled as exchange institutions with prices.

The aim of our research is to introduce models in which markets are no longer abstract aggregated institutions that veil underlying organizational structures. In our type of models, markets result from exogenously given structures of bilateral trade relationships, their specific trade rules, and compatible knowledge structures. Thus, we are not primairily interested in the outcomes generated by some market form as an aggregated organizational entity. Our main interest is to analyze which structures of bilateral trade relationships, their specific trade rules, and compatible knowledge structures lead to well known market forms. We aim at endogenizing market forms in making them a function of, amongst others, the underlying structure of trade relationships. To put it differently, we are interested in how specific market forms can be realized in our type of models.

The trade rules governing the bilateral trade relationships may change from relationship to relationship and are formalized by the institutional characteristic of the trade relationship under consideration. The institutional characteristic specifies the messages agents send through a given trade relationship. Thus, our approach of specifying the messages sent over (bilateral) trade relationships is along the lines of mod-

elling institutions as mechanisms as, e.g., discussed in Hurwicz (1989). A significant difference, however, is that we use "partial" mechanisms in specifying institutional characteristics of trade relationships. Thus, we "decentralize" the mechanism for the economy as a whole in several partial mechanisms which are independent with respect to their messages to be sent, as specified by the trade rules. Agents, however, may participate in a number of partial mechanisms. Therefore, the anticipations of any such agent concerning the reactions of the other agents participating in these partial mechanisms on a change in his actions establish the interaction of the partial mechanisms. Thus, the anticipations play a crucial role in the interaction of the partial mechanisms in an economy and their effects on the actions of the agents.

In the present paper we discuss a model of a pure exchange economy with an exogenously given structure of bilateral trade relationships. We assume the set of agents to be partitioned in hierarchical levels in such a way that the two agents in a trade relationship are of different hierarchical levels. Thus, the partition in hierarchical levels directs the trade relationships. For each of the resulting asymmetric trade relationships the trade rules determine what signals the agent in the relationship of the highest hierarchical level, the leader, can sent to the dominated agent, the follower, and how these signals restrict the set of net trades the dominated agent can choose from with respect to this trade relationship. More specific, we assume that every trade relationship has as its trade rule that the leader acts as a price setter with respect to this relationship, whereas the follower acts as a price taker. Finally, we specify the information of the individual agents in such a way that they have (aggregated) information about all agents of a lower hierarchical level that are relevant for them, but have no information about the remaining agents except themselves. Given the structure of the economy and the information the agents have, the anticipations (or conjectures) of each of the agents concerning the consequences of changing his actions are derived. The anticipations are constructed recursively, starting with the agents from the lowest hierarchical level, followed by the agents of the lowest hierarchical level but one, and so on. For a background on modelling economies this way, we refer to Gilles (1990) and Spanjers (1992).

In our economic models, the situation in which an agent has, say, two leaders setting different (normalized) vectors of prices for him plays a crucial role. If such a situation occurs, the agent may perform arbitrage by buying an arbitrary amount of a commodity that is relatively cheap from one leader and selling it to the other leader at a relatively high price. Thus, the agent can achieve an arbitrary high income. Furthermore, an agent may induce certain groups of agents to perform arbitrage if he sees a possibility for all agents concerned, including himself, to obtain arbitrary high incomes in doing so. Therefore, in an economy with sufficient potential possibilities for arbitrage, any equilibrium has the same vector of prices for every trade relationship. The assumption that "sufficient" potential possibilities for arbitrage are present in the economy is crucial for the results of this paper.

In Section 2, we present a formal model of our exchange economy, which we call a hierarchically structured economy. In Section 3, we prove a first theorem on the existence of equilibrium, a corollary stating additional conditions for equivalence with Walrasian equilibrium, and a corollary that states that agents of the highest hierarchical level may find themselves in a situation of two sided rationing. In Section 4, assuming that the highest hierarchical level contains only one agent, we give a theorem on the equivalence of equilibrium with the outcomes of a monopolistic market with or without price differentiation, depending on the structure of trade relationships in the economy. Furthermore, we give yet another corollary on equivalence with Walrasian equilibrium. Like the corresponding result in Section 3, it rests on the observation that the Walrasian auctioneer can be replaced by a non producing monopolist with zero initial endowments who cannot perform price differentiation. Thus, we can replace the artifical Walrasian auctioneer by a suitably choosen ordinary price setting agent. Finally, a second existence theorem is proven. A short summary of the results, some concluding remarks and some suggestions for further research are given in Section 5.

2 The Model

In this section we define a hierarchically structured economy. We describe such an economy by its hierarchical structure, by its agents and their individual characteristics, and by the institutional characteristics of the trade relationships in the economy. The hierarchical structure of the economy is described by a relationship structure that describes between which of the agents in the economy trade relationships exist, and by a complete echelon partition of the relationship structure that partitions the set of agents in hierarchical levels and describes which agents dominate which other agents. The agents in the economy are described by their individual characteristics. Since

we analyze a pure exchange economy, we describe each agent in the economy by his utility function and his initial endowments. Finally, we assume that every trade relationship in the economy has the institutional characteristic of what we call mono price setting between the dominating agent and the dominated agent, i.e., the dominating agent acts as a price setter and the dominated agent acts as a price taker with respect to their trade relationship. Then we define the information structure for a hierarchically structured economy. We use it to derive the anticipations of the agents about the consequences of their actions as a function of the individual characteristics of the other agents and of the institutional characteristics of the relationships.

We start by introducing the concepts we use to describe the hierarchical structure of our exchange economy. First, however, we introduce some terminology concerning graph theory. In this paper we restrict ourselves to graphs in which vertices have at most one direct connection.

A **(Simple) Undirected Graph** is a pair (A, R) consisting of a finite set of vertices A and a set of edges $R \subset \{\{i,j\} \subset A \mid i \neq j\}$. A **Path** $\gamma(a,b)$ from a to b in a undirected graph (A, R) is a non empty ordered set of distinct edges $(\{c_0,c_1\},\{c_1,c_2\},\ldots,\{c_{n-1},c_n\})$ with $c_0 = a$ and $c_n = b$, such that $\forall i,j \in \{0,1,\ldots,n\}$ $i \neq j: \; c_i \neq c_j$, with the possible exception that $c_0 = c_n$. We use $\Psi_{\mathcal{G}}$ to denote the set of paths in $\mathcal{G} := (A, R)$. A undirected graph (A, R) is **Connected** if $\forall a, b \in A$, with $a \neq b$ there exists a path $\gamma(a,b)$ from a to b in (A, R). With slight abuse of notation we write for a path $\gamma := (v_1,\ldots,v_n)$ that $v \in \gamma$ to denote $v \in \{v_1,\ldots,v_n\}$.

Definition 2.1 *A* **Relationship Structure** *is a connected (simple) undirected graph (A, R).*

Definition 2.2 *Let (A, R) be a relationship structure. The ordered partition $\xi := (S_1,\ldots,S_k)$ of the set A is a* **(Complete) Echelon Partition** *of (A, R) if for each $a \in \{1,\ldots,k\}$ it holds that there do not exist $i,j \in S_a$ such that $\{i,j\} \in R$.*

As is made precize in the below, the echolon partition determines the information structure of the economy.

Let $((A, R), \xi)$ be a pair consisting of a relationship structure and one of its complete echelon partitions. For each $i,j \in A$ we use $i \succ_\xi j$ to denote that $i \in S_a$ and $j \in S_b$ with $a < b$, which is interpreted as stating that i is of a higher hierarchical level then j. For each $i \in A$ we denote $L_i := \{h \in A \mid \{h,i\} \in R \;\; and \;\; h \succ_\xi i\}$ which interpreted as the set of **(Direct) Leaders** of i. Similarly, $F_i := \{j \in A \mid i \in L_j\}$ denotes the set of his **(Direct) Followers**.

Definition 2.3 *Let (A, R) be a relationship structure and ξ a complete echelon partition of (A, R). Let $r := \{i, j\} \in R$ with $i \succ_\xi j$. The* **Institutional Characteristic** *of r is the correspondence $\mathcal{T}_r : \tilde{X}_r \rightrightarrows \tilde{Y}_r$.*

The institutional characteristic $\mathcal{T}_r$ for a relationship $r := \{i, j\} \in R$ with $i \succ_\xi j$ is interpreted as specifying for each signal $s \in \tilde{X}_r$, chosen by agent i, the set $\mathcal{T}_r(s)$ of actions agent j can choose from with respect to the relationship r.

The institutional characteristic we are concerned with here is that of **Mono Price Setting**. Verbally, this institutional characteristic describes that the dominating agent acts as price setter and the dominated agent acts as a price taker with respect to their trade relationship. The price setting agent sets a (normalized) price for each of the l commodities in the economy. Let $S^{l-1} := \{p \in \mathbb{R}^l_+ \mid \sum_{i=1}^{l} p_i = 1\}$ denote the $(l-1)$-dimensional unit simplex. The institutional characteristic of mono price setting restricts the net trades of a follower $j \in F_i$ of an agent i over their trade relationship according to the correspondence $\mathcal{T}^{mon} : S^{l-1} \rightrightarrows \mathbb{R}^l$ such that for each vector of prices $p \in S^{l-1}$ we have

$$\mathcal{T}^{mon}(p) := \{d \in \mathbb{R}^l \mid p \cdot d \leq 0\} ,$$

i.e., the set $\mathcal{T}^{mon}(p)$ is the polar set of $\{p\}$. As in the Arrow-Debreu model, the price for buying and selling a commodity on a given trade relationship is the same. The follower determines the amounts that are traded, the leader has the obligation to buy or sell whatever amount the follower decides to trade at the given prices.

Definition 2.4 *A* **Hierarchically Structured Economy** *with l commodities is a tuple $\mathbf{E} = (((A, R), \xi), \{U_i, \omega_i\}_{i \in A}, \{\mathcal{T}_r\}_{r \in R})$ where:*

1. *(A, R) is a relationship structure.*

2. *ξ is a complete echelon partition of (A, R).*

3. *$U_i : \mathbb{R}^l_+ \to \mathbb{R}$ is the utility function of agent $i \in A$.*

4. *$\omega_i \in \mathbb{R}^l_+$ is the vector of initial endowments of agent $i \in A$.*

5. *$\mathcal{T}_r : \tilde{X}_r \rightrightarrows \mathbb{R}^l$ with $r := \{i, j\} \in R$ is the institutional characteristic of relationship r.*

An economy consists of a hierarchical structure that describes the position of the agents in the economy, a set of agents who have utility functions and initial endowments as their individual characteristics, and a set of trade relationships with their institutional characteristics.

We make the following assumption with respect to the individual characteristics of the agents and the institutional characteristics of the trade relationships in the economy.

Assumption 2.5 *Let $\mathbf{E}$ be a hierarchically structured economy. For every agent $i \in A$ the utility function U_i represents a neoclassical preference relation.* [1] *Furthermore, $\omega^{A \setminus S_1} := \sum_{i \in A \setminus S_1} \omega_i \gg 0$, $\forall i \in A : U_i(0) = 0$ and $U_i(\mathbb{R}^l_+) = \mathbb{R}_+$. Finally, $\forall r \in R : \mathcal{T}_r = \mathcal{T}^{mon}$.*

We use $L := \{1 \ldots, l\}$ to denote the set of commodities in the economy and with $X_i := \mathbb{R}^{L_i \times L} \times (S^{l-1})^{F_i}$ we denote the set of actions agent $i \in A$ can choose from. We denote $X := \prod_{i \in A} X_i$.

Definition 2.6 *A* **Trade-Price-Allocation Tuple** *in the hierarchically structured economy $\mathbf{E}$ is a tuple $(d, p, x) \in \mathbb{R}^{R \times L} \times (S^{l-1})^R \times \mathbb{R}^{A \times L}_+$ where:*

1. *$d_{ji} \in \mathbb{R}^l$ is the vector of net trades over the relationship $\{i, j\} \in R$ with $i \succ_\xi j$. We denote $d_i := (d_{ih})_{h \in L_i}$.*

2. *$p_{ij} \in S^{l-1}$ is the vector of prices on the trade relationship $\{i, j\} \in R$ with $i \succ_\xi j$. We denote $p_i := (p_{ij})_{j \in F_i}$.*

3. *$x_i \in \mathbb{R}^l_+$ is the consumption bundle of agent $i \in A$.*

[1] See also Aliprantis, Brown and Burkinshaw (1989, Def. 1.3.4). They say a continuous preference relation $\succeq$ on some $\mathbb{R}^l_+$ is neoclassical whenever either:

1. $\succeq$ is strictly monotone and strictly convex.
2. $\succeq$ is strictly monotone and strictly convex on int $\mathbb{R}^l_+$, and everything in the interior is preferred to anything on the boundary.

They call a preference $\succeq$ strictly monotone on a set X if $\forall x, y \in X : [x > y \Rightarrow x \succ y]$.

The set of indirect subordinates of some agent i, $\mathcal{S}(i)$, is the set of agents with whom agent i is connected through a path in which all agents are of a lower hierarchical level then agent i. It plays an important role in describing the information structure of the economy.

Definition 2.7 *Let (A, R) be a relationship structure, and let ξ be a complete echelon partition to this relationship structure. The set of* **(Indirect) Subordinates** *of agent i, denoted by $\mathcal{S}(i)$, is a subset of A such that $j \in \mathcal{S}(i)$ if and only if there exists a path $\gamma(i,j) := (\{i,c_1\},\{c_1,c_2\},\ldots,\{c_{n-1},c_n\})$ with $c_n = j$, such that for each $k \in \{1,\cdots,n\}$ we have that $i \succ_\xi c_k$.*

We denote $\mathcal{S}^+(i) := \mathcal{S}(i) \cup \{i\}$, and refer to it as the **Extended Set of (Indirect) Subordinates** of agent i.

The economy is assumed to have an information structure which enables each agent to construct his anticipations of the consequences of his actions for the agents in his set of (indirect) subordinates according to the recursive procedure described below.

We assume that each agent i knows the following about the economy for a given trade-price-allocation system. Firstly, agent i knows the individual characteristics of the agents in the set $\mathcal{S}^+(i)$, and the institutional characteristics of the trade relationships these agents are a part of. Second, agent i knows that the agents in $\mathcal{S}(i)$ construct their anticipated reactions correspondences and feasible actions correspondences in the way introduced below. Third, agent i knows the prices set on any trade relation between an agent in $\mathcal{S}^+(i)$ and an agent in $A \setminus \mathcal{S}^+(i)$, and take these prices as given and not to be influences by the actions of any subset of agents of $\mathcal{S}^+(i)$. Finally, agent i assumes that the agents in $\mathcal{S}(i)$, if they are indifferent between some "optimal" actions, choose the actions that suit agent i best. Thus, each agent in the economy has the information he needs to derive his anticipated reactions correspondence and his feasible actions correspondence through the recursive procedure described below.

One of the problems in deriving the anticipations of the agents is that for some tuples of price vectors some agents may want to engage in arbitrage. We say an agent engages in **Arbitrage** if he has at least two leaders who set different price vectors for him, and therefore he can obtain an arbitrary high income by buying a sufficiently large amount of a commodity that is relatively cheap in the trade with the one leader

and selling it to a leader where it has a relatively high price. For a nonsatiated utility maximizing agent, this results in "infinitely large" net trades, which are not in $\mathbb{R}^l$. We circumvent this type of problem by assuming these net trades are *anticipated* to be "sufficiently large" instead of "infinitely large" in a sense made precize later.

The correspondences that describe the net trades the agents anticipate to result with their followers as a consequence of a change in their actions are called anticipated net trade correspondences. These correspondences are defined by a recursive procedure. This recursive procedure starts with the agents of the lowest hierachical level. The anticipated net trade correspondence for some arbitrary agent $i \in A$ is constructed with the help of some auxiliary concepts used in the procedure, viz., feasible actions correspondences $(B_j)_{j\in\mathcal{S}(i)}$, which are constructed with the help of the anticipated trade correspondences $(t_j)_{j\in\mathcal{S}(i)}$ of the (indirect) subordinates of agent i, and the restricted reactions correspondence $\overline{B}_i$, the feasible consumption bundles correspondence $\overline{C}_i$ and the anticipated reactions correspondence τ_i of agent i.

First we introduce some notation.

$Q: A \times \mathbb{N} \rightrightarrows A$ with $\forall i \in A,\ a \in \mathbb{N}:$

$$Q(i,a) := \{h \in L_i \mid h \in S_b,\ b \leq a\}.$$

The correspondence Q gives for each agent i and each hierarchical level a the set of agents that have a trade relationship with agent i in which they dominate him, and that are of a hierarchical level not lower than a. [2] The set $Q(i,a)$ is the set of agents who are price setters with respect to agent i and who are at least of hierarchical level a.

For each $i \in A$ we denote:

$\tilde{x}_i:\ \mathbb{R}^{R\times L} \to \mathbb{R}^l$ with $\forall d \in \mathbb{R}^{R\times L}$:

$$\tilde{x}_i(d) := \omega_i + \sum_{h\in L_i} d_{ih} - \sum_{j\in F_i} d_{ji}.$$

$$Y^{\mathcal{S}(i)} := \textstyle\prod_{j\in\mathcal{S}(i)}(X_j \times \mathbb{R}^l).$$

The function $\tilde{x}_i$ assigns to each tuple of net trades $d \in \mathbb{R}^{R\times L}$ the resulting commodity bundle $\tilde{x}_i(d) \in \mathbb{R}^l$ for agent i. [3] We say the commodity bundle $\tilde{x}_i(d)$ results from by

[2] Notice that an agent $h \in S_b$ is of a hierarchical level higher then a if $b < a$.

[3] We refer to $\tilde{x}_i(d)$ as a commodity bundle instead of a consumption bundle since it may be in $\mathbb{R}^l \setminus \mathbb{R}^l_+$.

the system of net trades d. Finally, $Y^{S(i)}$ denotes the space of tuples prices, net trades and commodity bundles of the agents in the subgraph of agent i. Note that in this space commodity bundles with negative components are allowed for, contrary to the space of trade-price-allocation tuples.

In the recursive procedure, we start with an agent that has an empty set of (indirect) subordinates, i.e., an agent $j \in A$ such that $\mathcal{S}(j) = \emptyset$. The set of agents $\{j \in A \mid \mathcal{S}(j) = \emptyset\}$ contains the set S_k, the set of agents of the lowest hierarchical level, and therefore is not empty. Since agent j does not have any (indirect) subordinates, we need not define his anticipated reactions correspondence. Therefore we can directly define his feasible actions correspondence B_j by using Definition 2.12.

Suppose we have applied the procedure to construct the feasible actions correspondences of all (indirect) subordinates of agent i, i.e., the tuple of feasible actions correspondences $(B_j)_{j\in\mathcal{S}(i)}$ is given. We now construct the feasible actions correspondence B_i in applying the following definitions.

First we define the restricted reactions correspondence $\overline{B}_i$. In this definition we use, for the given agent i, the set $\mathcal{F}_i$ and for agent i and his (indirect) subordinate $j \in \mathcal{S}(i)$ the correspondence $\hat{B}_{ij}$.

We define:

$\mathcal{F}_i := \{(\tilde{e}_j, \tilde{q}_j, \tilde{y}_j)_{j\in\mathcal{S}(i)} \in Y^{\mathcal{S}(i)} \mid \forall j \in \mathcal{S}(i) :\ \tilde{y}_j \leq \omega_j + \sum_{h\in L_j} \tilde{e}_{jh} - \sum_{m\in F_j} \tilde{e}_{mj}\}$.

Thus, $\mathcal{F}_i$ is the set of tuples of net trades, prices and commodity bundles of the (indirect) subordinates of agent i that are such that for each (indirect) subordinate it holds that his commodity bundles does not exceed the bundle that results from his initial endowment and his trades with his leaders and followers as described by the tuple.

The correspondence $\tilde{B}_{ij} :\ X \times (S^{l-1})^{F_i} \times Y^{\mathcal{S}(i)} \times \mathbb{R}_+ \rightrightarrows X_j \times \mathbb{R}^l_+$ is introduced for notational convencience and is such that $\forall ((d,p), q_i, (e_m, q_m, y_m)_{m\in\mathcal{S}(i)}, \delta_j) \in X \times (S^{l-1})^{F_i} \times Y^{\mathcal{S}(i)} \times \mathbb{R}_+$ we have

$\tilde{B}_{ij}((d,p), q_i, (e_m, q_m, y_m)_{m\in\mathcal{S}(i)}, \delta_j) :=$
$B_j((d_g, p_g)_{g\in A\setminus[Q(j,b)\cap S^+(i)]}, (e_h, q_h)_{h\in Q(j,b)\cap\mathcal{S}(i)}, (d_i, q_i)) \cap$
$\{(\overline{e}_j, \overline{q}_j, \overline{y}_j) \in X_j \times \mathbb{R}^l_+ \mid U_j(y_j) \leq \delta_j\}$.

The correspondence $\tilde{B}_{ij}$ describes the set of trade-price-consumption tuples of agent j that are feasible for him and such that their consumption bundle yields him a utility level that does not exceed δ_j. The number δ_j can be interpreted as the aspiration level of agent j as assumed by agent i.

Finally, the correspondence $\hat{B}_{ij} :\ X \times (S^{l-1})^{F_i} \times Y^{\mathcal{S}(i)} \times \mathbb{R}_+ \rightrightarrows X_j \times \mathbb{R}^l_+$ is such that $\forall ((d,p), q_i, (e_m, q_m, y_m)_{m\in\mathcal{S}(i)}, \delta_j) \in X \times (S^{l-1})^{F_i} \times Y^{\mathcal{S}(i)} \times \mathbb{R}_+$:

$\hat{B}_{ij}((d,p), q_i, (e_m, q_m, y_m)_{m\in\mathcal{S}(i)}, \delta_j) :=$
$\operatorname{argmax}_{(\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)\in\tilde{B}_{ij}((d,p),q_i,(e_m,q_m,y_m)_{m\in\mathcal{S}(i)},\delta_j)} U_j(\tilde{y}_j)$.

The correspondence $\hat{B}_{ij}$ has as its values the sets of trade-price-consumption tuples for agent j that maximize his utility under the constraint that they are feasible for him and that their consumption bundle yields a utility level not exceeding δ_j. We denote $\delta := (\delta_j)_{j\in\mathcal{S}(i)}$.

Definition 2.8 *The* **Restricted Reactions Correspondence** $\overline{B}_i : X \times (S^{l-1})^{F_i} \times \mathbb{R}_+^{\mathcal{S}(i)} \rightrightarrows Y^{\mathcal{S}(i)}$ *of agent* $i \in A$ *is such that* $\forall ((d,p), q_i, \delta) \in X \times (S^{l-1})^{F_i} \times \mathbb{R}_+^{\mathcal{S}(i)}$ *we have:*

$\overline{B}_i((d,p), q_i, \delta) := \{(e_j, q_j, y_j)_{j\in\mathcal{S}(i)} \in \mathcal{F}_i \mid \forall j \in \mathcal{S}(i),\ b \in \mathbb{N},\ j \in S_b :$
1. $(e_j, q_j, y_j) = (d_j, p_j, \tilde{x}_j(d))$ if $\forall m \in S^+(j),\ h \in S^+(i)\setminus S^+(j) :\ p_{hm} = q_{hm}$
2. $(e_j, q_j, y_j) \in \hat{B}_{ij}((d,p), q_i, (e_m, q_m, y_m)_{m\in\mathcal{S}(i)}, \delta_j)$ else$\}$.

The restricted reactions correspondence $\overline{B}_i$ restricts the reactions agent i anticipates from his (indirect) subordinates, amongst others as a function of a vector δ of assumed aspiration levels of these agents. In order to describe these restrictions we distinguish between the (indirect) subordinates of agent i who notice a change in actions and those who do not notice this change.

An (indirect) subordinate $j \in \mathcal{S}(i)$ of agent i is said to notice a change if some agent $m \in S^+(j)$ in the extended set of (indirect) subordinates of agent i gets a price $q_{hm} \neq p_{hm}$ set by some agent $h \in S^+(i) \setminus S^+(j)$, i.e., agent h is in the extended set of (indirect) subordinates of agent i but not in the extended set of (indirect) subordinates of agent j. An (indirect) subordinate of agent i for whom the above does not hold is said not to notice the change.

Agent i anticipates actions of his (indirect) subordinates which are mutually compatible and such that the following holds.

Firstly, an agent $j \in \mathcal{S}(i)$ who does not notice a change is anticipated by agent i not to change the prices $(p_{jm})_{m\in F_j}$ he sets for his followers and the net-trades $(d_{jh})_{h\in L_j}$ with his leaders, even if such changes would be advantageous for agent j. Agent j is anticipated by agent i to obtain as his (possibly new) commodity bundle the bundle $\tilde{x}_j(d)$ resulting from the initial system d, even if $\tilde{x}_j(d)$ is negative in some components.

An agent $j \in \mathcal{S}(i)$ who notices a change is restricted to choose a feasible reaction. Furthermore, the restricted reactions correspondence restricts the set of feasible reactions of agent j to those actions that cannot be improved by a feasible action that yield a utility level not exceeding δ_j.

Definition 2.9 *The* **Feasible Consumption Bundles Correspondence** *of agent $i \in A$ is the correspondence $\overline{C}_i : Y^{\mathcal{S}(i)} \times (S^{l-1})^{L_i} \rightrightarrows \mathbb{R}^l_+$ where $\forall ((e_j, q_j, y_j)_{j\in\mathcal{S}(i)}, (p_{hi})_{h\in L_i}) \in Y^{\mathcal{S}(i)} \times (S^{l-1})^{L_i}$ we have*

$$\overline{C}_i((e_j,q_j,y_j)_{j\in\mathcal{S}(i)},(p_{hi})_{h\in L_i}) := \{y_i \in \mathbb{R}^l_+ \mid \exists e_i \in \prod_{h\in L_i} \mathcal{T}^{mon}(p_{hi}) : y_i \le \omega_i + \sum_{h\in L_i} e_{ih} - \sum_{j\in F_i} e_{ji}\}.$$

The feasible consumption bundles correspondence $\overline{C}_i$ of agent i denotes the set of consumption bundles attainable for agent i, given the net trades $(e_{ji})_{j\in F_i}$ of his direct followers and the prices $(p_{hi})_{h\in L_i}$ set by his direct leaders. The other arguments of the correspondence are void and are included for later notational convenience only.

Next we introduce for given $i \in A$ with a such that $i \in S_a$ and for $j \in \mathcal{S}(i)$ the functions V_j and $\tilde{\delta}_{ij}$ and for agent i the function $\tilde{\delta}_i$ and the correspondences $\overline{\tau}_i$ and $\hat{\tau}_i$ in order to define the anticipated reactions correspondence τ_i. The function V_j is used in defining the function $\tilde{\delta}_{ij}$. The functions $\tilde{\delta}_{ij}$ are used in defining the function $\tilde{\delta}_i$, which is used in defining the correspondence $\hat{\tau}_i$.

We define the function $V_j : S^{l-1} \to \mathbb{R}_+$ such that $\forall \overline{p} \in S^{l-1}$ we have

$$V_j(\overline{p}) := \max_{x \in \{y\in\mathbb{R}^l_+ \mid \overline{p}\cdot y \le \overline{p}\cdot(\sum_{m\in S^+(j)}\omega_m)\}} U_j(x).$$

For a given price vector $\overline{p} \in S^{l-1}$ the number $V_j(\overline{p})$ is the utility level agent j could achieve if he had the income $\overline{p}\cdot(\sum_{m\in S^+(j)}\omega_m)$ at his disposal. Thus, $V_j(\overline{p})$ is the value of the indirect utility function of agent j at prices $\overline{p}$ and income $\overline{p}\cdot\sum_{m\in S^+(j)}\omega_m$.

The function $\tilde{\delta}_{ij} : (S^{l-1})^R \times (S^{l-1})^{S^+(i)} \to \mathbb{R}_+$ is such that $\forall (p,(q_m)_{m\in S^+(i)}) \in (S^{l-1})^R \times (S^{l-1})^{S^+(i)}$ we have

$$\tilde{\delta}_{ij}(p,(q_m)_{m\in S^+(i)}) := \max_{\overline{p} \in \{p_{hj}\}_{h\in L_j\setminus S^+(i)} \cup \{q_{hj}\}_{h\in L_j\cap S^+(i)}} V_j(\overline{p}).$$

The value of $\tilde{\delta}_{ij}$ is used in the definition of $\hat{\tau}_i$ as a lowerbound on the aspiration level agent i assumes for agent j.

The function $\tilde{\delta}_i : (S^{l-1})^R \times (S^{l-1})^{S^+(i)} \to \mathbb{R}^{\mathcal{S}(i)}_+$ is such that $\forall (p,(q_m)_{m\in S^+(i)}) \in (S^{l-1})^R \times (S^{l-1})^{S^+(i)}$ we have

$$\tilde{\delta}_i(p,(q_m)_{m\in S^+(i)}) := (\tilde{\delta}_{ij}(p,(q_m)_{m\in S^+(i)}))_{j\in\mathcal{S}(i)}.$$

Thus $\tilde{\delta}_i$ has the vector of values of the functions $\tilde{\delta}_{ij}$ for the (indirect) subordinates $j \in \mathcal{S}(i)$ of agent i as its value.

The correspondence $\overline{\tau} : X \times X_i \rightrightarrows Y^{\mathcal{S}(i)}$ is such that $\forall ((d,p),(e_i,q_i)) \in X \times X_i$ we have

$$\overline{\tau}_i((d,p),(e_i,q_i)) := \{(\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)_{j\in\mathcal{S}(i)} \in Y^{\mathcal{S}(i)} \mid \exists \delta \in \mathbb{R}^{\mathcal{S}(i)}_+ : [(\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)_{j\in\mathcal{S}(i)} \in \overline{B}_i((d,p),q_i,\delta)] \text{ and } [\forall \hat{\delta} \ge \delta : (\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)_{j\in\mathcal{S}(i)} \in \overline{B}_i((d,p),q_i,\hat{\delta}) \Rightarrow \overline{C}_i((\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)_{j\in\mathcal{S}(i)},(p_{hi})_{h\in L_i}) = \emptyset]\}.$$

The values of $\overline{\tau}_i$ are the sets of trade-price-consumption tuples of the (indirect) subordinates of agent i that are "sufficiently large" and leave him with an empty set $\overline{C}_i$ of feasible consumption bundles. The correspondence describes the sets of unfavorable arbitrage flows agent i anticipates to be confronted with as a consequence of him choosing actions (e_i,q_i) for a given trade-price tuple (d,p).

Finally, the correspondence $\hat{\tau}_i : X \times X_i \rightrightarrows Y^{\mathcal{S}(i)}$ is such that $\forall ((d,p),(e_i,q_i)) \in X \times X_i$ we have

$$\hat{\tau}_i((d,p),(e_i,q_i)) := \{(\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)_{j\in\mathcal{S}(i)} \in Y^{\mathcal{S}(i)} \mid (\tilde{e}_j,\tilde{q}_j,\tilde{y}_j)_{j\in\mathcal{S}(i)} \in \overline{B}_i((d,p),q_i,\tilde{\delta}_i(p,(\tilde{q}_m)_{m\in S^+(i)})) \text{ and } \forall \delta \ge \tilde{\delta}_i(p,(q_m)_{m\in S^+(i)}) : \overline{C}_i((\hat{e}_j,\hat{q}_j,\hat{y}_j)_{j\in\mathcal{S}(i)},(p_{hi})_{h\in L_i}) \ne \emptyset\}.$$

The correspondence $\hat{\tau}_i$ is the set of trade-price-consumption tuples of the indirect subordinates of agent i such that for each (indirect) subordinate of agent i it holds that his actions either are optimal in his feasible actions set, and therefore the restriction that the utility level of the consumption bundle does not exceed the aspiration level agent i assumed for him is not binding, or these tuple are part of a sufficiently large arbitrage flow. Each such tuple of restricted reactions induces a feasible consumption bundle y_i for agent i, and for every vector of utilty levels $\delta \ge \tilde{\delta}_i(p,(q_m)_{m\in S^+(i)})$, there exists a tuple of restricted reactions $(\hat{e}_j,\hat{q}_j,\hat{y}_j)_{j\in\mathcal{S}(i)}$ such that the set $\overline{C}_i(\hat{e}_j,\hat{q}_j,\hat{y}_j)_{j\in\mathcal{S}(i)},(p_{hi})_{h\in L_i})$ of feasible bundles for agent i is not empty. Thus, $\hat{\tau}_i$ describes those reactions agent i anticipates from his (indirect) subordinates that either do not involve arbitrage flows, or describe sufficiently large arbitrage flows that do not harm (and may benefit) agent i. [4]

Definition 2.10 *The* **Anticipated Reactions Correspondence** *of agent $i \in A$ is*

[4] If the arbitrage flows would harm agent i, the for some δ sufficiently large, the induced set $\overline{C}_i(\cdot)$ would be empty.

the correspondence $\tau_i : X \times X_i \rightrightarrows Y^{S(i)}$ *such that* $\forall ((d,p),(e_i,q_i)) \in X \times X_i$:

$$\tau_i((d,p),(e_i,q_i)) := \bar{\tau}_i((d,p),(e_i,q_i)) \cup \hat{\tau}_i((d,p),(e_i,q_i)).$$

The anticipated reactions correspondence τ_i of agent i denotes the set of actions agent i anticipates the agents in $S(i)$ to take as a reaction on a change in (some of) the prices agent i sets for his followers.

The anticipated reactions correspondences as defined above describe what agent i anticipates to be the consequences of a change in his actions given a certain state of the economy. In determining his anticipations, agent i assumes that any agent who does not notice any changes as a direct or indirect consequence of the change in actions of agent i, does not change his actions. As is noted in Section 3, this assumption ensures that we do not assume away possible failures in coordination between agents, as we find in Theorem 3.5.

Definition 2.11 *The* **Anticipated Net Trade Correspondence** $t_i : X \times X_i \rightrightarrows \mathbb{R}^{F_i \times L}$ *of agent* $i \in A$ *is such that* $\forall ((d,p),(e_i,q_i)) \in X \times X_i$:

$$t_i((d,p),(e_i,q_i)) := \{(e_{ji})_{j\in F_i} \in \mathbb{R}^{F_i \times L} \mid \exists (\tilde{e}_j, \tilde{q}_j, \tilde{y}_j)_{j\in S(i)} \in \tau_i((d,p),(e_i,q_i)) \text{ such that } \forall j \in F_i : e_{ji} = \tilde{e}_{ji}\}.$$

In earlier models we have defined the anticipated net trade correspondences t_{ij} of each agent i for each of his followers $j \in F_i$, independent of his anticipations regarding his other followers. [5] Here we can no longer do so because the reactions of one follower $j_1 \in F_i$ can be anticipated by agent i to be influenced by the prices he sets for another of his followers $j_2 \in F_i$. It may also happen that, given a system of uniform prices, some (indirect) subordinate $m \in S(i)$ of agent i can choose to have his net trades at the same prices either with agent $j_1 \in F_i$ or with agent $j_2 \in F_i$. This decision of agent m does not influence the total net trades agent i anticipates from his set of (indirect) subordinates, but it does influence the amount of trade taking place over the trade relationships $\{i, j_1\}$ and $\{i, j_2\}$, respectively.

[5] This is the case in Spanjers (1992, Chapter 5 to Chapter 9).

Definition 2.12 *The* **Feasible Actions Correspondence** $B_i : X \rightrightarrows X_i \times \mathbb{R}^l_+$ *of agent* $i \in A$ *is such that* $\forall (d,p) \in X$:

$$B_i(d,p) := \{(e_i,q_i,y_i) \in X_i \times \mathbb{R}^l_+ \mid e_i \in \prod_{h\in L_i} T^{mon}(p_{hi}), \; y_i \le \omega_i + \sum_{h\in L_i} e_{ih} - \sum_{j\in F_i} e_{ji} \text{ such that } (e_{ji})_{j\in F_i} \in t_i((d,p),(e_i,q_i))\}.$$

It should be noted that neither the anticipated reactions correspondence nor the feasible actions correspondence need to be bounded. On the other hand, the feasible actions correspondences may be empty valued for some hierarchically structured economies, as, e.g., in Spanjers (1992, Chapter 6, Example 6.3.1). What we effectively need is that both the anticipated reactions correspondences and the feasible actions correspondences do not give problems in the hierarchical structures under consideration in this paper, with respect to deviations from equilibrium behavior, if an equilibrium exists at all. As we shall see, no such problems arise for the hierarchical structures in the theorems of this paper.

We assume each agent maximizes his utility over his budget set as it follows from the information structure of the economy. This leads to the following definition of an equilibrium in a hierarchically structured economy **E**. We define an equilibrium to be a state that is feasible and that is such that no agent anticipates to possibly be better off if he changes some of his actions.

Definition 2.13 *Let* **E** *be a hierarchically structured economy. A trade-price-allocation tuple* $(d^*, p^*, x^*) \in X \times \mathbb{R}^{A\times L}_+$ *is an* **Equilibrium** *in* **E** *if for any* $i \in A$:

$$(d_i^*, p_i^*, x_i^*) \in \operatorname{argmax}_{(e_i,q_i,y_i)\in B_i(d^*,p^*)} U_i(y_i).$$

It should be noted that because of the specification of the anticipations of the agents in the economy, in equilibrium, for each agent $i \in A$ we have

$$x_i^* \le \omega_i + \sum_{h\in L_i} d_{hi}^* - \sum_{j\in F_i} d_{ji}^*.$$

This property that the consumption bundle x_i^* that is anticipated by agent i to be feasible for him in equilibrium, is actually feasible for him in equilibrium does not automatically hold in models in which the conjectures of the agents are made endogenous. Typically, this property is one of the equilibrium conditions. [6]

[6] Clearly, it implies $\sum_{i\in A} x_i^* \le \sum_{i\in A} \omega_i$.

3 An Existence Result

In this section we, essentially, restrict ourselves to economies in which the highest echelon in the hierarchy, S_1, contains at least two agents. In the next section we turn to the case in which the highest echelon consists of one single agent. We show that in equilibrium agents of the highest hierarchical level, who have no leader, may be rationed. We also prove a corollary that states under what conditions we have equivalence with Walrasian equilibrium.

Let $\mathcal{G} := (A, R)$ be a undirected graph. The correspondence $\hat{\Psi}_{\mathcal{G}} : \Psi_{\mathcal{G}} \rightrightarrows A$ with $\forall\, \gamma(a,b) \in \Psi_{\mathcal{G}}$:

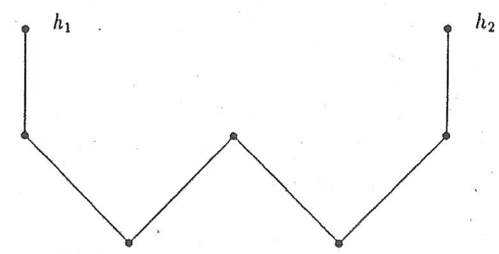
$$\hat{\Psi}_{\mathcal{G}}(\gamma(a,b)) := \{i \in A \setminus \{a,b\} \mid \exists j \in A : \{i,j\} \in \gamma(a,b)\}$$

assigns to each path $\gamma(a,b)$ from a to b the set of its vertices minus the starting point a and the endpoint b. Let $r := \{i,j\} \in R$ with $i \in L_j$. The we use p_r to denote p_{ij} and d_r to denote d_{ji}.

The first theorem we prove is a theorem on the existence of equilibrium, but first we prove a lemma and two corollaries.

The lemma and the first corollary are about particular paths in the relationship structure $\mathcal{G} := (A, R)$ of $\mathbf{E}$. They state conditions such that for each path γ that satisfies them, each edge $w \in \gamma$ has, in equilibrium, the same price vector $\overline{p} \in S^{l-1}$, i.e., $\forall\, w \in \gamma : \; p_w = \overline{p}$.

The proof of the lemma formalizes the notion that if some edges $v, w \in \gamma$ have different prices, i.e., $p_v \neq p_w$, then one of the following holds. Either some agent i in an edge in the path has the opportunity to engage in arbitrage, thus being able to improve any (finite) equilibrium consumption bundle. Or some agent i in the path can set a price vector which he anticipates to induce some agent(s) $j \in S(i)$ in his set of (indirect) subordinates to engage in arbitrage which he anticipates to also be profitable for himself.



Lemma 3.1 *Let* $\mathbf{E}$ *be a hierarchically structured economy with relationship structure* $\mathcal{G} := (A, R)$ *for which Assumption 2.5 holds. Let* (d, p, x) *be an equilibrium in* $\mathbf{E}$*. Let* $h_1, h_2 \in A$ *and* $\gamma(h_1, h_2) \in \Psi_{\mathcal{G}}$ *such that* $\emptyset \neq \hat{\Psi}_{\mathcal{G}}(\gamma(h_1,h_2))$ *and* $\forall\, j \in \hat{\Psi}_{\mathcal{G}}(\gamma(h_1,h_2))$: $[j \preceq_\xi h_1$ and $j \preceq_\xi h_2]$. *Then* $\exists\, p^* \in S^{l-1} : \; \forall\, z \in \gamma(h_1,h_2) : \; p_z = p^*$, *i.e.* p *has uniform prices on* $\gamma(h_1,h_2)$.

Proof

Suppose, for contradiction, that $\gamma(h_1,h_2)$ does not have uniform prices. Denote $\gamma(h_1,h_2) =: (\tilde{v}_1, \ldots, \tilde{v}_n)$. Choose $f_1, f_2, f \in A$, such that $\gamma(f_1,f_2) := (\tilde{v}_{\overline{a}}, \tilde{v}_{\overline{a}+1}, \ldots, \tilde{v}_{\overline{b}})$ with $f_1 \succeq_\xi f$, $f_2 \succeq_\xi f$, $f \in \hat{\Psi}_{\mathcal{G}}(\gamma(f_1,f_2))$ and $\forall\, g \in \hat{\Psi}_{\mathcal{G}}(\gamma(f_1,f_2)) \setminus \{f\} : \; g \prec_\xi f$. Furthermore, let $g_1, g_2 \in A$ be such that $\exists\, \overline{c} \in \{\overline{a}, \ldots, \overline{b}-1\}$ such that $\tilde{v}_{\overline{c}} =: r_1 = \{g_1, f\}$ and $\tilde{v}_{\overline{c}+1} =: r_2 = \{g_2, f\}$. Now, for suitably choosen f_1, f_2, f, g_1 and g_2, there exists a path $\gamma(f_1,f_2)$ as describe above such that

$$\overline{p} := p_{\tilde{v}_{\overline{a}}} = \ldots = p_{\tilde{v}_{\overline{c}}} = p_{r_1} \neq p_{r_2} = p_{\tilde{v}_{\overline{c}+1}} = \ldots = p_{\tilde{v}_{\overline{b}}} := \hat{p}.$$

For such a path one of the following cases holds.

(i) $(r_1 \cup r_2) \cap S(f) = \emptyset$.

In this case f is price-taker on both r_1 and r_2.

Since $\overline{p}, \hat{p} \in S^{l-1}$ we have $\exists\, a, b \in L : \; [\overline{p}_a > \hat{p}_a$ and $\overline{p}_b < \hat{p}_b]$. Now $\exists\, e_{r_1} \in \mathcal{T}^{mon}(\overline{p})$, $e_{r_2} \in \mathcal{T}^{mon}(\hat{p})$ such that $\exists\, y_f \in \mathbb{R}^l_+$ such that

$$y_f := \omega_f + (e_{r_1} - d_{r_1}) + (e_{r_2} - d_{r_2}) - \sum_{j \in F_i} d_{ji} + \sum_{h \in L_i} d_{ih} > x \geq 0,$$

with $y_{fa} > x_{fa}$ and $y_{fb} > x_{fb}$.[7]
Since $\overline{p}_a > \hat{p}_a \geq 0$ we have that $\exists \hat{y}_f \in \mathbb{R}^l_+ : \overline{p} \cdot \hat{y}_f = \overline{p} \cdot y_f$ with $U_f(\hat{y}_f) > U_f(x_f)$. Therefore agent f can improve his consumption bundle by choosing some $(e_f, p_f, \hat{y}_f) \in B_f(d,p)$, which contradicts (d,p,x) being an equilibrium. Therefore $\overline{p} = \hat{p}$.
(ii) $(r_1 \cup r_2) \cap S(f) \neq \emptyset$ *and* $(r_1 \cup r_2) \cap (A \setminus S^+(f)) \neq \emptyset$.
Thus, f is price-taker on, say, r_1 and price-setter on r_2.
Denote $\gamma(f_1, f_2) =: (v_1, \dots, v_n)$. Without loss of generality we may assume $v_1 = r_1$ and $v_2 = r_2$. Denote $\Gamma := \{v_1, \dots, v_n\}$ and $\Pi := \hat{\Psi}_G(\gamma(f_1, f_2))$.
Because of the choice of f_1, f_2, f, g_1 and g_2 as described above, the path $\gamma(f, f_2) := (v_2, \dots, v_n)$ has uniform prices, i.e., $\forall z \in \Gamma \setminus \{v_1\} : p_z = \hat{p}$.
Without restricting the scope of the proof, we may assume that $\exists a, b \in L : \forall z \in \Gamma, c \in L \setminus \{a,b\} : \overline{p}_{zc} = \hat{p}_{zc}$.
Now $\exists \tilde{p} \in (S^{l-1})^\Gamma : \forall z \in \Gamma,\ c \in L \setminus \{a,b\}$ we have $\tilde{p}_{zc} = \hat{p}_{zc}$ and

$$\overline{p}_a = \tilde{p}_{v_1 a} > \dots > \tilde{p}_{v_n a} = \hat{p}_a$$

$$\overline{p}_b = \tilde{p}_{v_1 b} < \dots < \tilde{p}_{v_n b} = \hat{p}_b.$$

Consider the set

$$\Sigma := \{e \in (\mathbb{R}^l)^R \mid \forall z \in \Gamma : e_z \in \mathcal{T}^{mon}(\tilde{p}_z) \text{ and } \forall z \in R \setminus \Gamma : e_z = d_z\}.$$

In a way similar as in case (i), we can choose $\tilde{e} \in \Sigma$ sufficiently large such that $\forall k \in \Pi : \hat{y}_k := \omega_k + \sum_{h \in L_k} \tilde{e}_{hk} - \sum_{j \in F_k} \tilde{e}_j \gg x_k \geq 0$.
Since we have neo-classical preferences by Assumption 2.5, we have $\forall k \in \Pi : U_k(\hat{y}_k) > U_k(x_k)$. So, agent f anticipates that changing the vector of prices $\overline{p}$ to $\tilde{p}_{r_2}$ may result in a consumption bundle $\hat{y}_f$ for him with $U_f(\hat{y}_f) > U_f(x_f)$. This contradicts (d,p,x) being an equilibrium. Therefore $\overline{p} = \hat{p}$.
(iii) $(r_1 \cup r_2) \cap (A \setminus S^+(f)) = \emptyset$.
Thus, f is price-setter on both r_1 and r_2.
Denote $\gamma(f_1, f) =: (v_n, \dots, v_1)$ with $v_1 = r_1$ and $\gamma(f, f_2) =: (w_1, \dots, w_m)$ with $w_1 = r_2$. Denote $\Gamma := \gamma(f_1, f_2) \cup \gamma(f_1, f_2)$ and $\Pi = \hat{\Psi}_G(\gamma(f_1, f) \cup \gamma(f, f_2))$.
Without restricting the scope of the proof we may assume that $\exists a, b \in L : \forall z \in \Gamma : \forall c \in L \setminus \{a,b\} : \overline{p}_{zc} = \hat{p}_{zc}$.

[7] Note that this does not suffice to have $U_f(y_f) > U_f(x_f)$ since preferences are neo-classical and we may have $y_f, x_f \in \mathbb{R}^l_+ \setminus \mathbb{R}^l_{++}$.

Let $p_{v_n a}\overline{p}_a < \hat{p}_a = p_{w_m b}$ and $p_{v_n b}\overline{p}_b > \hat{p}_b = p_{w_m b}$.
Choose $\tilde{p} \in (S^{l-1})^\Gamma$ such that $\forall z \in \Gamma,\ c \in L \setminus \{a,b\} : \tilde{p}_{zc} := \overline{p}_c = \hat{p}_c$ and

$$\overline{p}_a = \tilde{p}_{v_n a} < \tilde{p}_{v_{n-1} a} < \dots < \tilde{p}_{v_1 a} < \tilde{p}_{w_1 a} < \tilde{p}_{w_2 a} < \dots < \tilde{p}_{w_m a} = \hat{p}_a$$

$$\overline{p}_b = \tilde{p}_{v_n b} > \tilde{p}_{v_{n-1} b} > \dots > \tilde{p}_{v_1 b} > \tilde{p}_{w_1 b} > \tilde{p}_{w_2 b} > \dots > \tilde{p}_{w_m b} = \hat{p}_b.$$

Consider the set

$$\Sigma := \{e \in (\mathbb{R}^l)^R \mid \forall z \in \Gamma : e_z \in \mathcal{T}^{mon}(\tilde{p}_z) \text{ and } \forall z \in R \setminus \Gamma : e_z = d_z\}.$$

Now, as in case (i), we can choose $\tilde{e} \in \Sigma$ sufficiently large such that $\forall k \in \Pi : \exists y_k \in \mathbb{R}^l_+$:

$$y_k := \omega_k + \sum_{h \in L_k} \tilde{e}_{kh} - \sum_{j \in F_k} \tilde{e}_{jk} > x_k \geq 0,$$

with $y_{ka} > x_{ka}$ and $y_{kb} > x_{kb}$.

For each $k \in \Pi \setminus \{f\}$ define z^k to be the edge v_t or w_t containing k such that t is maximal. For f take, without loss of generality, $z^f := v_1$. For each $k \in \Pi$ we have $[\tilde{p}_{z^k a} > \overline{p}_a \geq 0$ or $\tilde{p}_{z^k b} > \hat{p}_b \geq 0]$, so $\forall k \in \Pi : \tilde{p}_{z^k} \cdot y_k > \tilde{p}_{z^k} \cdot x_k \geq 0$. For each $k \in \Pi$ consider the consumption bundle $\hat{y}_k \in \mathbb{R}^l_+$ such that

1. if $z^k \in \{v_1, \dots, v_n\}$, then $\hat{y}_{ka} = y_{ka} - \frac{1}{2}(y_{ka} - x_{ka})$, $\hat{y}_{kb} = y_{kb} > x_{kb}$ and $\forall c \in L \setminus \{a,b\} : \hat{y}_{kc} = y_{kc} + \varepsilon_{kc}$ where $\varepsilon_{kc} > 0$ and $\tilde{p}_{z^k c} \cdot \varepsilon_{kc} \leq \tilde{p}_{z^k a} \cdot \frac{1}{2l}(y_{ka} - x_{ka})$.
2. if $z^k \in \{w_1, \dots, w_m\}$, then $\hat{y}_{kb} = y_{kb} - \frac{1}{2}(y_{kb} - x_{kb})$, $\hat{y}_{ka} = y_{ka} > x_{ka}$ and $\forall c \in L \setminus \{a,b\} : \hat{y}_{kc} = y_{kc} + \varepsilon_{kc}$ where $\varepsilon_{kc} > 0$ and $\tilde{p}_{z^k c} \cdot \varepsilon_{kc} \leq \tilde{p}_{z^k b} \cdot \frac{1}{2l}(y_{kb} - x_{kb})$.

Clearly, $\forall k \in \Pi : \tilde{p}_{z^k} \cdot \hat{y}_k < \tilde{p}_{z^k} \cdot y_k$, and therefore $\hat{y}_k$ is affordable given x_k and net trades $\tilde{e}$. Since we also have $\hat{y}_k \gg x_k$ and since we have neo-classical preferences by Assumption 2.5, we have $U_k(\hat{y}_k) > U_k(x_k)$. Furthermore, for a suitable choice of $(y_k)_{k \in \Pi}$ and $\tilde{e}$, we have for each $k \in \Pi$ that $\hat{y}_k$ results from sufficiently large net trades.
Finally, note that the price $\tilde{p}_a$ of commodity a declines over the path $\gamma(f_1, f) = (v_n, \dots, v_1)$ and the price $\tilde{p}_c$ of any commodity $c \in L \setminus \{a,b\}$ is constant over this path. Thus, we find that transferring net trades $(\hat{y}_k - y_k)$ from some agent $k \in \Pi$ with $z^k \in \gamma(f_1, f)$ to agent f_1 over this path increases the "income" of all agents involved. The analogon holds for commodity b with respect to the path $\gamma(f, f_2) = (w_1, \dots, w_m)$.

So agent f anticipates that changing p_{v_1} to $\tilde{p}_{v_1}$ and p_{w_1} to $\tilde{p}_{w_1}$ may result in the consumption bundle $\hat{y}_f$ for him with $U_f(\hat{y}_f) > U_f(x_f)$. [8] This contradicts (d,p,x) being an equilibrium. Therefore $\overline{p} = \hat{p}$.

Q.E.D.

The following statement is a direct consequence of Lemma 3.1. It describes relationship structures and echelon partitions for hierarchical structured economies $\mathbb{E}$ for which in equilibrium the same price $\overline{p} \in S^{l-1}$ is set on every trade relationship w in a path between two (possibly identical) agents of the highest hierarchical level, if an equilibrium in $\mathbb{E}$ exists.

Corollary 3.2 [Uniform Prices I]
Let $\mathbb{E}$ be a hierarchically structured economy with relationship structure $\mathcal{G} := (A, R)$ and echelon partition ξ for which Assumption 2.5 holds. Let (d,p,x) be an equilibrium in $\mathbb{E}$. Suppose $\exists i, j \in S_1 : \exists \gamma(i,j) \in \Psi_{\mathcal{G}}$. Then p has uniform prices on $\gamma(i,j)$.

The corollary follows from applying Lemma 3.1 to the path $\gamma(i,j)$.

The next corollary gives conditions under which, if an equilibrium in the hierarchically structured economy exists, this equilibrium has uniform prices in the the whole of the economy.

Corollary 3.3 [Uniform Prices II]
Let $\mathbb{E}$ be a hierarchically structured economy with relationship structure $\mathcal{G} := (A, R)$ for which Assumption 2.5 holds. Let (d,p,x) be an equilibrium in $\mathbb{E}$. Suppose $\forall i \in A,\ j \in F_i : \exists h \in A,\ h \succeq_\xi i,\ h \neq i$, such that $\exists \gamma(i,h) \in \Psi_{\mathcal{G}}$ with $\hat{\Psi}_{\mathcal{G}}(\gamma(i,h)) \subset \mathcal{S}(i)$ and $\{i,j\} \in \gamma(i,h)$. Then p has uniform prices.

The corollary is a direct consequence of Lemma 3.1.

Now we give a theorem on the existence of equilibrium in hierarchically structured economy that has the information structure we assume in this paper. Essentially, it states that if there are sufficiently many potential possibilities for arbitrage in the economy, then an equilibrium exists.

We prove the existence of an equilibrium for a certain class of hierarchically structured economies by identifying one of the (possibly many) equilibria. In particular, in the economies under consideration a tuple $(d^*, p^*, x^*) \in X \times (\mathbb{R}^l_+)^A$ consistent with some agent $k \in S_1$ of the highest hierarchical level in the hierarchy behaving as a non producing monopolist (i.e., price setter) who cannot differentiate prices for the market constisting of the agents in $A \setminus \{k\}$ is an equilibrium tuple.

Theorem 3.4 [Existence Theorem I]
Let $\mathbb{E}$ be a hierarchically structured economy with relationship structure $\mathcal{G} := (A, R)$ for which Assumption 2.5 holds. Suppose $\forall i \in A,\ j \in F_i : \exists h \in A,\ h \succeq_\xi i,\ h \neq i$, such that $\exists \gamma(i,h) \in \Psi_{\mathcal{G}}$ with $\hat{\Psi}_{\mathcal{G}}(\gamma(i,h)) \subset \mathcal{S}(i)$ and $\{i,j\} \in \gamma(i,h)$. Then an equilibrium in $\mathbb{E}$ exists. Furthermore, every equilibrium in $\mathbb{E}$ is a uniform price equilibrium.

Proof
The second part of the statement is a direct consequence of Corollary 3.3.
We proceed by proving the first part of the theorem. Let $k \in S_1$. We prove that $\bar{p}$, a monopoly price for agent k for the market $A \setminus \{k\}$, is a uniform equilibrium price in $\mathbb{E}$. We start by proving that such a monopoly price $\overline{p} \in S^{l-1}$ exists.
(i) A monopoly price $\overline{p}$ for agent $k \in S_1$ exists.
Since each agent $i \in A$ has neoclassical preferences and $\sum_{i \in A \setminus S_1} \omega_i \gg 0$ by Assumption 2.5, we can without loss of generality restrict ourselves to the set $\operatorname{int} S^{l-1}$ of prices agent k can choose from. [9]

Let $\zeta^{A\setminus\{k\}} : \operatorname{int} S^{l-1} \rightrightarrows \mathbb{R}^l$ be such that $\forall p \in \operatorname{int} S^{l-1}$ we have

$$\zeta^{A\setminus\{k\}}(p) := \sum_{i \in A\setminus\{k\}} x_i(p) - \sum_{i \in A\setminus\{k\}} \omega_i,$$

where $x_i : S^{l-1} \to \mathbb{R}^l_+$ is the (price taking) individual demand function of agent $i \in A \setminus \{k\}$. Indeed, $\zeta^{A\setminus\{k\}}$ is the excess demand function of the (Walrasian) pure

[8] Note that agent f changes the price vector for two of his followers, inducing both of them to engage in arbitrage. By construction of the anticipated reaction correspondences, he anticipates the net trades with both of these followers to be such that they "cancel out", thus leaving agent f with an improved consumption bundle. Surely, agent f is heroic in anticipating the net trades with both followers to be compatible in this sense.

[9] See also Aliprantis, Brown and Burkinshaw (1989, Th. 1.6.5). Let $p \in \partial S^{l-1}$ with $c \in L$ such that $p_c > 0$. Since $\sum_{i \in A\setminus S_1} \omega_i \gg 0$, we have that $\exists i \in A \setminus S_1$ such that $\omega_{ic} > 0$. Therefore $p \cdot \omega_i > 0$. Since the utility function U_i is continuous and represents a neoclassical preference we find that the optimization problem of agent i does not have a solution for the price vector p, and therefore p can not be an equilibrium price.

exchange economy $\mathbf{E}^W := \{U_i, \omega_i\}_{i \in A \setminus \{k\}}$.
Since the individual demand functions are continuous under Assumption 2.5 (see, e.g., Aliprantis, Brown and Brukinshaw (1989, Th. 1.3.8)), we have that $\zeta^{A\setminus\{k\}}$ is a continuous function and, furthermore, $\zeta^{A\setminus\{k\}}$ is bounded from below by $-\sum_{i\in A\setminus\{k\}} \omega_i$. Therefore, the set $P^k := (\zeta^{A\setminus\{k\}})^{-1}([-\sum_{i\in A\setminus\{k\}} \omega_i, \omega_k])$, the $\zeta^{A\setminus\{k\}}$ – inverse of the box
$[-\sum_{i\in A\setminus\{k\}} \omega_i, \omega_k]$ is a closed set.
Since $P^k \subseteq \text{int}\, S^{l-1}$, we have that P^k is a compact set.
Since $\exists p \in P^k : \zeta^{A\setminus\{k\}}(p) = 0$ (see, e.g., Aliprantis, Brown and Burkinshaw (1989, Th. 1.4.8)) we have that $P^k \neq \emptyset$.

Thus, by the construction of P^k, the set of optimal (monopoly) prices for agent k when facing the market consisting of the agents $A \setminus \{k\}$ is

$$\text{argmax}_{p\in P^k} U_k(\omega_k - \zeta^{A\setminus\{k\}}(p)).$$

Since both $\zeta^{A\setminus\{k\}}$ and U_k are continuous functions and since P^k is a non empty compact set, the set of optimal (monopoly) prices for agent k is non empty.
(ii) The price $\overline{p}$ induces an equilibrium.
Let d^* be a system of net trade vectors in $\mathbf{E}$ such that the allocation x^* under a monopoly of k and monopoly price $\overline{p}$ results, and that the value of trade over each of the trade relationships equals zero given the prices $\overline{p}$. That is, x^* is such that $\forall i \in A \setminus \{k\} : x_i^* = x_i(\overline{p})$, and $x_k^* := \sum_{i\in A}\omega_i - \sum_{i\in A\setminus\{k\}} x_i(\overline{p})$, d^* is such that $\forall r \in R : d_r^* \in T^{\text{mon}}(\overline{p})$ and $\forall i \in A : x_i^* = \omega_i + \sum_{h\in L_i} d_{ih}^* - \sum_{j\in F_i} d_{ji}^*$, and $p^* := ((\overline{p})_{r\in R})$. From step *(i)* it follows that (d^*, p^*, x^*) is feasible.
It remains to show it is optimal from the point of view of each individual agent.
Assume, for contradiction, $\exists h_1 \in A \setminus \{k\} : \exists (e_{h_1}, q_{h_1}, y_{h_1}) \in B_{h_1}(d^*, p^*)$ with $U_{h_1}(y_{h_1}) > U_{h_1}(x_{h_1}^*)$. This implies that $q_{h_1} \neq p_{h_1}^*$. Let $i \in F_{h_1}$ be such that $q_{h_1 i} \neq \overline{p}$.
Because of the structure of $\mathcal{G}$ we have $\exists h_2 \in A,\ h_2 \neq h_1,\ h_2 \succeq_\xi h_1$, such that $\exists \gamma(h_1, h_2) \in \Psi_\mathcal{G}$, with $\{h_1, i\} \in \gamma(h_1, h_2)$ and $\hat{\Psi}_\mathcal{G}(\gamma(h_1, h_2)) \subset S(h_1)$.
Let $\gamma(h_1, h_2) := (v_1, \ldots, v_n)$ be such a path. By the definition of a path it follows that $v_1 = \{h_1, i\}$. Let $\gamma(h_2, i) := (v_n, \ldots, v_2)$. Denote $\Gamma := \{v_1, \ldots, v_n\}$ and $\Pi := \hat{\Psi}_\mathcal{G}(\gamma(h_1, h_2))$.
Since $q_{h_1 i} \neq \overline{p}$, we assume without restricting the scope of the proof, as we did in the proof of Lemma 3.1, that $\exists a, b \in L$ such that $\exists p \in (S^{l-1})^{\gamma(h_2,i)}$ with $\forall r \in \gamma(h_2, i),\ c \in L \setminus \{a, b\} : p_{rc} = p_{rc}^* = \overline{p}_c$, and

$$\overline{p}_a = p_{v_n a} > \ldots > p_{v_2 a} > q_{h_1 ia}$$

$$\overline{p}_b = p_{v_n b} < \ldots < p_{v_2 b} < q_{h_1 ib}.$$

Define $\tilde{p} \in (S^{l-1})^R$ such that $\forall r \in R$:

$$\tilde{p}_r := \begin{cases} p_r^* & \text{if } r \in R \setminus \Gamma \\ p_r & \text{if } r \in \gamma(h_2, i) \\ q_r & \text{if } r = \{h_1, i\} \end{cases}$$

Consider the set

$$\Sigma := \{e \in (\mathbb{R}^l)^R \mid \forall z \in \Gamma : e_z \in T^{mon}(\tilde{p}_z) \text{ and } \forall z \in R \setminus \Gamma : e_z = d_z\}.$$

As in the proof of Lemma 3.1 we can choose some $\tilde{e} \in \Sigma$ such that $\forall g \in \Pi$:

$$\hat{y}_g := \omega_g + \sum_{h\in L_g} \tilde{e}_{gh} - \sum_{j\in F_g} \tilde{e}_j \gg x_g^* \geq 0,$$

and $y_{h_1} = \omega_{h_1} + \sum_{h\in L_{h_1}} \tilde{e}_{h_1 h} - \sum_{j\in F_{h_1}} \tilde{e}_{jh_1}$. Since we have neoclassical preferences, we have $\forall g \in \Pi$ that $U_g(\hat{y}_g) > U_g(x_g^*)$. Therefore, we have $\sum_{g\in\Pi} \overline{p} \cdot \hat{y}_g > \sum_{g\in\Pi} \overline{p} \cdot x_g^*$. Since $\tilde{p}_{v_1} \cdot \tilde{e}_{v_1} = 0$, and $\tilde{p}_{v_n} \cdot \tilde{e}_{v_n} = 0$, it holds for the resulting consumption bundle y_{h_1} for agent h_1 that

$$\overline{p} \cdot y_{h_1} + \sum_{g\in\Pi} \overline{p} \cdot \hat{y}_g = \overline{p} \cdot x_{h_1}^* + \sum_{g\in\Pi} \overline{p} \cdot x_g^*$$

and therefore that $\overline{p} \cdot y_{h_1} < \overline{p} \cdot x_{h_1}^*$. Thus we have, by definition of x^*, that $U_{h_1}(y_{h_1}) < U_{h_1}(x_{h_1}^*)$, which yields a contradiction.

Q.E.D.

For the equilibria of which we proved the existence in the Theorem 3.4 we can derive some properties as in the following two corollaries.

The first corollary shows that even in equilibrium some agents may be "disappointed" by the outcome. Suppose that $\#S_1 \geq 2$. It may well be the case that some agent $k \in S_1$ finds that his equilibrium consumption bundle x_k^* does not correspond to a best element in the set $\{x \in \mathbb{R}_+^l \mid p^* \cdot x \leq p^* \cdot \omega_k\}$, his "price taking" budget set at uniform equilibrium prices p^*. This may happen since the agents in S_1 have no way to coordinate their trades with the rest of the economy. Still, no such agent anticipates to improve by changing the prices he sets for his followers since in that case he anticipates an arbitrage flow sufficiently large to make sure he cannot deliver. Furthermore, as follows from the definition of the anticipated reactions correspondences, if he does not change some of the prices he sets he anticipates that the trades he is confronted with do not change. This (possibly two sided) rationing occurs only for agents i such that $i \in S_1$. [10]

Corollary 3.5 [Rationing in Equilibrium]
Let $\mathbb{E}$ *be a hierarchically structured economy with relationship structure* $\mathcal{G} := (A, R)$, *for which Assumption 2.5 holds. Suppose* $\forall r \in R: \exists i, j \in S_1, i \neq j$, *such that* $\exists \gamma(i,j) \in \Psi_{\mathcal{G}} : r \in \gamma(i,j)$. *Let* $(d, p, x) \in X \times \mathbb{R}_+^{A \times l}$ *with* $p := (\overline{p})_{r \in R}$ *for some* $\overline{p} \in S^{l-1}$, *such that*

1. $\forall r \in R: d_r \in \mathcal{T}^{mon}(\overline{p})$.
2. $\forall i \in A: x_i := \omega_i + \sum_{h \in L_i} d_{ih} - \sum_{j \in F_i} d_{ji}$
3. $\forall i \in A \setminus S_1$ *it holds that:*
   $x_i \in \operatorname{argmax}_{y_i \in \{y \in \mathbb{R}_+^l \mid \overline{p} \cdot y \leq \overline{p} \cdot \omega_i\}} U_i(y_i)$.

Then (d, p, x) *is an equilibrium in* $\mathbb{E}$.

The corollary follows from the line of proof of Theorem 3.4, part (ii).

In some case, an equilibrium tuple of prices and consumption bundles may be supported by a continuum of tuples of net trade vectors. Furthermore, the above corollary indicates that, there may be a continuum of equilibrium prices is the economy $\mathbb{E}$. In some economies the set of equilibria in a hierarchically structured economy is more restricted. This is stressed by the following property which is on equivalence with Walrasian equilibrium.

Corollary 3.6 [Walrasian Equivalence I]
Let $\mathbb{E}$ *be as in Theorem 3.4, such that* $\forall k \in S_1 : \omega_k = 0$. *Then* $\overline{p}$ *is a (uniform) equilibrium price in* $\mathbb{E}$ *if and only if it is a Walrasian equilibrium price for the pure exchange economy* $\mathbb{E}^W := \{U_i, \omega_i\}_{A \setminus S_1}$.

[10] In settings more general then those described in Theorem 3.4, this kind of rationing may occur for any agent $i \in A$ with $L_i = \emptyset$.

Proof
The "if"-part of proof of this corollary follows directly by applying Corollary 3.5. The "only if"-part of the proof is as follows.
Suppose $\overline{p}$ is an equilibrium price. Instead of having some agent $k \in S_1$ optimizing over the set P^k, we consider the set of agents S_1 facing the excess demand function $\zeta^{A \setminus S_1} : \operatorname{int} S^{l-1} \to \mathbb{R}^l$ with $\forall p \in \operatorname{int} S^{l-1} : \zeta^{A \setminus S_1}(p) = \sum_{i \in A \setminus S_1} x_i(p) - \sum_{i \in A \setminus S_1} \omega_i$. Since $\overline{p} \in \operatorname{int} S^{l-1}$ is an equilibrium price, we have that $\zeta^{A \setminus S_1}(\overline{p}) \leq \sum_{k \in S_1} \omega_k$, and by assumption we have $\sum_{k \in S_1} \omega_k = 0$. Therefore $\zeta^A(\overline{p}) := \sum_{i \in A} x_i(\overline{p}) - \sum_{i \in A} \omega_i = \zeta^{A \setminus S_1}(\overline{p}) \leq 0$, and by Walras' law it follows that $\zeta^A(\overline{p}) = 0$. Thus, $\overline{p}$ is a Walrasian equilibrium price.

Q.E.D.

4 Results on Monopolistic Outcomes

In this section we state two main theorems. The first theorem states that for certain relationship structures in which one agent is the only agent of the highest hierarchical level in the economy, i.e., $\#S_1 = 1$, this agent sets prices as if he where a price differentiating (i.e., third degree price discriminating) monopolist facing some separated submarkets. The second theorem gives conditions on the relationship structure of the economy such that, no matter which complete echelon partition is chosen, the resulting hierarchical structure is such that an equilibrium in the corresponding hierarchically structured economy exists. In this second theorem we allow for echelon partitions in which $\#S_1 > 1$, as in the previous section.

Once again, we start by introducing some additional terminology on graphs.

Let $\mathcal{G} := (A, R)$ be a simple undirected connected graph. Let $A_a \subset A$. The **Restriction** of the graph $\mathcal{G}$ to A_a is the graph $\mathcal{G} \mid A_a := (A_a, R_a)$, where $R_a := \{r \in R \mid r \subset A_a\}$. A set $S \subset A$ is a **Separating Set** of $\mathcal{G}$ if the graph $\mathcal{G} \mid (A \setminus S)$ is disconnected. The **Connectivity** $\kappa(\mathcal{G})$ of $\mathcal{G}$ is the size of the smallest separating set in $\mathcal{G}$, if such a set exists. Furthermore, if for the graph $\mathcal{G}$ no separating set exists, then we define its connectivity $\kappa(\mathcal{G}) := \#A$. [11] The graph $\mathcal{G}$ is **k-connected** if $\kappa(\mathcal{G}) \geq k$. If the graph $\mathcal{G}$ is 2-connected, then $\forall a, b \in A, a \neq b, \exists \gamma(a,a) \in \Psi(\mathcal{G}) : b \in \gamma(a,a)$, i.e., for any two distinct points a and b there exist there exists a circuit in $\mathcal{G}$ containing

[11] Note that a complete graph has no separating set of vertices, since $(\emptyset, \{\emptyset\})$ is not a graph, and therefore certainly not a disconnected graph.

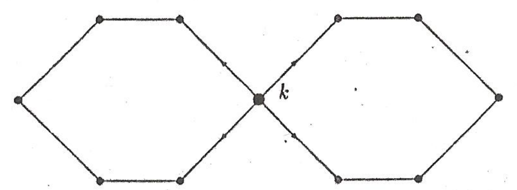
them. Thus, there exists two paths from a to b in $\mathcal{G}$ that are disjoint except for their begin and end points.

The next lemma states that in equilibrium each circuit that contains some agent of the highest hierarchical level has uniform prices.

Lemma 4.1 *Let* $\mathbb{E}$ *be a hierarchically structured economy with relationship structure* $\mathcal{G} := (A, R)$ *for which Assumption 2.5 holds. Let* (d^*, p^*, x^*) *be an equilibrium in* $\mathbb{E}$. *Let* $k \in S_1$ *and* $\gamma(k,k) \in \Psi_{\mathcal{G}}$ *Then* $\exists \bar{p} \in S^{l-1} : \forall z \in \gamma(k,k) : p^*_z = \bar{p}$.

This lemma follows directly from Corollary 3.2 and the definition of a circuit.

The following theorem describes hierarchical structures that have those and only those equilibria that are equilibria for the sole top agent in the economy behaving as a price differentiating, i.e., third degree price discriminating, non producing monopolist for the suitable set of submarkets. The submarkets consist of the maximal groups of agents for which the hierarchical structure has sufficient potential possibilities for arbitrage to ensure uniform prices. Formulated like this, the result is very intuitive indeed.



Theorem 4.2 [Price Differentiation]
Let $\mathbb{E}$ *be a hierarchically structured economy for which Assumption 2.5 holds and that has* $\mathcal{G} := (A, R)$ *as its relationship structure. Let* $\{G_a := (A_a, R_a)\}_{a \in T}$ *be a family of restrictions of* $\mathcal{G}$, *i.e.,* $\forall a \in T : G_a := \mathcal{G} \mid A_a$, *such that*

1. $\forall a, b \in T,\ a \neq b : A_a \cap A_b = \{k\} = S_1$.

2. $\cup_{a \in T} A_a = A$.

3. $\cup_{a \in T} R_a = R$.

4. $\forall a \in T, \forall i \in A_a \setminus \{k\} : \forall j \in F_i\ \exists h \in A_a,\ h \succeq_\xi i,\ h \neq i$, *such that* $\exists \gamma(i,h) \in \Psi_{\mathcal{G}}$ *with* $\hat{\Psi}_{\mathcal{G}}(\gamma(i,h)) \subset S(i)$ *and* $\{i,j\} \in \gamma(i,h)$. [12]

Suppose that for each $a \in T$ *we have* $\sum_{i \in A_a \setminus \{k\}} \omega_i \gg 0$. *The tuple* $p^* \in (S^{l-1})^R$ *is a tuple of equilibrium price vectors if and only if it consists of prices which are uniform within every* G_a, $a \in T$, *and these prices are a tuple of differentiated (i.e., third degree price discriminated) monopoly prices of agent* k *for the set of markets* $(A_a \setminus \{k\})_{a \in T}$.

The line of proof for this theorem is similar to that of Theorem 3.4.

To define the choice set $P^k \subset (\text{int}\, S^{l-1})^T$ of a price differentiating non producing monopolist $k \in S_1$ with respect tot the tuple of markets $(\{U_i, \omega_i\}_{i \in A_a} \setminus \{k\})_{a \in T}$ we consider for each $a \in T$ the function $\zeta^{A_a \setminus \{k\}} : \text{int}\, S^{l-1} \to \mathbb{R}^l$ with for each $p_a \in \text{int}\, S^{l-1}$:

$$\zeta^{A_a \setminus \{k\}}(p_a) := \sum_{i \in A_a \setminus \{k\}} x_i(p_a) - \sum_{i \in A_a \setminus \{k\}} \omega_i.$$

Now define $\zeta^{A \setminus \{k\}} : (\text{int}\, S^{l-1})^T \to \mathbb{R}^l$ such that for each $(p_a)_{a \in T} \in (\text{int}\, S^{l-1})^T$ we have $\zeta^{A \setminus \{k\}}((p_a)_{a \in T}) := \sum_{a \in T} \zeta^{A_a \setminus \{k\}}(p_a)$. Since a Walrasian equilibrium exists for the pure exchange economy $\mathbb{E}^{A \setminus \{k\}} := \{U_i, \omega_i\}_{i \in A \setminus \{k\}}$ we have that P^k is non empty. Furthermore, P^k is compact and therefore the optimization problem of agent k has a solution. For each $a \in T$ we have uniform prices in the graph G_a by a argument similar to that in step *(ii)* of the proof of Theorem 3.4. [13]

So, as in the case of Theorem 3.4, we find that an equilibrium in $\mathbb{E}$ exists, and, furthermore, that for each $a \in T$ this equilibrium has uniform prices on G_a.

As a direct consequence of this theorem we find the following corollaries on monopoly equivalence and on Walrasian equivalence. The corollary on monopoly equivalence

[12] Note that in this condition all information contained in ξ is used. However, the following stronger condition only uses that $S_1 = \{k\}$, thus making the result of the theorem more robust to changes in and misspecification of the echelon partition ξ. This alternative condition is
4'. $\forall a \in T$: G_a *is 2-connected* ~~or $\#A_a = 2$~~.

[13] In establishing that the solution of the optimization problem of agent k is an equilibrium, the minimal assumed aspiration levels as introduced by the functions δ_{ij} play a crucial role in ensuring that agent k does not anticipate to possibly improve by deviating form the solution of the optimization problem of the (price differentiating) monopolist. In Theorem 3.4 this was established through the existence of another agent in the set S_1 who sets prices which agent k takes as given.

states that if the hierarchical structure ensures uniform prices throughout the economy, then the top agent, $k \in S_1$, cannot engage in price differentiation and will optimally set a monopoly price for the market.

Corollary 4.3 [Monopoly Equivalence]
Let $\mathbf{E}$ *be a hierarchically structured economy with relationship structure* $\mathcal{G} := (A, R)$ *for which Assumption 2.5 holds. Let* $\mathcal{G}$ *be as in Theorem 4.2 with* $\#T = 1$ *and* $S_1 = \{k\}$. *Then* $\bar{p}$ *is a (uniform) equilibrium price in* $\mathbf{E}$ *if and only if it is a monopoly price for agent* k *for the market* $A \setminus \{k\}$.

This corollary is a direct consequence of Theorem 4.2 since $\sum_{i \in A \setminus \{k\}} \omega_i \gg 0$ by Assumption 2.5.

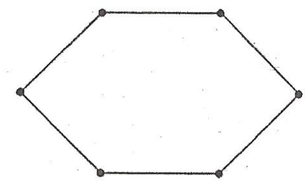
The following corollary is the direct analogue of Corollary 3.6. It states that if the only agent in the economy with real price setting power, i.e., agent $k \in S_1$, can not differentiate the prices he sets and has zero initial endowents, and therefore is not able to exploit his price setting power, then we have Walrasian equivalence. Thus, we do not need the "small agent assumption" to justify the price taking behavior of the agents in a Walrasian market.

Corollary 4.4 [Walrasian Equivalence II]
Let $\mathbf{E}$ *be a hierarchically structured economy with relationship structure* $\mathcal{G} := (A, R)$ *for which Assumption 2.5 holds. Let* $\mathcal{G}$ *be as in Theorem 4.2. Suppose that for* $k \in S_1 : \omega_k = 0$. *Then* p^* *is a uniform equilibrium price in* $\mathbf{E}$ *if and only if it is a Walrasian equilibrium price for the economy* $\mathbf{E}^W := \{U_i, \omega_i\}_{i \in A \setminus S_1}$. *Furthermore, the equilibrium allocation corresponding to* p^* *is the Walrasian equilibrium allocation corresponding to* p^*.

This corollary follows by applying the line of proof of Corollary 3.6 and using Corollary 4.3.

Corollary 4.4 illustrates that in exchange economies with utility functions representing neoclassical preferences, the Walrasian auctioneer can be interpreted as a non producing monopolist who cannot differentiate prices, and who has zero initial endowment. Since the set of prices P^k agent k can choose from contains Walrasian equilibrium prices only, his preference relation is irrelevant for the result.

Finally, we state the second main theorem of this section. It states that for any complete echelon partition of a relationship structure that is a 2-connected graph, an equilibrium in the induced hierarchically structured economy exists.



Theorem 4.5 [Existence Theorem II]
Let $\mathbf{E}$ *be a hierarchically structured economy with relationship structure* $\mathcal{G} := (A, R)$ *for which the utility functions are as in Assumption 2.5 and* $\exists \{i_1, i_2\} \in R : [\omega_{i_1} \gg 0$ *and* $\omega_{i_2} \gg 0]$. *Let* $\mathcal{G}$ *be a 2-connected graph* ~~*or #A = 2*~~. *Then an equilibrium in* $\mathbf{E}$ *exists. Furthermore, every equilibrium in* $\mathbf{E}$ *is a uniform price equilibrium.*

Proof
By the definition of a complete echelon partition, Definition 2.2, i_1 and i_2 can not be both of the highest hierarchical level, so for any complete echelon partition the condition $\sum_{i \in A \setminus S_1} \omega_i \gg 0$ of Assumption 2.5 holds. Since, by definition, $S_1 \neq \emptyset$, the theorem follows from applying Corollary 4.3 if $\#S_1 = 1$ and Theorem 4.2 otherwise.

Q.E.D.

5 Concluding Remarks

In this paper a model of a pure exchange economy with price setting agents and arbitrage is introduced. To achieve this, a hierarchical structure on the set of agents is used. The hierarchical structure describes between which pairs of agents trade relationships exist, and gives a partition of the set of agents in hierarchical levels. A trade relationship between two agents of different hierarchical levels is assumed to have the institutional characteristic of mono price setting, i.e., the agent of the higher hierarchical level behaves as a price setter, whereas the other agent behaves as a price taker with respect to this relationship. It is assumed that no trade relationships exist between agents of the same hierarchical level.

In this setting we have looked for equilibria which follow from the agents having anticipations found a recursive procedure, working from the bottom up, starting with the agents of the lowest hierarchical level.

For economies that have "sufficient" potential possibilities for arbitrage, we have proven theorems on the existence of equilibrium and we have found that in equilibrium, the agents of the highest hierarchical level may find themselves in a situation of two sided rationing. For specific hierarchical structures and assumptions on the initial endowments of particular agents, we have proven that monopolistic markets with and without price differentiation and the Walrasian market arise as special cases of our model. We find that, if sufficient potential possibilities for arbitrage exist, it suffices that the agents with real price setting power have zero initial endowments, in order to observe price taking behavior of all agents in the economy.

The model of this paper opens a range of interesting topics for further research.

First, one may want to incorporate production in the model of this paper. Interestingly, the possibility of having one price setting (and utility maximizing) firm and a number of price taking firms, opens the opportunity to model oligopolistic markets in a natural way in the context of our model. In the present version of the model this type of oligopoly does not emerge since we restricted attention to pure exchange economies.

Second, one may want to use different institutional characteristics of the trade relationships. In particular, institutional characteristics with price signals that allow only some of the commodities in the economy to be traded over certain relationships, and characteristics that do not use price signals seem to be interesting. They might turn out useful in modelling non-market institutions.

Finally, one may hope to, eventually, be able to model changes in the hierarchical structure, i.e., in the structure of trade relations and the partition of the set of agents in hierarchical levels, endogenously. Agents might negotiate over optimal hierarchical structures, taking into account the costs of maintaining relationships and allowing for side payments through the transfer of commodities.

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