THEORY AND APPLICATION OF RENEGOTIATION IN REPEATED GAMES

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Meiner Familie.

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Contents

CHAPTER 1

Introduction

Whenever economic agents interact in such a way that the choice of one agent affects the outcome of other agents, and all agents anticipate the responses to their actions, this can be modeled as a *game*. The interacting agents are the *players* of the game, and the mathematical study of the logical and strategic structures of these interdependent interactions is called *game theory*. The first formal game-theoretic analyses of such interactions date back to the early 19th century, when Cournot (1838) developed his thoughts on the theory of competition in a duopoly, but elements of game theory may have first appeared in ancient times.[1] The mathematically founded theory was introduced in the seminal work of Von Neumann and Morgenstern (1944), and game theory has since been used to study a wide variety of social, political, and economic questions, such as in the analysis of competition between firms, the study of cooperation in free-riding problems, and the evolution of populations.

The first game-theoretic models were only applicable in very limited settings, but both the conceptual and mathematical frameworks have since been extended in many directions. This doctoral thesis aims to contribute to extending the literature and is motivated by the simple and most famous of all games: the *Prisoner's Dilemma*.[2]

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[1] A thorough overview of the history of game theory can be found, for instance, in Ross (2016).

[2] The game it describes was first introduced and studied by Merrill Flood and Melvin Dresher in 1950, but they did not refer to it as a *Prisoner's Dilemma*. The famous title was given by

There are many interpretations of this game, but they mostly refer to the following situation. Suppose that the police have arrested two suspects who have committed a crime together. The district attorney interrogates the two separately but lacks the evidence to convict either. Each suspect has two options in the interrogation. He or she can either *defect* and confess the crime, thereby implicating the other, or the two can *cooperate* with each other and stay silent. If both keep silent, the prosecutor can only impose a mild sentence. If one of the suspects keeps silent, the other can improve his or her situation by confessing, and therefore receive favorable treatment. If one suspect anticipates that the other will confess, then the first suspect should also confess to reduce his or her own expected sentence. Thus, regardless of the other player's decision, a suspect can and will always improve his or her own position by confessing to the crime. Consequently, if both players are aware of this structure, they will both confess and, thereby, follow their private interests, even though the outcome is worse for both than if they stay silent.

In game-theoretic terminology, the outcome or solution of a game is called an *equilibrium*, referring to the stable or balanced state of a game. In the Prisoner's Dilemma, the state described above is such that no player can improve his or her respective outcome by changing his or her own decision, given that the decision of the other player remains as it is. This particular type of equilibrium is referred to as a *Nash equilibrium*, as it was introduced by Nobel Laureate John F. Nash Jr. (1950, 1951).

In many situations, players not only meet once but may interact repeatedly. The structures of these situations may stay the same over the duration of an entire interaction; therefore, it is a natural extension to study games that are played repeatedly. In contrast to single-stage games, where only one period of interaction is considered, repeated games may be played over a finite or infinite horizon, thus enlarging the set of decisions to be made. A strategy in such a repeated game can be interpreted to be a plan of action, which defines an action to be taken for every period of the repeated game, conditional on the actions that were taken by all players previously. Consequently, the set of possible outcomes (and, therefore, equilibria) of repeated games may be larger than if the game was only played once. Nash Jr. (1950, 1951) shows that every finitely repeated game has a Nash equilibrium, but there are other solutions of the game that can be characterized as stable outcomes. Moreover, the notion of a Nash equilibrium may not always be suitable to define a stable outcome, as it only regards the one-time decision for a complete plan of action and does not account for possible deviations at later points in the game.

Albert Tucker a little later, and the game has since been interpreted and applied ubiquitously (see Straffin, 1980, for more details on the history of the game).

Several other equilibrium notions exist for repeated games. The most frequently applied was introduced by Nobel Laureate Reinhard Selten (1965). He defined a *subgame perfect equilibrium* to be a Nash equilibrium not only in the repeated game as a whole, but in every *subgame* of the repeated game, where "subgame" refers to a part of the whole game that is played after a given history of decisions is established.

In the present thesis we are interested in infinitely repeated games and their stable outcomes. It has been shown by Fudenberg and Maskin (1986) that many outcomes of a single-stage game that are not a Nash equilibrium can be sustained as a subgame perfect equilibrium if the game is repeated over an infinite time horizon. In the Prisoner's Dilemma, for example, the outcome where both players do not confess but stay silent can also be sustained as a stable outcome. The underlying idea is that, to sustain a certain outcome as a subgame perfect equilibrium, players face the threat of punishment if they do not play according to the agreed-upon strategy. If the punishments are *credible*, players are deterred from single deviations and the strategy, thereby, sustains cooperative behavior in the repeated game.

For the infinitely repeated Prisoner's Dilemma game, mutual cooperation can be established as follows: The players cooperate as long as no one unilaterally deviates. After a single deviation, both players switch to defect for every subsequent period of the game. By the definition of the game, the mutual defection outcome is strictly worse than the mutual cooperation outcome and can, therefore, deter a player from deviating.[3]

The credibility of the threat of punishment is crucial to the success of the strategy. If, for example, a player would unilaterally and profitably deviate from the punishment, the agreed-upon strategy cannot be subgame perfect. This idea is the underlying credibility criterion used for subgame perfect equilibria: No single player should have the option to profitably deviate from the agreed-upon strategy in any subgame, thus the prescribed play in any subgame has to be a Nash equilibrium. The strategy discussed above regarding the Prisoner's Dilemma satisfies this condition.

In the late 1990s, however, a second thought on the credibility of punishments was sparked by several authors.[4] What if the prescribed punishment hurts both the previously deviating agent and the innocent players that are called upon to punish the deviation? Is it plausible that the players will stick to the punishment or, rather, *renegotiate* to a Pareto-improving outcome? In the strategy

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[3] The issue is slightly more complex than this, but this general idea suffices for the purposes of this introduction.

[4] For example Farrell and Maskin (1989), Bernheim and Ray (1989), Asheim (1997) discuss this issue; see also the review of literature given in Chapter 4.

already discussed for the repeated Prisoner's Dilemma, for example, the mutual defection outcome leaves both players worse off, and they could both improve their situations by ignoring the deviation and going back to mutual cooperation.

The literature on *renegotiation-proofness* studies these questions and has proposed several answers. We thoroughly review the existing literature in Chapter 4 of this thesis. One of the most frequently applied concepts is introduced in Farrell and Maskin (1989), where Eric Maskin is the third Nobel Laureate referred to in this introduction. In their seminal work, the authors define the credibility problem of subgame perfect punishments more precisely and argue that "when renegotiation is possible, players are unlikely to play, or to be deterred by, a proposed continuation equilibrium (whether on or off the equilibrium path) that is strictly Pareto-dominated by another equilibrium that they believe is available to them" (p. 328).

More generally, Farrell and Maskin (1989) argue that it is "inconsistent" (p. 328) if cooperation in a repeated game is sustained as a subgame perfect equilibrium by means of punishments that leave all players worse off than under cooperation, which is supposed to be the stable outcome of the game in the first place. In their view, such punishments are not credible, and a strategy that specifies such punishments cannot be a stable outcome of the repeated game. They therefore offer a refining equilibrium notion that they call *weak renegotiation-proofness.*

In the present thesis, we follow this train of thought and contribute to the literature on weakly renegotiation-proof equilibria. More generally speaking, this thesis contributes to our understanding of strategic interactions that go beyond the basic game-theoretic models and focuses on the characterization of stable outcomes in repeated multilateral interactions.

1.1 Contributions

Chapter 2 of this thesis is joint work with Tim Hellmann and is dedicated to an application of weakly renegotiation-proof equilibria in a real-world context. In this application, we consider a game in which the interacting agents are countries that produce externalities in the form of pollution that affects all countries. Any unilateral effort to reduce these externalities will be undermined by the others' incentive to free-ride on this effort. To overcome this dilemma, countries may strive for an outcome or agreement that fully internalizes the pollution effects. In this context, such agreements are called International Environmental Agreements (IEAs), which have been the subject of many studies in the recent past.[5]

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[5] For a good overview of the literature, we refer to Jørgensen et al. (2010), Benchekroun and Long (2012) and Hovi et al. (2015).

Several IEAs have been formed over the past decades and focus on varying issues with respect to pollution. The Kyoto Protocol of 1997, for instance, specifies a reduction of greenhouse gases and was agreed upon by 190 countries. All IEAs have in common that there is no global agency that can legally enforce an agreement; consequently, they need to be *self-enforcing.* To study and characterize self-enforcing IEAs in game-theoretic frameworks, the established literature predominantly uses the notion of Farrell and Maskin (1989).

Most existing game-theoretic models in the literature on IEAs assume that pollution has a global effect. However, many forms of pollution have additional negative effects on countries within the same region as the polluting source. The presence of these local spillovers hence plays a non-negligible role in the problem of establishing IEAs. In Chapter 2, we therefore ask which IEAs may be implemented if the negative externalities of pollution have both a local and a global component.

The local effects of pollution are represented by a network, where a link between two countries indicates whether one country's pollution locally affects the other. This could represent a common border or mean that the countries are within a certain geographic distance that is critical to the local externality. To study the stability of IEAs, we use a repeated game approach, where an IEA is interpreted as a strategy that coordinates the abatement efforts of its members to maximize joint utility. Formally, we define an IEA as stable if it is a weakly renegotiation-proof equilibrium.

Our main contribution is that we characterize necessary and sufficient conditions for the stability of an IEA when pollution has both a global and a local effect. We find that stable IEAs exist if the network structure is balanced. Too-large asymmetries in the degree of local spillovers may, however, lead to the non-existence of stable structures. We also discuss the implications of our results for welfare. The generality of our approach allows for several applications, in particular regarding the provision of public goods.

In the model of Chapter 2 we restrict our attention to strategies that are simple in terms of implementation in order to sustain full cooperation. In other settings, this may be more difficult or other payoffs may need to be sustained. For two-player games, Farrell and Maskin (1989) present in their Theorem 1 (p. 332) necessary and sufficient conditions for weakly renegotiation-proof payoffs. Given a strictly individual rational payoff and two action pairs that can be used for punishment, the authors construct specific sequences of actions to obtain the equilibrium payoff, and they also construct renegotiation-proof punishments. Yet, the proof of the sufficient conditions is not completely correct.

Chapter 3 is, therefore, dedicated to this proof. At first, we precisely identify the erroneous claim by Farrell and Maskin (1989) and offer a counterexample that illustrates the problem. In a nutshell, the authors assume more structure on the set of payoffs in two-player games than actually exists, and they do not, therefore, correctly distinguish between convexification and mixing of strategies. More precisely, in the construction of the sequence of actions that yields the equilibrium payoff, they claim to obtain a payoff with independent randomization that is only obtainable with correlated strategies, which they exclude from their model. Nevertheless, we provide an alternative result that yields a different sequence of actions to obtain the equilibrium payoff.

Given that the hypotheses of the original theorem are satisfied, we prove that for every strictly individual rational payoff in a two-player game, one can find two action pairs with the following properties: a convex combination of their respective payoffs yields the payoff in question, and the line segment between those two payoffs is of non-positive slope. These actions can be used for the sequence of actions that yields the equilibrium payoff, and no continuation payoff along this sequence can be Pareto-ranked.

To satisfy subgame perfection, one also needs to define punishment strategies that deter any unilateral deviation from the equilibrium path. Due to our alternative construction of the equilibrium payoff, we also need to modify the construction of punishment strategies compared to the one proposed in Farrell and Maskin (1989). While we also make use of the two action pairs that are given by the hypotheses of the theorem, we need to adjust the punishment path to ensure that there can be no Pareto-ranking across any continuation equilibria. This ultimately fixes the proof and yields that the proposed conditions of Farrell and Maskin (1989) are indeed sufficient for weakly renegotiation-proof payoffs.

In Chapter 4, we approach the equilibrium notion of weak renegotiation-proofness conceptually and elaborate on the shortcomings of the concept when it is applied to games with more than two players. In fact, Farrell and Maskin (1989) have only formally introduced the equilibrium notion for two-player games and, as we show, its application to n-player games may yield results that are against the intuition of stable outcomes in multilateral and repeated interactions.

Even though Farrell and Maskin (1989) already noted in their conclusion that, for an application of their notion to n-player games, a refinement is necessary, their concept has frequently been applied in n-player games by extending the condition of Pareto-undominated continuation equilibria from two to n players. This, however, excludes possible subgroup renegotiation and gives a single player the pivotal power to block a renegotiation. By means of several examples, we demonstrate counterintuitive outcomes that may already arise in three-player games. Moreover, we show that the characterization results of Farrell and Maskin (1989) do not generalize to n-player games as proposed by the authors (Farrell and Maskin, 1989, p. 355).

We therefore suggest a refinement of the weak renegotiation-proofness notion that precludes counterintuitive results in general n-player games. In doing so, a renegotiation protocol is specified that allows players to renegotiate after any single deviation from the agreed-upon strategy has occurred. We propose different renegotiation rules that determine which subgroups of players may jointly renegotiate a change of their strategies while leaving the others' strategies fixed. Depending on the specification of the renegotiation rule, the size and composition of these subgroups may range from the group of all innocent players, as in the original definition of Farrell and Maskin (1989), to any feasible subset of players.

Furthermore, we study the relationship between these different specifications and we elaborate on difficulties to obtain general characterization results. Finally, we return to our results from Chapter 2, discussing them in light of our additional refinements and, coming back to the Prisoner's Dilemma, show that full cooperation can always be sustained in a general n-player Prisoner's Dilemma.

Chapter 2

International Environmental Agreements for Local and Global Pollution

2.1 Introduction

Rising concerns about climate change has led politicians worldwide to rethink their countries' emission of greenhouse gases and air pollution. Doing what is best for their own countries' interest, however, does not fully internalize the global effects of the emissions and hence their optimal policy will not reduce pollution efficiently. In other words, countries free-ride on others' abatement efforts, similar to the case of private provision of public goods. To overcome this dilemma and achieve more efficient pollution abatement, several International Environmental Agreements (IEAs) have been proposed and formed in recent years.[1]

Besides their global effects, many forms of pollution have additional negative effects on countries within the same region of the polluting source. Air pollution, for instance, can cause smog, acid deposition and eutrophication which are mostly experienced locally while the global effects (e.g., global warming) are

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[1] Examples include the Oslo Protocol on sulfur reduction in Europe (also including other states) in 1994, the Montreal Protocol on the depletion of the ozone layer in 1987 and the Kyoto-Protocol on the reduction of greenhouse gases in 1997.

endured worldwide. Short-lived climate pollutants such as black carbon, methane and tropospheric ozone have both a local and global impact. Their effects on "regional and global climate, through both direct interaction with atmospheric radiation and indirect effects related to changes in cloud properties are a growing concern" (Committee Committee on the Significance of International Transport of Air Pollutants; National Research Council, 2009). As another example, a nuclear power plant causes higher negative effects in nearby regions by danger of malfunctioning compared to the global risk. The presence of these local spillovers hence plays a non-negligible role and adds heterogeneity to the problem of forming IEAs.

In this paper, we ask which IEAs may be implemented by purely self-interested countries when the negative externalities of pollution have a local and a global component. We use a repeated game approach of abatement efforts to study the stability of IEAs. An IEA, here, is a strategy profile in the repeated game which coordinates the abatement efforts of its members to maximize joint utility. We focus on simple strategies which give rise to punishment paths where the punishment lasts only one period and which can only be executed via higher emissions. By stability we mean that an IEA shall be self-enforcing in the sense that no member shall have an incentive to deviate from cooperation and renegotiation shall be prevented. Formally, we define an IEA as stable, if it is a weak renegotiation-proof equilibrium.

Which countries are affected by the local externality of pollution is represented by a network: a link between two countries indicates whether these countries' pollution affects each other locally. Here, a link could mean that two countries share the same border or are within some distance which is critical for the local externality. Given a local spillover structure, we derive optimal punishment strategies such that the grand coalition of all countries can be implemented as a subgame perfect equilibrium. In contrast to the general literature without a local spillover structure, global cooperation may fail to be a weakly renegotiation-proof equilibrium in very asymmetric networks, where the asymmetry is with respect to neighbors in the network. In other words, if the channels through which countries affect each other are very unevenly distributed, then global cooperation may fail. However, we also show that it can always be sustained in regular networks, i.e. networks such that all countries have the same number of neighbors.

The additional local spillover structure adds heterogeneity to the problem of stability of IEAs (in the sense of existence of a WRP equilibrium) and has interesting effects such that in some asymmetric structures, the global IEA is not a WRP equilibrium. As global pollution can be seen as a perfect public bad, the local side of it has the characteristics of a local public bad. Since reducing pollution has the characteristic of a public good, we also contribute to the problem of public good provision when the public good has both a local and a global component. To our knowledge, including both aspects in one model is also new to the literature of public goods.

Our results have important policy implications. When contemplating an IEA, strict rules have to be imposed in order to prevent deviation. These rules must specify the consequences of deviating from the agreed reductions and shall make use of the local spillover effects. With respect to welfare, we show in Section 2.6 that it is indeed better to first appoint neighbors for punishment of a deviation before non-neighbors shall punish. That is, neighbors of a deviator are more effective with their punishment since a deviator is punished through both the local and the global spillover channel and therefore the punishment path can be sustained more easily and requires fewer total emission.

More generally, the results may serve as a benchmark that can be useful in future analyses of IEAs. We point to several possible extensions in our Conclusion (Section 2.8). Moreover, the results can easily be transferred to other problems of public good provision and may support a better understanding of free-riding problems.

The paper is organized as follows: first, we further elaborate on the issue of local and global pollution and discuss related literature as well as our contributions. In Section 2.3 we introduce the basic model of a single-stage game. In Section 2.4 we extend the model to an infinitely repeated game and derive conditions on existence of weakly renegotiation-proof equilibria for several prominent networks. Section 2.5 focuses on the welfare-maximizing global IEA. In Section 2.6 we analyze welfare implications of different network structures. Finally, Section 2.8 concludes. All proofs are presented in the Appendix.

2.2 Background and Literature Review

International Environmental Agreements (IEAs) have been analyzed in various game-theoretic models over the past two decades. Starting with the seminal paper by Barrett (1994), several authors have studied the free-rider problem when joining an agreement by studying both one-shot and repeated games. For a good overview of the game-theoretic literature on environmental economics we refer to recent literature surveys such as for example Jørgensen et al. (2010) or Benchekroun and Long (2012). Formal models of climate cooperation are thoroughly reviewed in Hovi et al. (2015).

A majority of the models in the literature tackle the problem of air pollution, caused by the emission of greenhouse gases from fossil fuel combustion. While some models have at least abstracted from the stark assumption of homogeneous countries and introduced asymmetries to account for different impact and contribution levels of pollution (e.g., McGinty, 2007; Hannesson, 2010), the implications of geographical distance to the sources of air pollution have not been largely accounted for.

However, there is broad scientific evidence for the importance of regional characteristics for several air pollution effects. Most importantly, short-lived air pollutants, that include methane, black carbon and tropospheric ozone, have a significant local or regional impact besides contributing to global problems such as climate change (see, e.g., Kühn et al., 2013, for a study of emissions on local and global aerosol properties for China and India). Other examples for the regional effects of air pollution include the ozone level. For instance, the ozone level of the Mediterranean region is not only affected by local emissions but also perturbed by long-range pollution import from Northern Europe, North America and Asia (Richards et al., 2013).

Summarizing the above evidence we can conclude that the consideration of local spillover effects in addition to global externalities of emissions is crucial to better understand and represent the incentives that underlay the formation of IEAs and stability.[2] While Yang (2006) considers an optimal control problem where countries provide negatively (!) correlated local and global stock externalities (his example is CO_2 and SO_2), Dockner and Nishimura (1999) consider a dynamic game model where each country contributes to a domestic stock of pollution. Both, however, do not consider the possibility of an IEA to reduce pollution.

Hence, to our knowledge there exists no game-theoretic model that incorporates both a local and global spillover effect of air pollution in a standard coalition formation game for an IEA. This however seems to be crucial in understanding possible solutions to the problem of reducing pollution as for example Bollen et al. (2009) show in a cost–benefit analysis, concluding that "combined climate and local air pollution policy generates extra benefits in terms of climate change mitigation." They therefore recommend that policies need to be designed such that they jointly implement both global climate change and local air pollution strategies.

Considering only global pollution as a repeated game, several works have studied IEAs as a strategy profile of a coalition which maximizes members' utilities and which may punish possible deviators by returning to pollution strategies. Often in this literature, all members of an IEA are involved in the punishment of a deviator. This has been found to limit outcomes in terms of the number of cooperating countries in equilibrium (e.g., Barrett, 1994, 1999),

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[2] In our paper, we study stability of an IEA from a repeated game perspective: an IEA is a strategy profile in the repeated game of a coalition of signatories which maximizes the sum of members' utilities. It is stable if it constitutes a weakly renegotiation-proof (WRP) equilibrium.

as the more countries punish a deviator, the fewer countries cooperate in the punishment phase which then lowers the punishing countries' payoffs in this phase. To lower the incentives for renegotiation, several authors studied different punishment strategies that either limit the number of punishers or the duration of punishment. Among those are Asheim and Holtsmark (2009) and Froyn and Hovi (2008), who consider a "penance" strategy as in Farrell and Maskin (1989), but refine this to a "penance-k" strategy, where only a subgroup of k players punishes a deviator for a finite number of periods before reverting to cooperation. Another example is Asheim et al. (2006), where artificially two regions are introduced in order to restrict punishment to be executed only by a subset of IEA members. By incorporating the regional effects in our model, however, it comes very natural to use the regional structure for punishment patterns.

A different enforcement system for global pollution abatement was proposed by Heitzig et al. (2011). Their dynamic strategy of linear compensation does not use effective punishment but rather a redistribution scheme where abatement liabilities are distributed according to past compliance levels. This strategy can be shown to implement any given allocation of target contributions, therefore also the full cooperative and efficient solution. By keeping the global abatement level constant across periods, they avoid renegotiation and can show that any ex ante chosen allocation is a subgame perfect equilibrium.

We shall also mention that all these models treat pollution as a flow variable. This may be plausible for short-lived climate gases, but a debatable simplification for most greenhouse gases that accumulate in the atmosphere over time. The first models that study IEAs for stock pollutants were proposed by Rubio and Casino (2005) and Rubio and Ulph (2007). A detailed literature overview can be found in Calvo and Rubio (2012). More recently, Kratzsch et al. (2012) study the conditions for stable climate agreements where emissions build up over time and payoffs in every period depend on the accumulated stock level in the atmosphere. They also show that global cooperation can be enforced with a penance-k strategy. Although our model also treats pollution as a flow variable, we discuss how our results can be interpreted in light of the stock pollution literature in the Conclusion (Section 2.8).

The application of network theory to problems of public goods is not new to the literature. Several authors analyze the provision of public goods in a network and study a local spillover effect where players can only benefit from their direct neighbors' provisions (e.g., Allouch, 2015; Bramoullé and Kranton, 2007; Bloch and Zenginobuz, 2007; Elliott and Golub, 2013). However, none of these include a global spillover effect that would be necessary for an adequate representation of the pollution problem. We therefore contribute to the climate change literature by incorporating elements of the network theory, an issue that is becoming more and more interesting to researchers of that field (see Currarini et al., 2014).

2.3 A Pollution Game of Local and Global Spillovers

2.3.1 Model Setup

We consider an economy with a finite set of countries N, which are denoted by $i = 1, \ldots, n$. Countries are heterogeneous with respect to their size (i.e. satiation level of consumption) and their position in the local spillover network. We assume that countries are represented by one individual.[3] Each country derives benefits from consuming a good $x_i \in \mathbb{R}_+$ with marginal benefits assumed to be decreasing. Likewise, we assume decreasing returns from additional abatement efforts (i.e. consumption reduction). Benefits of consumption are therefore represented by the quadratic and concave function

$$B_i(x_i) = -\frac{1}{2}\left(\bar{x}_i - x_i\right)^2,$$

where $\bar{x}_i \in \mathbb{R}$ is an exogenously fixed satiation level which represents the first-best emission level if there would be no pollution effects of consumption – or at least there would be no concern for them.

Note. In the following we will make use of the following notation: the n-tupel $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ describes the output vector of all countries. For a subset $A = \{i_1, \ldots, i_l\} \subseteq N$, the vector $x_A = \left(x_{i_1}, \ldots, x_{i_l}\right)$ is the output vector of all countries in A. Also, we use the following abbreviation for the output vector of all countries but country i: $x_{-i} = x_{N\setminus\{i\}}$.

Consuming x_i emits air pollutants and thus contributes to the stock of pollution which is accumulated on a local and global level.[4] While benefits from consuming x_i are private, the emission of pollutants has spillover effects on all other countries. All countries equally suffer from the global level of pollution. In addition to the global impact, effects of emissions differ locally and are experienced by a certain subgroup of countries. For instance, short-lived climate pollutants such as black carbon, methane and tropospheric ozone have both a local and global impact.[5]

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[3] We leave out all issues related to opinion formation and political debate within a country but focus on the negotiations taking place at the global level.

[4] For example, Battaglini and Harstad (2012) interpret x_i to be the level of energy used to produce some good. For simplicity we assume one unit of consumption to generate one unit of pollution.

[5] Also, usually not only one single pollutant is released during production or consumption but others are emitted simultaneously and these might only impact certain, local areas. We abstract from this by summarizing all different pollutants in one representative emission flow x_i.

We model the local spillover effect by a network structure $g \in G^N$, where $G^N = \{g \mid g \subseteq g^N\}$ denotes the set of all possible networks on the set of players N, with g^N denoting the set of all subsets of N of size 2. A link $\{i, j\}$ between two countries i and j in the network g then describes the presence of a direct local spillover which could be due to geographical distance, common borders, sharing an ocean or a lake, or other underlying assumptions that we exclude from our model. We assume that local emission spillovers between countries are bidirectional and thus focus on undirected networks for simplicity. However, we show in Section 2.7 that this is not restrictive and adapting notation our results also hold for directed and weighted networks.

For a network $g \in G^N$, we denote by $N_i(g) := \{j \in N | i, j \in g\}$ the set of neighbors of country i in the local spillover network, i.e. those countries which affect i and are affected by i's emission not only with respect to the global externality but also through the local externality. Further, we denote by $\eta_i := |N_i(g)|$ the number of countries affected locally by i's emission, called i's degree. The network structure is captured via the indicator function g_{ij} which is equal to 1, if i and j are neighbors and 0 in all other cases. With respect to the spillover effect, every country suffers from its own emissions both through the local and the global effect. To account for this and to incorporate it into our model, we let $\bar{g}_{ij} = g_{ij}$ for all $i \neq j$ and $\bar{g}_{ii} = 1$.

We assume linear spillover effects on both the global and local level due to analytical tractability. The marginal impacts are weighted relative to benefits from consumption by $\beta > 0$ for the global spillover effect and $\gamma > 0$ for the local spillover effect.[6] The costs incurred from total pollution are then represented by the cost function

$$K_i(x_i, x_{-i}) = \beta \sum_{j \in N} x_j + \gamma \sum_{j \in N} \bar{g}_{ij} x_j .$$

The individual profit π_i of a country $i \in N$ can thus be represented as follows:

$$\begin{aligned} \pi_i \left(x_i, x_{-i}\right) &= B_i(x_i) - K_i(x_i, x_{-i}) \\ &= -\frac{1}{2} \left(\bar{x}_i - x_i\right)^2 - \beta \sum_{j \in N} x_j - \gamma \sum_{j \in N} \bar{g}_{ij} x_j . \end{aligned} \tag{2.1}$$

In the standard literature, an IEA is defined by the game-theoretic concept of a coalition. In the repeated setup that we focus on in this paper this corresponds to a strategy profile that defines agreement (and punishment) actions among coalition members. The common interpretation is that this coalition coordinates its member countries' emissions. Hence, let $C \subseteq N$ be a coalition of k countries

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[6] We shall mention that we abstract from heterogeneities with respect to marginal impacts to focus on the effect that is derived from the network position.

$i_1, \ldots, i_k$ that cooperate on the abatement level to maximize their utilitarian welfare.[7] A member of a coalition, called *signatory*, hence chooses a pollution level x_i such that it maximizes the sum of all signatories' utilities. Given a coalition C, we denote by $C + i := C \cup \{i\}$ the coalition obtained by i joining C, and, analogously $C - i := C \setminus \{i\}$. A main input factor for our results will be the number of intra-coalition links, i.e. the number of neighbors that are part of the coalition, which will be denoted by $k_i := |N_i \cap C|$.

2.3.2 Global Versus no Cooperation in the Single-Stage Game

To illustrate the asymmetries in the countries' incentives to cooperate on global abatement efforts, we look at the two extreme cases of either no or full cooperation, i.e. $C = \emptyset$ and $C = N$, in the single-stage game. In this game all countries simultaneously choose their level of emissions x_i.[8]

In the situation of no cooperation, i.e. $C = \emptyset$, every country myopically determines its emission level to maximize individual profit π_i as defined in (2.1). The first order conditions (subsequently abbreviated as FOCs) then directly yield the *non-signatory* Nash outcome

$$x_i^{NS} = \bar{x}_i - \beta - \gamma. \tag{2.2}$$

Hence, in absence of a full cooperation agreement, every country emits just slightly below its first-best level $\bar{x}_i$ by accounting for the own marginal emission effect $\beta + \gamma$.

Assumption 2.1. As we assume non-negative emissions, we impose the following condition for all $i \in N$: $\bar{x}_i \geq m_1\beta + m_2\gamma, \ \forall \ 0 \leq m_1, m_2 \leq n$.

Since signatories $C \subseteq N$ choose emissions to maximize the utilitarian welfare restricted to the members of the coalition, the maximization problem for those countries is given by

$$\max_{(x_i)_{i \in C}} \sum_{i \in C} \pi_i \left(x_C, x_{N \setminus C} \right). \tag{2.3}$$

The FOCs yield an optimal emission level for every signatory that depends on the size of the coalition, k, and the number of intra-coalition links, $k_i = |N_i \cap C|$,

$$x_i^S \left(C \right) = \bar{x}_i - \beta k - \gamma(k_i + 1), \tag{2.4}$$

---

[7] We abstract from the possibility of multiple agreements, thus only one coalition can form even though the consideration of a local spillover structure may naturally induce locally organized agreements and thus multiple coalitions. We discuss possible generalizations to multiple coalitions in Section 2.7.

[8] The assumption of simultaneous move is standard in the literature. There are, however, also papers that study the effects of a coalition that acts as a Stackelberg leader (e.g., Rubio and Ulph, 2006).

and all non-signatories $j \notin C$ choose the emission level x_j^{NS} given by (2.2). In the following, we will denote by $x(C) = \Big((x_i^S(C))_{i\in C}, (x_j^{NS})_{j\in N\setminus C}\Big)$ the vector of outputs when a coalition C is collaborating and denote the respective profits by $\pi_i(C) = \pi_i(x(C))$.

For the case of full, global cooperation, i.e. $C = N$, utilitarian welfare of all countries $\sum_{i\in N}\pi_i(x)$ is maximized. We obtain $x_i^S(N) = \bar{x}_i - \beta n - \gamma\,(\eta_i + 1)$, where every country takes into account the global effects from its pollution as well as the local spillovers to every respective neighbor. From a global perspective, this would be the first-best solution as all externalities are internalized. However, not every individual country is necessarily better off under full cooperation than under no cooperation. To demonstrate this, we compute the difference in individual profits between global and no cooperation. For this purpose we introduce the difference function $\Delta_i(A, B)$ for two sets $A, B \subseteq N$ such that

$$\Delta_i\,(A, B) := \pi_i(x(A)) - \pi_i(x(B)).$$

By inserting $x_i^S(N)$, respectively x_i^{NS}, into (2.1), we then obtain for the difference between full and no cooperation

$$\begin{aligned}\Delta_i\,(N, \emptyset) =& \pi_i(x(N)) - \pi_i(x(\emptyset))\\ =& -\frac{1}{2}\Big[(\beta n + \gamma(\eta_i + 1))^2 - (\beta+\gamma)^2\Big]\\ &+ \beta\Big[\beta n^2 + \gamma\sum_{j\in N}(\eta_j + 1) - \beta n - \gamma n\Big]\\ &+ \gamma\Big[(\eta_i + 1)\beta n + \sum_{j\in N}\bar{g}_{ij}\gamma(\eta_j + 1) - (\beta+\gamma)(\eta_i + 1)\Big],\end{aligned}$$

where the first bracket captures the additional cost from abatement when accounting for global cooperation rather than playing individually rational, and is hence negative, while the second and third brackets capture the positive effects of global and local emission reduction by the coalition on payoffs. We can simplify to get

$$\Delta_i\,(N, \emptyset) = \frac{\beta^2}{2}\Big(n-1\Big)^2 + \beta\gamma\Big(\sum_{j\neq i}\eta_j\Big) + \frac{\gamma^2}{2}\Big(\sum_{j\neq i} g_{ij}\eta_j - \eta_i^2\Big). \tag{2.5}$$

Thus, the potential gains from a full cooperation agreement are positive if and only if

$$\gamma^2\Bigg(\eta_i^2 - \sum_{j\in N} g_{ij}\eta_j\Bigg) \leq \beta\Bigg(\beta(n-1)^2 + 2\gamma\sum_{j\neq i}\eta_j\Bigg). \tag{2.6}$$

Since the right-hand side is positive, Condition (2.6) holds either if the value of $(\eta_i^2 - \sum_{j\in N} g_{ij}\eta_j)$ is small or if β is large relative to γ. Note that the left-hand side bracket is non-positive if and only if i's degree is below the average degree of its neighbors, i.e. $(\eta_i^2 - \sum_{j \in N} g_{ij}\eta_j) \leq 0$ if and only if $\eta_i \leq \frac{\sum_{j \in N_i} \eta_j}{\eta_i}$.

Thus, the potential gains from a full cooperation agreement are positive for all countries if either the network is not too asymmetric (i.e. where $\eta_i \simeq \eta_j$ for all $i, j \in N$) or in those cases where the global impact β is large compared to the local impact γ.[9] Instead, very asymmetric network structures and a high local spillover effect γ can lead to cases where some countries actually prefer no cooperation to full cooperation as we see in Example 2.1. The countries that prefer no cooperation over full cooperation are those who have a large degree compared to their neighbors' degree, $\eta_i > \frac{\sum_{j \in N_i} \eta_j}{\eta_i}$, since they have to abate most and receive relatively few abatement of their neighbors in return. This effect becomes smaller, the smaller the impact of local pollution γ relative to global pollution β becomes. Thus, the first observation that we can take away here is that the local spillover structure may yield asymmetries between countries' incentives which are difficult to overcome when forming IEAs.

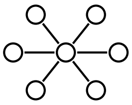

Figure 2.1: A star network with one central node and 6 peripheral nodes.

Example 2.1. Consider the star network $g^*(n)$ with one player connected to all other $n-1$ players who are only connected to the center, exemplarily shown in Figure 2.1 for 7 nodes. The center node, call it country 1, thus has $\eta_1 = n-1$, while all other countries $j \neq 1$ have $\eta_j = 1$. Country 1 prefers no cooperation over full cooperation if (2.6) is violated. We get

$$\Delta_1(N, \emptyset) = \tfrac{\beta^2}{2}(n-1)^2 + \beta\gamma(n-1) - \tfrac{\gamma^2}{2}(n-1)(n-3)$$

from (2.5) by plugging in the center and peripheral nodes' degrees and by noting that the center is connected to all other nodes, i.e. $g_{1j} = 1$ for all $j \neq 1$. Hence it follows that $\Delta_1(N, \emptyset) < 0$ for all values of $\gamma > \beta\frac{n-1}{n-3}$. For a peripheral node, no cooperation is clearly worse than the full-cooperative solution by (2.6) for all parameter values, since its degree $\eta_i = 1$ is smaller than its neighbor's degree $\eta_1 = n-1$. Nevertheless, global cooperation is not a Pareto-improving outcome to no cooperation if the local externality dominates the global externality, i.e. $\gamma > \beta\frac{n-1}{n-3}$.

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[9] Two examples of sufficient conditions for (2.6) to hold are that either the network is regular, i.e. $\eta_i = \eta_j$ for all $i, j \in N$ or that $\gamma \leq \beta$.

2.4 Stable IEAs in Infinitely Repeated Games

As the nature of pollution and production is rather of repeated form, a one-shot game may not be the accurate model to consider IEAs. Thus, we consider from now on the game described in Section 2.3 which is repeated infinitely often. As before, we think of an IEA as an agreement between countries who want to establish an outcome which maximizes welfare among them. In the one-shot case, this boils down to a simple coalition formation game. When the game is of repeated form, an IEA requires more: for each possible history of play the signing countries need to coordinate their actions. Thus, we will define an IEA as a strategy profile of the repeated game which aims to maximize the utilitarian welfare of the coalition.

Such an agreement is stable if it meets two requirements: First, stability requires that no signatory has an incentive to deviate from the strategy that maximizes the coalition's utilitarian welfare through threats of future punishments by the other signatories. These threats deter free-rider incentives and allow for the implementation of full cooperation as a subgame perfect equilibrium (subsequently abbreviated as SGP equilibrium) as long as the discount factor is high enough (see Fudenberg and Maskin, 1986). Second, stability requires execution of the punishment strategies such that they are not vulnerable to renegotiation. In other words, the punishers shall not have an incentive to *renegotiate* the terms of the agreement in a way that they do not carry out the punishment but strictly prefer to follow a different continuation equilibrium. We rule out this possibility by considering as stable outcomes only those equilibria of the repeated game that are weakly renegotiation-proof.[10]

Thereby in our analysis of IEAs, we do not ask how a group of signatories forms, but rather which coalition of countries can implement a stable IEA. Properties of a stable IEA such as the composition of signatories and the required punishment paths then crucially depend on the local spillover structure. Therefore, we also derive conditions on the local spillover structure that foster or harm stability of an IEA.

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[10] Note that there are two limitations of this concept in our subsequent analysis: First, weak renegotiation-proofness takes account of the possibility of a unilateral deviation of a single country but does not regard the possibility of a deviation of a subset of countries, which may very well occur as a result of coordinated action among some countries. Second, by deriving a WRP equilibrium we cannot answer the question of how coordination may be achieved, i.e. how countries agree on a particular IEA (see also the discussion in Asheim and Holtsmark, 2009).

2.4.1 The Infinitely Repeated Game

We briefly introduce a standard infinitely repeated game of the stage game as described in Section 2.3. Time is discrete and indexed by $t \in \mathbb{N}$. In each period, countries choose consumption (i.e. emission levels) $x_i(t)$ (with slight abuse of notation). In other words, the stage game is played in each period. At time t, country i's choice of emission may depend on the entire history of the game through period $t-1$, denoted

$$h^{t-1} = \Big((x_1(0), ..., x_n(0)), ..., (x_1(t-1), ..., x_n(t-1))\Big).$$

Thus, a strategy $\mathbf{s_i}$ for country i is a function that, for every date t and every possible history h^{t-1}, defines a period t action $x_i(t) \in \mathbb{R}_+$. Future payoffs are discounted with a common discount factor $\delta < 1$ such that each country i receives a discounted payoff for a sequence of emissions $\{(x_i(t), x_{-i}(t))\}_{t=0}^{\infty}$,

$$\Pi_i = (1-\delta)\sum_{t=0}^{\infty} \delta^t \pi_i(x_i(t), x_{-i}(t)). \tag{2.7}$$

In the infinitely repeated game, a weakly renegotiation-proof equilibrium is defined as a strategy profile of the repeated game $\mathbf{s} = (\mathbf{s_i})_{i \in N}$ such that it satisfies the following.

Definition 2.1. [Farrell and Maskin (1989)] A strategy profile $\mathbf{s}$ is a *weakly renegotiation-proof (WRP) equilibrium* of the infinitely repeated game if and only if (i) $\mathbf{s}$ is a subgame perfect equilibrium of the infinitely repeated game and (ii) there do not exist two continuation equilibria of $\mathbf{s}$ such that all players strictly prefer the one to the other.

Since we view an IEA as an agreement of a coalition of players $C \subseteq N$ to implement the signatory action x_C^S as defined by (2.4), the application of the concept of WRP equilibrium has exactly the conditions desired for stability of such an IEA.[11] Condition (i) of Definition 2.1 ensures that the IEA is stable with respect to unilateral deviations and (ii) implies stability with respect to renegotiations.

Hence, we ask whether the play of signatory actions can be observed as the equilibrium path of a WRP equilibrium. For this, *credible* punishment paths have to be designed to deter deviations. Due to renegotiation incentives, it may not be optimal that all other signatories punish. In fact, the harsher the punishment or the more countries punish, the higher is the incentive to renegotiate.[12] Another

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[11] Note that in the following we will use the notation $x_i^S \equiv x_i^S(C)$ unless otherwise stated. Also, to shorten notation we will omit the vector $x_{N \setminus C}$.

[12] It has been shown in other papers, such as Froyn and Hovi (2008) and Asheim et al. (2006), that the limitation of punishers to a subset of the cooperating players can decrease the

aspect of implementability of an IEA is the fact that strategies shall be simple. We use the notion of simple strategy profiles by Abreu (1988), defining an equilibrium path, n respective punishment paths and a transition rule that specifies that any single deviation by player i from an ongoing prescribed path is responded by imposing her punishment path. In our setting, we further simplify this by focusing on punishments that last only one period, restricting to time-invariant sets of punishers and punishment actions that are independent of the deviator. Hence a simple strategy, here, is such that for each player at any time there are only two possible actions played, equilibrium action or punishment action. The punishment lasts only one period and only the set of punishers depends on the identity of the deviator.

Formally, we denote $P_i(g) \subseteq C$ the set of players that punish deviator $i \in C$. Punishment for a deviating signatory $i \in C$ is thus carried out as follows: Each country $j \in P_i$ punishes a deviation of country i by emitting the punishment level x_i^P instead of the signatory emission x_i^S where

$$x_j^P = \bar{x}_j - p(\beta + \gamma)$$

in the period after the deviation. We assume that $p \geq 1$ such that the highest punishment level is the Nash output x_j^{NS}. As all non-signatories $l \notin C$ play their first-best action, i.e. the Nash equilibrium level x_l^{NS}, they will not punish a deviator but continue with their strategy.

An IEA by a coalition $C = \{i_1, \dots, i_k\}$ is then defined by a strategy profile $\mathbf{s}$ of the repeated game such that signatories use the agreement strategy $\mathbf{s^C}$, which is defined for period $t = 1$ by $\mathbf{s_i^C}(\emptyset, 1) = x_i^S$ for all $i \in C$ and for $t \in \mathbb{N}$ recursively defined by

$$\mathbf{s_i^C}(h^{t-1}, t) = \begin{cases} x_i^P, & \text{if } \exists! j \in C : x_j(t-1) \neq \mathbf{s_j^C}(h^{t-2}, t-1) \text{ and } i \in P_j \\ x_i^S, & \text{else} \end{cases}.$$

Non-signatories $j \in N \setminus C$ stick to their Nash emission level x_j^{NS} throughout the game and their strategy is simply given by $\mathbf{s_j^{N \setminus C}}(\cdot, t) = x_j^{NS}$ for all $t \in \mathbb{N}$. Such an IEA defines a simple strategy profile in the spirit of Abreu (1988), since it gives rise to the $(n+1)$-vector of paths

$$(\mathbf{a^C}, \mathbf{p_1^C}, \dots, \mathbf{p_n^C}),$$

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incentives for renegotiation. The same effect can be observed in our model. Here, however, players face heterogeneous costs from pollution through the local spillover channel. Thus, we have to derive individual punishment paths, that is individual sets of punishers, for any possible deviator. Whereas for example Asheim et al. (2006) artificially introduce two separated regions and let a deviating country be punished only by countries in the same region as the deviator, we allow for more flexible punishment sets and focus on the impact of the local spillover structure, represented by the network g, on possible equilibrium outcomes.

where $\mathbf{a^C}$ is the agreement path, such that

$$\mathbf{a^C} = \left\{ \left(x_C^S, x_{N \setminus C}^{NS} \right), \left(x_C^S, x_{N \setminus C}^{NS} \right), \dots \right\}$$

and $\mathbf{p_i^C}$ the punishment paths which are triggered if a single country $i \in N$ deviates, such that

$$\mathbf{p_i^C} = \left\{ \left(x_{P_i}^P, x_{C \setminus P_i}^S, x_{N \setminus C}^{NS} \right), \left(x_C^S, x_{N \setminus C}^{NS} \right), \left(x_C^S, x_{N \setminus C}^{NS} \right), \dots \right\}.$$

In other words, any single deviation of a country i results in a one-period punishment by the countries P_i who subsequently revert to their signatory strategies, while all others play as on the agreement path.[13] Moreover, only signatories may have an incentive to deviate from their cooperative output and thus must be threatened to be punished in order to deter deviation. Non-signatories, on the other hand, although benefiting from the abatement of others and being hurt by punishment, are not part of the IEA in the sense that they play their individual rational output. Thus, punishment paths do not have to be specified for non-signatories in order to enforce the equilibrium strategy. All together we ask whether $\mathbf{s} = (\mathbf{s^C}, \mathbf{s^{N \setminus C}})$ forms a WRP, i.e. whether the IEA $\mathbf{s}$ is stable.

It is worth remarking that we consider a very specific punishment rule and simple punishment strategies which can be seen as a refinement of a regular penance-k strategy (see e.g., Froyn and Hovi, 2008) since it determines for each signatory an individual set of punishers whereas in the penance-k strategy the number of punishers is set to be identical across all signatories. However, as stated before, it is still a strategy that is simple to implement and therefore suitable for the application in an IEA. Moreover, it is the one that has been frequently used in the repeated games literature on IEAs and our subsequent results show that this may not be sufficient to establish full cooperation as a WRP equilibrium. In Section 2.7 we also briefly discuss what changes if we allowed for other punishment strategies.

2.4.2 Weakly Renegotiation-Proof Equilibria

Since stability of an IEA by a coalition C requires existence of a WRP equilibrium such that signatory actions are played, optimal punishments sets P_i for each $i \in C$ and punishment level p have to be determined. As spillovers are heterogeneously distributed across the signatories due to their network position, this might be a quite complex task. In Theorem 2.1 we present necessary and sufficient conditions on these punishment sets.

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[13] Note that only single deviations are considered, that is multiple deviations in a single period are not punished.

In accordance with Definition 2.1, we have to check for subgame perfection and weak renegotiation-proofness. To satisfy the subgame perfectness requirement, it suffices to compare the short-term payoff gain from a one-shot deviation, given by

$$\frac{1}{2}\Big(\beta k+\gamma(k_i+1)\Big)^2-\frac{1}{2}\Big(\beta+\gamma\Big)^2-\Big(\beta+\gamma\Big)\Big(\beta(k-1)+\gamma k_i\Big)=\frac{1}{2}\left(\beta(k-1)+\gamma k_i\right)^2,$$

with the loss incurred by punishment for all $i \in N$. Since punishment is executed by other countries increasing their emissions in the period following her one-shot deviation, a deviator i suffers through both the global and local spillover channel: Every punisher $j \in P_i$ increases emissions by $\beta(k-p)+\gamma(k_j+1-p)$, which are experienced by the deviator i through the global channel, weighted with β, and, if j is a neighbor of i, also through the local channel, weighted with γ. Summing over all punishers $j \in P_i$ and rearranging terms yields the term in square brackets in Equation (2.8) in Theorem 2.1.

For weak renegotiation-proofness we only need to compare the punishers' payoff along a punishment path with their equilibrium payoffs. When all punishers renegotiate back to cooperation, every punisher j incurs an individual payoff loss, due to consumption reduction, given by $\frac{1}{2}\left[\Big(\beta(k-1)+\gamma k_j\Big)^2-(1-p)^2(\beta+\gamma)^2\right]$. On the other hand, payoffs increase due to spillovers from the other players' abatement, given by $\beta(k-p)+\gamma(k_l+1-p)$ which are experienced by j through the global channel, weighted with β, and if l is a neighbor also through the local channel, weighted with γ. Summing over all punishers $l \in P_i \setminus \{j\}$ and rearranging terms yields Equation (2.9) in Theorem 2.1.

Therefore, we obtain the following result.

Theorem 2.1. *An IEA* $\mathbf{s}$ *by a coalition* C *is stable if and only if for all* $i \in C$

$$\begin{aligned}\delta\Bigg[\beta^2|P_i|(k-p)+\beta\gamma\Bigg(|P_i|(1-p)+\sum_{m\in P_i}k_m+|P_i\cap N_i|(k-p)\Bigg)\\ +\gamma^2\Bigg(\sum_{m\in P_i\cap N_i}k_m+|P_i\cap N_i|(1-p)\Bigg)\Bigg]-\frac{1}{2}\left(\beta(k-1)+\gamma k_i\right)^2\geq 0 \quad (2.8)\end{aligned}$$

and for all $i \in C$ *there exists at least one* $j \in P_i$ *such that*

$$\begin{aligned}\beta^2(k-p)(|P_i|-p)+\beta\gamma\Bigg((|P_i|-p)(1-p)+\sum_{m\in P_i\setminus\{j\}}k_m+|P_i\cap N_j|(k-p)\Bigg)\\ +\frac{\gamma^2}{2}\Bigg(2\sum_{m\in P_i\cap N_j}k_m+\Big(2|P_i\cap N_j|+1-p\Big)(1-p)\Bigg)-\frac{1}{2}\Big(\beta(k-1)+\gamma k_j\Big)^2\leq 0.\end{aligned}$$

(2.9)

To sum up, P_i needs to be large enough while p must be low enough in order for Equation (2.8) to hold.[14] While punishment needs to be harsh enough to deter deviations, Equation (2.9) yields that punishment cannot be too harsh to prevent incentives for renegotiation.[15]

Hence, if both conditions are satisfied, the IEA that specifies for every signatory $i \in C$ a set of punishers P_i and a punishment level of the punishers p, is stable, i.e. play of signatory actions by members of the coalition C is an equilibrium path in a WRP equilibrium. Even with the simple strategies that we consider here as an IEA, the conditions in Theorem 2.1 seem quite complex. The complexity stems from the heterogeneous spillover channels represented by the network.

The intuition of the conditions can be better explained in the following when we focus only on one type of spillover (global respectively local, Section 2.4.3) and subsequently explore comparative statics with respect to changes in the network and punishing sets (Section 2.4.4). For the rest of the paper we will moreover assume that every country punishes a deviation by emitting its Nash output level, i.e. for all $i \in N$ we set $p = 1$ for all punishers $j \in P_i$.

2.4.3 WRP Conditions for Special Spillover Structures

First, suppose that there exists only the global spillover channel, as for example in Asheim and Holtsmark (2009). Hence, the underlying network plays no role and the only heterogeneity in the game stems from the exogenously given satiation levels $\bar{x}_i$. However, as these do not influence the results, the intuition alone implies that it is not important who punishes, but how many punish, i.e. it is not the composition of the punishment set that matters but the size. Indeed, setting $\gamma = 0$ in (2.8) and (2.9), one obtains the following.

Corollary 2.1. *For $\gamma = 0$, the conditions of Theorem 2.1 reduce to*

$$\frac{1}{2\delta}(k-1) \leq |P_i| \leq \frac{1}{2}(k+1) \qquad \forall i \in C.$$

Without the local spillover effect, the conditions of Theorem 2.1 determine the number of punishers allocated to each signatory in order to guarantee stability of an IEA. To give some intuition, the punishment set needs to be large enough in order to deter deviation (first inequality) while it cannot be too large in order

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[14] Note that the SGP condition would also entail that no player $j \in P_i$ has an incentive to unilaterally not carry out his punishment. This however is automatically satisfied as we assume $p \geq 1$ (see also Lemma 2.A.1 in the Appendix).

[15] Note that Equation (2.9) only has to hold for one element of the punishment set which may lead to results such that enlargement of the punishment group may actually benefit Condition (2.8). However, the results presented in this paper also hold for stronger versions of WRP (e.g., that (2.8) has to hold for all $j \in P_i$), which are not available in the literature so far.

to prevent renegotiation (second inequality). The conditions of Corollary 2.1 are equivalent to the conditions of Asheim and Holtsmark (2009), Theorem 1, with $s = k$ and $p = 1$.

Second, if we instead consider general spillover effects but very special networks, then similar observations can be made. For example the empty network $g = g^{\emptyset}$ is trivially equivalent to the case where no local spillover effects exist. Further, consider $g = g^N$, i.e. the complete network. Then, all countries experience the local spillover from a given country which immediately implies that this is equivalent to the case where the magnitude of the global spillover is $\beta + \gamma$ while there are no local spillovers. Hence, also for the case of the complete networks, the conditions for a WRP equilibrium are equivalent to the ones from Corollary 2.1.

Third, consider the case when the global spillover channel vanishes, that is $\beta \to 0$. Then, the game essentially boils down to a local spillover game where countries can only free-ride on the actions of their direct neighbors.

Corollary 2.2. *Let $\beta \to 0, \beta \neq 0$. The conditions of Theorem 2.1 on the punishment set P_i, $i \in C$ reduce to the following:*

$$\forall\ i \in C \qquad \sum_{m \in P_i \cap N_i} k_m \geq \frac{k_i^2}{2\delta},$$

$$\forall\ i \in C,\ \exists\ j \in P_i,\ s.t. \quad \sum_{m \in P_i \cap N_j} k_m \leq \frac{k_j^2}{2}.$$

When the global externalities vanish, the composition of the punishment sets becomes important. Since the global spillover effect becomes negligible, only neighbors have a deterring effect which implies that the set of punishers must have enough neighbors in C. The left-hand side of the first equation is then due to the fact that emission is increased from $x_j^S = \bar{x}_j - k_j\gamma$ to $x_j^P = \bar{x}_j - \gamma$ for $j \in P_i$ while the right-hand side, which is the incentive to deviate for country i, is determined by k_i. On the other hand, punishers shall not have an incentive to renegotiate, which is presented in the second condition. Incentives to renegotiate occur if the neighborhood structure of the punishers overlaps too much. Thus, given a punishment level, the set of punishers should be constructed such that their local spillover channels interfere minimally.[16]

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[16] Note that Corollary 2.2 does *not* apply to a pure local spillover game, that is for $\beta = 0$. The reason is that Condition (2.9) of Theorem 2.1, thus the condition for weak renegotiation-proofness, is no longer necessary: In the general model with strictly positive global spillovers, that is also for $\beta \to 0$, every country other than the punishers would strictly profit from a renegotiation away from punishment back to cooperation. For zero global spillovers, however, any country that is not among the punishers and not connected to a punisher will not strictly profit, therefore block renegotiation in accordance with Definition 2.1. This may seem rather counter-intuitive but is in accordance with the widely-accepted WRP notion given in Farrell and Maskin (1989). A slight adaption of this notion to avoid blocking of unaffected players allows also for applicability to the case where $\beta = 0$.

2.4.4 Comparative Statics

Abstracting from the special cases of only global respectively local spillovers, we further explore the meaning of the conditions for existence of a stable IEA by the coalition $C \subseteq N$ given in Theorem 2.1 by means of comparative statics. To understand the effect of the local spillover channel, i.e. the underlying network, we study the effect of additional links in the network on the conditions of subgame perfection and weak renegotiation-proofness. Further, we ask how an enlargement of the punishment group may impact these conditions, or more precisely, what the marginal effect of an additional punishing country is.

The effect of the spillover structure

First, consider the condition on subgame perfection (see Theorem 2.1, Equation 2.8). Take $i \in C$ and define the function $f_i(\delta, C, g, P_i)$ as the left-hand side of (2.8). The marginal effect of an additional link (currently not in the network) $lm \notin g$, $l, m \neq i$, on the subgame perfect condition of player i can be calculated to be,

$$f_i(\delta, C, g + lm, P_i) - f_i(\delta, C, g, P_i) = \delta \left(\beta\gamma \Big(\mathbb{1}_{P_i}(l) + \mathbb{1}_{P_i}(m) \Big) + \gamma^2 \Big(\mathbb{1}_{P_i \cap N_i}(l) + \mathbb{1}_{P_i \cap N_i}(m) \Big) \right),$$

where $\mathbb{1}_A(i)$ denotes the indicator function such that $\mathbb{1}_A(i) = 1$ if $i \in A$ and $\mathbb{1}_A(i) = 0$ else. The marginal effect is positive as long as the link lm involves at least one of i's punishers (i.e. $\mathbb{1}_{P_i}(l) = 1$ or $\mathbb{1}_{P_i}(m) = 1$), meaning that Condition (2.8) is more likely to hold for $i \in C$ after link addition since (2.8) requires $f_i(\delta, C, g, P_i) \geq 0$. Thus, the marginal effect of an additional spillover channel is largest if the link is between two punishers of i who are also neighbors with i and lowest if both are neither. The reason is that punishment increases if a punisher has an additional spillover channel, since emission reduction is higher in the non-punishment case. Moreover, neighbors cause a larger marginal effect for country i through the additional spillover channel.

While the marginal effect of link addition between two countries other than i on i's incentive to play the signatory action is unambiguously non-negative, the same is not so clear for the marginal effect if i itself is involved in the additional link:

$$f_i(\delta, C, g + im, P_i) - f_i(\delta, C, g, P_i) = \beta\gamma \Big(\mathbb{1}_{P_i}(m)\delta k - (k-1) \Big) + \frac{\gamma^2}{2} (\mathbb{1}_{P_i}(m) 2\delta - (2k_i + 1)).$$

Obviously, if a deviator i has additional spillover channels to non-punishers, i.e. $\mathbb{1}_{P_i}(m) = 0$, the effect is negative, since i is required to reduce more of its emission in the signatory action and, thus, more tempted to deviate. If instead the additional link is to a punishing player, i.e. $\mathbb{1}_{P_i}(m) = 1$, then punishment also increases. This has an additional deterring effect which clearly depends on δ such that the effect on subgame perfection is negative as long as the discount factor δ is small enough, i.e. $\delta \leq \bar{\delta}(g) = \frac{k-1+\frac{\gamma}{\beta}(k_i+\frac{1}{2})}{k+\frac{\gamma}{\beta}}$. Note that the marginal effect is negative for all discount factors if $\bar{\delta}(g) \geq 1$, which holds for large enough k_i and marginal local spillovers γ.

Next, we turn to the second condition of stability of Theorem 2.1, i.e. the condition that ensures weak renegotiation-proofness. Considering a deviator $i \in C$ and a punisher $j \in P_i$, we define $h_{ij}(\delta, C, g, P_i)$ as the left-hand side of (2.9). The marginal effect of an additional link $lm \notin g$ on the incentives of a punisher j of deviator i is then given by

$$\begin{aligned} h_{ij}(\delta, C, g+lm, P_i) - h_{ij}(\delta, C, g, P_i) = \beta\gamma\Big(\mathbb{1}_{P_i}(l) + \mathbb{1}_{P_i}(m)\Big) \\ + \gamma^2\Big(\mathbb{1}_{P_i \cap N_j}(l) + \mathbb{1}_{P_i \cap N_j}(m)\Big). \end{aligned}$$

Since the marginal effect is positive if at least one link involves a punisher of i, the condition preventing renegotiation of country $j \in C$, (2.9), is less likely to hold after link addition since (2.9) requires $h_{ij}(\delta, C, g, P_i) \leq 0$. The marginal effect of an additional link between l and m on the incentives of $j \in P_i$ to renegotiate is largest, when both l and m are punishers and neighbors of j.[17] If there is an additional link between two countries that are not in the punishing group P_i, this has obviously no impact on the WRP condition for j. Thus, an overlapping spillover structure of punishing group P_i makes renegotiation more attractive (and thus makes the IEA vulnerable to renegotiation) since the profit under cooperation increases. This effect is fostered if there is also a connection to the punisher j, as decreasing costs through local spillovers increase j's incentives to renegotiate and not carry out the punishment.

Since we study the incentives for j to renegotiate, it makes a difference if j itself is part of the additional spillover. We obtain

$$h_{ij}(\delta, C, g+jm, P_i) - h_{ij}(\delta, C, g, P_i) = \begin{cases} -\beta\gamma(k-1) - \gamma^2(k_j + \frac{1}{2}), & m \notin P_i \\ \beta\gamma - \gamma^2(k_j - k_m - \frac{1}{2}), & m \in P_i \end{cases}.$$

For j's incentive itself to renegotiate, the effect of additional links is ambiguous. First, if the additional link leads to a non-punisher of i, then j has lower benefits from cooperation compared to her Nash action making renegotiation less attractive. If instead the additional link is to a punisher of i, then j also suffers

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[17] Note that here $m = i$ is not excluded. But since i cannot be part of the punishment group, we always have $\mathbb{1}_{P_i}(i) = 0$.

from the punishment level of the additional neighbor during the punishment phase, which harms j through the local spillover channel and hence works in the opposite direction to make renegotiation more attractive.

We can conclude: Given the punishment group, higher density of the spillover structure within the coalition facilitates subgame perfection while it harms the renegotiation-proofness condition for most countries. Note, however, that this does not necessarily hold for the potential deviator respectively a potential punisher who is involved in the additional link. So while the subgame perfection condition (2.8) has to hold for all $i \in C$ and the renegotiation-proofness condition (2.9) for at least one $j \in P_i$, the overall effect of link addition on both stability conditions of an IEA may be ambiguous.

Additional punishers

In order to determine an individual, optimal punishment group for every possible deviator of a coalition, we also have to understand what the marginal effect of an additional punisher is for the two stability conditions for an IEA. First, we study the effect on the SGP condition (2.8). We have the following marginal deterring effect of an additional punisher l on the deviator i:

$$f_i(\delta, C, g, P_i \cup \{l\}) - f_i(\delta, C, g, P_i) = (\beta + \mathbb{1}_{N_i}(l)\gamma)(\beta(k-1) + \gamma k_l).$$

Obviously, any additional punisher will decrease the incentive of country $i \in C$ to deviate from the signatory strategy and if the punisher is one of i's neighbors, then the additional spillover channel increases this effect. For the WRP condition (2.9), we have the following marginal effect of an additional punisher l on the incentive to renegotiate for a punisher $j \in C$:

$$h_{ij}(\delta, C, g, P_i \cup \{l\}) - h_{ij}(\delta, C, g, P_i) = (\beta + \mathbb{1}_{N_j}(l)\gamma)(\beta(k-1) + \gamma k_l).$$

Here, the more punishers the higher incentives to renegotiate, and, again, this effect is enhanced if the punishers are also neighbors.

The comparative statics have shown that there is a trade-off in characterizing the optimal punishment group for each coalition-member: The more punishers and the higher the connectedness among them, the higher the threat of punishment and the easier to sustain an SGP equilibrium. In turn, incentives to renegotiate increase with the size of the punishment group and its clustering.

2.5 The Stability of a Global IEA

Having determined general conditions for the stability of an IEA, the question of existence of such a stable IEA has not yet been answered. We focus here on the stability of a worldwide IEA, i.e. an IEA where all countries play the signatory strategy and have no incentive to deviate or renegotiate. However, it is rather obvious that not all network structures allow for stability of a global IEA. For instance, from Example 2.1 we already know that the center player of a star network prefers no cooperation to global cooperation if the local externality is large enough, i.e. $\gamma \geq \beta \frac{n-1}{n-3}$. This immediately implies that a subgame perfect equilibrium supporting global cooperation cannot exist in the repeated game for $\gamma \geq \beta \frac{n-1}{n-3}$. Clearly, adding the WRP condition (2.9) to this makes the existence of a stable global cooperation even more restrictive. In fact it can be shown that for large star networks, a WRP supporting global cooperation fails to exist.

Proposition 2.1. *Consider the star network $g^*(n)$ and let γ and β be independent of n. Then for $n \to \infty$, there does not exist a stable global IEA.*

The intuition behind this result is that as n grows, the players become more and more heterogeneous in terms of degree. While the center player of the star has to reduce her emission increasingly in the number of her neighbors, the set of punishers has to grow as well in order to deter deviation by the center player. This, however, gives increasing incentives to renegotiate – implying non-existence of a stable global IEA.[18] This is a fundamental difference to the existing models in the literature that study the possibility of global cooperation as a stable outcome of the climate game. Unlike in Asheim and Holtsmark (2009), we have shown that for very asymmetric networks such as the star network, global cooperation may fail to be a WRP equilibrium for the specific punishment rule.

Hence, it seems to be the asymmetry of the network – in particular the asymmetry of degrees – which leads to failure of a global IEA. Instead, we may also look at the other extreme case of spillover networks where there are no heterogeneities in terms of degree, i.e. a network where the number of neighbors of all players are the same. Such a structure is defined as a *regular* network.

Example 2.2. Consider a regular network of $n = 12$ players with $\eta_i = 4 \; \forall \; i \in N$, illustrated in Figure 2.2. Let $\beta = \gamma$ and for simplicity $\delta \to 1$. Which punishment sets P_i sustain full cooperation as a WRP equilibrium?

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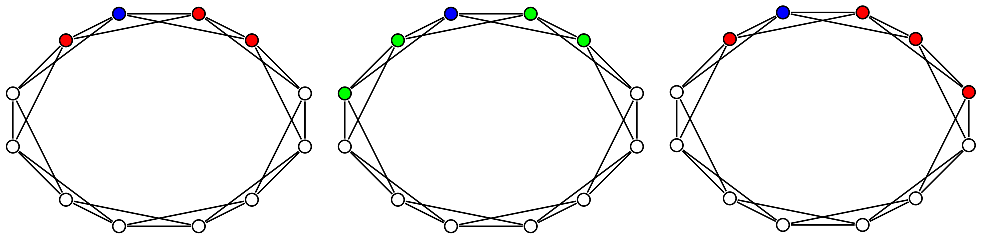
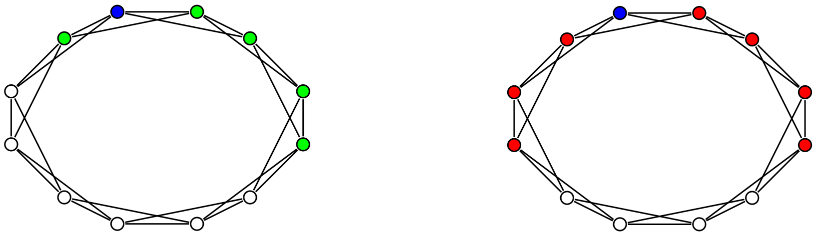
[18] This result is not restricted to the star network only. Suppose the parameter setting is such that we can sustain full cooperation as a WRP equilibrium in the star network. Then, as seen in the comparative statics section above, the addition of one single link may change the marginal incentives such that the punishment structure needs to be redesigned and may end up to not sustain full cooperation as a WRP equilibrium. For instance, in the 5-player star network with $\gamma = 2\beta$, full cooperation is a WRP equilibrium. However, if two peripheral nodes are linked, this is no longer the case.



$P_i \subset N_i$ $P_i = N_i$ $P_i = N_i \setminus \{j_1\} \cup \{l\}$



$P_i = N_i \setminus \{j_1\} \cup \{l_1, l_2\}$ $P_i = N_i \cup \{l_1, l_2, l_3\}$

Figure 2.2: Different scenarios in a circle network. Green nodes represent a WRP punishing set.

The conditions for the global IEA to be a WRP equilibrium (see Theorem 2.1) read

$$\forall\ i \in N: \quad |P_i| + |P_i \cap N_i| \geq 7.5, \tag{2.10}$$

$$\forall\ i \in N, \exists\ j \in P_i: \quad |P_i| + |P_i \cap N_j| \leq 8.5. \tag{2.11}$$

Denote the neighbors of player i by $N_i = \{j_1, j_2, j_3, j_4\}$. While for $P_i \subset N_i$, $P_i \neq N_i$, the punishment can be calculated to be too low, choosing $P_i = N_i$ both Conditions (2.10) and (2.11) are satisfied:

$$(2.10) \Leftrightarrow 4 + 4 = 8 > 7.5,$$

$$(2.11) \Leftrightarrow 4 + 1 = 5 < 8.5.$$

Another possible punishment set of a stable global IEA can be calculated to be $P_i = N_i \setminus \{j_1\} \cup \{l_1, l_2\}$ with $l_1, l_2 \notin N_i$. However, punishment sets like $P_i = N_i \setminus \{j_1\} \cup \{l_1\}$ with $l_1 \notin N_i$ do not satisfy (2.10) while $P_i = N_i \cup \{l_1, l_2, l_3\}$ with $l_1, l_2, l_3 \notin N_i$ do not satisfy (2.11). The different scenarios are displayed in Figure 2.2.

While for very asymmetric network structures a stable global IEA may fail to exist (see Proposition 2.1), a symmetric network structure as given in Example 2.2 allows for stability of the global IEA. If symmetry in the degree is given, these findings can indeed be generalized by considering regular networks such that $\eta_i = \eta_j = \eta$ for all $i, j \in N$.

Focusing on the global IEA in regular networks yields that $k_i = \eta$ for all $i \in N$, implying that Conditions (2.8) and (2.9) simplify to

$$\forall\ i \in N: \qquad \delta\left(|P_i \cap N_i|\gamma + |P_i|\beta\right) \geq \tfrac{1}{2}\left(\beta(n-1) + \gamma\eta\right), \quad (2.12)$$

$$\forall\ i \in N, \exists\ j \in P_i: \quad \Big(|P_i \cap N_j|\gamma + (|P_i| - 1)\beta\Big) \leq \tfrac{1}{2}\left(\beta(n-1) + \gamma\eta\right). \quad (2.13)$$

Using the results from the comparative statics analysis, we can now directly derive an existence result for a stable global IEA in regular networks.

Proposition 2.2. *Let $\delta \to 1$. Then, for every regular network there exists a stable global IEA.*

This result underlines the intuition that in very symmetric settings it is easier to achieve cooperation than in very asymmetric network settings such as the star. More specifically, this proposition yields that if countries are sufficiently patient, we can always find a punishment set P_i such that the simple strategy profile $\mathbf{s^N}$ where all countries play the signatory strategy constitutes a WRP equilibrium of the infinitely repeated game, meaning that such a global IEA is stable.[19]

Note that the condition $\delta \to 1$ is not binding. It is impossible to consider all possible punishment sets for all possible regular networks, but it is easy to argue that the complete network is the most restrictive case, since there the spillover structure maximally overlaps as all spillover channels are present.[20] For the complete network, the threshold value for the discount factor δ can be easily determined and coincides with the the threshold in Asheim and Holtsmark (2009) since full cooperation can be established as a WRP equilibrium if the discount factor δ fulfills the following conditions:

$$\delta \geq \frac{n-1}{n+1} \quad \text{for } n \text{ odd,}$$
$$\delta \geq \frac{n-1}{n} \quad \text{for } n \text{ even.}$$

Thus, if $\delta \geq 1 - \frac{1}{n}$, a global IEA is stable in the complete network.

We conclude that whenever countries are homogeneous with respect to the number of neighbors in the network, global cooperation can be a WRP equilibrium. As derived in the comparative statics section, the more asymmetric the network becomes, the harder it is to restrain countries from renegotiation. For

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[19] Note that in the proof of Proposition 2.2 we even show that it is possible to find a stable global IEA such that none of the punishers want to renegotiate. Obviously demanding (2.9) to hold for all $j \in P_i$ is more restrictive and, if applied to the definition of WRP equilibria, would yield a stronger equilibrium concept. In particular it prevents unreasonable equilibria to appear for instance by adding isolated countries to the punishment sets.

[20] In network terminology, clustering is equal to 1.

example, in the very asymmetric case of the star network, no global IEA can be a WRP equilibrium and hence stable.

However, when the marginal local impact becomes negligible, the network structure, even if very asymmetric, becomes less significant, implying that global cooperation can be sustained in all networks.

Proposition 2.3. *Let $\delta \to 1$ and the local spillover parameter γ be small enough. Then for every network there exists a stable global IEA.*

The result comes without proof since it follows immediately from Asheim and Holtsmark (2009), where $\gamma = 0$ is assumed and holds by continuity in γ.

2.6 Social Benefits and Costs

2.6.1 Social Benefits

While we have shed some lights on the conditions for individual rational behavior with respect to membership in a coalition, it is important to know for, e.g., policy implications what the collective or total welfare effect of an International Environmental Agreement is. We consider the utilitarian welfare composed of the sum of all countries' utilities when the equilibrium path is followed, which is given by

$$\begin{aligned}\mathcal{W}(x(C)) =& n\Big(\frac{1}{2}(\gamma^2 - \beta^2)\Big) - n\beta \sum_{i\in N} \bar{x}_i + \gamma\beta\Big(\sum_{i\in N} \eta_i + n^2\Big) + \beta^2 n^2 + \gamma^2 \sum_{i\in N} \eta_i \\ & + \beta^2 k(k-1)\Big(n - \frac{1}{2}(k+1)\Big) + \beta\gamma \sum_{i\in N} k_i(k-1) + \gamma^2 \sum_{i\in N}\sum_{m\in C} \bar{g}_{im} k_m \\ & + \beta\gamma n \sum_{m\in C} k_m - \frac{1}{2}\gamma \sum_{m\in C} k_m(2\beta k + \gamma k_m + 2\gamma) - \gamma \sum_{i\in N}\sum_{j\in N} \bar{g}_{ij}\bar{x}_j.\end{aligned}$$

Since in the global IEA all countries already maximize $\mathcal{W}$, it is immediate to see that the global IEA maximizes welfare. Further, because emission reduction always has a positive effect on all and the members of a coalition maximize their sum of utilities, it is also quite immediate to see that every IEA by a coalition C yields higher welfare than an IEA of a subset of C. Given an IEA of a set of signatories C, the marginal effect of an additional member m on welfare can be calculated to be

$$\begin{aligned}&\mathcal{W}(x(C+m)) - \mathcal{W}(x(C)) = \sum_{i\in N} \Delta_i(C+m, C) \\ =&\beta^2 k\Big(2n - \frac{3}{2}(k+1)\Big) + \beta\gamma\Big(\sum_{i\notin C} k_i + k_m(2n-2-k)\Big) + \frac{\gamma^2}{2} k_m(k_m - 1)\end{aligned}$$

which is obviously positive.

Further, it is easy to see that increasing spillovers, either through the relative effect β or γ, or the spillover structure (by adding links to the network) have negative effects on welfare.

2.6.2 Social Costs of Punishment

Besides the social benefits of an IEA by a coalition C there are also social costs whenever a country needs to be punished. Although this is off-equilibrium, one might ask what the welfare effect of punishment is and who should punish in cases when there is more than one possible punishment group that sustains global cooperation (see e.g., Example 2.2).

Therefore, assume a player j has deviated and the set of punishers P_j is called upon to punish. To study the effect of an additional punisher, denote by $\Delta\mathcal{W}\left(P_j + i, P_j\right)$ the marginal effect on welfare when player i joins the set of punishers P_j.

Lemma 2.1. *Suppose player j has deviated. Then,*

$$\Delta\mathcal{W}\left(P_j + i, P_j\right) = -\frac{1}{2}\Big(\beta(n-1) + \gamma\eta_i\Big)^2.$$

Now when allocating the set of punishers, the question arises who should punish; neighbors or non-neighbors? Recall that the marginal deterring effect of an additional punisher $i \in N$ on deviator $j \in N$ is given by

$$f_j(\delta, C, g, P_j + i) - f_j(\delta, C, g, P_j) = (\beta + \mathbb{1}_{N_j}(i)\gamma)(\beta(k-1) + \gamma k_i).$$

Then it is clear that in order to achieve an equal deterring effect, a non-neighbor $m \notin N_j$ must punish more, i.e. have more neighbors than a neighbor $i \in N_j$, i.e. $\eta_m > \eta_i$ which implies higher social costs. Hence, consider the case that two instead of one non-neighbor punish.[21] Similarly to above, if we have $\eta_m > \eta_i$ for a country $m \in P_j$, $m \notin N_j$, then the social cost of punishment will be larger when the non-neighbors punish. Instead, consider the case where both non-neighbors have smaller degree than a punishing neighbor, but together achieve the same deterring effect. The following result characterizes conditions on β and γ such that it is socially optimal to have a neighbor with higher degree punish.

Proposition 2.4. *Suppose that $\beta \leq (1+\sqrt{2})\gamma$. Then, punishment of a deviator by one of its neighbors is socially preferred to punishment by one or two non-neighbors such that the deterring effect is the same.*

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[21] Of course, this may also have negative effects on the WRP condition since potentially two punishers' neighbors instead of one join the set of punishers. Here, however, we are only interested in the social cost of punishment.

Thus, if the global spillover effect β is not too large relative to the local spillover effect γ, it will be better in terms of welfare to have neighbors punish instead of non-neighbors since to achieve the same deterring effect, total punishment emission is higher when non-neighbors punish.

2.7 Extensions

2.7.1 Other Punishment Strategies

Besides the very specific penance punishment strategy we consider in our analysis in the main article, there are of course numerous other ways to punish a possible deviator. Here, we want to discuss two possible variations of punishment strategies and their implications for the existence of stable IEAs in the repeated game.

Stronger Punishment

We have seen that in very asymmetric networks, such as the star, there is no stable global IEA with punishment levels $x_j^P = x_j^{NS}$. Let us now consider what happens if the punishment level that is emitted by the punishers is larger than their respective Nash output, i.e. what if $x_j^P > x_j^{NS}$ holds?

First, consider again the example of the star network.

Example 2.3. Let $\gamma = 1.5\beta$, $n = 5$, $\delta \to 1$ and $g = g^*(n)$. Suppose we want only three peripheral nodes to punish the center node i, i.e. $|P_i| = 3$, and let us now determine the required punishment level p^* that yields a punishment strategy that sustains full cooperation as a WRP equilibrium. The center i has no incentive to unilaterally deviate from the signatory emission level if

$$\begin{aligned} (2.8) \quad &\Leftrightarrow \quad 3(5-p) + 1.5(3(7-2p)) + 2.25(3(2-p)) \geq 50 \\ &\Leftrightarrow \quad p^* \leq 0.53. \end{aligned} \tag{2.14}$$

is fulfilled. For the WRP conditions we have

$$\begin{aligned} (2.9) \quad &\Leftrightarrow \quad (5-p)(3-p) + 1.5((3-p)(1-p)+2) + \frac{2.25}{2}(1-p)^2 \leq 15.125 \\ &\Leftrightarrow \quad p^* \geq 0.6, \end{aligned}$$

which contradicts the necessary condition for subgame perfection given by (2.14). Thus, $|P_i| = 3$ does not yield a different result.

The equivalent considerations for $|P_i| = 2$ and $|P_i| = 1$ also yield that the necessary conditions from subgame perfection conflict with the conditions for weak renegotiation-proofness. Therefore, in this setting global cooperation fails to be a WRP equilibrium for any punishment level if only emitted for one period.

A more general statement, though, is not possible as the comparative static effects on the conditions for subgame perfection and weak renegotiation-proofness work in opposite directions for decreasing p. We therefore conclude that our restriction on Nash punishment levels is not too critical. Moreover, as mentioned before, it is obvious that even with these simple strategies the design of suitable punishment strategies is anything but straightforward as proposed in previous papers.

Longer Punishment

As a second variation, consider punishment strategies that punish a deviator for more than one period. More specifically, we change the simple strategy profile $\mathbf{s}$ to $\mathbf{s}^T$ such that we allow punishments over multiple but finite periods $T \in N$, i.e. the punishment path is given by

$$\mathbf{p_i^C}(T) = \Big\{ \underbrace{\Big(x^P_{P_i}, x^S_{C\setminus P_i}, x^{NS}_{N\setminus C}\Big), \ldots, \Big(x^P_{P_i}, x^S_{C\setminus P_i}, x^{NS}_{N\setminus C}\Big)}_{T \text{ periods}}, \Big(x^S_C, x^{NS}_{N\setminus C}\Big), \ldots \Big\}.$$

We assume that if during a punishment phase a new deviation occurs, either by the same or by another player, punishment switches to the beginning of the punishment path of that player. The conditions for subgame perfection and weak renegotiation-proofness then read as follows. First, there are no unilateral deviations from the equilibrium if

$$\sum_{t=1}^{T} \delta^t \Big(\pi_i(x^S_C) - \pi_i(x^S_i, x^P_{P_i}, x^S_{C\setminus P_i})\Big) \geq \pi_i(x^{NS}_i, x^S_{C\setminus\{i\}}) - \pi_i(x^S_C) \tag{2.15}$$

is satisfied for all $i \in N$. Furthermore, deviations from the punishment are deterred if

$$\begin{aligned}\delta^T \Big(\pi_j(x^S_C) - \pi_j(x^S_j, x^P_{P_j}, x^S_{C\setminus P_j})\Big) + \sum_{t=1}^{T-1} \delta^t \Big(\pi_j(x^P_{P_i}, x^S_{C\setminus P_i}) - \pi_j(x^S_j, x^P_{P_j}, x^S_{C\setminus P_j})\Big) \\ \geq \pi_j(x^S_j, x^P_{P_i\setminus\{j\}}, x^S_{C\setminus P_i}) - \pi_j(x^P_{P_i}, x^S_{C\setminus P_i})\end{aligned} \tag{2.16}$$

is fulfilled for all $j \in P_j$ and for all $i \in N$. Finally, for weak renegotiation-proofness, we need for all $i \in N$ at least one $j \in P_i$ such that

$$\sum_{t=0}^{T-1} \delta^t \Big(\pi_j(x^P_{P_i}, x^S_{C\setminus P_i}) - \pi_j(x^S_C)\Big) \geq 0$$

is satisfied, which is obviously equivalent to the original Condition (9) of Theorem 1 with only one punishment period, i.e. $T = 1$. Consequently, the extension of the punishment period has no effect on the renegotiation incentives of the punishers.

Meanwhile, an increase in punishment periods does influence the conditions for subgame perfection: The series on the left-hand side of (2.15) is equal to

$$\frac{\delta(1-\delta^T)}{1-\delta}\Big(\pi_i(x_C^S)-\pi_i(x_i^S,x_{P_i}^P,x_{C\setminus P_i}^S)\Big),$$

which obviously increases for larger T but is bounded from above by

$$\frac{\delta}{1-\delta}\Big(\pi_i(x_C^S)-\pi_i(x_i^S,x_{P_i}^P,x_{C\setminus P_i}^S)\Big).$$

We receive that independent of the parameters, an extension of the punishment yields that fewer punishers are sufficient to deter a country from deviating from the signatory strategy. In line with the folk theorem we can conclude that in any given network g, if players are patient enough, i.e. if δ is sufficiently large, we can always find a duration of punishments T such that for (2.15) to be fulfilled, a single punisher is sufficient. Also, this punisher does not need to be a neighbor.

Additionally, for small enough P_i, the left-hand side of (2.16) is always positive such that we can have that for a large enough T, (2.16) is always satisfied, too. Thus, we can achieve full cooperation as a subgame perfect equilibrium. As WRP is not affected and for $|P_i| = 1$ it is always satisfied, we can conclude without proof the following proposition:

Proposition 2.5. *For δ sufficiently large, for every network g there exists a duration of punishments T such that the simple strategy profile $\mathbf{s}^T$ is a stable global IEA.*

Note, however, that this result requires very long punishment and hence lots of pollution. Such a threat might not be credible, if e.g. the negative consequences of accumulated pollution are increasing in the amount of pollution. We therefore conclude again that our restriction on the simple strategy profile is not too restrictive and already offers several interesting insights into the structure of the model.

Example 2.4. Let $\gamma = 1.5\beta$, $n = 5$, $\delta \to 1$ and $g = g^*(n)$. We have seen that for $p = 1$, the global IEA is not stable and also harsher punishment has not changed this result due to the large asymmetry between the center and peripheral nodes. Yet, by Proposition 2.5, it is possible to find $T > 1$ such that the global IEA is stable: To see this, let $T = 2$. Then, as WRP remains unchanged, for the center node i the punishment group must not be larger than 3. Condition (2.15) for the center node yields $P_i \geq 2$ so it remains to check Condition (2.16). Let us choose $P_i = 3$. For any peripheral node j, condition (2.15) yields that the center node is sufficient to punish, i.e. $|P_j| = 1$. Then, with these punishment sets given, (2.16) is satisfied for the center and also for the peripheral nodes. Thus, when the punishment phase is extended to two periods, the global IEA is stable in this network.

2.7.2 More General Networks

For the sake of simplicity, we have only considered undirected networks, i.e. local spillovers were assumed to be bidirectional. Of course, this may often not be the case. For example, the direction of wind plays an important role for the effects of air pollution and the direction a river flow influences the pollution effects along the stream.

Additionally, we can also consider heterogeneities with respect to the scale of local spillover effects between countries. While some countries may have a very strong local spillover effect, others may only have little impact on its neighbors' environmental costs. For instance, these heterogeneities may be connected to the size of the country.

Does generalizing the unweighted network assumption to weighted and directed networks qualitatively change our results? To illustrate this, let $W = (w_{ij})_{i,j \in N}$ with $w_{ij} \in \mathbb{R}_+$ denote the weighted network such that w_{ij} denotes the impact of the spillover from country i to country j which is allowed to be different to w_{ji}. Note that our undirected, unweighted setup in the paper can be expressed in terms of these matrices W such that W is symmetric and all entries are either 0 or 1, while unweighted but directed networks do not require the symmetry property. Assuming a general W, the environmental costs are given by

$$\tilde{K}_i(x_i, x_{-i}) = \beta \sum_{j \in N} x_j + \gamma \sum_{j \in N} w_{ij} x_j.$$

Note that in accordance to our model assumptions we assume $w_{ii} = 1$.

While in unweighted networks, the number of intra-coalition links k_i is crucial for the outcomes as k_i determines the abatement efforts, this naturally translates to the sum of weights within a coalition C. Let us denote this number by $w_i^C = \sum_{j \in C, j \neq i} w_{ij}$. We therefore receive a signatory's output to be $x_i = \bar{x}_i - \beta k - \gamma(w_i^C + 1)$ while a non-signatory's output remains unchanged, since $w_{ii} = 1$. Further, the impact of a punishment from a punishing country j to a deviator i matters for both conditions of Theorem 1, since for weighted networks the local component of the punishment is determined by the impact of the spillover w_{ji}. Hence, analogous calculations as in Theorem 1 show that for weighted networks the WRP conditions read:

$$(2.8) \Rightarrow \delta \left[\beta^2 |P_i|(k-p) + \beta\gamma \left(|P_i|(1-p) + \sum_{m \in P_i} \left(w_m^C + w_{mi}(k-p) \right) \right) \right.$$

$$\left. + \gamma^2 \left(\sum_{m \in P_i} w_{mi}(w_m^C + 1 - p) \right) \right] - \frac{1}{2} \left(\beta(k-1) + \gamma w_i^C \right)^2 \geq 0$$

$$(2.9) \Rightarrow \beta^2 (k-p)(|P_i| - p) + \beta\gamma \left((|P_i| - p)(1-p) + \sum_{m \in P_i \setminus \{j\}} \left(w_m^C + w_{mj}(k-p) \right) \right)$$

$$+ \frac{\gamma^2}{2} \left(2 \sum_{m \in P_i} w_{mj}(w_m^C + 1 - p) + (1-p)^2 \right) - \frac{1}{2} \left(\beta(k-1) + \gamma w_j^C \right)^2 \leq 0.$$

The total effect of both incoming and outgoing local spillovers will now determine a country's incentives to sign an IEA or not. If there are large asymmetries, the global IEA may not be stable but if all countries suffer equally i.e. $w_i^N = w$ for all $i \in N$, the global IEA can be stable for sufficiently large discount factors δ (Proposition 6). Thus, our results nicely generalize to directed and weighted networks.

2.7.3 Formation of an IEA

In the main article we have restricted the analysis to the question of stability of IEAs by means of WRP equilibria without modeling the formation of an IEA. We briefly and informally outline here, how such an IEA could come into place. First, of course, we could always imagine a climate conference where the local spillover structure is taken into account. Since there are potentially many equilibria of the repeated game even for a given coalition, it is difficult to model the strategies used by the countries to select among the WRP equilibria of the repeated game.

In fact we could also imagine that a small subset of all countries (e.g. the US and Canada, or the countries within the EU) start out with a coalition to obtain a critical mass and then approach countries outside the coalition, particularly those exposed to local spillovers of the coalition by offering those countries to reduce emission if they themselves do so. In other words, they threaten punishment by non-implementation of an IEA (which is equivalent to business as usual) if other countries do not reduce themselves. Thus, given a coalition C_1, rank all other countries $i \in N \setminus C_1$ by the ratio $\frac{|N_i \cap C_1|}{|N_i|}$. Coalition C_1 then agrees to the terms of an IEA conditional on additional countries joining. Those with large ratio $\frac{|N_i \cap C_1|}{|N_i|}$ are the ones that are most likely to join C_1 since for them the SGP condition (9) is easiest to be satisfied by the threat of non-implementation of signatory strategies of C_1. Thus i_1 would join C_1 if there exists a punishment set $P_{i_1} \subset C_1$ such that the conditions of Theorem 1 are satisfied. After acquiring the highest-ranked country i_1 to C_1, for $C_2 = C_1 \cup \{i_1\}$ repeat the procedure for C_2 etc.

Since small coalitions are easy to sustain even without threats of future punishment (see Section 3.2), it may be the case that such a procedure actually leads to the implementation of a global IEA (if stable for the given spillover structure). In this way, an initially small IEA may spread to a large IEA, i.e. from an initially local IEA can emerge a global IEA (see also Section 3.4.2 in Currarini et al., 2014).

The idea of small coalitions supporting the build up of global climate cooperation is also present in the recently growing literature on climate clubs. In Victor (2011), a "carbon club" is proposed which may be small at the start but could grow over time. A crucial factor for the success of such climate clubs is that all members derive an exclusive benefit and that non-members can be excluded from the benefits or even be penalized for not participating. As emissions are a public bad and in our model the only independent variable, our model does not allow for this exclusiveness and therefore our outline above can only be seen as an approximation of a climate club approach. For a more precise treatment of climate clubs, other, exclusive benefits must be introduced to incentivize countries to join or so-called "external penalties", unconnected to the emission game, must be adopted to punish non-signatories, for instance by border (carbon) tax adjustments. This is not considered in our model and we refer to Nordhaus (2015) for a more formal approach.

2.8 Conclusion

We have merged local and global pollution spillovers into one model by introducing a network structure. In the repeated game, we define an IEA as a strategy profile where signatories aim to play in each stage the action which maximizes their utilitarian welfare. Using a specific punishment strategy, weakly renegotiation-proof agreements can be achieved via the threat of punishments. If the punishing countries suffer too much from punishment themselves, they may want to renegotiate. To account for this, we characterize an individual group of punishing countries for each signatory and therefore decrease the incentives to renegotiate. However, when the network is very asymmetric, as for example in the star network, full cooperation may not be a weakly renegotiation-proof equilibrium. In turn, when players are symmetric with respect to their spillover impacts and sufficiently patient, a global IEA can be stable.

We have also studied welfare implications of the network structure. More links in the network have a negative impact on global welfare as the local spillover effects outweigh the higher efforts by signatories that internalize the additional externality. Furthermore, we extend our model in several ways. First, we consider other punishment strategies. We show that if countries can choose stronger punishments as their Nash output level, fewer countries are necessary to achieve the same deterrence effect. In the star network, however, this does not change the results. If, instead, countries are allowed to punish for more than one period, then less countries are necessary to punish a deviation implying that for large enough discount factors, we can always find a duration of punishments such that in any network full cooperation is a WRP equilibrium. Secondly, we abstract from the undirected and unweighted network setting, and show that our results nicely generalize to directed and weighted networks. Thirdly, we discuss the formation of an IEA and elaborate how the local spillover structure can be taken into account to achieve a stable global IEA.

Due to the generality of our approach, our model can serve as benchmark model which should be extended and refined in the future. Yet we can already see that, along the lines of Bollen et al. (2009), a pollution policy that takes account of the effects of both global and local (air) pollution can help sustain global cooperation and ultimately increase global welfare. By taking into account the local spillover structure, punishment mechanisms can be designed more appropriately and therefore help deter countries from free-riding without making the agreement vulnerable to renegotiation. In this way, a few countries (e.g., US and Canada or EU countries), who initially agree to certain terms of reduction conditional on others joining them, may achieve global cooperation by particularly taking the spillover structure into account. Our model is also not limited to the application in the strive for joint emission reduction. It can easily be adapted to other problems in the provision of public goods.

It can also be extended to analyze a stock pollutant rather than a flow pollutant as we do in this paper. Assuming a linear law of motion for the stock of pollution, as it is standard in the literature, and using the same profit-function of the stage game, we obtain a linear state differential game. Thus, equilibrium strategies will be constant with respect to the state variable and the results for non-cooperative and full-cooperative outputs will not change. Note that to capture the local and global externalities, we would need to include one state variable capturing the global stock of emissions, and n state variables that capture the local emission stock for each country.[22]

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[22] See for example Mason et al. (2016) and Kratzsch et al. (2012), who analyze self-enforcing IEAs in a differential game and show that a penance punishment strategy, can yield full cooperation for discount factors large enough.

Finally, let us remark, that we have restricted our study of IEAs such that we allowed countries to have only one choice variable: their emission level. Therefore in the repeated game framework, punishment can only be executed in terms of increased emission. In reality, it is not so clear that countries are actually flexible enough to adjust their emission levels accordingly. However, countries usually also have other threats to deter deviation. One example are trade sanctions imposed on other countries, respectively signatories of an IEA. In a quite elaborated model, Barrett (1997) shows that trade sanctions may be an efficient tool since the threat of trade sanctions may be sufficient to incentivize countries to join the IEA and thereby sustain full cooperation. We have shown in our reduced model that even though countries are only able to punish by increasing their emissions, it is still possible to sustain cooperation via a WRP equilibrium (if the network structure is not too asymmetric, or if punishment is long enough). Allowing for trade sanctions by also using the network structure, could be a valuable extension of our model. In this case, the network could also represent an established trade structure that enables countries to link pollution and trade strategies (*issue-linkage*). We expect such an extension to strengthen our results and to make full cooperation even more likely to emerge.

There are various other ways how our model may be extended. As noted in Currarini et al. (2014), there is a large potential for network economics to be applied in environmental economics. In the following, we want to discuss several of the aspects that could emerge from our model.

First, some simplifications we have taken may be relaxed. Of course, further heterogeneities imply less analytical tractability but as it is frequently done in the IEA literature, simulations could be considered to compare the outcomes of our model to other existing ones. Furthermore, other punishment strategies and the possibility of multiple coalitions may be worth studying, too.

Regarding multiple coalitions, as long as we strive for the socially optimal outcome of global cooperation, there is no need for more than a single agreement. However, whenever global cooperation cannot be sustained as a self-enforcing equilibrium, one could study what happens if multiple coalitions formed. In the case of linear costs of pollution, one could reach an outcome where every country is a signatory in a possibly only very small coalition. Then, individual contributions to abatement may not substantially improve the business as usual outcome as all countries only account for very few externalities of their coalition members.

The extension of the model to incorporate transfers and side-payments is also very natural. One could then interpret the underlying network structure, i.e. the links between countries, also as established ways of communication or negotiation through which countries can offer side-payments to incentivize non-cooperators to join the coalition.

Moreover, while reduction of emissions is one way to contribute to the global effort of fighting climate change, investments in R&D are another possibility to mitigate pollution. And as it is standard in the (network) literature, spillovers from R&D play an important role in the decision of optimal investments. Thus, bringing together the literature of R&D spillovers and the mitigation of pollution through an IEA is another possible extension of our model.

Finally, there already exist some models that study local and regional agreements that may lead to global cooperation. Methods from Evolutionary Game Theory have been used to study whether or not local agreements may facilitate the formation of global cooperation.[23] By applying results from opinion formation in a network, our model may serve as an approach to better understand the chances of such a formation process. In our benchmark model we only consider the formation of a single IEA, but the extension to multiple coalitions should be natural and thus offer a promising area of future research.

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[23] Regional agreements and initiatives have been formed to tackle the problem of regional pollution effects (visit the Global Atmospheric Pollution Forum online for a list of regional initiatives worldwide). One example is the “Climate and Clean Air Coalition” that strives for a reduction of short-lived air pollutants and has been gaining influence over the past years.

Appendix

2.A Proofs

Proof of Theorem 2.1. Let **s** be an IEA by a coalition $C \subset N$. To be stable, **s** needs to be a subgame perfect (SGP) equilibrium which is weakly renegotiation-proof.

First, consider the SGP condition and note that there does not exist a non-signatory $j \in N \setminus C$ who has an incentive to deviate from playing its Nash action. Hence, in order for **s** to be SGP, the signatories $i \in C$ shall not have an incentive to deviate. For this, there are two conditions that need to be fulfilled:

(i) No signatory $i \in C$ has an incentive to deviate from x_i^S

(ii) Given country $i \in C$ deviates, no punishing country $j \in P_i$ has an incentive to not punish country i

For Condition (i) to be satisfied, we have to ensure that the potential gain from deviating from the signatory action x_i^S is outweighed by the payoff loss due to execution of punishment in the following period. Thus we have to derive conditions such that the following holds for all $i \in C$:

$$\begin{aligned} & \pi_i(x_C^S) + \delta\pi_i(x_C^S) \geq \pi_i(x_i^{NS}, x_{C\setminus\{i\}}^S) + \delta\pi_i(x_i^S, x_{C\setminus P_i}^S, x_{P_i}^P) \\ \Leftrightarrow \quad & \delta\left(\pi_i(x_C^S) - \pi_i(x_i^S, x_{C\setminus P_i}^S, x_{P_i}^P)\right) \geq \pi_i(x_i^{NS}, x_{C\setminus\{i\}}^S) - \pi_i(x_C^S) \qquad (2.A.1) \end{aligned}$$

If all signatories $i \in C$ play the signatory action x_i^S as agreed upon, the stage payoff for i is

$$\begin{aligned}\pi_i(x_C^S) =& -\frac{1}{2}\left(\beta k + \gamma(k_i+1)\right)^2 - \beta \sum_{m\in C}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right)\\ &-\beta \sum_{l\notin C}(\bar{x}_l - \beta - \gamma) - \gamma \sum_{m\in C} \bar{g}_{im}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right)\\ &-\gamma \sum_{l\notin C} \bar{g}_{il}(\bar{x}_l - \beta - \gamma).\end{aligned}$$

Consider now a situation when country $i \in C$ deviates from $\mathbf{s_i}$ in period t. Then, by $\mathbf{s}$, in the next period $t+1$ we have

$$x_j(C, t+1) = \begin{cases} \bar{x}_j - \beta k - \gamma(k_j+1), & \text{if } j = i\\ \bar{x}_j - p(\beta+\gamma), & \text{if } j \in P_i \cup (N \setminus C)\\ \bar{x}_j - \beta k - \gamma(k_j+1), & \text{if } j \in C \setminus P_i \end{cases}.$$

This yields the stage payoff during punishment

$$\begin{aligned}\pi_i(x_i^S, x_{C\setminus P_i}^S, x_{P_i}^P) =& -\frac{1}{2}\left(\beta k + \gamma(k_i+1)\right)^2 - \beta \sum_{m\in C\setminus P_i}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right)\\ &-\beta \sum_{l\in P_i}\left(\bar{x}_l - p(\beta+\gamma)\right) - \beta \sum_{l\notin C}(\bar{x}_l - \beta - \gamma)\\ &-\gamma \sum_{l\notin C} \bar{g}_{il}(\bar{x}_l - \beta - \gamma) - \gamma \sum_{m\in C\setminus P_i} \bar{g}_{im}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right)\\ &-\gamma \sum_{l\in P_i} \bar{g}_{il}\left(\bar{x}_l - p(\beta+\gamma)\right),\end{aligned}$$

and we receive for the payoff loss from punishment

$$\begin{aligned}\pi_i(x_C^S) - \pi_i(x_i^S, x_{C\setminus P_i}^S, x_{P_i}^P) =& -\beta \sum_{m\in C}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right)\\ &-\gamma \sum_{m\in C} \bar{g}_{im}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right) + \beta \sum_{m\in C\setminus P_i}\left(\bar{x}_m - \beta k - \gamma(k_m+1)\right)\\ &+\beta \sum_{l\in P_i}\left(\bar{x}_l - p(\beta+\gamma)\right) + \gamma \sum_{m\in C\setminus P_i} \bar{g}_{im}(\bar{x}_m - \beta k - \gamma(k_m+1))\\ &+\gamma \sum_{l\in P_i} \bar{g}_{il}(\bar{x}_l - p(\beta+\gamma))\\ =& -\beta \sum_{l\in P_i}\left[(\bar{x}_l - \beta k - \gamma(k_l+1)) - (\bar{x}_l - p(\beta+\gamma))\right]\\ &-\gamma \sum_{l\in P_i} g_{il}\left[(\bar{x}_l - \beta k - \gamma(k_l+1)) - (\bar{x}_l - p(\beta+\gamma))\right]\\ =&\beta \sum_{l\in P_i}\left[\beta(k-p) + \gamma(k_l+1-p)\right] + \gamma \sum_{l\in P_i} g_{il}\left[\beta(k-p) + \gamma(k_l+1-p)\right].\end{aligned} \tag{2.A.2}$$

Furthermore we have for the short-term payoff gain from a one-shot deviation

$$\begin{aligned}\pi_i(x_i^{NS}, x_{C\setminus\{i\}}^S) - \pi_i(x_C^S) &= \frac{1}{2}\Big(\beta k + \gamma(k_i+1)\Big)^2 - \frac{1}{2}\Big(\beta+\gamma\Big)^2 \\ &\qquad - \Big(\beta+\gamma\Big)\Big(\beta(k-1)+\gamma k_i\Big) \\ &= \frac{\beta^2}{2}(k-1)^2 + \beta\gamma(k_i(k-1)) + \frac{\gamma}{2}k_i^2 \\ &= \frac{1}{2}\left(\beta(k-1)+\gamma k_i\right)^2 \geq 0. \qquad (2.A.3)\end{aligned}$$

Multiplying with δ and rewriting Equation (2.A.2), then substracting (2.A.3), we obtain that Condition (i) is satisfied if (2.8) holds.

Let us now consider Condition (ii). Suppose country $i \in C$ deviated in period $t-1$. In order to ensure that all $j \in P_i$ actually punish the deviator, the following condition has to hold for all $j \in P_i$:

$$\begin{aligned}&\pi_j(x_{P_i}^P, x_{C\setminus P_i}^S) + \delta\pi_j(x_C^S) \geq \pi_j(x_j^S, x_{P_i\setminus\{j\}}^P, x_{C\setminus P_i}^S) + \delta\pi_j(x_j^S, x_{C\setminus P_j}^S, x_{P_j}^P) \\ \Leftrightarrow\ &\delta\left(\pi_j(x_C^S) - \pi_j(x_j^S, x_{C\setminus P_j}^S, x_{P_j}^P)\right) \geq \pi_j(x_j^S, x_{P_i\setminus\{j\}}^P, x_{C\setminus P_i}^S) - \pi_j(x_{P_i}^P, x_{C\setminus P_i}^S)\end{aligned} \qquad (2.A.4)$$

For the single-stage payoffs we have

$$\begin{aligned}\pi_j(x_j^S, x_{P_i\setminus\{j\}}^P, x_{C\setminus P_i}^S) = &-\frac{1}{2}\Big(\beta k + \gamma(k_j+1)\Big)^2 - \beta\sum_{m\in P_i\setminus\{j\}}\left(\bar{x}_m - p(\beta+\gamma)\right) \\ &-\beta\sum_{m\notin C}(\bar{x}_m - \beta - \gamma) - \beta\sum_{l\in C\setminus(P_i\setminus\{j\})}\left(\bar{x}_l - \beta k - \gamma(k_l+1)\right) \\ &-\gamma\sum_{m\in P_i\setminus\{j\}}\bar{g}_{jm}\left(\bar{x}_m - p(\beta+\gamma)\right) - \gamma\sum_{m\notin C}\bar{g}_{jm}(\bar{x}_m-\beta-\gamma) \\ &-\gamma\sum_{l\in C\setminus(P_i\setminus\{j\})}\bar{g}_{jl}(\bar{x}_l - \beta k - \gamma(k_l+1))\end{aligned}$$

and

$$\begin{aligned}\pi_j(x_{P_i}^P, x_{C\setminus P_i}^S) = &-\frac{1}{2}\left(p(\beta+\gamma)\right)^2 - \beta\sum_{m\in P_i}\left(\bar{x}_m - p(\beta+\gamma)\right) \\ &-\beta\sum_{m\notin C}(\bar{x}_m-\beta-\gamma) - \beta\sum_{l\in C\setminus P_i}\left(\bar{x}_l - \beta k - \gamma(k_l+1)\right) \\ &-\gamma\sum_{m\in P_i}\bar{g}_{jm}\left(\bar{x}_m - p(\beta+\gamma)\right) - \gamma\sum_{m\notin C}\bar{g}_{jm}(\bar{x}_m-\beta-\gamma) \\ &-\gamma\sum_{l\in C\setminus P_i}\bar{g}_{jl}\left(\bar{x}_l - \beta k - \gamma(k_l+1)\right).\end{aligned}$$

We will show that (2.A.1) already implies (2.A.4). To prove this, we need the following lemma.

Lemma 2.A.1. *For all β, γ, k, k_j and $p \geq 1$ it always holds*

$$\begin{aligned}
&-\frac{1}{2}(\beta k + \gamma(k_j+1))^2 + \frac{1}{2}(p(\beta+\gamma))^2 + (\beta+\gamma)(\beta(k-p) + \gamma(k_j+1-p)) \\
&\qquad \leq \frac{1}{2}(\beta k + \gamma(k_j+1))^2 - \frac{1}{2}(\beta+\gamma)^2 - (\beta+\gamma)(\beta(k-1)+\gamma k_j). \quad (2.A.5)
\end{aligned}$$

Proof. As $x_j^P \geq x_j^S$, we have $p(\beta+\gamma) \leq \beta k + \gamma(k_j+1)$ for all j in P_i and thus we have

$$\begin{aligned}
0 &\leq \Big(\beta(k-1)+\gamma k_j\Big)^2 - \frac{1}{2}\left((p-1)(\beta+\gamma)\right)^2 \\
&= \beta^2\Big((k-1)^2 - \frac{1}{2}(p-1)^2\Big) + \gamma^2\Big(k_j^2 - \frac{1}{2}(p-1)^2\Big) + \beta\gamma\Big(2k_j(k-1) + (1-p)^2\Big) \\
&= \beta^2\Big(k^2 - \frac{1}{2}(1+p^2) - (2k-p-1)\Big) + \gamma^2\Big((k_j+1)^2 - \frac{1}{2}(1+p^2) \\
&\quad - (2k_j+1-p)\Big) + \beta\gamma\Big(2k(k_j+1) - p^2 - (2k-p-1+2k_j+1-p)\Big) \\
&= \Big(\beta k + \gamma(k_j+1)\Big)^2 - \frac{1}{2}\Big((\beta+\gamma)^2 + (p(\beta+\gamma))^2\Big) \\
&\quad - (\beta+\gamma)\Big(\beta(2k-p-1) + \gamma(2k_j+1-p)\Big),
\end{aligned}$$

which is nothing else but (2.A.5) and proves the lemma. ☐

We can now rewrite the left-hand side of (2.A.4) and receive

$$\begin{aligned}
&\pi_j(x_j^S, x_{P_i\setminus\{j\}}^P, x_{C\setminus P_i}^S) - \pi_j(x_{P_i}^P, x_{C\setminus P_i}^S) \\
&= -\frac{1}{2}\Big(\beta k + \gamma(k_j+1)\Big)^2 + \frac{1}{2}\left(p(\beta+\gamma)\right)^2 - \beta\Big(\bar{x}_j - \beta k - \gamma(k_j+1)\Big) \\
&\quad + \beta\Big(\bar{x}_j - p(\beta+\gamma)\Big) - \gamma\Big(\bar{x}_j - \beta k - \gamma(k_j+1)\Big) + \gamma\Big(\bar{x}_j - p(\beta+\gamma)\Big) \\
&= -\frac{1}{2}\Big(\beta k + \gamma(k_j+1)\Big)^2 + \frac{1}{2}\left(p(\beta+\gamma)\right)^2 + (\beta+\gamma)\Big(\beta(k-p) + \gamma(k_j+1-p)\Big) \\
&\leq \frac{1}{2}\Big(\beta k + \gamma(k_j+1)\Big)^2 - \frac{1}{2}(\beta+\gamma)^2 - (\beta+\gamma)\Big(\beta(k-1)+\gamma k_j\Big) \\
&= \frac{1}{2}\Big(\beta(k-1)+\gamma k_j\Big)^2 \\
&= \pi_i(x_i^{NS}, x_{C\setminus\{i\}}^S) - \pi_i(x_C^S).
\end{aligned}$$

Thus, whenever (2.A.1) is satisfied, (2.A.4) has no bite and **s** therefore constitutes an SGP equilibrium if and only if (2.A.1) holds.

Let us now turn to the condition of weak renegotiation-proofness. As given in Definition 2.1, a subgame perfect equilibrium **s** is weakly renegotiation-proof (WRP) if there do not exist two continuation equilibria such that all players strictly prefer the one to the other. That is, we have to derive conditions such that all punishing countries $j \in P_i$ will actually punish instead of ignoring the deviation and continuing with another equilibrium path, e.g., renegotiating to playing cooperate again.

For any period t, there are $k+1$ possible continuation equilibria that implement either the agreement path $\mathbf{a^C}$ or the punishment path $\mathbf{p_j^C}$ for any signatory $j \in C$.

Assume that the strategy profile $\mathbf{s}$ is an SGP equilibrium, thus Condition (2.8) is satisfied. In accordance with the definition, for weak renegotiation-proofness we now need to consider all continuation equilibria and the respective incentives of each player.

Obviously, a deviating country prefers the agreement continuation equilibrium to the one generated by its respective punishment path $\mathbf{p_i^C}$, i.e. $\pi_i(x_C^S) > \pi_i(x_{C\setminus P_i}^S, x_{P_i}^P) \;\; \forall\, i \in C$. Thus, any country that is punished would not block a renegotiation to the agreement path.

All non-signatories $j \notin C$ will continue to free-ride on the others' efforts in any continuation equilibrium. They will not block a renegotiation either. Also, all signatories $j \in C \setminus P_i$ that do not punish prefer the equilibrium path with payoffs $\pi_j(x_C^S)$ to a continuation equilibrium from following the punishment path $\mathbf{p_i^C}$.

Thus, it remains to check the incentives of the punishers. If $\pi_j(x_C^S) > \pi_j(x_{P_i}^P, x_{C\setminus P_i}^S)$ holds for all $j \in P_i$, all punishing countries prefer the continuation equilibrium when no punishment is carried out to the one where i deviated. Thus, all players strictly prefer the agreement path to the punishment path and therefore $\mathbf{s}$ would not be weakly renegotiation-proof.

Hence, for $\mathbf{s}$ to be a WRP equilibrium the following condition needs to be satisfied for at least one $j \in P_i$:

$$\pi_j(x_{P_i}^P, x_{C\setminus P_i}^S) - \pi_j(x_C^S) \geq 0. \tag{2.A.6}$$

We have

$$\begin{aligned}
\pi_j(x_{P_i}^P, x_{C\setminus P_i}^S) - \pi_j(x_C^S) =& -\frac{1}{2}\left(p(\beta+\gamma)\right)^2 + \frac{1}{2}\Big(\beta k + \gamma(k_j+1)\Big)^2 \\
& - \beta \sum_{l\in P_i} \left(\bar{x}_l - p(\beta+\gamma)\right) + \beta \sum_{m\in P_i} \left(\bar{x}_m - \beta k - \gamma(k_m+1)\right) \\
& - \gamma \sum_{l\in P_i} \bar{g}_{jl} \left(\bar{x}_l - p(\beta+\gamma)\right) \\
& + \gamma \sum_{m\in P_i} \bar{g}_{jm} \left(\bar{x}_m - \beta k - \gamma(k_m+1)\right) \\
=& -\frac{1}{2}\left(\left(p(\beta+\gamma)\right)^2 - \Big(\beta k + \gamma(k_j+1)\Big)^2\right) \\
& - \beta \sum_{m\in P_i} \left(\beta(k-p) + \gamma(k_m+1-p)\right. \\
& - \gamma \sum_{m\in P_i} \bar{g}_{jm} \left(\beta(k-p) + \gamma(k_m+1-p)\right)
\end{aligned}$$

and thus (2.A.6) is equivalent to

$$\begin{aligned}&\frac{\beta^2}{2}\left((k-p)(k+p-2|P_i|)\right)\\&\quad+\beta\gamma\left(kk_j-|P_i\cap N_j|(k-p)-\sum_{m\in P_i}(k_m+1-p)+p(1-p)\right)\\&\qquad+\frac{\gamma^2}{2}\left(k_j^2-2\sum_{m\in P_i}g_{jm}(k_m+1-p)-1+p(2-p)\right)\geq 0, \quad (2.A.7)\end{aligned}$$

which we can rewrite such that we obtain (2.9).

Concluding, if the strategy **s** satisfies (2.8) for all $i\in C$, i.e. is subgame perfect, and additionally is such that for any $i\in C$ there is a punishment set P_i such that there exists at least one $j\in P_i$ that satisfies Condition (2.9), **s** is weakly renegotiation-proof. □

Proof of Corollary 2.1. For $\gamma=0$ we have that the IEA **s** by the coalition C is a subgame perfect equilibrium if and only if for all $i\in C$

$$\delta\beta^2|P_i|(k-1)\geq\frac{1}{2}\left(\beta(k-1)\right)^2$$

holds. Furthermore, it is weakly renegotiation-proof if and only if for all $i\in C$

$$\beta^2(k-1)(|P_i|-1)\leq\frac{1}{2}\left(\beta(k-1)\right)^2.$$

For $k\geq 2$ this gives

$$\begin{aligned}&|P_i|\geq\frac{k-1}{2\delta}\quad\wedge\quad|P_i|\leq\frac{k+1}{2}\\\Leftrightarrow\quad&\frac{1}{2\delta}(k-1)\leq|P_i|\leq\frac{1}{2}(k+1).\end{aligned}$$

□

Proof of Proposition 2.1. As we have already seen in Example 2.1, global cooperation cannot be supported as an SGP equilibrium in the star network if $\gamma>\beta\frac{n-1}{n-3}$ holds.

Let us now suppose $\gamma\leq\beta\frac{n-1}{n-3}$ and consider again the center i of the star network. Then, we can find a punishment group P_i such that full cooperation can be sustained as a subgame perfect equilibrium, that is we can find P_i such that $|P_i|\geq\frac{(n-1)^2(\beta+\gamma)}{2(\beta(n-1)+\gamma)}$ is satisfied (compare Condition (2.8) of Theorem 2.1).

Clearly, whenever this lower bound is larger than the upper bound from the WRP condition (2.9), full cooperation cannot be a WRP equilibrium. We have

$$\frac{(n-1)^2(\beta+\gamma)}{2(\beta(n-1)+\gamma)} > \frac{\beta(n-1)+\gamma}{2\beta}$$
$$\Leftrightarrow \quad n > n_1 := 2 + \frac{\beta}{\gamma}\left(1+\sqrt{1+2\frac{\gamma}{\beta}+3(\frac{\gamma}{\beta})^2+(\frac{\gamma}{\beta})^3}\right). \qquad (2.A.8)$$

Hence, given parameters γ and β, for n large enough (2.A.8) is always satisfied and global cooperation fails to be a WRP equilibrium. ☐

Proof of Proposition 2.2. Denote by Ψ_i the set of permutations of players $\psi_i : N\setminus\{i\} \to N\setminus\{i\}$ such that $\psi_i(j) < \psi_i(m)$ for all $j \in N_i$, $m \notin N_i$. Further, given a permutation $\psi_i \in \Psi_i$, let $M_{\psi_i}(\nu)$ denote the first ν elements of the permutation, i.e. $M_{\psi_i}(\nu) := \{\psi_i(1), \ldots, \psi_i(\nu)\}$. This defines a possible punishing set P_i.

Let $\nu^* := \arg\min_{1\le\nu\le n-1}\{P_i = M_{\psi_i}(\nu) \text{ satisfies (2.12)}\}$ be the lowest integer such that the set $M_{\psi_i}(\nu^*)$ deters a signatory from deviating.

First suppose that $\nu^* \le \eta$. Then by construction we have $M_{\psi_i}(\nu^*) \cap N_i = M_{\psi_i}(\nu^*)$ and for any $j \in M_{\psi_i}(\nu^*)$, j is also in N_i. Thus, $j \notin M_{\psi_i}(\nu^*) \cap N_j$ and therefore we get for any $\psi_i \in \Psi_i$ that

$$|M_{\psi_i}(\nu^*) \cap N_j| \le |M_{\psi_i}(\nu^*) \cap N_i| - 1 \qquad (2.A.9)$$

holds for all $j \in M_{\psi_i}(\nu^*)$.

Hence, the following holds for all $\psi_i \in \Psi_i$:

$$|M_{\psi_i}(\nu^*) \cap N_j|\gamma + (|M_{\psi_i}(\nu^*)|-1)\beta \le (|M_{\psi_i}(\nu^*) \cap N_i|-1)\gamma + (|M_{\psi_i}(\nu^*)|-1)\beta. \qquad (2.A.10)$$

Suppose now the opposite, i.e. $\nu^* > \eta$. We show that there still exists a permutation $\psi_i^* \in \Psi_i$ such that (2.A.10) holds for all $j \in M_{\psi_i^*}(\nu^*)$.

From (2.12) we get that $\nu^* \ge \frac{1}{2}(n-1) - \frac{1}{2}\frac{\gamma}{\beta}\eta$ since for $P_i = M_{\psi_i}(\nu^*)$ we have that $M_{\psi_i}(\nu^*) \cap N_i = N_i$. Moreover, because of minimality and ν^* being an integer, we have

$$\nu^* = \left\lceil \frac{1}{2}(n-1) - \frac{1}{2}\frac{\gamma}{\beta}\eta \right\rceil. \qquad (2.A.11)$$

For all neighbors $j \in M_{\psi_i}(\nu^*) \cap N_i$ of the deviator i, (2.A.9) still holds and there is nothing to show.

From (2.A.11) we receive that additional to the neighbors of i there are

$$\nu^* - \eta = \left\lceil \frac{1}{2}(n-1) - \frac{\gamma}{2\beta}\eta \right\rceil - \eta$$

non-neighbors in the punishing set $M_{\psi_i}(\nu^*)$.

Denote by $\tilde{\psi}_i \in \Psi_i$ the permutation which minimizes the number of those non-neighbors $j \in M_{\psi_i}(\nu^*) \setminus N_i$ that have all their links within the set $M_{\psi_i}(\nu^*)$, i.e. such that $\eta_j(g|_{M_{\psi_i}(\nu^*)}) = \eta$ holds.[24] Suppose that this number is different from zero, i.e. at least one country in $M_{\psi_i}(\nu^*) \setminus N_i$ has all neighbors in $M_{\psi_i}(\nu^*)$. Then, by (2.A.11), from the set $M_{\tilde{\psi}_i}(\nu^*)$ there are at most $\eta \left\lceil \left(\frac{1}{2}(n-1) - \frac{1}{2}\frac{\gamma}{\beta}\eta \right) \right\rceil - 2\eta$ links into the set $N \setminus \left\{ M_{\tilde{\psi}_i}(\nu^*) \cup \{i\} \right\}$.

As $|N \setminus \left\{ M_{\tilde{\psi}_i}(x^*) \cup \{i\} \right\}| = n - 1 - \nu^*$, we have that the sum of degrees of members of the set $N \setminus \left\{ M_{\tilde{\psi}_i}(x^*) \cup \{i\} \right\}$ satisfies

$$\begin{aligned}
\eta \left| N \setminus \left\{ M_{\tilde{\psi}_i}(\nu^*) \cup \{i\} \right\} \right| &= \eta \left(n - 1 - \left\lceil \left(\frac{1}{2}(n-1) - \frac{1}{2}\frac{\gamma}{\beta}\eta \right) \right\rceil \right) \\
&= \eta \left\lceil \left(\frac{1}{2}(n-1) + \frac{1}{2}\frac{\gamma}{\beta}\eta \right) \right\rceil \\
&> \eta \left\lceil \left(\frac{1}{2}(n-1) - \frac{1}{2}\frac{\gamma}{\beta}\eta \right) \right\rceil - 2\eta.
\end{aligned}$$

Thus, the number of all links of members of the set $N \setminus \left\{ M_{\tilde{\psi}_i}(\nu^*) \cup \{i\} \right\}$ exceeds the maximum amount of links coming into the set from its complement, meaning that there has to exist a link lm between members l, m of the set $N \setminus \left\{ M_{\tilde{\psi}_i}(\nu^*) \cup \{i\} \right\}$.

Considering a permutation $\hat{\psi}_i \in \Psi_i$ that is obtained from $\tilde{\psi}_i$ by switching a member of $M_{\tilde{\psi}_i}(\nu^*)$, who has all her links within $M_{\tilde{\psi}_i}(\nu^*)$, with $l \in N \setminus \left\{ M_{\tilde{\psi}_i}(\nu^*) \cup \{i\} \right\}$, who has a link $lm \in g|_{N \setminus \left\{ M_{\tilde{\psi}_i}(\nu^*) \cup \{i\} \right\}}$, contradicts the assumption that $\tilde{\psi}_i$ yields the minimal number of j with $\eta_j(g|_{M_{\psi_i}(\nu^*)}) = \eta$. Hence, there exists a permutation $\psi_i^* \in \Psi_i$ such that for all $j \in M_{\psi_i}(\nu^*)$ we have $|M_{\psi_i^*}(\nu^*) \cap N_j| \leq |M_{\psi_i^*}(\nu^*) \cap N_i| - 1$, implying that (2.A.10) holds.

Finally, choosing $P_i := M_{\psi_i^*}(\nu^*)$ yields first that trivially (2.12) is satisfied. Moreover, because of minimization we have that (2.12) cannot be satisfied by any subset of P_i. Thus $((|P_i \cap N_i| - 1)\gamma + (|P_i| - 1)\beta) < \frac{1}{2}(\beta(n-1) + \gamma\eta)$ and since (2.A.10) holds for $P_i = M_{\psi_i^*}(\nu^*)$, we get that (2.13) is satisfied. Hence, both conditions of Theorem 2.1 are satisfied by choosing a punishment set $P_i := M_{\psi_i^*}(\nu^*)$ for every $i \in N$, implying that there exists a WRP equilibrium in which all countries play a signatory strategy, i.e. a stable global IEA exists.

Note that we have shown here a slightly more general result since we have shown that the WRP condition holds for all punishers. □

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[24] Note that all permutations $\psi_i \in \Psi_i = \left\{ \psi_i : N \setminus \{i\} \to N \setminus \{i\} \text{ s.t. } \psi_i(j) < \psi_i(m) \; \forall j \in N_i, m \notin N_i \right\}$, deliver the same ν^* due to regularity of the network.

Proof of Lemma 2.1. Suppose a player j has deviated and players P_j are called upon to punish. When a player i joins the punishment group P_j, the marginal effect on total welfare $\mathcal{W}$ can be calculated to be

$$\begin{aligned}
\Delta\mathcal{W}(P_j+i,P_j) =& -\Big(\beta n+\gamma(\eta_i+1)\Big)\Big(\beta(n-1)+\gamma\eta_i\Big)\\
&-\frac{1}{2}\Big((\beta+\gamma)^2-(\beta n+\gamma(\eta_i+1))^2\Big)\\
=& -\Big(\beta n+\gamma(\eta_i+1)\Big)\Big(\beta(n-1)-\frac{1}{2}\beta n+\gamma\eta_i-\frac{1}{2}\gamma(\eta_i+1)\Big)\\
&-\frac{1}{2}(\beta+\gamma)^2\\
=& -\frac{1}{2}\Big(\beta n+\gamma(\eta_i+1)\Big)\Big(\beta(n-1)+\gamma\eta_i-\beta-\gamma\Big)-\frac{1}{2}(\beta+\gamma)^2\\
=& -\frac{1}{2}\Big(\beta(n-1)+\gamma\eta_i\Big)^2+\frac{1}{2}(\beta+\gamma)\left(\beta n+\gamma(\eta_i+1)\right.\\
&\left.-\beta(n-1)+\gamma\eta_i\right)-\frac{1}{2}(\beta+\gamma)^2\\
=& -\frac{1}{2}\Big(\beta(n-1)+\gamma\eta_i\Big)^2+\frac{1}{2}(\beta+\gamma)\left(\beta n+\gamma(\eta_i+1)-\beta(n-1)\right.\\
&\left.+\gamma\eta_i-\beta-\gamma\right)-\frac{1}{2}\Big(\beta(n-1)+\gamma\eta_i\Big)^2.
\end{aligned}$$

☐

Proof of Proposition 2.4. Suppose a player j has deviated and players P_j are called upon to punish. Let $i\in N_j$ and $l,m\notin N_j\cup\{j\}$ be such that $f_j(\cdot,P_j+i)\le f_j(\cdot,P_j+l+m)$. That is, we have that

$$\begin{aligned}
&\beta\Big(\beta(n-1)+\gamma\eta_l+\beta(n-1)+\gamma\eta_m\Big)\ge(\beta+\gamma)\Big(\beta(n-1)+\gamma\eta_i\Big)\\
\Leftrightarrow\quad&\beta^2\Big(\beta(n-1)+\gamma\eta_l+\beta(n-1)+\gamma\eta_m\Big)^2\ge(\beta+\gamma)^2\Big(\beta(n-1)+\gamma\eta_i\Big)^2\\
\Leftrightarrow\quad&\underbrace{\frac{\beta^2}{(\beta+\gamma)^2}}_{=:a}\Bigg(\underbrace{\Big[\beta(n-1)+\gamma\eta_l\Big]^2+\Big[\beta(n-1)+\gamma\eta_m\Big]^2}_{=:\xi_1}\\
&\quad+\underbrace{2\Big[\left(\beta(n-1)+\gamma\eta_l\right)\left(\beta(n-1)+\gamma\eta_m\right)\Big]}_{=:\xi_2}\Bigg)\ge\underbrace{\Big[\beta(n-1)+\gamma\eta_i\Big]^2}_{=:\xi_3}.
\end{aligned}$$

Next, note that $\xi_1\ge\xi_3$ if $\xi_1\ge a(\xi_1+\xi_2)$, i.e. $\xi_1-\frac{a\xi_2}{1-a}\ge 0$. This is equivalent to

$$\begin{aligned}
0\le&\Big[\beta(n-1)+\gamma\eta_l\Big]^2+\Big[\beta(n-1)+\gamma\eta_m\Big]^2\\
&-\frac{2\beta^2\Big[(\beta(n-1)+\gamma\eta_l)(\beta(n-1)+\gamma\eta_m)\Big]}{2\beta\gamma+\gamma^2}.
\end{aligned}$$

Choosing $\gamma\eta_l = \Big(\beta(n-1) + \gamma\eta_m\Big)\frac{\beta^2}{2\beta\gamma+\gamma^2} - \beta(n-1)$ minimizes the right-hand side and thus above is implied by

$$\begin{aligned}\Leftarrow 0 \leq \Big[\beta(n-1) + \gamma\eta_m\Big]^2 \left[1 + \left(\frac{\beta^2}{2\beta\gamma + \gamma^2}\right)^2\right] \\ - 2\left(\frac{\beta^2}{2\beta\gamma + \gamma^2}\right)^2 \Big[\beta(n-1) + \gamma\eta_m\Big]^2,\end{aligned}$$

which in turn is equivalent to $\beta^2 \leq 2\beta\gamma + \gamma^2$. For positive values, we obtain $\beta \leq (1+\sqrt{2})\gamma$, which concludes the proof. □

Chapter 3

A Note on Renegotiation in Repeated Games

3.1 Introduction

In Farrell and Maskin (1989), the concept of *weak renegotiation-proofness* (subsequently abbreviated as WRP) is introduced and the authors provide a characterization of WRP payoffs for general two-player games. In their Theorem 1 (p. 332), the authors give both sufficient and necessary conditions for a strictly individual rational payoff to be weakly renegotiation-proof. In this note, we use a counterexample to show that their proof of the sufficient conditions fails at a particular step. While Farrell and Maskin (1989) are very careful in many steps of the proof, they implicitly assume more of a structure on the set of payoffs than actually exists. More specifically, they claim to obtain a payoff with *independent randomization*, which is only obtainable with correlated strategies. However, if correlated strategies were allowed, large parts of the proof would be unnecessary.

First, we introduce the basic notation as given in Farrell and Maskin (1989). Then, we go through the arguments of the original proof before we point to the crucial and erroneous claim in that proof. We use a counterexample to illustrate the problem, and then prove an alternative result that replaces the erroneous claim and ultimately fixes the proof.

3.2 Basics and Original Result

We adopt most of the original notation from Farrell and Maskin (1989), but denote sets by calligraphy instead of regular letters since we need a more elaborate notation for our proof.

Consider a two-player, single-stage game with players $i = 1, 2$. Each player i possesses a finite set of actions, and we denote the simplex consisting of player i's mixed actions by $\mathcal{A}_i$. We denote the set of both players' actions by $\mathcal{A} \equiv \mathcal{A}_1 \times \mathcal{A}_2$. Let $g : \mathcal{A} \longrightarrow \mathbb{R}^2$ be the vector of continuous payoff functions $g_i : \mathcal{A}_i \longrightarrow \mathbb{R}$. The single-stage game g is then defined by the set of payoffs and actions. We will denote the set of mixed-strategy payoffs, i.e., the image of g, by

$$\mathcal{U} = \Big\{ (v_1, v_2) \in \mathbb{R}^2 \,|\, \exists\, a \in \mathcal{A} \text{ with } g\left(a\right) = (v_1, v_2) \Big\}$$

and the set of feasible payoffs in the repeated game by

$$\mathcal{V} = \operatorname{co}\left(\mathcal{U}\right).$$

For player i, the profit-maximizing deviation from action pair $a = (a_1, a_2)$ is defined by $c_i(a) = \max_{a_i} g_i(a_i, a_j)$, $i \neq j$, the minimax payoff[1] is defined by $\underline{v}_i = \min_{a_j} \max_{a_i} g_i(a_i, a_j)$ and $v_i^{max} = \max_{a_1, a_2} g_i(a_1, a_2)$ is the maximal attainable payoff. The set of strictly individual rational payoffs in the repeated game is given by

$$\mathcal{V}^* = \left\{ (v_1, v_2) \in \mathcal{V} \,|\, v_1 > \underline{v}_1, v_2 > \underline{v}_2 \right\}.$$

In the repeated game, we consider the infinite repetition of the single-stage game g, which will be denoted by g^*. Let $t = 1, 2, \ldots, \infty$ denote the periods and the sequence $\{a_i\left(t\right)\}$ denote a player's action profile with $a_i\left(t\right) \in \mathcal{A}_i^t$. Note that we assume constant action spaces $\mathcal{A}_i^t = \mathcal{A}_i$ for all t. A t-history will be denoted by $h^t = \big(a\left(1\right), \ldots, a\left(t\right)\big)$, and $\mathcal{H}$ is the set of all such possible t-histories. A strategy σ_i for player i in the repeated game is a function that defines an action $a^i \in \mathcal{A}_i$ for every date t and history $h^t \in \mathcal{H}$.[2] In every period, players receive the stage-game payoffs. Player i's discounted average payoff at time t is then given by $(1-\delta) \sum_{\tau=t}^{\infty} \delta^{\tau - t} g_i \big(a_1\left(\tau\right), a_2\left(\tau\right)\big)$, where $\delta < 1$ is the common discount factor for all players. The expected payoffs of strategy σ with discount factor δ will be denoted by $g^*(\sigma, \delta)$, but we often omit δ and simply write $g^*(\sigma)$.

A weakly renegotiation-proof equilibrium is defined as follows.

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[1] While Farrell and Maskin (1989) normalize the minimax payoff to zero for both players, we omit this normalization in the subsequent sections for a better illustration. This is immaterial to our results.

[2] Note that by the definition of a strategy σ, we ultimately assume that players can not only observe the realized actions, but also the mixed strategies in the repeated game. Players can therefore condition their strategies on all past *private randomizations.* This assumption is also made by Farrell and Maskin (1989), but they remark that it is not strictly necessary (see their footnote 2 on p. 329).

Definition 3.1 (Farrell and Maskin, 1989)**.** A subgame perfect equilibrium σ is *weakly renegotiation-proof* if there do not exist continuation equilibria σ^1, σ^2 of σ such that σ^1 strictly Pareto-dominates σ^2. If an equilibrium σ is WRP, then we also say that the payoffs $g^*(\sigma)$ are WRP.

3.2.1 Sufficient Conditions for Weakly Renegotiation-Proof Payoffs

Let us cite the conditions that Farrell and Maskin (1989) propose as sufficient for WRP payoffs, which is the first part of their Theorem 1 (p. 332).

Theorem 3.1 (Farrell and Maskin, 1989)**.** *Let* $v = (v_1, v_2)$ *be in* $\mathcal{V}^*$. *If there exist action pairs* $a^i = (a_1^i, a_2^i)$ *(for* $i = 1, 2$*) in* g *such that (i)* $c_i(a^i) < v_i$*, while (ii)* $g_j(a^i) \geq v_j$ *for* $j \neq i$*, then the payoffs* (v_1, v_2) *are WRP for all sufficiently large* $\delta < 1$.

To prove this result, two steps have to be completed. First, one needs to construct a sequence of actions to obtain v as a payoff of the repeated game such that no two continuation payoffs along this path can be strictly Pareto-ranked. If the players could use correlated strategies, this step would be trivial. As they can only use independent randomizations, and the set of mixed-strategy payoffs is a peculiar subset of feasible payoffs, this is not straightforward, as we show in the following section. Given this sequence of actions for the normal phase of the game, one then needs to design punishment paths such that v is a subgame perfect equilibrium and no continuation payoffs of the equilibrium strategy can be strictly Pareto-ranked.

3.3 The Error in the Proof of Farrell and Maskin (1989)

In the following text, we will go along the original proof of Theorem 3.1 and first discuss the simple cases where the proof of Farrell and Maskin (1989) works. Then, we will give a counterexample for the crucial step in their proof and offer a correction.

Clearly, for a mixed-strategy payoff $v \in \mathcal{U}$, i.e., if there exists an action a such that $g(a) = v$, there is not much to do as v can be obtained by playing action a in every period, and trivially, all continuation payoffs along the path are equal to v. For a payoff $v \in \mathcal{V}^* \setminus \mathcal{U}$, the folk theorem for observable mixed strategies without public randomization given in Fudenberg and Maskin (1991, p. 434) yields that for a sufficiently large δ, we can find a sequence of action pairs $\{\hat{a}(t)\}$ such that $(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} g\left(\hat{a}(t)\right) = v$. However, this does not ensure that any two continuation payoffs of this sequence are Pareto-undominated, as required by the definition of weak renegotiation-proofness.

Therefore, Farrell and Maskin (1989) construct normal-phase actions using one of the action pairs a^1 or a^2 that are given by the hypotheses of the theorem. Given the vectors $g(a^1)$ and v, one can construct the line l^1 that starts in $g(a^1)$ and runs through v. If all payoffs of the sequence $\{\hat{a}(t)\}$ were on this line, Lemma 1 of Farrell and Maskin (1989, p. 355, subsequently denoted as Lemma FM1) yields that the Pareto condition is satisfied. If, however, not all payoffs lie on l^1, there must be actions a^* and a^{**} with payoffs $g(a^*)$ above and $g(a^{**})$ below l^1.

So far, everything is true and works in all two-player games. However, on page 334, Farrell and Maskin (1989) implicitly claim the following.

Claim 3.1. *Let $v \in \mathcal{V}^* \setminus \mathcal{U}$ and suppose that a^1, a^2 in $\mathcal{A}$ satisfy the hypotheses of Theorem 3.1; that is, a^i satisfies*

(i) $g_j(a^i) \geq v_j$, $j \neq i$,

(ii) $c_i(a^i) < v_i$,

then, without loss of generality, there exists an action pair $\tilde{a}$ in $\mathcal{A}$ that satisfies

(a) $g_1(a^1) < v_1 < g_1(\tilde{a})$, $g_2(a^1) > v_2 > g_2(\tilde{a})$,

(b) v is a convex combination of $g(a^1)$ and $g(\tilde{a})$.

If the players were able to play correlated strategies, they could easily randomize between $g(a^*)$ and $g(a^{**})$ to obtain a payoff on l^1. However, as there is no public randomization device, players cannot play correlated strategies and can only randomize independently. As Farrell and Maskin (1989) rightly continue, if players randomize independently between a^* and a^{**} with probabilities $p \in (0, 1)$ and $1 - p$, the obtained payoffs, denoted by $\Gamma(p) = (\Gamma_1(p), \Gamma_2(p))$, will lie above l^1 for a sufficiently large p and below l^1 for a low p. As $\Gamma(p)$ is continuous in p, they argue correctly that there must exist a p^* such that $\Gamma(p^*)$ lies on l^1. Clearly, if $\Gamma(p^*) = v$, the normal phase can be implemented by requiring randomization between a^* and a^{**}, but as we assumed $v \in \mathcal{V}^* \setminus \mathcal{U}$, this is not relevant here.

To obtain v as a convex combination of $\Gamma(p^*)$ and $g(a^1)$, we must have $\Gamma_1(p^*) > v_1$. However, in the following counterexample, we show that there is no mixed-strategy payoff $\Gamma(p^*)$ on l^1 with $\Gamma_1(p^*) > v_1$. Moreover, contrary to the claim by Farrell and Maskin (1989) in their footnote 6 on page 334, the analogous construction with $g(a^2)$ and l^2 does not work either, which ultimately rejects Claim 3.1.

3.3.1 Counterexample to Claim 3.1

Consider the two-player game where Players 1 and 2 can choose between two pure actions $\{u, d\}$ and $\{l, r\}$, and the stage-game payoffs of the pure strategies are given by the payoff matrix shown in Table 3.1.

	l	r
u	$(0, 0)$	$(2, 2)$
l	$(4, 0)$	$(0, 0)$

Table 3.1: Payoff matrix of the two-player strategic game.

For $p, q \in [0, 1]$, we denote by $a = (p, q)$ the mixed strategy in which Player 1 randomizes between u and d with probabilities $1 - p$ and p, respectively, and Player 2 randomizes between l and r with probabilities $1 - q$ and q.

The set of feasible payoffs $\mathcal{V}$ is the convex hull of the payoff vectors $(0, 0), (2, 2)$ and $(4, 0)$; i.e.,

$$\mathcal{V} = co\left(\{(0, 0), (2, 2), (4, 0)\}\right)$$

and the set of strictly individually rational payoffs is given by

$$\mathcal{V}^* = \left\{v \in \mathcal{V} | v_1 > \tfrac{4}{3}, v_2 > 0\right\}.$$

In Figure 3.1, we illustrate how the set of mixed-strategy payoffs $\mathcal{U}$ is included in the set of feasible payoffs $\mathcal{V}$.

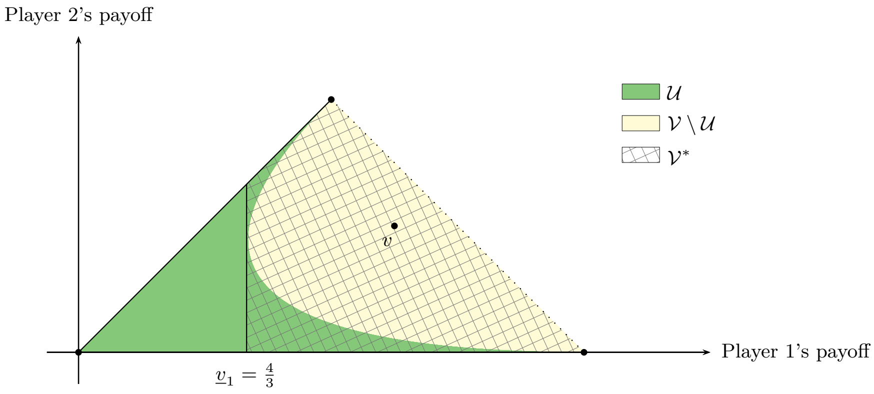

Figure 3.1: Illustration of $\mathcal{U}$ and $\mathcal{V}^*$.

Let us consider the strictly individually rational payoff $v = (\frac{5}{2}, 1)$, which is not obtainable with mixed strategies, i.e., $v \in \mathcal{V}^* \setminus \mathcal{U}$. Then, consider the action pairs $a^1 = (\frac{1}{4}, \frac{7}{8})$ and $a^2 = (1, \frac{5}{16})$. First, we show that a^1 and a^2 satisfy the conditions of Theorem 3.1. For action a^1, $g(a^1) = (\frac{23}{16}, \frac{21}{16})$, and thus $g_2(a^1) > 1$. For Player 1, the maximal deviation payoff is given by $c_1(a^1) = \frac{7}{4}$, which is also in accordance with the conditions. For the action pair a^2, we obtain $g(a^2) = (\frac{11}{4}, 0)$, and therefore $g_1(a^2) > \frac{5}{2}$. Finally, Player 2's maximal deviation payoff is given by $c_2(a^2) = 0$. Thus, the two actions both satisfy Conditions (i) and (ii) of the theorem.

Next, as proposed by Farrell and Maskin (1989), we construct the line l^1 and find that if all payoffs of the normal phase sequence $\{\hat{a}(t)\}$ lie on l^1, the average payoff would not be v since there is no stage-game payoff x on l^1 with $x_1 > v_1$. Graphically speaking, there are no stage-game payoffs to the right of v as l^1 does not intersect with $\mathcal{U}$ right of v (see Figure 3.2). Thus, if we select two action pairs a^* and a^{**} with payoffs $g(a^*)$ above and $g(a^{**})$ below l^1, and if players randomize between these two actions with parameters p and q, respectively, the resulting payoff $\Gamma(p, q)$ will certainly lie in $\mathcal{U}$, in the dark-gray area in Figure 3.2. While there exist p^*, q^* such that $\Gamma(p^*, q^*)$ lies on l^1, in our example, this will certainly be to the left of v, that is, $\Gamma_1(p^*, q^*) < v_1$.

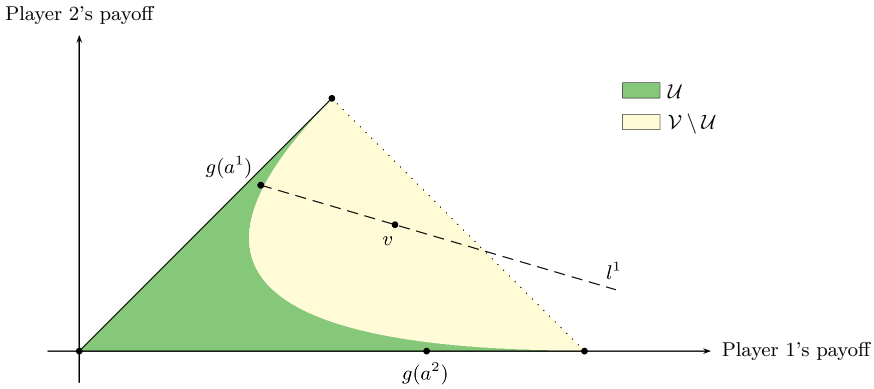

Figure 3.2: Construction with payoffs $g(a^1)$.

Thus, as Farrell and Maskin (1989) claim erroneously in their footnote 6 on page 334, the analogous construction should work for Player 2 and l^2. But, as one can clearly see in Figure 3.3, this does not hold. There are no payoffs x on line l^2 such that $x_1 < v_1$; graphically speaking, l^2 does not intersect with $\mathcal{U}$ left of v. Therefore, it is not clear how to obtain v, and the proof is not correct at this step.

In general, the proof by Farrell and Maskin (1989) fails whenever the two action profiles a^1 and a^2 are such that the constructed vectors $l^1 = g(a^1) + \lambda\left(v - g(a^1)\right)$ and $l^2 = g(a^2) + \lambda\left(v - g(a^2)\right)$ do not intersect with the set of mixed-strategy payoffs $\mathcal{U}$ for $\lambda > 1$. This is not to say, though, that there are no games where the proposed construction works and Claim 3.1 holds true.

Note. In this example, v can still be constructed as required. If we choose the action pair $\tilde{a}^1 = (0, 1)$ that corresponds to the payoffs $(2, 2)$, an extreme point of $\mathcal{V}$, this action pair satisfies the conditions of the theorem. Furthermore, if we

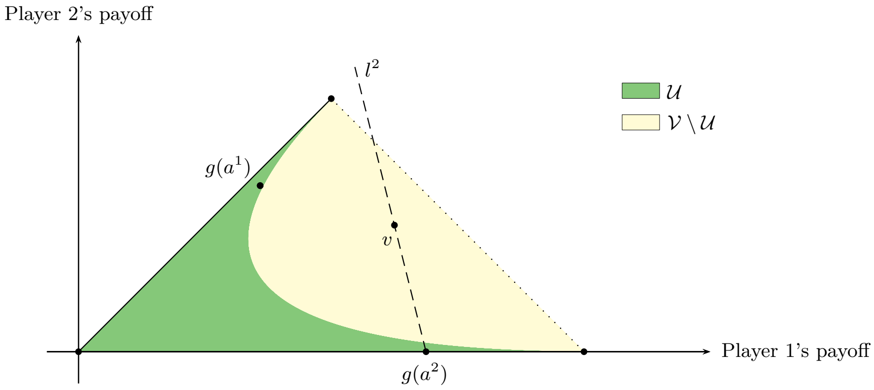

Figure 3.3: Construction with payoffs $g(a^2)$.

construct the line $\tilde{l}^1$ that starts in $g(\tilde{a}^1)$ and runs through v, it intersects with $\mathcal{U}$ right of v, and therefore Claim 3.1 holds. This, however, does not conflict with our point as the sufficient conditions of Theorem 3.1 are stated to hold for *any* action pairs a^1, a^2 that satisfy the conditions of the theorem. Moreover, in general $n \times m$ games, one cannot always find such an alternative action pair $\tilde{a}$ that satisfies the conditions of the theorem. Nevertheless, to fix the proof of Theorem 3.1, we will show that we can always find two action pairs to obtain v as a convex combination, and that this already suffices if we also modify the subsequent steps in the original proof of Farrell and Maskin (1989).

3.4 Corrected Proof of Theorem 3.1

For the proof of Theorem 3.1, we replace Claim 3.1 with the following proposition.

Proposition 3.1. *Let $v \in \mathcal{V}^* \setminus \mathcal{U}$. If there exist action pairs a^1, a^2 in $\mathcal{A}$ that satisfy the hypotheses of Theorem 3.1, that is, a^i satisfies*

(i) $g_j(a^i) \geq v_j$, $j \neq i$,

(ii) $c_i(a^i) < v_i$,

then there exist action pairs a^{1*} and a^{2*} in $\mathcal{A}$ that satisfy

(a) $g_1(a^{1*}) < v_1 < g_1(a^{2*})$, $g_2(a^{1*}) > v_2 > g_2(a^{2*})$,

(b) *v is a convex combination of $g(a^{1*})$ and $g(a^{2*})$.*

Given the result of Proposition 3.1, we can continue with the proof of Theorem 3.1 as follows. By Lemma FM1, we obtain that for a sufficiently large δ there exists a sequence of actions $\{a(t)\}$ with $a(t) \in \{a^{1*}, a^{2*}\}$ that yields discounted average payoffs v. To conclude the proof, we need to show that v can be established as a WRP equilibrium. Therefore, one needs to define punishments to sustain v as a subgame perfect equilibrium and that are such that there is no Pareto-ranking across any continuation equilibria of the strategy.

If a^{i*} satisfies the hypotheses of Theorem 3.1, it can be used to construct a penance punishment strategy for player i, as suggested by Farrell and Maskin (1989, p. 335), and the rest of the proof then follows their outline.[3] In general, however, this is not the case, and we need to construct a different punishment strategy to sustain v as a WRP equilibrium.

Given the actions a^{1*} and a^{2*} from Proposition 3.1, we define

$$l^* = \Big\{ v \in \mathcal{V} | v = (1-\lambda)g(a^{1*}) + \lambda g(a^{2*}), \lambda \in [0,1] \Big\}$$

as the set of payoffs that lie on the line segment between $g(a^{1*})$ and $g(a^{2*})$. We will first construct Player 1's punishment and assume, without loss of generality, that $g_2(a^1) > v_2$ holds.[4]

As $c_1(a^1) < v_1$, there exists $\delta < 1$ such that

$$(1-\delta)v_1^{max} + \delta c_1(a^1) < v_1$$

and $\epsilon_1 > 0$ such that

$$c_1(a^1) < v_1 - \epsilon_1.$$

Since $g_2(a^1) > v_2$, there also exists $\epsilon_2 > 0$ such that $g_2(a^1) \geq v_2 + \epsilon_2$, and therefore we can find $\hat{\lambda} \in [0,1]$ that satisfies

$$\begin{aligned} v_1 - \frac{\epsilon_1}{2} \leq (1-\hat{\lambda})g_1(a^{1*}) + \hat{\lambda} g_1(a^{2*}) < v_1, \\ v_2 \leq (1-\hat{\lambda})g_2(a^{1*}) + \hat{\lambda} g_2(a^{2*}) \leq v_2 + \epsilon_2. \end{aligned} \tag{3.1}$$

Let $\tilde{\lambda} = \min_{\hat{\lambda} \in [0,1]} \{\hat{\lambda} \text{ satisfies (3.1)}\}$ be the minimal value for such $\hat{\lambda}$ and denote the corresponding payoff on l^* by $\tilde{v} = (1-\tilde{\lambda})g(a^{1*}) + \tilde{\lambda} g(a^{2*})$. According to Lemma FM1, there exists a sequence of $g(a^{1*})$ and $g(a^{2*})$, and a discount factor $\delta^p < 1$ such that for all $\delta \in [\delta^p, 1)$, the expected average payoff is $\tilde{v}$ and all continuation payoffs along this sequence can be limited to the line segment between $\tilde{v}$ and v. We denote the sequence of actions by $\{a^p(t)\}$ with $a^p(t) \in \{a^{1*}, a^{2*}\}$ for all t.

Analogously, there exists a sequence of $g(a^{1*})$ and $g(a^{2*})$, and a discount factor $\delta^n < 1$ such that for all $\delta \in [\delta^n, 1)$, the expected average payoff is v and all continuation payoffs along this sequence can be limited to the line segment

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[3] It can be shown that for every 2×2 game, one can always identify such an action pair a^{1*}.

[4] If $g_2(a^1) = v_2$ but $g_1(a^2) > v_1$, the subsequent punishment construction can be carried out for Player 2. The case where both $g_2(a^1) = v_2$ and $g_1(a^2) = v_1$ hold is discussed in Appendix 3.A.

between v and $\tilde{v}$. This shall be the normal phase of the equilibrium strategy $\sigma(v)$, and we denote the sequence of actions by $\{a^n(t)\}$ with $a^n(t) \in \{a^{1*}, a^{2*}\}$ for all t. Note that, in general, $\delta^n \neq \delta^p$, and we shall therefore take the maximum of the two in the following steps to ensure that the continuation payoffs of $\{a^p(t)\}$ and $\{a^n(t)\}$ are limited to the line segment between $\tilde{v}$ and v. In the following, therefore, let $\delta > \bar{\delta} = \max\{\delta^n, \delta^p\}$.

The punishment of Player 1 shall be carried out as follows: In the first period, play action a^1. Then, from period $t = 1$ on, follow the sequence $\{a^p(t)\}$ with average payoff $\tilde{v}$. If Player 1 cheats during her punishment, restart with action a^1. The payoffs at the beginning of her punishment are then given by $p^1 = (1 - \delta)g(a^1) + \delta\tilde{v}$, and all subsequent continuation payoffs lie on the line segment between $\tilde{v}$ and v. This construction is illustrated in Figure 3.4.

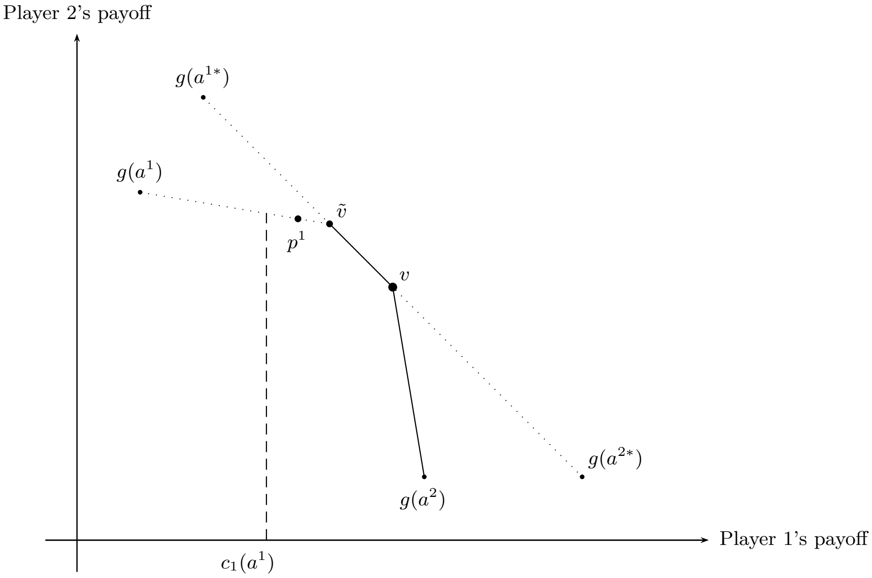

Figure 3.4: Construction of punishment for Player 1.

For the punishment of Player 2, we can adapt the construction for a penance punishment strategy, as suggested by Farrell and Maskin (1989, p. 335): After a single deviation of Player 2, play action a^2 for a suitable number of periods t_2 before returning to the normal phase with expected average payoff v. If Player 2 cheats on her punishment, restart with action a^2 (for details see Farrell and Maskin, 1989).

If Player 1 cheats during the normal phase, she receives $p_1^1 < v_1$, which satisfies

$$(1 - \delta)v_1^{max} + \delta p_1^1 < v_1$$

for a sufficiently large δ. As also $p_1^1 > v_1 - \epsilon_1$ for δ sufficiently large, Player 1 has no incentive to cheat in the normal phase or on her own punishment. For Player 2, all her continuation payoffs along Player 1's punishment path are weakly greater than her equilibrium payoff v_2, and thus she has no incentive to deviate from punishing Player 1. As the same holds for Player 2's punishment, this strategy constitutes a subgame perfect equilibrium. Formally, we can define the equilibrium strategy $\sigma(v)$ as follows:

> Play begins in the normal phase, in which players are to follow the sequence $\{a^n(t)\}$. If Player 1 cheats in the normal phase, the continuation equilibrium is "play a^1 for 1 period, then follow the sequence $\{a^p(t)\}$". If Player 2 cheats in the normal phase, the continuation equilibrium is "play a^2 for t_2 periods, then return to the normal phase". If a player cheats during her punishment, the punishment begins again. If player i cheats during the opponent's punishment, then player i's punishment begins immediately.

All continuation payoffs of the normal phase and all continuation payoffs of Player 1's punishment after the first punishment period lie on the line segment between $\tilde{v}$ and v. Furthermore, all continuation payoffs of Player 2's punishment lie on the line segment between between v and $g(a^2)$, and therefore there is no Pareto-ranking between those three equilibrium paths. Finally, as $p_1^1 < \tilde{v_1}$ and $p_2^1 > \tilde{v_2}$, none of the continuation payoffs of $\sigma(v)$ are Pareto-ranked, and therefore v is a WRP equilibrium.

Note. If the punishment for Player 1 as suggested by Farrell and Maskin (1989) was followed, which is to play a^1 for a suitable number of periods and then revert to the sequence $\{a^n(t)\}$, all continuation payoffs of the punishment phase would lie on the line segment between $g(a^1)$ and v, and therefore below the line l^*. Then, even for a sufficiently large δ, it cannot be excluded that there is a strict Pareto-improvement from Player 1's punishment to the normal phase (see our discussion in Appendix 3.B).

3.5 Proof of Proposition 3.1

As the proof of Proposition 3.1 is quite intricate, we will first give an elaborate outline of the proof before we formally proof the result in Subsections 3.5.1–3.5.3.

We start with the trivial observation that for every v in $\mathcal{V} \setminus \mathcal{U}$, and therefore in the interior of $\mathcal{V}$, one can always find two payoffs $v^{'}$ and $v^{''}$ in $\mathcal{U}$ such that v is a convex combination. However, it is not straightforward that the line segment between $v^{'}$ and $v^{''}$ has a negative slope to satisfy Condition (a). This will generally depend on the payoff structure of the game, and we therefore have to complete several steps to show that Condition (a) is always satisfied, given the hypotheses of the theorem.

In a first step, we will prove Proposition 3.1 for 2×2 games. As we are interested only in those games where $\mathcal{U}$ is a strict subset of $\mathcal{V}$, we will first give a general characterization result of the set of mixed-strategy payoffs $\mathcal{U}$ in Subsection 3.5.1. While the set of feasible payoffs $\mathcal{V}$ is generically a quadrilateral whose extreme points correspond to pure-strategy payoffs, we show in Lemma 3.1 that any payoff $v \in \mathcal{V} \setminus \mathcal{U}$ will be in a convex set that can be characterized by an edge of $\mathcal{V}$ and a convex curve between the two end-points of this edge. To show that Proposition 3.1 holds for 2×2 games, we must distinguish between the following two cases.

If the edge is of a positive slope, the construction given by Farrell and Maskin (1989) to show Claim 3.1 does not fail. That is, for at least one of the two action pairs a^1 or a^2 given by the hypotheses of the theorem, there exists an action pair $\tilde{a} \in \mathcal{A}$ such that v is a convex combination of $g(\tilde{a})$ and $g(a^i)$, and thus Proposition 3.1 holds immediately.

If the edge is of a negative slope, we can use a parallel line that passes through v. Due to the shape of $\mathcal{V} \setminus \mathcal{U}$, this line must intersect with two edges of $\mathcal{V}$ whose points correspond to payoffs in $\mathcal{U}$. That is, the two points of intersection yield action pairs a^{1*} and a^{2*} that fulfill the conditions of Proposition 3.1.

In the second step in Subsection 3.5.3, we extend the results for 2×2 games to general $n \times m$ games. While the set of payoffs is generically a polygon, we can identify for every $v \in \mathcal{V} \setminus \mathcal{U}$ a 2×2 game such that v is in its respective convex hull of payoffs. Then, we can use the result for 2×2 games to finally complete the proof of Proposition 3.1.

3.5.1 Characterization of $\mathcal{U}$ in 2×2 games

Consider a general 2×2 game with a payoff-matrix of the following form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{3.2}$$

where $A, B, C, D \in \mathbb{R}^2$. For mixed strategies, we assign probabilities $(1-p)$, p to rows and $(1-q)$, q to columns of (3.2). To characterize the set of mixed-strategy payoffs $\mathcal{U}$, we will distinguish between four different cases. These cases will be determined by the shape of $\mathcal{V}$, i.e., the convex hull of the pure-strategy payoffs A, B, C and D. In the following text, we will therefore shift our analysis from the set of actions and payoff matrices to the space of payoffs; that is, we will study the graphs produced by the payoff function g.[5]

We will frequently make use of the following definitions.

Definition 3.2. For $A, B \in \mathbb{R}^2$, we will denote by $\overline{AB}$ the edge or line segment that connects A and B. The infinite line through points A and B will be denoted by $\overleftrightarrow{AB}$, and the vector that starts in A and connects A with B will be denoted by $\overrightarrow{AB}$. The triangle with extreme points A, B and C will be denoted by ΔABC.

Definition 3.3. We call $a = (a_1, a_2) \in \mathcal{A}$ a *semi-pure strategy* if one player plays a pure strategy, while the other chooses a mixed strategy, in which either a_1 or a_2 is equal to a standard unit vector. The set of payoffs from a semi-pure strategy is called an *inducement correspondence*.[6]

It is straightforward that for two payoffs of the matrix (3.2) that are either in the same row or column, all payoffs on the edge between these two payoffs are obtainable with a semi-pure strategy. Therefore the payoff matrix (3.2) yields six edges, $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{BC}$, $\overline{BD}$ and $\overline{CD}$, and four inducement correspondences, $\overline{AB}$, $\overline{AC}$, $\overline{BD}$ and $\overline{CD}$.

Generically, the convex hull $\mathcal{V}$ of the four payoffs will be a quadrilateral in the payoff space. As we are interested in those cases where $\mathcal{U}$ is a strict subset of $\mathcal{V}$, the cases where $\mathcal{V}$ is not a two-dimensional object are of no interest for the proof of Proposition 3.1. It is straightforward to see that if all four points of (3.2) are equal, $\mathcal{V}$ is a singleton, and therefore $\mathcal{U} = \mathcal{V}$. Also, if the four payoffs are such that $\mathcal{V}$ is a line, $\mathcal{U} = \mathcal{V}$ holds.

If $\mathcal{V}$ is a two-dimensional object that is defined by at least three extreme points, the issue is more complicated and $\mathcal{U} = \mathcal{V}$ is generally not true. The set of mixed-strategy payoffs is given by

$$\begin{aligned}\mathcal{U} = \{v \in \mathcal{V} \mid \\ & v = (1-p)(1-q)A + (1-p)qB + p(1-q)C + pqD; p, q \in [0,1]\},\end{aligned}$$

where v can be rewritten such that

$$\begin{aligned}\mathcal{U} = \{v \in \mathcal{V} \mid \\ & v = A + p(1-q)(C-A) + q(1-p)(B-A) + pq(D-A); p, q \in [0,1]\}.\end{aligned}$$

Without loss of generality we assume that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are linearly independent, i.e., A, B and C are not on a line. Then we can find parameters $\beta, \gamma \in \mathbb{R}$ such

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[5] For example, see Robinson and Goforth (2005) for an elaborate discussion of this approach.

[6] It is the set of payoffs that one player can "induce" by playing a pure strategy. See also Robinson and Goforth (2005).

that we can construct the point D as follows:

$$D = \beta(B - A) + \gamma(C - A).$$

For $\beta, \gamma \in [0, 1]$ and $\beta + \gamma \leq 1$, $\mathcal{V}$ is a triangle. That is, the point D is either on the boundary or in the interior of the triangle defined by the extreme points A, B and C. In all other cases, $\mathcal{V}$ will be a quadrilateral defined by the four extreme points A, B, C and D. Two edges between these extreme points will necessarily lie in the interior of $\mathcal{V}$, and each one of these edges subdivides $\mathcal{V}$ into two triangles.

Depending on the parameters β and γ, i.e., on the position of D, there are three different cases. For $\beta < 0$, $\gamma > 0$ and $\beta + \gamma < 1$, $\overline{AC}$ is an interior edge of $\mathcal{V}$. For $\beta > 0$,$\gamma < 0$ and $\beta + \gamma < 1$, $\overline{AB}$ is an interior edge of $\mathcal{V}$. For $\beta, \gamma > 0$ and $\beta + \gamma > 1$, $\overline{AD}$ is an interior edge of $\mathcal{V}$. As we show in Lemma 3.1, we can neglect the last case in the following definition.

Definition 3.4. Let $\mathcal{V}$ be a quadrilateral with extreme points A, B, C and $D = \beta(B - A) + \gamma(C - A)$ with $\beta + \gamma < 1$. If $\beta < 0$, $\gamma > 0$, $\overline{AC}$ divides $\mathcal{V}$ into two *subtriangles*, ΔABC and ΔACD. If $\beta > 0$, $\gamma < 0$, $\overline{AB}$ divides $\mathcal{V}$ into two *subtriangles*, ΔABC and ΔABD. We denote these subtriangles by

$$\mathcal{V}^1 = \Delta ABC, \qquad \mathcal{V}^2 = \begin{cases} \Delta ACD, & \beta < 0, \gamma > 0 \\ \Delta ABD, & \beta > 0, \gamma < 0 \end{cases}.$$

Note that by definition, we have $\mathcal{V}^1 \cup \mathcal{V}^2 = \mathcal{V}$. Furthermore, for $\beta < 0 < \gamma < 1 + \beta$, we have $\mathcal{V}^1 \cap \mathcal{V}^2 = \overline{AC}$, and for $\gamma < 0 < \beta < 1 + \gamma$, we have $\mathcal{V}^1 \cap \mathcal{V}^2 = \overline{AB}$. Using this subdivision we obtain the following result.

Lemma 3.1. *Let $\mathcal{V}$ be the convex hull of payoffs A, B, C and D and let A, B and C not be on a line. Let $\beta, \gamma \in \mathbb{R}$ be such that $D = \beta(B - A) + \gamma(C - A)$.*

1. *In the following cases, $\mathcal{V}$ is a triangle and $\mathcal{V} \setminus \mathcal{U}$ is a convex set at the boundary of $\mathcal{V}$.*
   - *(a) $\beta, \gamma \geq 0$ and $\beta + \gamma < 1$*
   - *(b) $\gamma \leq 0$ and $\beta + \gamma \geq 1$*
   - *(c) $\beta \leq 0$ and $\beta + \gamma \geq 1$*
   - *(d) $\beta < 0$ and $\gamma < 0$*
2. *If $\beta, \gamma > 0$ and $\beta + \gamma \geq 1$, $\mathcal{U} = \mathcal{V}$.*
3. *If $\beta + \gamma < 1$ and $\beta < 0$ or $\gamma < 0$, $\mathcal{V}$ can be characterized such that $\mathcal{V}^1 \setminus \mathcal{U}$ and $\mathcal{V}^2 \setminus \mathcal{U}$ are convex sets at the boundary of $\mathcal{V}^1$ and $\mathcal{V}^2$, respectively.*

If $\mathcal{V}$ is a triangle and $\mathcal{U} \neq \mathcal{V}$, we obtain that $\mathcal{U} \subset \mathcal{V}$ is the set of payoffs between the two edges that are inducement correspondences and a convex curve between their distinct endpoints. For a payoff matrix (3.2) and the parameters $\beta, \gamma \geq 0$ with $\beta + \gamma < 1$, this is the area between $\overline{AB}$ and $\overline{AC}$ and the curve between B and C that is below $\overline{BC}$. An exemplary graph is given in Figure 3.5, and in the proof of the lemma (in Appendix 3.A), we give an analytical expression for the boundary.

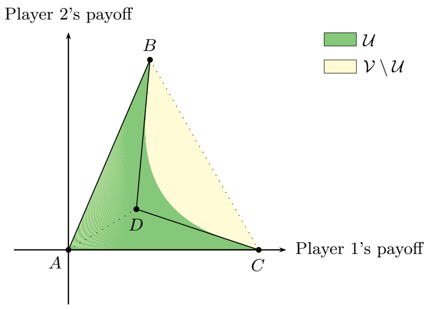

Figure 3.5: $\beta, \gamma \in (0, 1)$ and $\beta + \gamma < 1$.

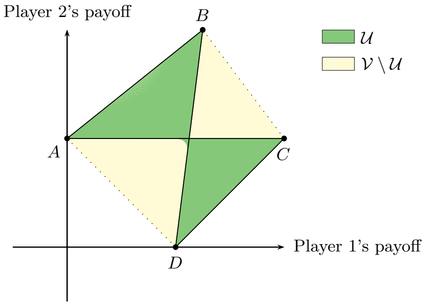

Figure 3.6: $\beta < 0$,$\gamma > 0$ and $\beta + \gamma < 1$.

If $\mathcal{V}$ is a quadrilateral and $\mathcal{U} \neq \mathcal{V}$, as illustrated in Figure 3.6, the characterization of $\mathcal{U}$ in the two subtriangles $\mathcal{V}^1$ and $\mathcal{V}^2$ is similar to the characterization of $\mathcal{U}$ where $\mathcal{V}$ is a triangle. In the proof of the lemma (in Appendix 3.A), we give an analytical expression for the boundaries of $\mathcal{U}$ in the subtriangles.

Given this characterization for the set of mixed-strategy payoffs, we can now proof Proposition 3.1 for 2×2 games. As for the characterization, we will first consider those games where $\mathcal{V}$ is a triangle, and then we will consider the general case where $\mathcal{V}$ is a quadrilateral.

3.5.2 Proposition 3.1 for 2×2 games

Lemma 3.2. *Let $\mathcal{V}$ be the convex hull of payoffs A, B, C and D. Let $\beta, \gamma \in \mathbb{R}$ be such that $D = \beta(B - A) + \gamma(C - A)$. Then Proposition 3.1 holds.*

Proof. Those cases where $\mathcal{U} = \mathcal{V}$ can be excluded here. First, we will proof the lemma for the case where $\mathcal{V}$ is a triangle. From Lemma 3.1, we have that one of the edges on the boundary of $\mathcal{V}$ is not an inducement correspondence, and that this edge is also a boundary of the set $\mathcal{V}\setminus\mathcal{U}$. In the following, we will distinguish between different cases for the slope of this edge.

Without loss of generality, we assume that A, B and C are not on a line and that $\beta, \gamma \geq 0, \beta + \gamma < 1$. Then, the edge of $\mathcal{V}$ that is not an inducement correspondence is $\overline{BC}$ (see also Figure 3.5). For the characterization, we first normalize A to zero and assume, without loss of generality, that for the payoffs $B = (B_1, B_2)$ and $C = (C_1, C_2)$, we have $B_1 \leq C_1$.

First, consider those cases with $B_1 < C_1$, $B_2 > C_2$, where $\overleftrightarrow{BC}$ has a negative slope, as illustrated in Figure 3.7. As $v \in \mathcal{V}$, the line l^* that is parallel to $\overleftrightarrow{BC}$ and runs through v will intersect with both inducement correspondences $\overline{AB}$ and $\overline{AC}$. Let these points of intersections be $v^{'} = l^* \cap \overline{AB}$ and $v^{''} = l^* \cap \overline{AC}$. Since $v^{'}, v^{''}$ are mixed-strategy payoffs, there are actions a^{1*} and a^{2*} such that $g(a^{1*}) = v^{'}$ and $g(a^{2*}) = v^{''}$. Clearly a^{1*} and a^{2*} satisfy the conditions of Proposition 3.1. The same holds true when $\overleftrightarrow{BC}$ has an infinite slope, i.e., when $B_1 = C_1$ and $B_2 > C_2$ or $B_2 < C_2$.

Second, assume that $B_1 < C_1$ and $B_2 \leq C_2$, so that $\overleftrightarrow{BC}$ has a non-negative slope. Assume first that $\overleftrightarrow{BC}$ lies above the origin; that is, it crosses the x-axis to the left of the origin or is constant above the x-axis, as, for example, in Figure 3.8. Then, as in the original proof, construct the line l^1 that starts in $g(a^1)$ and runs through v. By the hypothesis of Theorem 3.1, l^1 has a negative slope and will therefore intersect with $\overline{AC}$ at a point $v^{''}$. Hence, the construction for Claim 3.1 works, and therefore Proposition 3.1 follows immediately with $a^{1*} = a^1$ and a^{2*} such that $g(a^{2*}) = v^{''}$. For the case that $\overleftrightarrow{BC}$ lies below the origin, i.e., it crosses the x-axis to the right of the origin or is constant below the x-axis, the analogous construction with l^2 works, and therefore Proposition 3.1 holds as well.

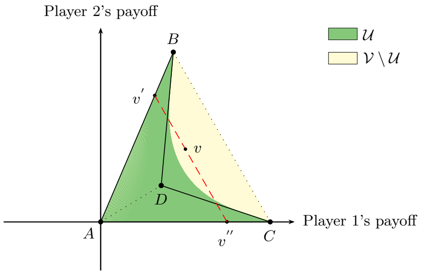

Figure 3.7: $\mathcal{V}$ for $\overleftrightarrow{BC}$ with negative slope.

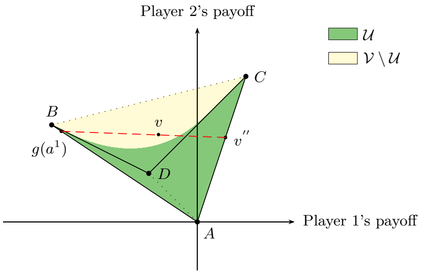

Figure 3.8: $\mathcal{V}$ for $\overleftrightarrow{BC}$ with positive slope.

The proof for the case where $\mathcal{V}$ is a quadrilateral uses the same approach. As in Lemma 3.1, we consider the two subtriangles $\mathcal{V}^1$ and $\mathcal{V}^2$. Without loss of generality, assume that $v \in \mathcal{V}^1$. Then, one of the edges on the boundary of $\mathcal{V}^1$ is not an inducement correspondence, and this edge is also a boundary of the set $\mathcal{V}^1 \setminus \mathcal{U}$. We can now duplicate the arguments from the case where $\mathcal{V}$ is a triangle to the subtriangle $\mathcal{V}^1$ to show that Proposition 3.1 holds. $\square$

3.5.3 Generalization to $n \times m$ games

To completely prove Proposition 3.1, we have to consider general $n \times m$ games with $n, m \geq 2$. The resulting convex hull of payoffs $\mathcal{V}$ in these games will generally be a polygon, as in Example 3.1 below. As in the proof for 2×2 games, we first characterize the set $\mathcal{U}$. To do so, we will make use of the following definition.

Definition 3.5. Let $\Pi = (\pi_{ij}) \in \mathbb{R}^{n \times m}$ be the payoff matrix of the $n \times m$ game g. Then, every two elements π_{ij} and π_{kl} of Π with $i \neq k$, $j \neq l$, will induce a unique 2×2 submatrix

$$\Pi_{ijkl} = \begin{pmatrix} \pi_{ij} & \pi_{il} \\ \pi_{kj} & \pi_{kl} \end{pmatrix}.$$

We define the *induced* 2×2 *game* $g|_{ijkl}$ of g as the 2×2 game restricted to those pure actions that yield payoff matrix Π_{ijkl}. The set of mixed-strategy payoffs obtainable in $g|_{ijkl}$ will be denoted by $\mathcal{U}|_{ijkl}$.

By definition, we have $\mathcal{U}|_{ijkl} \subseteq \mathcal{U}$ for every induced 2×2 game $g|_{ijkl}$ of g, and also

$$\mathcal{U}_{\{2\times2\}} := \bigcup_{\substack{i,j,k,l \\ i\neq k, j\neq l}} \mathcal{U}|_{ijkl} \subseteq \mathcal{U}. \tag{3.3}$$

This characterization, together with Lemma 3.2, suffices to prove Proposition 3.1.

Proof of Proposition 3.1. Let $v \in \mathcal{V}^* \setminus \mathcal{U}$. By (3.3) we have

$$\mathcal{V}^* \setminus \mathcal{U} \subseteq \mathcal{V}^* \setminus \mathcal{U}_{\{2\times2\}}.$$

Therefore, there exists an induced 2×2 game $g|_{ijkl}$ of g such that $v \in \mathcal{V}^* \setminus \mathcal{U}|_{ijkl}$. By Lemma 3.2, we have that for every $v \in \mathcal{V}^* \setminus \mathcal{U}|_{ijkl}$ with a^1 and a^2 as given in the hypotheses of Theorem 3.1, there are always action pairs a^{1*} and a^{2*} that satisfy Conditions (a) and (b) of Proposition 3.1. ☐

Example 3.1. Consider the following 2×4 game with pure-strategy payoffs $A = (0, 2), B = (1, 3), C = (5, 3), D = (5, 1), E = (6, 2), F = (3, 4), G = (1, 1)$ and $H = (3, 0)$ according to the payoff matrix

$$\Pi = \begin{pmatrix} A & B & C & D \\ E & F & G & H \end{pmatrix}. \tag{3.4}$$

The resulting convex hull of payoffs $\mathcal{V}$ and the set $\mathcal{U}_{\{2\times2\}}$ are illustrated in Figure 3.9. Consider payoff $v = (5.1, 2.75)$. As illustrated in Figure 3.9, v is not a mixed-strategy payoff. It is close to the edge $\overline{CE}$, which is not an inducement correspondence. This edge induces the 2×2 game $g|_{ACEG}$ with payoff matrix

$$\Pi_{ACEG} = \begin{pmatrix} A & C \\ E & G \end{pmatrix}.$$

As illustrated in Figure 3.9, v is in the convex hull of A, C, E and G. For this 2×2 game, we can apply Lemma 3.2 to show that Proposition 3.1 holds. Using the punishment strategy developed in Section 3.4, v can be sustained as a WRP equilibrium.

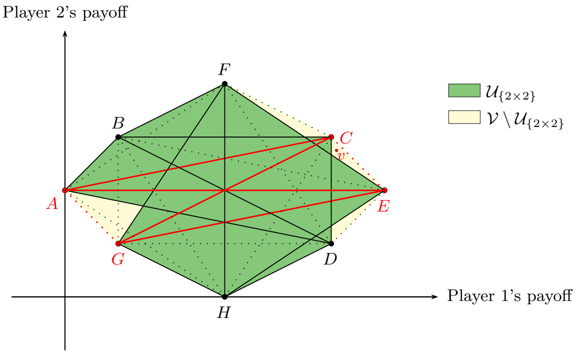

Figure 3.9: $\mathcal{V}$ is a polygon and v induces the 2×2 game $g|_{ACEG}$.

3.6 Conclusion

We have shown by means of a counterexample that the proposed proof of Theorem 1 in Farrell and Maskin (1989) may fail. Given a strictly individually rational payoff v and two action pairs a^1 and a^2 that satisfy the hypotheses of Theorem 3.1, these action pairs cannot always be used to construct a sequence that yields an average payoff v. Nevertheless, as we have shown in Proposition 3.1, given such action pairs a^i, we can always find two alternative actions such that their convex combination yields payoff v and that can be used to define the normal phase of the game. For the punishment strategies, we can use the action pairs a^i, although we need to design a different punishment as in the original proof to ensure that no continuation payoffs of the strategy can be strictly Pareto-ranked. Therefore, we prove that the sufficient conditions of Farrell and Maskin (1989) continue to hold, and that an equilibrium strategy exists that sustains v as a WRP equilibrium.

Appendices

3.A Proofs

In the proof of Theorem 3.1 in Section 3.4, we assumed that for $i \neq j$, at least one of the inequalities $g_j(a^i) \geq v_j$ is strict. In the following segment, we show that this is indeed without loss of generality by discussing the case where $g_2(a^1) = v_2$ and $g_1(a^2) = v_1$ hold, as illustrated in Figure 3.A.1.

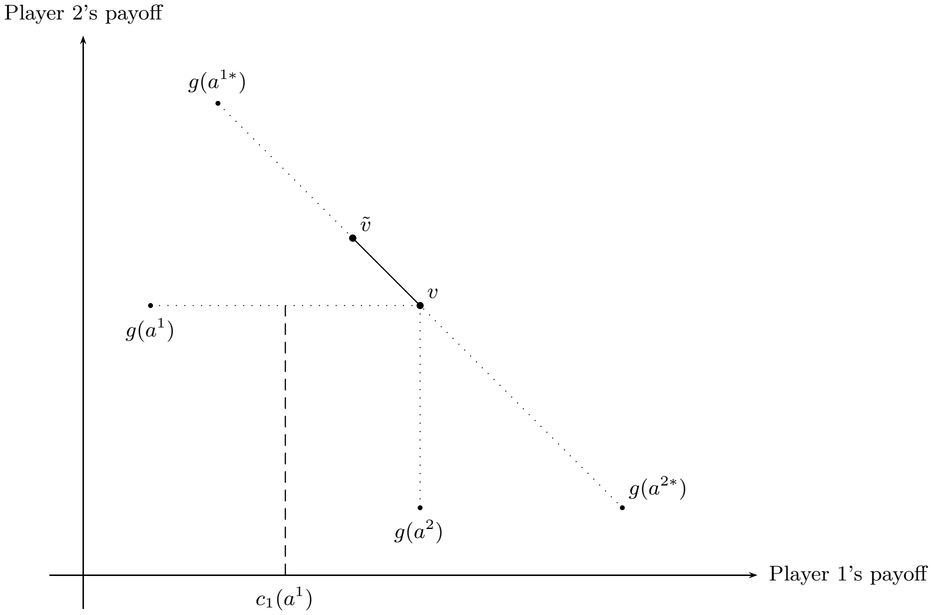

Figure 3.A.1: Boundary case of Theorem 3.1.

Let $a^1 = (p^1, q^1) \in [0,1]^{n\times m}$, where p^1 and q^1 are vectors of probabilities over pure actions such that $p^1 = (p^1_1, \ldots, p^1_n)$ with $\sum_{i=1}^n p^1_i = 1$, and $q^1 = (q^1_1, \ldots, q^1_m)$

with $\sum_{i=1}^{m} q_i^1 = 1$. By the hypotheses of the theorem, we have that $g_1(p, q^1) < v_1$ for every probability vector $p \in [0,1]^n$. If for Player 1 there is a probability vector $\tilde{p} \in [0,1]^n$ such that $g_2(\tilde{p}, q^1) > v_2$, we can use $\tilde{a}^1 = (\tilde{p}, q^1)$ to construct the punishment of Player 1. This is illustrated in Figure 3.A.2.

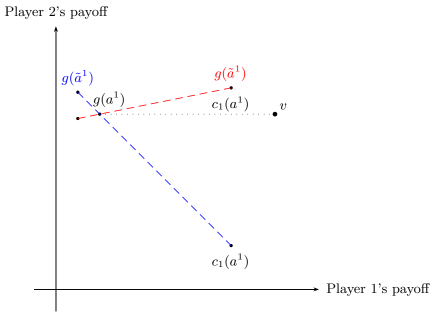

Figure 3.A.2: The alternative payoff $g(\tilde{a}^1)$ lies on $g(\cdot, q^1)$.

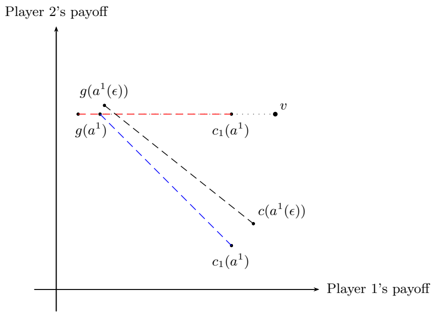

Figure 3.A.3: The alternative action $a^1(\epsilon)$ is a perturbation of a^1.

If there is no such $\tilde{p}$, as illustrated in Figure 3.A.3, we need to find a different action for Player 1's punishment. We will therefore slightly perturb Player 2's mixed strategy q^1 by $\epsilon > 0$ to obtain an action pair $a^1(\epsilon)$ that we can use to construct Player 1's punishment. We will need the following definition.

Definition 3.A.1. Let $q \in [0,1]^m$ be a probability vector. For $\epsilon > 0$, we define $Q(\epsilon)$ to be the set of probability vectors $q(\epsilon)$ that differ from q in every entry by at most ϵ:

$$Q(\epsilon) = \left\{ q(\epsilon) \in [0,1]^m \;\middle|\; |q_j(\epsilon) - q_j| \leq \epsilon \text{ for all } j \in \{1, \dots, m\}, \sum_{j=1}^{m} q_j(\epsilon) = 1 \right\}.$$

Since $c_1(a^1) < v_1$, there is an entry $i \in \{1, \dots, n\}$ such that $p_i = 1$ is a best response to q^1 and $\sum_{j=1}^{m} q_j^1 g_1(a_{ij}) < v_1$. Then, there exists an $\epsilon > 0$ such that for all $q^1(\epsilon) \in Q^1(\epsilon)$, we have

$$\sum_{j=1}^{m} q_j^1(\epsilon) g_1(a_{ij}) < v_1.$$

Furthermore, we have that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} p_i q_j^1(\epsilon) g_1(a_{ij}) < v_1$$

for all $p \in [0,1]^n$, and therefore $c_1(\cdot, q(\epsilon)) < v_1$.

If $g_2\big(\cdot, q^1(\epsilon)\big) \leq v_2$ for all $q^1(\epsilon) \in Q^1(\epsilon)$, then $g(\cdot, q^1)$ must be either an edge or an extreme point of $\mathcal{V}$, and consequently the construction for Claim 3.1 holds true. Otherwise, there exists $p \in [0,1]^n$ and $q^1(\epsilon) \in Q^1(\epsilon)$ with $g_2(p, q^1(\epsilon)) > v_2$, and we can use $a^1(\epsilon) = (p, q^1(\epsilon))$ to construct the punishment of Player 1.

Proof of Lemma 3.1. For the proof of Lemma 3.1, we first derive the characterization result for those cases where $\mathcal{V}$ is a triangle. For better readability, we state this in a separate lemma.

Lemma 3.A.1. *Let $\mathcal{V}$ be the convex hull of payoffs A, B, C and D, and let A, B and C not be on a line. Let $\beta, \gamma \in [0,1]$ with $\beta + \gamma \leq 1$ be such that $D = \beta(B - A) + \gamma(C - A)$.*

1. *If $\beta + \gamma = 1$, $\mathcal{U} = \mathcal{V}$.*

2. *If $\beta, \gamma \in (0,1)$ and $\beta + \gamma < 1$, $\mathcal{V} \setminus \mathcal{U}$ is a convex set at the boundary of $\mathcal{V}$.*

Proof of Lemma 3.A.1. First, and without loss of generality, we assume the payoff A to be normalized to zero, i.e., $A = (0,0)$. For $D = \beta B + \gamma C$ and with abuse of notation, we can rewrite $\mathcal{U}$ as

$$\begin{aligned}\mathcal{U} &= Cp\left((1-q) + q\gamma\right) + Bq\left((1-p) + p\beta\right)\\ &= Cp\left(1 - q(1-\gamma)\right) + Bq\left(1 - p(1-\beta)\right).\end{aligned}$$

Next, we define two functions $x, y : [0,1]^2 \longrightarrow [0,1]$ that are defined as $x(p,q) = p\left(1 - q\left(1-\gamma\right)\right)$ and $y(p,q) = q\left(1 - p\left(1-\beta\right)\right)$ such that $\mathcal{U}$ can be rewritten as

$$\mathcal{U} = x(p,q)C + y(p,q)B.$$

In order to determine the set $\mathcal{U}$, we will characterize its boundaries. Clearly, $\mathcal{U}$ is a subset of the convex hull of $\mathcal{V}$, and the sides $\overline{AB}$ and $\overline{AC}$ are obviously boundaries of $\mathcal{U}$. To completely characterize the shape of $\mathcal{U}$, we need to determine the remaining boundary of $\mathcal{U}$ between the two extreme points B and C. Depending on β and γ, $\mathcal{U}$ may not reach the side $\overline{BC}$, but rather lie below this edge. We can characterize this boundary that is as close as possible to $\overline{BC}$ by determining the maximal value of y for each x. Geometrically speaking, for every distance from A along the vector $\overrightarrow{AC}$, we want to find the maximal distance that we can go along the vector $\overrightarrow{AB}$.

In formal terms, we will solve the optimization problem that yields the maximal value of y for every given value of x , subject to p and q being from the unit interval. Given a value x and $\gamma > 0$, p is implicitly defined as a function of x and q by $p(x,q) = \frac{x}{1-q(1-\gamma)}$. Therefore, we can express the optimization problem only in x and q, i.e., $\max_q y(x,q)$, and as we will make use of the

Karush–Kuhn–Tucker (KKT) Theorem, we state it in the following standard form:

$$\begin{aligned} \max_q q\left(1-\frac{x(1-\beta)}{1-q(1-\gamma)}\right) & \\ \text{s.t.}\quad & q \geq 0 \\ & 1-q \geq 0 \\ & \frac{x}{1-q(1-\gamma)} \geq 0 \\ & 1-\frac{x}{1-q(1-\gamma)} \geq 0 \end{aligned} \tag{3.A.1}$$

With the Lagrange multipliers $\alpha_1, \ldots, \alpha_4 \geq 0$, the necessary conditions for a solution of (3.A.1) are given by

$$\begin{aligned} 1-\frac{x\,(1-\beta)}{\left(1-q\,(1-\gamma)\right)^2}+\alpha_1-\alpha_2+\alpha_3\frac{x(1-\gamma)}{\left(1-q\,(1-\gamma)\right)^2} & \\ -\alpha_4\frac{x(1-\gamma)}{\left(1-q\,(1-\gamma)\right)^2} &= 0 \qquad (3.A.2) \\ q &\geq 0 \qquad (3.A.3) \\ \alpha_1 q &= 0 \qquad (3.A.4) \\ 1-q &\geq 0 \qquad (3.A.5) \\ \alpha_2(1-q) &= 0 \qquad (3.A.6) \\ \frac{x}{1-q(1-\gamma)} &\geq 0 \qquad (3.A.7) \\ \alpha_3\left(\frac{x}{1-q(1-\gamma)}\right) &= 0 \qquad (3.A.8) \\ 1-\frac{x}{1-q(1-\gamma)} &\geq 0 \qquad (3.A.9) \\ \alpha_4\left(1-\frac{x}{1-q(1-\gamma)}\right) &= 0 \qquad (3.A.10) \end{aligned}$$

As $y(x,q)$ is concave in q and all inequality constraints are linear, these necessary conditions are also sufficient. Let us first discuss the general case for $\beta \in (0,1)$ and $\gamma \in (0,1)$, that is, D is in the interior of $\mathcal{V}$, as illustrated in Figure 3.5.

(1) Assume $\alpha_1 > 0$ holds.

From (3.A.4) we obtain $q = 0$ as a possible solution and from (3.A.6) it follows that $\alpha_2 = 0$. Condition (3.A.7) is equivalent to $x \geq 0$, and from (3.A.9) we obtain that $x \leq 1$ must hold. Assume $\alpha_3 > 0$. Then, by (3.A.8) $x = 0$ must hold, but (3.A.2) yields a contradiction and thus $\alpha_3 = 0$ must hold. For $\alpha_4 = 0$, (3.A.2) is equivalent to $1 - x(1-\beta) + \alpha_1 = 0$, which is again a contradiction.

Therefore, it remains to check $\alpha_4 > 0$ and hence $x = 1$. Condition (3.A.2) yields no contradiction, and therefore $q = 0$ is a solution if $x = 1$.

(2) Assume that $\alpha_1 = 0$ and $\alpha_2 > 0$ hold.

From (3.A.6) we obtain that $q = 1$ is a possible solution and from (3.A.7) we obtain that $x \geq 0$ must be satisfied. Also, by (3.A.9) we have that $x \leq \gamma$ has to hold. Assume $\alpha_3 > 0$. From (3.A.8) we have that $x = 0$, and therefore $\alpha_4 = 0$ must be satisfied. Condition (3.A.2) becomes $1 - \alpha_2 = 0$, and is therefore satisfied for $\alpha_2 = 1$.

For $\alpha_3 = 0$, assume first $\alpha_4 > 0$. That is, $x = \gamma$ needs to hold. However, (3.A.2) then becomes $1 - \frac{1-\beta}{\gamma} - \alpha_2 - \alpha_4 \frac{1-\gamma}{\gamma} = 0$, which is a contradiction as $\beta + \gamma < 1$. Therefore, $\alpha_4 = 0$ needs to hold. Condition (3.A.2) then reads $1 - x\frac{1-\beta}{\gamma^2} - \alpha_2 = 0$ and yields $x = \frac{(1-\alpha_2)\gamma^2}{1-\beta}$. This is not in conflict with (3.A.7) and (3.A.9) for $\alpha_2 \leq 1$, and therefore $q = 1$ is a solution if $x \in [0, \frac{\gamma^2}{1-\beta})$.

(3) Finally, assume that $\alpha_1 = 0$ and $\alpha_2 = 0$ hold.

First, assume $\alpha_3 > 0$. Then, by (3.A.8) $x = 0$ has to hold, but this yields a contradiction of (3.A.2). Therefore, $\alpha_3 = 0$ must hold. If we assume $\alpha_4 = 0$, we receive from (3.A.2) that $q^* = \frac{1-\sqrt{x(1-\beta)}}{1-\gamma}$ is a possible solution. Condition (3.A.9) is only satisfied for $x \leq 1-\beta$. We now check whether this is in accordance with conditions (3.A.3) and (3.A.5), i.e., that $q^* \in [0,1]$. First, $q^* \geq 0$ if and only if $x \leq \frac{1}{1-\beta}$, which is already implied by $x \leq 1 - \beta$, and which is therefore no additional constraint. Second, $q^* \leq 1$ if and only if $x \geq \frac{\gamma^2}{1-\beta}$, and therefore q^* is a solution for $x \in [\frac{\gamma^2}{1-\beta}, 1 - \beta]$.

Finally, assume $\alpha_4 > 0$. Then, (3.A.10) yields $q^{**} = \frac{1-x}{1-\gamma}$ as a solution candidate. Inserting this into (3.A.2) yields that $x > 1 - \beta$ must hold. Condition (3.A.3) is satisfied if and only if $x \leq 1$, and (3.A.5) holds if and only if $x \geq \gamma$. The latter is already implied by $x > 1-\beta$, and therefore q^{**} is a solution for $x \in (1 - \beta, 1]$.

Summarizing, we have obtained the following optimal q^{max} as a function of the parameter $x \in [0,1]$

$$q^{max}(x) = \begin{cases} 1, & x \in [0, \frac{\gamma^2}{1-\beta}) \\ \frac{1-\sqrt{x(1-\beta)}}{1-\gamma}, & x \in [\frac{\gamma^2}{1-\beta}, 1-\beta] \\ \frac{1-x}{1-\gamma}, & x \in (1-\beta, 1] \end{cases},$$

which yields for a given x the optimal y^{max} defined by

$$y^{max}(x) = \begin{cases} 1 - \frac{x(1-\beta)}{\gamma}, & 0 \leq x < \frac{\gamma^2}{1-\beta} \\ \frac{\left(1-\sqrt{x(1-\beta)}\right)^2}{1-\gamma}, & \frac{\gamma^2}{1-\beta} \leq x \leq 1-\beta \\ \frac{(1-x)\beta}{1-\gamma}, & 1-\beta < x \leq 1 \end{cases}.$$

Therefore the remaining boundary of $\mathcal{U}$, denoted by $\mathcal{U}^{max}$, is defined as

$$\mathcal{U}^{max} = \Big\{ v \in \mathcal{U} \;\Big|\; xC + y^{max}(x)B, x \in [0,1] \Big\}. \tag{3.A.11}$$

That is, we can describe this boundary of $\mathcal{U}$ between the points B and C as a tripartite curve in the Cartesian plane with two linear parts, where either $q = 1$ or $p = 1$ holds, and therefore the edge $\overline{BD}$ or $\overline{CD}$ is the boundary, respectively, and a non-linear part defined by the curve

$$xC + \frac{\Big(1 - \sqrt{x(1-\beta)}\Big)^2}{1-\gamma} B$$

for $x \in [\frac{\gamma^2}{1-\beta}, 1-\beta]$.

In the following boundary cases, some of the previous steps are not necessary or need to be considered differently. We will discuss them briefly.

If D is equal to one of the two extreme points B or C, the problem simplifies. For $\beta = 0, \gamma = 1$, that is, $D = C$, the function $x(p,q)$ reduces to p and $y(p,q) = q(1-p)$. Thus, given a fixed value p, $q = 1$ is always the maximizer that corresponds to the edge $\overline{BD} = \overline{BC}$. Therefore, the third boundary of $\mathcal{U}$ corresponds to the third side of $\mathcal{V}$, and therefore $\mathcal{U} = \mathcal{V}$ holds. Analogously, for $\beta = 1, \gamma = 0$, that is, $B = D$, the same approach yields $\mathcal{U} = \mathcal{V}$.

If $\beta + \gamma = 1$ holds, the point D lies on the edge $\overline{BC}$, and again $\mathcal{U} = \mathcal{V}$ holds. For values of $x \in [0, 1-\beta]$, $q = 1$ is the feasible maximizer of y, and therefore the edge $\overline{BD}$ is the boundary of $\mathcal{U}$. For $x \in (1-\beta, 1]$, $q = \frac{1-x}{\beta}$ is the feasible maximizer of y that corresponds to $p = 1$. Therefore, the edge $\overline{CD}$ is the boundary of $\mathcal{U}$ and consequently $\mathcal{U} = \mathcal{V}$.

If $\beta = 0$ and $\gamma \in (0,1)$, the point D lies on the edge $\overline{AC}$. The tripartite boundary $\mathcal{U}^{max}$ reduces to a bipartite one. For $x \in [0, \gamma^2)$, $q = 1$ is the feasible maximizer of y, and therefore the edge $\overline{BD}$ is the boundary of $\mathcal{U}$. For $x \in [\gamma^2, 1]$, $q^* = \frac{1-\sqrt{x}}{1-\gamma}$ determines the boundary of $\mathcal{U}$.

Finally, if $\gamma = 0$, the implicit function $p(x,q)$ is not well-defined for $q = 1$. Only if $x = 0$ is this the case, and this directly yields that $q = 1$ is a feasible maximizer of y if and only if $x = 0$. If also $\beta = 0$, then for $x \in (0,1]$, $q^* = 1 - \sqrt{x}$ is the maximizer, and therefore the boundary is completely determined by the corresponding curve. If $\beta \in (0,1)$ holds, for $x \in (0, 1-\beta]$, $q^* = 1 - \sqrt{x(1-\beta)}$ and for $x \in (1-\beta, 1]$, $q^{**} = 1 - x$ are the respective solutions of (3.A.1).

It now remains to be shown that $\mathcal{V} \setminus \mathcal{U}$ is a convex set for $\beta + \gamma < 1$. First, note that $\mathcal{V} \setminus \mathcal{U}$ is determined by the edge $\overline{BC}$ and the boundary $\mathcal{U}^{max}$ derived above. The set $\mathcal{V} \setminus \mathcal{U}$ can therefore be interpreted as a simple closed curve.[7]

---

[7] A curve is a simple closed curve if it is a connected curve that does not cross itself and ends at the same point where it begins.

A closed regular plane simple curve is convex if and only if its signed curvature is either always non-negative or always non-positive (see, for example, Gray et al., 2006, pp. 163–165). If we interpret $\mathcal{U}^{max}$ as a vector function

$$f : [0,1] \longrightarrow \mathbb{R}^2, f(x) = xC + y^{max}(x)B,$$

we can easily show that $f(x)$ is $\mathcal{C}^2$. Also, $\frac{\partial^2 y^{max}(x)}{\partial^2 x} \leq 0$ holds for all $x \in [0,1]$, and therefore the signed curvature $\kappa(x) = \frac{f''(x)}{(1+[f'(x)]^2)^{3/2}}$ is non-positive for all $x \in [0,1]$. As the signed curvature of the edge $\overline{BC}$ is also non-positive, the set $\mathcal{V} \setminus \mathcal{U}$ is convex. □

Now, we turn to the proof of Lemma 3.1. For those cases where $\mathcal{V}$ is a triangle, we can use the characterization of the previous Lemma 3.A.1. For the remaining cases, we use an analogous approach for the two triangles $\mathcal{V}^1$ and $\mathcal{V}^2$.

Without loss of generality, we assume A to be normalized to zero, that is, $A = (0,0)$ and $D = \beta B + \gamma C$ with $\beta, \gamma \in \mathbb{R}$. First, we show that all parameter constellations given in 1.(a) – 1.(d) yield that $\mathcal{V}$ is a triangle.

In 1.(a), for $\beta, \gamma \in [0,1]$ with $\beta + \gamma \leq 1$, the convex hull of payoffs $\mathcal{V}$ is a triangle, and therefore we can apply Lemma 3.A.1. For $\gamma = 0, \beta \in \mathbb{R}$, the point D lies on the straight line $\overleftrightarrow{AB}$. For $\beta = 0, \gamma \in \mathbb{R}$, D lies on $\overleftrightarrow{AC}$ and for $\beta + \gamma = 1$, D lies on $\overleftrightarrow{BC}$.

In 1.(b), $\frac{1}{\beta}, \frac{-\gamma}{\beta} \in [0,1]$ and $\frac{1}{\beta} - \frac{\gamma}{\beta} < 1$. Therefore, $B = \frac{1}{\beta}D - \frac{\gamma}{\beta}C$ is in the interior of the triangle ΔADC. For 1.(c), analogous considerations yield that $C = \frac{1}{\gamma} - \frac{\beta}{\gamma}B$ is in the interior of the triangle ΔABD.

Finally, in 1.(d), if $\beta, \gamma \leq 0$, the origin, that is, $A = (0,0) = D - \frac{\beta}{1-\beta-\gamma}(B - D) - \frac{\gamma}{1-\beta-\gamma}(C - D)$ is in the interior of the triangle ΔDBC.

Next, we consider case 2 with $\beta, \gamma > 0$ and $\beta + \gamma \geq 1$. If $\beta + \gamma = 1$, Lemma 3.A.1 yields that $\mathcal{U} = \mathcal{V}$. If $\beta + \gamma > 1$, D is clearly above the edge $\overline{BC}$, and this edge is an interior edge of the quadrilateral $\mathcal{V}$. Analogously to the proof of Lemma 3.A.1, we solve the optimization problem (3.A.1) to show that in this case $\mathcal{U} = \mathcal{V}$ holds.

First, for $\gamma < 1$, we obtain

$$q^{max}(x) = \begin{cases} 1, & x \in [0,\gamma) \\ \frac{1-x}{1-\gamma}, & x \in [\gamma, 1] \end{cases}$$

as the solution for (3.A.1), which yields the following optimal y^{max} for a given x:

$$y^{max}(x) = \begin{cases} 1 - \frac{x(1-\beta)}{\gamma}, & 0 \leq x < \gamma \\ \frac{(1-x)\beta}{1-\gamma}, & \gamma \leq x \leq 1 \end{cases}.$$

If $\beta < 1$, the function $y(x, q)$ is concave in q, and therefore the Karush–Kuhn–Tucker (KKT) conditions (3.A.2) through (3.A.10) are both necessary and sufficient. If $\beta \geq 1$, the KKT conditions yield only necessary, but not sufficient, conditions. More specifically, $y(x, q)$ is strictly increasing in q, and therefore $q = 1$ is the feasible solution as long as $x < \gamma$. For all $x \geq \gamma$, the maximal value is determined by the linear function $q(x) = \frac{1-x}{1-\gamma}$, and therefore coincides with $y^{max}(x)$. Thus, the sides $\overline{BD}$ and $\overline{DC}$ are also boundaries of $\mathcal{U}$, and therefore $\mathcal{U} = \mathcal{V}$.

If $\gamma = 1$, $x(p, q)$ reduces to p and $y(p, q) = q(1-p)$. Thus, given a fixed value p, q=1 is always the maximizer that corresponds to the edge $\overline{BD}$. Therefore, the sides $\overline{BD}$ and $\overline{DC}$ are also boundaries of $\mathcal{U}$, and therefore $\mathcal{U} = \mathcal{V}$.

For $\gamma > 1$, we obtain for $x \in [0, 1]$

$$q^{max}(x) = 1$$

as the solution for (3.A.1), which yields the following optimal $y^{max}(x)$ for a given x:

$$y^{max}(x) = 1 - \frac{x(1-\beta)}{\gamma}.$$

If $\beta > 1$, the function $y(x, q)$ is concave in q, and therefore the KKT conditions (3.A.2) through (3.A.10) are both necessary and sufficient. If $\beta \leq 1$, $y(x, q)$ is strictly decreasing in q, and therefore we need to compare $y^{max}(x)$ with $y(0, x)$. As $1 - \frac{x(1-\beta)}{\gamma} > 0$ for all $x \in [0, 1]$, $y^{max}(x)$ is the solution of the optimization problem (3.A.1). Thus, the sides $\overline{BD}$ and $\overline{DC}$ are also boundaries of $\mathcal{U}$, and thus $\mathcal{U} = \mathcal{V}$.

Finally, let us now consider the remaining parameter constellations of case 3, which we will group into four different cases: i) $\beta < 0, \gamma \in (0,1]$, ii) $\beta < 0, \gamma > 1$ and $\beta + \gamma < 1$, iii) $\beta \in (0,1], \gamma < 0$ and iv) $\beta > 1, \gamma < 0$ and $\beta + \gamma < 1$. Four exemplary graphs for the resulting quadrilaterals are given in Figures 3.A.4–3.A.7. We will only discuss the cases i) and ii), as iii) and iv) are obviously analogous.

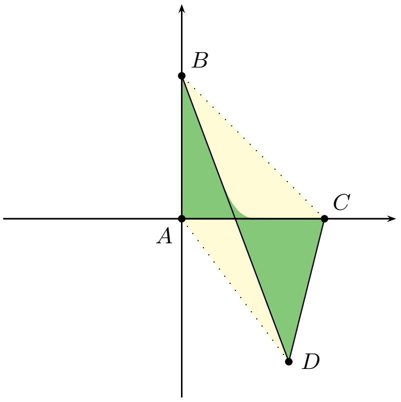

Figure 3.A.4: $\beta < 0, \gamma \in (0,1)$

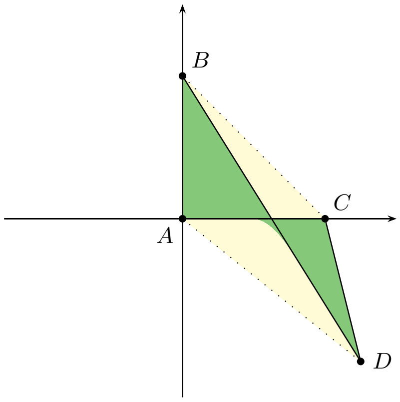

Figure 3.A.5: $\beta < 0, \gamma > 1$

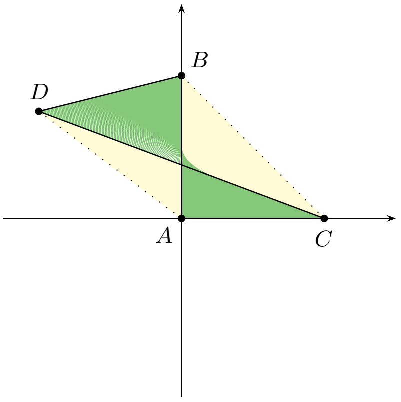

Figure 3.A.6: $\beta \in (0,1), \gamma < 0$

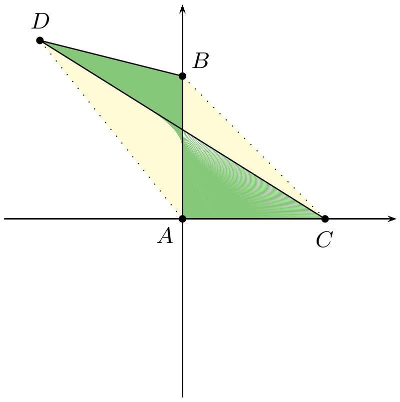

Figure 3.A.7: $\beta > 1, \gamma < 0$

i) First, let $\beta < 0$ and $\gamma \in (0,1]$. In principle, we will follow the same approach as in the proof of Lemma 3.A.1, but we must add several considerations. First, we note that $y(\frac{1}{1-\beta}, q) = 0$ for all $q \in [0,1]$, and since $\beta < 0$, we have to separately study $\mathcal{U}$ for values of $p < \frac{1}{1-\beta}$, $p > \frac{1}{1-\beta}$ and $p = \frac{1}{1-\beta}$. Thus, we effectively split up the convex hull of $\mathcal{V}$ into the two subtriangles $\mathcal{V}^1 = \Delta ABC$ and $\mathcal{V}^2 = \Delta ADC$, and therefore study $\mathcal{U}$ above and below $\overline{AC}$. Clearly, the edge $\overline{AC}$ is an inducement correspondence and is therefore included in $\mathcal{U}$.

We will first determine $\mathcal{U}$ for $p \in [0, \frac{1}{1-\beta})$ and $\gamma \in (0,1)$. We can follow the same steps as in the proof of Lemma 3.A.1, and therefore solve (3.A.1), but for $p \in [0, \frac{1}{1-\beta})$ instead of $p \in [0,1]$. Then, given the Lagrange multipliers

$\alpha_1, \dots, \alpha_4 \geq 0$, the necessary conditions for a solution of the new optimization problem can be stated as follows:

$$1 - \frac{x\,(1-\beta)}{\left(1-q\,(1-\gamma)\right)^2} + \alpha_1 - \alpha_2 + \alpha_3 \frac{x(1-\gamma)}{\left(1-q(1-\gamma)\right)^2}$$

$$-\alpha_4 \frac{x(1-\gamma)}{\left(1-q(1-\gamma)\right)^2} = 0 \qquad (3.A.12)$$

$$q \geq 0 \qquad (3.A.13)$$

$$\alpha_1 q = 0 \qquad (3.A.14)$$

$$1 - q \geq 0 \qquad (3.A.15)$$

$$\alpha_2(1-q) = 0 \qquad (3.A.16)$$

$$\frac{x}{1-q(1-\gamma)} \geq 0 \qquad (3.A.17)$$

$$\alpha_3 \left(\frac{x}{1-q(1-\gamma)} \right) = 0 \qquad (3.A.18)$$

$$\frac{1}{1-\beta} - \frac{x}{1-q(1-\gamma)} \geq 0 \qquad (3.A.19)$$

$$\alpha_4 \left(\frac{1}{1-\beta} - \frac{x}{1-q(1-\gamma)} \right) = 0 \qquad (3.A.20)$$

As $y(x,q)$ is a concave function of q, these conditions are also sufficient and we obtain the optimal $y^{max}(x)$ for $x \in [0, \frac{1}{1-\beta})$ as follows

$$y^{max}(x) = \begin{cases} 1 - \frac{x(1-\beta)}{\gamma}, & 0 \leq x < \frac{\gamma^2}{1-\beta} \\ \frac{\left(1-\sqrt{x(1-\beta)}\right)^2}{1-\gamma}, & \frac{\gamma^2}{1-\beta} \leq x < \frac{1}{1-\beta} \end{cases}.$$

Therefore, for $p < \frac{1}{1-\beta}$ and $\gamma \in (0,1)$,

$$\mathcal{U}^{max} = \left\{ v \in \mathcal{U} \;\middle|\; xC + y^{max}(x)B, x \in [0, \tfrac{1}{1-\beta}) \right\}$$

is the boundary of $\mathcal{U}$ between the points B and C. That is, we can describe the boundary of $\mathcal{U}$ between the points B and C as a bipartite curve in the Cartesian plane with a linear part, where $q = 1$ holds, and therefore $\overline{BD}$ is the boundary, and with a non-linear part defined by the curve

$$xC + \frac{\left(1 - \sqrt{x(1-\beta)}\right)^2}{1-\gamma} B$$

for $x \in [0, \frac{1}{1-\beta})$. For $\gamma = 1$, as discussed in the special cases for triangles, $x(p,q) = p$, and therefore $q = 1$ is the maximizer of $y(p,q)$ for all $p < \frac{1}{1-\beta}$. Thus, $\mathcal{U}$ is the triangle between $\overline{AB}$, $\overline{AC}$ and $\overline{BD}$. We have now completely characterized $\mathcal{U}$ in $\mathcal{V}^1$, i.e., above $\overline{AC}$ for $\gamma \in (0,1]$.

For $p > \frac{1}{1-\beta}$, still with abuse of notation, we rewrite $\mathcal{U}$ as follows:

$$\mathcal{U} = C - (1-p)\Big(1 - \frac{q}{\beta}(1-\gamma)\Big)C + \frac{q}{\beta}\Big(1 - p(1-\beta)\Big)(D - C)$$

Next, and analogously to the proof of Lemma 3.A.1, we define two functions $\tilde{x}(p,q) = (1-p)\left(1 - \frac{q}{\beta}(1-\gamma)\right)$ and $\tilde{y}(p,q) = \frac{q}{\beta}\left(1 - p(1-\beta)\right)$ such that

$$\mathcal{U} = C - \tilde{x}(p,q)C + \tilde{y}(p,q)(D - C).$$

As in the proof of Lemma 3.A.1, we now determine for each $\tilde{x}$ the maximal $\tilde{y}$ such that $C - \tilde{x}(p,q)C + \tilde{y}(p,q)(D - C)$ is as close as possible to the edge $\overline{AD}$. Geometrically speaking, for every distance from C along the vector $\overrightarrow{CA}$, we want to find the maximal distance that we can go in the direction of vector $\overrightarrow{CD}$.

In formal terms, we will solve the optimization problem that yields the maximal value of $\tilde{y}$ for every given value of $\tilde{x}$, subject to $p > (\frac{1}{1-\beta}, 1]$ and $q \in [0,1]$. Given a value $\tilde{x}$, p is implicitly defined as a function of the two parameters $\tilde{x}$ and q by $p(\tilde{x}, q) = 1 - \frac{\beta\tilde{x}}{\beta - q(1-\gamma)}$.[8]

Therefore, $\tilde{y} = q\Big(1 + \frac{\tilde{x}(1-\beta)}{\beta - q(1-\gamma)}\Big)$, and we can express the optimization problem only in $\tilde{x}$ and q:

$$\begin{aligned} &\max_{q} \tilde{y}(\tilde{x}, q) \\ &\quad \text{s.t.} \quad q \in [0,1] \\ &\qquad\qquad p(\tilde{x}, q) \in (\tfrac{1}{1-\beta}, 1] \end{aligned} \tag{3.A.21}$$

However, for $p > \frac{1}{1-\beta}$, $\tilde{y}$ is a strictly convex function in q, and we therefore only need to consider the two boundary points $q = 0$ and $q = 1$. We have that $\tilde{y}(\tilde{x}, 0) = 0$ and $\tilde{y}(\tilde{x}, 1) = \frac{1}{\beta}\Big(\beta + \frac{\tilde{x}(1-\beta)}{1 - \frac{1-\gamma}{\beta}}\Big) > 0$ for all $\tilde{x} \in [0, 1 - \frac{\gamma}{1-\beta})$. This is equivalent to $p \in (\frac{1}{1-\beta}, 1]$, and thus $q = 1$ is the maximizer. This corresponds to the inducement correspondence $\overline{BD}$, and hence in $\mathcal{V}^2$, $\mathcal{U}$ is the triangle between $\overline{AC}$, $\overline{CD}$ and $\overline{BD}$.

Finally, for $p = \frac{1}{1-\beta}$, we have that $y(p,q) = 0$ and $x \in [\frac{\gamma}{1-\beta}, \frac{1}{1-\beta}]$. That is, the obtainable mixed-strategy payoffs for this value of p is a subset of the inducement correspondence for $q = 0$, i.e., the edge $\overline{AC}$.

In conclusion, we have characterized $\mathcal{U}$ in $\mathcal{V}$ by splitting up $\mathcal{V}$ into two triangles $\mathcal{V}^1$ and $\mathcal{V}^2$ such that $\mathcal{U} \cap \mathcal{V}^1$ is analogous to Lemma 3.A.1 and $\mathcal{U} \cap \mathcal{V}^2$ is a triangle. Therefore, $\mathcal{V}^1 \setminus \mathcal{U}$ and $\mathcal{V}^2 \setminus \mathcal{U}$ are convex sets at the boundary of $\mathcal{V}$.

ii) Now, let $\beta < 0, \gamma > 1$ and $\beta + \gamma < 1$. For this parameter constellation, we can follow the same approach as in i) and split up $\mathcal{U}$ into values of $p < \frac{1}{1-\beta}$, $p > \frac{1}{1-\beta}$ and $p = \frac{1}{1-\beta}$.

---

[8] As $\beta < 0$ and $\beta + \gamma < 1$, the implicit function $p(\tilde{x}, q)$ is well-defined for all $\tilde{x}$ and q.

For $p < \frac{1}{1-\beta}$ and $\gamma > 1$, $y(x,q)$ is a quadratic, convex function in q, and therefore the KKT conditions (3.A.2)–(3.A.10) do not yield sufficient conditions. Thus, it suffices to check the two boundary points $q = 1$ and $q = 0$. We have that $y(x,0) = 0$ and $y(x,1) = 1 - x\frac{1-\beta}{1-\gamma} > 0$ for all $x \in [0, \frac{y}{1-\beta})$. This is equivalent to $p \in [0, \frac{1}{1-\beta})$, and thus, for this range of p, in $\mathcal{V}^1$, $\mathcal{U}$ is the triangle between $\overline{AB}$, $\overline{AC}$ and $\overline{BD}$.

For $p > \frac{1}{1-\beta}$, $\tilde{y}(\tilde{x},q)$ is a concave function in q. Thus, the KKT Theorem yields sufficient and necessary conditions for a solution of the optimization problem (3.A.21). Given the Lagrange multipliers $\alpha_1, \ldots, \alpha_4 \geq 0$, these are:

$$\begin{aligned} 1 + \frac{\tilde{x}\,(1-\beta)\,\beta}{\left(\beta - q\,(1-\gamma)\right)^2} + \alpha_1 - \alpha_2 - \alpha_3 \frac{\tilde{x}\beta(1-\gamma)}{\left(\beta - q(1-\gamma)\right)^2} & \\ + \alpha_4 \frac{\tilde{x}\beta(1-\gamma)}{\left(1 - q(\beta-\gamma)\right)^2} &= 0 \qquad (3.A.22) \\ q &\geq 0 \qquad (3.A.23) \\ \alpha_1 q &= 0 \qquad (3.A.24) \\ 1 - q &\geq 0 \qquad (3.A.25) \\ \alpha_2(1-q) &= 0 \qquad (3.A.26) \\ \frac{1}{1-\beta} + \frac{\tilde{x}}{\beta - q(1-\gamma)} &\geq 0 \qquad (3.A.27) \\ \alpha_3 \left(\frac{1}{1-\beta} + \frac{\tilde{x}}{\beta - q(1-\gamma)} \right) &= 0 \qquad (3.A.28) \\ \frac{\beta\tilde{x}}{\beta - q(1-\gamma)} &\geq 0 \qquad (3.A.29) \\ \alpha_4 \left(\frac{\beta\tilde{x}}{\beta - q(1-\gamma)} \right) &= 0 \qquad (3.A.30) \end{aligned}$$

(1) Assume $\alpha_1 > 0$.

From (3.A.24) we obtain $q = 0$ as a possible solution, and from (3.A.26) it follows that $\alpha_2 = 0$. Condition (3.A.27) is equivalent to $\tilde{x} \leq \frac{\beta}{\beta-1}$, and from (3.A.29) we obtain that $\tilde{x} \geq 0$ must hold. Assume $\alpha_3 > 0$, then $\tilde{x} = \frac{\beta}{\beta-1}$ must hold and (3.A.22) is satisfied for suitable $\alpha_1, \alpha_3, \alpha_4 \geq 0$. For $\alpha_3 = 0$ and $\alpha_4 > 0$, $\tilde{x} = 0$ must hold, but then (3.A.22) yields a contradiction as $1 + \alpha_1 > 0$. For $\alpha_3 = 0$ and $\alpha_4 = 0$, $0 \leq \tilde{x} \leq \frac{\beta}{\beta-1}$ must hold, but then $1 + \alpha_1 + \tilde{x}(1-\beta) > 0$, which also conflicts with (3.A.22). Therefore, $q = 0$ is a solution if $\tilde{x} = \frac{\beta}{\beta-1}$.

(2) Assume that $\alpha_1 = 0$ and $\alpha_2 > 0$ hold.

From (3.A.26) we obtain that $q = 1$ is a possible solution. As $\beta + \gamma < 1$, (3.A.29) yields that $\tilde{x} \geq 0$ must be satisfied, and by (3.A.27) we have that $x \leq \frac{\beta-1+\gamma}{\beta-1}$ has to hold. Assume $\alpha_4 > 0$. From (3.A.30) we have that $\tilde{x} = 0$ must hold, and therefore $\alpha_3 = 0$. Condition (3.A.22) is satisfied for $\alpha_2 = 1$.

For $\alpha_4 = 0$, assume first that $\alpha_3 > 0$. That is, $\tilde{x} = \frac{\beta-1+\gamma}{\beta-1}$ needs to hold. But then, (3.A.22) becomes $1 - \frac{\beta}{\beta-1+\gamma} - \alpha_2 + \alpha_3 \frac{\beta(1-\gamma)}{(1-\beta)(\beta-1+\gamma)} = 0$, which is a contradiction as $\beta + \gamma < 1$ and $\gamma > 1$. Therefore, $\alpha_3 = 0$ needs to hold and Condition (3.A.22) reads $1 - \frac{\tilde{x}(1-\beta)\beta}{(\beta-1+\gamma)^2} - \alpha_2 = 0$. Thus, $\tilde{x} = \frac{(1-\alpha_2)(1-\beta-\gamma)^2}{\beta(\beta-1)}$, and for $\alpha_2 \leq 1$, (3.A.27) and (3.A.29) are satisfied. Therefore, $q = 1$ is a solution if $\tilde{x} \in [0, \frac{(1-\beta-\gamma)^2}{\beta(\beta-1)})$.

(3) Finally, assume that $\alpha_1 = 0$ and $\alpha_2 = 0$ hold.

First, assume that $\alpha_4 > 0$. Then, (3.A.30) yields that $\tilde{x} = 0$ has to hold, which conflicts with (3.A.22). Therefore, $\alpha_4 = 0$ must hold.

If we assume that $\alpha_3 > 0$, we receive from (3.A.28) that $q^{**} = \frac{\beta-\tilde{x}(\beta-1)}{1-\gamma}$ is a possible solution. For (3.A.23) and (3.A.25) to be fulfilled, $\tilde{x}$ must satisfy $\tilde{x} \leq \frac{\beta}{\beta-1}$ and $\tilde{x} \geq \frac{\beta-1+\gamma}{\beta-1}$. However, for this range of $\tilde{x}$, (3.A.22) is not fulfilled, and therefore q^{**} is not a feasible solution.

It remains to check whether $\alpha_3 = 0$. In this case, (3.A.22) yields two possible candidates: $q^{*a} = \frac{\beta+\sqrt{\tilde{x}(\beta-1)\beta}}{1-\gamma}$ and $q^{*b} = \frac{\beta-\sqrt{\tilde{x}(\beta-1)\beta}}{1-\gamma}$. The candidate q^{*a} does not satisfy (3.A.25), but q^{*b} is in the unit interval for $\tilde{x} \in [\frac{(1-\beta-\gamma)^2}{\beta(\beta-1)}, \frac{\beta}{\beta-1}]$.

Summarizing, we have obtained the following optimal $q^{max}(x)$ for $x \in [0, \frac{\beta}{\beta-1}]$:

$$q^{max}(x) = \begin{cases} 1, & \tilde{x} \in [0, \frac{(1-\beta-\gamma)^2}{\beta(\beta-1)}) \\ \frac{\beta-\sqrt{\tilde{x}(\beta-1)\beta}}{1-\gamma}, & \tilde{x} \in [\frac{(1-\beta-\gamma)^2}{\beta(\beta-1)}, \frac{\beta}{\beta-1}] \end{cases},$$

which yields

$$\tilde{y}^{max}(x) = \begin{cases} 1 + \frac{\tilde{x}(1-\beta)}{\beta-1+\gamma}, & \tilde{x} \in [0, \frac{(1-\beta-\gamma)^2}{\beta(\beta-1)}) \\ \frac{\beta-\sqrt{\tilde{x}(\beta-1)\beta}}{1-\gamma}(1 + \frac{\tilde{x}(1-\beta)}{\sqrt{\tilde{x}(\beta-1)\beta}}), & \tilde{x} \in [\frac{(1-\beta-\gamma)^2}{\beta(\beta-1)}, \frac{\beta}{\beta-1}] \end{cases}.$$

Therefore, for $p > \frac{1}{1-\beta}$ we can describe the boundary of $\mathcal{U}$ below $\overline{AC}$ between the points B and C as a function with a linear part, where $q = 1$, and a non-linear part, described by the curve

$$C + \tilde{x}C + \frac{\beta - \sqrt{\tilde{x}(\beta-1)\beta}}{1-\gamma}\left(1 + \frac{\tilde{x}(1-\beta)}{\sqrt{\tilde{x}(\beta-1)\beta}}\right)(D - C)$$

for $\tilde{x} \in [\frac{(1-\beta-\gamma)^2}{\beta(\beta-1)}, \frac{\beta}{\beta-1}]$.

Finally, for $p = \frac{1}{1-\beta}$, we have that $y(p, q) = 0$ and $x \in [\frac{\gamma}{1-\beta}, \frac{1}{1-\beta}]$. That is, the obtainable mixed-strategy payoffs for this value of p is a subset of the inducement correspondence for $q = 0$, i.e., the edge $\overline{AC}$.

In conclusion, we have characterized $\mathcal{U}$ in $\mathcal{V}$ by splitting up $\mathcal{V}$ into the two subtriangles $\mathcal{V}^1$ and $\mathcal{V}^2$ such that $\mathcal{U} \cap \mathcal{V}^1$ is a triangle and $\mathcal{U} \cap \mathcal{V}^2$ is analogous to Lemma 3.A.1. Therefore, $\mathcal{V}^1 \setminus \mathcal{U}$ and $\mathcal{V}^2 \setminus \mathcal{U}$ are convex sets at the boundary of $\mathcal{V}$. $\square$

3.B On The Punishment for Player 1

Consider the following 2×2 game where Players 1 and 2 can choose between two pure actions $\{u, d\}$ and $\{l, r\}$ and mix between them with probabilities $(1-p), p$ and $(1-q), q$, respectively. Recall that we denote a mixed-strategy action by $a = (p, q)$. The stage-game payoffs of the pure strategies are given by the payoff matrix shown in Table 3.B.1.

	l	r
u	$(0,0)$	$(4,0)$
l	$(0,4)$	$(0,0)$

Table 3.B.1: Payoff matrix of the two-player strategic game.

Assume that $v = (1.5, 1.5)$. Then, $a^1 = (\frac{1}{2}, 0)$, $a^2 = (0, \frac{1}{2})$ satisfy the hypotheses of Theorem 3.1: $g(a^1) = (2, 0)$ and $g(a^2) = (0, 2)$. However, as illustrated in Figure 3.B.1, these actions cannot be used to construct the normal phase. Nevertheless, we can easily show that $a^{1*} = (1, \frac{1}{4})$ and $a^{2*} = (0, \frac{3}{4})$ satisfy the conditions of Proposition 3.1, and that we can use these actions to construct the normal phase with expected average payoff v.

If we were to construct the penance punishment for Player 1 according to Farrell and Maskin (1989, p. 335), the respective continuation payoffs of the punishment phase are on the line segment between $g(a^1)$ and v, and the continuation payoff at time t is given by

$$p^1(t) = (1 - \delta^{t_1 - t})g(a^1) + \delta^{t_1 - t}v \tag{3.B.1}$$

for $\delta < 1$ and a sufficiently large t_1.

Let $\epsilon > 0$ and $v(\epsilon)$ be the point on the line segment between $g(a^{1*})$ and $g(a^{2*})$ with $v_1(\epsilon) = v_1 - \epsilon$. Then, by Lemma FM1, there exists a sequence of actions a^{1*} and a^{2*}, and a $\hat{\delta} < 1$ such that for all $\delta > \hat{\delta}$, the average expected payoff of the sequence is v, and all continuation payoffs are limited to the line segment between $v(\epsilon)$ and v.

However, all continuation payoffs along Player 1's punishment path will be on the line segment between $g(a^1)$ and v, which lies strictly below the line segment $g(a^{1*})$ and v. That is, when $\epsilon > 0$ is too large, Player 2's payoff on Player 1's punishment path may be smaller than in the normal phase. Since Player 1 is also better off in the normal phase, there may be a period t such that the punishment payoff $p^1(t)$ is strictly Pareto-dominated by a continuation payoff of the normal phase. That is, there is a renegotiation incentive from Player 1's punishment path to the normal phase which therefore contradicts the WRP condition. This is illustrated in Figure 3.B.1 for $\epsilon = 0.5$.

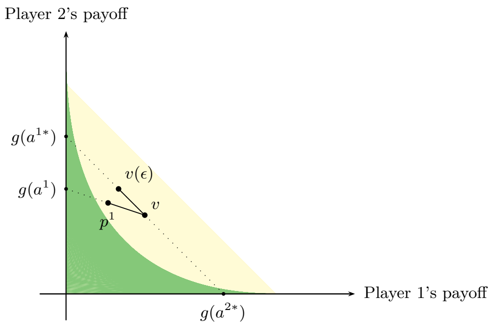

Figure 3.B.1: Punishment- and normal-phase payoffs in the game.

If we decrease ϵ, according to Lemma FM1, we consequently need to increase δ. That is, for $\epsilon \longrightarrow 0$, we have that $v(\epsilon) \longrightarrow v$ but also $\delta \longrightarrow 1$. This in turn implies that, by the construction of p^1 in (3.B.1), we also have that $p^1(t) \longrightarrow v$. Hence, it is not clear whether in the limit there is still a Pareto-ranking between the punishment and the normal phase. In fact, we show in the following analysis that in our example, there may always be an incentive to renegotiate from the punishment to the normal phase for all $\epsilon > 0$.

According to Lemma FM1, all continuation payoffs of the normal phase satisfy

$$v_1 \in [1.5 - \epsilon(\delta), 1.5], v_2 \in [1.5, 1.5 + \gamma(\delta)]$$

for $\epsilon(\delta), \gamma(\delta) > 0$. Note that due to the selection of a^{1*} and a^{2*}, we have that $\epsilon(\delta) = \gamma(\delta)$, and from the proof of Lemma FM1 (Farrell and Maskin, 1989, p. 356) we can determine the value of $\epsilon(\delta)$, which is given by

$$\epsilon = \left(\tfrac{1}{\delta} - 1\right)\left(g_1(a^{2*}) - g_1(a^{1*})\right) = 3\left(\tfrac{1}{\delta} - 1\right).$$

Let $\delta > 0.9$. Then, from the proof of Farrell and Maskin (1989, p. 335), we obtain that $t_1 = 3$ is sufficient for punishment. By (3.B.1), Player 1's continuation payoff on her respective punishment path at time t is given by $p_1^1(t) = 1.5\delta^{3-t}$, while Player 2 receives $p_1^2(t) = 2(1 - \delta^{3-t}) + 1.5\delta^{3-t}$. Then, for any $\delta > 0.9$, we have that

$$\begin{aligned} p_1^1(0) &= 1.5\delta^3 < 1.5 - \epsilon(\delta) \\ p_2^1(0) &= 2(1 - \delta^3) + 1.5\delta^3 < 1.5 + \epsilon(\delta) \end{aligned}$$

holds. Thus, there is an incentive to renegotiate from Player 1's punishment before its start to the continuation payoff $v(\epsilon)$ of the normal phase, as illustrated in Figure 3.B.1.

CHAPTER 4

Extending Weak Renegotiation-Proofness to n-Player Games

4.1 Introduction

In infinitely repeated non-cooperative games, when binding agreements are not possible, threats of punishment are used to sustain an outcome as a subgame perfect equilibrium. These punishments can harm both the punisher(s) and the deviator(s), and may therefore not be efficient. Incentives to renegotiate the initial terms of an agreement may arise and consequently undermine the credibility of the threats. If one requires an outcome to be ex post efficient, the set of credible threats and, ultimately, the set of self-enforcing equilibria outcomes may be limited.

In the late 1980s and the early 1990s, several authors developed concepts that addressed this problem of credibility, and two main approaches have since been established in the literature. The first approach, initiated by the contemporary work of Farrell and Maskin (1989) and Bernheim and Ray (1989), requires credible punishments, and therefore Pareto-undominated continuation equilibria. The second strand of literature, started by Pearce (1987) and Abreu et al. (1993), considers limitations to the set of possible deviations that need to be self-enforcing in order to oppose an equilibrium path credibly.[1]

In the present paper, we reason along the lines of the first strand of literature, and focus on the concept of *weak renegotiation-proofness* by Farrell and Maskin (1989) (subsequently abbreviated as WRP), which is frequently applied in various game-theoretic problems.[2] We argue that the application of this equilibrium concept does not always truly capture the intuitive concept of renegotiation in multilateral negotiations. More specifically, weak renegotiation-proofness has only been introduced and defined properly for two-player games, but is also applied and adopted in larger games with more than two players. We will show that this is not always unproblematic.

In a game with two players, the behavior in equilibrium is fairly simple, as players either cooperate or they do not, and renegotiation is blocked if the innocent party benefits from implementing the agreed-upon punishment. For more than two players, this is no longer straightforward: Some may cooperate independently of the others, and therefore also consider coalitional renegotiation or deviations (Farrell and Maskin, 1989). However, as Asheim and Holtsmark (2009) also concluded, “there exists no refinement of the concept of weakly renegotiation-proof equilibrium that takes into account that also a subset of players can gain by implementing a coordinated deviation” (p. 526).

In this paper, we motivate and develop such a refinement. First, we discuss the sufficient and necessary conditions for WRP equilibria that are presented in Farrell and Maskin (1989) and show that it is not possible to extend the characterization results to general n-player games as these authors proposed. For three players, the conditions will already fail, and we explain how a single player can already hinder the proof.

Second, we provide three concise examples that further motivate the need for a refinement. In our model, in addition to the entire group of players, we also allow subgroups to renegotiate over continuation equilibria. In order to keep the model tractable, we do not allow all possible subgroups of players to renegotiate in every period of the game, but offer a precise protocol that defines when renegotiations may take place and who is allowed to renegotiate.

Our proposed concept strengthens the original WRP definition by excluding those continuation equilibria that are enforceable and weakly efficient for the group of renegotiators. These additional constraints ultimately limit the equilibrium payoffs in certain games, but we also show that in an n-player game of the

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[1] For a discussion and comparison between these two approaches, we refer to Fudenberg and Tirole (1991).

[2] For instance, the large amount of literature on International Environmental Agreements relies mainly on this concept; see, for example, Chapter 2 of this thesis and Finus (2000) for a survey.

Prisoner's Dilemma type, full cooperation withstands our refinements and can still be sustained as an equilibrium.

Several other equilibrium concepts have addressed renegotiation-proofness and built upon the WRP concept. However, as we elaborate in Section 4.2, these notions have also failed to capture the situation in n-player games appropriately. Either a single player or the group of all players can decide upon a deviation or renegotiation. We therefore conclude that there is no suitable concept that addresses the issue of subgroup renegotiation in n-player games in a satisfying way.

The paper is organized as follows. In Section 4.2, we discuss the related literature, and Section 4.3 introduces the standard notation for infinitely repeated games. In Section 4.4, we motivate our equilibrium refinement, and discuss three concise examples that demonstrate how the original WRP notion may yield counterintuitive results in n-player games. We then model the game and introduce our additional restrictions in Section 4.5. Characterization results of the new equilibrium concept and two applications thereof can be found in Section 4.6, before Section 4.7 provides the conclusion. All proofs are presented in Appendix 4.A.

4.2 Related Literature

The focus of the present paper is on the consideration of renegotiation incentives in multilateral negotiations. The first contributions to this strand of literature were made by Farrell and Maskin (1989) and Bernheim and Ray (1989), who study how far renegotiation-proofness conditions limit the set of ex ante efficient subgame perfect equilibria in two-player games. Their proposed notions of *weak renegotiation-proofness* and *internal consistency*, respectively, are widely applied in different game-theoretic settings in which the lack of an enforcing agency implies the need for self-enforcing equilibria to achieve cooperation.[3]

Several refinements to these equilibrium notions exist. Farrell and Maskin (1989) and Bernheim and Ray (1989) themselves extend their notions to *strong renegotiation-proofness* and *strong internal consistency*, respectively, and impose that only those continuation equilibria that are themselves renegotiation-proof, may be an issue in renegotiation, and not only subgame-perfect as in their weaker notion. However, these equilibria may not always exist, and these refinements

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[3] Applications can be found, for example, in the literature on International Environmental Agreements (see, for example, Chapter 2 of this thesis), international trade (e.g., Kletzer and Wright, 2000), and other forms of international cooperation (e.g., McGillivray and Smith, 2000), as well as in dynamic agency models (e.g., Chassang, 2013), oligopoly models that include renegotiation (e.g., McCutcheon, 1997), and family constitution models (e.g., Cigno, 2006).

have therefore not been applied as widely. Ray (1994) offers a modification of the internal consistency equilibrium notion, but does not deliver an existence result either. Rabin (1991) argues that those continuation equilibria that rely on the cooperation of the previous deviator can no longer be subject of renegotiation. He therefore rules out these paths, and calls his concept *reneging-proof*. We refer to Bergin and MacLeod (1993) for a detailed survey and for a harmonization of the standard theories concerning renegotiation-proofness.

More recently, Farrell (2000) extends the weak-renegotiation proofness equilibrium to n-player games, and focuses on symmetric Cournot and Bertrand oligopolies. He remarks that, to sustain collusion as a renegotiation-proof outcome, firms can use highly asymmetric punishments, such that at least one player blocks renegotiation. However, he argues that one would expect innocent firms to be treated symmetrically during punishment, and therefore requires that, after any single deviation, every innocent player should receive weakly higher payoffs than if no one had deviated (see also Aramendía et al., 2005, for a similar approach). Thus, every player would block renegotiation, and not only one single player as required by the definition of WRP. He calls his refinement *quasi-symmetrically weakly renegotiation-proofness* and shows that full cooperation cannot be sustained by too many firms in symmetric oligopolies. Horniaček (2011) defines a *strict renegotiation-proof* equilibrium as a subgame perfect equilibrium with all of its continuation equilibria being strictly Pareto-efficient, not only against other continuation equilibria, but against all strategies.

Thus, to our knowledge, all the relevant literature on n-player games refers either to a single player or to the group of all players that can decide upon a deviation or renegotiation, and there is no consideration of subgroup incentives in the WRP context. Several papers stress this limitation but do not offer an alternative; for example, Barrett (2005) emphasizes “that the concept of a renegotiation-proof equilibrium was developed for the two-player game only [...]. When there are more than two players, some [...] may cooperate independently of the others [...]. To underline the possible importance of this assumption [...], [he refers] to such treaties as being ‘collectively rational’ rather than renegotiation-proof” (pp. 1490–1491).

Somewhat misleadingly, there are some concepts with similar sounding names that seem to address renegotiation as mentioned above, but which take different approaches. For example, Asheim (1997) introduces an equilibrium concept called *revision proofness*. This concept refines subgame perfection in such a way that strategies need to be robust against joint deviations by multiple players in any subgame by imposing an internal stability criterion, as in Bernheim et al. (1987) for coalition proofness, and by adding an external stability criterion. However, it omits any issues related to ex post renegotiation (see also Ales and

Sleet, 2014). *Renegotiation perfection* is introduced by Jamison (2014), and uses elements of tournament theory to define renegotiation-proof sets in general games axiomatically. Nonetheless, he also imposes additional criteria (such as optimality and external stability) that do not coincide with the original idea of renegotiation-proofness. Xue (2000) calls his concept *negotiation-proof*, but only considers a one-shot game with explicit preplay communication to refine Nash equilibria.

There is also established literature that does not implicitly assume the negotiation and renegotiation process at the collective level, but offers an explicit model of communication or bargaining before or during the game. Among the first is Blume (1994), who models intraplay communication using an explicit model of communication that allows for bargaining over continuation payoffs. More recently, Miller and Watson (2013) model the agreement on an equilibrium as a bargaining process that is embedded in the infinitely repeated game. Safronov and Strulovici (2016) model an additional stage after any period of the game in which renegotiation is carried out according to an explicit protocol.

For completeness, we also mention that there is a large amount of literature on renegotiation in finite games (e.g., Benoît and Krishna, 1993; Wen, 1996), as well as in contract theory (e.g., Fudenberg and Tirole, 1990; Hart and Moore, 1988; Battaglini, 2005). Lately, renegotiation-proof networks have also been analyzed (e.g., Jackson et al., 2012).

4.3 Notation and Standard Concepts in Repeated Games

In this section, we introduce the standard notation and concepts that we will use in the following sections. We adopt most of the notations proposed by Farrell and Maskin (1989), but need some more elaborate concepts to capture the subgroup behaviors that we will address in our equilibrium refinement.

Let $N = \{1, \dots, n\}$ denote the set of players with typical element i, and the set of proper subsets of N will be denoted by $\mathcal{P}(N)$. Each player i possesses a finite set of actions and we denote by A_i the simplex consisting of player i's mixed actions. We denote by $A \equiv A_1 \times \dots \times A_n$ the set of all players' actions. Let $g : A \longrightarrow \mathbb{R}^n$ be the vector of continuous payoff functions $g_i : A_i \longrightarrow \mathbb{R}$. The single-stage game $g \equiv [g, A]$ is then defined by the set of payoffs and actions.

For any n-tuple $(x_1, ..., x_n)$, we define $x_M \equiv \{x_i\}_{i \in M}$ and $x_{-M} \equiv \{x_i\}_{i \notin M}$ for an arbitrary subset of players $M \subset N$. For player i, the profit maximizing deviation from $a \in A$ is defined by $c_i(a) = \max_{a_i'} g_i(a_{-i}, a_i')$, and the minimax payoff by $\underline{v}_i = \min_{a_{-i}} \max_{a_i} g_i(a_{-i}, a_i)$.

In the repeated game, we consider the infinite repetition of the single-stage game g, which will be denoted g^*. Let $t = 1, 2, \dots, \infty$ denote the periods and

$\alpha_i = \{a_i(t)\}_{t\in\mathbb{N}}$ a player's action profile with $a_i(t) \in A_i$. Note that we assume constant action spaces, i.e., $A_i^t = A_i$ for all periods t.

A t-history will be denoted by $h^t = (a(1), \ldots, a(t))$, and H is the set of all such possible t-histories. In each period, the game g is played, and $g^*|_{h^t}$ is the subgame of g^* defined by the history $h^t \in H$.

A strategy σ_i for player i in the repeated game is a function that defines an action $a^i \in \mathcal{A}_i$ for every date t and history $h^t \in H$. We call Σ_i the set of all strategies for player i, and we denote by $\sigma \equiv (\sigma_i)_{i\in N} \in \Sigma \equiv \Sigma_1 \times \ldots \times \Sigma_n$ a strategy profile of g^*.[4] An action profile or path induced by σ is denoted by $\alpha(\sigma)$, and $\alpha(\sigma, h^t)$ is the action profile starting from time $t+1$, which is induced by the t-history h^t and the subsequent application of σ. Finally, the subgame of g^* after history h^t is denoted by $g^*|_{h^t}$, and $\sigma|_{h^t}$ denotes the restriction of σ to this subgame; this will subsequently be referred to as a continuation strategy of σ. The set of continuation strategies of σ is denoted by $\Sigma(\sigma) = \{\sigma|_{h^t}, h^t \in H\}$.

In each period, players receive the stage game payoffs. Given a sequence of actions $\{a_i(t)\}_{t\in\mathbb{N}}$, player i's discounted payoff at time t is given by

$$(1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}g_i\left(a_i(\tau), a_{-i}(\tau)\right),$$

where $\delta < 1$ is the common discount factor for all players. Let us denote by $g_i^*(\sigma)$ player i's expected payoff from the strategy profile σ. The payoffs of a continuation strategy $\sigma|_{h^t}$ will be referred to as the continuation payoffs in subgame $g^*|_{h^t}$. The repeated game $g^* \equiv [g, \Sigma]$ is then defined by the set of payoffs and strategies, and we denote the set of attainable payoffs in the repeated game by

$$V = \text{co}\left(\left\{(v_1, \ldots, v_n) \mid \exists\, a \in A \text{ with } \pi(a) = (v_1, \ldots, v_n)\right\}\right).$$

The set of strictly individually rational payoffs is given by

$$V^* = \left\{(v_1, \ldots, v_n) \in V \mid v_i > \underline{v}_i \;\forall\; i \in N\right\}.$$

In the following, we will identify strategies by so-called simple strategy profiles. Using the results from Abreu (1988), we can identify any subgame perfect equilibrium by a simple strategy profile that defines the strategy $\sigma \in \Sigma$ by $n+1$ paths and a transition rule between these paths. Let us denote the equilibrium path p^* and individual punishment paths $p^1, \ldots, p^n$. The associated continuation equilibria will be denoted $\sigma^1, \ldots, \sigma^n$, where $\sigma^i = \sigma|_{h^t}$, and where h^t is a

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[4] Note that, by the definition of a strategy σ, we ultimately assume that players can not only observe the realized actions, but also the mixed strategies in the repeated game. Players can therefore condition their strategies on all past *private randomizations*. This assumption is also adopted by Farrell and Maskin (1989), but they remark that it is not strictly necessary (see their footnote 2 on p. 329).

history in which player i has deviated unilaterally from the equilibrium strategy in period t.

4.4 Motivation

Many complex issues can arise when considering repeated interactions among multiple players. Subgame perfect equilibria determine those outcomes that can be sustained by imposing threats of punishment for possible unilateral deviations. For situations involving only two players, the equilibrium strategy can be characterized by a normal phase in which all players cooperate, and two punishment phases, in which the innocent player punishes the deviator after her unilateral deviation from the agreed-upon strategy.

If, for example, during the punishment phase of one player, the innocent player is also worse off than in the normal phase, it is reasonable to assume that both players would favor, and thus renegotiate to, the normal phase. Farrell and Maskin (1989) argue that players can be assumed to be “competent negotiators” (p. 330) to support their intuitive approach. They extend their argument and propose that, whenever there is a possibility of renegotiation, “players are unlikely to play, or to be deterred by, a proposed continuation equilibrium (whether on or off the equilibrium path) that is strictly Pareto-dominated by another equilibrium that they believe is available to them” (p. 328). They suggest the following definition to capture this:

Definition 4.1 (Farrell and Maskin, 1989)**.** A subgame perfect equilibrium $\sigma \in \Sigma$ is *weakly renegotiation-proof* (WRP) if there do not exist continuation equilibria σ', σ'' of σ such that σ' strictly Pareto-dominates σ''. If an equilibrium σ is WRP, then we also say that the payoffs $g^*(\sigma)$ are WRP.

This definition is given in the context of two-player games. Of course, it can be “naturally generalized” (Farrell and Maskin, 1989, p. 355) to n-player games by simply extending the criterion of strict Pareto-dominance among the players to the entire group (we will subsequently refer to this as the *natural extension of WRP.*). This is what has been done in the literature thus far (see, for example, the literature mentioned in Section 4.2). In their conclusion, however, Farrell and Maskin (1989) note that, for games with more than two players, their concept would need to be strengthened. In fact, already for games with three players, two of them may cooperate independently of the third and may therefore also consider coalitional renegotiation or deviations. This consequently makes continuation equilibria more vulnerable to renegotiation, which is in stark contrast to the strict Pareto criterion imposed by the WRP generalization, as it allows a single, possibly non-involved player to decide the outcome of a renegotiation.

We elaborate on this discrepancy in the following two subsections. First, we show that the addition of a single player makes it impossible to extend the characterization results given by Farrell and Maskin (1989) to n-player games. We thereby refute their assertion that their results have "natural and immediate generalizations to games with three or more players" (p. 328). Second, we discuss three different examples in which the pivotal power of a single player yields counterintuitive outcomes.

4.4.1 Characterization of WRP Payoffs in n-Player Games

As one of the main results of Farrell and Maskin (1989), their Theorem 1 (p. 332) yields sufficient and necessary conditions for WRP payoffs in games with two players. To increase readability, we divide the theorem into two parts and state either condition as its "natural extension" for n players. Claim 4.1 covers the sufficient conditions, extended to n-player games.

Claim 4.1. *Let $v \in V^*$. If for all $i \in N$, there exist actions $a^i \in A$ such that $c_i(a^i) < v_i$, while $g_j(a^i) \geq v_j$ for all $j \neq i$, then v is a WRP payoff for all sufficiently large $\delta < 1$.*

This claim holds true for two players. For the proof, a sequence of actions for the normal phase, such that no continuation payoff along this sequence can be strictly Pareto-ranked, must first be found. The proof offered by Farrell and Maskin (1989) uses the action pairs a^1 or a^2 to construct this sequence but, as we elaborate in Chapter 3, this construction may fail. In Chapter 3, we therefore suggest an alternative approach to obtain v without Pareto-rankable continuation equilibria. The idea is to select two action pairs a^{1*} and a^{2*} such that v is a convex combination of their respective payoffs, and the line segment connecting these payoffs has a negative slope. If the hypotheses of the claim hold true, these action pairs can always be found in a game with two players. For a game with more than two players, as the following example demonstrates, this is no longer the case; therefore, Claim 4.1 must be rejected.

Example 4.1. Consider a game among three players $i = 1, 2, 3$ in which each has two actions, A and B, and the stage-game payoffs are given in Figure 4.1.

The minimax payoff is given by $\underline{v} = (0, 0, 0)$, and we consider the symmetric payoff $v = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. The actions $a^1 = (B, A, A), a^2 = (A, B, A)$ and $a^3 = (A, A, B)$ satisfy the conditions of Claim 4.1. Can v be a weakly renegotiation-proof equilibrium payoff?

Analogous to the proof for two-player games, we first need to construct the normal phase actions. That is, we need to find a sequence of actions $\{\hat{a}(t)\}_{t \in \mathbb{N}}$

Pl. 2

		A	B
Pl. 1	A	$(0,0,0)$	$(1,0,1)$
	B	$(0,1,1)$	$(0,0,0)$

A

Pl. 2

	A	B
A	$(0,0,0)$	$(0,0,0)$
B	$(0,0,0)$	$(1,1,0)$

B

Figure 4.1: A three-player strategic game in which Player 3 chooses matrix A or B.

such that no continuation payoff along this sequence can be strictly Pareto-ranked.

One can show that the construction suggested by Farrell and Maskin (1989) does not work in this example. Moreover, we can show that, for any $\delta < 1$, there is no sequence of actions that yields continuation payoffs that cannot be strictly Pareto-ranked. Assume to the contrary that $\{\hat{a}(t)\}_{t\in\mathbb{N}}$ is a sequence in which no two continuation payoffs can be strictly Pareto-ranked. Then clearly, any action pair that yields payoffs $(0,0,0)$ cannot be played along $\{\hat{a}(t)\}_{t\in\mathbb{N}}$. Consequently, there must be $\lambda_1, \lambda_2, \lambda_3 \in [0,1]$ such that $\lambda_1 + \lambda_2 + \lambda_3 = \sum_{t=0}^{\infty} \delta = \frac{1}{1-\delta}$, and the discounted payoff v can be rewritten as

$$\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right) = (1-\delta)\left(\lambda_1(0,1,1) + \lambda_2(1,0,1) + \lambda_3(1,1,0)\right).$$

This yields $\lambda_i = \frac{1}{4(1-\delta)}$, $i = 1,2,3$, which conflicts with $\sum_{i=1}^{3} \lambda_i = \frac{1}{1-\delta}$, and therefore Claim 4.1 must be rejected. Thus, the sufficient conditions do not generalize to games with more than two players, as asserted in Farrell and Maskin (1989).

The necessary conditions for weakly renegotiation-proof payoffs in two-player games are extended to n-player games in Claim 4.2.

Claim 4.2. *Let $v \in V^*$ be a WRP payoff for a discount factor $\delta < 1$. Then there exists an action $a^i \in A_i$ for all $i \in N$ such that $c_i(a^i) \leq v_i$, while $g_j(a^i) \geq v_j$ for all $j \neq i$.*

This claim holds true for two-player games, as is shown in Farrell and Maskin (1989). For n-player games, however, this no longer holds. In fact, the following example shows that the claim is already incorrect for a three-player game.

Example 4.2. Consider the following infinitely repeated game among three players with discount factor $\delta \in (\frac{1}{2}, \frac{2}{3})$. Player 1 chooses rows A, B or C, and Player 2 columns A, B or C. Player 3 chooses between the payoff matrix L, given in Table 4.1, and the payoff matrix R, which has the same payoffs as L for Players 1 and 2, but Player 3's payoffs are all reduced by ϵ. Thus, Player 3 has a strictly dominant action L, and we can therefore neglect her strategy in the subsequent discussion. Nonetheless, her payoffs are crucial for our finding.[5]

Pl. 2

		A	B	C
	A	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$
Pl. 1	B	$(1,1,1)$	$(0,0,0)$	$(1+\epsilon,0,0)$
	C	$(1+\epsilon,0,0)$	$(0,0,0)$	$(\frac{1}{2},4,-2)$

L

Table 4.1: A three-player strategic game in which Player 3's strictly dominant action is L.

For any $\epsilon \leq \frac{1}{8}$, the payoff vector $v = (1,1,1)$ can be established as a WRP equilibrium with the following strategy: Play (B,A) as long as no unilateral deviation occurs. If Player 1 deviates, follow punishment path p^1 such that the players play $(A,B),(C,C)$ before reverting to the equilibrium path. There is always one player blocking renegotiation along the punishment path p^1; therefore, v is a WRP payoff. However, contrary to Claim 4.2, for Player 1 and any action a^1 with $c_1(a^1) \leq 1$, that is, for $a^1 \in \{(A,B),(B,B),(C,B)\}$, there is no other player $j \neq 1$ with $g_j(a^1) \geq 1$.

This clearly disproves Claim 4.2. The underlying reason is that, due to the strict Pareto-criterion in Definition 4.1, it is always sufficient to have a single player who can block renegotiation. In Appendix 4.A, we elaborate on why the proof of Farrell and Maskin (1989) fails for three-player games.

In conclusion, the two counterexamples show that Theorem 1 of Farrell and Maskin (1989) does not extend to n-player games, which provides the first indication that the extension of WRP to n-player games may not be unproblematic.

4.4.2 Counterintuitive Equilibrium Strategies in n-Player Games

In addition to the failure of the characterization results in n-player games, there are more reasons that indicate the importance of a WRP refinement in n-player games. If we accept the assumption of competent negotiators, the following three examples illustrate that the natural extension of WRP can yield counterintuitive results that are in conflict with this assumption.

First, consider a game among $n > 2$ players, in which a subgame perfect equilibrium $\sigma \in \Sigma$ is sustained via the threat of future punishments, and is

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[5] In fact, it would be convenient to neglect the active role of Player 3 in this game to obtain the same result. However, we want to stress that it is not only an inactive player that can cause a contradiction with the intuition of Farrell and Maskin (1989).

defined by the respective paths $p^*, p^1, \dots, p^n$. Let player i receive a constant and equal payoff along every path of the equilibrium; hence, she is indifferent at any stage. Suppose player $j \neq i$ deviates and the group is supposed to follow punishment path p^j. In accordance with the WRP definition or, more precisely, due to the strict Pareto-criterion therein, player i can block renegotiation to any other continuation equilibrium even though she is always indifferent, i.e. neither affected by a deviation nor worse off during punishment.[6]

The immediate thought would be to relax the condition of strict Pareto-dominance and only require weak Pareto-dominance. In fact, some authors have interpreted the condition in this way, even though Farrell and Maskin (1989) stated clearly that they require a strictly Pareto-dominating continuation equilibrium to which to be renegotiated; in other words, a strictly improving continuation equilibrium for all players. Moreover, by applying the weaker criterion, the above issue would be resolved but, as we will show in the following, this does not resolve every problem associated with WRP equilibria in n-player games.

As a second example, imagine a game among n players, in which a subgame perfect equilibrium is again sustained via threats of punishment. Further assume that this is not a WRP equilibrium according to Definition 4.1. Suppose that an additional player, who receives higher payoffs during punishment phases than when on the equilibrium path is added to the game, and that all other players' payoffs remain unaffected. This additional player will block renegotiation and will therefore make the subgame perfect equilibrium a WRP equilibrium, even though she may not have the means to actually block renegotiation. Hence, an indifferent player, or the simple addition of a dummy player to the game, can already yield counterintuitive WRP equilibria.

Finally, other situations in simple form games that yield results questioning the intuition of competent negotiators can arise. To illustrate this, consider the following example.

Example 4.3. Consider a three-player game in which Players 1, 2 and 3 can choose between two pure actions $\{A, B\}$, and the stage-game payoffs are given as follows, in which Player 3 chooses between matrix A or B:

To avoid cases of indifference, let $\epsilon > 0$. The payoff vector $v = (1, 1, 1)$ can easily be implemented as a subgame perfect equilibrium as follows. Since A is a best-reply for Players 1 and 2, the only punishment threat needed to support $(1, 1, 1)$ is that for a possible deviation of Player 3. Let her punishment path be

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[6] In principle, this could also occur in two-player games. However, if the non-deviating player does not suffer from the opponent's deviation, the original subgame perfect equilibrium cannot be Pareto-efficient; therefore, competent negotiators would not follow this equilibrium in the first place. In games with more than two players, the situation is no longer as clear-cut.

		Pl. 2			Pl. 2	
		A	B		A	B
Pl. 1	A	$(1, 1, 1)$	$(1 + \epsilon, \frac{1}{2}, 0)$	A	$(-\frac{1}{2}, -\frac{1}{2}, 2)$	$(1 + \epsilon, -\frac{1}{2}, 0)$
	B	$(\frac{1}{2}, 1 + \epsilon, 0)$	$(0, 0, 0)$	B	$(-\frac{1}{2}, 1 + \epsilon, 0)$	$(0, 0, 0)$
		A			B	

Figure 4.2: A three-player strategic game in which Player 3 chooses matrix A or B.

$p^3 = \{(A, B, B)\}_{t \in \mathbb{N}}$, and the equilibrium path be $p^* = \{(A, A, A)\}_{t \in \mathbb{N}}$; then, for any $\delta \geq \frac{1}{2}$, v is a subgame perfect equilibrium.

Obviously, not only is Player 3 hurt, Player 2 is also hurt on p^3, and they would both prefer renegotiating to the equilibrium path $p^* = \{(A, A, A)\}_{t \in \mathbb{N}}$. However, Player 1 has no interest in doing so because $\epsilon > 0$, and will therefore block this renegotiation. Hence, this equilibrium is also weakly renegotiation-proof. But can Player 1 really block this renegotiation? In fact, she has no means of blocking the joint renegotiation by Players 2 and 3, as she does not need to change her action at all and, since A is her weakly dominant action, has no interest in doing so once Players 2 and 3 have decided to play (A, A, A). Therefore, we do not think that one can call such an equilibrium renegotiation-proof.[7]

To summarize the findings of the three scenarios above, the crucial problem that arises when the WRP concept is applied in n-player games is that it suffices to have one single player, who can block renegotiation even if she is indifferent and does not participate actively in a change of actions. Thus, the proposed intuitiveness of the WRP notion does not fully carry over to n-player games; therefore, in the following section, we will strengthen the concept and offer a refinement that is better at capturing the behavior in repeated games with more than two players.

4.5 The Model

To be as precise as possible, we will model a protocol that covers the negotiation and renegotiation processes taking place in the game. It should be stressed that we will still model a purely non-cooperative game, but assume that there is some cooperative behavior or a *gentleman's agreement* among the players that allows us to not only model the actions, but to also include some sort of communication and negotiation in the repeated interactions.[8] We therefore agree with Bernheim

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[7] Note that we omit all welfare considerations at this point and focus on the individuals' incentives.

[8] This is also present in the original WRP concept: Farrell and Maskin (1989) argue that "the assumption that players reach an equilibrium at all seems to postulate that there is a preplay negotiation in which the credible agreements are the subgame perfect equilibria" (p. 328).

et al. (2015), who argue that "one cannot formulate a theory of renegotiation without first specifying which agents take part in the negotiation" and without a "description of the various plans over which they might negotiate" (p. 1897).

Let us consider an infinitely repeated game g^*, as introduced in Section 4.3. All players act non-cooperatively, but can communicate and agree upon an equilibrium before the game begins. In order to model the negotiation and renegotiation precisely, we assume the following protocol for the game.

Before the game begins, players meet and agree upon an equilibrium strategy, σ. They consider all possible single-player deviations and, if necessary, design a punishment path p^i for each player. Furthermore, the players already take into account that they should not agree upon an equilibrium that has continuation equilibria that are strictly Pareto-dominated. That is, they agree upon a weakly renegotiation-proof equilibrium.

Once the game has started, and a player i deviates unilaterally in period t, the equilibrium prescribes that players switch to the punishment path p^i. Before the game continues, and according to a specific renegotiation protocol, different groups of players may get together with the deviator and renegotiate the continuation of the game. We will develop this protocol in the following.

We denote by $R_i(h^t) \equiv R_i^t \subset N$ a group of players, the *renegotiators*, who can renegotiate with the deviator i after her deviation in period t. We denote this union by $\bar{R}_i^t$, i.e. $\bar{R}_i^t = R_i^t \cup \{i\}$. Let $\mathcal{R}_i(h^t) \equiv \mathcal{R}_i^t \subset \mathcal{P}(N)$ be the set of all feasible groups R_i^t after player i's deviation in period t. This set is history-dependent in the sense that, for any history h^t and deviator i, the set $\mathcal{R}_i^t$ can be different. We can now define the renegotiation protocol formally.

Definition 4.2. We denote by $\mathcal{R} : \Sigma \times H \times N \longrightarrow \mathcal{P}(N)$ the mapping that assigns for a strategy σ, history h^t and player i's deviation in period t the set of feasible renegotiation groups $\mathcal{R}_i^t \subset \mathcal{P}(N)$, and we call $\mathcal{R}$ the *renegotiation protocol.*

The complete specification of the renegotiation protocol will be provided in Subsection 4.5.1, and we will now determine the continuation equilibria over which R_i^t may renegotiate. As is standard in the coalition-proofness literature, see for instance Bernheim et al. (1987), we let any group R_i^t negotiate only over those continuation equilibria that are available to them. That is, given a history h^t, only those continuation equilibria $\tilde{\sigma} \in \Sigma(\sigma)$ can be a subject of renegotiation that $\bar{R}_i^t$ can attain by changing its strategies while leaving the other players' strategies fixed. We will denote this set of available continuation equilibria by $\Sigma_{\bar{R}_i^t}(\sigma)$. More formally, then, the group $\bar{R}_i^t$ plays the subgame $g^*|_{h^t}$, limited to the available continuation equilibria $\tilde{\sigma} \in \Sigma_{\bar{R}_i^t}(\sigma)$ that are induced by the strategies of the other players $N \setminus \bar{R}_i^t$.

For a WRP equilibrium, Farrell and Maskin (1989) propose that players can renegotiate over any continuation equilibrium that “they believe is available to them” (p. 328). In other words, it does not matter whether all or just one player needs to change her action to achieve the favorable continuation equilibrium.[9] In our model, we allow for subgroup renegotiation as proposed by Asheim and Holtsmark (2009), and do not require either consent or an action on the part of all players to implement a renegotiation.[10]

The group $\bar{R}_i^t$ will agree to renegotiate to another continuation equilibrium if there is at least one player $j \in \bar{R}_i^t, j \neq i$, who is strictly better off while all other players in $\bar{R}_i^t$ are not worse off. This implies that a player who takes part in the renegotiation process, but is indifferent between two continuation equilibria, can no longer block the renegotiation from one to the other. Thus, we eliminate the problem of dummy or indifferent players that we discussed in the previous section.

Next, we introduce a notation for weak Pareto-dominance among a subgroup, which will become useful in the following analysis.

Definition 4.3. Given a subset of players $M \subseteq N$ and for any two strategy profiles $\sigma, \tilde{\sigma} \in \Sigma$, we let the weak dominance relation $\succeq_M$ be defined by

$$\sigma \succeq_M \tilde{\sigma} \quad \Leftrightarrow \quad \begin{cases} g_i^*(\sigma) \geq g_i^*(\tilde{\sigma}) \ \forall \ i \in M \\ \text{with "}>\text{" for at least one } i \in M \end{cases} . \tag{4.1}$$

Whenever $M = N$, we will simply write $\sigma \succeq \tilde{\sigma}$.

Given that $\bar{R}_i^t$ weakly prefers the continuation equilibrium $\tilde{\sigma} \in \Sigma_{\bar{R}_i^t}(\sigma)$ to σ^i, i.e., $\sigma^i \succeq_{\bar{R}_i^t} \tilde{\sigma}$, players change their strategies accordingly and switch from σ^i to $\tilde{\sigma}$. Thus, σ^i is not a credible continuation equilibrium, and the equilibrium strategy is therefore not renegotiation-proof with regard to subgroup R_i^t. However, if only a single player changes her strategy, that is, if $\left|\{j \in N | \tilde{\sigma}_j \neq \sigma_j^i\}\right| = 1$, renegotiation is only feasible if $\bar{R}_i^t = N$. We will discuss this in the following.

If only a single player changes her strategy as a result of a renegotiation, the question of whether this can still be considered to be a feasible renegotiation outcome or whether it is a unilateral deviation that induces a punishment according to the subgame perfect equilibrium strategy arises. In Farrell and Maskin (1989),

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[9] There are indeed other approaches (e.g., Rabin, 1991) that limit the set of attainable continuation equilibria, and therefore receive different results for the set of stable equilibria.

[10] Note that we only allow for intraplay meetings in the event of a deviation. This can be justified by the potential need for additional communication among the punishers due to a change in the game plan. Alternatively, we can argue that, unless a player deviates unilaterally, subgroup meetings are not feasible as there is no specified protocol that determines the setup of the subgroup and therefore only all players can meet.

the former is the case in a two-player game, and unilateral changes of strategies are not sanctioned if they lead to a strictly Pareto-improving continuation equilibrium for both players. The authors do not discuss this issue, but one can argue that, in a two-player game according to our protocol, both the deviator and the punisher meet and renegotiate. Thus, there can be no misunderstanding about this unilateral deviation. If there are more players in the game, however, this may change: Those players not present at the renegotiation meeting may indeed misinterpret the unilateral deviation as an act that calls for punishment.

Therefore, we propose the following rule. If all players unite and renegotiate, i.e., $\bar{R}_i^t = N$, a unilateral change of strategy to implement the outcome of a renegotiation will not be regarded as an act to be punished. However, if at least one player does not participate in the negotiation meeting, she may misinterpret a unilateral deviation and therefore proceed with punishment. Thus, such a renegotiation would not be feasible.

This finally leads us to our new equilibrium concept.

Definition 4.4. Let g^* be the infinitely repeated game played among players $N = \{1, \ldots, n\}$. Let $\mathcal{R}$ be a renegotiation protocol; a WRP equilibrium $\sigma \in \Sigma$ of g^* is then *$\mathcal{R}$-weakly renegotiation-proof* ($\mathcal{R}$-WRP) if, for any time t, any history $h^t \in H$, and any unilateral deviation of player i in period t, there is no set $R_i^t \in \mathcal{R}_i^t$ and no continuation equilibrium $\tilde{\sigma} \in \Sigma_{\bar{R}_i^t}(\sigma)$ such that

$$\tilde{\sigma} \succeq_{R_i^t} \sigma^i$$

and, if $\bar{R}_i^t \neq N$, also

$$\left| \left\{ j \in \bar{R}_i^t \mid \tilde{\sigma_j} \neq \sigma_j^i \right\} \right| \geq 2$$

holds.

Our definition essentially considers Pareto-dominance not only among all players, but also among specific subgroups of players that are determined by the renegotiation protocol for each unilateral deviation. To agree upon renegotiation and to implement a new continuation equilibrium before the start of the punishment, it suffices for a group to have a continuation equilibrium available that constitutes a weak Pareto-improvement to the original one. More specifically, if one of the players outside of this group is not weakly better off, this does not prevent the renegotiation from taking place. Moreover, if there are players outside of the group who are worse off in the new continuation equilibrium, this constitutes no objection to the renegotiation outcome.

4.5.1 Specification of the Renegotiation Protocol

To complete the model, we have to specify the renegotiation protocol $\mathcal{R}$. There are certainly various approaches to this, and Definition 4.4 is held sufficiently general such that every specification of $\mathcal{R}$ can be adopted. In the following, we will introduce and discuss five different specifications of $\mathcal{R}$.

Specification 1: WRP with weak Pareto-dominance

In the first specification, let us assume that all players meet after any deviation, i.e., the renegotiation protocol $\mathcal{R}$ assigns

$$\mathcal{R}_i^t = \{N\}$$

for all players $i \in N$ and histories $h^t \in H$. The $\mathcal{R}$-WRP equilibrium of Definition 4.4 is then similar to a WRP equilibrium in which the strict Pareto-dominance of Definition 4.1 is replaced by weak Pareto-dominance. Note that this specification is not equivalent to WRP with weak Pareto-dominance as, according to our definition, the players only meet after a previous deviation, not in every period.

Specification 2: Every feasible subset of players can renegotiate

The strongest refinement of WRP in our model is the renegotiation protocol that allows every feasible subgroup of N to renegotiate after any deviation. That is, we let $\mathcal{R}$ be such that it assigns

$$\mathcal{R}_i^t = \mathcal{P}(N)$$

for all players $i \in N$ and histories $h^t \in H$. This condition resembles the equilibrium condition for a perfect strong Nash equilibrium, but recall that we only allow for ex post renegotiation after a deviation, and not for multilateral deviations at any stage of the game.

Specification 3: The active players can renegotiate

In this specification, we reason along the lines of Subsection 4.4.2, and limit the set of feasible renegotiators to those players who are actively involved in the change of strategies. We therefore exclude those players from a renegotiation between two continuation equilibria who would not change their actions if the game switched from one to the other. Thus, we eliminate the problem of inactive players who can block renegotiation without taking any action, as seen in our Example 4.3.

More specifically, suppose player i deviated from σ in period t, and let $\tilde{\sigma} \in \Sigma(\sigma)$ be a continuation equilibrium of σ. The set of players that can then renegotiate between σ and $\tilde{\sigma}$ will be defined by the set of players who play different actions in σ and $\tilde{\sigma}$. That is, we let $\mathcal{R}$ be such that, for the equilibrium strategy σ, the set of feasible renegotiation groups after a deviation by player i and history h^t is given by

$$\mathcal{R}_i^t(\sigma) = \Big\{ R_i^t \in \mathcal{P}(N) \;\Big|\; \exists\ \tilde{\sigma} \in \Sigma(\sigma) \text{ with } \alpha_j(\sigma^i) \neq \alpha_j(\tilde{\sigma})\ \forall\ j \in R_i^t, \\ \alpha_k(\sigma^i) = \alpha_k(\tilde{\sigma})\ \forall\ k \in N \setminus \bar{R}_i^t \Big\}.$$

Thus, the renegotiation protocol $\mathcal{R}$ determines a subgroup of players for every history and continuation equilibrium. As there are countably infinite continuation equilibria of σ, there are up to countably infinite many subgroups $R_i^t \in \mathcal{R}_i^t$ that can renegotiate away from σ^i.[11]

In the following example, we show that the proposed WRP equilibrium strategy does not withstand the additional restriction of Definition 4.4 when incorporating this specification.

Example 4.4. Consider a three-player game in which every player can choose between pure strategies A and B; the stage-game payoffs are given in Figure 4.3. Let $\delta \geq \frac{1}{2}$ and $\epsilon \in (0, 1]$.

		Pl. 2	
		A	B
Pl. 1	A	$(1,1,1)$	$(0,0,0)$
	B	$(2,0,1)$	$(0,0,0)$

A

	Pl. 2	
	A	B
A	$(0,1,2)$	$(0,0,0)$
B	$(0,0,0)$	$(\epsilon, 1+\epsilon, 0)$

B

Figure 4.3: A three-player strategic game in which Player 3 chooses matrix A or B.

The payoff $v = (1, 1, 1)$ can be sustained as a WRP equilibrium as follows: As long as no player deviates, play (A, A, A), i.e. $p^* = \{(A, A, A)\}_{t\in\mathbb{N}}$. If Player 1 deviates, play (A, A, B) forever, i.e. $p^1 = \{(A, A, B)\}_{t\in\mathbb{N}}$. If Player 3 deviates, play (B, B, B) forever, i.e. $p^3 = \{(B, B, B)\}_{t\in\mathbb{N}}$.

However, on the punishment path p^1, the continuation payoffs of Player 1 and Player 2 are strictly below their respective continuation payoffs on p^3. By Specification 3, Players 1 and 2 can now agree to renegotiate from p^1 to p^3. Player 3 is not needed for a change in action and can therefore not block this renegotiation, even though she is strictly worse off. Thus, this WRP equilibrium is not $\mathcal{R}$-WRP with Specification 3.

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[11] The problem with the cardinality of renegotiation alternatives is also present in the original WRP notion.

Specification 4: The punishing players renegotiate

We further limit the number of possible renegotiators in this specification, and allow only those players to renegotiate who are involved in the deviator's punishment and take an active role in it, i.e., the punishers. Formally, we define the group of punishers as follows.

Definition 4.5. Suppose the game follows a prescribed strategy $\sigma \in \Sigma$ with history $h^t \in H$. For every possible deviator $i \in N$ at time t, let $P_i^t \subset N \setminus \{i\}$ be the set of players that punish player i by changing their action profiles according to the punishment strategy σ^i. Let h^t be the history in which player i deviated in period t, and $\bar{h}^t$ the history in which all players complied with the strategy σ in t. The set of punishers is then defined by

$$P_i\left(h^t\right) \equiv P_i^t \equiv \left\{ j \in N \setminus \{i\} \;\middle|\; \alpha_j\left(\sigma, h^t\right) \neq \alpha_j\left(\sigma, \bar{h}^t\right) \right\}.$$

That is, we distinguish between those players who do not react to the deviation and those who actively change their actions due to the prescribed punishments, and who may therefore also reconsider their actions, i.e. possibly renegotiate. The punishment set is history-dependent; i.e., the path the game follows and the specific moment when the deviation occurs are decisive to the composition of the set of respective punishers. According to this motivation, we propose the renegotiation protocol $\mathcal{R}$ that assigns

$$\mathcal{R}_i^t = \{P_i^t\}$$

for all players $i \in N$ and histories $h^t \in H$. This is clearly a weaker condition for renegotiation-proofness than is Specification 3, as there is always only one subgroup that may renegotiate from σ^i at time t.

In the following example, the proposed WRP equilibrium strategy is not an $\mathcal{R}$-WRP with Specification 4.

Example 4.5. Consider a three-player game in which each player can choose between pure strategies A and B; the stage-game payoffs are given in Figure 4.4. Let $\delta \geq \frac{1}{2}$ and $\epsilon \leq 1$.

The payoff $v = (1, 1, 1)$ can be sustained as a WRP equilibrium as follows: As long as no player deviates, play (A, A, A), i.e. $p^* = \{(A, A, A)\}_{t \in \mathbb{N}}$. If Player 3 deviates, play (A, B, B) forever, i.e. $p^3 = \{(A, B, B)\}_{t \in \mathbb{N}}$.

However, on the punishment path p^3, the continuation payoffs of Player 2 and Player 3 are strictly below their equilibrium payoff $v_2 = v_3 = 1$. Let h^t be such that Player 3 deviated from σ in period t. Then $P_3 = \{2\}$, $\bar{R}_3^t = \{2, 3\}$, and Player 1 is thus excluded from the renegotiation meeting by Specification 4.

Pl. 2

		A	B
Pl. 1	A	$(1, 1, 1)$	$(0, 0, 0)$
	B	$(0, 0, 0)$	$(0, 1 + \epsilon, 0)$

A

Pl. 2

	A	B
A	$(0, 0, 1 + \epsilon)$	$(1 + \epsilon, 0, 0)$
B	$(0, 0, 0)$	$(0, 0, 0)$

B

Figure 4.4: A three-player strategic game in which Player 3 chooses matrix A or B.

Therefore, Players 2 and 3 can jointly renegotiate back to the equilibrium path p^*. Thus, this WRP equilibrium is not $\mathcal{R}$-WRP with Specification 4.

Nevertheless, the players can still obtain $v = (1, 1, 1)$ as their equilibrium payoff by changing Player 3's punishment to $\tilde{p}^3 = \{(B, B, A)\}_{t \in \mathbb{N}}$. Players 1 and 3 would now favor a renegotiation to the equilibrium path but, as $P_3 = \{1, 2\}$ and $\bar{R}_3^t = \{1, 2, 3\}$, Player 2 would block the renegotiation.

Remark 4.1. The interpretation of active punishers may be somewhat misleading. Of course, players who do not change their strategy after a deviation may also be interpreted as punishers. In fact, not changing the strategy may be the actual punishment.[12] Nonetheless, we want to pinpoint those players who need to actively change actions due to the punishment strategies, and who may therefore be in a situation in which additional communication is helpful.

Specification 5: A majority renegotiates

As we have elaborated above, it may be the case that a majority of players strictly prefers a renegotiation, but a single player can block this according to the WRP notion. Therefore, it is another natural assumption to specify the renegotiation protocol such that it allows for renegotiation from the prescribed continuation equilibrium σ^i to another continuation equilibrium $\tilde{\sigma}$ if a majority is weakly better off. Formally, we let $\mathcal{R}$ be such that

$$\mathcal{R}_i^t = \left\{ R_i^t \in \mathcal{P}(N) \;\middle|\; |R_i^t| > \tfrac{|N|}{2} \right\}$$

for all players $i \in N$ and histories $h^t \in H$. As a result, for renegotiation to be successful, it suffices for two players who would change their strategies in order to be better off in continuation equilibrium $\tilde{\sigma}$ to find a sufficient number of players (at least $\frac{|N|}{2} - 1$) who do not necessarily need to change their strategies, but who are weakly better off in $\tilde{\sigma}$.

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[12] Note also that we do not exclude any players but the deviator from being among the punishers. Even though one may find good reasons that players who are not affected by a deviation should not be able to punish, or also players who have no impact on the deviator through their change of action should be excluded, this is beyond the scope of this paper.

4.6 Characterization of $\mathcal{R}$-WRP Equilibrium Payoffs

We will now proceed to characterize the new equilibrium for each specification given in the previous section. To do so, we will not characterize strategies, but will focus on the equilibrium payoffs, as is common practice in the repeated games literature. Therefore, let $\mathcal{W}_\delta$ denote the set of WRP payoffs for a given discount factor $\delta < 1$. The set of all WRP payoffs in the infinitely repeated game g^* will be denoted by $\mathcal{W} = \cup_{\delta<1} (\mathcal{W}_\delta, \delta)$. Equivalently, we denote by $\mathcal{W}^s = \cup_{\delta<1} \left(\mathcal{W}^s_\delta, \delta\right)$ the set of $\mathcal{R}$-WRP payoffs with specification $s = 1, 2, 3, 4, 5$.

Obviously, by imposing our additional condition we can (weakly) limit the set of possible punishment strategies, and therefore ultimately (weakly) reduce the set of possible equilibrium outcomes in comparison to the original WRP notion. That is, for every specification s, we have $\mathcal{W}^s \subseteq \mathcal{W}$. Nevertheless, we still have existence as the trivial WRP equilibrium is also an $\mathcal{R}$-WRP equilibrium for every specification: A strategy that assigns playing the Nash equilibrium in each subgame for every player has no other continuation equilibrium than itself, and is therefore clearly an $\mathcal{R}$-WRP equilibrium (see Farrell and Maskin, 1989).

For a two-player game, we return to the original definition of WRP equilibria. Intuitively, if in the period after a unilateral deviation there were a continuation equilibrium $\tilde{\sigma}$ that the non-deviator strictly prefers to the punishment one, WRP yields that the deviating player can, at best, be indifferent in $\tilde{\sigma}$. However, $\tilde{\sigma}$ can then also be used as punishment. This is shown in the following proposition.

Proposition 4.1. *If $N = \{1, 2\}$, Definition 4.4 is equivalent to Definition 4.1, i.e., $\mathcal{W}^s = \mathcal{W}$ for $s = 1, 2, 3, 4, 5$.*

We can also compare the different specifications with regard to their equilibrium payoffs. It is immediately obvious from the specification of the renegotiating groups that Specification 2, which allows every feasible subgroup to renegotiate, is the strongest refinement of WRP. That is, we have $\mathcal{W}^2 \subseteq \mathcal{W}^1, \mathcal{W}^3, \mathcal{W}^4, \mathcal{W}^5$. Furthermore, we have that the set of punishers P_i^t is included in the set of active players, as introduced in Specification 3. This directly yields that $\mathcal{W}^4 \subseteq \mathcal{W}^3$, i.e., Specification 4 is stricter than is Specification 3. Even though the set of players $\mathcal{R}_i^t$ specified in the different renegotiation protocols may seem comparable, they cannot be further ranked with regard to their equilibrium payoffs (see Appendix 4.A).

Frankly, the question arises whether the additional conditions we impose in Definition 4.4 have any limiting effect in games with more than two players. Intuitively, allowing for renegotiation by subgroups should generate more threats of renegotiation, and therefore less stable equilibria. In fact, the following example of a three-player game demonstrates that not every WRP equilibrium can be sustained as an $\mathcal{R}$-WRP equilibrium, independent of the specification of $\mathcal{R}$.

Example 4.6. Consider the infinitely repeated game among three players with discount factor $\delta \in (0,1)$, where the single-stage profits are represented by the matrix in Figure 4.5 and where $\epsilon > 0$.

		Pl. 2	
		A	B
Pl. 1	A	$(1,1,1)$	$(0,0,0)$
	B	$(0,-\epsilon,-\epsilon)$	$(0,-\epsilon,-\epsilon)$
		A	

	Pl. 2	
	A	B
A	$(\frac{1}{10},0,2)$	$(1+\epsilon,0,0)$
B	$(0,-\epsilon,-\epsilon)$	$(0,-\epsilon,-\epsilon)$
	B	

Figure 4.5: A three-player strategic game in which Player 3 chooses matrix A or B.

As in Example 4.3, the players can establish the welfare-maximizing payoff vector $v = (1,1,1)$ using the following strategy σ: Follow the equilibrium path $p^* = \{(A,A,A)\}_{t\in\mathbb{N}}$ as long as no unilateral deviation occurs. If Player 3 deviates, play punishment action $a^3 = (A,B,B)$ forever. This then constitutes a weakly renegotiation-proof equilibrium for $\delta \geq \frac{1}{2}$, as it is subgame perfect and Player 1 will always block renegotiation back to the equilibrium path p^* for $\epsilon > 0$.

We will first exclude Specification 1 from our considerations and consider Specifications 2, 3, 4 and 5. If we impose our additional conditions from Definition 4.4, we have Player 1 being unable to block a renegotiation as she is not in the group of renegotiators R_i^t. Thus, the proposed WRP strategy is not $\mathcal{R}$-WRP, and we show that there is in fact no strategy that sustains v as a $\mathcal{R}$-WRP equilibrium.

First note that there is no other strategy that yields the equilibrium payoff $v = (1,1,1)$, as Player 2 receives strictly less than 1 for all action triples other than (A,A,A). Thus, p^* is the unique path to yield v, and it therefore suffices to focus on Player 3's punishment, as A is a weakly dominant strategy for both Players 1 and 2. We show that there is no punishment path that sustains v as an $\mathcal{R}$-WRP equilibrium.

Suppose to the contrary that there is another strategy $\tilde{\sigma}$ with punishment path $\tilde{p}^3$ for Player 3's deviation, which sustains v as an $\mathcal{R}$-WRP equilibrium, i.e. $\tilde{\sigma}$ is WRP and no subgroup R_i^t can renegotiate. As previously, Player 2 will always receive less than her equilibrium payoff $v_2 = 1$ unless (A,A,A) is played forever. Since $\{(A,A,A)\}_{t\in\mathbb{N}}$ cannot be a subgame perfect punishment for Player 3's deviation, Player 2 will always have an incentive to renegotiate from $\tilde{p}^3$ to p^*.

Recall from Definition 4.4 that, if only a single player carries out the renegotiation, this change of strategy is not feasible according to the renegotiation process. That is, if only Player 2 plays different actions after Player 3's deviation, she would not be able to renegotiate from the punishment path $\tilde{p}^3$, and the respective strategy could thus be $\mathcal{R}$-WRP. Therefore, assume that this is the case, and that only Player 2 changes her action profile when the game moves from $\tilde{p}^3$ to p^*. Then, $\tilde{p}^*$ must be such that action (A, B, A) is played for a suitable number of periods to achieve subgame perfection. This, however, yields payoffs strictly below v, and therefore renders this strategy not WRP in the first place. Thus, it cannot be $\mathcal{R}$-WRP.

Therefore, those subgame perfect punishment paths for Player 3 in which Player 1 blocks renegotiation to p^* and is also involved in the punishment remain to be examined. That is, there must be a time $\bar{t}$ such that Player 1 plays B on the punishment path $\tilde{p}^3$.

On the punishment path $\tilde{p}^3$, the continuation payoffs in period $\bar{t}$ are then given by

$$g_1(\tilde{\sigma}^3, h^{\bar{t}-1}) = (1-\delta)0 + \delta g_1(\tilde{\sigma}^3, h^{\bar{t}}) \leq g_1(\tilde{\sigma}^3, h^{\bar{t}}), \tag{4.2}$$

$$g_2(\tilde{\sigma}^3, h^{\bar{t}-1}) = (1-\delta)(-\epsilon) + \delta g_2(\tilde{\sigma}^3, h^{\bar{t}}) \leq g_2(\tilde{\sigma}^3, h^{\bar{t}}), \tag{4.3}$$

$$g_3(\tilde{\sigma}^3, h^{\bar{t}-1}) = (1-\delta)(-\epsilon) + \delta g_3(\tilde{\sigma}^3, h^{\bar{t}}) \leq g_3(\tilde{\sigma}^3, h^{\bar{t}}). \tag{4.4}$$

According to WRP, at least one of the inequalities must not be strict. Suppose therefore that (4.2) is an equality; $g_1(\tilde{\sigma}^3, h^{\bar{t}-1}) = g_1(\tilde{\sigma}^3, h^{\bar{t}}) = 0$ must then hold. Based on the definition of the game, we have $g_2(\tilde{\sigma}^3, h^{\bar{t}}) \leq 0$ and $g_3(\tilde{\sigma}^3, h^{\bar{t}}) \leq 0$, and therefore the continuation equilibrium $\tilde{\sigma}^3|_{h^{\bar{t}}}$ is strictly Pareto-dominated by the normal phase equilibrium $\tilde{\sigma}$. Analogous arguments hold for equations (4.3) and (4.4), which renders $\tilde{p}^3$ not WRP; therefore, $\tilde{\sigma}$ is not $\mathcal{R}$-WRP.

If we consider a renegotiation protocol $\mathcal{R}$ with Specification 1, the pivotal power of Player 1 in the WRP notion is carried over to $\mathcal{R}$-WRP. Given that all players meet after Player 3's deviation, Player 1 will block renegotiation to the normal phase of the equilibrium. For $\epsilon = 0$, however, this no longer holds true, as Player 1 cannot block the renegotiation, and v is not $\mathcal{R}$-WRP with Specification 1 either. Nevertheless, we assume $\epsilon > 0$ to avoid the simple modification—or misinterpretation—from strict to weak Pareto-dominance in Definition 4.1 rendering our example useless, as σ would then not even be WRP. Clearly, our findings for Specifications 2, 3, 4 and 5 also hold true for $\epsilon = 0$.

A more general characterization of $\mathcal{R}$-WRP payoffs, however, is not easy to obtain. In fact, not even WRP equilibria can be easily characterized in games with more than two players, as we have shown in Subsection 4.4.1. We therefore return to the game that has been most frequently studied in the context of WRP equilibria in n-player games: the Prisoner's Dilemma. As shown in Van Damme (1989) for two-player games, we show that, for symmetric n-player Prisoner's Dilemma games, full cooperation can also be obtained as a renegotiation-proof equilibrium that withstands our additional restrictions of Definition 4.4. Before we proceed, we first need to define this game for more than two players formally.

In the following Definition 4.6, assumptions (i)–(iv) are standard and generally accepted for a game of the Prisoner's Dilemma type. However, there is no universal definition of the game, as there is always certain degree of freedom in terms of the different payoffs from multiplayer defections. We therefore introduce an additional assumption (*).

Definition 4.6. The game G is of the Prisoner's Dilemma type if the following conditions are satisfied:

(i) Every player has the same two pure actions, C (*cooperate*) and D (*defect*)

(ii) D is the strictly dominant action

(iii) $g_i(C,\ldots,C) > g_i(D,\ldots,D)$ for all players i, $\sum_i g_i(C,\ldots,C) > \sum_i g_i(a)$ for all other actions $a \in A$

(iv) The more players defect, the less profit a cooperator receives

(*) For $a \in A$ such that at least two players play D, we have $g_i(a) < g_i(C,\ldots,C)$ for all players i, i.e., if more than one player defects, every player receives less than she would in the event of full cooperation

Given this definition, we can now state our result for $\mathcal{R}$-WRP equilibria in symmetric Prisoner's Dilemmas.

Proposition 4.2. *Let G be a symmetric n-player game of the Prisoner's Dilemma type. If δ is sufficiently large, full cooperation can be established as an $\mathcal{R}$-WRP payoff for all specifications $s = 1, 2, 3, 4, 5$.*

The intuition is straightforward: For every individual deviation, one can construct a penance punishment strategy whereby one player switches to defect while all the others play cooperate for a suitable duration before returning to the full cooperation equilibrium. As only one player is effectively involved in the punishment, she has no incentive to renegotiate, and none of the others has the means to do so. Moreover, no matter which group gets together, there will be no possibility that this group could enforce a renegotiation to the full cooperative outcome. Obviously, other punishment paths are not enforceable either. Therefore, the full cooperative solution is stable against any subgroup renegotiation in the sense of Definition 4.4.

4.6.1 Application

As mentioned in our introduction and in the discussion of the related literature, the WRP equilibrium notion is widely applied in various game-theory models. In situations in which the interacting agents cannot make binding agreements, self-enforcing agreements must be found to enforce cooperation. In such settings, the concept of WRP equilibrium often has exactly the conditions desired for such self-enforcing agreements. As we have shown above, for symmetric games of the Prisoner's Dilemma type, there is no need for a refinement of these conditions when the game is played among more than two players. If this is not the case, however, there are some weaknesses of WRP that our refinement $\mathcal{R}$-WRP resolves.

In the following example, we elaborate such a case. We reconsider the model in Chapter 2 of this thesis, which discusses International Environmental Agreements (IEAs) among asymmetric players. We will apply the notation of that model in order to relate the results, and refer to Chapter 2 for all details. Most importantly, the instantaneous payoff function will be denoted by π, and g will refer to a network.

We will first repeat the basic model of Chapter 2 before we discuss an example that leads directly to the additional $\mathcal{R}$-WRP conditions. In the IEA game, in each period, players choose an action $x_i \in \mathbb{R}_+$, and their instantaneous payoffs are given by $\pi_i\left(x_i, x_{-i}\right) = -\frac{1}{2}\left(\bar{x}_i - x_i\right)^2 - \beta \sum_{j \in N} x_j - \gamma \sum_{j \in N} \bar{g}_{ij} x_j$, where β and γ are the spillover parameters and $\bar{g}_{ij}$ is the indicator function, which is equal to 1 if players i and j are linked, or $i = j$ and 0 in all other cases. The set of neighbors is denoted by N_i. An IEA refers to a strategy profile $\mathbf{s}$ by a coalition C, and the parameter k denotes the size of the coalition that implements the IEA. The integer k_l denotes the number of links that player l has in that coalition.

To implement an IEA in the repeated game, all coalition members play the signatory action $x_i^S = \bar{x}_i - \beta k - \gamma(k_i + 1)$ as long as no single deviation occurs. After a single deviation by player j, all punishers $i \in P_j$ play punishment action $x_i^p = \bar{x}_i - p(\beta + \gamma), p \geq 1$ for one period before the signatory action is played again. The agreement path will be denoted by $\mathbf{a^C}$, and the punishment path of player i by $\mathbf{p_i^C}$.

In the following example, we demonstrate the renegotiation opportunities that are available to the players when the grand coalition is implemented as a WRP equilibrium. For simplicity, we will focus on the pure local spillover game, i.e. we let $\beta = 0$ and assume that $p = 1$.[13]

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[13] Note that, for $\beta > 0$ but $\gamma = 0$, the game would be symmetric and would be reduced to a game of the Prisoner's Dilemma type. Thus, our refinement does not yield additional insights.

Example 4.7. Consider a game among nine players $i = 1, \ldots, 9$ who are linked to each other according to the network g, as illustrated in Figure 4.6. According to Theorem 2.1 in Chapter 2, full cooperation is a subgame perfect equilibrium if there is a set of punishers P_i for all player i, such that

$$\delta \left[\gamma^2 \left(\sum_{m \in P_i \cap N_i} k_m \right) \right] - \frac{1}{2} (\gamma k_i)^2 \geq 0.$$

Due to the local spillover structure, only the neighbors of a player can punish her deviation effectively. Let $\delta = 0.6$; it can then easily be seen that $P_i = N_i$ for all i yields that full cooperation is a subgame perfect equilibrium.

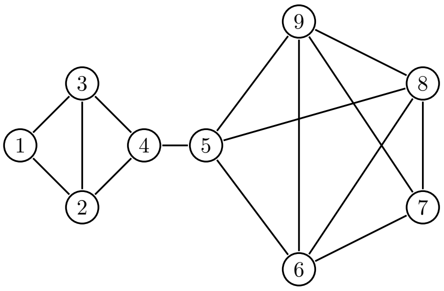

Figure 4.6: Local spillover network in the example.

Moreover, due to the network structure in this example, for every punishment path $\mathbf{p_i^C}$, there is always a player who is not involved in the punishment and who is not linked directly to a punisher. This player is therefore indifferent between $\mathbf{p_i^C}$ and the cooperation continuation equilibrium $\mathbf{a^C}$, and will henceforth block renegotiation from the former to the latter. Thus, WRP follows immediately (see also Corollary 2.2 of Chapter 2). However, we can show that this equilibrium does not withstand our additional refinement, and is therefore not $\mathcal{R}$-WRP.

We will discuss three possible renegotiation options. First, consider the punishment of Player 7. The set of punishers is given by $P_7 = \{6, 8, 9\}$, and every punisher $j \in$ receives

$$\begin{aligned} \pi_j(x^p_{P_7}, x^s_{N \setminus P_7}) &= -\tfrac{1}{2}(\bar{x}_j - x^p_j)^2 - \gamma \sum_{l \in N} \bar{g}_{jl} x_l \\ &= \frac{23}{2}\gamma^2 - \gamma \sum_{l \in N} \bar{g}_{jl} \bar{x}_l \end{aligned}$$

in the period of punishment. Their respective stage-game payoffs on the agreement path $\pi_j(x^s_N)$ are given by

$$\begin{aligned} \pi_j(x^s_N) &= -\tfrac{1}{2}(\bar{x}_j - x^s_j)^2 - \gamma \sum_{l \in N} \bar{g}_{jl} x^s_l \\ &= \frac{23}{2}\gamma^2 - \gamma \sum_{l \in N} \bar{g}_{jl} \bar{x}_l \end{aligned}$$

and therefore, $\pi_j(x^p_{P_7}, x^s_{N\setminus P_7}) = \pi_j(x^s_N)$. That is, all punishing players $j \in P_7$ are indifferent between the punishment path $\mathbf{p}_7^{\mathbf{C}}$ and the agreement continuation equilibrium $\mathbf{a}^{\mathbf{C}}$. If, according to Specification 4, the renegotiation rule assigns $\mathcal{R}^t_7 = P_7$, this yields no contradiction to $\mathcal{R}$-WRP. However, all non-punishing players $j \in N \notin P_7$ are strictly better off. Therefore, in all other specifications, any other group of renegotiators R^t_7 (sufficiently large for Specification 5) that includes the punishers, i.e. $P_7 \subset R^t_7$, will renegotiate from $\mathbf{p}_7^{\mathbf{C}}$ to $\mathbf{a}^{\mathbf{C}}$.

Second, consider the punishment of Player 6. For the punishing players $P_6 = \{5, 7, 8, 9\}$, we have

$$\begin{aligned}
\pi_5(x^p_{P_6}, x^s_{N\setminus P_6}) &= \frac{23}{2}\gamma^2 - \gamma \sum_{l\in N} \bar{g}_{5l}\bar{x}_l = \pi_5(x^s_N) \\
\pi_7(x^p_{P_6}, x^s_{N\setminus P_6}) &= \frac{15}{2}\gamma^2 - \gamma \sum_{l\in N} \bar{g}_{7l}\bar{x}_l < \pi_7(x^s_N) \\
\pi_8(x^p_{P_6}, x^s_{N\setminus P_6}) &= \frac{17}{2}\gamma^2 - \gamma \sum_{l\in N} \bar{g}_{8l}\bar{x}_l < \pi_8(x^s_N) \\
\pi_9(x^p_{P_6}, x^s_{N\setminus P_6}) &= \frac{17}{2}\gamma^2 - \gamma \sum_{l\in N} \bar{g}_{9l}\bar{x}_l < \pi_9(x^s_N)
\end{aligned}$$

Thus, punisher $j = 5$ is indifferent between the punishment and the agreement paths, while all the other punishers would prefer a renegotiation to the agreement path. According to Specification 4, the punishers would thus renegotiate from $\mathbf{p}_6^{\mathbf{C}}$ to $\mathbf{a}^{\mathbf{C}}$, which renders the WRP equilibrium not $\mathcal{R}$-WRP. As before, all other specifications yield the same result.

Finally, consider the punishment of Player 1. The punishing players $j \in P_1 = \{2, 3\}$ receive $\pi_j(x^p_{P_1}, x^s_{N\setminus P_1}) = \frac{17}{2}\gamma^2 - \gamma \sum_{l\in N} \bar{g}_{jl}\bar{x}_l$ during the punishment period. As Players 2 and 3 are not linked to Player 5, they are not affected by a change of action on the part of this player. Thus, their punishment payoff from punishing Player 1 or Player 4 is equal, i.e. $\pi_j(x^p_{P_1}, x^s_{N\setminus P_1}) = \pi_j(x^p_{P_4}, x^s_{N\setminus P_4})$ for $j = 2, 3$. Player 5, however, is strictly better off on $\mathbf{p}_4^{\mathbf{C}}$ than on $\mathbf{p}_1^{\mathbf{C}}$, as the punishment allows this player to reduce abatement efforts while still free-riding on her neighbors' efforts. Thus, if the renegotiation rule is specified according to Specifications 2 or 3, then $R^t_1 = \{2, 3, 5\} \in \mathcal{R}^t_1$, and therefore the players will renegotiate from $\mathbf{p}_4^{\mathbf{C}}$ to $\mathbf{p}_1^{\mathbf{C}}$.

We can generalize our observations of the example and propose additional conditions that are necessary and sufficient for an $\mathcal{R}$-WRP equilibrium in the IEA game. Clearly, the conditions of Theorem 2.1 in Chapter 2 remain, since they are necessary and sufficient for WRP. For Definition 4.4 to be satisfied, it is therefore sufficient to determine whether any subgroup R^t_i can renegotiate from the prescribed punishment path to another continuation equilibrium after a deviation by player i.

Suppose player i deviated in period $t-1$. Players are then supposed to follow punishment path $\mathbf{p_i^C}$. For a coalition of size k, there are consequently $k+1$ different continuation equilibria at time t: the agreement path $\mathbf{a^C}$ and the punishment paths $\mathbf{p_i^C}$ for all signatories $i \in C$.

According to the definition of the spillover game, all players who are not participating in the coalition will favor more cooperation over less cooperation. The same holds for the players who are members of the coalition but who are not punishing player i. The previous deviator herself strictly prefers the agreement path to her punishment path. Furthermore, she may also prefer a different punishment path $\mathbf{p_j^C}$ to her own punishment. By Condition (2.9) of Theorem 2.1 in Chapter 2, there is at least one punisher who is weakly better off on the punishment than on the agreement path. However, she may only be indifferent between the two, and all the other punishers are strictly better off in $\mathbf{a^C}$. Thus, Condition (2.9) must be strict.

There could also be a different punishment path $\mathbf{p_j^C}$ for $j \notin P_i$ that all punishers $l \in P_i$ weakly prefer to $\mathbf{p_i^C}$. Hence, there are various renegotiation possibilities at time t, and the following conditions ensure that an agreement is an $\mathcal{R}$-WRP equilibrium. As the conditions differ for the five specifications of $\mathcal{R}$, instead of presenting five different results, we will only provide the results for Specifications 2, 3 and 4, and will place the results for the other specifications in the appendix.

First note that, for all specifications, we need to replace the WRP condition (2.9) in Chapter 2 with its strict version to exclude those cases in which only one indifferent player j blocks renegotiation from $\mathbf{p_i^C}$ to $\mathbf{a^C}$, while all the other punishers would prefer to renegotiate. If player $j \in P_i$ satisfies the strict condition (4.5), she can block renegotiation to the agreement path. However, there are other continuation equilibria to which a subgroup R_i^t may renegotiate.

Proposition 4.3. *An IEA* $\mathbf{s}$ *by a coalition* C *is an* $\mathcal{R}$*-WRP equilibrium with Specifications 2, 3 or 4, if and only if for all* $i \in C$

$$\delta \left[\beta^2 |P_i|(k-p) + \beta\gamma \left(|P_i|(1-p) + \sum_{m \in P_i} k_m + |P_i \cap N_i|(k-p) \right) \right.$$
$$\left. + \gamma^2 \left(\sum_{m \in P_i \cap N_i} k_m + |P_i \cap N_i|(1-p) \right) \right] - \frac{1}{2}\left(\beta(k-1) + \gamma k_i\right)^2 \geq 0,$$

for all $i \in C$ there exists at least one $j \in P_i$, such that

$$\beta^2(k-p)(|P_i|-p) + \beta\gamma \left((|P_i|-p)(1-p) + \sum_{m \in P_i \setminus \{j\}} k_m + |P_i \cap N_j|(k-p) \right)$$
$$+\frac{\gamma^2}{2}\left(2 \sum_{m \in P_i \cap N_j} k_m + \Big(2|P_i \cap N_j| + 1 - p\Big)(1-p) \right) - \frac{1}{2}\Big(\beta(k-1) + \gamma k_j\Big)^2 < 0. \tag{4.5}$$

and for all $j \notin P_i$, at least one of the following two conditions is satisfied:

(a) there exists an $l \in P_j \setminus P_i$ such that

$$\sum_{m \in P_i \setminus P_j} \Big(-\beta(k-p) - \gamma(k_m + 1 - p)\Big)(-\beta - \gamma\bar{g}_{lm}) + \sum_{m \in P_j \setminus P_i} \Big(-\beta(k-p) - \gamma(k_m + 1 - p)\Big)(\beta + \gamma\bar{g}_{lm}) - \frac{1}{2}(p(\beta+\gamma))^2 + \frac{1}{2}\Big(\beta k + \gamma(k_l + 1)\Big)^2 > 0$$

(b) there exists $l \in P_i \setminus P_j$ such that

$$\sum_{m \in P_i \setminus P_j} \Big(-\beta(k-p) - \gamma(k_m + 1 - p)\Big)(-\beta - \gamma\bar{g}_{lm}) + \sum_{m \in P_j \setminus P_i} \Big(-\beta(k-p) - \gamma(k_m + 1 - p)\Big)(\beta + \gamma\bar{g}_{lm}) + \frac{1}{2}(p(\beta+\gamma))^2 - \frac{1}{2}\Big(\beta k + \gamma(k_l + 1)\Big)^2 > 0$$

The additional conditions (a) and (b) extend Theorem 2.1 of Chapter 2, such that there is no renegotiation from player i's punishment $\mathbf{p_i^C}$ to player j's punishment $\mathbf{p_j^C}$. Clearly, we only need to consider those cases in which $j \notin P_i$, as every punisher $j \in P_i$ will certainly block the renegotiation to her own punishment. For a derivation of the additional conditions (a) and (b), we refer to Appendix 4.A.

In summary, we have clearly demonstrated how the application of the WRP equilibrium concept in our model in Chapter 2 may yield international environmental agreements that are stable in the WRP sense, but may not precisely fit the condition of being self-enforcing, as there can be subgroups that have a strong incentive to renegotiate at certain contingencies. The outcome of such a subgroup renegotiation can even improve welfare; for example, when the WRP condition is only satisfied because there is one player who is indifferent between two continuation equilibria while all the others are weakly better off. In Proposition 4.3, we provided necessary and sufficient conditions that guarantee an IEA to satisfy our additional constraints and make it $\mathcal{R}$-WRP with Specifications 2, 3 and 4.

4.7 Conclusion

In this paper, we have clearly demonstrated the necessity for a refinement of the renegotiation concepts for games with more than two players. In general games, counterintuitive strategies may be supported as a weakly renegotiation-proof (WRP) equilibrium whereby indifferent and/or inactive players can block renegotiation without the actual means or incentives to do so. We have therefore proposed additional constraints on WRP equilibria that exclude the renegotiation of subgroups that are entitled to renegotiate according to a given renegotiation protocol $\mathcal{R}$. In an $\mathcal{R}$-weakly renegotiation-proof ($\mathcal{R}$-WRP) equilibrium, there is no feasible history in which a single player has deviated, and a subgroup defined by $\mathcal{R}$ can renegotiate to a continuation equilibrium other than the one prescribed by the equilibrium strategy.

We have proposed five different specifications for the renegotiation protocol, and have partially characterized their respective equilibrium payoffs. They range from a very weak refinement of the original WRP equilibrium to a very strict condition that effectively limits the set of attainable payoffs. However, $\mathcal{R}$-WRP equilibria always exist, but are difficult to characterize. Nevertheless, this is already the case for WRP equilibria in games with more than two players, as we have also demonstrated. Furthermore, we have shown that, in games of the Prisoner's Dilemma type, full cooperation can always be supported as an $\mathcal{R}$-WRP equilibrium for every specification.

Needless to say, we have not resolved every open issue in games involving renegotiation. In fact, we have incorporated most weaknesses inherent in the WRP notion. First, we only consider single-player deviations and do not allow for group behavior ex ante. As elaborated in Appendix 4.B, coalition-proof behavior in infinitely repeated games is an open issue in the literature, and has not yet been resolved. Our approach can therefore be seen as a step towards this goal. Second, WRP and $\mathcal{R}$-WRP lack external stability in the sense that the question of why players would not renegotiate to subgame perfect equilibria outside the set of continuation equilibria is not answered. Furthermore, the notion does not require that the equilibrium that may be renegotiated to is itself renegotiation-proof.

The WRP notion has nonetheless been frequently applied in the literature, as it offers intuitive and analytically tractable conditions for equilibria in infinitely repeated games with two players. Our refinement can therefore be seen as an extension of this intuitive approach to general n-player games and incorporates group behavior ex post.

Finally, there are certainly more ways to specify a renegotiation protocol, but we believe that we have addressed the most intuitive ones. Nevertheless, one could certainly consider different specifications, and could allow for cooperative elements such as the core solution. Together with the refinement of the ex ante equilibrium concept, we leave this to future research.

Appendices

4.A Proofs and Additional Results

On the necessary conditions of Theorem 1 in Farrell and Maskin (1989).

In Example 4.2, we show that the necessary conditions for WRP payoffs, given in Farrell and Maskin (1989), do generalize to games with more than two players. In the following, we go along the proof of Farrell and Maskin (1989) for two players and show why it fails for three and more players.

Let $v \in V^*$ and let $\sigma(v)$ be the weakly renegotiation-proof strategy that yields payoff v. Without loss of generality, we can consider Player 1 and her worst continuation equilibrium $\underline{\sigma}^1$, with a first period action $\underline{a}^1$ and continuation $\hat{\underline{\sigma}}^1$. This worst continuation equilibrium obviously yields payoffs for Player 1, which are less or equal v_1. If there is no unique $\underline{\sigma}^1$, pick the one that is best for Player 2. If it is still not unique, proceed with the next players and if this does not yield a unique one, take the first in the list.

If $g_1^*(\underline{\sigma}^1) < v_1$, there must then exist at least one player $j \neq 1$ with $g_j^*(\underline{\sigma}^1) \geq v_j$ because of the WRP condition. If $g_1^*(\underline{\sigma}^1) = v_1$, then there is at least one player $j \neq 1$ with $g_j^*(\underline{\sigma}^1) \geq v_j$ due to the selection process of $\underline{\sigma}^1$. For two players, this ultimately yields that $g_2^*(\underline{\sigma}^1) \geq v_2$. For more than two players, we have that for $g_1^*(\underline{\sigma}^1) \leq v_1$, there exists a player $j \neq 1$ with

$$g_j^*(\underline{\sigma}^1) \geq v_j. \tag{4.A.1}$$

Next, Farrell and Maskin (1989) claim that $g_k(\underline{a}^1) \geq g_k^*(\underline{\sigma}^1)$ for a player $k \neq 1$. Suppose the contrary holds, this then implies that $g_k^*(\hat{\sigma}^1) > g_k^*(\underline{\sigma}^1)$. Thus, because of the WRP condition, there must exist a player $l \neq k$ with $g_l^*(\hat{\sigma}^1) \leq g_l^*(\underline{\sigma}^1)$, as otherwise $\hat{\sigma}^1$ would strictly Pareto-dominate $\underline{\sigma}^1$. For the two-player game, this contradicts with the selection process of $\underline{\sigma}^1$. For more than two players, however, we don't obtain this contradiction and cannot deduce that for a player $k \neq 1$ it has to hold that

$$g_k(\underline{a}^1) \geq g_k(\underline{\sigma}^1). \tag{4.A.2}$$

Moreover, even if we showed (4.A.2) for a player $k \neq 1$, this player k from (4.A.2) and j from (4.A.1) are not necessarily the same players, as it must be the case in the two-player game, and we therefore cannot conclude that $g_j(\underline{a}^1) \geq v_j$ for a $j \neq 1$. In fact, this is why Example 4.2 works: The worst continuation equilibrium for Player 1 is the punishment path σ^1. Along this path, Player 2 satisfies (4.A.1) but not (4.A.2), while for Player 3 it is vice versa.

Proof of Proposition 4.1. We only need to show $\mathcal{W} \subseteq \mathcal{W}^i$. This is trivial, if $w \in \mathcal{W}$ is the Nash equilibrium payoff. For all other equilibrium payoffs $w \in \mathcal{W}$, for any time t and history h^t, there do not exist two continuation equilibria that can be strictly Pareto-ranked. We show by contradiction that $w \in \mathcal{W}^s$ for all specifications $s = 1, 2, 3, 4, 5$. Clearly, after a deviation of player i in period t, the set $\bar{R}_i^t$ is equal for all specifications, i.e., $\bar{R}_i^t = \{i, j\}$ and thus $\mathcal{W}^s = \mathcal{W}^{s^{'}}$ for all $s, s^{'} = 1, \ldots, 5$. Note, that there must always be a punisher of a deviation if $w \in \mathcal{W}$ is not a Nash equilibrium payoff and thus $R_i^t \neq \emptyset$.

Suppose $w \notin \mathcal{W}^s$. There then exists at least one contingency h^t, such that deviator i has deviated in t, and there exists a continuation equilibrium $\tilde{\sigma}$, such that $j \neq i$ strictly prefers $\tilde{\sigma}$ to σ^i. As $w \in \mathcal{W}$, player i cannot strictly prefer $\tilde{\sigma}$ to σ^i, that is, it must hold that $g_i^*(\tilde{\sigma}) \leq g_i^*(\sigma^i)$. By the definition of σ, there are four possible types of continuation equilibria that are available to player j at time $t+1$. Either $\tilde{\sigma}$ is a continuation equilibrium on one of the punishment paths, it is the other player's punishment continuation equilibrium, or it is a continuation equilibrium from the normal phase of the game.

The latter case can be directly excluded by the subgame perfection requirement. If $g_i^*(\sigma^i) \geq g_i^*(\sigma|_{h^\tau})$ for any history $h^\tau \in H$ such that no player has deviated, σ cannot be subgame perfect. If $\tilde{\sigma} = \sigma^j$, then this contradicts with the subgame perfection condition for player j.

It therefore remains to check any continuation equilibria on the two punishment paths p^1 and p^2. Thus, consider any history $h^\tau \in H$, $h^\tau \neq h^t$, and suppose first that $\tilde{\sigma} = \sigma^i|_{h^\tau}$. We can then sustain w with a new punishment strategy $\tilde{\sigma}^i$, which uses this continuation equilibrium to punish player i. That is, if we define $\tilde{\sigma}^i = \sigma^i|_{h^\tau}$, then this renegotiation option is excluded. Next, assume $\tilde{\sigma} = \sigma^j|_{h^\tau}$. Again, we can sustain w with a punishment strategy $\tilde{\sigma}^i = \sigma^j|_{h^\tau}$, such that there is no renegotiation option. Hence, we can sustain w without any continuation equilibria that j would renegotiate to, which contradicts our assumption that $w \notin \mathcal{W}^s$, and therefore yields that $w \in \mathcal{W}^s$ for $s = 1, 2, 3, 4, 5$. □

On the relation of equilibrium payoffs between the different specifications of $\mathcal{R}$.

The different specifications of $\mathcal{R}$, given in Subsection 4.5.1 all yield different sets of equilibrium payoffs. We have already elaborated that Specification 2 is the strongest among the five, i.e., $\mathcal{W}^2 \subseteq \mathcal{W}^s, s = 1, 3, 4, 5$, and that $\mathcal{W}^4 \subseteq \mathcal{W}^3$. The question arises, whether we can make general statements about the relationship of the other equilibrium payoffs, but in the following we show that this is not possible.

First, it is clear that in general $\mathcal{W}^2 \neq \mathcal{W}^s, s = 1, 3, 4, 5$. A renegotiation protocol $\mathcal{R}$ with Specification 2 can generally impose more threats of renegotiation than under any other specification. Second, it may be intuitive to assume that Specification 1 is the weakest, i.e., that also $\mathcal{W}^3, \mathcal{W}^4, \mathcal{W}^5 \subseteq \mathcal{W}^1$. To show that this is generally not true, consider Specification 3. Let $w \in \mathcal{W}^3$, and let player i deviate in period t. Assume that every group $R_i^t \in \mathcal{R}_i^t$ is indifferent between the proposed continuation equilibrium σ^i and every other available continuation equilibrium $\tilde{\sigma} \in \Sigma_{\bar{R}_i^t}(\sigma)$. Let $R_i^t \neq N$, there can then exist a player that is not in R_i^t but strictly better off in $\tilde{\sigma}$. This does not conflict with Specification 3, but yields $w \notin \mathcal{W}^1$. For $\mathcal{W}^4$ and $\mathcal{W}^5$, analogous arguments yield that Specification 1 is not necessarily weaker than Specifications 3, 4 and 5, i.e., $\mathcal{W}^3, \mathcal{W}^4, \mathcal{W}^5 \not\subseteq \mathcal{W}^1$.

Third, it is immediate to see that Specification 1 is not necessarily stronger than Specifications 3, 4 and 5, i.e., also $\mathcal{W}^1 \not\subseteq \mathcal{W}^3, \mathcal{W}^4, \mathcal{W}^5$. Fourth, Specification 5 can also not be related to Specifications 3 and 4. That is, we have that generally $\mathcal{W}^5 \not\subseteq \mathcal{W}^3, \mathcal{W}^4$, and $\mathcal{W}^3, \mathcal{W}^4 \not\subseteq \mathcal{W}^5$.

It remains to compare Specifications 3 and 4. Let $w \in \mathcal{W}^4$. Then for all players i and periods t, the group of renegotiators $R_i^t = P_i^t$ cannot renegotiate to another continuation equilibrium $\tilde{\sigma} \in \Sigma_{P_i^t}(\sigma)$. For Specification 3, however, $\mathcal{R}$ allows more subgroups to renegotiate at period t and this may consequently yield that $w \notin \mathcal{W}^3$.

Proof of Proposition 4.2. Let the full cooperation equilibrium strategy σ be defined as follows: As long as no single player deviates, all players play C, i.e., $p^* = \{(C, \ldots, C)\}_{t \in \mathbb{N}}$. After a single deviation by player i in period $t = \bar{t}$, the player $j = \begin{cases} i+1, & \text{if } j < n \\ 1, & \text{else} \end{cases}$, plays D while all others play C for t_i periods, before all come back to playing C, i.e.

$$p^i = \{(C, \ldots, C, D, C, \ldots, C)\}_{t=\bar{t}+1}^{\bar{t}+t_i}, \{(C, \ldots, C)\}_{t=\bar{t}+t_i+1}^{\infty}.$$

First, we show that for any $\delta < 1$, we can always find a duration t_i such that σ^* is a subgame perfect equilibrium. As the game is symmetric, we assume without loss of generality that $i = 1$ and player $j = 2$ is the punisher. Player 1's deviation payoff is given by

$$(1-\delta)\left(g_1(D, C, \ldots, C) + \sum_{t=\bar{t}+1}^{\bar{t}+t_1} \delta^{t-1} g_1(C, D, C, \ldots, C)\right) + \delta^{\bar{t}+t_1} g_1(C, \ldots, C).$$

As $g_1(C, D, C, \ldots, C) < g_1(C, \ldots, C)$, for any $\delta < 1$, we can determine $t_1 > 0$, such that player 1 has no incentive to deviate from full cooperation.

Furthermore, by condition (*) of Definition 4.6, it holds that no other player has an incentive to deviate from her strategy during the punishment, since

$$c_j(C, D, C, \ldots, C) = g_j(C, D, C \ldots, C) \ \forall\, j \neq 2.$$

And since D is the strictly dominant action for Player 2, no player will deviate from Player 1's punishment path. Therefore, this strategy is certainly subgame perfect.

Second, there are no two continuation equilibria that can be strictly Pareto-ranked. The only continuation equilibria to compare are the ones along the full cooperation path p^*, and the punishment continuation equilibria along p^1. Clearly, Player 2 will always block renegotiation from p^1 to p^* and to her own punishment p^2. As all players are symmetric, there is no other continuation equilibrium to consider; thus, full cooperation is WRP. For $\mathcal{R}$-WRP, in all of the specifications given, there are obviously no subgroups that can enforce a renegotiation: No other continuation equilibrium is available without Player 2 and she will always block the renegotiation. ☐

Proof of Proposition 4.3. Let σ be WRP strategy, that is, it satisfies both the subgame perfection and WRP condition of Theorem 2.1 in Chapter 2. To guarantee that σ satisfies Definition 4.4, the original WRP condition needs to be changed to its strict version to exclude those continuation equilibria of σ which make all punishers P_i weakly better. That is, if there is one punisher $j \in P_i$ such that $\pi_j(x^p_{P_i}, x^s_{N \setminus P_i}) > \pi_j(x^s_N)$, she will always block renegotiation from the punishment path $\mathbf{p_i^C}$ to the agreement path $\mathbf{a^C}$. This is Condition 4.5.

To exclude a renegotiation from a punishment $\mathbf{p_i^C}$ to a different punishment $\mathbf{p_j^C}$, we need to compare the payoffs for all relevant groups of players. In all specifications, those players who are involved in punishing player i but not player j, as well as those who are involved in punishing player j but not player i, can block a renegotiation. That is, we need to have at least one player $l \in P_i \setminus P_j$ or $l \in P_j \setminus P_i$ who satisfies $\pi_l(x^p_{P_i}, x^s_{N \setminus P_i}) > \pi_l(x^p_{P_j}, x^s_{N \setminus P_j})$. This yields Conditions (a) and (b). ☐

On Proposition 4.3 for Specifications 1 and 5.

For Specification 1, players outside the punishment groups P_i and P_j can also block renegotiation. Thus, if there is one player $l \notin P_i \cup P_j$ such that $\pi_l(x^p_{P_i}, x^s_{N\setminus P_i}) > \pi_l(x^p_{P_j}, x^s_{N\setminus P_j})$, she will block any renegotiation from a punishment $\mathbf{p_i^C}$ to a different punishment $\mathbf{p_j^C}$. Thus, to guarantee that an IEA $\mathbf{s}$ by a coalition C is an $\mathcal{R}$-WRP equilibrium with Specification 1, the conditions of Proposition 4.3, together with the following condition (c), are sufficient and necessary:

(c) there exists $l \notin P_i \cup P_j$ such that

$$\sum_{m\in P_i\setminus P_j} \Big(-\beta(k-p) - \gamma(k_m+1-p)\Big)(-\beta - \gamma\bar{g}_{lm}) + \sum_{m\in P_j\setminus P_i} \Big(-\beta(k-p) - \gamma(k_m+1-p)\Big)(\beta + \gamma\bar{g}_{lm}) > 0$$

For Specification 5, this condition only applies if this player $l \notin P_i \cup P_j$ is needed for R_i^t to constitute a simple majority.

4.B Coalitional Behavior in Non-Cooperative Games

The standard theory of renegotiation-proofness tackles collective dynamic consistency among the entire group of players. The refinement introduced in this paper allows for subgroup renegotiation ex post, but does not cover subgroup or coalitional behavior ex ante. Quite naturally, one may strive for a concept that incorporates both ideas, but this has to our understanding not been achieved yet. In fact, there is a wide literature that studies group behavior in infinitely repeated games, but the problem of ex post renegotiation is not covered adequately. We will discuss the most relevant ones in the following.

The first to tackle coalitional behavior in non-cooperative games is Aumann (1959); *strong Nash equilibria* are robust against every conceivable coalition in the single-stage game. Rubinstein (1980) extends this to infinitely repeated games, but the conditions are often found to be too harsh, such that existence of strong Nash equilibria fails. The seminal work of Bernheim et al. (1987) (subsequently abbreviated as BPW) relaxes these conditions and proposes the equilibrium notion of *perfect coalition-proofness* for finitely repeated games—interestingly developed at the same time as Farrell and Maskin (1989). They impose an internal consistency condition on the equilibrium set in the sense that they exclude any deviations which do not fulfill the criteria of the original agreement. The agreement must therefore no longer be stable against any cooperative deviation but only those, that are self-enforcing in the sense that no subcoalition can deviate in a self-enforcing way. However, they can only consider finitely repeated games as the recursive definition of their equilibrium notion prevents the application to infinite horizon games. Further refinements of their work can be found, for instance, in Chakravorti and Kahn (1991) or Kahn and Mookherjee (1992) but neither of them consider renegotiation.

The problem of recursive definitions has led several authors to use the approach proposed by Greenberg (1989, 1990), who applies von Neumann and Morgenstern abstract stable sets to define coalition-proof equilibrium notions and *standards of behavior*. Such standards of behavior can be found in DeMarzo (1992), Asheim (1997), Ferreira (2001) and Xue (2002).

DeMarzo (1992) introduces an exogenously defined leader who suggests a behavior that is then followed by the group of players; thereby motivating collective consistent behavior. Asheim (1997) assumes that "in each subgame, any coalition can coordinate on any strategy profile, taking into account that in the current and each later subgame, any subcoalition can in turn do so" (p. 437), and refers to this as the perfectly coalition-proof situation (this is indeed the setting for BPW's definition). In addition to the internal consistency condition imposed by Farrell and Maskin (1989) and Bernheim and Ray (1989), he requires external consistency for the equilibrium set (as in his earlier paper Asheim, 1991), and argues that in the renegotiation situation it is left "completely unexplained why the players do not renegotiate to a subgame perfect equilibrium outside the set of continuation equilibria". Nevertheless, he also incorporates the recursive definition of BPW.

Another refinement or extension of the BPW notion is suggested in Ferreira (1996). Here, the purpose is to capture both problems of coalition- and renegotiation-proofness in one equilibrium notion. When considering such an equilibrium, the author requires it to be "immune to deviations by coalitions that not only take as fixed the actions of the complementary coalition in the current period, but also consider reactions by any coalition in future periods" (p. 250). He argues that, while perfectly coalition-proof Nash equilibria do not incorporate this, his proposed concept does. Xue (2002) discusses *stable agreements* and claims to capture the problem of collective dynamic consistency by also limiting the set of possible deviations, even further than BPW do. Both authors therefore impose a weaker internal consistency condition than in the renegotiation-proof concept.

Finally, Chung (2004) imposes a recursive, internal consistency condition that excludes many possible deviations which are not self-enforcing. He can therefore obtain an existence result, but is too strict to allow for proper renegotiation in the sense of WRP.

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