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R&D Competition versus R&D Cooperation in Oligopolistic Markets with Evolving Structure*

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Abstract

This paper considers investment behavior of duopolistic firms subject to technological progress. It is assumed that initially both firms offer a homogeneous product, but after a stochastic waiting time they are able to realize a product innovation. Production capacities of both firms are product specific. It is shown that firms anticipate a future product innovation by under-investing (if the new product is a substitute to the established product) and higher profits, and over-investing (in case of complements) and lower profits, compared to the corresponding standard capital accumulation game. This anticipation effect is stronger in the case of R&D cooperation. Furthermore, since due to R&D cooperation firms introduce the new product at the same time, this leads to intensified competition and lower firm profits right after the new product has been introduced. In addition, we show that under R&D competition the firm that innovates first, overshoots in new-product capacity buildup in order to exploit its temporary monopoly position. Taking into account all these effects, the result is that, if the new product is neither a close substitute nor a strong complement of the established product, positive synergy effects in R&D cooperation are necessary to make it more profitable for firms than R&D competition.

1 Introduction

The overall aim of this paper is to develop and exploit a dynamic framework of analysis that allows studying the optimal product development, the choice between competition and cooperation in R&D, and investment strategies of oligopolistic firms under consideration of the uncertainty about future changes

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in the market structure. The considered changes in the market structure are due to changes in the range of products offered on the market triggered by product innovations of incumbent firms. The considered market environment captures in a stylized way the dynamic emergence of new submarkets as can be observed repeatedly in established oligopolistic markets. For example, producers of personal computers developed new related submarkets by introducing portable MP3 players starting in the early 2000s or tablet PCs in 2010. In both submarkets the major competitors are established producers of PCs and the new products influence demand for PCs, where in case of MP3 players the relationship is complementary, whereas tablet PCs are substitutes[1]. Other examples include the TV industry, where major producers of standard CRT television sets have started the production of flatscreens around the year 2000, and the (time-phased) introduction of hybrid cars by many car manufacturers, which opened a new submarket in this industry co-existing with the established ones.

The question whether innovation projects should be carried out in cooperation with partners, including competitors on the market, is a key issue and has attracted substantial interest from a theoretical (e.g. D'Aspremont and Jacquemin (1988) or Choi (1993)) and an empirical perspective (e.g. Hagedoorn (2002), Roijakkers and Hagedoorn (2006)). Also in the industries mentioned above examples for cooperation, like the Global Hybrid Cooperation (GM and Daimler) for the development of hybrid cars, the cooperation between Sony and Samsung for the development of TFT-LCD screens, or a cooperation between Lenovo and NEC to develop tablet computers play an important role. On the other hand, several of the innovations opening new submarkets in these industries were introduced by incumbents who did not engage in R&D cooperation.

In order to capture the main implications of R&D cooperations in industry settings like the ones discussed above, a dynamic framework is needed[2], which takes into account that firms potentially involved in R&D cooperations are at the same time competitors on the established product markets. In particular, they should realize that the introduction of new products will affect profitability of their current product range. Existing theoretical studies of incentives for R&D cooperations typically are not dynamic and/or do not study interdependencies with existing products.

This paper attempts to fill this gap. To do so, we consider a dynamic oligopoly model, where incumbent firms offer an established homogeneous product. At some ex-ante unknown point in time the range of products is enlarged, because one or several of the competing firms obtain the option to introduce a new product, which is vertically and horizontally differentiated from the existing product. Capacities cannot be (fully) transferred between the production of

[1]To illustrate, one of the main producers on the PC and tablet market, Samsung Electronics, stated that it expects the global personal computer market to shrink by 5 percent in 2013 as consumer demand continues to shift to mobile devices such as tablet computers (see India Times (2013))

[2]The importance of a dynamic perspective on oligopoly markets has been stressed in Cabral (2012, p.278), who argues that, "... dynamic oligopoly models are an area where much work needs to be done and much work can be done.", and further, "... dynamic oligopoly model provide considerable value added with respect to static models."

different products, and therefore the introduction of the new product reduces (in case of substitutes) or increases (in case of complements) the value of the existing capacities. The firms' objectives are to maximize their total discounted profits by optimally selecting their investments in production capacities for the different products they offer. Firms can enter an R&D cooperation with the competitor in order to develop the new product. Entering such a cooperation reduces the expected time till the firm can introduce the new product because the firm gets access to the R&D results of the partner and the cooperation might also generate synergies in the R&D activities (see e.g. Link et al. (1996) or Link(1998) for empirical evidence for the existence of synergies in R&D cooperations). The drawback of such a cooperation is that the partner will be able to introduce the new product at the same time, thereby intensifying competition.

The developed model has the form of a piecewise deterministic differential game with different modes. In the initial mode none of the firms has introduced the new product. Any introduction of new products results in a switch to a new mode opening a broader range of investment possibilities for the innovating firm. To our knowledge this is the first paper to study dynamic strategic interactions in an oligopoly where the product range evolves and also the first use of a multi-mode game in the field of Industrial Organization.

Using this framework we address the following main research questions:

- Under which circumstances is it profitable for firms to cooperate in the product innovation project?
- How does the *anticipation* of the emergence of a new sub-market affect capacity dynamics of both firms prior to the innovation?
- How is the build-up of capacities after the emergence of the new sub-market influenced by cooperation/non-cooperation in the innovation project?
- How do the answers to the questions above depend on the degree of substitutability between the established and the new product?

Characterizing Markov Perfect Equilibria (MPE) of the game leads to the following answers. Although the model abstracts from explicit cooperation costs and knowledge spillover considerations, R&D cooperation has implicit costs due to strategic interactions on the market. We identify two main effects responsible for these costs: i) R&D cooperation implies that after a successful innovation the new product is available for market introduction to all cooperation partners. Hence, market competition emerges immediately after the introduction of the new product (we denote this as the *synchronization effect*). ii) In case the new product is a complement to the established one, the anticipation of the eventual introduction of the new product induces an investment increase in capacities of the established product already *prior* to the new product introduction. The increased investment brings the price for the established product closer to the competitive level, thereby reducing firm profits. This *anticipation effect* is stronger if firms cooperate in R&D, since the innovation is accelerated.

This results in additional implicit costs of R&D cooperation. In particular, if the new product is neither a close substitute nor a strong complement of the established product, these implicit costs outweigh the gains of cooperation. These gains consist not only of reduction of expected innovation time, but also of a reduction in capacity adjustment costs for the new product (*adjustment cost effect*). We show that under R&D competition the (intertemporally optimal) capacity trajectory of the innovator is non-monotonous. Directly after the new product introduction the innovator builds capacities in order to exploit the temporary market power. After the competitor has also introduced the new product, the innovator scraps parts of this installed capacity to keep product prices at a sufficiently high level. The costs associated with such 'overshooting' of capacity build-up are avoided in case of an R&D cooperation, because in this case we observe monotone capacity adjustments on both markets after the innovation.

Another main insight from our dynamic analysis is that the anticipation of the addition of a substitute to the established product to the market acts as a collusion device. If firms expect that a substitute for their current product will be introduced, the expected return from current capacity investment decreases. Hence, such expectations lead to lower capacities and a higher price for the established product. This moves the price closer to the monopoly price and increases current total industry profits compared to a situation without such an anticipation.

The paper is organized as follows. The next section provides a review of related streams of literature. The model is presented in Section 3, which is followed by a characterization of the Markov Perfect Equilibria of the game in Section 4. Section 5 discusses the economic implications of our analysis and Section 6 concludes.

2 Literature Review

Three streams of literature are related to our research. First, several studies have analyzed the incentives for entering R&D cooperations with competitors in oligopolistic markets. A common theme in this literature is that, whereas R&D cooperation facilitates the innovation process, the fact that a larger number of firms gets access to the innovation increases the intensity of product market competition. In an influential paper Goyal and Yoshi (2003) characterize shapes of stable R&D networks between competitors in static oligopolistic markets, where the marginal production costs depend negatively on the number of R&D partners. Martin (2002) considers a patent race model with spillovers for cost saving innovations in a quantity-setting duopoly. He assumes that the innovator provides the cost-reducing technology to the follower in exchange for a licensing fee. He finds that for large parts of the parameter space there are no private incentives to form R&D cooperations, which would be socially beneficial. In a related contribution Miyagiwa and Ohno (2002) consider a similar innovation race, where it is assumed that the profits of innovators decrease with the number of firms having access to the innovation. Furthermore, they assume that the innovation becomes available to all firms a fixed period after its initial market introduction. They find that firms have incentives to form an R&D joint venture if total industry (flow) profits are higher if all firms have access to the innovation compared to a scenario with only one innovator. This work has been extended in Erkal and Piccinin (2010) by assuming free market entry and allowing for R&D joint ventures which do not include all firms in the market.

Few contributions in this literature explicitly focus on product innovation. One such contribution is Bourreau and Dogan (2010), who consider market competitors who can partially cooperate. The level of cooperation determines the fraction of product parts developed jointly, which implies that increasing this level, on the one hand, reduces development cost for each firm, but, on the other hand, also reduces the degree of differentiation between their products. Furthermore, firms may reduce production costs for the parts by process R&D. The focus of this paper lies on the complementarity between the degrees of cooperation in product and process innovation.

Whereas the trade-off between faster innovation and lower post innovation profits due to more intensive product market competition is also an important aspect for the evaluation of R&D cooperations in our framework, we address several issues that are not covered in this stream of literature so far. First, we explicitly model the effects of R&D cooperation on the behavior on established markets. Second, we explicity consider the implications of capacity adjustment for pre and post innovation profitability. Third, in our dynamic setting we explicitly capture that R&D cooperations for product innovation have a synchronizing effect on market introduction dates across competitors.

The second related stream of research is the well-established literature on capital accumulation games (see e.g. Jun and Vives 2004), where capacity investments of single-product firms engaged in oligopolistic competition have been characterized both in the framework of open-loop Nash equilibria and Markov-perfect Nash equilibria. As pointed out, for example, in Dockner (1992) intertemporal strategic effects present in Markov Perfect Equilibria imply higher (lower) investments compared to open-loop scenarios if the products of competitors are substitutes (complements). Dawid et al. (2010) extend these considerations to a setting where one competitor offers multiple products and show that due to the intertemporal strategic effect the single product firm invests more aggressively. Besanko and Doraszelski (2004) introduce a stochastic capacity accumulation game in discrete time to describe an evolving industry structure and to explain persistent differences in firm size (see also Doraszelski and Pakes (2007)). Chen (2009) provides an extension to explain the short- and long-run effects of mergers in (near-homogeneous product) industries. Besanko et al. (2010a) study capacity accumulation patterns in a dynamic duopoly game with strategic uncertainty (about the rival's cost). They find that the occurrence of preemption races (where excess capacity is built up) and capacity coordination (where capacities are close to cartel levels) depend on the degree of product differentiation, investment sunkness, and capacity depreciation. Besanko et al. (2010b) extend this model to include demand uncertainty. Our contribution extends this stream of literature by considering multi-product firms, as well as the effects of the (anticipation of the) introduction of new products on investment behavior, and the analysis of R&D cooperation and competition.

Finally, this paper is related to differential game models dealing with R&D in dynamic oligopolies. Cellini and Lambertini (2002) consider such a game where firms over time influence the degree of differentiation of their products. They find that this degree is higher if the number of firms is larger. Furthermore, the existence of an R&D cartel leads to higher product differentiation. Cellini and Lambertini (2009) consider a dynamic version of the static process innovation model with spillovers (see d'Aspremont and Jaquemin, 1988), and, in contrast to the static framework, find that independent of the degree of spillovers, R&D cooperation is preferable to noncooperative R&D from both a private and a social perspective. Furthermore, cooperative R&D efforts over time are higher than in the fully noncooperative game. Lambertini and Mantovani (2010) study R&D activities, process or product R&D, in Cournot and Bertrand oligopolies. They demonstrate that in a dynamic setting, the two types of innovation are not necessarily complementary. For an analysis of the differentiated Bertrand case with process innovation, see Cellini and Lambertini (2011). In contrast to the present paper, these contributions mainly focus on single-product firms and do not study the issue of incumbents who face the problem of launching new products and have to invest/disinvest in capacity for established and new products over time.

3 The Model

We consider a duopoly where both firms, denoted Firm A and Firm B, have initial capacities $K_{1f}(0)$ $(f = A, B)$ available for production of an established product, denoted as product 1. At $t = 0$ both firms start an innovation project aiming at the development of a new differentiated product (product 2). The completion time of the innovation project of firm i is denoted by $\tau_i,\ i = A, B$. The project might be carried out independently or jointly. In the independent case, which we will refer to as R&D competition, both firms in general have different project completion times. The new product becomes available only to the firm which has finished its project at that time. Completion times are exponentially distributed with arrival rate $\lambda^{comp} > 0$ and independent across firms. In order to focus the analysis on the capacity dynamics of both firms, the distribution of the project completion times is assumed to be exogenous.

Alternatively, in the case of R&D cooperation, the firms engage in a joint innovation project, whose completion time τ^{coop} is exponentially distributed with arrival rate $\lambda^{coop} > 0$. At this completion time, the new product becomes available for both firms to produce. Since in the R&D competition case the minimal completion time $min[\tau_A, \tau_B]$ is exponentially distributed with arrival rate $2\lambda^{comp}$, we assume λ^{coop} to be greater than or equal to this value. We interpret $\lambda^{coop} - 2\lambda^{comp} \geq 0$ as the degree of synergies that arise due to cooperation of the two firms.

In order to be able to produce positive quantities of this new product, the firms have to build up production capacities, where initial capacities are $K_{2f}(0) = 0, f = A, B$. At the same time they can adjust their capacities for the established product. Production capacities here involve physical capital at production facilities as well as the specific know-how, supply chains, and distribution channels. Production capacities are specific to the production of either the established or the new product.[3]

It is assumed that both firms at each point in time fully exploit their production capacities. [4] Prices are given by the linear inverse demand system

$$p_1(t) = 1 - (K_{1A}(t) + K_{1B}(t)) - \eta(K_{2A}(t) + K_{2B}(t)), \tag{1}$$

$$p_2(t) = 1 + \theta - \eta(K_{1A}(t) + K_{1B}(t)) - (K_{2A}(t) + K_{2B})). \tag{2}$$

This type of inverse demand system can be derived from a representative consumer model with a quality-augmented version of the standard quadratic utility function (see Symeonidis (2003), Vives (1999)). The parameter η determines the degree of horizontal differentiation, where $-1 < \eta < 1$. The parameter $\theta \geq 0$ measures the degree of vertical differentiation between the two products, where it is assumed that the new product is of higher or equal quality. We assume that production costs are linear, and for reasons of simplicity we assume that marginal production costs for both products are normalized to zero.

Investment costs are assumed to be linear-quadratic and symmetric across firms, i.e. $C(I_{if}(t)) = b_i I_{if} + \frac{cI_{if}(t)^2}{2}$, where $I_{if}(t)$ denotes the investment of firm $f \in \{A, B\}$ for product $i = 1, 2$ at time t. The parameter $b_i \geq 0$ is the unit price of capacity for good i and $c > 0$ is the adjustment cost parameter. We allow for disinvestment of firms and hence $I_{if}(t)$ may be any real number for any t taking into account the non-negativity of the capacities. Investments (or disinvestments) in capacities to produce the new product are only possible at t if the new product has become available at some $\tau \leq t$. To capture this constraint formally we define four modes of the game (see Dockner et al. (2000), Chapter 8 for a general description of multi-mode differential games referred to as 'Piecewise deterministic games'). The first mode, labeled as m_1, corresponds to the periods where the new product has not become available yet. In the modes m_2 and m_3 the new product is available only to one of the two firms, where in mode m_2 firm A is the one that innovates first, whereas firm B is the innovation leader in mode m_3. One of these two modes always occurs under

[3] It could be argued that in many industries capacities built for the production of the old product can be partly transferred to the production of the new product. Relaxing our assumption that capacities are completely product specific along these lines would however not alter our qualitative conclusions as long as a sufficiently large fraction of old market capacities is lost when transferred to the production of the new product. Moreover, in the examples like flatscreen TVs mentioned in the Introduction, the production processes for the established and the new product are based on very different technologies. In such situations the capacities devoted to the established product typically can hardly be transferred to the production of the new product.

[4] The assumption that capacities are fully used by firms has been adopted in large parts of the literature, see for example Goyal and Netessine (2007) for an extensive discussion and related references.

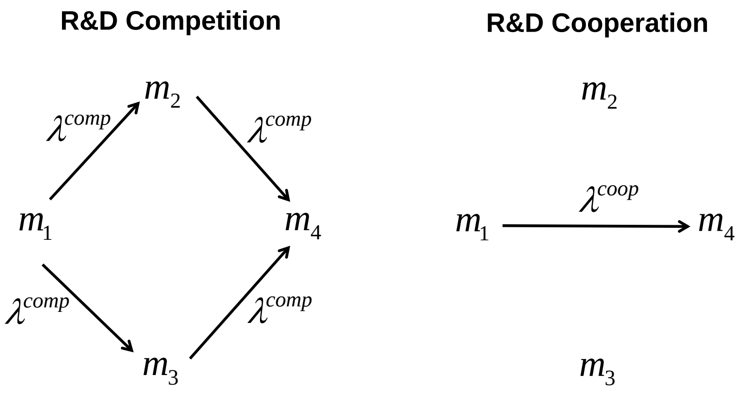


Figure 1: Transition rates between the different modes under R&D competition and R&D cooperation.

R&D competition in the time interval between $\min[\tau_A, \tau_B]$ and $\max[\tau_A, \tau_B]$. Under R&D cooperation none of these modes can occur. Instead, with R&D cooperation there is a direct transition from mode m_1 to mode m_4, where both firms have access to the new product. Also under R&D competition mode m_4 is eventually reached after the second firm has completed its innovation project. See figure 1 for a graphical summary of these transitions.

In mathematical terms we define a Markov process $m(t)$ on the set $M := \{m_1, m_2, m_3, m_4\}$ with $m(0) = m_1$ and the transition rates given in figure 1. The restrictions on investment are then captured by the constraints

$$\begin{aligned} I_{2f}(t) &= 0 \qquad \forall t \text{ s.t. } m(t) = m_1, f = A, B. \\ I_{2B}(t) &= 0 \qquad \forall t \text{ s.t. } m(t) = m_2 \\ I_{2A}(t) &= 0 \qquad \forall t \text{ s.t. } m(t) = m_3 \end{aligned} \tag{3}$$

Overall there are four relevant capacities, which evolve according to the state dynamics

$$\dot{K}_{if} = I_{if}, \qquad i = 1, 2,\ f = A, B. \tag{4}$$

with initial conditions

$$K_{1f}(0) = K_{1f}^{ini} \geq 0, \quad K_{2f}(0) = 0, \qquad f = A, B. \tag{5}$$

Capacities have to be non-negative and are bounded above by some (large) value $\bar{K}_{if},\ i = 1, 2, f = A, B$. For simplicity no depreciation of capital is considered.

Each of the two firms chooses its investments in order to maximize its discounted profits net of investment costs over an infinite horizon. Denoting by

$r > 0$ the discount rate, the objective functions of the two firms are given by

$$J_f = \mathbb{E}\left[\int_0^\infty e^{-rt}\left[(1-(K_{1A}+K_{1B})-\eta(K_{2A}+K_{2B}))K_{1f}+(1+\theta-\eta(K_{1A}+K_{1B})\right.\right.$$
$$\left.\left.-(K_{2A}+K_{2B}))K_{2f}-b_1 I_{1f}-\frac{c}{2}I_{1f}^2-b_2 I_{2f}-\frac{c}{2}I_{2f}^2\right]dt\right], \qquad (6)$$

subject to (3), (4), (5), where the expectation is taken with respect to the stochastic process $m(t)$.

We assume that firms use stationary Markovian feedback strategies. Put formally, we define by $\mathcal{I}$ the set of all piecewise continuous[5] functions from $[0,\bar{K}_1]^2 \times [0,\bar{K}_2]^2 \times M$ to $\mathcal{R}$. Note that due to the fact that we have a multi-mode game, a Markovian feedback strategy has to depend on the current states and the current mode. The strategy space of each firm is then given by $\mathcal{I}^2$.

Defining a Markov-perfect equilibrium in a multi-mode game is not completely standard. For a given strategy profile $(\hat{I}_{jB})_{j=1,2}$ we denote as $\hat{P}_A$ the (stochastic) optimal control problem firm A faces if the feedback rules $(\hat{I}_{1B},\hat{I}_{2B})$ are inserted for (I_{1B},I_{2B}). Note that after insertion of these strategies, the right hand side of the state dynamics depends on the mode $m(t)$. Following Dockner et al. (2000) we call a control path $i_f = (i_{1f}, i_{2f}) : [0,\infty) \times \Xi \mapsto \mathcal{R}^2$, where $f \in \{A, B\}$, feasible if i) i_f is non-anticipating with respect to the realization of the project completion time. In the case of R&D competition this means that $i_f(t,\tau_A,\tau_B) = \tilde{i}_f(t)\ \forall t < \min[\tau_A,\tau_B]$, $i_f(t,\tau_A,\tau_B) = \tilde{i}_f(t,\tau_A)$ for $\tau_A = \min[\tau_A,\tau_B]$ and $t \in [\tau_A,\tau_B)$ and $i_f(t,\tau_A,\tau_B) = \tilde{i}_f(t,\tau_B)$ for $\tau_B = \min[\tau_A,\tau_B]$ and $t \in [\tau_B,\tau_A)$ for some $\tilde{i}_f(.)$. Analogously for R&D cooperation ; ii) the piecewise deterministic process $(K_{1A}(.),K_{1B}(.),K_{2A}(.),K_{2B}(.),m(.))$ is well-defined; iii) the constraints $K_{jf} \geq 0$ and investment constraints (3) are satisfied with probability 1, and iv) the objective integral is well-defined. A feasible control path i_A^* is optimal if it maximizes the objective integral within the set of all feasible control paths. Analogously we define the optimal path i_B^*.

Definition 1 *A strategy profile $(I_{1A}^*, I_{2A}^*, I_{1B}^*, I_{2B}^*) \in \mathcal{I}^4$ is Markov-perfect equilibrium (MPE) of the game if the following condition holds for each firm $f = A, B$:*

- *If the feedback strategies (I_{1f}^*, I_{2f}^*) are applied to the control problem $\hat{P}_f$ determined by inserting $(I_{1g}^*, I_{2g}^*), g \neq f$, then the resulting (stochastic) investment path $(I_{1f}^*(K_{1A}(t;\tau),K_{2A}(t;\tau),K_{1B}(t;\tau),K_{2B}(t;\tau),m(t;\tau))$, $I_{2f}^*(K_{1A}(t;\tau),K_{2A}(t;\tau),K_{1B}(t;\tau),K_{2B}(t;\tau),m(t;\tau)))$ concides with the optimal path $i_f^*(t,\tau)$ with probability 1, where $\tau = (\tau_A,\tau_B)$ under R&D competition and $\tau = \tau^{coop}$ under R&D cooperation.*

[5]In the considered model the restriction of attention to piecewise continuous feedback strategies turns out to be innocuous.

4 Characterization of the Markov Perfect Equilibria

We characterize the Markov-perfect equilibria of the described dynamic capacity game using a dynamic programming approach. As pointed out in Dockner et al. (2000), for multi-mode games the value functions of the players depend not only on the states, but also on the mode of the game. Formally, the value function of firm $f = A, B$ is a mapping $V_f : [0, \bar{K}_1]^2 \times [0, \bar{K}_2] \times M \mapsto \mathcal{R}$, where in mode m_1 the value functions only have to be determined on the subset of the state space where $K_{2A} = K_{2B} = 0$ and in mode m_2 (m_3) only on the subset where $K_{2B} = 0$ ($K_{2A} = 0$). The Hamilton-Jacobi-Bellman equations differ between modes. In particular in mode m_1 we have under R&D competition

$$\begin{aligned} & r\ V_f(K_{1f}, 0, K_{1g}, 0, m_1) \\ &\quad = \max_{I_{1f}} \Bigg[(1-(K_{1A}+K_{1B}))K_{1f} - b_1 I_{1f} - \frac{c}{2} I_{1f}^2 + \frac{\partial V_f(\cdot, m_1)}{\partial K_{1f}} I_{1f} \\ &\qquad + \frac{\partial V_f(\cdot, m_1)}{\partial K_{1g}} I_{1g}^* + \lambda^{comp} \left(V_f(\cdot, m_2) - V_f(\cdot, m_1) \right) + \lambda^{comp} \left(V_f(\cdot, m_3) - V_f(\cdot, m_1) \right) \Bigg], \\ &\qquad f, g = A, B,\ g \neq f, \end{aligned} \tag{7}$$

whereas for R&D cooperation we obtain

$$\begin{aligned} & r\ V_f(K_{1f}, 0, K_{1g}, 0, m_1) \\ &\quad = \max_{I_{1f}} \Bigg[(1-(K_{1A}+K_{1B}))K_{1f} - b_1 I_{1f} - \frac{c}{2} I_{1f}^2 + \frac{\partial V_f(\cdot, m_1)}{\partial K_{1f}} I_{1f} \\ &\qquad + \frac{\partial V_f(\cdot, m_1)}{\partial K_{1g}} I_{1g}^* + \lambda^{coop} \left(V_f(\cdot, m_4) - V_f(\cdot, m_1) \right) \Bigg], \quad f, g = A, B,\ g \neq f. \end{aligned} \tag{8}$$

In mode m_2 we have for the innovator firm A

$$\begin{aligned} & r\ V_A(K_{1A}, K_{2A}, K_{1B}, 0, m_2) \\ &\quad = \max_{I_{1A}, I_{2A}} \Big[(1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1A} \\ &\qquad + (1+\theta - K_{2A} - \eta(K_{1A}+K_{1B}))K_{2A} - b_1 I_{1A} - \frac{c}{2} I_{1A}^2 - b_2 I_{2A} - \frac{c}{2} I_{2A}^2 \\ &\qquad + \frac{\partial V_A(\cdot, m_2)}{\partial K_{1A}} I_{1A} + \frac{\partial V_A(\cdot, m_2)}{\partial K_{1B}} I_{1B}^* + \frac{\partial V_A(\cdot, m_2)}{\partial K_{2A}} I_{2A} + \lambda^{comp} \left(V_A(\cdot, m_4) - V_A(\cdot, m_2) \right) \Big] \end{aligned} \tag{9}$$

and for the laggard firm B

$$\begin{aligned} & r\ V_B(K_{1A}, K_{2A}, K_{1B}, 0, m_2) \\ &\quad = \max_{I_{1B}} \Big[(1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1B} - b_1 I_{1B} - \frac{c}{2} I_{1B}^2 \\ &\qquad + \frac{\partial V_B(\cdot, m_2)}{\partial K_{1A}} I_{1A}^* + \frac{\partial V_B(\cdot, m_2)}{\partial K_{1B}} I_{1B} + \frac{\partial V_B(\cdot, m_2)}{\partial K_{2A}} I_{2A}^* + \lambda^{comp} \left(V_B(\cdot, m_4) - V_B(\cdot, m_2) \right) \Big]. \end{aligned} \tag{10}$$

Symmetric equations arise in mode m_3, where firm B is the innovator and firm A is the laggard. Finally, in mode m_4 the HJB-equation reads

$$\begin{aligned} r\ & V_f(K_{1A}, K_{2A}, K_{1B}, K_{2B}, m_4) \qquad (11) \\ = & \max_{I_{1f}, I_{2f}} \Big[(1 - (K_{1A} + K_{1B}) - \eta(K_{2A} + K_{2B}))K_{1f} \\ & +(1 + \theta - (K_{2A} + K_{2B}) - \eta(K_{1A} + K_{1B}))K_{2f} - b_1 I_{1f} - \frac{c}{2} I_{1f}^2 - b_2 I_{2f} - \frac{c}{2} I_{2f}^2 \\ & + \frac{\partial V_f(\cdot, m_4)}{\partial K_{1f}} I_{1f} + \frac{\partial V_f(\cdot, m_4)}{\partial K_{1g}} I_{1g}^* + \frac{\partial V_f(\cdot, m_4)}{\partial K_{2f}} I_{2f} + \frac{\partial V_f(\cdot, m_4)}{\partial K_{2g}} I_{2g}^* \Big], \\ & f, g = A, B,\ f \neq g. \end{aligned}$$

Considering the first order conditions, it follows directly from these equations that the equilibrium investment strategies are of the form

$$I_{if}^*(\mathbf{K}, m) = \frac{1}{c} \left(\frac{\partial V_f(\mathbf{K}, m)}{\partial K_{if}} - b_i \right) \quad i = 1, 2,\ f = A, B,\ m \in M,$$

with $\mathbf{K} = (K_{1A}, K_{2A}, K_{1B}, K_{2B})$. Since the game is symmetric with respect to the two firms, we will concentrate on the characterization of symmetric Markov-perfect equilibria. Due to the linear quadratic structure of the game the following form for the value functions can be assumed:

$$\begin{aligned} V_f(\mathbf{K}, m_4) &= \alpha^4 + \beta^4 K_{1f} + \chi^4 K_{1g} + \epsilon^4 K_{2f} + \phi^4 K_{2g} + \varphi^4 K_{1f}^2 + \gamma^4 K_{1g}^2 \\ &\quad + \iota^4 K_{2f}^2 + \kappa^4 K_{2g}^2 + \mu^4 K_{1f} K_{1g} + \nu^4 K_{1f} K_{2f} + \sigma^4 K_{1f} K_{2g} \\ &\quad + \omega^4 K_{1g} K_{2f} + \psi^4 K_{1g} K_{2g} + \xi^4 K_{2f} K_{2g}, \\ V_A(K_{1A}, K_{2A}, K_{1B}, 0, m_2) &= \alpha_A^2 + \beta_A^2 K_{1A} + \chi_A^2 K_{1B} + \epsilon_A^2 K_{2A} + \varphi_A^2 K_{1A}^2 + \gamma_A^2 K_{1B}^2 \\ &\quad + \iota_A^2 K_{2A}^2 + \mu_A^2 K_{1A} K_{1B} + \nu_A^2 K_{1A} K_{2A} + \omega_A^2 K_{1B} K_{2A}, \\ V_B(K_{1A}, K_{2A}, K_{1B}, 0, m_2) &= \alpha_B^2 + \beta_B^2 K_{1B} + \chi_B^2 K_{1A} + \phi_B^2 K_{2A} + \varphi_B^2 K_{1B}^2 + \gamma_B^2 K_{1A}^2 \\ &\quad + \kappa_B^2 K_{2A}^2 + \mu_B^2 K_{1A} K_{1B} + \psi_B^2 K_{1A} K_{2A} + \sigma_B^2 K_{1B} K_{2A}, \\ V_f(K_{1A}, K_{2A}, 0, 0, m_1) &= \alpha^1 + \beta^1 K_{1f} + \chi^1 K_{1g} + \varphi^1 K_{1f}^2 + \gamma^1 K_{1g}^2 + \mu^1 K_{1A} K_{1B}, \end{aligned}$$

with $f, g = A, B,\ f \neq g$. The value functions in mode m_3 are completely symmetric to those in mode m_2 with reversed firm roles and $\alpha_f^3 = \alpha_g^2, \beta_f^3 = \beta_g^2, \ldots$ for $f, g = A, B,\ f \neq g$.

In order to determine the coefficients of these value functions we follow a standard approach by inserting the equilibrium investment rules and the quadratic value functions for the different modes into the HJB-equations. In particular, for the case of R&D competition the value functions for all four modes are inserted into (7) and (9) - (11), whereas for the case of R&D cooperation the value functions for modes m_1 and m_4 are inserted into (8) and (11). Comparing the coefficients of the capital stock terms of different degrees yields 41 nonlinear equations in 41 unknows under R&D competition and 21 nonlinear equations in 21 unknowns under R&D cooperation. Solving these systems is facilitated by the fact that they can be solved recursively, starting with mode

m_4 and then proceeding to modes m_2 and m_1. In all results to be presented below it has been checked that (globally) stable steady states exist in all modes, which implies that transversality conditions are satisfied.

5 Economic Analysis

In what follows we address the economic questions posed in the Introduction by carrying out a numerical analysis. We depart from the following default set of parameters

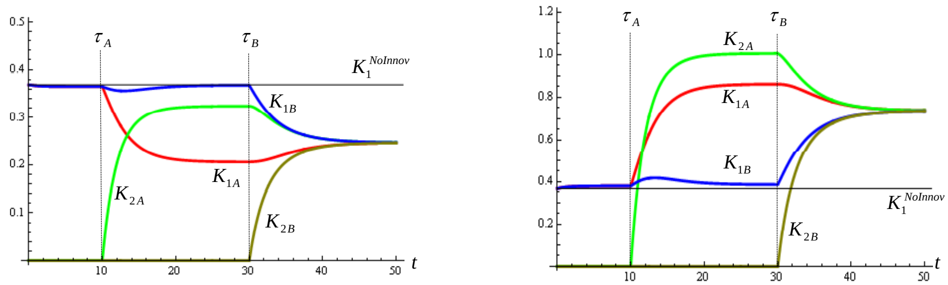
$$\lambda^{comp} = 0.05, \lambda^{coop} = 0.1, \theta = 0, b_1 = b_2 = 0, c = 5, r = 0.04,$$

where we will present robustness checks of the qualitative findings with respect to variations of these parameters at the end of the section. Without loss of generality we always assume that the price of capital is normalized to zero, i.e. $b_i = 0$. In the default setting we have $\lambda^{coop} = 2\lambda^{comp}$, which means that we consider the case without R&D synergies. Results differ qualitatively between the cases where the new product is a substitute or a complement to the established product. Hence, we always separately discuss results for positive ($\eta = 0.5$) and negative ($\eta = -0.5$) values of the degree of horizontal differentiation. In the default setting we consider the case where the new product is of equal quality to the established one ($\theta = 0$), in subsection 5.4 we show that results change only marginally if the new product is assumed to be also vertically differentiated.

5.1 Capacity dynamics

In all the analyses, the initial levels of the capital stocks of the established product are set to the steady state levels arising in the Markov-perfect equilibrium of the standard capital accumulation game where both firms produce only the established product forever. In some of our discussion below we will refer to this capacity level as the no-innovation benchmark. Setting such initial conditions can be interpreted in a way that before the start of the R&D project(s) firms ignored the possibility of new products being introduced into the market.

Figure 2 shows the equilibrium dynamics of the four capital stocks with R&D competition under the assumptions that innovations occur exactly at their expected time and that firm A is the first innovator. After the new product is introduced by firm A, the investment patterns differ substantially between the cases of substitutes and complements. If the new product is a substitute the innovator reduces investment in the established product. Also the innovation laggard does so directly after the introduction of the new product. This is caused by the increased competition on the market, since the introduction of the new product reduces the price of the established product for a given quantity. The innovator has an additional incentive to reduce quantities of the established product, because this increases the price of the new product. Once the new product is also introduced by firm B ($t > \tau_B$) capital stocks quickly converge to symmetric steady state. Due to the absence of vertical differentiation ($\theta = 0$)



(a) (b)

Figure 2: Equilibrium dynamics of the four capital stocks under R&D competition (a) if the new product is a substitute, (b) if the new product is a complement of the established product.

the steady state levels of both products coincide. It should be noted that the dynamics of the capital stocks of firm A have a non-monotone pattern. During the period where this firm is the only producer of the new product ($\tau_A < t < \tau_B$), it increases its capacity for the new product and decreases that of the established product. After the introduction of the new product by firm B, exactly the opposite dynamics arise. Firm A reduces K_{2A} and increases K_{1A}. This is a reaction to i) the increase in new product capacity of its competitor, which reduces the price of the new product, and ii) the decrease of K_{1B}, which provides incentives for firm A to expand capacity on the established market. In this way the sequential introduction of the new product by the two firms creates an overshooting pattern of production capacities of firm A.

In the case of complements, the introduction of the new product triggers an increase in the capacity investments of the established products. This is because the introduction of the new product creates additional demand for the established product. The symmetric steady state reached after the introduction of the new product by firm B in this case induces a larger capacity for the established product than in the no-innovation benchmark. Like in the case of substitutes, overshooting occurs with respect to the capital stocks of the innovator firm A. In the complements case the capacities for both products increase between τ_A and τ_B and then decrease after the introduction of the new product by firm B.

In addition to these general patterns two observations should be stressed, which genuinely rely on the intertemporal considerations of the firms. These two effects are illustrated in figure 3, where the dynamics of the capacity and the instantaneous payoff of the laggard firm B in modes m_1 and m_2 are shown, i.e. for all $t < \tau_B$. First, it can be seen that starting the R&D projects influences the capacity levels for the established product even before the new product is introduced. We refer to this as the *anticipation effect.* In particular, if the anticipated new product is a substitute the firm invests less. The reason is, that the firm takes into account that after the introduction of the new product the revenue generated by one unit of capital stock of the established product will decrease. This reduces the value of the capital stock already before the new product arrives. Hence, the firms invest less in mode m_1 compared to the no-innovation benchmark. Due to this underinvestment the price of the established product moves closer to the monopoly price and the instantaneous payoffs of both firms go up compared to the no-innovation benchmark[6].

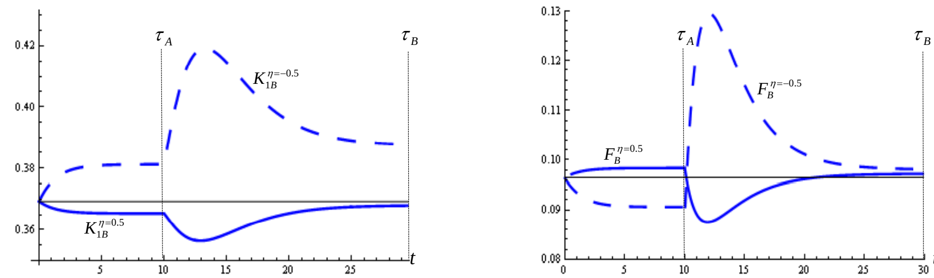
If the new product is a complement for the established product, then its anticipated introduction has a positive effect on the value of the established product's capital stock. Therefore, firms invest more (prior to the product introduction) compared to the no-innovation benchmark. This leads to lower prices and lower instantaneous payoffs.

The second dynamic effect arises in mode m_2. As can be seen in panel (b) of figure 3 the instantaneous payoffs of firm B in mode m_2, where firm A (but not firm B) already offers the new product, settle down at a level above the no-innovation benchmark. This holds true both for the cases of substitutes and complements. Whereas it is intuitively understandable that the introduction of a complementary product also increases the payoffs of the laggard, this observation is at first sight counter-intuitive for the substitute case. This result is driven by an intertemporal strategic incentive emerging from the feedback structure of the Markov-perfect equilibrium. Firm B takes into account that an increase in K_{1B} induces a reduction of the future investments of firm A in both of its capital stocks. The resulting reduction in K_{1A} and K_{2A} leads to an increase in the price of the established product, which increases the payoff of firm B. The fact that firm A has two capital stocks makes the intertemporal strategic incentives for firm A smaller and for firm B larger compared to the no-innovation benchmark. This asymmetry generates the relatively high payoffs of the laggard (see also Dawid et al. (2010)).

Having discussed the main properties of the dynamics emerging under R&D competition we now turn to the case of R&D cooperation. Since both firms introduce the new product at the same time τ^{coop}, the trajectories of both capital stocks are symmetric across firms. Figure 4 shows the developments of both capital stocks over time. Overall, it can be seen that after the introduction of the new product both capital stocks quickly converge to the steady-state of mode m_4.

Comparing the dynamics with that under R&D competition we see that the size of the anticipation effect in mode m_1 is larger under R&D cooperation than under competition. The reason is that under R&D cooperation the change of the value of the established product's capital stock due to the anticipated innovation is larger. In the case of substitutes we get a larger reduction in the value for the

[6] Although we only depict capacity and payoffs of firm B here, identical statements hold for firm A, because in mode m_1 the two firms act completely symmetrically.



(a) (b)

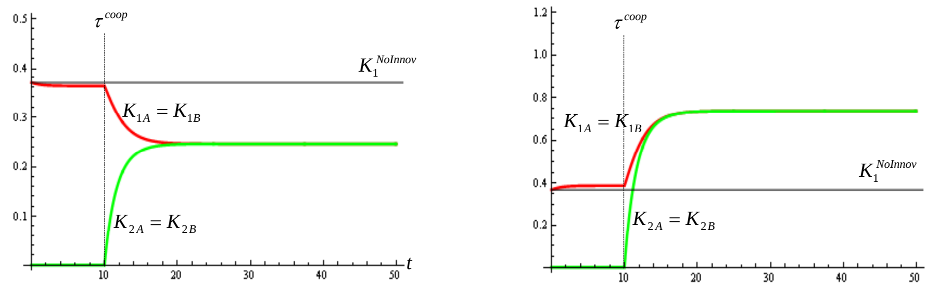
Figure 3: Equilibrium dynamics of the capital stock on the established market (a) and instantaneous payoffs for firm B (b) under R&D competition (solid line: substitutes; dashed line: complements).

following two reasons. On the one hand, under R&D cooperation both firms introduce the new product at τ^{coop} which implies a stronger price decrease for the established product compared to R&D competition, where only one firm introduces at the time when the new product first reaches the market. On the other hand, under R&D cooperation for each firm the expected time until it introduces the new product is lower than under competition. Once the firm offers both products, the value of the established product's capital stock further decreases due to a cannibalization effect. Analogous arguments show that in the case of complements the overinvestment is larger under R&D cooperation because the value of the established product's capital stock increases more under R&D cooperation compared to R&D competition. It should be noted that the presence of synergies enhances the difference in the size of the anticipation effect between the cases of R&D cooperation and R&D competition, because the expected time to the new product introduction is then shorter under R&D cooperation.

Also after the introduction of the new products there are qualitative differences in the dynamics between the two R&D scenarios. Contrary to the case of R&D competition, under cooperation there is no overshooting of capacities and all capital stocks approach the long run steady state monotonously. Consequently, total capital adjustments costs are smaller under R&D cooperation.

5.2 Incentives for R&D cooperation

In order to examine the incentives of firms to engage in R&D cooperation, we consider in Figure 5 the relative difference of the value functions of the firms under R&D competition and under R&D cooperation evaluated at the initial capital stocks. It can be clearly seen that in the absence of synergies there is



(a) (b)

Figure 4: Equilibrium dynamics of the capital stocks on both markets under R&D cooperation for the case of substitutes (a) and complements (b).

no incentive to cooperate for almost all values of the horizontal differentiation parameter η. This is quite remarkable given that we do not assume that cooperation induces any explicit costs for the firms. Furthermore, it can be seen that the relative advantage of R&D competition crucially depends on the degree of horizontal differentiation. In particular R&D cooperation is attractive if the new product is either a close substitute or a strong complement to the established product.

To explain these observations three effects have to be considered. First, as already discussed above, the *anticipation effect* is stronger under cooperation than under competition. For complements this results in lower payoffs under cooperation in mode m_1, whereas for substitutes it is the other way round. As can be seen in panel (a) of Figure 6, where the established product's capacity in the steady state of mode m_1 is depicted, the size of the anticipation effect and of the resulting difference between R&D cooperation and competition is larger for complements than for substitutes and increases substantially as η approaches -1. This is quite intuitive, because the anticipation of a strong complement to the established product induces an increased price expectation for the established product. Therefore, the value of the capacity of that product increases more substantially leading to stronger overinvestment.

Second, an important implication of R&D cooperation is that the introduction of the new product is synchronized among the two firms. We refer to this as the *synchronization effect*. It implies that immediately after the innovation there is duopolistic competition also on the market for the new product. Under R&D competition there is always a time interval, where only one firm is active on the new market. During this interval the innovator is gaining a relatively high payoff due to its market power with respect to the new product. The laggard, however, is worse off than it would be in a situation where both firms introduce the new product simultaneously. Ex-ante firms do not know which

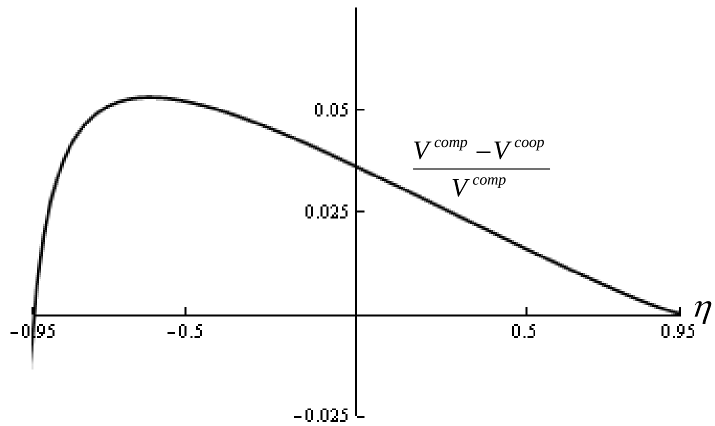
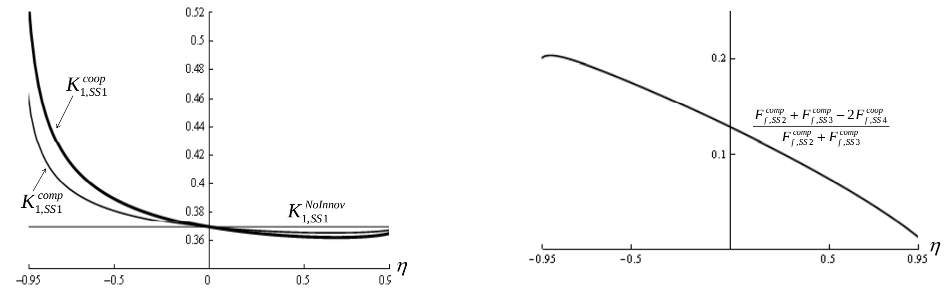


Figure 5: The relative difference in game values between R&D competition and R&D cooperation when no synergies from cooperation are present .

will be the first innovator, however the expected market profit (net of capital adjustment costs) under R&D competition in the time interval between the two innovation times is higher than in the duopolistic scenario under R&D cooperation. This is an implication of the fact that industry profits are larger under monopoly than under duopoly. Panel (b) of figure 6 illustrates this observation. It compares the average instantaneous payoff of a firm in the steady states of modes m_2 and m_3 with its payoff in the steady state of mode m_4. It should be noted that in the steady state no capacity investments occur, which means that investment costs are irrelevant. It can be clearly seen that in the considered range of the degree of horizontal differentiation indeed the average payoff in the steady states of the asymmetric modes under R&D competition is larger than the steady state payoff in mode m_4, where both firms offer the new product. Furthermore, the relative difference increases as η becomes smaller.

The third effect is denoted as the *adjustment cost effect*. It arises from the difference in the capital adjustment patterns between the two R&D scenarios. As discussed above, overshooting of the capacities on both markets of the innovation leader arises under R&D competition, whereas no such phenomenon occurs under R&D cooperation. For this reason expected capital adjustment costs are larger in the R&D competition case. Furthermore, it follows from the discussion above that this effect is more pronounced if the new product is a (strong) complement of the established product.



(a) (b)

Figure 6: (a) Capital stocks in the steady of mode m_1 under R&D cooperation (bold) and R&D competition (solid); (b) Difference between average instantaneous payoffs in the steady state of modes m_2/m_3 under R&D competition and the steady state of mode m_4 under R&D cooperation.

Considering figure 5 we conclude that the synchronization effect (plus the anticipation effect in case of complements), which favors R&D competition, is dominant. Avoiding synchronization of the time of the introduction of the new product enhances expected market profits to a sufficient degree so that the extra adjustment costs are more than compensated. Consequently, firms should decide for R&D competition in the absence of synergies.

5.3 Impact of R&D synergies

In the discussion so far it was assumed that R&D cooperation does not generate any synergies in the sense that the expected time until the new product can be first introduced to the market is the same under R&D cooperation and R&D competition. As mentioned in the Introduction, empirical work suggests that cooperation between different firms in the market might lead to synergies in the R&D activities of these firms. In our model positive synergies are captured by assuming that $\lambda^{coop} > 2\lambda^{comp}$ holds. In the case of such positive synergies, a fourth effect is added to the three effects just discussed. Under synergies the expected time span until the first introduction of the new product is shorter under R&D cooperation and hence firms expect the corresponding increase in their instantaneous payoffs to occur earlier. This makes R&D cooperation more attractive. We refer to this effect as the *synergy effect*.

To explore this issue more deeply, we define by λ^* the minimal value of the innovation arrival rate such that the firms' value functions under R&D cooperation exceed that of R&D competition for all $\lambda^{coop} \geq \lambda^*$. In figure 7 this threshold value is depicted for varying degrees of horizontal differentiation and compared to the value $2\lambda^{comp}$ corresponding to no synergies. Consistent with our discussion above, we observe that for all values of η apart from the

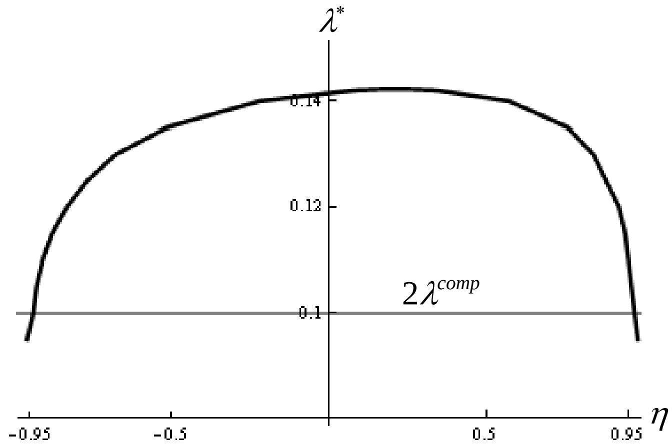


Figure 7: The threshold value λ^* for varying degree of horizontal differentiation
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extreme degrees of differentiation positive synergies are needed to make R&D cooperation attractive. Since the payoff increase resulting from the introduction of the new product is larger for complements than for substitutes, the synergy effect is more pronounced for negative values of η. Hence, as confirmed in the figure, particularly high synergies are needed if the two products are (not too strong) substitutes. The reason that only weak synergies are needed to make R&D cooperation attractive in the case of close substitutes is that in such a scenario the synchronization effect becomes very small. The sole producer of the new product has very little market power because the new product is almost homogeneous to the established one. The relatively low values of λ^* for values of η close to -1 are due to the strength of the adjustment cost effect in the case of strong complements.

5.4 Robustness

Our discussion of the different qualitative effects resulting from the choice between R&D cooperation and R&D competition has been based on a particular parameter constellation. It is important to check robustness of the qualitative statements with respect to variations of the key parameters of the model. Such a sensitivity check has been carried out and to illustrate the robustness of our results we depict in figure 8 the changes in the λ^* curve if the discount rate, the

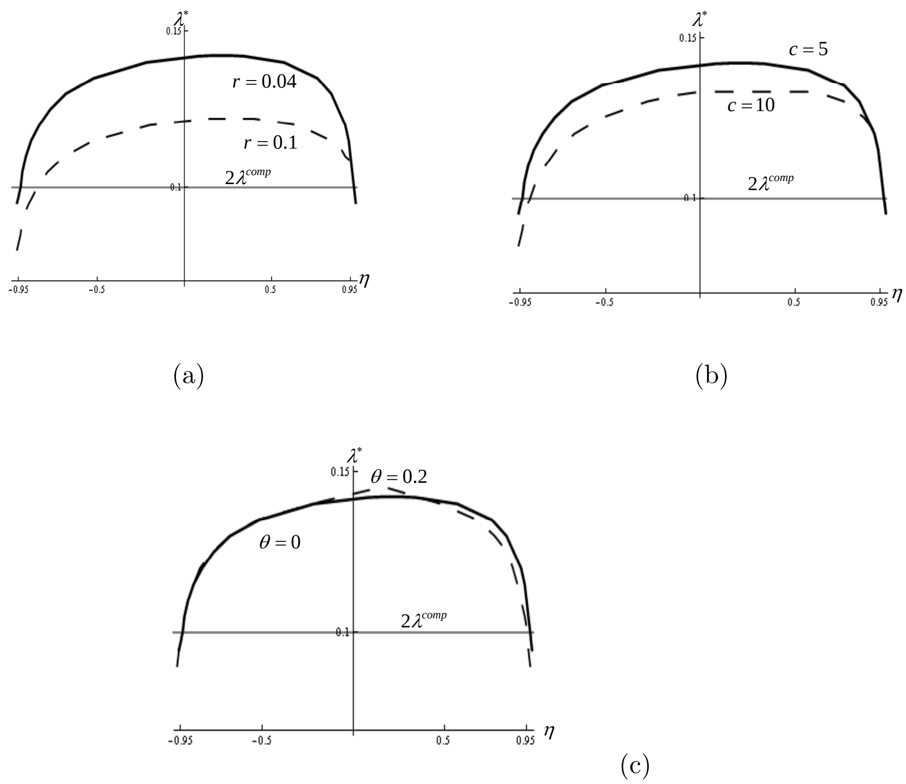

Figure 8: Effect of the changes in the discount rate (a), the capital adjustment cost parameter (b) and the vertical differentiation parameter (c) on the arrival rate threshold λ^*.

capital adjustment cost parameter and the degree of vertical differentiation is varied.

It can be seen that the qualitative features of this curve, in particular the inverse U-shape and the requirement for positive synergies for almost the entire range of η values, stay intact. The fact that higher discounting moves the curve downwards can be explained by the increased relevance of the synergy effect. The expected reduction of the arrival time of the innovation caused by an increase of the arrival rate is valued more if the discount rate is higher. Hence, a smaller advantage with respect to the arrival rate is needed to make R&D cooperation as attractive as competition. Also an increase in the capital adjustment costs leads to a downward shift of the curve, because the adjustment cost effect, which favors R&D cooperation, becomes more pronounced. Finally, it can be seen that an increase in the level of vertical differentiation of the new product does not seriously affect the threshold value λ^*.

5.5 Welfare Effects

To conclude our analysis we briefly consider the welfare[7] implications of R&D cooperation. Intuition suggests that at any point in time in mode m_1, prior to the introduction of the new product, an increase of the total quantity of the established product, relative to the no-innovation benchmark, is welfare enhancing. The deviation of the output from the no-innovation benchmark in m_1 stems from the anticipation effect and, as discussed above, this effect is stronger under R&D cooperation than under R&D competition. If the new product is a substitute of the established one, this reasoning implies that R&D cooperation has a negative effect on (instantaneous) welfare, whereas R&D cooperation is welfare enhancing if the new product is a complement. Considering the modes after the innovation, it is easy to see that welfare in modes m_2 and m_3, where only one firm offers both products, is smaller than in mode m_4, where both firms offer both products. This is due to larger total output and smaller capacity adjustment costs in mode m_4. Since R&D cooperation leads to a direct transition from m_1 to m_4, (instantaneous) welfare after the introduction of the new product is always larger under R&D cooperation than under R&D competition. These qualitative statements have been confirmed by numerical analysis. This analysis also shows that for the parameter settings considered above, in the case of substitutes the negative welfare effects of R&D cooperation prior to innovation is dominated by the positive effects after innovation such that also in the absence of synergies the discounted welfare stream under R&D cooperation is larger than under competition regardless of the degree of differentiation. In case of complements R&D cooperation increases welfare before and after the innovation. Therefore, also in the complements case the effect of R&D cooperation on the discounted welfare stream is clearly positive.

6 Conclusions

This paper analyzes the interplay between capacity dynamics and technological progress in an oligopolistic market. We focus on product innovation and study how anticipated and actual introduction of new products influence market dynamics. In particular, we characterize the implications of R&D cooperations in product innovations on the investment behavior of established firms in an industry. We identify three effects which influence the relative profitability of R&D cooperation: the anticipation effect, the synchronization effect, and the adjustment cost effect. We show that the sign and the size of these effects depend on the degree of product differentiation. We find that, although R&D competition is associated with overshooting of the capacity dynamics for both products and, in case of substitutes also leads to smaller firm profits than R&D cooperation prior to the innovation date, still firms prefer not to to cooporate in R&D in the absence of sufficiently strong synergy effects. Given that R&D cooperation is preferred from a welfare perspective, these insights call for policy

[7] As usual we define welfare as the sum of total industry profit and consumer surplus.

measures stimulating R&D cooperation.

In order to concentrate the analysis on capacity investment dynamics we have assumed in this paper that innovation arrival rates are exogenously given. Therefore, an important extension is to introduce knowledge stocks of the firms into the analysis, which can be increased by R&D investments, and to assume that arrival rates depend on knowledge stocks (see e.g. Doraszelski (2003)). Since such a formulation leads to a differential game which does not have a linear quadratic structure, numerical methods like collocation will have to be employed in such an analysis. An important issue that can be addressed in such a setting is the impact of market share on the established market on the incentives to invest in product innovation.

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