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Abstract

In this paper we study the impact of environmental pollution in an endogenous growth model with endogenous structural change. The paper allows for both horizontal and vertical innovations where newer technologies are less polluting compared to older ones. The analysis shows that the presence of environmental externalities stimulates structural change but reduces the growth rate of the economy. Further, comparing the models with and without structural change demonstrates that the latter implies stronger environmental damages and, consequently, a lower growth rate than the first one. Finally, levying a tax on the polluting output speeds up structural change, thus, reducing environmental pollution and spurring economic growth.

JEL classification: Q55, O31, O44

Keywords: Environmental Damages, Endogenous Growth, Creative Destruction, Endogenous Structural Change

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1 Introduction

The question of how economic evolution and environmental degradation are interrelated has a long tradition in economics. Seminal work in this field has been undertaken by Forster (1973), Mäler (1974) or Gruver (1976), for example. Forster analyzed the Ramsey growth model where environmental pollution occurs as a by-product of capital accumulation and can be reduced by abatement spending. He shows that this model is characterized by a stationary state in the long-run with all variables being constant, unless the economy is hit by an exogenous shock. Mäler analyzes several aspects associated with environmental degradation in different frameworks, such as a general equilibrium model and a model of economic growth with environmental damages. However, in contrast to Forster, he is less interested in the long-run evolution of the economy but assumes a finite time horizon.

With the emergence of endogenous growth theory in the late 1980's and 1990's, the research focus has moved to the interrelation between environmental policies, such as taxes and quotas, on the one hand, and the long-run growth rate and welfare, on the other hand. Examples of such studies are Bovenberg and Smulders (1995), Smulders and Gradus (1996), Greiner (2005) or Grimaud and Rougey (2014).[1] In those papers, the economy is characterized by ongoing growth with the long-run growth rate being an endogenous variable. That property results from the fact that the marginal product of capital does not decline as capital grows which, for its part, may be a result of human capital accumulation, of the creation of new technologies or from productive public investment, for example. However, to our knowledge none of those contributions deals with the relationship between environmental pollution and endogenous strucural change in a growth context.

In this paper we analyze the effects of environmental pollution within an endogenous growth model allowing for structural change that results from the introduction of new technologies that make old ones obsolete, giving rise to creative destruction as already described by Schumpeter (1942). Starting point of our analysis is the model

[1]For a survey, see also the book by Greiner and Semmler (2008).

without environmental pollution presented in Bondarev and Greiner (2014). There, new technologies are permanently developed as a result of R&D investment replacing old technologies. Simultaneously, existing technologies are improved through vertical innovations as in the seminal paper by Aghion and Howitt (1992). Newer technologies have a higher productive potential and, therefore, can attain a higher productive efficiency although initially all new technologies are identical as in the model by Peretto and Conolly (2007).

We take up the benchmark model by Bondarev and Greiner (2014) and extend this model by assuming that goods production implies negative environmental externalities that are a pure public good (or bad) that exerts a negative impact on the production of each sector in the economy. Further, the emissions intensity of each new technology is smaller than the one of the preceeding technolgy implying that newer technologies are less polluting than older ones. Our goal, then, is to compare the effects of environmental degradation in the growth model with structural change to those obtained in a model without structural change. Further, we analyze the effects of environmental pollution by contrasting the benchmark model, where environmental considerations are absent, with the model including environmental damages. Finally, we integrate an ad-valorem tax on revenues of the manufacturing firm, with the tax rate equal to the emissions intensity, and study its effects on the growth rate of the economy and its implications with respect to the environment.

The rest of the paper is organized as follows. The next section briefly presents the structure of the growth model and shows how the environment has been integrated into the benchmark model. Section 3 gives the solution of the model and section 4 derives the impacts of environmental pollution. Section 5, finally, concludes.

2 The growth model with environmental pollution

We briefly describe the structure of the growth model with environmental degradation. For more details concerning the model without the environment, which serves as the benchmark model, the reader is referred to Bondarev and Greiner (2014).

2.1 The economy

The economy is decentralised with a household sector, a productive sector and a R&D sector that invests in horizontal and vertical innovations. The representative household maximizes[2]

$$J^H = \int_0^\infty e^{-\rho t} \ln C \, dt, \tag{1}$$

with ρ the discount rate and C a continuum of differentiated products from existing sectors,

$$C = \left[\int_{N_{min}}^{N_{max}} C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

with ε the elasticity of substitution between goods and N_{max} is the range of manufacturing sectors with positive operating profit and N_{min} is the range of sectors, which disappeared from the economy up to time t. The range of developed sectors is growing over time reflecting the expansion in the variety of products. However, the range of existing sectors, given by $N_{max} - N_{min}$, may grow decrease or stay constant in time, depending on the characteristics of the process of variety expansion of technologies, $\dot{N}$, with N the total number of technologies that have been invented up to time t. The budget constraint of the household is given by,

$$\dot{K} = rK + wL - E, \tag{3}$$

with E denoting consumption expenditures, K capital, r return to capital, w the wage rate that serves as the numéraire , $w \equiv 1$, and L labor. Expenditures are given by,

$$E = \int_{N_{min}}^{N_{max}} P_i C_i di, \tag{4}$$

with P_i the price of good i.

The solution of this optimization problem leads to the standard Euler equation,

$$\frac{\dot{E}}{E} = r - \rho. \tag{5}$$

[2]We delete the time argument t as long as no ambiguity arises.

The market form of the manufacturing sector is characterized by monopolistic competition where firms produce different goods, Y_i, with the help of a patented technology i from the available spectrum. Firms use technology, A_i, and labor for production and there is a negative effect from environmental pollution that is a pure public good (or bad),

$$Y = \int_{N_{min}}^{N_{max}} Y_i di, \; Y_i = \left(\frac{1}{1+T}\right) A_i^{\alpha} L_i, \tag{6}$$

with $\int Y_i di$ aggregate output and T gives environmental damages, with $T = 0$ standing for the unpolluted state of the nature.[3] Profits of firms in the manufacturing sector are,

$$\Pi_i = P_i Y_i - L_i - \Psi, \tag{7}$$

with Ψ a fixed operating cost. Profit maximization of firms, then, determines prices and labor demands in a standard way as in the benchmark model.

The technology is described by vertical and horizontal innovations undertaken by the R&D sector exactly in the same way as in the benchmark model, with investments set optimally by R&D firms. This leads to:

$$\dot{A}_i = \gamma g_i - \beta A_i; \tag{8}$$

$$\dot{N} = \delta u. \tag{9}$$

The R&D activities are unaffected by the state of the environment and are identical to those in the benchmark model. Hence, the overall influence of the environment on the economy consists solely in the symmetric reduction of output of all existing sectors in this economy.

[3]For example, T could be interpreted as the deviation of the average surface temperature on earth from its pre-industrial level.

2.2 The environment

The natural environment is affected by aggregate output in a usual fashion, as in Bréchet et al. (2011) for example,

$$\dot{T} = -\mu T + eY, \tag{10}$$

where:

- T is some aggregate measure of the environment (deviation from the average global surface temperature);
- μ is the regeneration rate of the environment;
- e is the intensity of emissions, defined by the state of technology;
- Y is the aggregate output of the economy.

The intensity of emissions is a function of an effective mix of technologies being used for production at a given point in time. We assume that each of the technologies has a different intensity of emissions or environmental impact. For simplicity we assume a hyperbolic decrease of the emissions intensity across the space of technologies (since a linear decrease is not applicable to the unrestricted space $\mathbf{N}$):

$$\forall i \in \mathbf{N} : \ \iota(i) = 1/i; \tag{11}$$

where $\iota(i)$ is the function of the environmental impact for technology i. Then, the average emissions intensity of the economy at any point in time is given by,

$$e(t) = e_0 \frac{\int\limits_{N_{min}}^{N_{max}} (1/i) di}{N_{max} - N_{min}}, \tag{12}$$

where N_{min}, N_{max} are the ranges of outdated and of operational technologies, respectively, and e_0 is the base emissions intensity of the economy. Following Bréchet et al. (2011) and Bondrev et al. (2014), we set this parameter to $e_0 = 0.0475$.

In the decentralised economy where the environmental externality (10) is not taken into account by profit maximizing firms, the range of operational technologies is constant over time.[4] Thus, we have,

$$N(t)_{max} - N(t)_{min} = \mathcal{O} = const. \tag{13}$$

In this situation the aggregate emissions intensity of the economy can be easily computed as a function of one variable:

$$e(t) = e_0 \frac{\int_{N_{min}}^{N_{max}} (1/i)di}{\mathcal{O}} = e_0 \frac{\ln(N_{min}(t) + \mathcal{O}) - \ln(N_{min}(t))}{\mathcal{O}}, \tag{14}$$

Given that $N_{min}(t)$ is a linear function of time for a homogeneous technological space as in the benchmark model, the average emissions intensity is a hyperbolically decreasing function of time for the decentralised economy. The speed of decrease depends on the size of the core of the economy $\mathcal{O}$ as demonstrated in Figure 1.

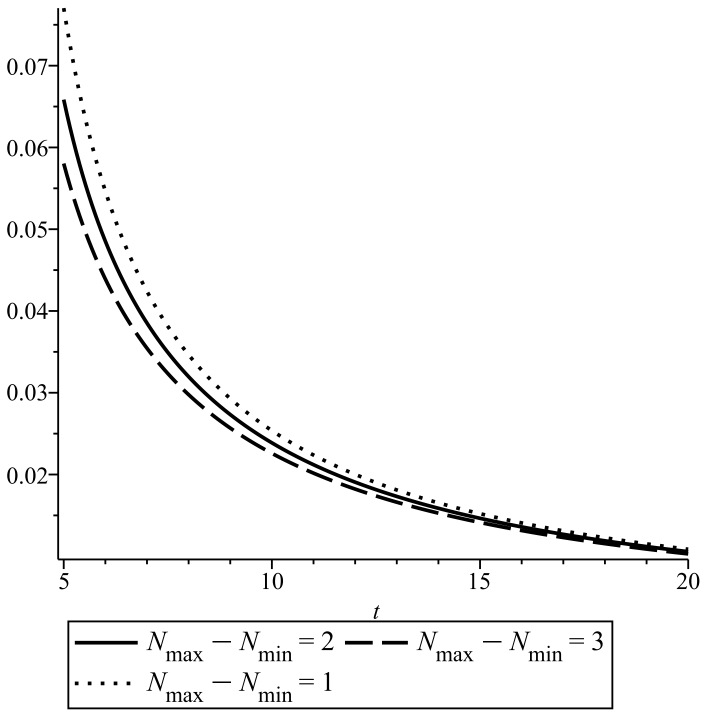


Figure 1: Emissions intensity and size of the core of the economy

[4]A formal proof of that property can be found in Bondarev and Greiner (2014).

However, a more accurate estimate of the environmental impact of the actual technology mix includes the fraction of output being generated with the use of a certain technology. Such a function cannot be computed without knowing the evolution of output of every sector and is done next. Thus, the emissions intensity is formulated as,

$$e(t) = e_0 \frac{\int_{N_{min}}^{N_{max}} (1/i) y_i di}{\mathcal{O}} \tag{15}$$

where y_i is the share of each technology output relative to total output:

$$y_i = \frac{Y_i}{Y}, \tag{16}$$

with output given by (6). The overall dynamics of the joint system, then, is given by:

- Capital accumulation, (3);
- Productivities evolution for each sector, (8);
- Expansion of a variety of technologies, (9);
- Evolution of the environment, (10).

3 Solution of the model

The solution procedure follows the same steps as for the benchmark model. Optimal R&D investments for each sector are proportional to the capital stock minus horizontal investments:

$$g_i^* = \frac{K - u}{N - N_{min}}. \tag{17}$$

Horizontal innovations are linear and proportional to the expected profit of the next technology:

$$u^* = \delta \pi^R(i)|_{i=N}. \tag{18}$$

Since the technologies are homogeneous, the expected profit is the same for all technologies and the variety expansion is a linear function of time:

$$N(t) = \delta^2 \pi^R t + N_0. \tag{19}$$

The obsolescence of technologies and the entrance of new technologies into the profitable phase are also linear processes yielding a constant size of the core $\mathcal{O}$:

$$N_{min} = \delta^2 \pi^R (t - t_{min}) + N_0; \tag{20}$$

$$N_{max} = \delta^2 \pi^R (t - t_{max}) + N_0, \tag{21}$$

with

- $t_{min} = N_{min}^{-1}(i)$, the time when product (technology) i becomes outdated;
- $t_{max} = N_{max}^{-1}(i)$, the time when product (technology) i becomes profitable;
- $t_0 = N^{-1}(i)$, the time when product (technology) i is invented.

From now on we denote quantities for the economy with environmental spillovers by the superscript T and the quantities from the benchmark model by the superscript O. Recalling that the output of each sector is affected uniformly by environmental pollution, $Y_i = A_i^\alpha L_i/(1+T)$, one immediately gets that prices and labor demand are changing proportionally:

$$\begin{aligned} P_i^T &= \left(\frac{\epsilon}{\epsilon - 1}\right) A_i^{-\alpha}(1+T) = (1+T)P_i^O, \\ L_i^T &= E\left(\frac{\epsilon - 1}{\epsilon}\right)\left(\frac{1}{1+T}\right)\frac{A_i^{-\alpha(1-\epsilon)}}{\int\limits_{N_{min}}^{N_{max}} A_j^{-\alpha(1-\epsilon)} dj} = \left(\frac{1}{1+T}\right) L_i^O, \end{aligned} \tag{22}$$

leaving expenditures unchanged relative to the benchmark model:

$$E^T = \int\limits_{N_{min}}^{N_{max}} P_i Y_i di = \frac{\epsilon}{\epsilon - 1}\int\limits_{N_{min}}^{N_{max}} \left(A_i^{-\alpha}(1+T)\frac{L_i^O A_i^\alpha}{1+T}\right) di = \left(\frac{\epsilon}{\epsilon - 1}\right) L^O = E^O. \tag{23}$$

However, the total labor income changes since the total output of the economy is lower because of the environmental degradation. Consider the labor market clearing condition:

$$L^T = \int_{N_{min}}^{N_{max}} L_i^T di = \frac{1}{1+T} \int_{N_{min}}^{N_{max}} L_i^O di = \left(\frac{1}{1+T}\right) L^O < L^O. \tag{24}$$

It follows that employment in the economy under environmental pollution is decreasing compared to the benchmark model. This gap seems to follow quite naturally the notion of environmental unemployment:

$$U^T = L^O - L^T = \left(\frac{1}{1+T}\right) L^O. \tag{25}$$

It should be noted that the discrepancy between labor demand in the benchmark economy and in the economy with environmental spillovers will rise in time if environmental degradation continues. This will decrease the labor income of the households and, thus, slow down capital accumulation:

$$\dot{K}^T = rK - E^O + \frac{1}{1+T} < \dot{K}^O = rK - E^O + 1, \tag{26}$$

But the dynamics of R&D is the same in the model with environmental spillovers as for the benchmark one since the environment only affects final goods production.

The state of the environment depends on output and on the technology mix. We start with computing the share of each technology in total output. The output of each individual sector is given by,

$$Y(i,t) = \left(\frac{1}{1+T}\right) \frac{A(i,t)^{\alpha\epsilon}}{\int_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)}} \tag{27}$$

and the fraction of output of each (operational) technology is:

$$y_i = \frac{Y_i}{Y} = \frac{A(i,t)^{\alpha\epsilon}\left(\int\limits_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)}\right)^{-1}}{\int\limits_{N_{min}}^{N_{max}} Y(i,t)di} = \frac{A(i,t)^{\alpha\epsilon}}{\int\limits_{N_{min}}^{N_{max}} A(i,t)^{\alpha\epsilon}di}, \tag{28}$$

where $A(i,t)$ is the productivity level of technology i at time t.

Thus, the evolution of the environment can be expressed as a function of productivities and of the environment:

$$\dot{T} = -\mu T + \frac{\int\limits_{N_{min}}^{N_{max}}(1/i)y_i di}{\mathcal{O}} Y(t) = \frac{1}{\mathcal{O}}\left(\frac{1}{1+T}\right)\frac{\int\limits_{N_{min}}^{N_{max}}\left(\frac{1}{i}\right)A(i,t)^{\alpha\epsilon}di}{\int\limits_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)}} - \mu T =$$

$$= \frac{1}{\mathcal{O}}\left(\frac{1}{1+T}\right)\int\limits_{N_{min}}^{N_{max}}\left(\frac{1}{i}\right)A(i,t)^{\alpha}di - \mu T. \tag{29}$$

Equation (29) shows that the larger the operational range of technologies $\mathcal{O}$ (core) is, the lower is the environmental impact in the economy. The economic intuition behind this fact is as follows: the higher the range of technologies, the lower is the fraction of output produced by each of them and, consequently, the lower is the share of dirty technologies. Since the capital is distributed evenly across all technologies (they are homogeneous in this respect), a rise in the operating range of technologies is always shifting capital towards cleaner technologies, thus raising the relative share of less polluting technologies.

The lower stock of capital decreases productivity growth but not the variety expansion. The latter is linear and depends on the potential profit of the next technology. Let us consider the creation of new technologies in the environmental spillovers model compared to the benchmark economy. This is governed by the profits resulting from the development of a new technology,

$$\pi^R(i,t) = p_A(i) - \frac{1}{2}\int\limits_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} g^2(i,t)dt, \tag{30}$$

which depends on the price of the patent, $p_A(i)$, and on accumulated investments.

The prices of patents will be higher, since the lower output is counterbalanced by the higher prices, and the manufacturing sector profits are larger than in the benchmark model due to lower labor costs,

$$\Pi_i^T = P_i^T Y_i^T - L_i^T - \Psi = \left(\frac{\epsilon}{\epsilon-1} - \frac{1}{1+T}\right) L_i^O - \Psi; \tag{31}$$

$$\Pi_i^O = P_i^O Y_i^O - L_i^O - \Psi = \left(\frac{\epsilon}{\epsilon-1} - 1\right) L_i^O - \Psi; \tag{32}$$

$$T > 0 : \; \Pi_i^T > \Pi_i^O. \tag{33}$$

Thus, the patent price under environmental degradation will be higher and depends on the state of the environment at the time when technology i becomes operational and on the time when it becomes outdated. But, this factor affects all technologies in the same way and also influences investments (through capital accumulation).

Accumulated investments at the same time are lower for every technology due to slower capital accumulation compared to the benchmark model:

$$\int\limits_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} \left((g^T(i,t))^2\right) dt = \int\limits_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} \left(\frac{K^T - u^T}{N^T - N^T_{min}}\right)^2 dt \tag{34}$$

Assuming the same linear variety expansion process for the economy with environmental spillovers, it follows that the dynamics is governed by the K^T term which is always lower than the capital in the benchmark model, see (26). Then, the accumulated investments into productivity development (vertical innovations) are indeed lower for every technology by the factor of environmental damages, $1/(1+T)$. This gives,

$$\pi^{R,T} > \pi^{R,O}, \tag{35}$$

and variety expansion (and thus structural change) is boosted in the economy with environmental spillovers, Thus, we obtain

$$\dot{N}^T > \dot{N}^O \rightarrow \mathcal{O}^T > \mathcal{O}^O. \tag{36}$$

A full analytic solution for our model economy cannot be obtained, but we can analyze the behaviour of its main variables compared to the model with a fixed range of sectors, i.e. without horizontal innovations, to illustrate the impact of structural change on the environment. That is done in the next section.

4 Analysis of the model

4.1 Comparison with the model without structural change

First, we compare the evolution of the environment with and without structural change. The economy without structural change is identical to the one with structural change but operates with a fixed range of sectors. This implies that newer technologies just replace older ones as in the quality ladders model of Aghion and Howitt (1992). Without horizontal innovations the emissions intensity in the economy, $e(t)$, is constant and determined by the existing composition of the technology.

It is straightforward to see that for this model there is no slowdown of environmental degradation in the economy at all. Consider the differential equation describing environmental degradation for the case where all technologies grow at the same speed as the average technology $\bar{A}$, i.e. $\dot{A}_i = \dot{\bar{A}}$. The rate of environmental degradation is then determined by the average technology $\bar{A}$:

$$\dot{T} = \frac{1}{\mathcal{O}}\left(\frac{1}{1+T}\right)\int\limits_{N_{min}}^{N_{max}}\left(\frac{1}{i}\right)A(i,t)^{\alpha}di - \mu T = \frac{\bar{A}^{\alpha}\ln(N_{max}/N_{min})}{\mathcal{O}^2(1+T)} - \mu T, \tag{37}$$

In the economy without structural change the term $\ln(N_{max}/N_{min})$ is constant, while under structural change it decreases over time such that the emissions intensity declines as illustrated in Figure 1 (as the boundaries of new and outdated mass of technologies move forward). Thus, other things equal the structural change slows down environmental degradation.

Next, consider capital accumulation and productivity growth. Since environmental degradation is less drastic in the economy with structural change, capital accumulation is higher. Given a higher total stock of capital, the available capital that can be used to raise productivities of existing technologies is larger. Therefore, productivity growth will also be higher.[5] Thus, we can establish

Proposition 1 (Effects of the structural change)
In the economy with endogenous structural change the following holds true:

1. *The environmental degradation is lower than in the economy with a constant range of technologies;*

2. *The economy exhibits a higher capital accumulation and a higher productivity growth because of lower environmental damages.*

To illustrate that proposition, we consider a numerical example with some plausible parameter values given in table 1.

Table 1: Parameters used in illustrations

Parameter	Value
α	0.4
δ	0.5
β	0.1
μ	0.2
r	0.05
N_0	1

Given these parameter values, the evolution of the environment is illustrated in figure 2 for the economy with structural change and without, i.e. for a fixed range of technologies. The state of the environment under structural change is stabilized in the medium-run because of the introduction of cleaner technologies and because of the

[5] In the model without structural change, horizontal innovations are absent, tending to raise investment in productivity growth. However, the negative effect of a rising environmental degradation will always dominate sooner or later since it increases over time.

out-dating of older ones. In the long-run, however, the environmental pollution rises again because the effect of cleaner technologies is dominated by the strong increase in the (average) productivity and the ensuing output growth that exerts a negative impact on the environment.

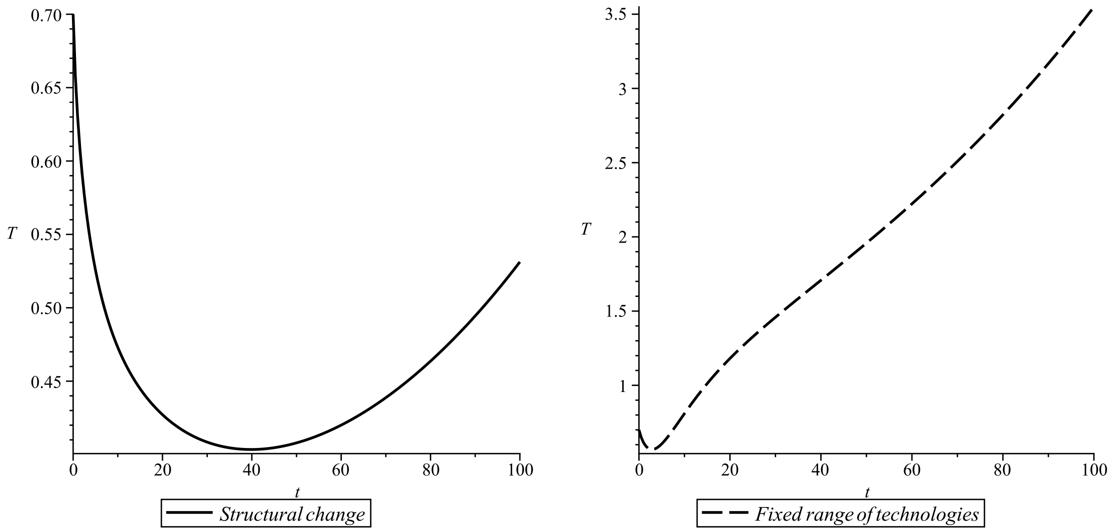
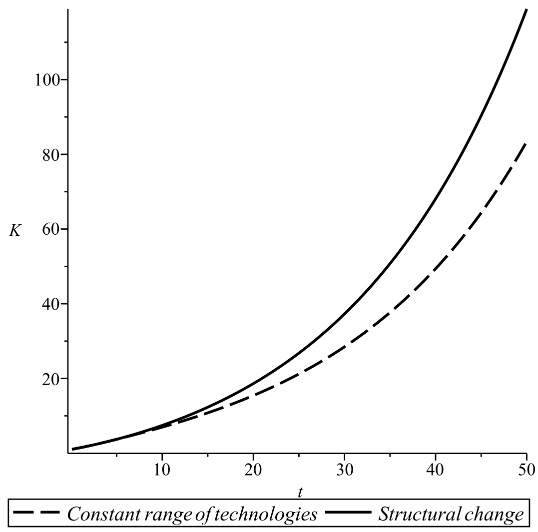
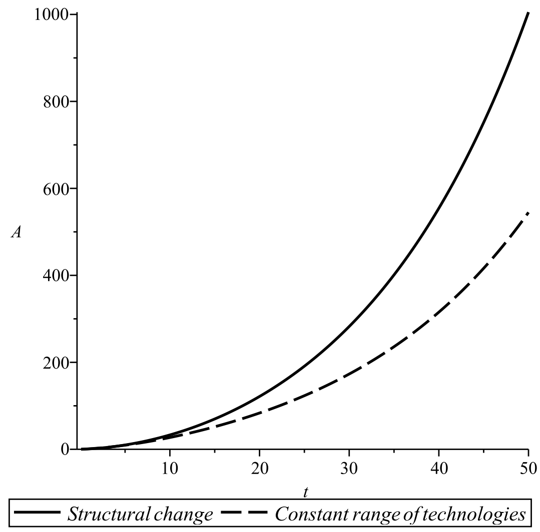


Figure 2: Influence of structural change on the environment

The economic evolution is shown in figure 3 where the evolution of the capital stocks and of the average productivities are depicted. It can be seen that both capital and productivity are higher in the case of structural change. It is then straightforward to conclude that the output growth is also higher in the economy with structural change compared to the economy with a fixed range of sectors. It should be noted that this is not the consequence of a different size of the economies in terms of the spectrum of technologies used (as this is constant in both cases), but rather a result of the composition of this range determined through the speed of structural change.



(a) Capital



(b) Productivity

Figure 3: Influence of structural change on the economy

Thus, it can be stated that the economy with structural change is characterized by smaller environmental degradation, by a faster capital accumulation and by a higher productivity growth leading to higher output growth.

It should also be pointed out that environmental degradation continues in the long-run as output grows unless resource are used for abatement. The simplest way to achieve a constant level of environmental pollution would be to levy a lump-sum tax and to use the tax revenue for abatement, for example. The question of how environmental pollution can be stabilized in growing economies has been the subject of a great many studies (see e.g. the models in Greiner and Semmler, 2008). Therefore, we do not treat this problem but, rather, focus on the relation between structural change, economic growth and environmental pollution with the latter determined by the economic system alone, neglecting abatement activities.

4.2 Comparing the model to the one without environment

To consider the impact of environmental pollution on the economy with structural change we compare the benchmark economy of Bondarev and Greiner (2014) with the one described in this paper. First, it should be noted that capital accumulation and, thus, productivity growth is slower under environmental pollution due to the presence of the damage function reducing output. This follows from (26) and from the evolution of productivity, (8), with g_i given by (17). Next, since output is affected identically by the environment in all sectors, the output growth with environmental pollution is

$$\left(\dot{Y}/Y\right)^T = \alpha\left(\frac{1}{1+T}\right)\frac{\dot{\bar{A}}^T}{\bar{A}^T}(N^T_{max} - N^T_{min}) - \frac{\dot{T}}{(1+T)^2}, \tag{38}$$

where the first component is the same as in the benchmark model multiplied by $1/(1+T)$ and the second is determined by environmental degradation given in (37). It should also be pointed out that the productivity growth is slower in the economy with environmental pollution, but the structural change is faster, see the discussion preceeding equation (36).

As long as the environmental degradation continues, that is as long as $\dot{T} > 0$, the output growth is slower than without environmental spillover. However, what distinguishes our model from other endogenous growth models is that the environmental degradation slows down because of structural change since the latter implies that the emissions intensity declines. Thus, after some point in time the environment starts to regenerate and it is possible to have $\dot{T} < 0$. This happens when the core of the economy includes only technologies with very small environmental impact, $i \in \mathcal{O} : \iota(i) \to 0$, and the regeneration rate μ of the environment is higher than the impact of emissions. Hence, in the medium-run the economic growth of the economy under environmental spillovers may be even higher, than that of the benchmark model.

However, in the long-run the output will slow down, since the temperature starts to increase again due to the higher productivity growth rate that exceeds the decrease of emissions intensity. The length of the period during which the recovery of the environment is observed depends on the relationship between μ, the regeneration rate of the environment, and α, the elasticity of output with respect to technology that is the same for all sectors. Figure 2 illustrates the stabilization and the regeneration of the environment during 50 years (periods) for the model with structural change.

Substituting equation (37) into (38) provides the foundation for the comparison of output growth rates:

$$\left(\dot{Y}/Y\right)^T - \left(\dot{Y}/Y\right)^O = \frac{\alpha}{1+T}\frac{\dot{\bar{A}}^T}{\bar{A}^T}\mathcal{O}^T - \frac{(\bar{A}^T)^\alpha \ln(N^T_{max}/N^T_{min})}{(\mathcal{O}^T)^2(1+T)^2} + \frac{\mu T}{1+T} - \alpha\frac{\dot{\bar{A}}^O}{\bar{A}^O}\mathcal{O}^O =$$

$$\frac{\mu T}{1+T} - \frac{(\bar{A}^T)^\alpha \ln(N^T_{max}/N^T_{min})}{(\mathcal{O}^T)^2(1+T)^2} + \frac{\mathcal{O}^T - \mathcal{O}^O}{1+T}\left(\frac{\dot{\bar{A}}^T}{\bar{A}^T}\right) - \alpha\mathcal{O}^O\left(\frac{\dot{\bar{A}}^O}{\bar{A}^O} - \frac{1}{1+T}\frac{\dot{\bar{A}}^T}{\bar{A}^T}\right) \quad (39)$$

As long as the environmental state is stabilized, $\dot{T} \leq 0$, the regeneration rate of the temperature is equal to or higher than emissions from output and the growth rate of the economy is actually boosted. In the long-run, when the growth of productivity and, thus, of total output dominates the effects of cleaner technologies, the degradation of the environment starts again and the output growth diminishes to zero. The following proposition summarizes our results.

Proposition 2 (Environmental impact on the economy)
In the economy with structural change environmental spillovers lead to the following:

1. *The environment recovers in the medium-run boosting output growth;*
2. *Capital and productivity of each sector grows at a lower rate than in the benchmark model without environmental degradation;*
3. *The economic growth rate is almost always lower than in benchmark economy and becomes negative in the long-run;*
4. *Structural change is faster than in the benchmark economy, but the core is still constant.*

Proposition 2 demonstrates the consequences of the market failure in internalizing environmental spillovers under structural change. The decentralised economy responds to the environmental pollution by speeding up structural change, compared to the benchmark model, but the higher variety of technologies cannot offset the negative impact of environmental damages without any government intervention. Therefore, environmental policy, such as a tax on the polluting output, is necessary to correct the market failure. That is the contents of the next subsection.

4.3 Environmental policy and impact on growth

Given the results from the previous sections, it is straightforward to note that the government should stimulate an increase in the range of technologies to slow down environmental degradation. This can be done by internalizing the environmental impact caused by each technology. The latter can be achieved by levying a tax on the revenue of the firms in the manufacturing sector with the tax rate, τ^E, determined by the degree of environmental damages caused by the respective firm. Thus, the tax rate can be written as,

$$\tau^E(i) = \iota(i) = 1/i, \tag{40}$$

where the superscript E denotes the situation with the tax rate τ^E. At this stage we do not study where the taxes are going to since competitive uses of environmental taxation (R&D subsidies, consumption subsidies, etc.) may constitute an interesting follow up study. Our main concern is to demonstrate that under such a tax system the resulting outcome is better both for the economy and for the environment.

Given the tax specified in (40) the profit function for the manufacturing firm is written as,

$$\Pi_i = (1 - \frac{1}{i})P_i Y_i - L_i - \Psi. \tag{41}$$

Then, the price demanded for the product i is obtained as,

$$P_i^E = \frac{\epsilon}{\epsilon - 1}(1 + T)\frac{i}{i - 1}A_i^{-\alpha} \tag{42}$$

and labor demand is proportionally reduced:

$$L_i^E = \frac{1}{1 + T}\frac{i - 1}{i}L_i^O. \tag{43}$$

This will change capital accumulation and, thus, total productivity growth by the factor $\mathcal{O}$ because labor and, therefore labor income, takes a different form:

$$L^E = \frac{1}{1+T} \int\limits_{N_{min}}^{N_{max}} \frac{i-1}{i} L_i^O di = \frac{1}{1+T} \left(\mathcal{O} - \ln(N_{max}/N_{min})\right) L. \tag{44}$$

Depending on the dynamics of the operational range in this regulated economy, capital accumulation may be faster or slower than in the economy without taxation. Now, turn to the changes in profits of R&D. Since a higher index of the sector implies a lower tax burden, the profits for R&D are now increasing in i,

$$\frac{\partial \pi^R(i)}{\partial i} > 0. \tag{45}$$

Because of that, horizontal innovations are no longer constant but increase in time making variety expansion a non-linear convex function. Since the processes N_{min} and N_{max} are proportional to the variety expansion, they are also non-linear. The core $\mathcal{O}$ is then an increasing function of time and not constant any longer.

It is difficult to obtain the analytic form of optimal investments for each technology since the shadow costs of these investments are no longer identical. The reason for that is that shadow costs, which determine the investments into vertical innovations, depend on the derivative of the patent price with respect to productivity and are no longer constant across the technologies. Indeed, they now depend on i because profits are different across sectors:

$$\Pi^E(i) = P_i^E Y_i^E - L_i^E - \Psi = \left(\frac{\epsilon}{\epsilon - 1} - \frac{1}{1+T}\frac{i-1}{i}\right) L_i^O - \Psi; \tag{46}$$

$$p_A(i) = \int\limits_{t_{max}(i)}^{t_{min}(i)} \mathrm{e}^{-r(t-t_0(i))} \Pi^E(i) dt. \tag{47}$$

The shadow costs are then decreasing in i making investments into newer technologies more attractive. The resulting economy is characterized by a higher variety expansion speed and a higher productivity growth, while environmental pollution is significantly decreased because of an increasing core $\mathcal{O}$ over time.

We summarize the results in proposition 3.

Proposition 3 (Effects of environmental policy)
In the decentralised economy with structural change and environmental pollution, the introduction of environmental taxes $\tau(i) = \iota(i)$ *will lead to the following:*

1. *The operational range of sectors* $\mathcal{O}$ *is an increasing function of time,* $\dot{\mathcal{O}} > 0$*;*

2. *Environmental degradation is slowed down compared to the economy without taxation,* $\dot{T}^E < \dot{T}^T$*;*

3. *Economic growth is faster than in the deregulated economy:*

$$\frac{\dot{Y}^E}{Y^E} > \frac{\dot{Y}^T}{Y^T}. \tag{48}$$

The last statement follows from the fact that the core $\mathcal{O}$ is constant and environmental degradation is higher in the deregulated case, i.e. in the economy without environmental taxation.

Thus, proposition 3 demonstrates that the introduction of an environmental tax leads both to higher growth and to smaller environmental degradation. This shows that the internalization of the negative externalities does not go along with a reduction in production but rather leads to higher output. Hence, taxing the polluting output is clearly Pareto improving.

5 Conclusion

In this paper we have studied the consequences of environmental pollution in a growing economy taking into account endogenous structural change. The model with unified horizontal and vertical innovations allows to consider different environmental damages caused by different technologies rather than positing an ad hoc emissions intensity in the economy. It turns out that the mix in the emissions intensity is crucial with respect to the environment and for the economy. The decentralised economy without regulation cannot cope with environmental degradation, even if newer and less polluting technologies are continously introduced, since it lacks sufficient incentives for boosting fundamental research that generate less polluting technologies. To achieve both positive long-run growth of the economy and to avoid a permanently deteriorating environment, it is necessary to speed up structural change, i.e. the expansion of the range of operational technologies. Such an expansion can counterbalance the negative influence of productivity growth on the environment and can be achieved through environmental taxation.

In particular, we have seen that environmental pollution enhances structural change but reduces output growth. On the other hand, allowing for structural change weakens the negative impact of pollution, thus, fostering economic growth. In the medium-run, environmental pollution can even decline because structural change leads to the replacement of older more polluting technologies by newer and cleaner ones. However, in the long-run that effect is dominated by the productivity increase that leads to a rising output that pollutes the environment. Finally, taxing the polluting output is both beneficial for the environment as well as for economic growth and, therefore, yields a Pareto superior outcome compared to the economy without taxation.

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