Optimal Policy and the Role of Social Contacts in a Search Model with Heterogeneous Workers

Yuliia Stupnytska, Anna Zaharieva

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Abstract

This paper develops a search model with heterogeneous workers and social networks. High ability workers are more productive and have a larger number of professional contacts. Firms have a choice between a high cost vacancy in the regular labour market and a low cost job opening in the referral market. In this setting the model predicts that a larger number of social contacts is associated with a larger wage gap between high and low ability workers and a larger difference in the equilibrium unemployment rates. Next we demonstrate that the decentralized equilibrium is inefficient for any value of the bargaining power. There are two reasons for the inefficiency. First, the private gain from creating a job in the referral market is always below the social gain, so the equilibrium unemployment of high ability workers is above its optimal value. Moreover, high ability workers congest the market for low ability workers, so the equilibrium wage inequality is inefficiently large. This is in contrast to the result of Blazquez and Jansen (2008) showing that the distribution of wages is compressed in a search model with heterogeneous workers. Finally, we show that a combination of taxes and subsidies can restore the optimal allocation.

Keywords: social capital, social networks, referrals, wage dispersion, wage compression

JEL Classification: J23, J31, J38, J64

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1 Introduction

The purpose of this paper is to investigate social welfare and optimal policy in a search model with heterogeneous workers. In our model workers differ with respect to their productivity (high and low ability workers) and the structure of social networks, in particular, there is a positive correlation between ability and the number of professional contacts. In this setting, when both types of workers are mixed in the regular labour market, the decentralized equilibrium is inefficient as high ability workers congest the market for workers with low abilities. Moreover, this inefficiency is increasing in the number of social contacts and is associated with a larger wage gap between the two groups of workers. This finding questions the traditional view that social contacts increase efficiency by mitigating the problem of adverse selection (see Montgomery (1991)). It is also different from the literature on heterogeneous worker groups where wages are generally compressed when two types of workers are simultaneously searching for jobs in the same labour market (see Blazquez and Jansen (2008)).

There is strong empirical evidence that $30 - 60\%$ of new hires find jobs through personal contacts (see for example Staiger (1990), Granovetter (1995), Pistaferri (1999), Kugler (2003), Pelizarri (2010), Bentolila et. al. (2010) for different countries). In addition, Hensvik and Skans (2013) report that incumbent workers with a high test score are more likely to be linked to the new hires than low ability employees. In particular, in their data firms rely on referrals from high-ability workers in order to attract applicants with higher unobserved ability. To incorporate these empirical findings into the model and keep it tractable we assume that only high ability workers are linked in a network and have the same exogenous number of professional contacts who can give a reference for the job. In contrast, low ability workers are restricted to search for jobs in the regular labour market. Therefore there is a tight connection in the model between the productivity of the worker and the amount of social capital.

The choice of search methods by firms is endogenous. When deciding to enter the labour market, firms face a trade-off between a high cost vacancy in the regular job market and a low cost job opening in the referral market. The pool of job applicants in the regular labour market is mixed as both types of unemployed workers apply for the publicly advertised positions. On the contrary, the pool of applicants in the referral market is limited to unemployed workers with high ability as only these workers are connected in a network. Therefore, the cost difference between the markets is explained by the necessity to screen the applicants. This assumption is in line with the original idea of Montgomery (1991) that workers hired through social networks are, on average, more productive and the formal channel of search is more expensive for firms.

Due to screening, firms in the regular market are informed about the productivity of the applicant. Thus it is natural to assume that wages are negotiated ex-post between the firm and the applicant by means of Nash-bargaining. Equilibrium is determined by two free-entry conditions in the regular and the referral market. Depending on the parameters, there are two types of equilibria. If the number of social contacts is low it is not optimal for firms to rely solely on referrals. Therefore there exists a unique equilibrium without referrals where both types of workers are mixed in the regular labour market. In contrast, if the number of social contacts is sufficiently large, then some firms save on the screening cost and use referrals in the hiring process. This is the equilibrium with two channels of job search.

High-ability workers are better paid than low ability workers. On the one hand high ability workers are more productive which leads to higher wages. On the other hand, their reservation wages are high due to the additional possibility of finding jobs through the network of contacts. In this setting, the model predicts that a larger number of social contacts puts an upward pressure on wages of high ability workers and reduces the equilibrium unemployment of these workers. Low ability workers are negatively effected: their wages fall and the unemployment rate is increased. This implies that a more intensive use of referrals is associated with an increased wage dispersion between the two groups of workers. Thus a more important role of social networks in the modern society provides an additional explanation for the increased income inequality in the United States in the recent decade. Next our model predicts that the decision of firms to use referrals may be inefficient from a social perspective. The job-filling rate in the referral market does not depend on the number of other informal vacancies in this market. It is rather that the hiring probability depends on the architecture of the social network. So firms hiring through referrals do not impose a negative search externality on other firms which is the case in the regular market. From a social perspective this means that vacancies in the referral market should be created up to the point where the expected cost of an open position is equal to the expected surplus of a filled job. In contrast, in the decentralized equilibrium firms start using referrals at the point where the expected cost is equal to the expected profit. This means that the optimal threshold number of contacts which is necessary for firms to use referrals is lower than the minimum number of contacts in the decentralized economy. In the paper we show that this inefficiency may be mitigated by means of employment subsidies in the referral market. In reality such subsidies can take the form of referral bonuses which are reimbursed by the state.

Finally, we identify a pooling inefficiency in the regular labour market. High ability workers are more productive but they also bargain a higher wage. Which of these two effects is dominating for profits strongly depends on the productivity difference between workers and the number of social contacts. If the productivity (wage) effect is dominating then the expected profit of firms in the regular market is increasing (decreasing) in the proportion of high ability workers. In a numerical example we show that the effect of higher wages is dominating already for a small number of social contacts and therefore high ability workers impose a negative externality on low ability workers. This effect generates an equilibrium wage dispersion which is inefficiently large. The optimal policy in this case is associated with increasing (decreasing) the reservation wage of low (high) ability workers and reducing the equilibrium inequality in wages. Moreover, we show that this pooling inefficiency is an artefact of referrals and does not exist in the labour market without social contacts.

This paper is closely related to the literature on heterogeneous workers and social networks. Albrecht and Vroman (2002) is the first study analysing an economy with skill differences across workers and varying skill requirements of firms. Gautier (2002) extends their framework to allow for on-the-job search and Blazquez and Jansen (2008) analyse welfare in a model economy of Albrecht and Vroman (2002). Our comparative static result is similar to the one reported in Gautier (2002), namely that mixing two types of workers in the same labour market may congest the market for low ability workers and their unemployment may increase with a higher proportion of high ability workers. One deviation is that we do not consider the possibility of

on-the-job search, instead the focus of our paper is on social welfare and policy, these issues are not addressed in Gautier (2002).

From the perspective of welfare our paper is close to Blazquez and Jansen (2008). They find that wage bargaining when agents are matched at random compresses the wage distribution relative to workers' shadow values and doesn't lead to the efficient outcome. This means that low (high) ability workers receive more (less) in the decentralized equilibrium than in the socially efficient allocation. We show that this situation is a special case of our model when productivity differences are large and the number of social contacts is small. However, when the number of social contacts is moderate the direction of inefficiency is reversed and the gap in wages is inefficiently large. This is the situation we report in a benchmark model of our paper.

Other related studies include Sattinger (1995) and Shimer and Smith (2001). The purpose of the former study is to show that the uncoordinated matching decisions of agents may give rise to multiple equilibria with inefficient matching sets. Shimer and Smith (2001) reach a stronger conclusion. They consider a model with endogenous search intensity decisions and show that the equilibrium is never efficient because bargaining distorts the marginal returns from search. Finally, Shi (2002) and Shimer (2005a) also develop a model of directed search in which jobs attract applications from several types of workers. However, in their models the allocation is always efficient because firms can commit to the posted wage offers.

From the perspective of social networks our study is most closely related to Cahuc and Fontaine (2009) and Zaharieva (2013). Cahuc and Fontaine (2009) restrict workers to choose between the two job search methods, so in their model, decentralized decisions by workers and firms to use networks can suffer from a coordination failure. On the contrary, the choice of search methods is not limited in our model and thus both search channels are simultaneously used by high ability workers to find a job. Zaharieva (2013) considers a matching model with family networks and wage posting and examines welfare in this model. Wage posting and directed search lead to the ex-ante separation of unemployed workers in the regular labour market. Consequently the decentralised equilibrium is constrained efficient.

Early economic studies on social contacts include Montgomery (1991, 1992, 1994) and Mortensen and Vishwanath (1994). The focus of Montgomery (1991) is on the effect of asymmetric information on wage inequality in the presence of the "inbreeding bias", implying clustering of workers with respect to their ability type. As a result the equilibrium is characterized by the positive correlation between ability and wages. Mortensen and Vishwanath (1994) consider the population of workers differing with respect to the probability of receiving job offers through personal contacts, they show that wages paid in jobs obtained through personal contacts are more likely to be higher than wage offers obtained through a direct application. This conclusion is questioned in the recent empirical literature, and moreover, "both the models of Montgomery (1991) and Mortensen and Vishwanath (1994) ignore what may be the most important role for network: to increase the job offer arrival rate."(p. 7, Margolis and Simonnet (2003)).

Recent theoretical studies emphasizing the positive effect of referrals on wages include Kugler (2003) and Galenianos (2012). Specifically, Kugler (2003) finds that the benefit of using referrals for firms is that they lower monitoring costs, because workers can exert peer pressure on co-workers. As a result, firms relying on referrals find it cheaper to elicit effort by paying efficiency wages than firms using formal hiring methods. Galenianos (2012) extends the original idea of

Montgomery (1991) and shows a positive link between the intensity of referrals and the job finding rate. Other studies investigating the link between referrals and the job-finding rate are Calvo-Armengol and Jackson (2004, 2007) as well as Fontaine (2004, 2007, 2008). A larger overview of this literature can be found in Ioannides and Datcher Loury (2004).

The plan of the paper is as follows. Section 2 explains notation and the general economic environment. Section 3 deals with the existence of the decentralized equilibrium. Section 4 contains welfare analysis of the decentralized equilibrium. Section 5 illustrates our theoretical results by means of a numerical example, while section 6 concludes the paper.

2 Labour market modeling framework

The labour market is characterized by the following properties. There is a unit mass of infinitely lived risk neutral workers and an endogenous number of firms, both workers and firms discount the future at rate r. Workers are ex-ante heterogeneous with respect to their ability and social capital. Let μ denote the fraction of low ability workers, once employed these workers produce the flow output y_0. The fraction of high ability workers is $1-\mu$, these workers are more productive and generate the flow output $y_1 > y_0$ when employed. To simplify the model we assume that low ability workers do not have professional contacts, in contrast high ability workers have an equal number of professional contacts $l > 0$. This assumption reflects the positive correlation between worker's ability and the number of contacts in the labour market who are willing to refer this worker to the potential employer. Workers can be either employed and producing output or unemployed and searching for a job. Let u_1 and u_0 denote the total numbers of unemployed workers with high and low ability, so that $u \equiv u_0 + u_1$. Unemployed workers enjoy the flow value of leisure z.

Every firm entering the labour market can choose between a high cost vacancy in the regular submarket and a low cost job opening in the referral submarket. The corresponding flow costs of open vacancies are denoted by $c + p$ and c. The flow cost p here is an additional screening cost in the regular labour market. Since the outcome of screening is a perfect signal about the applicant's productivity there is no problem of asymmetric information in the model.

Further, let v and v_2 be the numbers of vacancies in the two submarkets respectively. Due to the absence of social contacts workers with low ability are restricted to search in the regular job market. On the contrary, high ability workers can simultaneously search in both submarkets. This means that high and low ability workers are mixed in the regular job market so that firms don't know the type of a worker until the meeting takes place. The matching function in the regular job market is then given by $m(u, v)$, and the market tightness is $\theta = v/u$. This matching function is assumed to be increasing in both arguments, unemployment and vacancies, concave, and exhibiting constant returns to scale. Therefore, the job finding rate $\lambda(\theta)$ and the vacancy filling rate $q(\theta)$ in the regular job market are given by:

$$q(\theta) = m(u, v)/v = m\theta^{-\eta} \qquad \lambda(\theta) = \theta q(\theta) = m\theta^{1-\eta}$$

where $0 < \eta < 1$ is the elasticity of the job filling rate.

Matching function in the referral job market is based on a simplified urn-ball matching process

(e.g. Albrecht, Gautier, and Vroman (2003), Cahuc and Fontaine (2009)), in which every high ability worker has an urn and every job offer is a ball which is sent into urns. First, firms with open vacancies contact high ability employees at rate a per unit time. Second, every employee transmits vacancy information to exactly one randomly chosen unemployed social contact out of a pool of l contacts. Here we assume that job information is only transmitted to the direct social links, so the job offer is lost if all l contacts are employed. The matching function in the referral job market is therefore equal to $m_1(u_1, v_2) = av_2[1 - (1 - \frac{u_1}{1-\mu})^l]$. The term in brackets is the probability to meet an employee with at least one unemployed social contact (as $(1 - \frac{u_1}{1-\mu})^l$ is the probability that all l contacts are employed). Therefore this matching function can be understood as the number of vacancies in the referral job market sent to the employees with at least one unemployed contact at rate a. The job finding rate λ_2 and the vacancy filling rate q_2 in the referral job market are given by:

$$q_2 = m_1(u_1, v_2)/v_2 = a[1 - (1 - \frac{u_1}{1 - \mu})^l] \qquad \lambda_2 = m_1(u_1, v_2)/u_1 = av_2[1 - (1 - \frac{u_1}{1 - \mu})^l]/u_1$$

Any job can be destroyed for exogenous reasons with a Poisson destruction rate δ. Upon a separation the worker becomes unemployed and the firm may open a new job.

3 The decentralized equilibrium

3.1 Bellman equations

Let U_i, $i = 0, 1$ denote the present values of being unemployed and, similarly, W_i, $i = 0, 1, 2$ – the present values of being employed. The subindex 0 refers to low ability workers. The subindex 1 refers to high ability workers obtaining jobs in the regular market, while the subindex 2 stands for the present values of workers finding jobs in the referral market. The structure of the labour market is illustrated in figure 1. In addition, let variables τ_0 and τ_1 denote the flow values of transfers that unemployed workers receive from the public budget. The present values U_0 and U_1 for the unemployed can be written as:

$$rU_0 = z + \tau_0 + \lambda(\theta)(W_0 - U_0) \qquad rU_1 = z + \tau_1 + \lambda(\theta)(W_1 - U_1) + \lambda_2(W_2 - U_1) \qquad (1)$$

where the latter equation incorporates the fact that high ability workers can simultaneously search for jobs in both submarkets. The present values W_i for the employed are given by:

$$rW_0 = w_0 - \delta(W_0 - U_0) \qquad rW_i = w_i - \delta(W_i - U_1), \quad i = 1, 2 \qquad (2)$$

Next consider firms and let J_i, $i = 0, 1, 2$ denote the present values of profits. Bellman equations for filled jobs are then given by:

$$rJ_i = y_i - w_i - \delta J_i, \quad i = 0, 1 \qquad rJ_2 = y_1 - w_2 - \delta J_2 \qquad (3)$$

Further, we describe firms with open vacancies. In the regular labour market, let $\gamma = u_0/u$ denote the probability for the firm to meet a low ability worker, so that $1 - \gamma = u_1/u$ is the probability to meet a high ability worker. Besides, let s denote the flow values of transfers that
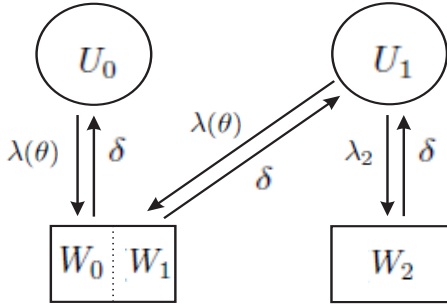
Figure 1: Labour market flows

firms in the referral market obtain from the public budget. For example, these transfers can cover the traveling expenses of job applicants and the costs of accommodation at the place of the job interview. In the next section we consider the optimal policy of the social planner, so the vector of policy instruments $\{\tau_0, \tau_1, s\}$ will allow the social planner to affect wages and the job creation. The present values of open vacancies V and V_2 in the regular and the referral market respectively can then be written as:

$$rV = -(c+p) + q(\theta)(\gamma J_0 + (1-\gamma)J_1 - V) \qquad rV_2 = -c + s + q_2(J_2 - V_2) \qquad (4)$$

where the term $\gamma J_0 + (1-\gamma)J_1$ is the expected present value of firm profits in the regular labour market. In the following we investigate the economy in the steady state. Hence, the equilibrium unemployment for both types of workers reads:

$$u_0 \lambda(\theta) = \delta(\mu - u_0) \qquad u_1(\lambda(\theta) + \lambda_2) = \delta(1 - \mu - u_1) \qquad (5)$$

Each of these equations implies that the inflow of workers into unemployment (on the right-hand side) is equal to the outflow of workers from this state (on the left-hand side). It is easy to see therefore that u_0 decreases in θ and that workers with low ability face a higher equilibrium unemployment rate: $u_0/\mu > u_1/(1-\mu)$.

The steady state conditions (5) allow us to express the equilibrium probability of being in contact with a low ability worker in the following way:

$$\gamma \equiv \frac{u_0}{u_0 + u_1} = \frac{\mu(\delta + \lambda(\theta) + \lambda_2)}{\delta + \lambda(\theta) + \mu\lambda_2} = \frac{\delta\mu}{\delta\mu + (\delta + \lambda(\theta))u_1}$$

This means that $\gamma(\theta, u_1)$ is decreasing in both θ and u_1. Intuitively, a higher market tightness θ reduces the equilibrium unemployment of low ability workers u_0, so the probability that a randomly chosen applicant is of low ability is decreasing in θ. Similarly, more high-skilled unemployed workers u_1 reduce the chances of meeting a low ability worker. Finally, note that $\gamma > \mu$ in the presence of social contacts, while $\gamma = \mu$ otherwise. Networks reduce unemployment of high ability workers, so in the equilibrium with social contacts firms are less likely to meet these workers in the regular market: $(1 - \gamma) < (1 - \mu)$. Next section investigates existence and uniqueness of the decentralized equilibrium with social contacts.

3.2 Wage determination and the free-entry conditions

This section investigates the labour market without policy instruments ($\tau_0 = \tau_1 = s = 0$). Both the efficient resource allocation and the optimal policy are later addressed in section 4.

The equilibrium wages are determined by means of Nash bargaining. When bargaining over w_0 unemployed low ability workers act to maximize the total job rent $W_0 - U_0$ which is an increasing function of w_0. Similarly, unemployed high ability workers act to maximize the rent $W_i - U_1$, where the subindex i takes values 1 or 2 depending on the type of search channel. Firms are maximizing the surplus value J_i, $i = 0, 1, 2$ so the rent sharing conditions become:

$$J_0 = \frac{(1-\beta)}{\beta}(W_0 - U_0) \qquad J_i = \frac{(1-\beta)}{\beta}(W_i - U_1), \quad i = 1, 2 \tag{6}$$

where we impose the free-entry conditions $V = V_2 = 0$, therefore in the equilibrium firms are indifferent between a formal vacancy in the regular market and an informal vacancy through referrals. The corresponding wage equations are given by:

$$w_0 = \beta y_0 + (1-\beta)rU_0 \qquad w_1 = w_2 = \beta y_1 + (1-\beta)rU_1 \tag{7}$$

Denote $S_0 = J_0 + W_0 - U_0$ the total job surplus in a match between a firm and a low ability worker, similarly let $S = J_i + W_i - U_1$, $i = 1, 2$ the total job surplus in a match between a firm and a high ability worker. Note that S is independent of the search channel, so that $J_1 = J_2$ and $W_1 = W_2$. This is because bargaining is an ex-post wage setting mechanism so the sunk costs of open vacancies are not directly reflected in wages. Surplus values S_0 and S are given by the following system of equations:

$$S_0(\theta) = \frac{y_0 - z}{r + \delta + \beta\lambda(\theta)} \qquad S(u_1) = \frac{y_1 - z}{r + \delta + \beta\delta(1 - \mu - u_1)/u_1}$$

where in the last expression we make use of the steady-state condition $\lambda(\theta) + \lambda_2 = \delta(1 - \mu - u_1)/u_1$. Intuitively, a higher market tightness θ improves the outside opportunities of low ability unemployed workers $rU_0 = z + \beta\lambda(\theta)S_0$, so the total job surplus $S_0(\theta)$ is decreasing in θ. At the same time, in the equilibrium a higher number of unemployed high ability workers u_1 can only be attributed to a lower job-finding rate $\lambda(\theta) + \lambda_2$. In this latter case the reservation wage of high ability workers is also lower $rU_1 = z + \beta(\lambda(\theta) + \lambda_2)S$, hence the total job surplus $S(u_1)$ is increasing in u_1.

The free-entry conditions in each of the two submarkets are then given by:

$$\frac{c}{q_2(u_1)} = (1-\beta)S(u_1) \qquad \frac{c+p}{q(\theta)} = (1-\beta)[\gamma(\theta, u_1)S_0(\theta) + (1 - \gamma(\theta, u_1))S(u_1)]$$

Both of these equations suggest that the expected cost of an open vacancy in the equilibrium should be equal to the expected present value of profits. Consider first the referral market. Expression $c/q_2(u_1)$ is decreasing in u_1 since more high ability unemployed workers make it easier for firms to fill informal vacancies. At the same time a higher unemployment level u_1 worsens the bargaining position of workers. This leads to lower wages w_2 and higher profits $J_2 = (1-\beta)S(u_1)$ in the referral market. Consequently the free-entry condition in the referral market defines a unique equilibrium value of u_1 if $c/a < (1-\beta)(y_1 - z)/(r+\delta)$. In the following

we assume that this condition is satisfied. Now consider the regular labour market. The right hand side of the corresponding free-entry condition is an expected firm profit from an open vacancy in the regular job market. Indeed, bargaining implies that firms obtain a fraction $1 - \beta$ of the total surplus and with probability $\gamma(\theta, u_1)$ the firm is in contact with a low ability worker.

Define the equilibrium with social contacts in the following way:

Definition 1. *Search equilibrium with social contacts is a vector of variables $\{U_0, U_1, W_i, J_i, V, V_2, w_i, \theta, u_1\}$, $i = 0, 1, 2$ satisfying the asset value equations for workers (1) and (2), for firms (3) and (4), the rent-sharing equations (6) as well as the free-entry conditions $V = 0$ and $V_2 = 0$.*

Further note that existence of the equilibrium with social contacts implies

$$v_2 \geq 0 \quad \Leftrightarrow \quad \lambda_2 \geq 0 \quad \Leftrightarrow \quad \lambda(\theta) \leq \delta(1 - \mu - u_1)/u_1 \quad \text{for a given} \quad u_1$$

which imposes an upper bound on the equilibrium market tightness θ. This means that existence of the equilibrium with referrals is not always guaranteed. Our results concerning this question are summarized in proposition 1.

Proposition 1. *Define $\lambda(\bar{\theta}(u_1)) \equiv \delta(1 - \mu - u_1)/u_1$. There exists an equilibrium with referrals if the following condition is satisfied:*

$$\textbf{\textit{Condition A:}} \qquad \frac{c + p}{q(\bar{\theta}(u_1))} > (1 - \beta)\Big(\frac{\mu y_0 + (1 - \mu)y_1 - z}{r + \delta + \beta\lambda(\bar{\theta}(u_1))}\Big)$$

where u_1 is determined from the job creation condition in the referral market (JC$_2$):

$$\frac{c}{a[1 - (1 - \dfrac{u_1}{1 - \mu})^l]} = \frac{(1 - \beta)(y_1 - z)}{r + \delta + \beta(1 - \mu - u_1)\delta/u_1}$$

and $\theta \leq \bar{\theta}(u_1)$ is given by the job creation condition in the regular market (JC):

$$\frac{c + p}{q(\theta)} = (1 - \beta)\Big(\frac{\gamma(\theta, u_1)(y_0 - z)}{r + \delta + \beta\lambda(\theta)} + \frac{(1 - \gamma(\theta, u_1))(y_1 - z)}{r + \delta + \beta(1 - \mu - u_1)\delta/u_1}\Big)$$

Moreover, wage dispersion $\Delta w = w_2 - w_0 = w_1 - w_0$ is decreasing in θ and u_1.

Proof: Appendix I.

Suppose condition A is satisfied for some $l > 0$, which means there exists an equilibrium with referrals. A higher number of social contacts makes information transmission more efficient in the referral market. Therefore the equilibrium unemployment of high ability workers is unambiguously decreasing in the number of contacts. Moreover in the limiting case $l \to \infty$ the job-filling rate in the referral market approaches its upper bound $q_2 \to a$, hence u_1 asymptotically converges to its minimum value $c\beta\delta(1 - \mu)/[a(1 - \beta)(y_1 - z) - c(r + \delta(1 - \beta))]$.

In the opposite case a lower number of social contacts raises the equilibrium unemployment of high ability workers u_1. So there is a negative impact on the upper bound of the equilibrium market tightness $\bar{\theta}(u_1)$. With respect to condition A this means that the difference between the left hand side and the right hand side is diminishing with a lower number of social contacts l

9

(see figure 2). Therefore there exists a threshold value $l_0 > 0$ such that condition A is satisfied with a strict equality. This automatically implies that the equilibrium with referrals does not exist for $l \leq l_0$. These results are summarized in corollary 1:

Corollary 1: *For $l < l_0$ there exists a unique search equilibrium without referrals, where the market tightness θ^* is given by:*

$$\frac{c+p}{q(\theta^*)} = (1-\beta)\left(\frac{\mu y_0 + (1-\mu)y_1 - z}{r + \delta + \beta\lambda(\theta^*)}\right)$$

The threshold number of social contacts l_0 is then given by $\theta^* = \bar{\theta}(u_1(l_0))$.
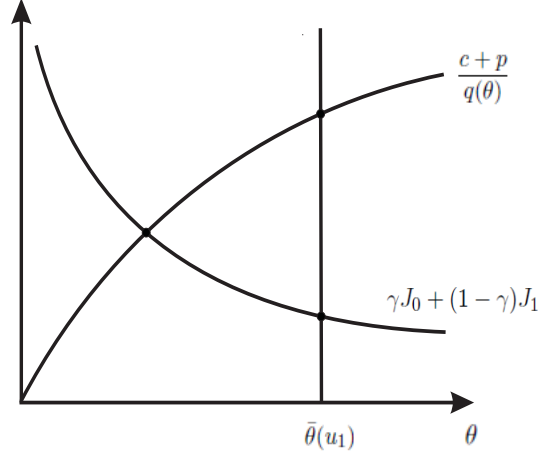

Figure 2: Existence of the decentralized equilibrium

Intuitively, if the number of social contacts is low $l \leq l_0$ it is not profitable for firms to rely solely on referrals. This means that social contacts are not valuable and wage dispersion is purely attributed to differences in the productivity: $\Delta w = \beta(y_1 - y_0)$. Moreover, the equilibrium unemployment is the same for both types of workers: $u_0/\mu = u_1/(1-\mu) = \delta/(\delta + \lambda(\theta^*))$.

4   Social optimum

This section investigates efficiency properties of the decentralized equilibrium. Consider the problem of a social planner, whose objective is to maximize the present discounted value of output minus the costs of job creation:

$$\max_{\theta, v_2} \int_0^\infty e^{-rt}((1 - \mu - u_1)y_1 + (\mu - u_0)y_0 + (z - (c+p)\theta)(u_0 + u_1) - cv_2)dt \tag{8}$$

In addition, the social planner is subject to the same matching constraints as firms and workers, therefore the dynamics of unemployment is described by the following differential equations $\dot{u}_0 = \lambda(\theta)u_0 - \delta(\mu - u_0)$ and $\dot{u}_1 = (\lambda(\theta) + \lambda_2)u_1 - \delta(1 - \mu - u_1)$. The next proposition presents solution of the planner's optimization problem.

Proposition 2. *Consider a social planner choosing the market tightness θ in the regular market and the number of vacancies v_2 in the referral market. Let $\phi = (\partial m_1(u_1, v_2)/\partial u_1)\cdot(u_1/m_1(u_1, v_2))$*

10

be the elasticity of the matching function $m_1(u_1, v_2)$. Then the optimal job creation is:

$$\frac{c+p}{q(\theta)} = (1-\eta)(\gamma k_0 + (1-\gamma)k_1) \quad and \quad \frac{c}{q_2} = k_1 \qquad (9)$$

where the costate variables k_0 and k_1 ($\Delta k = k_0 - k_1$) are obtained as:

$$k_0 = \frac{y_0 - z - \lambda(\theta)\Delta k(1-\eta)(1-\gamma)}{r+\delta+\eta\lambda(\theta)} \qquad k_1 = \frac{y_1 - z + \lambda(\theta)\Delta k(1-\eta)\gamma + (\eta-\phi)\lambda_2 k_1}{r+\delta+\eta\lambda(\theta)+\eta\lambda_2}$$

Proof: Appendix II.

The costate variables k_0 and k_1 are the shadow prices of the surplus values S_0 and S respectively. Consider first the situation when $k_0 > k_1$ which means that high ability workers create a lower job surplus than low ability workers. This situation is possible since high ability workers have an additional possibility of employment in the referral market. Hence their outside opportunities are better and their reservation wages are higher. Therefore, if $k_0 > k_1$ every additional high ability worker searching in the regular market reduces the expected profits of firms. To see this let $\bar{J} = (1-\eta)(\gamma k_0 + (1-\gamma)k_1)$ be the expected firm profit at the optimum, so that $\partial\bar{J}/\partial(1-\gamma) = -(1-\eta)(k_0-k_1) < 0$. This result implies that high ability workers impose a negative externality on low ability workers in the regular labour market. This conclusion is similar to the result by Gautier (2002) who shows that unemployment of low-skilled workers may increase if both high- and low-skilled workers are pooled in the job market for simple jobs. However, Gautier (2002) does not consider welfare and referrals in his work and so our result may serve as an extension of his finding.

Next consider the opposite case when $k_0 < k_1$ which means that high ability workers are significantly more productive and create a higher surplus than low ability workers. Then the external effect is reversed. Every additional high ability worker searching in the regular market increases the expected profits of firms and so high ability workers impose a positive externality on low ability workers. From proposition 2 the surplus difference Δk can be expressed as follows:

$$\Delta k = \frac{y_0 - y_1 + \phi c\theta_2}{r+\delta+\lambda(\theta)} \quad where \quad \theta_2 = \frac{v_2}{u_1}$$

Therefore Δk is positive if $y_1 - y_0 < c\phi\theta_2$ and negative otherwise. Intuitively, a lower difference in productivities and a larger number of social contacts (which increase the market tightness θ_2) make the first case more likely. In contrast, a large productivity difference and a low number of social contacts contribute to the occurrence of the second case. In addition, the above equation implies that $\Delta k > 0$ if productivity differences between workers are negligibly small, that is $y_0 = y_1$. This means that high ability workers unambiguously impose a negative externality on low ability workers in the regular labour market.

Finally, consider the labour market without contacts, so that $v_2 = 0$. For the traditional Hosios value of the bargaining power ($\beta = \eta$) it is than true that: $\gamma k_0 + (1-\gamma)k_1 = \gamma S_0 + (1-\gamma)S$, so the externality is neutralized and the market tightness θ coincides with the optimal choice of the social planner. If $v_2 = 0$ it follows that $\Delta k < 0$ so more productive high ability workers unambiguously impose a positive externality on agents with low ability. Nevertheless, in the equilibrium both externalities offset each other as low ability workers produce less output which

explains a negative external effect on high ability workers. In a more general framework with social contacts the two externalities can be internalized by means of an optimal redistribution policy, which is characterized in the proposition below:

Proposition 3. *Let the Hosios condition be satisfied, so that $\beta = \eta = \phi$. For $l > l_0$ the equilibrium with social contacts is inefficient but there exists a policy scheme $\{\tau_0^*, \tau_1^*, s^*\}$ that can restore the optimal allocation:*

$$\tau_0^* = \lambda(\theta)\Delta k(1-\eta)(1-\gamma) \qquad \tau_1^* = -\lambda(\theta)\Delta k(1-\eta)\gamma \qquad s^* = \eta c$$

Proof: Appendix III.

Next observe that firms in the referral market do not impose a negative externality on other firms because q_2 does not depend on v_2. Indeed a new job opening in the referral market does not change the hiring probability of other firms. Therefore, the optimal job creation in the referral market is obtained at the point where the total surplus of the job k_1 is equal to the expected cost c/q_2. In contrast, in the decentralized economy firms capture a fraction $(1-\beta)$ of the total surplus S so the job creation is distorted downwards. The optimal policy then includes paying employment subsidies s to firms in the referral market. One immediate implication of this policy should be a lower unemployment of high ability workers u_1 and a higher job-finding rate λ_2. The situation is different in the regular market. These firms impose a standard search externality on other firms which is neutralized for $\beta = \eta$.

Further consider an economy with the optimal employment subsidy s^*. Proposition 3 describes a system of Pigouvian taxes τ_0^* and τ_1^*. When $\Delta k > 0$ high ability workers impose a negative congestion externality on low ability workers, so the optimal policy implies a negative value of $\tau_1^* < 0$. These transfers are supposed to reduce the reservation wage of high ability workers and increase the expected profit $\bar{J}$. In a similar way, low ability workers create more profits and impose a positive congestion externality on high ability workers. So the optimal policy implies a positive value of $\tau_0^* > 0$, these transfers are supposed to increase the reservation wage of low ability workers. Finally, note that this policy should increase the wage w_0 and decrease both wages w_1 and w_2, so a lower equilibrium wage dispersion is a positive side effect of this policy.

For $\Delta k < 0$, proposition 3 implies $\tau_0^* < 0$ and $\tau_1^* > 0$. In this case the bargained wage of low ability workers is too high and the wage of high ability workers is too low. Wages are then compressed in the decentralized equilibrium and the planner needs to raise the reservation wage of high ability workers and reduce the reservation wage of low ability workers. These predictions coincide with the results of Blazquez and Jansen (2008), however they do not describe the possibility of the reverse policy when $\Delta k > 0$. Identifying and characterising this latter case is the primary contribution of this paper.

To complete this section we also compare the minimum number of contacts l_0 in the decentralized equilibrium and l_0^* in the social optimum. In the equilibrium without referrals the labour market tightness is equal to θ^* (see corollary 1) and the corresponding unemployment of high ability workers is $u_1 = \delta(1-\mu)/(\delta + \lambda(\theta^*))$. In this economy opening a vacancy in the referral market is associated with a present value of profits $J_2 = (1-\beta)(y_1 - z)/(r + \delta + \beta\lambda(\theta^*))$ and is independent of the number of social contacts l (see figure 3).
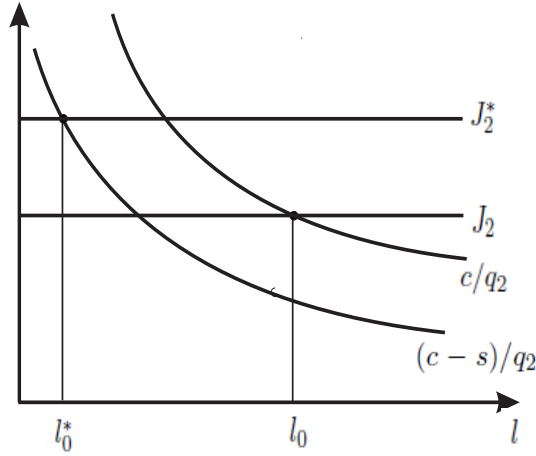
Figure 3: Choice of l_0 in the decentralized equilibrium and in the social optimum

In contrast, expected costs from an open referral vacancy are equal to c/q_2 which is a decreasing function of l. Intuitively, expected duration of an open vacancy is lower with a larger number of social links. The threshold value l_0 can then be found as a minimum number of contacts with positive net profits from referral vacancies $J_2 = c/q_2$, which is equivalent to $\theta^* = \bar{\theta}(u_1(l_0))$:

$$l_0 = \frac{\ln\left(a(1-\beta)(y_1 - z) - c(r + \delta + \beta\lambda(\theta^*))\right) - \ln a(1-\beta)(y_1 - z)}{\ln\lambda(\theta^*) - \ln(\delta + \lambda(\theta^*))}$$

If the optimal policy is implemented, firms' expected profits are larger as $y_1 - \tau_1^* - z > y_1 - z$, while the expected costs are lower ($c - s < c$). Therefore referral vacancies become attractive for firms at a lower number of social contacts $l_0^* < l_0$. Therefore, the decentralized decision of firms not to use referrals may be inefficient from a social perspective.

5   Numerical example

This section parameterizes the model to match the average labour market indicators in the OECD countries. Without loss of generality, we normalize the productivity parameter y_0 to 1. The productivity of high ability workers y_1 is taken to be 1.25 for the benchmark case and we also consider the cases $y_1 = 1$ (workers differ only in social capital) and $y_1 = 1.5$. For comparison, Gautier (2002) uses the value of 0.5 for the productivity of the low-skilled workers and 1 for the high-skilled. In Albrecht and Vroman (2002) productivity values of the high-skilled workers are set in the interval from 1.25 to 1.6 which is similar to our range.

We choose a unit period of time to be one quarter and set $r = 0.012$ which corresponds to the annual discount rate of 5%. Further, we follow Shimer (2005b) and set the flow value of leisure z equal to 0.4. Fontaine (2008) uses the value of 0.15 for the U.S. economy and 0.4 for the French economy. Gautier (2002) and Cahuc and Fontaine (2009) set z equal to 0.2. At the same time, Hall and Milgrom (2008) obtain a larger value of 0.71. Therefore, our choice of z is in the middle range of values in the literature. We also take $\delta = 0.1$ and $\eta = 0.72$ as in Shimer (2005b). This choice of δ implies an average employment duration of 2.5 years. Shimer (2005b) obtains these estimates from the monthly US transition data for the period 1960-2004. The same value of the separation rate is also used in Pissarides (2009).

The cost of an open referral vacancy c is chosen to be 0.2. Intuitively, this parameter captures the costs of traveling and accommodation of job applicants at the place of the interview. Also note that the cost of an open vacancy in the regular market is higher due to the screening expenses of firms, therefore, we set $c + p = 0.4$. This exactly coincides with the Cahuc and Fontaine's (2009) values of these parameters. Shimer (2005b) has chosen the value of 0.213 for the cost of vacancies in the regular market while Fontaine (2008) uses the value of 0.3. Next, the fraction of low-ability workers is set to 60% of the overall population so that $\mu = 0.6$. Albrecht and Vroman (2002) choose a similar value of 0.67 for the proportion if low-skilled workers in their model, while Gautier (2002) uses the value of 0.5 for this parameter.

With respect to the bargaining power, we assume $\beta = \eta = 0.72$ to satisfy the Hosios condition. Moreover, we make a similar assumption in the referral market and set $\phi = \eta$ in the benchmark case where ϕ is the elasticity of the job-finding rate in the referral market. Combining $\phi = \eta$ with $a = 2$ and solving equations (9) for l, we find that $l^* = 40$. Therefore, the implied number of professional contacts in a network of high-ability workers is equal to 40. Cahuc and Fontaine (2009) use $l = 50$, while Fontaine (2008) uses $l = 40$ in a benchmark model of his paper. These numbers are in line with the empirical evidence, for example, in their recent study Cingano and Rosolia (2008) find that the median number of professional contacts in Italy is equal to 32. This number is higher in Germany and is equal to 43 according to Glitz (2013).

y_0	y_1	r	δ	z	η	β	ϕ	m	μ	c	p	a	l
1	1.25	0.012	0.1	0.4	0.72	0.72	0.72	1.22	0.6	0.2	0.2	2	40

Table 1: Values of the model parameters

Finally, we set m to 1.22 which is an efficiency multiplier in the Cobb-Douglas matching function: $\lambda(\theta) = m\theta^\eta$. This parameter yields the following equilibrium unemployment rates: $u_0/\mu = 0.0924$ and $u_1/(1-\mu) = 0.0388$ for the two groups of workers. So the average unemployment rate in the economy is equal to 0.07 which is close to the long-term unemployment rate in the U.S. For comparison, Blazquez and Jansen (2008) set m equal to 1, while Shimer (2005b) uses the value of 1.355.

5.1 Comparative statics

First, the model shows that it is not profitable for firms to open vacancies in the referral job market when the number of workers' contacts is low enough. Numerically solving the system of equations (JC), (JC_2) and $\lambda_2 = 0$ we can find the threshold value l_0 after which firms begin to create vacancies in the referral job market. In the benchmark case, l_0 is approximately equal to 5 and it is decreasing in y_1 or a. Hence when the number of contacts is less than 5 it is not profitable for firms to use referrals.

In all our simulations the decentralized equilibrium is unique as can be seen from figure 4. The curve (JC) is decreasing for low values of u_1 and then increasing, while (JC_2) is parallel to the θ-axis. It can also be shown that $S_0 > S$ which means that firms obtain higher profits in a match with low ability workers. When u_1 is low and increases, the probability of hiring a low ability worker $\gamma(\theta, u_1)$ falls, the average firm profits decrease and so the market tightness θ is reduced. In contrast, when u_1 is already high and increases further, the fall in $\gamma(\theta, u_1)$ is dominated by

the increase in the total surplus value S. Intuitively, a more pronounced unemployment u_1 puts a downward pressure on the reservation wage of high ability workers. This dampens the wage w_1 and leads to a higher profit $J_1 = (1 - \beta)S$.
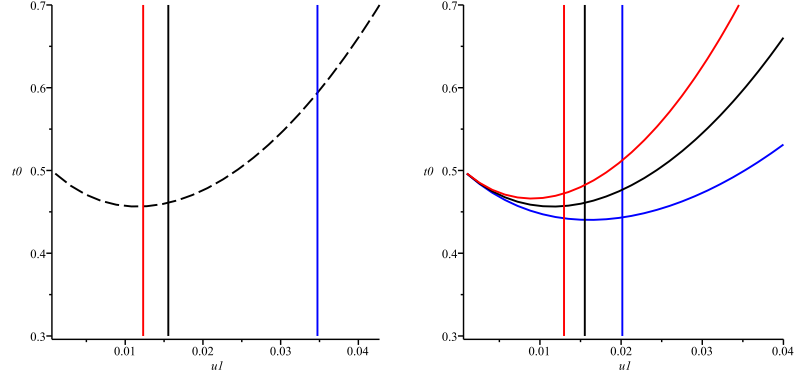

Figure 4: Left panel: Job creation curves determining the equilibrium values of θ and u_1 when $l = 5$ (blue), $l = 40$ (black) and $l = \infty$ (red). Right panel: Job creation curves determining the equilibrium values of θ and u_1 when $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red).

What are the implications of a higher productivity y_1 for θ (u_0) and u_1? The model predicts that both unemployment rates decrease. This result is intuitive as firms expect higher profits and open more vacancies in both job markets (because high ability workers search in both markets). Figure 4 (right) illustrates this effect for the benchmark case $\Delta y = 0.25$ and the other two cases when $\Delta y = 0$ and $\Delta y = 0.5$: (JC_2) moves to the left and (JC) to the up-left with the increase in Δy. This result is similar to Gautier (2002) where the author finds that low-skilled workers gain from the increased productivity of high-skilled workers in simple jobs.

What is the impact of the increase in the number of social contacts l on the equilibrium unemployment? First, the model predicts that a larger number of contacts reduces u_1 and raises u_0. This effect is illustrated in figure 4 (left) where the decentralized equilibrium values of u_1 and θ for $l = 5$, $l = 40$ and $l = \infty$ are compared. As only (JC_2) depends on the number of contacts, there is a parallel shift of this line to the left (right) with the increase (decrease) in l. The larger is the number of contacts the smaller is the shift. We can also calculate the asymptotic value of u_1 which is equal to 0.012.
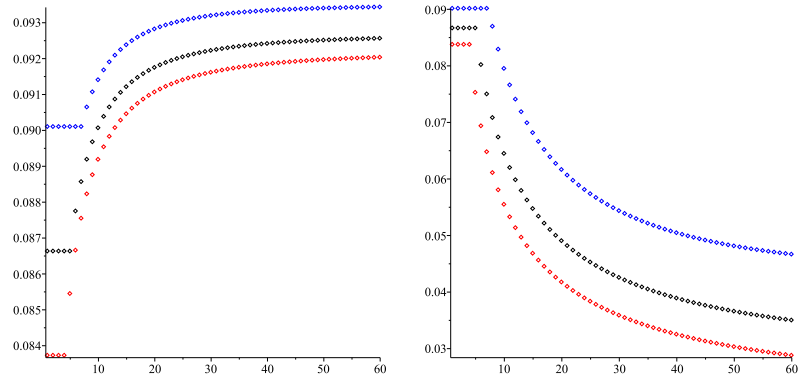

Figure 5: Left panel: Change in u_0/μ with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red). Right panel: Change in $u_1/(1 - \mu)$ with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red)

15

Changes in the unemployment rates u_0 and u_1 with the increase in the number of contacts are illustrated in figure 5. For $l \leq 5$ there exists a unique equilibrium without referrals and so the two unemployment rates coincide. However, if the number of social links is more than 5 firms rely on social contacts to fill their open vacancies. The two unemployment rates are than diverging. On the one hand, as high ability workers are better connected, their equilibrium rate of unemployment is reduced. On the other hand, there is an adverse effect on the equilibrium unemployment of low ability workers which is increasing in l. If high ability workers are better connected, their outside opportunities are improved as finding jobs becomes easier. At the same time, better outside opportunities strengthen the bargaining position of these workers and increase their wages in the regular market. Therefore, firms' profits from regular vacancies and the number of such vacancies are both reduced. Finally, a lower number of vacancies in the regular market worsens the bargaining position of low ability workers and reduces their employment and wages. This latter change is illustrated in figure 6.
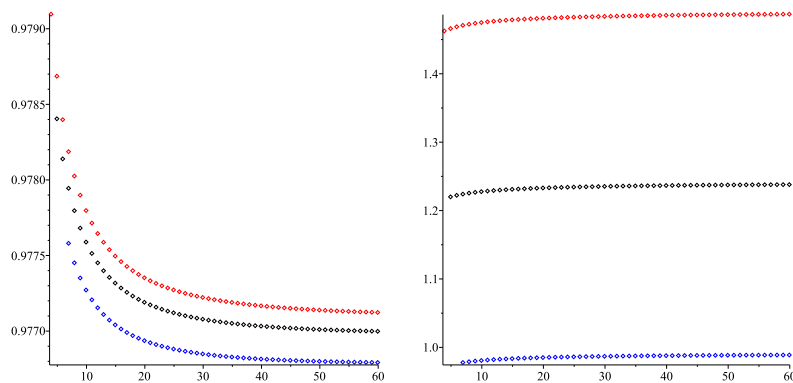

Figure 6: Left panel: Change in w_0 with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red). Right panel: Change in $w_1 = w_2$ with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red)

	u_0	u_1	Δw	l_0
$\Delta y \uparrow$	-	-	+	-
$l \uparrow$	+	-	+	
$a \uparrow$	+/-	-	+	-

Table 2: Comparative statics

Our comparative statics results are summarized in table 2. Overall, the model predicts that a larger number of social links is associated with a more pronounced wage dispersion between the two groups of workers and a higher unemployment rate of low ability workers. Such predictions are compatible with the observed empirical evidence in the U.S. documenting an increase in the inequality of earnings (see Autor, Katz and Kearney (2008)). This allows us to conclude that a part of this inequality may be generated by a stronger growth and utilization of social networks in the U.S. and other countries in the recent decade[1].

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[1]For example, there exist labor market policies trying to encourage the establishment or the improvement of social networks (McClure (2000), OECD (2001)). One of them is Australian Working Together program which aims at providing people the incentives to stay together with their communities even if they are economically disadvantaged (OECD (2003)).

5.2 Optimal policy

This subsection investigates the effect of optimal policy $\{s, \tau_0, \tau_1\}$ on endogenous variables in the labour market. Consider first the case $y_1 = y_0 = 1$ when the productivity of the job is not sensitive to worker's ability. The optimal vector of policy instruments is given by: $s = 0.144$, $\tau_0 = 0.017$, $\tau_1 = -0.160$. Table 3 presents the results. For the ease of exposition policy changes are implemented in two steps: only employment subsidies $s = \eta c$ in the referral market and the final policy. In addition, we introduce two new variables Ω_0 and Ω_1. These variables denote the expected income of low- and high-ability workers respectively:

$$\Omega_0 = u_0(z + \tau_0) + (\mu - u_0)w_0 \qquad \Omega_1 = u_1(z + \tau) + (1 - \mu - u_1)w_1$$

where the first term is the flow income of unemployed workers and the second term is the flow income of the employed. Also note that high ability workers earn the same wage $w_1 = w_2$ independent of the search channel.

Optimal policy	θ	u_0/μ	$u_1/1 - \mu$	w_0	$w_1 = w_2$	Ω_0	Ω_1
Without policy	0.4432	0.0933	0.0504	0.9768	0.9872	0.5538	0.3830
Only subsidy $\{s\}$	0.4527	0.0928	0.0211	0.9769	0.9945	0.5540	0.3928
Final policy $\{s, \tau_0, \tau_1\}$	0.4475	0.0931	0.0182	0.9775	0.9940	0.5552	0.3921

Table 3: Optimal policy $s = 0.144, \tau_0 = 0.017, \tau_1 = -0.160$ in a labour market with $y_1 = 1$

As expected, employment subsidies s reduce the equilibrium unemployment of high ability workers $u_1/1 - \mu$ and raise their wages. The unemployment rate of low ability workers u_0/μ is slightly decreased as a consequence of a higher market tightness θ. This is an outcome of a lower competition between agents in the regular labour market (lower $1 - \gamma$). Overall, one can conclude that subsidizing referrals is associated with a large welfare gain for high ability workers (Ω_1 is higher) and a minor rise in the welfare of low ability workers (Ω_0 is higher).

Table 3 further shows that the optimal transfers $\tau_0 = 0.017$ and $\tau_1 = -0.160$ internalize congestion externalities in the regular market. This finding is in contrast to Blazquez and Jansen (2008) as we find that $\Delta k > 0$ for the chosen parameter values. Firm profits are lower in a match with high ability workers and so every additional high ability unemployed imposes a negative externality on workers with low abilities making it more difficult for them to find a job. Therefore, the optimal tax policy favours low ability workers at the cost of the other group (Ω_0 is higher at the second step, while Ω_1 is lower). The wage of low (high) ability workers becomes higher (lower), so the wage inequality is slightly reduced. In addition, observe that this second policy is purely redistributive as it holds that $u_0\tau_0 + u_1\tau_1 = 0$. Similar tables for $y_1 = 1.25$ and 1.5 are presented in Appendix IV and confirm our predictions.

As follows from table 3, imposing taxes τ_0, τ_1 is associated with a welfare gain of 0.2% for workers without connections. In order to understand whether this amount is significant, let us discount the approximate net yearly average wage in Germany (2200*12.5=27500 EUR)[2] over 50 years with an annual discount rate of 5%. We get an amount of 502000 EUR and, therefore, this increase in welfare is approximately equivalent to the lump sum transfer of 1000 EUR per worker. Next consider high ability workers. Although the redistributive tax scheme is associated with a

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[2]Approximate average monthly salary (Statistisches Bundesamt yearly report, 2013), multiplied by 12.5 months

value loss, the overall increase in the welfare of these workers is about 2.4% which is equivalent to the lump sum amount of 12000 EUR per worker.

In the next step we investigate the robustness of our results. This can be done by considering the sign of $\Delta k = k_0 - k_1$ with a variation in the key parameters y_1, l and a. Figure 7 confirms our theoretical conclusion from section 4 that the case $\Delta k < 0$ is more likely with a higher difference in productivities and a lower number of social contacts. Every curve on this figure shows the values of Δk for l in the range from 5 (lower line) to 14 (higher line) and for y_1 taking values from 1.05 to 1.95. Moreover $a = 1$ on the left panel of the graph and $a = 2$ on the right panel. It is easy to see, therefore, that Δk is non-negative when $a = 2$ and $y_1 \leq 1.95$. Although, when $a = 1$, Δk can become negative for low values of l and a high productivity y_1. This is precisely the case when our result is in line with the finding of Blazquez and Jansen (2008) and the equilibrium exhibits a wage compression in the regular labour market.
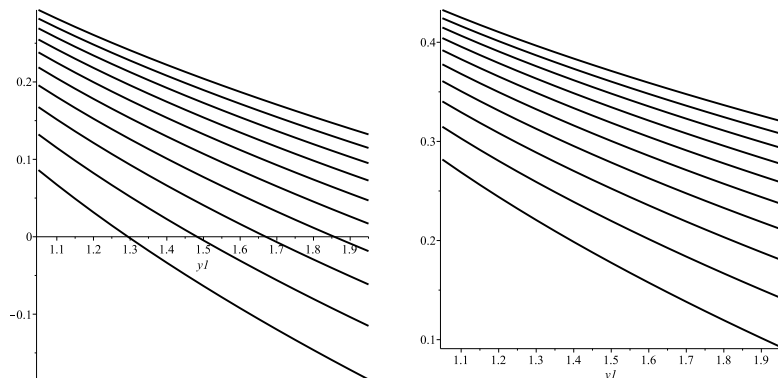

Figure 7: Left panel: Values of Δk for $l = [5...14]$, $y_1 = [1.05...1.95]$ and $a = 1$. Right panel: Values of Δk for $l = [5...14]$, $y_1 = [1.05...1.95]$ and $a = 2$.

Figure 8 compares the threshold number of social contacts in the decentralized economy l_0 with the optimal planner's choice l_0^*. In particular, it illustrates our result from section 4 that l_0^* is always lower than l_0 for every a from 1 to 10. Thus, the decentralized decision not to use referrals may be suboptimal from the social perspective. Cahuc and Fontaine (2009) have already found in their setting that for low values of β formal search methods can be used instead of social networks and this allocation can be inefficient. Our paper extends this result in the sense that it holds for every β and depends on the number of contacts in the networks.
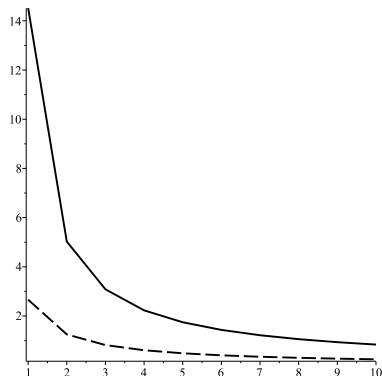

Figure 8: Comparison between l_0 (solid) and l_0^* (dashed) for $a = [1...10]$.

18

6 Conclusion

This paper develops a labour market matching model with heterogeneous workers and two channels of job search: formal methods and social networks. Entering firms have an option to post a vacancy in the regular job market or in the referral market, where job information is exclusively transmitted by employees. The paper proves existence of the decentralized equilibrium in this framework and then shows that this equilibrium is inefficient. There are two reasons for the inefficiency. First, firms obtain a fixed fraction of the total job surplus in the referral market which is below the social gain. Therefore, the number of referral vacancies is low and the equilibrium unemployment of high ability workers is inefficiently high. This inefficiency can be corrected by means of employment subsidies in the referral market. Second, high ability workers congest the market for low ability workers. Moreover, this congestion externality is increasing in the number of social contacts. The optimal policy then includes a positive income transfer to low ability workers and a negative transfer to high ability workers. This policy reduces the equilibrium wage dispersion which is different from the result of Blazquez and Jansen (2008) reporting a compressed equilibrium wage distribution in a similar framework without contacts.

Finally, we examine the effect of a larger number of social contacts. High ability workers rely strongly on their networks, thus their unemployment falls and their wages rise with a larger density of the network. In contrast, low ability workers are adversely effected by this change. Their wages fall and the unemployment rate is increased. Overall, the equilibrium wage dispersion is increasing in the number of social contacts.

7 Acknowledgements

8 Appendix

Appendix I: Proof of Proposition 1. The right-hand side of equation (JC_2) is monotonically increasing in u_1, while the left-hand side of (JC_2) is monotonically decreasing in u_1:

$$\lim_{u_1 \to 1-\mu} \frac{(1-\beta)(y_1-z)}{(r+\delta+\beta\delta\frac{1-\mu-u_1}{u_1})} = (1-\beta)\frac{y_1-z}{r+\delta} \qquad \lim_{u_1 \to 1-\mu} \frac{c}{a[1-(\frac{1-\mu-u_1}{1-\mu})^l]} = \frac{c}{a}$$

Therefore, (JC_2) will determine a unique value of u_1 when $\frac{c}{a} < (1-\beta)\frac{y_1-z}{r+\delta}$.

Condition A: the left-hand side of condition A is monotonically increasing in θ:

$$\lim_{\theta \to 0} \frac{c+p}{q(\theta)} = 0 \qquad \lim_{\theta \to \bar\theta(u_1)} \frac{c+p}{q(\theta)} = \frac{c+p}{q(\bar\theta(u_1))}$$

The right-hand side of this condition is not necessarily monotonic in θ, however:

$$\lim_{\theta \to 0} \frac{\gamma(\theta,u_1)(y_0-z)}{r+\delta+\beta\lambda(\theta)} + \frac{(1-\gamma(\theta,u_1))(y_1-z)}{r+\delta+\beta\lambda(\theta)+\beta\lambda_2} = \frac{\mu(y_0-z)}{(\mu+u_1)(r+\delta)} + \frac{u_1(y_1-z)}{(\mu+u_1)(r+\delta+\beta\lambda_2)} > 0$$

19

where we use that $\gamma(0, u_1) = \mu/(\mu + u_1)$ and

$$\lim_{\theta \to \bar{\theta}(u_1)} \frac{\gamma(\theta, u_1)(y_0 - z)}{r + \delta + \beta\lambda(\theta)} + \frac{(1 - \gamma(\theta, u_1))(y_1 - z)}{r + \delta + \beta\lambda(\theta) + \beta\lambda_2} = \frac{\mu y_0 + (1 - \mu)y_1 - z}{r + \delta + \lambda(\bar{\theta}(u_1))}$$

where $\gamma(\bar{\theta}(u_1)) = \mu$. Then there exists an equilibrium with $\theta(u_1) < \bar{\theta}(u_1)$ if:

$$(1 - \beta)\frac{y_0\mu + y_1(1 - \mu) - z}{r + \delta + \beta\lambda(\bar{\theta}(u_1))} < \frac{c + p}{q(\bar{\theta}(u_1))}$$

Wage dispersion Δw is given by:

$$\Delta w = \beta(y_1 - y_0) + (1 - \beta)\left(\frac{\beta(y_1 - z)(1 - \mu - u_1)\delta}{(r + \delta)u_1 + \beta(1 - \mu - u_1)\delta} - \frac{\beta(y_0 - z)\lambda(\theta)}{r + \delta + \beta\lambda(\theta)}\right)$$

Differentiation of Δw with respect to u_1 and θ gives

$$\frac{\partial \Delta w}{\partial u_1} = -(1 - \beta)\frac{\beta(y_1 - z)\delta(ru_1 + \delta(1 - \mu))}{((r + \delta)u_1 + \beta(1 - \mu - u_1)\delta)^2} < 0 \quad \frac{\partial \Delta w}{\partial \lambda(\theta)} = -(1 - \beta)\frac{\beta(y_0 - z)(r + \delta)}{(r + \delta + \beta\lambda(\theta))^2} > 0$$

Appendix II: Proof of Proposition 2.

The Hamiltonian for the problem of the social planner:

$$H = (1 - \mu - u_1)y_1 + (\mu - u_0)y_0 + (z - (c + p)\theta)(u_0 + u_1) - cv_2 + k_0(\lambda(\theta)u_0 - \delta(\mu - u_0))$$
$$+ k_1(\lambda(\theta)u_1 + av_2[1 - (1 - \frac{u_1}{1 - \mu})^l] - \delta(1 - \mu - u_1))$$

where k_0, and k_1 are costate variables corresponding to u_0 and u_1 respectively. The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial \theta} = -(c + p)(u_0 + u_1) + \lambda'(\theta)(k_0u_0 + k_1u_1) = 0 \quad \Rightarrow \quad \frac{c + p}{q(\theta)} = (1 - \eta)(\gamma k_0 + (1 - \gamma)k_1)$$

$$\frac{\partial H}{\partial v_2} = -c + ak_1[1 - (1 - \frac{u_1}{1 - \mu})^l] = 0 \quad \frac{\partial H}{\partial u_0} = -y_0 + (z - (c + p)\theta) + k_0(\lambda(\theta) + \delta) = -rk_0$$

$$\Rightarrow k_0 = \frac{y_0 - z + (1 - \eta)\lambda(\theta)(\gamma k_0 + (1 - \gamma)k_1)}{r + \delta + \lambda(\theta)} \quad \Rightarrow \quad k_0 = \frac{y_0 - z - (1 - \eta)\lambda(\theta)(1 - \gamma)\Delta k}{r + \delta + \eta\lambda(\theta)}$$

$$\frac{\partial H}{\partial u_1} = -y_1 + (z - (c + p)\theta) + k_1(\lambda(\theta) + \frac{alv_2}{1 - \mu}(1 - u_1/(1 - \mu))^{l-1} + \delta) = -rk_1$$

$$\Rightarrow k_1 = \frac{y_1 - z + (1 - \eta)\lambda(\theta)(\gamma k_0 + (1 - \gamma)k_1)}{r + \delta + \lambda(\theta) + \frac{alv_2}{1 - \mu}(1 - \frac{u_1}{1 - \mu})^{l-1}} \quad \Rightarrow \quad k_1 = \frac{y_1 - z + (1 - \eta)\lambda(\theta)\gamma\Delta k}{r + \delta + \eta\lambda(\theta) + \phi\lambda_2}$$

Appendix III: Proof of Proposition 3. Consider the decentralized economy with a vector of policy instruments $\{s = \eta c, \tau_0 = \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma), \tau_1 = -\lambda(\theta)\Delta k(1 - \eta)\gamma\}$:

$$S_0 = \frac{y_0 - z - \tau_0}{r + \delta + \beta\lambda(\theta)} = \frac{y_0 - z - \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma)}{r + \delta + \beta\lambda(\theta)}$$

$$S = \frac{y_1 - z - \tau_1}{r + \delta + \beta\lambda(\theta) + \phi\lambda_2} = \frac{y_1 - z + \lambda(\theta)\Delta k(1 - \eta)\gamma}{r + \delta + \beta\lambda(\theta) + \phi\lambda_2}$$

For the case when $\beta = \eta = \phi$ the free-enrty conditions become:

$$\frac{c+p}{q(\theta)} = (1-\eta)[(1-\gamma)S_0 + \gamma S] \quad \text{and} \quad \frac{c-\eta c}{q_2} = (1-\eta)S \quad \Rightarrow \quad \frac{c}{q_2} = S$$

therefore, it follows that $S_0 = k_0$ and $S = k_1$ and the optimal allocation can be implemeted.

Appendix IV: Tables for model parameters after imposing the policy for $y_1 = 1.25$ and $y_1 = 1.5$

Optimal policy	θ	u_0/μ	$u_1/1-\mu$	w_0	$w_1 = w_2$	Ω_0	Ω_1
Without policy	0.4609	0.0924	0.0388	0.9770	1.2359	0.5542	0.4814
Only subsidy $\{s\}$	0.4634	0.0923	0.0171	0.9770	1.2437	0.5543	0.4917
Final policy $\{s, \tau_0, \tau_1\}$	0.4562	0.0926	0.0158	0.9775	1.2433	0.5552	0.4912

Table 4: Optimal policy $s = 0.144, \tau_0 = 0.014, \tau_1 = -0.124$ in a labour market with $y_1 = 1.25$

Optimal policy	θ	u_0/μ	$u_1/1-\mu$	w_0	$w_1 = w_2$	Ω_0	Ω_1
Without policy	0.4722	0.0918	0.0324	0.9772	1.4847	0.5545	0.5798
Only subsidy $\{s\}$	0.4702	0.0919	0.0147	0.9771	1.4930	0.5545	0.5908
Final policy $\{s, \tau_0, \tau_1\}$	0.4623	0.0923	0.0141	0.9775	1.4928	0.5552	0.5905

Table 5: Optimal policy $s = 0.144, \tau_0 = 0.012, \tau_1 = -0.084$ in a labour market with $y_1 = 1.5$

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