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Endogenous Structures of Trade Relationships in Hierarchically Structured Economies with Bid and Ask Prices and Two Commodities

by

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Abstract

The purpose of this paper is to introduce a model of a pure exchange economy with two commodities in which the (fixed) structure of exchange relationships is endogenously determined.

The model starts from a structure of bilateral communication links between agents and a suitable partition of the set of agents in hierarchical levels. Each agent not of the highest hierarchical level chooses one agent from a higher hierarchical level with whom he has a communication link to transform this communication link into a trade relationship. The trade relationship is endowed with the institutional characteristic of bid and ask prices, i.e., the dominating agent sets (possibly different) prices for buying and selling for each commodity with respect to the trade relationship under consideration. The dominated agent takes these prices as given and determines the amounts to be traded over this trade relationship.

We formulate two games describing the choice of trade partners and consider Nash equilibrium in mixed strategies in the first, and subgame perfect equilibrium in very nice plays is the second game. We formulate theorems of the existence of equilibrium for both games.

JEL-Classification: L14, D40, C72.

1 Introduction

The problem of allocating scarce resources whilst facing unlimited needs is the basic problem of economics.

In 1776, Adam Smith suggested that for certain goods competitive markets allocate scarce resources to the well-being of all, an "invisible hand" coordinating the ruthless persuit of self interest by the participants in the market.

This notion of Smith's has been translated into formal models of competitive markets in, e.g., Walras (1874) and Debreu (1959). They model competitive markets as markets in which all trade takes place at a single vector of prices which holds for all market participants, and which is assumed by each of them not to be influenced by his trades on the market. As stated in the First Theorem on Welfare Economics, in economies with a complete system of interconnected competitive markets, every equilibrium allocation is Pareto efficient. Thus, in the context of these models, Pareto efficiency can be interpreted as describing the notion of "the well-being of all".

Having established this result and recognizing its limitations, a natural next question is whether one can think of more general organizational forms, not necessarily being markets, that lead to desired allocations, not necessarily according to the criterion of Pareto efficiency, for a wide class of different types of agents that may possibly be active in the economy.

In the first half of this century this question was the theme of the socialism debate between, a.o., Lange, v. Mises and v. Hayek. The debate arose over the question whether or not socialist economies are capable of realizing market allocations.

The formal counterpart of this discussion took place over the last two decades, focussing on the question if and how organizational forms, called mechanisms, can be found that realize arbitrary goals for a large class of different environments they may have to cope with. In answering this question, a complicating factor is that agents typically have private information concerning their own characteristics, e.g., concerning their initial endowments and preferences. Such agents may be tempted not to disclose this private information truthfully, hoping to benefit from misrepresenting their characteristics. Therefore, a mechanism must both persuade agents to, directly or indirectly, reveal the relevant information truthfully, and coordinate the actions of the agents as to realize given goals. Unfortunately, the results obtained in trying to answer this question are typically impossibility results, stating that it is impossible to find satisfactory organizational forms that perform those tasks. For a survey we refer to Groves and Ledyard (1987). Lately, emphasis has increasingly shifted to interpreting mechanisms as describing (hypothetical) institutional and organizational structures. Institutions and organizations that have been observed in past and present societies have been formally described as mechanisms, possibly with the hope to find a new angle at the problems described above.

One may want to interpret mechanisms as describing (the results of) the institutional structure of a society as a whole, this being in line with the spirit of the models of Walras and Debreu, and with the socialism debate. In this interpretation, a mechanism describes, implicitly or explicitly, laws, habits, norms of behaviour and the like, as present in a (hypothetical) society. Thus, a change in some of the laws of a society typically results in a different mechanism describing its institutional structure. Having this in mind and being aware of the inertia of habits and norms, it seems impossible to, in the short run, make substantial changes in the institutional structure of a society as to change its functioning from realizing one mechanism to realizing a different mechanism. Rather, except for some revolutions, changes in the institutional structure of a society are gradual changes, trying to repair some unwanted effects occuring in some part of the society. Unfortunately, in the present type of formal models of mechanisms it is next to impossible to describe such phenomena, the concepts only allow for "changes" from one mechanism to another as if a revolution would occur.

The aim of this research is the development of formal models that describe the institutional structure of a society consisting of a number of "partial" mechanisms. Partial mechanisms are to describe separate parts of the organizational form that function, in their formal rules, independent from each other. Agents, however, will typically participate in a number of partial mechanisms. Each such agent will coordinate his actions with respect to the different partial mechanisms to his best of interests, given his anticipations regarding the results of his actions on the outcomes of these partial mechanisms. Thus, the anticipations of the agents determine the way the partial mechanisms interact through the behaviour of the agents. Consequently, the anticipations of the agents, which in our models are determined endogenously, play a crucial role in "aggregating" the partial mechanisms to a mechanism describing the functioning of the society as a whole. In the present paper we introduce a model that endogenizes the institutional structure of an economy with two commodities,

making it a function of the actions of the agents.

In theories of industrial organization, some attention is paid to models in which the structure of the market is endogenized. This is, amongst others, the case in models on product differentiation as in Furth (1990), and in models on horizontal and vertical integration as in Krelle (1976) and, more recently, Emons (1991). In Gehrig (1990) the structure of networks of intermediaries is endogenized, whereas in Gehrig (1991) both the location of market places and the product differentiation in each market place is, to some extent, endogenized.

A model closer to endogenizing the institutional structure of an economy as we describe in this paper can be found in Gilles (1989, 1990, Chapter 5) and in Spanjers (1992, Chapter 4).

The model of Gilles (1989, 1990) has two main features. Firstly, it is intended to be a model of an economy with "official" trade relationships as well as "informal" black market activities. The trade relationships are modelled as price setting relationships. The black market activities are modelled as exchanges within coalitions of agents. The second main feature is that the structure of trade relationships in the economy is endogenously determined. As is argued in Spanjers (1992), the model has some weaknesses. Firstly, the problem of existence of equilibrium is not paid due consideration and is circumvented in an ad hoc manner. Secondly, both the interpretation of coalitions as representing black market activities, and the integration of these coalitions within the framework of the model are unsatisfactory.

In Gilles (1989, 1990), the setup of the model, with respect to endogenizing the structure of trade relationships, is along the following lines. The model starts from a finite economy with a given structure of communication links and a suitable partition of the set of agents in hierarchical levels, and ends, after a number of steps, with an equilibrium structure of official trade relationships with equilibrium prices, equilibrium net trades and an equilibrium allocation. Given the partition of the set of agents in hierarchical levels, and given the structure of communication links, each agent (except the agent of the highest hierarchical level) chooses a single agent of a higher hierarchical level with whom he has a communication link and establishes an official trade relationship with this agent. Any such trade relationship is assumed to be endowed with the trade rules of mono price setting, i.e., the agent of the higher hierarchical level sets a vector of prices with respect to this trade relationship, such that for every commodity the price for buying equals the price for selling. The dominated agent takes this vector of prices as given and determines the net trades over the trade relationship.

The aim of the present paper is to develop a model in which the structure of the trade relationships is endogenously determined, as is a tuple of equilibrium prices, equilibrium net trades and an equilibrium allocation.

In the first stage of the model, which is described by an economy with choice of trade partners, the structure of trade relationships emerges. This structure can be represented by a hierarchical tree. In the second stage of the model, the trade relationships are assumed to have the institutional characteristic of bid and ask prices. Thus, the second stage of the model is described by a hierarchically structured economy with bid and ask prices. Given, amongst others, the structure of trade relationships, an equilibrium is defined in the economy with bid and ask prices. It is assumed that the choice of trade partners in the first stage of the model is in line with what the agents anticipate to be the consequences of this choice in the resulting second stage of the model. Since we may have multiple equilibria in the economy with bid and ask prices, in the description of the economy with choice of trade partners, the agents are assumed to have subjective probability distributions concerning which of these equilibria arise. For the economy with choice of trade partners, we consider two equilibrium concepts. Firstly, in the normal form of the corresponding game, we look for Nash equilibrium in mixed strategies. Secondly, in the extensive form of the game associated to the economy with choice of trade partners where the agents higher in the hierarchy move before agents of a lower hierarchical level, we look for subgame perfect equilibrium in very nice plays. [1]

The two stage model in this paper is build along the lines of Gilles (1989, 1990). In order to solve the model, however, we make a number of significant modifications. Firstly, we do not allow for "black market activities" of coalitions. This implies that the ad hoc solution to the problem of existence of equilibrium in the second stage of the model no longer holds. Now, if we have the institutional characteristic of mono price setting in the second stage of the model, then an equilibrium may fail to exist, as is shown in Spanjers (1992, Example 6.3.1). As is shown in Spanjers (1994), the existence of equilibrium in the second stage of the model is guaranteed if we endow the trade relationships with the institutional characteristic of bid and ask prices and allow for only two commodities. The trade rules to the institutional characteristic of

[1] For a definition of very nice plays, see Hellwig and Leininger (1987).

bid and ask prices state that the dominating agent in a trade relationship sets two prices for each commodity, one price at which he buys, the bid price, and one at which he sells, the ask price. The dominated agent acts as a price taker with respect to this trade relationship. Finally, in the first stage of the model, concerning the choice which communication link to transform into a trade relationship, we consider Nash equilibrium in mixed strategies and subgame perfect equilibrium in very nice plays, wheras in Gilles (1989, 1990), only Nash equilibrium in pure strategies is considered.

The structure of the paper is as follows. In Section 2 we introduce a pre-economy, which is the starting point of our model. In Section 3 we describe a hierarchically structured economy with bid and ask prices and two commodities, which describes the second stage of our model. This section is to a large extent similar to Spanjers (1994, Section 2) and is included mainly to make the present paper self-contained. We also cite the theorem on existence of equilibrium from it. In Section 4, the first stage of our model is described by an economy with choice of trade partners. We state theorems on the existence of equilibrium for the two different equilibrium concepts. Finally, in Section 5 we make some concluding remarks and give some suggestions for further research.

2 A Pre-Economy

In this section we discuss the basics of the model. We define the concepts of a pre-economy and of a hierarchical tree.

A pre-economy consists of a set of agents, a set of bilateral relationships between agents, a partition of the set of agents in hierarchical levels and the individual characteristics of agents, being their von Neumann–Morgenstern utility functions and initial endowments. Thus, some of the basic concepts we need to build a model of an economy are present. Still, some other basic building blocks are absent. In the pre-economy no institutional structure is present that can be used to exchange commodities. Since we interpret the institutional structure of an economy as describing the possibilities to trade and cooperate, rather then the restrictions on the possibilities to trade and cooperate, not having specified an institutional structure implies the *absence* of possibilities to trade or cooperate. [2]

[2] Note that in, e.g., North (1990) the opposite position is taken and institutions are interpreted as restricting the possibilities to trade and cooperate. Thus, an institution free environment is considered to be the ideal case, since in such an environment every form of trade or cooperation is

In building the model, the communication links, which are going to be potential trade relationships, thus far have no meaning. Neither has the concept "potential trade relationship" as long as the rules of trade, formulated as the institutional characteristic, of the trade relationships are not specified. Once again, not having specified rules of trade is interpreted to mean that no rules of trade apply and therefore no trade can take place.

Similarly, the partition of the set of agents in hierarchical levels is, for the time being, without any economic content. It states that some kind of dominance relation is (implicitly) present in pre-economy, but as yet it does not have any economic meaning. In the model of an economy with choice of trade partners and in the model of a hierarchically structured economy with bid and ask prices, however, it is endowed with economic contents. In those derived models, it influences the sets of actions available to the individual agents, it determines the order in which the individual agents move, and, finally, the information the individual agents are assumed to have about the economy as a whole will depend upon their respective places in the hierarchy.

These considerations seem to justify using the term pre-economy instead of economy.

We start by introducing some terminology on graph theory, which we need to describe the set of communication links in the pre-economy.

A **(Simple) Undirected Graph** is a pair (A, R) consisting of a non-empty finite set of vertices and a set of edges $R \subset \{\{i,j\} \subset A \mid i \neq j\}$. A (simple) undirected graph (A, R) is **Connected** if there do not exist two (simple) undirected graphs (A_1, R_1) and (A_2, R_2) such that $A = A_1 \cup A_2$ and $R = R_1 \cup R_2$.

Definition 2.1 *A* **Relationship Structure** *is a connected, undirected, simple graph (A, R).*

Definition 2.2 *Let (A, R) be a relationship structure. The ordered partition $\xi := (S_1, \ldots, S_k)$ of the set A is an* **Echelon Parition** *of (A, R) if for each $i \in S_a$ with $a > 1$ there exists some $h \in S_b$ with $b < a$ and $\{i, h\} \in R$, and $\#S_1 = 1$.*

Let (A, R) be a relationship structure and let $\xi := (S_1, \ldots, S_k)$ be an echelon partition to (A, R). For $i, j \in A$ we denote $i \succ_\xi j$ if $i \in S_a$ and $j \in S_b$ with $a < b$, i.e., agent i is of a higher hierarchical level than agent j in the echelon partition ξ.

feasible.

Definition 2.3 *A* **Pre-Economy** *with l commodities is a tuple $\mathbf{E}_P := (((A, R), \xi), \{U_i, \omega_i\}_{i \in A})$ where:*

1. *(A, R) is a relationship structure.*
2. *ξ is an echelon partition of (A, R).*
3. *$U_i : \mathbb{R}^l_+ \to \mathbb{R}$ is the von Neumann–Morgenstern utility function of agent $i \in A$.*
4. *$\omega_i \in \mathbb{R}^l_+$ is the initial endowment of agent $i \in A$*

The first part of the definition of a pre-economy consists of a relational structure (A, R), where A describes the set of agents and R describes the set of (undirected) bilateral communication links between agents. The echelon partition ξ of the relationship structure (A, R) describes how the set of agents A is partitioned in hierarchical levels. It is defined to be such that each agent not in S_1 has a communication link with at least one agent of a higher hierarchical level. The echelon partition ξ is —in economies with choice of trade partners and in hierarchically structured economies with bid and ask prices derived from the pre-economy— used, amongst others, to describe the information of the agents about the other agents. The second part of the definition describes the individual characteristics of the agents in the pre-economy. Each agent is described by his von Neumann–Morgenstern utility function and his initial endowment. The von Neumann–Morgenstern utility function is such that in a situation with uncertainty the preference relation of the agent with respect to the induced lotteries is equivalently described by his expected utility with respect to this utility function. In our defintion l is the number of commodities in the economy, the set of commodities being $L := \{1, \ldots, l\}$.
We make the following assumption throughout this paper.

Assumption 2.4 *Let $\mathbf{E}_P = (((A, R), \xi), \{U_i, \omega_i\}_{i \in A})$ be a pre-economy. For each agent $i \in A$ the von Neumann–Morgenstern function U_i is continuous, strictly increasing and strictly quasi concave. Furthermore, $\forall i \in A \setminus S_1 : \omega_i \gg 0$. Finally, $l = 2$, i.e., there are only two commodities in $\mathbf{E}_P$.*

On the basis of a pre-economy we build a two stage model. The first stage of this model is described by an economy with choice of trade partners, and the second stage by a hierarchically structured economy with bid and ask prices and two commodities, which we discuss at length in Section 3.

In the first stage of the model, as described by an economy with choice of trade partners, the agents choose their trade partners. Each agent except the top agent chooses a single trade partner from the set of agents that are higher in the hierarchical structure and with whom he has a communication link. Thus, in this stage of the model some communication links are transformed, by the choice of trade partner by the agent, in trade relationships. It should be noted that an agent h being choosen by some agent i to transform their communication link into a trade relationship is obliged to accept this transformation. In the context of the model under consideration agent h can always choose a vector of bid and ask prices which enforces zero trades. Thus, he has no reason to oppose the transformation of a communication link into a trade relationship. [3]

We introduce the following terminology on graph theory to describe the situations that may arise as a consequence of the choices of trade partners as described above.

A **(Simple) Directed Graph** is a pair (A, D) consisting of a finite non-empty set of vertices A and a set of arcs $D \subset \{(i,j) \in A \times A \mid i \neq j\}$. The **Indegree** of a point $a \in A$ in a (simple) directed graph (A, D) is the number of ingoing arrows of the point a and is denoted as $\overleftarrow{\rho}(a) := \#\{(i,j) \in D \mid j = a\}$. A (simple) directed graph is **(Weakly) Connected** if it cannot be expressed as the union of two (simple) directed graphs, i.e., there do not exist two (simple) directed graphs (A_1, D_1) and (A_2, D_2) such that $A = A_1 \cup A_2$ and $D_1 \cup D_2$. A (simple) directed graph has a **Tree Structure** if it is (weakly) connected and $\#A - \#D = 1$.

Definition 2.5 *A* **Hierarchical Tree** *is a (weakly) connected, simple directed graph $T := (A, W)$ that has a tree structure and has exactly one agent $i \in A$ such that $\overleftarrow{\rho}(i) = 0$.*

Definition 2.6 *A directed graph $T := (A, W)$ is a* **Hierarchical Tree** to $((A, R), \xi)$ *if it is a hierarchical tree and for each $w := (i,j) \in W$ it holds that $\{i,j\} \in R$, and $i \prec_\xi j$ and for each $i \in A \setminus S_1$ we have $\#\{j \in A \mid (j,i) \in W\} = 1$. We use $T^{((A,R),\xi)}$ to denote the set of hierarchical trees to $((A, R), \xi)$.*

[3] Note that a leader may have reason to oppose the transformation of a communication link into a trade relationship if the trade relationship would be endowed with the institutional characteristic of mono price setting. In the case of mono price setting, the leader does not have a no-trade option, and he may be forced to unfavorable trades. See also Spanjers (1992, Example 6.3.1).

The following property states that to every tuple of choices of trade partners (be it randomized or not) as described above, a hierarchical tree of trade relationships emerges from the first stage of the model as described by an economy with choice of trade partners. Thus, in the second stage of the model, which is described by a hierarchically structured economy with bid and ask prices, it suffices to have existence of equilibrium for economies that have a hierarchical tree as their hierarchical structure.

Property 2.7 *Let E_P be a pre-economy. Let for each agent $i \in A \setminus S_1$ agent $h_i \in L_i$ be the agent with whom agent i establishes a trade relationship. Then the directed graph (A, W) where $W := \{(h_i, i) \in A \times A \mid i \in A \setminus S_1\}$ is a hierarchical tree to $((A, R), \xi)$.*

By definition of an echelon partition ξ to a relationship structure (A, R) we have that $\forall i \in A \setminus S_1 : L_i \neq \emptyset$ and $\#S_1 = 1$. Thus it follows (A, W) is a (weakly) connected directed graph. Since $\#W = \#A \setminus S_1 = \#A - 1$, and because for the top agent $k \in S_1$ we have $\overleftarrow{\rho}(k) = 0$, it follows that (A, W) is a hierarchical tree to $((A, R), \xi)$.

3 A Model of Actual Trade

In this section we define a hierarchically structured economy with bid and ask prices and two commodities. We describe such an economy by its hierarchical structure, by its agents and their individual characteristics, and by the institutional characteristics of the trade relationships. The hierarchical structure of the economy is described by a hierarchical tree, which describes between which agents bilateral asymmetric trade relationships exist and which agent dominates the other agent in such a trade relationship. Each agent is described by his utility function and his initial endowments. We assume that every trade relationship has the institutional characteristic of bid and ask prices. This means that for every asymmetric trade relationship the dominated agent sets two prices for each commodity. One price, the bid price, is the price at which he buys the commodity. The other price, the ask price, is the price at which he sells the commodity. The dominating agent has the obligation to buy and sell any amount the dominated agent wants to trade at the corresponding prices. Finally, we assume that the economy only has two commodities.

We start by introducing an additional concept we use to describe the hierarchically structured economy.

Definition 3.1 *Let (A, W) be a hierarchical tree. Let $w := (i, j) \in W$. The* **Institutional Characteristic** *of w is the correspondence $T_w : \tilde{X}_w \rightrightarrows \tilde{Y}_w$.*

The institutional characteristic T_w for a relationship $w \in W$ is interpreted as specifying for each signal $s \in \tilde{X}_w$, chosen by agent i, the set $T_w(s) \subset \tilde{Y}_w$ of actions agent j can choose from with respect to the relationship w.

Definition 3.2 *A* **Hierarchically Structured Economy** *with 2 commodities is a tuple $\mathbb{E} = ((A, W), \{U_i, \omega_i\}_{i\in A}, \{T_w\}_{w\in W})$, where:*

1. *(A, W) is a hierarchical tree.*
2. *$U_i : \mathbb{R}^2_+ \to \mathbb{R}$ is the utility function of agent $i \in A$.*
3. *$\omega_i \in \mathbb{R}^2_+$ is the initial endowment of agent $i \in A$.*
4. *$T_w : \tilde{X}_w \rightrightarrows \mathbb{R}^2$ is the institutional characteristic of trade relationship $w \in W$.*

As stated before, we only consider economies with the institutional characteristic of bid and ask prices.

In order to introduce the institutional characteristic of bid and ask prices, let S^3 be the 3-dimensional unit simplex defined by

$$S^3 := \{q \in \mathbb{R}^4_+ \mid \sum_{a=1}^{4} q_a = 1\}.$$

Define

$$P := \{q := (\underline{q}, \overline{q}) := (\underline{q}_1, \underline{q}_2, \overline{q}_1, \overline{q}_2) \in S^3 \mid 0 \le \underline{q} \le \overline{q}\}.$$

The institutional characteristic of **Bid and Ask Prices** is the correspondence $T^{bap} : P \rightrightarrows \mathbb{R}^l$ such that for each $p := (\underline{p}, \overline{p}) \in P$ we have

$$T^{bap}(p) := \{d \in \mathbb{R}^2 \mid \sum_{c=1}^{2} \underline{p}_c \cdot \min\{0, d_c\} + \sum_{c=1}^{2} \overline{p}_c \cdot \max\{0, d_c\} \le 0\}.$$

Now we are in a position to introduce the following definition.

Definition 3.3 *A* **Hierarchically Structured Economy with Bid and Ask Prices** *(with two commodities) is a hierarchically structured economy with two commodities, such that $\forall\, w \in W$ we have $T_w = T^{bap}$.*

For each agent $i \in A$ we use $L_i := \{h \in A \mid (h, i) \in W\}$ to denote the set of (direct) leaders of agent i, and we use $F_i := \{j \in A \mid i \in L_j\}$ to denote the set of (direct) followers of agent i. Since we have assumed that (A, W) is a hierarchical tree we have for one agent $k \in A$ that $L_k = \emptyset$. We refer to this agent as the **Top Agent** in the economy. We denote the set that has the top agent k as its only element by $S_1 := \{k\}$. For all other agents $i \in A \setminus S_1$ we have that $\#L_i = 1$. Finally, we use $L := \{1, 2\}$ to denote the set of commodities in $\mathbb{E}$.
We make the following assumption throughout this section.

Assumption 3.4 *Let $\mathbb{E}$ be a hierarchically structured economy with bid and ask prices. For each agent $i \in A$ the utility function U_i is continuous, strictly increasing and strictly quasi concave. Furthermore, $\forall\, i \in A \setminus S_1 : \omega_i \gg 0$.*

The next step in completing the model is to describe the choice spaces of the agents in the economy. We can, without loss of generality, restrict ourselves to a compact price space Q contained in P such that the vector of prices for selling is strictly larger than zero. Let $\hat{P} := \{(\underline{p}, \overline{p}) \in P \mid \overline{p} \gg 0\}$. Define the correspondence $\hat{B} : \hat{P} \times \mathbb{R}^2_+ \rightrightarrows \mathbb{R}^2_+$ such that $\forall\, (p, z) \in \hat{P} \times \mathbb{R}^2_+$, where p is a vector of bid and ask prices and z is a given commodity bundle, we have the set of attainable allocations:

$$\begin{aligned} \hat{B}(p, z) &:= \{y \in \mathbb{R}^2_+ \mid (y - z) \in T^{bap}(p)\} \\ &= \{y \in \mathbb{R}^2_+ \mid \sum_{c\in L} \underline{p}_c \cdot \min\{0, y_c - z_c\} + \sum_{c\in L} \overline{p}_c \cdot \max\{0, y_c - z_c\} \le 0\}. \end{aligned}$$

For each agent $i \in A$ the optimal trade function $x^*_i : \hat{P} \times \mathbb{R}^2_{++} \to \mathbb{R}^2_+$ is defined such that for every $p = (\underline{p}, \overline{p}) \in \hat{P}$ and $\omega_i \in \mathbb{R}^2_{++}$:

$$x^*_i(p, \omega_i) \in \operatorname{argmax}_{x_i \in \hat{B}(p,\omega_i)} U_i(x_i).$$ [4]

Define for $\varepsilon > 0$:

$$Q(\varepsilon) := \{(\underline{q}, \overline{q}) \in P \mid \overline{q} \ge \varepsilon \cdot 1_2\}.$$

We choose some $\varepsilon^* > 0$ such that for each $p \in \hat{P} \setminus Q(\varepsilon^*)$ we have

$$\exists\, q \in Q(\varepsilon) : x^*_i(q, \omega_i) = x^*_i(p, \omega_i)$$

[4]Note that by the properties of Assumption 3.4 we have that for each $p \in \hat{P}$ and $\omega_i \in \mathbb{R}^2_{++}$, the set $\operatorname{argmax}_{x_i \in \hat{B}_i(p,\omega_i)} U_i(x_i)$ contains only one element.

or

$$\forall i \in A \setminus S_1 : \exists c \in L : x^*_{ic}(p, \omega_i) > \omega_c,$$

where $\omega := \sum_{i \in A} \omega_i$. Such an ε^* exists because the initial endowments of the agents in $A \setminus S_1$ are strictly positive and the utility functions are strictly monotonic on $\mathbb{R}^2_{++}$. Clearly, the set $Q(\varepsilon^*)$ is compact.
Denote:

$$Q := Q(\varepsilon^*).$$

$$\tilde{\omega} := (\tilde{\omega}_c)_{c \in L}, \quad \text{where} \quad \tilde{\omega}_c := \sum_{i \in A} \max_{q \in Q} x^*_{ic}(q, \omega).$$

$$Y := \{d \in \mathbb{R}^2 \mid -\tilde{\omega} \leq d \leq \tilde{\omega}\}.$$

$$Y_+ := Y \cap \mathbb{R}^2_+.$$

$$X_i := Y^{L_i} \times Q^{F_i}.$$

$$X := \prod_{i \in A} X_i.$$

We give the following definition of a trade-price-allocation tuple in a given hierarchically structured economy with bid and ask prices.

Definition 3.5 *A* **Trade-Price-Allocation Tuple** *in the economy* $\mathbb{E}$ *is a tuple* $(d, p, x) \in Y^W \times Q^W \times Y_+^A$ *where:*

1. $d_{ji} \in Y$ *is the vector of net trade on the trade relationship* $(i, j) \in W$. *We denote* $d_i := (d_{ih})_{h \in L_i}$.

2. $p_{ij} = (\underline{p}_{ij}, \overline{p}_{ij}) \in Q$ *is the price vector denoting the bid prices* $\overline{p}_{ij}$ *and ask prices* $\underline{p}_{ij}$ *charged on the trade relationship* $(i, j) \in W$. *We denote* $p_i := (p_{ij})_{j \in F_i}$.

3. $x_i \in Y_+$ *is the consumption bundle for agent* $i \in A$.

Agents assume that their actions do not influence the prices their leader sets for them. On the other hand, we assume that each agent correctly anticipates the net trade with each of his followers as a function of the prices he sets. We assume he takes into account the consequences of possible changes in the prices his followers

set for their followers etc. These anticipations may result from each agent having aggregated information about the part of the economy that is downstream from him and knowing nothing but the prices his leader sets for him about the rest of the economy. We may assume that the downstream part of the economy for each agent is perfectly transparent for this agent, or at least that he has the information that summarizes what net trades to anticipate as a function of the prices he may set for his followers. Thus, we assume the **Downstream Information Structure** to hold. The anticipations of the agents are described by their anticipated net trade correspondences.

For any agents $i \in A$ and $j \in F_i$ the anticipated net trade correspondence $t_{ij} : Q \rightrightarrows Y$ is defined. This correspondence describes for each vector $q \in Q$ of bid and ask prices agent i may set on the trade relationship $(i, j) \in W$ the set of net trades $t_{ij}(q) \subset Y$ agent i anticipates to have with agent j. The anticipated net trade correspondences are defined recursively, using the following process. We start with the agents $j \in A_0 := \{j \in A \mid F_j = \emptyset\}$, who do not have any followers. Clearly, since those agents have no followers we do not need to define any anticipated net trade correspondence for them. Then, given the anticipated net trade correspondences for some non-empty set of agents A_n with $n \in \mathbb{N}$, with their followers, if they have any followers, we derive the anticipated net trade correspondences for the of agents $C_{n+1} := \{i \in A \setminus A_n \mid F_i \subset A_n\}$. Then we define $A_{n+1} := A_n \cup C_{n+1}$ etc. We stop this procedure when we reach an n^* such that $A_{n^*} = A$. Since (A, W) is an hierarchical tree, such an n^* exists. The anticipated net trade correspondences are derived using the following definition.

Definition 3.6 *Let* $i \in A$ *with* $F_i \neq \emptyset$. *Let for each agent* $j \in F_i$ *the tuple of anticipated net trade correspondences* $(t_{jm})_{m \in F_j}$ *be given.*[5] *The* **Anticipated Net Trade Correspondence** $t_{ij} : Q \rightrightarrows Y$ *of agent* i *with respect to agent* $j \in F_i$ *is given by:*

$$t_{ij}(q_{ij}) := \operatorname{argmax}_{e_{ji} \in T^{bop}(q_{ij}) \cap Y} \{ \max_{y_j \in \Psi_j(e_{ji})} U_j(y_j) \},$$

[5] If $F_j = \emptyset$, then the definition of the correspondence $\Psi_j : Y \rightrightarrows \mathbb{R}^2_+$ below reduces to:

$$\forall e_{ji} \in Y : \quad \Psi_j(e_{ji}) := \{y \in \mathbb{R}^2_+ \mid y \leq \omega_j + e_{ji}\}.$$

where $\Psi_j : Y \rightrightarrows \mathbb{R}^2_+$ *such that* $\forall e_{ij} \in Y$.

$$\Psi_j(e_{ji}) := \{y \in \mathbb{R}^2_+ \mid y \leq \omega_j + e_{ji} - \sum_{m \in F_j} e_{mj} \quad \text{where } \forall m \in F_j : \exists q_{jm} \in Q : e_{mj} \in t_{jm}(q_{jm})\}.$$

In the above definition agent $j \in F_i$ is assumed by agent i to be "optimistic" with respect to the reactions of his followers $m \in F_j$, i.e., it is assumed by agent i that a follower $m \in F_j$ of agent j, when he is indifferent between two actions at prices as set by agent j, chooses those actions that are the best for agent j. This implies that certain coordination problems between agent j and his followers are solved to the benefit of agent j. [6]

Given the anticipated net trade correspondences we define the correspondences that describe the actions an individual agent anticipates to be feasible for him. We refer to these correspondences as the feasible actions correspondences of the agents in the economy.

Definition 3.7 *The* **Feasible Actions Correspondence** $B_i : Q \rightrightarrows X_i \times Y_+$ *of agent* $i \in A \setminus S_1$ *with* $L_i = \{h\}$ *is such that* $\forall p_{hi} \in Q$

$$B_i(p_{hi}) := \{(e_{ih}, q_i, y_i) \in X_i \times Y_+ \mid e_{ih} \in T^{bap}(p_{hi}) \quad y_i \leq \omega_i + e_{ih} - \sum_{j \in F_i} e_{ji} \quad \text{where } \forall j \in F_i : e_{ji} \in t_{ij}(q_{ij})\}.$$

The feasible actions set B_k *for the top agent* $k \in S_1$ *is given by*

$$B_k = \{(q_k, y_k) \in X_k \times Y_+ \mid y_k \leq \omega_k - \sum_{j \in F_k} e_{jk} \quad \text{where } e_{jk} \in t_{kj}(q_{kj})\}.$$

The optimization problem of agent $i \in A \setminus S_1$ who has agent $h \in L_i$ as his direct leader is to maximize his utility over his feasible actions set. Here his feasible actions

[6] See also Spanjers (1994).

set depends on the prices p_{hi} his leader sets for him. Therefore agent i solves the following optimization problem

$$\max_{(e_{ih}, q_i, y_i) \in B_i(p_{hi})} U_i(y_i).$$

The optimization problem of the top agent $k \in S_1$ in the economy is given by

$$\max_{(q_k, y_k) \in B_k} U_k(y_k).$$

Now that we have described the individual optimization problem for each agent $i \in A$ we define an equilibrium in the hierarchically structured economy.

Definition 3.8 *Let* $\mathbf{E}$ *be a hierarchically structured economy with bid and ask prices. A tuple* $(d^*, p^*, x^*) \in X \times Y_+$ *is an* **Equilibrium** *in* $\mathbf{E}$ *if* $\forall i \in A \setminus S_1$ *such that* $\exists h \in A$ *with* $L_i = \{h\}$:

1. $(d_i^*, p_i^*, x_i^*) \in \text{argmax}_{(e_i, q_i, y_i) \in B_i(p_{hi}^*)} U_i(y_i)$
2. $x_i^* \leq \omega_i + d_{ih}^* - \sum_{j \in F_i} d_{ji}^*$

and for the top agent $k \in S_1$ *we have that:*

1. $(p_k^*, x_k^*) \in \text{argmax}_{(q_k, y_k) \in B_k} U_k(y_k)$
2. $x_k^* \leq \omega_k - \sum_{j \in F_k} d_{jk}^*$.

The above definition of equilibrium amounts to considering subgame perfect equilibria in very nice plays in the game corresponding to the hierarchically structured economy as described in this section. For a definition of very nice plays in the context of subgame perfect equilibrium, we refer to Hellwig and Leininger (1987).

As noted in Section 2, the hierarchical tree of the economy results from an economy with choice of trade partners, which describes the first stage of the model. For any tuple of pre-economy and hierarchical tree, we can define the resulting hierarchically structured economy.

Definition 3.9 *Let* $\mathbf{E}_P = (((A, R), \xi), \{U_i, \omega_i\}_{i \in A})$ *be a pre-economy. Let* $(A, W) \in T^{((A,R),\xi)}$. *Let* $\mathbf{E} := ((A, W), \{U_i, \omega_i\}_{i \in A}, \{T^{bap}\}_{w \in W})$ *be the hierarchically structured economy with bid and ask prices that results from* $\mathbf{E}_P$ *and* (A, W). *We use* $X^*_{(A,W)}$ *to denote the set of equilibrium allocations in* $\mathbf{E}$.

The following theorem states that for any hierarchical tree that may arise from the economy with choice of trade partners in the first stage of the model, an equilibrium in the second stage of the model exists.

Theorem 3.10 [Spanjers (1994, Theorem 3.5)]
Let $\mathbf{E}_P = (((A,R),\xi),\{U_i,\omega_i\}_{i\in A})$ *be a pre-economy for which Assumption 2.4 holds. Then it holds for each* $(A,W) \in T^{((A,R),\xi)}$ *that* $X^{*}_{(A,W)} \neq \emptyset$.

4 The Choice of Trade Partners

In this section we formalize the first stage of our model as an economy with choice of trade partners. Consider a pre-economy $\mathbf{E}_P$. In the fist stage of the model, each agent except the top agent chooses an agent of a higher hierarchical level with whom he has a communication link, to institutionalize this communication link as a trade relationship. This trade relationship is then endowed with the institutional characteristic of bid and ask prices. Since each agent from $A \setminus S_1$ turns exactly one of his communication links into a trade relationship, we end up with a (directed) graph of trade relationships which has a tree structure, and the number of trade relationships is the minimal number that is needed for this graph to be (weakly) connected.

An economy with choice of trade partners consists of two components. The first component is a pre-economy which describes its basic structure. The second component (indirectly) describes the (expected) payoffs the agents assign to the consequences of their combined choices. We understand the economy with choice of trade partners to be such that a hierarchical tree of trade relationships results. Furthermore, we understand the combination of this hierarchical tree and the individual characteristics of the agents to characterize a hierarchically structured economy with bid and ask prices as described in the previous section. Since the agents are described by their von Neumann–Morgenstern utility functions, to describe the payoffs the agents assign to the consequences of their combined choices, it suffices to describe the subjective probability distributions of the individual agents over the sets of equilibria in the hierarchically structured economies with bid and ask prices to the various hierarchical trees in $T^{((A,R),\xi)}$.

Definition 4.1 *Let* $\mathbf{E}_P := (((A,R),\xi),\{U_i,\omega_i\}_{i\in A})$ *be a pre-economy. Let* $T^{((A,R),\xi)}$ *be the set of hierarchical trees to* $\mathbf{E}_P$. *The* **Economy with Choice of Trade Partners** *to the pre-economy* $\mathbf{E}_P$ *is the tuple* $\mathbf{E}_C := (\mathbf{E}_P,\{\{\mathcal{F}^i_{(A,W)}\}_{(A,W)\in T^{((A,R),\xi)}}\}_{i\in A})$ *where for each* $i \in A$ *and* $(A,W) \in T^{((A,R),\xi)}$ *we have that* $\mathcal{F}^i_{(A,W)}$ *is a probability distribution over the set* $X^{*}_{(A,W)}$.

In this definition the probability distribution $\mathcal{F}^i_{(A,W)}$ is interpreted as the subjective probability distribution of agent i with respect to the actual occurence of the equilibrium allocations in $X^{*}_{(A,W)}$ in the hierarchically structured economy to $\mathbf{E}_P$ that has (A,W) as its hierarchical tree. The induced expected utility function of an agent gives for each hierarchical tree $(A,W) \in T^{((A,R),\xi)}$ the expected utility of the agent, as it follows from his subjective probability distribution over the set of equilibria in the hierarchically structured economy with bid and ask prices that results from the hierarchical tree $(A,W) \in T^{((A,R),\xi)}$ and his von Neumann–Morgenstern utility function as described by the pre-economy.

Definition 4.2 *Let* $\mathbf{E}_C := (\mathbf{E}_P,\{\{\mathcal{F}^i_{(A,W)}\}_{(A,W)\in T^{((A,R),\xi)}}\}_{i\in A})$ *be an economy with choice of trade partners. The function* $\hat{U}_i : T^{((A,R),\xi)} \to \mathbb{R}$ *is the* **Induced Expected Utility Function** *of agent* i *over the set* $T^{((A,R),\xi)}$ *of hierarchical trees to* $\mathbf{E}_P$, *where* $\forall (A,W) \in T^{((A,R),\xi)}$:

$$\hat{U}_i((A,W)) := \int_{X^{*}_{(A,W)}} U_i(\mathrm{proj}_i(x))\, d\mathcal{F}^i_{(A,W)}(x).$$

The induced expected utility function gives the payoffs of the agent over all combinations of choices of trade partners by the agents in $A \setminus S_1$.

In the next two subsections we introduce two different equilibrium concepts for the economy with choice of trade partners. In the first subsection, we consider Nash equilibrium in mixed strategies in a normal form game corresponding to the economy with choice of trade partners.

It should be noted that in the resulting hierarchically structured economy with bid and ask prices, which describes the second stage of the model, uncertainty no longer plays a role. The structure of trade relationships is given, as it results form the realization of the random variables that characterize the mixed strategies of the agents in the case of Nash equilibrium with mixed strategies, or as the realization of the random variable of the agent of the highest but one level in the hierarchy in case he is indifferent between actions he has to break ties over.

The subjective probabilities of the agents with respect to the occurence of one of the possibly many equilibria are assumed to no longer play a role in the resulting hierarchically structured economy with bid and ask prices. [7]

4.1 Nash Equilibrium in Mixed Strategies

In this subsection we consider equilibrium in the economy with choice of trade partners that is equivalent to Nash equilibrium in mixed strategies in the normal form game corresponding to the economy. The advantage from considering this equilibrium compared to Nash equilibrium in pure strategies is that equilibrium no longer fails to exist.

Definition 4.3 *Let* $\mathbf{E}_C$ *be an economy with choice of trade partners. The* **Set of Mixed Strategies** *of agent* $i \in A \setminus S_1$ *is*

$$C_i := \{r \in \mathbb{R}_+^{\hat{C}_i} \mid \sum_{h \in \hat{C}_i} r_h = 1\},$$

where

$$\hat{C}_i := \{h \in A \mid \{i,h\} \subset R \text{ and } h \succ_\xi i\}.$$

We denote $C := \prod_{i \in A \setminus S_1} C_i$ *to be the set of mixed strategies in the economy* $\mathbf{E}_C$.

The choices of mixed strategies by the agents in $A \setminus S_1$ leads to a vector of probabilities with which the hierarchical trees in $\mathcal{T}^{((A,R),\xi)}$ may occur. This is formalized by the function of probabilities in the following definition.

Definition 4.4 *Let* $\mathbf{E}_C$ *be an economy with choice of trade partners. Let* C *be the set of mixed strategies in the economy* $\mathbf{E}_C$. *The* **Set of Probabilities over** $\mathcal{T}^{((A,R),\xi)}$, *is given by*

$$D := \{r \in \mathbb{R}_+^{\mathcal{T}^{((A,R),\xi)}} \mid \sum_{(A,W) \in \mathcal{T}^{((A,R),\xi)}} r_{(A,W)} = 1\}.$$

[7] This assumption can be justified by assuming that the equilibrium is obtained through a dynamic exchange process as described in Spanjers (1994, Section 5). Attempts of agents to establish the equilibrium that they prefer by threatening to deviate from the equilibrium actions if any equilibrium different from this is played are not credible within the framework of this process, as long as it is recognized that deviating from the net-trades proposed by the leader leads to a breakdown of the trade process with a positive probability.

The **Function of Probabilities over** $\mathcal{T}^{((A,R),\xi)}$ *as a function of the actions of the agents in* $A \setminus S_1$ *is the function* $f : C \to D$ *such that* $\forall r := ((r_{hi})_{h \in L_i})_{i \in A \setminus S_1} \in C$ *we have* $f(r) := (f_{(A,W)}(r))_{(A,W) \in \mathcal{T}^{((A,R),\xi)}}$ *where*

$$f_{(A,W)}(r) := \prod_{(h,i) \in W} r_{hi}.$$

For each vector of probabilities over $\mathcal{T}^{((A,R),\xi)}$ we can, under Assumption 2.4, define the corresponding payoffs of the agents. This is achieved through the payoff functions in the following definition.

Definition 4.5 *Let* $\mathbf{E}_C := (\mathbf{E}_P, \{\{\mathcal{F}^i_{(A,W)}\}_{(A,W) \in \mathcal{T}^{((A,R),\xi)}}\}_{i \in A})$ *be an economy with choice of trade partners such that Assumption 2.4 holds for* $\mathbf{E}_P$. *Let* D *be the set of probabilities over the set of hierarchical trees. The* **Payoff Function** *of agent* $i \in A \setminus S_1$ *is a function* $V_i : D \to \mathbb{R}_+$ *where for any* $f \in D$:

$$V_i(f) := \sum_{(A,W) \in \mathcal{T}^{((A,R),\xi)}} f_{(A,W)} \cdot \hat{U}_i((A,W)).$$

Now that we have defined the sets from which the agents can choose their actions, the way these choices interact, and what resulting payoffs for the agents are, we can define an equilibrium for the economy with choice of trade partners. The definition is yields the, randomly determined, hierarchical tree for the hierarchically structured economy with bid and ask prices that describes second stage of our model.

Definition 4.6 *Let* $\mathbf{E}_C$ *be an economy with choice of trade partners. Let for each* $i \in A \setminus S_1$ *the function* V_i *be his payoff function. An* **Equilibrium** *in* $\mathbf{E}_C$ *is a vector* $r^* \in C$ *such that for each* $i \in A \setminus S_1$ *it holds that* $\nexists r_i \in C_i$ *such that:*

$$V_i(f(r^*_{-i}; r_i)) > V_i(f(r^*)).$$

Our definition of equilibrium is that of Nash equilibrium in mixed strategies in the normal form game corresponding to the economy with choice of trade partners. The following theorem holds.

Theorem 4.7 [Existence of Equilibrium]
Let $\mathbf{E}_C := (\mathbf{E}_P, \{\{\mathcal{F}^i_{(A,W)}\}_{(A,W) \in \mathcal{T}^{((A,R),\xi)}}\}_{i \in A})$ *be an economy with choice of trade partners. Such that Assumption 2.4 holds for* $\mathbf{E}_P$. *Then an equilibrium in the economy* $\mathbf{E}_C$ *exists.*

The Proof of this Theorem is along the lines of Nash (1950).

4.2 Subgame Perfect Equilibrium in Very Nice Plays

In this subsection, an alternative to the equilibrium concept discussed in the previous subsection is considered. A weakness of the equilibrium concept of Nash equilibrium in mixed strategies in the context of our model is that, strictly speaking, the outcome of the equilibrium actions is not a hierarchical tree but a random variable over the set $T^{((A,R),\xi)}$. Some of the trees occuring with positive probability may be occurring only because of the failures in coordination that are characteristic for (non degenerated) mixed strategies. In that case, however, it is hard to argue that the structure of trade relationships is going to be maintained, once it has resulted from the first stage of the model. Typically, one would expect some change in the hierarchical tree to occur to correct some of the unfortunate realizations of the random variable.

To circumvent this problem, we consider subgame perfect equilibrium in very nice plays in a game in extended form associated to the economy with choice of trade partners $\mathbb{E}_C$. This equilibrium concept has two characteristics that serve our purposes well. Firstly, we only have to consider equilibrium in pure strategies, since an equilibrium in pure strategies is shown to exist under certain restrictions. Second, by definition of very nice plays, if an agent is indifferent between a number of actions, he chooses (from these actions) the action that suits the last player that moved before him best. If this player is also indifferent, then he chooses the action that suits the player that moved two moves befor him best etc. Thus, no problems occur from a player choosing the "wrong" action of the actions between which he is indifferent, as far as the choices of the players later in the game tree affect those moving earlier. The other way around, however, need not be the case. Still, the problems related to the unfortunate realization of a random variable are reduced significantly, and may only occur in the case of multiple equilibria.

We start by defining a suitable game in extensive form to the economy with choice of trade partners. In this game, the agent of the highest hierarchical level in the economy does not appear as a player. As a technical restriction, we assume each hierarchical level in the economy with choice of trade partners to contain only one agent. Then, somewhat arbitrarily, we assume the agent of the highest hierarchical level but one to move first, the agent of the highest hierarchical level but two to move second and so on. This leads to the following game in extended form associated to the economy with choice of trade partners $\mathbb{E}_C$.

Definition 4.8 *Let $\mathbb{E}_C := (\mathbb{E}_P, \{\{\mathcal{F}^i_{(A,W)}\}_{(A,W)\in T^{((A,R),\xi)}}\}_{i\in A})$ be an economy with choice of trade partners where $\xi := (S_1, \ldots, S_k)$ with $\forall c \in \{1, \ldots, k\} : \#S_c = 1$. The* **Game $\Gamma^{\mathbb{E}_C}$ associated to $\mathbb{E}_C$** *is $\Gamma^{\mathbb{E}_C} := (\tilde{N}, (\tilde{I}, \tilde{D}), \tilde{S}, (\tilde{H}_n)_{n\in\tilde{N}})$ where:*

1. $\tilde{N} := A \setminus S_1$
2. $\tilde{I} := \{0\} \cup \bigcup_{j=2}^{k} \{(h_c)_{c=1}^{j-1} \in \prod_{c=1}^{j-1} A \mid \forall c \in \{1, \ldots, j-1\} : \exists i \in S_{c+1} : [\{i, h_c\} \in R \wedge h_c \succ_\xi i]\}$
3. $\tilde{D} := \{(0,k) \in \{0\} \times S_1\} \bigcup_{j=2}^{k-1} \{((h_c)_{c=2}^{j-1}; h) \in \tilde{I} \times A \mid i \in S_j \wedge [\{i, h\} \in R \wedge h \succ_\xi i]\}$
4. *$\tilde{S}$ is the set of functions $s : \tilde{I} \to A$ such that $\forall j \in \{2, \ldots, k-1\} : \forall (h_c)_{c=1}^{j-1} \in \tilde{I}$: $s((h_c)_{c=1}^{j-1}) := \{h \in L_i \mid i \in S_j \wedge h \succ_\xi i\}$.*
5. *$\forall i \in \tilde{N}$ we have $H_i : \tilde{I} \to \mathbb{R}$ such that $\forall (h_c)_{c=1}^{k-1} \in \tilde{I}$: $H_i((h_c)_{c=1}^{k-1}) := \hat{U}_n((A, D((h_c)_{c=1}^{k-1}))$ where $D((h_c)_{c=1}^{k-1}) := \{(h_c, m) \in A \times A \mid m \in S_{c+1}\}$*

In the above definition of the game $\Gamma^{\mathbb{E}_C}$ we have that $\tilde{N}$ denotes the set of players, the tuple $(\tilde{I}, \tilde{D})$ is the game tree, where $\tilde{I}$ is the class of information sets and $\tilde{D}$ is the set of moves. The set $\tilde{S}$ is the set of (pure) strategies, such that for each information set we have one move of the player that is to move at this information set. Finally, for each player $i \in \tilde{N}$ we have that H_i is the payoff function which is defined over the terminal nodes of the game tree.

Definition 4.9 *Let $\mathbb{E}_C$ be an economy with choice of trade partners and $\Gamma^{\mathbb{E}_C}$ the game associated with $\mathbb{E}_C$. An* **Equilibrium** *in an economy with choice of trade partners $\mathbb{E}_C$ is defined to be a subgame perfect equilibrium in very nice plays in the game $\Gamma^{\mathbb{E}_C}$.*

For a definition of very nice plays we refer to Hellwig and Leininger (1987).

Theorem 4.10 *Let $\mathbb{E}_C$ be an economy with choice of trade partners and $\Gamma^{\mathbb{E}_C}$ the game associated with $\mathbb{E}_C$. Then there exists an equilibrium in $\mathbb{E}_C$.*

This theorem can be proven by applying backward induction and using that the choice set of each agent is compact.

In the case of multiple equilibria, we have by the definition of the equilibrium concept that the agent of the highest hierarchical level but one in the economy with choice of trade partners is indifferent between which of the equilibria occurs. Since, however, we need a unique hierarchical tree for the hierarchically structured economy with bid and ask prices we may assume that the agent chooses one of these equilibria according to some (possibly degenerate) random variable. Thus, we have a unique hierarchical tree resulting from this stage of the economy.

5 Concluding Remarks

In this paper a restrictive model is introduced in which (a part of) the institutional structure of an economy, viz. the structure of trade relationships, is endogenized. The main problem with the model is the difficulty to prove the existence of equilibrium in the hierarchically structured economy that results from the choice of trade partners. Clearly, if we would be able to prove the existence of equilibrium in a broad class of hierarchically structured economies, we would be able to use the basics of the model to endogenize not only the structure of trade relationships, but also the institutional characteristics of the trade relationships. We could have each agent determining which communication link(s) he wants to turn into trade relationships for which he determines the institutional characteristics from the class of institutional characteristics for which an equilibrium exists. Once again, as long as the dominating agent has the possibility to enforce zero trades over the trade relationship he does not have a reason to oppose such choices. Such a model could be developed further if different institutional characteristics of the trade relationships lead to different costs of operating this partial mechanism, which are paid for by the dominated agent in the trade relationship. Considerations concerning the trade off between the results a mechanism establishes and its costs of operation can be found in, e.g., Hurwicz (1972) and in Karmann (1981). There a model with local competitive markets and individualized transportation costs is considered, which can be interpreted as describing the user costs of operating the local competitive markets. More recently, Gilles, Diamantaras and Ruys (1994) consider the costs of trade infra-structures in large economies.

If existence of equilibrium is also guaranteed for a class of hierarchical structures that contains structures of trade relationships that are not hierarchical trees,

this opens yet another possibility. We could have the agents choosing any number of communication links with agents of a higher hierarchical level to become trade relationships, where the dominated agent has to pay the costs of maintaining the trade relationships. Thus, agents face a trade off between more trade relationships, and thus more potential options to trade, and higher costs of maintaining the trade relationships.

Having this in mind, it seems that trying to solve the problem of existence of equilibrium in the second stage of the model for a large class of institutional characteristics and a large class of hierarchical structures is a most interesting and most pressing theme for further research.

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