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Essays on Communication and Information Transmission

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Université Paris 1 Panthéon-Sorbonne	and	Universität Bielefeld

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This thesis has been written within the
European Doctorate in Economics - Erasmus Mundus (EDEEM),
with the purpose to obtain a
Joint Doctorate Degree in

Applied Mathematics	**and**	**Economics**
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Jury

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Abstract

This Ph.D dissertation addresses different issues concerning communication and information transmission in a game theoretical framework. I analyze different dilemmas that a player who sends information has to deal with. These dilemmas correspond to the following questions: "Should I invest into a verifiable message?", "When should I pass my information?" and "Is it better if I do not send my information, but collect information from others?".
This thesis includes an introduction and three chapters. The introduction contains a general motivation for the three different problems that I model in this thesis. I give a detailed overview of all the chapters, survey the related literature and compare it to my results.

The first chapter, "Communication Games with Optional Verification" studies a Sender-Receiver game where the Sender can not only choose between cheap-talk messages, but can also select a costly verifiable message. I provide conditions under which the Receiver can enforce the Sender to tell the truth in all states of the world. Furthermore, I deal with the situation when full revelation is impossible and apply my results to the often used quadratic loss function.

The second chapter, "Information Transmission in Hierarchies" deals with a different problem of communication. Players in a hierarchy have to transmit information to their superiors. They face a communication dilemma, because they have two contrary incentives: On the one hand, they like to pass their information as early as possible, on the other hand, they can get an additional reward if they are the last player to submit. In this framework, I give a detailed analysis of a model where all players are directly connected to a principal. Furthermore, I study hierarchies with several layers. I compare different hierarchical structures and show that the speed of the centralization process does not only depend on the parameters of the model, but also on structure of the hierarchy.

The third chapter, "Centralizing Information in Endogenous Networks" analyzes the transmission of information in team projects when all players compete for the leadership of that project. In this setting a different communication dilemma arises: The players want that one of them centralizes all information as fast as possible and at the same time they have strong incentives to be the one that centralizes the information. I prove that in a connected network there is always a single player who becomes the winner. Not only the network structure, but also the discount factor and the decision order determine who centralizes all information. Furthermore, I show that only minimally connected networks can be pairwise stable. I state further conditions to find the set of pairwise stable networks for all values of the parameters.

Keywords: cheap-talk, communication, costly disclosure, dynamic network game, full revelation, hierarchical structure, information transmission, information centralization, network formation, pairwise stability, Sender-Receiver game, verifiable information.

Resumé

Cette thèse de doctorat traite de différentes questions concernant la communication et la transmission d'informations dans le cadre de la théorie des jeux. J'analyse différents dilemmes auxquels peut être confronté un joueur qui envoie des informations. Ces dilemmes correspondent aux questions suivantes: "Devrais-je investir dans un message vérifiable?", "Quand dois-je transmettre mon information?" et "Est-il préférable de ne pas envoyer mon information et uniquement de recueillir l'information des autres?".
Cette thèse comprend une introduction et trois chapitres. L'introduction contient une motivation générale pour les trois problèmes que je présente dans cette thèse. Je donne une vue d'ensemble détaillée de tous les chapitres, j'examine la littérature relative au sujet et je la compare à mes résultats.

Le premier chapitre intitulé "Jeux de communication avec vérification facultative" étudie un jeu entre un émetteur et un Récepteur lorsque l'Émetteur peut choisir entre un message vérifiable coûteux ou un message non vérifiable (au sens de la conversation libre ou cheap talk). Je fournis des conditions dans lesquelles le Récepteur peut imposer à l'Émetteur de dire la vérité dans tous les états du monde. De plus, je traite la situation dans laquelle une révélation complète est impossible et applique mes résultats au cas classique d'une fonction de coûts quadratique.

Le deuxième chapitre intitulé "Transmission d'informations dans les hiérarchies" traite d'un autre problème de communication. Les joueurs dans une hiérarchie doivent transmettre leur information à leurs supérieurs. Ils sont confrontés à un dilemme parce qu'ils ont deux motivations opposées: d'une part, ils souhaitent transmettre leur information le plus tôt possible afin que le projet aboutisse; d'autre part, ils peuvent obtenir une récompense supplémentaire s'ils sont le dernier joueur à soumettre leur information. Dans ce cadre, j'analyse de façon détaillée le cas où tous les joueurs sont directement connectés à un directeur de projet. J'étudie également les hiérarchies à plusieurs couches. Je compare différentes structures hiérarchiques et je montre que la vitesse du processus de centralisation ne dépend pas seulement des paramètres du modèle, mais aussi de la structure de la hiérarchie.

Le troisième chapitre intitulé "Centraliser l'information dans les réseaux endogènes" analyse la transmission d'information dans des projets d'équipe où tous les joueurs rivalisent pour la direction de ce projet. Dans ce contexte, un dilemme différent intervient: les joueurs veulent qu'un unique joueur centralise toutes les informations aussi rapidement que possible, tout en étant fortement incité à être ce joueur. Je prouve que dans un réseau connecté, il y a toujours un seul joueur qui devient le gagnant. Non seulement la structure du réseau, mais aussi le taux d'actualisation et l'ordre des décisions déterminent l'acteur qui centralise toutes les informations. En outre, je montre que seuls les réseaux minimalement connectés peuvent être stables par paires. J'indique d'autres conditions pour trouver l'ensemble de réseaux stables par paires pour toutes les valeurs des paramètres.

Contents

Introduction (English version)

"Scientia est potentia"
– Sir Francis Bacon

"Power is power"
– Cersei Lannister

Knowledge and information are key components of our every day life. We use our knowledge in most decisions we make. The information about the weather determines how we dress, our knowledge about traffic defines which route we take and we go to buy food when we know that the shops are open. How do we get all this information? We were either taught by parents, family and friends or learned by ourselves by experience.
Communication, verbal or written, is the way to exchange information. We ask our friends about the weather, we ask them if there is a lot of traffic or if they know whether a shop is open. We prefer to get the answers from a trustworthy person and for that we address our friends or ask people on the street that seem trustworthy.
This thesis deals with different aspects of information transmission and communication. In different models I analyze the problem that players (in a game) may have an incentive to give false information or hoard information for themselves. The main aspect of the first chapter is communication when the Sender and the Receiver of a message have different preferences. What should I believe about the message concerning the weather if I know that the Sender prefers to go to the cinema over laying on the beach? Which route should I take when I know that the person who navigates does not like highways? When do I go shopping if I know that my friend who gave me the information about opening hours wants me to go there as early as possible? A Receiver of a message has to deal with those questions if the Sender cannot verify any of the transmitted information.

The early literature on communication in Sender-Receiver games can be divided into two different strains. Crawford and Sobel (1982) introduce cheap-talk communication. The information exchanged can be the truth, partially true or completely wrong. The Sender has no way to verify his message and the Receiver has to take into account the Sender's preferences when he makes his decision whether to believe the message or what information to extract from it.
This is also the type of information exchange that happens most in our life: We hear from people and even though they cannot provide hard evidence we may believe their message.
On the other hand, it might be possible for the Sender to verify his message. Grossman (1981) and Milgrom (1981) model this such that the Sender can only tell the truth, but he does not have to reveal the entire truth. In the examples the partial truth are messages such as "it is less than 20°C outside", "this is not the fastest route" or "I am sure the shop closes before 20:00".
In Chapter 1 I combine those two different strains of communication literature into one model. I consider a Sender-Receiver game in which the Sender has some private information about the state

of the world. He sends a message to the Receiver who then chooses an action. The action yields to a payoff for both players which depends on the state. While in some states both players could have similar interests, the players preferences can be completely different in other states.
In this model the Sender's message can be either cheap-talk or verifiable. I assume that the Sender has several cheap-talk messages to choose from, one for each possible state of the world. While sending a cheap-talk message is free, the Sender has to pay some costs if he sends the verifiable message.
The Receiver has a certain belief about the state of the world and updates his belief after receiving the message. The verifiable message reveals the true state of the world and in that case the Receiver can choose the action he likes most. On the other hand, after receiving a cheap-talk message, the Receiver has to take into account that the Sender may lie. The Receiver knows about the Sender's preferences and just believes the cheap-talk message if he knows that the Sender has no incentive to lie.
Without any strong assumptions, I state conditions for equilibria in which the Receiver learns the truth in each state. Those fully revealing equilibria are optimal for the Receiver as he can choose the action he prefers. Under certain conditions the Receiver can enforce the Sender to tell the truth: As long as the Sender likes the Receiver's preferred action more than another action, the Receiver can threaten to select that other action and by that he can force the Sender to tell the truth or use the verifiable message. Full revelation can either be achieved just by verifiable messages, just by cheap-talk or by different types of messages in different states. All those different equilibria come with different conditions on the players preferences. I derive those conditions for a discrete setting. Furthermore, I state conditions for different partial revealing equilibria. Even if the Receiver does not learn the entire truth, the message might reveal some information to him so that he can reduce the set of possible states. I show that in a continuous state space, the conditions for full revelation are not strong enough. Furthermore, if the utility functions are continuous as well, full revelation is impossible in the continuous setting.
Applications for the model can be found in many different settings. Often it happens that a Sender can choose to verify his message, but usually this comes with some costs. Going back to the previous examples the verifiable message corresponds to a weather-forecast, online traffic-information or the detailed opening hours of a shop from their website. Another application that most people experience, is the situation of a job interview and the documentation of skills. Most technical or language skills are verifiable, but usually it is not only time intensive but also expensive to get a certificate, e.g. TOEFL test or Cambridge Certificate. Those tests give an exact result about the abilities of the applicant and it is uncommon that an applicant hands in those certificates if they do not improve his situation. Even if we see recommendation letters as verifiable messages, I argue that those are costly. In this case the costs are not monetary but investments into effort and time. The result from my model is that the interviewing firm could not take into account any non-certified skills to enforce the verifiable message. We often experience that job offers come with a list of certified requirements. Our result is that if all applications that do not fulfill this requirements are not considered, the applicants that fulfill the requirements hand in proofs.

Chapter 2 deals with another setting of information transmission. In organizations many different people work on the same project. If information from all those people is necessary to complete the project, they have no incentive to lie or only tell part of the truth. Still, the information transmission often comes with a problem: A worker might have an incentive to hold back his information at the beginning. This happens especially if this player is waiting for an additional incentive that is given by a later transmission of his piece of information, for example if he can deliver the final piece of a puzzle.

I model the organizations structure as a hierarchy, where the principal wants to collect all the information. A player has several periods of time to pass his information to his superior. I assume that the player gets a payoff as soon as he has passed his part and that he prefers to get this payoff as soon as possible. In addition the player has an incentive to hold his information private, because he knows that his superior gives an additional reward to the player who submits the last piece of information. Once a superior has collected the information from all of his agents, he faces the same problem when he competes over a reward with other players from his level of the hierarchy.

The model focuses on the time it takes the principal of the organization to get all information. Many different parameters have an impact on that duration: Not only does the payoff of the players matter, but also their discount factor and the additional reward. Furthermore, the hierarchical structure plays an important role. Depending on the arrangement of players, the duration varies between several periods. I show how to determine the optimal hierarchical structure for a given information distribution and for a given reward. While in some cases it is fastest if everybody reports directly to the principal, other structures can arise for certain sets of parameters. There are many different ways to arrange players in hierarchies: If we sort the players by their utility they get from passing, we can either arrange those who get a higher utility at the top or at the bottom. Furthermore, we can have a symmetric or asymmetric structure and we can install intermediaries with information or uninformed intermediaries.

I start the analysis with a simple model in which one principal is connected to all other players. In this setting I can derive all equilibria and focus on the duration. I show the impact of all the parameters and also compare different information distributions. With this as a foundation, I extend the setting to a multi-layer model. I state the equilibria for each sub-hierarchy and in examples I compare different hierarchies. Even though the optimal structure depends highly on the parameters, I prove that certain structures can be excluded from consideration, because they can never be optimal.

Applications can be found in many different fields. One can think about a researcher working on an important mathematical proof. He requires information from fellow researchers, who themselves might require information from others and who want to benefit from the important proof. While their utility comes from citations, they can also get the principals' gratitude if they help him with the final step of the proof. So, withholding the information first might be beneficial for them afterwards. The additional reward can be seen as a joint future project, co-authorship or an invitation to a conference.

Also in politics we see this kind of information transmission. One example is the European Union and its hierarchy of countries and regional parliaments. In some cases all the parliaments need to give their agreement which relates to passing the information. One small region can block the entire process by not signing the law and by that this region can cause a delay. This has happened in 2016 when Wallonia has blocked the CETA negotiations.

In Chapter 3, I work on a different angle of information transmission. While in firms most of the time a hierarchical structure exists, this is different in many team projects. Often teams consist of members that are on the same hierarchical level. In those teams it happens that, even though everybody works together on a project, there is a large competition. All players want to represent their team to the outside and become the leader of the project.
We can see this happening in many examples: Usually multi-lateral agreements between countries just get represented by one country or a small subset of the countries. For the climate agreement in Paris 2015 nearly 200 countries attend the meeting and more than 170 countries have signed the agreement. At the same time we know that only a small number of countries are seen as the face of this agreement.
Similar situations happen in research teams: Many people work together on a project, but single researchers are rewarded with awards or even the Nobel Prize. The same competition for leadership takes place in R&D cooperation between companies or in joint ventures.
In this model I assume that the different players are linked in a network and can share their information only with the players they are directly connected to. At the beginning all players have a unique piece of information and they work in that project for a certain time. Until this deadline is reached they can pass the information to all their neighbors in the network. The game ends as soon as one player has collected all the information and the project is successful. In that case all players receive a certain payoff, while the player who has centralized the information gets an additional reward. I assume that the players discount over time and would like to have their payoff as soon as possible.
The model consists of two parts: In the first part I study how the players share information. In each period they can choose between holding the information for themselves or sharing all information with all their neighbors. I assume that there exists a certain order in which the players make their decision. This order can be seen as the result of previous team projects. Since the players want to represent the project to the outside, they might hoard information for themselves so that they centralize everything and get an additional reward. At the same time they want to finish the project as fast as possible. I show that in equilibrium only a single player will centralize all the information. Who this winner is going to be depends not only on the decision order and the network structure, but also on the discount factor. If the players discount strongly, they care less about the reward and the player that can centralize the information fastest will do so in equilibrium. On the other hand, if the discount factor is higher, the network structure plays an important role. I provide a way to find the winner for all possible discount factors and all network structures. Furthermore, I characterize the time it takes them to complete the project.
In the second part of the model I analyze the network formation. I assume that the players are arranged in a connected network and then show which additional costly links between players get formed or which links get cut. These changes happen until the network is pairwise stable, i.e. if

no two linked players want to cut their connection and if no two not-linked players want to build a link. I prove that only minimally connected networks can be pairwise stable. In a minimally connected network all pairs of players are just connected by a single path through the network. This result shows that the information will only flow through this single path and that is the main reason why the players cut all other links.
Which of the minimally connected networks are pairwise stable depends on the arrangement of players and the discount factor. I provide a way to check a network for stability. In addition, I show that certain structures can never be stable.
In a last part I prove the robustness of the model: Once a player does not need all information, but can do without the information of one player, the pairwise stable networks are those where one player is disconnected. All other players build a connected component for which all the previous results hold.

Related literature

This section provides a short overview of the related literature in order to position the contribution of this thesis in the literature. Firstly, I present the literature on cheap-talk communication and some applications. Secondly, I review another strain of communication literature, dealing with verifiable messages. The third part contains a discussion of other models of communication and of those models that combine cheap-talk and verifiable messages. I point out the main differences between these models and my setting in Chapter 1.
Fourthly, I provide an overview of the literature dealing with information transmission, which is closely related to the second and third chapter of this thesis. The fifth part focuses on literature on leadership. Finally, I review models on network formation and stability.
Additional literature reviews can be found in the introductions of each chapter.

Cheap-talk

The foundation for all the literature dealing with cheap-talk is the paper by Crawford and Sobel (1982). In their model a Sender observes his private type and then sends a message to the Receiver. The Receiver takes the message into account and chooses an action that generates a payoff for both players. The authors use a parameter b to model the differences in the players preferences. They prove the existence of different equilibria depending on b and give additional conditions to ensure the uniqueness. If both players have at least some interest in common, cheap-talk can be credible. Their example of a quadratic loss model is also one of the examples I use in the first chapter of this thesis. The authors point out that commitments to tell the truth would benefit the players, but those commitments are impossible, because the Sender would have an incentive to deviate. This is caused by the fact that he can not verify any of his messages.

The work of Crawford and Sobel has been extended into many different directions and also found applications in several fields. A survey on this literature can be found in Sobel (2009).

In this part I focus on the importance of cheap-talk in different models.

Farrell and Rabin (1996) use the results of Crawford and Sobel for different examples. They show that if the Sender's preferences over the Receiver's beliefs are correlated, then cheap-talk can be used to improve the utility of both players. The uninformative equilibrium is more plausible if the players preferences do not coincide. The authors go through different applications and discuss the most plausible equilibria. In another paper Farrell (1987) comes up with similar results for a game in which potential entrants into a market cooperate. This can be achieved by cheap-talk as long as there is no conflict in their interests. As soon as their preferred actions differ, the communication does not increase the players' utilities.

Farrell and Gibbons (1989a) add a round of cheap-talk communication before negotiations start in a bargaining model. They prove that this communication yields to a new type of equilibrium, which cannot occur without cheap-talk communication. Depending on the communication, the seller and bidder change their behavior. If both parties show their interest in the deal, they give up some of their bargaining power to ensure that longer negotiations take place.

A different approach is done by Sobel (1985). He focuses on a repeated game in which trust is established between Sender and Receiver. His analysis shows how reliability is built up and destroyed. His motivation is the communication between a spy and a representative of the government. Players with similar interests either cooperate until one of them decides to betray the other and even players with different preferences first create some trust only to use it for their later benefit.

Stein (1989) applies the problem of trust and cheap-talk to the problem of the Federal Reserve and its announcement about future policies. The Federal Reserve cannot reveal all their information, because it would change expectations and manipulate speculators. Still they want to pre-announce their policies so that the markets can react fast. Cheap-talk communication allows the Federal Reserve to make some announcements and yields to faster reactions of the market, while manipulation is no longer possible.

Cheap-talk is often used in experiments. Croson et al. (2003) conduct an experiment in which participants of an experiment can use cheap-talk communication before starting to bargain. The authors show that in these experiments cheap-talk influences the short-term and long-term outcomes. In their experiment, the players relayed more on non-verified information then the authors expected them to do according to theoretical results.
The results of several experiments and related theories are summed up in Crawford (1998).

Verifiable messages

The economic work on verifiable communication started with the two papers of Grossman (1981) and Milgrom (1981).

Milgrom (1981) introduces a way to model the arrival of good news for a firm and its effects for example on the stock price. He applies his results in four different examples. For us the most relevant application is on "games of persuasion": A salesman can reveal different pieces of information to a buyer. All of these pieces are truthful, but they all just reveal a part of the relevant information. This allows the buyer to enforce full revelation. To do so the buyer follows the simple strategy in which he assumes that all withhold is bad and signals low quality. In Chapter 1 I observe something similar if the Sender can choose between cheap-talk and verifiable messages.

Grossman (1981) addresses a similar problem from a different point of view. He models a way for a seller of a product with good quality to distinguish himself from a bad seller. The additional information on the quality of products stops the buyer to pay the same price for items of different qualities. The author comes up with the same result as Milgrom: If the seller has the possibility to fully reveal all information, he does so in equilibrium. Otherwise the buyer assumes the worst quality and is only willing to pay a low price.

While in the work of Milgrom (1981) and Grossman (1981) the Sender has no incentive to hold back information, this is different in the setting of Verrecchia (1983) in which costs for disclosure of information are introduced. With this addition there is an incentive for a seller to withhold information if the costs are high. The buyer does not know if the costs of disclosure is too high or if the quality is such that the seller prefers not to reveal it. The author determines a threshold level of disclosure if the costs are constant. If the quality is above the threshold, the seller reveals the information, while if it is below, he keeps the information private. The author shows that his results are the same as the results of Grossman and Milgrom if the costs are zero. Furthermore, he extends his model to quality-dependent costs, even though it makes it impossible to ensure the existence of the threshold level of disclosure.

Another reason why information is not always revealed is analyzed by Dye (1985b). The author argues that firms have proprietary information and if a firm would reveal these information, then potential competitors have more information about the market. Dye shows that if a firm is forced to reveal more information, because of new regulations, the investors not necessarily learn more about the firm's perspectives. One possible reason for this is given by less voluntary reports as voluntary and mandatory reports may work as substitutes. In another paper Dye (1985a) provides two theories about why firms also withhold non-proprietary information. The author argues that previous theories lead to the result that all information should be revealed, even though this does not happen in reality. One reason he gives is that the investors may not know what information the management has and so they cannot enforce the disclosure.

In addition, Darrough and Stoughton (1990) show that even if proprietary information gives potential competitors an advantage on their decision to enter the market, a fully revealing equilibrium exists if the market entry cost is low.

Francis et al. (2008) use the reports of 677 firms to analyze the correlation between voluntary disclosure, earning qualities and cost of capital. They point out that the voluntary disclosure of information depends mainly on the earning qualities. Firms with better earning qualities reveal more information voluntarily.
Skinner (1994) examines the data of 93 NASDAQ firms and gives reasons why firms disclosure information voluntarily. The author provides evidence that managers have incentives to inform the public early about large negative earning surprises.

The paper of Cho and Kreps (1987) is not directly related to verifiable messages, but deals with one problem arising from these models. In many settings there exist several equilibria. The authors provide additional criteria to eliminate equilibria that are not intuitive.

A survey on disclosure and verifiable messages is given by Verrecchia (2001).

Other communication models

Not all communication models are based on cheap-talk or verifiable messages. In this part I provide some examples of papers that have created their own way of modeling communication and of papers that have a similar aim as the first chapter of this thesis. Especially for those papers that combine different types of communication, I point out the main differences to the results of Chapter 1.

Kartik (2009) analyzes communication between a Sender and a Receiver where the Sender has to pay costs if he lies in his message. In this setting, there exists no fully revealing equilibrium: The Sender always reports a better type than he actually is. The author gives conditions under which the low types are fully revealed but the high types are sending the same message. Furthermore, Kartik applies his model to the example of the quadratic loss function. He shows the existence of the equilibrium described above and gives an additional characterization of the cutoff. Players with a type below the cutoff send all different messages, while all types above the cutoff send the message that they are of the highest possible type.

Dewatripont and Tirole (2005) model communication in a completely different way. In their setting both players, the Sender and the Receiver, have to invest into effort. The effort determines if the Receiver understands the message. Their motivation is that not only the Sender has to submit an understandable message, but that also the Receiver has to invest time and effort to understand the message. They differ between two cases: In the first the Receiver does not need to understand the message to take a certain action, while in the second case the Receiver can only implement an action if he understands the message. The authors show the differences between the two cases and give conditions for the optimal effort levels.

Eső and Galambos (2013) combine cheap-talk and verifiable messages in a similar way as I do in Chapter 1. They make assumptions about the players payoff (strictly concave) and that the players optimal actions are strictly increasing in the type. Furthermore, they assume that for higher states of the world the difference in the utility of the optimal actions of Sender and Receiver becomes larger. The authors provide conditions under which the costly, verifiable message is used in equilibrium. Under their assumptions they show that the type space can be separated into intervals and that in each interval either all types send the same message or they all verify their message.
In Chapter 1 I do a similar combination of costly verifiable messages and cheap-talk, but I do not make the same assumptions as Eső and Galambos. Instead I give conditions for fully revealing equilibria without any strong assumption on the preferences and utilities of Sender and Receiver. It is to point out that their assumptions allow for an extension to a multi-dimensional state-space, while in my setting this does not seem easily possible.

Information transmission and processing in networks

The inspiration for the second and third chapter is Hagenbach (2011). She models a game of information transmission in networks. All players work together on a joint project and get a payoff as soon as one player has centralized all information, that winner gets an additional reward. The paper proves the existence of several equilibria with different winners. The author shows how the set of potential winners depends on the deadline.
In Chapter 2, I model the setting in a similar way, but I focus on hierarchies and argue that the last player to pass the information gets an additional reward.
Even closer related is Chapter 3. In the setting of Hagenbach (2011) one cannot model network formation, because for almost all networks there is no unique winner. To change this, I focus on sequential decisions and achieve a unique equilibrium outcome. I am convinced that in many projects the participants do not make their decision simultaneously, but that there is a certain order. Given this order I can get several additional results about the winner and the duration. Furthermore, I can analyze the pairwise stability of networks.

Bonacich (1990) conducts an experiment on information transmission in networks. He gives all players in a network a certain part of a quotation and incentives them to complete it. The setting is close to the one I use and to the one of Hagenbach. The players want to centralize the information so that everyone gets a payoff, but at the same time they try to collect all information to get an additional reward. The author shows how the communication changes between different network structures. The main difference between the experiment and my setting is that players in the experiment can make guesses about missing parts.

An important foundation and good overview about the diffusion of innovations in networks can be found in Rogers (2003). The communication of innovations is close to my models of information transmission. One aspect that cannot be found in this book is the communication dilemma of the players when they have to decide between hoarding or passing information.

Radner (1993) deals with a question related to Chapter 2. He models a hierarchy in which players work as information processors and he tries to minimize the time it takes the principal to get information. Information arrives repeatedly and has to be processed by players. The author shows different ways to arrange players in a hierarchy. The efficiency of the hierarchy not only depends on the rate of information arrival, but also on the number of players. Radner proves that at a certain point the delay does not decrease further even if more players are available.
An important difference to the model in Chapter 2 is the arrival and processing of information. I just deal with a single centralization of information. While in Radner (1993) the delay is caused by the limited processing capacities, I create a model in which the players have an incentive to withhold information and cause delay.

Bolton and Dewatripont (1994) see the organization of a firm as a communication network. In their model a firm deals with the arrival of information. Given that it is too much information to be processed by any group of players, the organization has to be structured such that the flow of information can be handled most efficiently. Players who work on the same kind of problem repeatedly become better at solving it. This incentives the organization to build up certain structures. Similar to the models in Chapters 2 and 3 the aim is that one agent centralizes all the information. One difference between the model of Bolton and Dewatripont and my model is the handling of information. I do not assume that the players need to solve problems to get the information, but they have them from the beginning. The delay in my model is caused by the incentives of the players to hold information back and not by slow problem solving.

Jehiel (1999) provides a setting in which a principal has to make a decision based on the information he gets from his workers. The decision is reviewed by a board which then decides about the employment of the decision maker: If he made a wrong decision, he gets fired. All other players have an incentive to transmit their information to the principal, because they get a share of the surplus generated by the decision of the principal. The players are arranged in a hierarchy and the author gives conditions for the optimal arrangement of players. The problem the players face here is that information might get lost in transmission.
Clearly, the problem faced in this paper is a different then in Chapter 2. In Jehiel (1999) there exists the risk that information is lost, but the time it takes until the principal gets the information plays no role.

Calvó-Armengol and de Marti Beltran (2009) take a different point of view. In their model the players face a common decision problem and can share their private information through a network. The authors introduce a knowledge index on which the player's optimal decision depends in equilibrium. They show that additional links not always benefit the organization. In their setting they find an optimal player arrangement which depends also on their knowledge index.

Garicano (2000) studies the solution of problems in a hierarchy. The author shows how to arrange players in a hierarchy when all players are specialized on a certain task. Players on lower levels of the hierarchy work on simple problems, while the complexity of the problems increases for higher hierarchical levels. A complication arises when the problems are not observable to the organization. Then additional incentives must be given to make sure that each player works on the right problem.

Leadership

The economic theory of teams goes back to Marschak and Radner (1972), but the modeling of leadership got neglected for a long time. Hermalin (1998) analyzes the incentives for players to follow a leader, who might mislead the players. The author discusses two different ways of leadership: Leadership by example or leader sacrifice. The first corresponds to what one can observe in many organizations: The leaders are a good example for the workers and they spend many hours on hard work. By this the leader can convince the other agents that it is worth to put a lot of effort. The other possibility is that the leader sacrifices some of his payoff and uses it for gifts. Even if the gifts do not generate additional utility for the players, the gifts can show the agents that it is worthwhile to put in long hours of activity. I focus on another aspect of leadership in Chapter 3. If players work together on a project, they compete for the role as the leader of the project.

In Dewan and Squintani (2017) a leader gets elected. He gets advice from agents in form of cheap-talk messages. The leader has to decide which of the players he can trust and whom he should not trust. The players have different information about the state of the world, but also different incentives. The authors show that the quality of a leader's decision depends on the number of his trustworthy advisers.

A survey on leadership is given by Ahlquist and Levi (2011). To the best of my knowledge, there is no economic literature dealing with the competition for leadership in teams, like I model it in Chapter 3.

Network formation and stability

Jackson and Wolinsky (1996) analyze the concepts of stability for networks and introduce pairwise stability. Under this stability concepts a link between two players only gets formed if both players want to have this link. A link between two players gets cut if at least one player likes to cut it. This means that two players can link only if they both want it, but once two players are linked, both players can decide on their own to destroy the link.

This concept has many applications and is used in many different models. I use this concept in Chapter 3 under the additional assumption that links are costly. In that case both players only form

a link if they receive a strictly higher payoff in the network including that link and a player cuts a link if his payoff does not decrease.
This stability concept is used also in our every day life. It takes two persons to build a friendship, but only one person to destroy it. Similar, a cooperation between firms only develops if both firms profit from it, while it is enough that one firm stops cooperating to destroy the link. Furthermore, this is also how friendships on social media platforms such as Facebook work.

Several other stability concepts and games on network formation have been introduced in the last twenty years. A survey is given by Jackson (2005). A more detailed overview on the economics of networks can be found in the books of Goyal (2012) and Bramoullé et al. (2016).

Introduction (Version française)

"Scientia est potentia"
– Sir Francis Bacon

"Power is power"
– Cersei Lannister

Les connaissances et l'information sont des éléments clés de notre vie quotidienne. Nous utilisons nos connaissances dans la plupart des décisions que nous prenons. L'information sur la météo détermine la façon dont nous nous habillons, les informations sur le trafic définissent la route que nous prenons et nous allons faire les courses lorsque nous savons que les magasins sont ouverts. Comment obtenons-nous toutes ces informations? Soit elles nous ont été enseignées par nos parents, notre famille et nos amis, soit nous avons appris par nous-mêmes grâce à notre expérience.
La communication, verbale ou écrite, est la manière d'échanger des informations. Nous demandons à nos amis comment est le temps, nous leur demandons s'il y a beaucoup de trafic ou s'ils savent si un magasin est ouvert. Nous préférons obtenir les réponses d'une personne digne de confiance donc nous nous adressons à nos amis ou à des gens dans la rue qui semblent dignes de confiance.
Cette thèse porte sur différents aspects de la transmission d'information et de la communication. J'analyse dans différents modèles les problèmes posés lorsque les joueurs (dans un jeu) peuvent être incités à donner une information fausse ou à garder une information pour eux-mêmes. Le cœur du premier chapitre est la communication entre un Émetteur et un Récepteur lorsqu'ils ont des préférences différentes. Que devrais-je croire au sujet du message concernant la météo si je sais que l'Émetteur préfère aller au cinéma au lieu d'aller à la plage? Quelle route devrais-je prendre quand je sais que la personne qui conduit n'aime pas les autoroutes? Quand devrais-je faire les courses si je sais que l'ami qui m'a donné l'information sur les heures d'ouverture veut que j'y aille le plus tôt possible? Le Récepteur d'un message doit traiter ces questions si l'Émetteur ne peut vérifier aucune des informations transmises.

La littérature initiale sur la communication dans les jeux entre un Émetteur et un Récepteur peut être divisée en deux catégories. Crawford and Sobel (1982) ont introduit la notion de conversation libre. L'information échangée peut être vraie, partiellement vraie ou complètement fausse. L'Émetteur n'a aucun moyen de vérifier son message et le Récepteur doit prendre en compte les préférences de l'Émetteur lorsqu'il décide s'il croit le message ou quelle information il peut en extraire.
Ceci est le type d'échange d'informations qui se produit la plupart du temps dans notre vie: nous écoutons des personnes et même si elles ne peuvent pas fournir des preuves tangibles, nous pourrions croire leur message.
D'autre part, l'Émetteur peut avoir la possibilité d'envoyer un message vérifiable. Grossman

(1981) et Milgrom (1981) modèlent ceci de telle sorte que l'Émetteur ne puisse dire que la vérité, mais il peut ne pas révéler toute la vérité. Dans les exemples, des vérités partielles sont des messages telles que "Il fait moins de 20°C à l'extérieur", "Ce n'est pas le chemin le plus rapide" ou "Je suis sûr que le magasin ferme avant 20h00".

Dans le chapitre 1, je combine ces deux approches de la littérature de la communication en un seul modèle. Je considère un jeu entre un Émetteur et un Récepteur dans lequel l'Émetteur a une information privée sur l'état du monde. Il envoie un message au Récepteur qui choisit ensuite une action. L'action produit un bénéfice pour les deux joueurs qui dépend de l'état. Alors que dans certains états, les deux joueurs peuvent avoir des intérêts similaires, les préférences des joueurs peuvent être complètement différentes dans d'autres états.

Dans ce modèle, le message de l'Émetteur peut être non vérifiable ou vérifiable. Je suppose que l'Émetteur a plusieurs signaux non vérifiables, un pour chaque état possible du monde. Alors qu'envoyer un de ces signaux non vérifiable est gratuit, l'Émetteur doit payer s'il envoie le message vérifiable.

Le Récepteur a une certaine croyance sur l'état du monde et actualise sa croyance après avoir reçu le message. L'utilisation du message vérifiable révèle l'état réel du monde et, dans ce cas, le Récepteur peut choisir l'action qu'il préfère. En revanche, après avoir reçu un message non vérifiable, le Récepteur doit prendre en compte le fait que l'Émetteur puisse mentir. Le Récepteur connaissant les préférences de l'Émetteur, il croit simplement le message non vérifiable lorsque l'Émetteur n'a aucune raison de mentir.

Sans aucune hypothèse forte, j'énonce les conditions d'équilibre dans lesquelles le Récepteur apprend la vérité dans chaque état. Ces équilibres entièrement révélateurs sont optimaux pour le Récepteur car il peut choisir l'action qu'il préfère. Sous certaines conditions, le Récepteur peut obliger l'Émetteur à dire la vérité: tant que l'Émetteur apprécie l'action préférée du Récepteur plus qu'une autre, le Récepteur peut menacer de choisir l'autre action et peut ainsi obliger l'Émetteur à dire la vérité ou à utiliser le message vérifiable. Une révélation complète peut être atteinte par des messages vérifiables uniquement, par des messages non vérifiables uniquement ou par différents types de messages dans différents états. Tous ces différents équilibres viennent avec de conditions différentes sur les préférences des joueurs que je détaille dans un cadre discret. En outre, j'énonce les conditions pour différents équilibres révélateurs partiels. Dans ces équilibres, le Récepteur n'apprend pas la vérité entière mais le message peut révéler certaines informations de telle façon que le Récepteur réduit l'ensemble des états possibles. Je démontre que dans un espace en état continu, les conditions pour la révélation complète dans le cas discret ne sont pas assez fortes pour une révélation complète. Par ailleurs, si les fonctions d'utilité sont continués, la révélation complète est impossible dans le cadre continu.

Les applications de ce modèle peuvent se trouver dans de nombreux contextes différents. Il arrive très souvent qu'un Émetteur puisse certifier son message mais ceci comporte généralement certains coûts. En revenant aux exemples précédents, le message vérifiable correspond à une prévision météorologique, à un renseignement en ligne sur le trafic ou aux heures d'ouverture détaillées d'un commerce publiées sur leur site web. Une autre application vécue par la plupart des gens est le cas d'un entretien d'embauche et de la liste des compétences. La plupart des compétences techniques ou linguistiques sont vérifiables mais généralement, cela prend non seulement du temps mais il est également coûteux d'obtenir un certificat, par exemple le TOEFL ou le Certificat Cambridge. Ces tests donnent un résultat exact des capacités du candidat et il est rare qu'un candidat remette ce genre de certificat si ce dernier n'améliore pas sa situation. Même si nous considérons les lettres de recommandation comme étant des messages vérifiables, j'argumente que ceux-ci sont coûteux. Dans ce cas, les coûts ne sont pas monétaires mais les investissements en efforts et en temps. Le résultat de mon modèle est que l'entreprise qui organise l'entretien d'embauche ne peut pas prendre en compte des compétences non certifiées pour faire valoir le message vérifiable. Nous voyons très souvent des offres d'emploi accompagnées d'une liste de requis certifiés. Notre résultat montre que si toutes les applications ne remplissant pas ces conditions ne sont pas prises en compte, les candidats qui remplissent ces conditions fournissent alors des preuves.

Le chapitre 2 aborde un autre contexte de transmission d'informations. Dans les organisations, de nombreuses personnes travaillent sur un même projet. Lorsque les informations venant de toutes les personnes sont nécessaires pour compléter le projet, les agents n'ont pas d'intérêt à mentir ou à ne dire qu'une partie de la vérité. Toutefois, la transmission d'informations implique souvent un problème : au début, un employé peut avoir intérêt à garder ses informations pour lui-même. Ceci arrive si ce joueur attend une récompense supplémentaire donnée par la transmission ultérieure de son information, par exemple en livrant la pièce finale du puzzle.

Je présente la structure des organisations comme étant une hiérarchie où le supérieur veut rassembler toutes les informations. Un joueur a plusieurs périodes pour transmettre ses informations à son supérieur. Je suppose qu'un joueur obtient une récompense dès qu'il a transmis sa partie et qu'il préfère obtenir cette récompense le plus tôt possible. Par contre, le joueur a une incitation à garder son information privée parce qu'il sait que son supérieur donne une récompense supplémentaire au joueur qui envoie la dernière information. Une fois que le supérieur a rassemblé toutes les informations de tous ses agents, il est confronté au même problème de transmettre ces informations à son propre supérieur en compétition avec les autres joueurs au même niveau hiérarchique. Le modèle se concentre sur le temps nécessaire pour que le supérieur rassemble toutes les informations. De nombreux paramètres différents ont un impact sur cette durée : non seulement la récompense des joueurs mais aussi leur taux d'actualisation et la récompense supplémentaire. De plus, la structure hiérarchique joue un rôle important. En fonction de la disposition des joueurs, la durée varie entre plusieurs périodes. Je montre comment déterminer la structure hiérarchique optimale pour une certaine distribution d'informations et pour une certaine récompense. Alors que, dans certains cas, il est plus rapide si tout le monde reporte directement au supérieur, d'autres structures peuvent apparaître pour certains ensembles de paramètres. Il y a plusieurs façons différentes d'ordonner les joueurs dans les hiérarchies : si nous trions les joueurs par l'utilité qu'ils reçoivent pour avoir transmis leur information, nous pouvons soit placer ceux qui ont une plus grande utilité au-dessus ou en-dessous. De plus, nous pouvons avoir une structure symétrique ou asymétrique et nous pouvons installer des intermédiaires avec des informations ou des intermédiaires non informés.

Je commence l'analyse avec un modèle simple dans lequel un supérieur est connecté à tous les

autres joueurs. Dans ce contexte, je peux calculer tous les équilibres et me concentrer sur la durée. Je montre l'impact de tous les paramètres et compare également les différentes distributions d'informations. Ayant ce résultat comme base, j'élargis le contexte pour avoir un modèle à plusieurs couches. J'indique les équilibres pour chaque sous-hiérarchie etje compare, sur des exemples, différentes hiérarchies. Même si la structure optimale dépend fortement des paramètres, je prouve que certaines structures peuvent être exclues car elles ne peuvent jamais être optimales.
Nous pouvons trouver des applications dans plusieurs domaines. Cela peut nous faire penser à un chercheur travaillant sur une preuve mathématique importante. Il a besoin d'informations de ses collègues chercheurs qui eux-mêmes peuvent demander des informations à d'autres personnes et qui veulent bénéficier de cette preuve importante. Bien que leur utilité provienne des citations, ils peuvent également obtenir la reconnaissance des supérieurs s'ils l'aident pour l'étape finale de la preuve. Ainsi, garder l'information en premier pourrait être bénéfique pour eux par la suite. La récompense supplémentaire peut être considérée comme un futur projet commun, une co-rédaction ou une invitation à une conférence.
En politique, nous voyons également ce genre de problème de transmission d'informations. Un exemple est l'Union européenne et sa hiérarchie de pays et de parlements régionaux. Dans certains cas, tous les parlements doivent donner leur accord concernant la transmission des informations. Une petite région peut bloquer l'ensemble du processus en ne signant pas la loi, provoquant ainsi un retard. Cela s'est produit en 2016 lorsque la Wallonie a bloqué les négociations pour le traité CETA.

Dans le chapitre 3, je travaille sur un autre angle de la transmission d'information. Alors qu'une structure hiérarchique existe dans les entreprises la plupart du temps, ceci n'est pas le cas dans beaucoup de projets d'équipe. Souvent, les équipes sont composées de membres qui se situent au même niveau hiérarchique. Dans ces équipes, même si tout le monde travaille ensemble sur un projet, il peut y avoir une grande concurrence. Tous les joueurs veulent représenter leur équipe à l'extérieur et devenir le chef du projet.
Nous pouvons observer ceci dans plusieurs exemples : généralement, les accords multilatéraux entre pays sont représentés par un seul pays ou un petit sous-ensemble de pays. Pour l'accord de Paris sur le climat en 2015, près de 200 pays ont assisté à la réunion et plus de 170 pays ont signé cet accord. En même temps, nous savons que seul un petit nombre de pays sont considérés comme le visage de cet accord.
Des situations similaires se produisent dans les équipes de recherche: de nombreux chercheurs travaillent ensemble sur un projet mais les prix, tels que le prix Nobel par exemple, sont souvent attribué à un chercheur unique. La même concurrence pour la direction de projet a lieu dans la coopération R&D (Recherche et Développement) entre organisations ou lors de projet entre entreprises.
Dans ce modèle, je suppose que les différents acteurs sont connectés par un réseau et ne peuvent partager leurs informations qu'avec les joueurs auxquels ils sont directement connectés. Au début, tous les joueurs ont une information unique et ils travaillent sur le projet pendant un certain temps. Jusqu'à la date finale du projet, ils peuvent transmettre leurs informations à tous leurs voisins du

réseau. Le jeu se termine dès qu'un joueur a recueilli toutes les informations et le projet est ainsi réussi. Dans ce cas, tous les joueurs reçoivent une certaine récompense, tandis que le joueur qui a centralisé les informations obtient une récompense supplémentaire. Je suppose que le nombre des joueurs réduit au fur et à mesure du temps et que les joueurs escomptent la récompense.
Le modèle se compose de deux parties: dans la première partie, j'étudie comment les joueurs partagent les informations. A chaque période, ils choisissent soit de garder l'information pour eux-mêmes, soit de la partager entièrement avec tous leurs voisins. Je suppose qu'il existe un certain ordre dans lequel les joueurs prennent leurs décisions. Cet ordre peut être vu comme étant le résultat de projets d'équipe antérieurs. Étant donné que les joueurs veulent représenter le projet à l'extérieur, ils peuvent décider de garder les informations pour eux-mêmes afin d'espérer centraliser l'information et obtenir la récompense supplémentaire. En même temps, ils veulent finir le projet aussi vite que possible à cause du taux d'escompte. Je démontre qu'en situation d'équilibre, seul un joueur centralisera toutes les informations. Le gagnant va dépendre non seulement de l'ordre des décisions et de la structure du réseau, mais aussi du taux d'actualisation. Si les joueurs escomptent fortement, ils se soucient moins de la récompense finale et, à l'équilibre, c'est l'information est centralisé parle joueur qui peut centraliser les informations le plus vite. D'autre part, si le taux d'actualisation est plus élevé, la structure du réseau joue un rôle important. Je fournis un moyen de trouver le gagnant pour tous les taux d'actualisation et toutes les structures de réseau. En outre, je détermine le temps qu'il faut aux joueurs pour terminer le projet.
Dans la deuxième partie du modèle, j'analyse la création du réseau. Je suppose que les joueurs sont initialement disposés dans un réseau connexe et peuvent construire de nouveaux liens couteux ou couper des liens. Des changements ont lieu jusqu'à ce que le réseau soit stable par paires, c'est-à-dire lorsque aucun joueur ne souhaite couper un de ces liens et lorsque aucune paire de joueur non liés ne souhaite créer un lien. Je prouve que seuls les réseaux minimalement connexe peuvent être stables par paires. Dans un réseau minimalement connecté, toutes les paires de joueurs sont connectées par un seul chemin à travers le réseau. Ce résultat montre que l'information ne circulera qu'à travers ce chemin unique et les joueurs coupent donc tous les autres liens.
Les minimalement connexe qui sont stables par paires dépendent de la disposition des joueurs et du taux d'actualisation. Je fournis un moyen de vérifier la stabilité d'un réseau. De plus, je montre que certaines structures ne peuvent jamais être stables.
Dans la dernière partie, je prouve la solidité du modèle: une fois qu'un joueur n'a pas besoin de toutes les informations, mais peut se passer des informations d'autres joueurs, les réseaux stables par paires sont ceux où un joueur est déconnecté. Tous les autres joueurs construisent une composante connexe pour laquelle tous les résultats précédents sont valables.

CHAPTER 1

Communication Games with Optional Verification

This chapter is based on the paper "Communication Games with Optional Verification".

Abstract: We analyse a Sender-Receiver game in which the Sender can choose between a costless cheap-talk message and a costly verifiable message. The Sender knows the true state of the world, while the Receiver only learns about the state through the message of the Sender. The utility of both players depends on an action the Receiver chooses. We keep the assumptions about the utility functions and about the messages to a minimum and state conditions for fully revealing equilibria. Under the assumption of "smooth" preferences and utility functions we show that a fully revealing equilibrium in which the Sender uses both her message types can only exist as long as the state space and action space are discrete. We illustrate this result for the classical example of quadratic loss utilities.

Keywords : cheap-talk, communication, costly disclosure, full revelation, increasing differences, Sender-Receiver game, verifiable information.
JEL Classification : C72, D82.

1.1 Introduction

This paper studies a Sender-Receiver game in which the Sender can choose between costless cheap-talk messages and a costly verifiable messages that reveals the true state. The Sender knows the true state of the world while the Receiver has just a belief about its distribution. Depending on the message of the Sender and his own beliefs the Receiver chooses an action from an action space. Both players get utility depending on the true state of the world and the action the Receiver selects. We assume that the utility functions of both players are common knowledge. The message of the Sender can either be a cheap-talk message or a verifiable message. The set of cheap-talk messages correlates to the set of states of the world and sending a cheap-talk message is costless for the Sender. When the Sender uses a cheap-talk message she does not have to tell the truth which yields to the problem that the Receiver might not belief the Sender, depending on the differences in their preferences. On the other hand, if the Sender chooses the verifiable message the Receiver learns the true state of the world. The Sender cannot lie in this message and it reveals the entire information about the state. For sending the verifiable message the Sender has to pay some costs.
We start our analysis of this model in a discrete setting in which the state space and the action space are finite. In this framework we give conditions for fully revealing equilibria. Those are the equilibria in which the Receiver always learns the true state after reading the message of the Sender. We show that there are three different ways to achieve full revelation. If the preferences of Sender and Receiver are similar, the Sender may have the incentive to send the information about the true state by cheap-talk in each state of the world. In that case the Receiver knows that the Sender has no incentive to lie, because both players have similar preferences and the Receiver can implement the action he prefers most. On the other hand, there can also exist a fully revealing equilibrium in which the Receiver enforces the Sender to use the costly verifiable message in all states. The Receiver can do so by choosing a certain action as a reply to all cheap-talk messages. If the Sender dislikes this action in all states of the world, she always prefers to pay for the verifiable message. The third possibility of full revelation is a combination of both message types. In a subset of states in which the Receiver and Sender have similar interests, the Sender uses cheap-talk messages, while in all other states the Receiver enforces the usage of the verifiable message. We state detailed conditions for all three cases and give examples. Furthermore, we take a closer look at partially revealing equilibria. In those equilibria the Receiver learns more about the true state of the world, but not necessarily which the true state is.
In a second step we extend our model and allow for a continuous action and state space. The main result is that there cannot exist a fully revealing equilibrium in which the Sender uses different message types in different states of the world as long as the utility functions of both players are continuous. We illustrate this result for the case that the utility functions follow quadratic loss functions. Furthermore, we show which additional conditions are necessary to achieve this type of fully revealing equilibrium.
This model can be applied to several classical examples. In Spence (1973) the Receiver of a message is an employer and the Sender an agent who is looking for employment. The interests of both players may differ, but the Sender likes to get an offer from the Receiver. The working effort of the

Sender cannot be observed, but she reports it to the Receiver and he chooses an action according to the message. Our model gives the Sender the additional possibility to pay for a certified report of her effort. In the example of a job interview this verifiable message corresponds to a certification of skills by showing credentials or reports of courses or training. In our model the Receiver also learns more about the Sender when she does not use the verifiable message.

Another well known example is the lemon market by Akerlof (1970). A seller has private information about the quality of the good she is selling. The buyer has to decide whether to buy the product or not. The seller can tell the quality of the good, but her messages are just cheap-talk and the buyer cannot rely on it. In our model the seller can pay to get the quality of her good certified. This allows her to prove the level of quality to the buyer. Obviously, the seller will never pay for the certification if the quality of the product is very low.

One real-life example for this is the market for used-cars. Most often advertisements of sellers just contain information that the seller provides and that the possible buyer should believe. At the same time there exist many ways the seller could certify these information, for example by paying an independent consultant. Clearly, this is costly for the seller and she prefers to sell her car without that certificate. For expensive or classic cars where the costs of verification are comparably low to the selling price, we observe the usage of certificates more often.

We know turn to the related literature. Crawford and Sobel (1982) introduce cheap-talk. In their model the content of a message of the Sender can be whatever the Sender wants it to be. She does not have to tell the truth and so the message may not change the Receiver's beliefs at all. The authors show that there are different types of equilibria. In the babbling equilibrium the Sender uses the same message in each state of the world (or for each of her types). In the informative equilibrium the Receiver learns more about the true state of the world, because the Sender uses different cheap-talk messages in different states. In the setting of Crawford and Sobel (1982) the Sender has no possibility to verify that she tells the truth.

On the other hand, there exist the models of Grossman (1981) and Milgrom (1981). Even though the two authors use different applications, their models are very similar. The Sender can decide how much information she likes to reveal about an item that she wants to sell. She can not lie or fake these information, but she can choose what she wants to reveal. In these models the Receiver can enforce full revelation. He assumes that all properties of the object for sale are the worst and he only changes his beliefs if information is revealed by the Sender. This unraveling argument yields to full revelation in equilibrium. In our model the Sender only has one verifiable message that completely reveals the state. We argue that if the Sender could verify as in Grossman (1981) and Milgrom (1981), the unraveling argument would hold and in equilibrium the Sender will reveal the complete information.

To the best of our knowledge there exists only one paper that follows the same idea as this paper and combines the two different strains of communication literature. Eső and Galambos (2013) start with a similar idea, but it is to point out that there are many differences in the settings. Eső and Galambos assume that the players' utility functions are strictly concave and that the players' optimal actions are strictly increasing in the state. Furthermore, they assume that the Sender's utility only depends on the Sender's ideal action and the action the Receiver chooses, but not on the state of the world. Under these assumptions they find that in equilibrium the state space can be split into different intervals and that the Sender uses either the same message for all states of an interval or that she uses the verifiable message in the entire interval. This confirms the result we derive in the continuous setting. Even under their additional assumptions there is no fully revealing equilibrium in which the Sender uses different type of messages in different states of the world. In comparison to Eső and Galambos (2013) this paper starts with less assumptions and focuses more on conditions for full revelation. In addtion, this paper also allows for a finite state space and action space and we show that in this setting there can exist a fully revealing equilibrium in which the Sender uses both message types.

Cheap-talk communication has been added to many different setting and the original model of Crawford and Sobel (1982) has been extended in several directions. Farrell and Gibbons (1989b) introduce an additional Receiver. They observe how his existence changes the report of the Sender. McGee and Yang (2013) and Ambrus and Lu (2014) do a similar step with multiple Senders. While McGee and Yang (2013) focus on two Senders with complementary information, Ambrus and Lu (2014) model several Senders who observe a noisy state. Noise is also added to the signaling game by Haan et al. (2011). The authors derive equilibria depending on the level of noise and confirm their results by an experiment. A different extension of cheap-talk is done by A. Blume and Arnold (2004). They model learning in cheap-talk games and derive a learning rule depending on common interest.

Bull and Watson (2007) and Mookherjee and Tsumagari (2014) deal with communication and mechanism design. While Bull and Watson (2007) focus on costless disclosure, Mookherjee and Tsumagari (2014) add communication costs to prevent full revelation of information. Communication costs are also introduced by Hedlund (2015) in a persuasion game. The author derives two types of equilibria: For high costs there exists a pooling equilibrium, while for not too high costs a separating equilibrium exists.

Other models focus more on disclosure of information and costly communication. Hagenbach et al. (2014) analyse a game with a set of players, where each player can tell the truth about his type or can masquerade as some other type. As usual, the player who deviates (from telling the truth) is punished by the other players. If a player masquerades, the other players assume a worst case type and punish him by choosing the action this type of player dislikes. The authors state conditions for full revelation depending on the possible masquerades of each player.

An overview over cheap-talk models and models with verifiable messages can be found in Sobel (2009). The author describes several models and gives some economic examples. Most of these examples can be extended to fit our setting by adding a reasonable verifiable message. Verrecchia (2001) provides an overview over different models of disclosure, which is extended by Dye (2001).

It remains to mention that there are several papers in which the authors have created their own way of modeling communication. Kartik (2009) introduces a model, where the Sender sends a message about her type, but has the incentive to make the Receiver believe that her type is higher than it actually is. If the Sender lies in her message, she has to pay some costs for lying, which depend on the distance between the true type and the type stated in the message.

Dewatripont and Tirole (2005) analyse the communication of Sender and Receiver when both players have to invest effort. The effort of the Sender is to make the message understandable, while the effort of the Receiver corresponds to him paying attention while reading the message. The authors motivate this model by the idea that very unclear messages and reading messages without paying a lot of attention yield to misunderstandings. The probability of understanding the message is influenced by the effort of both players.
Austen-Smith and Banks (2000) introduce the possibility for the Sender to send a costly message with the same content as a costless message. By this way of burning money the Sender has an additional possibility of signaling. The authors show that conditions exist under which both message types are used.

The paper is organized as follows. In Section 1.2 we introduce the discrete model and state results for this setting. In Section 1.2.1 we give conditions for fully revealing equilibrium. Section 1.2.2 focuses on partial revelation. In Section 1.3 we extended our setting to a continuous model and show that we cannot reproduce the previous results. We analyze the continuous model where the utility functions of both players are quadratic loss functions in Section 1.3.1. In Section 1.4 we discuss extensions. Finally, in Section 1.5 we conclude. All proofs are relegated to the appendix.

1.2 Discrete model

In this part we focus our attention on a model with a finite set of states of the world and a finite set of actions the Receiver can choose from. Let $\Omega = \{\omega_1, \ldots, \omega_L\}$ denote the set of the L different states of the world, where each state ω_i has the probability $\mathbb{P}[\omega_i]$.
The timing is as follows: The Sender learns the true state of the world and then she sends a message to the Receiver. We assume that the set of possible cheap-talk messages M corresponds to the states of the world Ω and that verifiable message v is unique in each state of the world. The Sender chooses a message from $M \cup \{v\}$, so either sends a cheap-talk message or the verifiable message v. There is no possibility for partial disclosure. While sending any cheap-talk message is free, the Sender has to pay costs $c > 0$ if she sends the verifiable message. An economic explanation for these costs can be either a payment for a certificate or the investment into effort. For simplicity we assume that the costs are state independent, but state dependent costs are discussed in a later part as an extension.
The Receiver reads the message and chooses an action from $A = \{a_1, \ldots a_N\}$. By $\Delta(A)$ we denote the set of mixed strategies. Both players have preferences about the actions, resulting in different von Neumann-Morgenstern utility functions for both players, depending on the action and state of the world.
For the Sender it is given by $\tilde{u}_S : A \times M \cup \{v\} \times \Omega \to \mathbb{R}$ with $\tilde{u}_S(a, m, \omega) = u_S(a, \omega)\ \forall m \in M$, $\tilde{u}_S(a, v, \omega) = u_S(a, \omega) - c$ and $u_S : A \times \Omega \to \mathbb{R}$. So we can split the utility function of the Sender up into two parts: First a utility function depending on action and state of the world. Additionally we have to subtract the costs for the message if there are any.
For the Receiver the utility function is not depending on the type of the message, but just on the

action and state: $u_R : A \times \Omega \to \mathbb{R}$. The utility functions show that there is neither a punishment for lying nor a direct reward for telling the truth. We assume that these utility functions are common knowledge. Let $a_R^*(\omega_i)$ denote the action the Receiver prefers in the state ω_i. We will make some more assumptions about this function later. We denote the Sender's behavior by the function $f : \Omega \to M \cup \{v\}$. This function f maps each state of the world to the message she sends. We assume that the Sender does not mix different messages.
The Receiver chooses the action, depending on the message he received: $g : M \cup \Omega \to \Delta(A)$. In equilibria we have to define the behavior of the Sender for every state, so $f(\omega)$ and the Receiver's action after each message, i.e. $g(m)\ \forall m \in M$ and $g(v)$.
The equilibrium concept we use is Perfect Bayesian Equilibrium.

Definition 1.1. *A Perfect Bayesian Equilibrium in a dynamic game of incomplete information is a strategy profile (f^*, g^*) and a belief system μ^* for the Receiver such that*

- *The strategy profile (f^*, g^*) is sequentially rational.*
- *The belief system μ^* is consistent whenever possible, given (f^*, g^*).*

In other words each equilibrium consists of optimal strategies for Sender and Receiver, which are sequentially rational. Furthermore the Receiver has a belief system over the true state of the world depending on the message he receives. This belief system is updated by Bayes rule whenever possible. For Perfect Bayesian Equilibria the actions off the equilibrium path have to be the best actions for the Receiver for at least one belief system. We can neglect this if we limit our attention to actions that are undominated for the Receiver.
We are specially interested in equilibria with full revelation:

Definition 1.2. *A Perfect Bayesian Equilibrium is fully revealing, if the Receiver knows the true state of the world, after reading the Sender's message.*

There is full revelation if the Sender either sends different cheap-talk messages in each state, or just verifiable messages, or different cheap-talk messages in some states and verifiable messages in the other states.
For the entire paper we make two assumptions:

Assumption 1.1. *Let us assume that for each action $a_j \in A$ there exists at least one belief system μ such that a_j is the Receiver's best response under the belief system μ.*
By $\hat{\Delta}(A) \subseteq \Delta(A)$ we denote the set of mixed strategies that satisfy this assumption, i.e.

$$\forall \hat{a} \in \hat{\Delta}(A) : \exists \mu : \hat{a} \in \arg\max_a \sum_{\omega \in \Omega} \mu(\omega) \cdot u_R(a, \omega)$$

Assumption 1.1 requires that each action is optimal for the Receiver at least under one belief system, which means that there are no dominated actions. Our results depend on the idea that the Receiver uses an action as a threat and so enforces the Sender to send verified messages. The threat is only credible, if this action is an element of $\hat{\Delta}(A)$.

We can think about different equilibrium refinements as introduced in several papers. The most common ones are the Divinity Criterion by Banks and Sobel (1987) and the Intuitive Criterion by Cho and Kreps (1987). Using any of them adds more conditions for the threat points, so the set $\hat{\Delta}(A)$ gets smaller and the Receiver has less possibilities to make a threat, but the conditions stay the same. In addition such refinements may rule out other equilibria, but this paper focuses on the existence of equilibria and not on the uniqueness.

Assumption 1.2. *Let us assume that in every state of the world the Receiver has strict preferences.*

Under Assumption 1.2, $a_R^*(\omega_i)$ is a single action, which will help for the upcoming results. This is to avoid the situation that the Receiver is indifferent between two actions.

1.2.1 Full revelation

In this first part we focus our attention on fully revealing equilibria. We will state conditions for full revelation, where the Sender just uses the cheap-talk messages, conditions where she uses only verified messages and conditions where she uses both types of message, depending on the state. Even if conditions for one of these fully revealing equilibria are satisfied, there might be other equilibria at the same time. By examples we show that the existence of these different types of full revelation are independent of each other.

Assumption 1.3. *Let us assume that for all states $\omega_i \neq \omega_j$ the Receiver prefers different actions, i.e. $a_R^*(\omega_i) \neq a_R^*(\omega_j)$.*
In other words the function $a_R^* : \Omega \to A$ has to be injective.

This assumption assures that there can be a fully revealing equilibrium, even if the Sender uses cheap-talk messages in several states. For the case that the Sender just uses the cheap-talk messages and we still have full revelation, the Sender is not allowed to have any incentive to deviate to another cheap-talk message. It is not necessary that the preferences in all states are the same for Sender and Receiver. Crucial is that the action the Receiver chooses when he knows the true state $a_R^*(\omega_i)$ generates a higher utility for the Sender than the Receiver's most preferred action of any other state $a_R^*(\omega_j)$. There is also the possibility that there exists an action the Sender prefers, but which is never included by the Receiver as long as he knows the true state of the world.

Theorem 1.1 (Full Revelation just by Cheap-Talk Messages)**.**
Let Assumption 1.3 hold. There is a fully revealing equilibrium with only costless messages sent if and only if:

$$\forall \omega_i \in \Omega : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i) \; \forall \omega_j \neq \omega_i \tag{1.1}$$

If Assumption 1.3 does not hold, i.e. if there exist two states ω_i, ω_j such that $a_R^*(\omega_i) = a_R^*(\omega_j)$, there is no fully revealing equilibrium. Still the Receiver can get the highest possible utility in every state, while the Sender just sends cheap-talk messages. If the Receiver just learns that he is either in state ω_i or ω_j his best response is the same and generates the highest possible utility for him.

Theorem 1.2 (Full Revelation just by Verifiable Messages)**.**
There is a fully revealing equilibrium with only verifiable messages sent if and only if:

$$\exists \hat{a} \in \hat{\Delta}(A) : 1)\ \forall \omega_i : \hat{a} \neq a^*_R(\omega_i)$$
$$2)\ \forall \omega_i : u_S(a^*_R(\omega_i), \omega_i) - c > u_S(\hat{a}, \omega_i)$$

The idea behind Theorem 1.2 is that the Sender replies to cheap-talk messages with an action $\hat{a}$ the Sender really dislikes. With this threat point $\hat{a}$ the Receiver forces the Sender to use the verified message. The same idea can be found in several existing papers dealing with verifiable messages, e.g. in Hagenbach et al. (2014). We can combine both theorems and get conditions for full revelation, where the Sender uses both types of messages.

Theorem 1.3 (Full Revelation by Cheap-Talk and Verifiable Messages)**.**
Let Assumption 1.3 hold. There is a fully revealing equilibrium with both message types used if and only if there exists $\hat{\Omega} \subsetneq \Omega$ with $\hat{\Omega} \neq \emptyset$ such that

$$\exists \hat{a} \in \hat{\Delta}(A) : 1)\ \forall \omega_i \notin \hat{\Omega} : u_S(a^*_R(\omega_i), \omega_i) - c > u_S(a^*_R(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}$$
$$2)\ \forall \omega_i \in \hat{\Omega} : u_S(a^*_R(\omega_i), \omega_i) > u_S(a^*_R(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}, \omega_j \neq \omega_i$$

Theorem 1.3 allows that the Receiver trusts the Sender in some states ($\hat{\Omega}$), but in the other states he enforces the use of verifiable messages as in Theorem 1.2. To have both message types used $\hat{\Omega}$ has to be a subset of Ω, not equal to Ω and non-empty, otherwise just one message type is used. The two conditions in this theorem are similar to those of the previous theorems. Instead of a single threat point $\hat{a}$, every $a^*_R(\omega_j)$ with $\omega_j \in \hat{\Omega}$ has to work as a threat. In addition the Sender is not allowed to have an incentive to deviate to another cheap-talk message if the true state is an element of $\hat{\Omega}$.
There might be several possibilities for the set of states $\hat{\Omega}$, where the Receiver trusts the cheap-talk messages. Those possibilities do not have to be subsets of each other, but also can be disjoint. For the case that there are several subsets we can say that for smaller sets Condition 1) has to hold for more states, but Condition 2) for less states.

Remarks.

- *For the result of Theorem 1.3 we need Assumption 1.3 just for the states in $\hat{\Omega}$. So even if there exist two states $\omega_i, \omega_j \in \Omega/\hat{\Omega}$ such that $a^*_R(\omega_i) = a^*_R(\omega_j)$, Theorem 1.3 still holds.*
- *If Assumption 1.3 does not hold and there exist two states $\omega_i, \omega_j \in \hat{\Omega}$ such that $a^*_R(\omega_i) = a^*_R(\omega_j)$, Theorem 1.3 does not hold, but under the conditions of that theorem, the Receiver still gets the highest possible utility in every state.*

We can also simplify the first condition of Theorem 1.3 which yields to the following corollary.

Corollary 1.1 (Full Revelation by Cheap-Talk and Verifiable Messages)**.**
Let Assumption 1.3 hold. There is a fully revealing equilibrium with both message types used if there exists $\hat{\Omega} \subsetneq \Omega$ with $\hat{\Omega} \neq \emptyset$ such that

$$\exists \hat{a} \in \hat{\Delta}(A) : 1)\ \forall \omega_i \notin \hat{\Omega} : c < \min_{\omega_j \in \hat{\Omega}} u_S(a_R^*(\omega_i), \omega_i) - u_S(a_R^*(\omega_j), \omega_i)$$
$$2)\ \forall \omega_i \in \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}, \omega_j \neq \omega_i$$

Theorems 1.1, 1.2 and 1.3 give conditions for different types of fully revealing equilibria. It can happen that there is no fully revealing equilibrium just by cheap-talk or just by verifiable messages, but one by a combination of both message types:

Example 1.1. *Assume that there are two states (ω_1, ω_2) and two actions (a_1, a_2).*
The Receiver prefers a_1 in ω_1 and a_2 in ω_2, while the Sender always prefers a_1. Obviously there is no fully revealing equilibrium just with cheap-talk, because the Sender always wants the action a_1 and so she would lie. Furthermore there is no equilibrium just with verifiable messages, because there is no threat available:
For the mixed strategy that plays a_1 with probability p and a_2 with probability $(1-p)$, we use the notation $pa_1 + (1-p)a_2$. Denote $\hat{a} = pa_1 + (1-p)a_2$. For $p = 0$, the Sender will not use the verifiable message in ω_2, because she gets the same action by sending cheap-talk, but verifiable messages are costly. Also for $p > 0$ the Sender will not use the verifiable message in ω_2, because she prefers a_1 over a_2 and so she also prefers $\hat{a}$ over a_2.
Still there is full revelation possible if c is low enough. Let us assume that costs c are small, i.e. $c < u_S(a_1, \omega_1) - u_S(a_2, \omega_1)$. If the Receiver answers every cheap-talk message by a_2, the Sender will use the verifiable message in ω_1, yielding to action a_1. The utility the Sender gets is $u_S(a_1, \omega_1) - c > u_S(a_2, \omega_1)$, while her utility would be $u_S(a_2, \omega_1)$ if she sends the cheap-talk message. In the second state ω_2, the Sender will use the cheap-talk message. Both message types will result in action a_2, so the Sender prefers the costless message.

Even though we stated conditions for full revelation, it might happen that there is no revelation at all. The easiest example can be done just by two states and two actions:

Example 1.2. *Assume that the Receiver prefers a_1 in ω_1 and a_2 in ω_2 and the Sender's preferences are switched, i.e. she prefers a_2 in ω_1 and a_1 in ω_2. Clearly there is no full revelation just by cheap-talk, because the Sender will always lie. Furthermore there can be no revelation just by verifiable messages. Assume that the threat point is $\hat{a} = pa_1 + (1-p)a_2$, with the notation used as in the previous example.*
For $p = 0$, the Sender will not use the verifiable message in ω_1, because she prefers a_2 over a_1. The same argument holds even for $p > 0$: Using the cheap-talk message resulting in $\hat{a}$ gives the Sender at least a little chance of a_2. Therefore $u_S(\hat{a}, \omega_1) > u_S(a_1, \omega_1)$ and this implies $u_S(\hat{a}, \omega_1) > u_S(a_1, \omega_1) - c$.
So the only possibility is to have a fully revealing equilibrium with both message types used. Doing the same steps again for Theorem 1.3 proves that there is no full revelation. So in this example

where the preferences of Sender and Receiver differ a lot, the Receiver has no possibility to enforce the full revelation.

1.2.1.1 Increasing and Decreasing Differences

The previous results have to be checked for every state, which might be not easy to do. If the utility function of the Sender satisfies either the increasing or decreasing differences property, we can state weaker conditions. The idea is that we just need to check the previous conditions, for one state and then can easily get full revelation for all states, if some additional properties are satisfied.

Definition 1.3. *We say $u_S(a,\omega)$ has increasing (decreasing) differences in (a,ω), if*
$\forall a' \geq a, \forall \omega' \geq \omega : u_S(a',\omega') - u_S(a,\omega') \geq (\leq)\, u_S(a',\omega) - u_S(a,\omega)$.

To have an order over the states of the world, let us assume that $\Omega \subset \mathbb{Q}$ holds. In addition, we sort the action state A as described in the following proposition.

Proposition 1.1 (Full revelation under increasing differences)**.**
Let $\Omega = \{\omega_1,\ldots,\omega_L\} \subset \mathbb{Q}$ hold and let us sort A such that $A = \{a^*_R(\omega_1),\ldots,a^*_R(\omega_L)\}$ with $a^*_R(\omega_i) > a^*_R(\omega_{i+1})\ \forall i \in \{1,\ldots,L-1\}$. We can ignore all actions, which are never the best reply for the Receiver in a single state.
There is a fully revealing equilibrium with $\hat{a} = a^*_R(\omega_j)$ if:

$$\begin{aligned}
&1)\ u_S \text{ has increasing differences}\\
&2.1)\ u_S(a^*_R(\omega_{j+1}),\omega_{j+1}) - c > u_S(a^*_R(\omega_j),\omega_{j+1})\\
&2.2)\ u_S(a^*_R(\omega_{j-1}),\omega_{j-1}) - c > u_S(a^*_R(\omega_j),\omega_{j-1})\\
&3.1)\ \forall \omega_i > \omega_j : u_S(a^*_R(\omega_i),\omega_i) \geq u_S(a^*_R(\omega_{i-1}),\omega_i)\\
&3.2)\ \forall \omega_i < \omega_j : u_S(a^*_R(\omega_i),\omega_i) \geq u_S(a^*_R(\omega_{i+1}),\omega_i)
\end{aligned}$$

The fully revealing equilibrium is such that the Sender sends cheap-talk in ω_j and verifiable messages in all other states.

Corollary 1.2.
We can replace Condition 3.1) by

$$3.1')\ \forall \omega_i > \omega_j : u_S(a^*_R(\omega_i),\omega_i) \geq u_S(a^*_R(\omega_{j+1}),\omega_i)$$

and Condition 3.2) by

$$3.2')\ \forall \omega_i < \omega_j : u_S(a^*_R(\omega_i),\omega_i) \geq u_S(a^*_R(\omega_{j-1}),\omega_i)$$

An interpretation of these properties can be done easily, if we look at the following corollary.

Corollary 1.3.
Let $\Omega = \{\omega_1, \dots, \omega_L\}$ and sort A such that $A = \{a^*_R(\omega_1), \dots, a^*_R(\omega_L)\}$. We can ignore all actions, which are never the best reply for the Receiver in a single state.
There is a fully revealing equilibrium with $\hat{a} = a^*_R(\omega_1)$ if:

$$1)\ u_S \text{ has increasing differences}$$
$$2)\ u_S(a^*_R(\omega_2), \omega_2) - c > u_S(a^*_R(\omega_1), \omega_2)$$
$$3)\ \forall \omega_i \in \{\omega_2, \dots, \omega_L\} : u_S(a^*_R(\omega_i), \omega_i) \geq u_S(a^*_R(\omega_{i-1}), \omega_i)$$

The fully revealing equilibrium is such that the Sender sends cheap-talk in ω_1 and verifiable messages in all other states.

The threat point here is $a^*_R(\omega_1)$. Condition 2) ensures that the Sender prefers the verifiable message in the state after, which is ω_2. Increasing Differences mean that the gains from a higher action increase, if the state gets higher. With Condition 3) combined, we get that the Sender also prefers the verifiable message in all states higher than ω_2. We can get a similar result with $a^*_R(\omega_L)$, where we have to replace ω_2 in Condition 2) by ω_{L-1} and adjust Condition 3) as well. An application can be found in Section 1.3.1.1.
Similar changes for decreasing differences can be made easily:

Proposition 1.2 (Full revelation under decreasing differences)**.**
Let Ω and A be as in Proposition 1.1. There is a fully revealing equilibrium with $\hat{a} = a^*_R(\omega_j)$ if:

$$1)\ u_S \text{ has decreasing differences}$$
$$2.1)\ u_S(a^*_R(\omega_L), \omega_L) - c > u_S(a^*_R(\omega_j), \omega_L)$$
$$2.2)\ u_S(a^*_R(\omega_1), \omega_1) - c > u_S(a^*_R(\omega_j), \omega_1)$$
$$3.1)\ \forall \omega_i > \omega_j : u_S(a^*_R(\omega_i), \omega_i) \geq u_S(a^*_R(\omega_{i+1}), \omega_i)$$
$$3.2)\ \forall \omega_i < \omega_j : u_S(a^*_R(\omega_i), \omega_i) \geq u_S(a^*_R(\omega_{i-1}), \omega_i)$$

The fully revealing equilibrium is such that the Sender sends cheap-talk in ω_j and verifiable messages in all other states.

Changing the conditions as in Corollary 1.2 and Corollary 1.3 is possible.

1.2.2 Maximization without full revelation

Whenever there is no full revelation, the Receiver cannot get the highest possible utility in all states. Depending on the preferences, utility and costs, there might be partial revelation or no revelation at all. We start our analysis by an example with just three states and give conditions for partial revelation and no revelation. In a numerical example we will show that each of this possibilities can be the best strategy for the Receiver.
In the second part we generalize: If there are more than three states, partial revelation can be of one of three different types: Verifiable messages in some states, revealing the true state. Unique

cheap-talk messages can have the same effect, but cheap-talk messages can also partial reveal information to the Receiver, such that it splits the state space into disjoint subsets. In that case the Receiver might just know whether he is in the first or second state, or in the third or fourth state.
We give conditions for all the different possibilities of partial revelation and also for combinations of those. Furthermore, we again use utility functions with increasing or decreasing differences to simplify these conditions.

1.2.2.1 Three state examples

Assume $|\Omega| = 3$ and assume that the Receiver prefers different actions in different states.
In case of partial revelation the Receiver can maximize his utility by three different possibilities:

1. Use the same action in every state.
2. Reply with one action to one cheap-talk message and with another to the remaining messages.
3. Use the same action as a reply to any cheap-talk message, enforcing the Sender to send the verifiable message in exactly one state, i.e. revelation of one state.

First possibility

$$\max_{\hat{a}} \sum_{i=1}^{3} \mathbb{P}[\omega_i] u_R(\hat{a}, \omega_i)$$
$$\text{s.t. } \forall \omega_i : u_S(\hat{a}, \omega_i) > u_S(a_R^*(\omega_i), \omega_i) - c$$

Second possibility (Revelation in ω_1 (wlog) by cheap-talk)

$$\max_{\hat{a}} \mathbb{P}[\omega_1] u_R(a_R^*(\omega_1), \omega_1) + \sum_{i=2}^{3} \mathbb{P}[\omega_i] u_R(\hat{a}, \omega_i)$$
$$\text{s.t. } u_S(a_R^*(\omega_1), \omega_1) > u_S(\hat{a}, \omega_1)$$
$$u_S(\hat{a}, \omega_i) > u_S(a_R^*(\omega_1), \omega_i) \; \forall \omega_i, i \in \{2, 3\}$$

Third possibility (Revelation in ω_1 (wlog) by a verifiable message)

$$\max_{\hat{a}} \mathbb{P}[\omega_1] u_R(a_R^*(\omega_1), \omega_1) + \sum_{i=2}^{3} \mathbb{P}[\omega_i] u_R(\hat{a}, \omega_i)$$
$$\text{s.t. } u_S(a_R^*(\omega_1), \omega_1) - c > u_S(\hat{a}, \omega_1)$$
$$u_S(\hat{a}, \omega_i) > u_S(a_R^*(\omega_i), \omega_i) - c \; \forall \omega_i, i \in \{2, 3\}$$

These are the different maximization problems the Receiver has to solve to find the best strategy. In the second case we do not need to state a condition that the Sender uses the verifiable message, because that condition (similar to the last condition of the third case) is weaker than the last condition of the second case.
With different example we will show that either of the strategies can be the best choice. For that

we have to keep in mind that the Receiver cannot commit to any strategies, but plays his best possibility given his beliefs. Especially in the case where he knows the true state, the Receiver will always play the action that yields the highest utility for him.

Example 1.3. *Assume* $c > 2, |\Omega| = |A| = 3$*,*

$$\begin{aligned} u_R(a_3, \omega_1) &= 4 \quad u_R(a_1, \omega_1) = 2 \quad u_R(a_2, \omega_1) = 1 \\ u_R(a_1, \omega_2) &= 4 \quad u_R(a_2, \omega_2) = 2 \quad u_R(a_3, \omega_2) = 1 \\ u_R(a_2, \omega_3) &= 4 \quad u_R(a_1, \omega_3) = 2 \quad u_R(a_3, \omega_3) = 1 \end{aligned}$$

and $u_S(\cdot, \omega_1) = u_R(\cdot, \omega_1)$*, but* $u_S(\cdot, \omega_2) = u_R(\cdot, \omega_3)$ *and* $u_S(\cdot, \omega_3) = u_R(\cdot, \omega_2)$*. So Sender and Receiver have the same preferences in* ω_1*, but switched between* ω_2 *and* ω_3*.*

No full revelation

Clearly there is no full revelation just by cheap-talk messages. The proof why there is no full revelation just by verifiable messages and also not by both message types used, follows the same idea: Assume that $\hat{a} = \pi_1 a_1 + \pi_2 a_2 + (1 - \pi_1 - \pi_2) a_3$*, therefore*

$$\begin{aligned} u_S(a_3, \omega_1) - c &> u_S(\hat{a}, \omega_1) \\ u_S(a_1, \omega_2) - c &> u_S(\hat{a}, \omega_2) \\ u_S(a_2, \omega_3) - c &> u_S(\hat{a}, \omega_3) \end{aligned}$$

have to hold. Rewriting this yields to

$$\begin{aligned} 4 - c &> \pi_1 \cdot 1 + \pi_2 \cdot 2 + (1 - \pi_1 - \pi_2) \cdot 4 \\ 2 - c &> \pi_1 \cdot 2 + \pi_2 \cdot 4 + (1 - \pi_1 - \pi_2) \cdot 1 \\ 2 - c &> \pi_1 \cdot 4 + \pi_2 \cdot 2 + (1 - \pi_1 - \pi_2) \cdot 1 \end{aligned}$$

and finally to

$$\begin{aligned} c &< 3\pi_1 + 2\pi_2 \\ 1 - c &> \pi_1 + 3\pi_2 \\ 1 - c &> 3\pi_1 + \pi_2 \end{aligned}$$

This is impossible for $c \geq 1$*. For the full revelation by both message types,* $\hat{a}$ *can also be equal to* a_1*,* a_2 *or* a_3*, but all these possibilities still contradict at least one condition.*

Maximization

1.) No revelation:

The Receiver solves $\max \frac{1}{3} \cdot (2\pi_1 + 1\pi_1 + 4(1 - \pi_1 - \pi_2)) + \frac{1}{3} \cdot (4\pi_1 + 2\pi_1 + 1(1 - \pi_1 - \pi_2)) + \frac{1}{3} \cdot (2\pi_1 + 4\pi_1 + 1(1 - \pi_1 - \pi_2))$*, which yields to* $\hat{a} = a_1$ *and expected utility for the Receiver* *given by* $\mathbb{E}[u_R] = \frac{1}{3}(2+4+2) = \frac{8}{3}$. *The conditions for the Sender not to deviate are:*

$$\begin{aligned} u_S(\hat{a},\omega_1) > u_S(a_3,\omega_1) &\Leftrightarrow 2 > 4-c \\ u_S(\hat{a},\omega_2) > u_S(a_1,\omega_1) &\Leftrightarrow 2 > 4-c \\ u_S(\hat{a},\omega_3) > u_S(a_2,\omega_1) &\Leftrightarrow 4 > 2-c. \end{aligned}$$

All these conditions hold for $c > 2$.
2.) Partial revelation of ω_1 *by cheap-talk:*
The Receiver has to answer with a_3 *to one cheap-talk message and with* $\hat{a}$ *to the others. The maximization problem yields that* $\hat{a} = \pi_1 a_1 + (1-\pi_1)a_2$, *with* $\pi_1 \in [0,1]$. *The conditions for the Sender's utility are*

$$\begin{aligned} u_S(a_3,\omega_1) > u_S(\hat{a},\omega_1) &\Leftrightarrow 4 > u_S(\hat{a},\omega_1) \\ u_S(\hat{a},\omega_2) > u_S(a_3,\omega_1) &\Leftrightarrow u_S(\hat{a},\omega_2) > 1 \\ u_S(\hat{a},\omega_3) > u_S(a_3,\omega_1) &\Leftrightarrow u_S(\hat{a},\omega_3) > 1, \end{aligned}$$

which are clearly satisfied. Here the Receiver's expected utility is $\mathbb{E}[u_R] = \frac{1}{3}(4+6) = \frac{10}{3}$.
3.) Partial revelation of ω_1 *by a verifiable message:*
For example we can take $\hat{a} = a_2$. *The conditions for the Sender's utility are*

$$\begin{aligned} u_S(a_3,\omega_1) - c > u_S(a_2,\omega_1) &\Leftrightarrow 4-c > 1 \\ u_S(a_2,\omega_2) > u_S(a_1,\omega_1) - c &\Leftrightarrow 4 > 2-c \\ u_S(a_2,\omega_3) > u_S(a_2,\omega_1) - c &\Leftrightarrow 2 > 2-c, \end{aligned}$$

which all hold for $c < 3$. *The Receiver's expected utility is* $\mathbb{E}[u_R] = \frac{1}{3}(4+6) = \frac{10}{3}$.
In this example it is possible to get partial revelation in ω_1 *either by cheap-talk or by verifiable message if* $c \in (2,3)$. *For* $c > 3$ *partial revelation is just possible by cheap-talk.*

Example 1.4. *Assume* $c > 2, |\Omega| = |A| = 3$,

$$\begin{aligned} &u_R(a_3,\omega_1) = 4 \quad u_R(a_1,\omega_1) = 2 \quad u_R(a_2,\omega_1) = 1 \\ &u_R(a_1,\omega_2) = 4 \quad u_R(a_2,\omega_2) = 2 \quad u_R(a_3,\omega_2) = 1 \\ &u_R(a_2,\omega_3) = 4 \quad u_R(a_1,\omega_3) = 2 \quad u_R(a_3,\omega_3) = 1 \end{aligned}$$

and

$$\begin{aligned} &u_S(a_3,\omega_1) = 4 \quad u_R(a_1,\omega_1) = 2 \quad u_R(a_2,\omega_1) = 1 \\ &u_S(a_3,\omega_2) = 4 \quad u_R(a_2,\omega_2) = 2 \quad u_R(a_1,\omega_2) = 1 \\ &u_S(a_3,\omega_3) = 4 \quad u_R(a_1,\omega_3) = 2 \quad u_R(a_2,\omega_3) = 1. \end{aligned}$$

No full revelation

Obviously there is no fully revealing equilibrium just by cheap-talk message, because Sender and Receiver prefer different actions in two states. To see that full revelation just by verifiable messages is impossible, we take a closer look at ω_2: *To have an incentive to send the verifiable information the Sender has to prefer* $u_s(a_1,\omega_2) - c$ *over* $u_S(\hat{a},\omega_2)$. *Since the left part equals* $1-c$ *and the right part something larger than* 1, *this is impossible. The same arguments contradict the full revelation by both message types for mixed strategies.*

For $\hat{a} = a_1$, the Sender does not use the verifiable message in ω_3 and for $\hat{a} = a_2$ she uses cheap-talk in ω_2. So there is no full revelation possible.
Maximization
1.) No revelation:
The maximization here is the same as in the previous example. It is possible for $c > 2$ and the expected utility is $\mathbb{E}[u_R] = \frac{8}{3}$.
2.) Partial revelation by cheap-talk
Getting partial revelation in ω_1 is impossible, because the Sender will use the same cheap-talk message in both other states. To get partial revelation in ω_2, $u_S(a_1, \omega_2) > u_S(\hat{a}, \omega_2)$ has to hold, which is impossible for any $\hat{a} \neq a_1$. Same arguments work with a_2 in ω_3. This means in this example it is not possible to achieve partial revelation by different answers to cheap-talk.
3.) Partial revelation of ω_1 by a verifiable message:
For example we can take $\hat{a} = a_2$. The conditions for the Sender's utility are

$$\begin{aligned} u_R(a_3, \omega_1) - c > u_S(a_2, \omega_1) &\Leftrightarrow 4 - c > 1 \\ u_R(a_2, \omega_2) > u_S(a_1, \omega_1) - c &\Leftrightarrow 2 > 1 - c \\ u_R(a_2, \omega_3) > u_S(a_2, \omega_1) - c &\Leftrightarrow 1 > 1 - c, \end{aligned}$$

which again all hold for $c < 3$. The Receiver's expected utility is $\mathbb{E}[u_R] = \frac{1}{3}(4 + 6) = \frac{10}{3}$.
In this example partial revelation is only possible by verifiable information and just if $c \in (2, 3)$ holds. For $c > 3$ partial revelation is impossible and the Receiver maximizes his utility as he would do without communication.

1.2.2.2 General results

Again we would like to underline that the Receiver cannot commit to any actions, but maximizes his utility. Then it should be obvious that the Receiver always prefers partial revelation over no revelation at all. If there is a partial revelation of one state, the Receiver will maximize his expected utility in the remaining states. It might happen that several actions (pure or mixed) solve this maximization problem.

Definition 1.4. *For $\Omega' \subseteq \Omega$ we define $\dot{A}(\Omega') = \arg\max_a \sum_{\omega \in \Omega'} \mu[\omega] u_R(a, \omega)$.*

$\dot{A}(\Omega')$ is the set of actions which maximize the Receiver's utility on a given state space Ω' according to the Receiver's belief system μ.
In a general model with more than three states, there can be different types of partial revelation: Partial revelation can be either achieved by verifiable messages, which then fully reveal a subset of states or by cheap-talk messages. Partial revelation by cheap-talk creates a partition of state subsets, each element of the partition can contain a single state or several states. Elements with just a single state have the same effect as verifiable messages: The Receiver knows whether specific state is the true state. For simplicity we split partial revelation by cheap-talk up into two cases.

1 Partial revelation by verifiable messages $\Rightarrow$ Full Revelation of a subset of states

2 A Partial revelation by cheap-talk $\Rightarrow$ Full Revelation of a subset of states
B Partial revelation by cheap-talk $\Rightarrow$ Dividing the state space into disjoint subsets.

The case 2A contains just the special cases, in which the partition consists of some subsets with just one element and a subset containing the remaining states. Note that also in case 2 there can be a full revelation of subsets of states.
In a world with four states $\{\omega_1, \dots \omega_4\}$ partial revelation by type 2B for example means that the Receiver just knows whether the true state is in $\{\omega_1, \omega_2\}$ or in $\{\omega_3, \omega_4\}$. Of course there can also be a combination of type 1 with 2A or with 2B.
Even with just 4 states most often it is impossible to see, which partial revelation is possible without calculating all possibilities. We state conditions for each of the different types and their combinations. With these conditions it is easy to write an algorithm and let a computer check all the possibilities.

Partial Revelation by one message type

Proposition 1.3 (Partial Revelation just by Verifiable Messages)**.**
There is a partial revealing equilibrium in which the Sender uses verifiable messages only if the true state $\hat{\omega}$ in an element of $\Omega^{vI} \subsetneq \Omega$ with Ω^{vI} such that:

$$\begin{aligned} \exists \hat{a} \in \dot{A}(\Omega \setminus \Omega^{vI}) : &1)\ \forall \omega \in \Omega^{vI} : u_S(a_R^*(\omega), \omega) - c > u_S(\hat{a}, \omega) \\ &2)\ \forall \omega \in \Omega \setminus \Omega^{vI} : u_S(a_R^*(\omega), \omega) - c < u_S(\hat{a}, \omega). \end{aligned}$$

With this proposition we can define the family of subsets in which partial revelation by verifiable information is possible.

Definition 1.5.

$$\begin{aligned} \mathbf{\Omega}^{vI}(\Omega) = \Big\{ \Omega^{vI} \subsetneq \Omega \;\Big|\; &\exists \hat{a} \in \dot{A}(\Omega \setminus \Omega^{vI}) \textit{ such that} \\ &\forall \omega \in \Omega^{vI} : u_S(a_R^*(\omega), \omega) - c > u_S(\hat{a}, \omega) \textit{ and} \\ &\forall \omega \in \Omega \setminus \Omega^{vI} : u_S(a_R^*(\omega), \omega) - c < u_S(\hat{a}, \omega) \Big\} \end{aligned}$$

This implies that partial revelation by verifiable information is impossible if $\mathbf{\Omega}^{vI}(\Omega) = \{\emptyset\}$. We can also define the set of all tuples of actions and subsets of states $(\hat{a}, \Omega^{vI})$, where the action maximizes the Receiver's utility on $\Omega \setminus \Omega^{vI}$, but for the states in Ω^{vI} this action works as a threat to enforce the Sender to use the verifiable message.

Definition 1.6.

$$\begin{aligned} \mathbf{\Omega}_A^{vI}(\Omega) = \Big\{ (\hat{a}, \Omega^{vI}) \;\Big|\; &1)\ \hat{a} \in \dot{A}(\Omega \setminus \Omega^{vI}) \\ &2)\ \forall \omega \in \Omega^{vI} : u_S(a_R^*(\omega), \omega) - c > u_S(\hat{a}, \omega) \\ &3)\ \forall \omega \in \Omega \setminus \Omega^{vI} : u_S(a_R^*(\omega), \omega) - c < u_S(\hat{a}, \omega) \Big\} \end{aligned}$$

This definition will help to combine different types of partial revelation. We can make similar statements for partial revelation of type 2A:

Proposition 1.4 (Partial Revelation just by Cheap-Talk)**.**
There is a partial revealing equilibrium in which the Sender uses cheap-talk messages to reveal the true state only if the true state $\hat{\omega}$ is an element of $\Omega^{ct} \subsetneq \Omega$ with Ω^{ct} such that:

$$\begin{aligned} \exists \hat{a} \in \dot{A}(\Omega \setminus \Omega^{ct}) : & 1)\ \forall \omega \in \Omega^{ct} : u_S(a_R^*(\omega), \omega) > u_S(\hat{a}, \omega) \\ & 2)\ \forall \omega \in \Omega \setminus \Omega^{ct} : u_S(a_R^*(\omega), \omega) < u_S(\hat{a}, \omega). \end{aligned}$$

Definition 1.7.

$$\begin{aligned} \mathbf{\Omega}^{ct}(\Omega) = \Big\{ \Omega^{ct} \subsetneq \Omega \ \Big| \ & \exists \hat{a} \in \dot{A}(\Omega \setminus \Omega^{ct}) \text{ such that} \\ & \forall \omega \in \Omega^{ct} : u_S(a_R^*(\omega), \omega) > u_S(\hat{a}, \omega) \text{ and} \\ & \forall \omega \in \Omega \setminus \Omega^{ct} : u_S(a_R^*(\omega), \omega) < u_S(\hat{a}, \omega) \quad \Big\} \end{aligned}$$

Definition 1.8.

$$\begin{aligned} \mathbf{\Omega}_A^{ct}(\Omega) = \Big\{ (\hat{a}, \Omega^{ct}) \ \Big| \ & 1)\ \hat{a} \in \dot{A}(\Omega \setminus \Omega^{ct}) \\ & 2)\ \forall \omega \in \Omega^{ct} : u_S(a_R^*(\omega), \omega) > u_S(\hat{a}, \omega) \\ & 3)\ \forall \omega \in \Omega \setminus \Omega^{ct} : u_S(a_R^*(\omega), \omega) < u_S(\hat{a}, \omega) \quad \Big\} \end{aligned}$$

For partial revelation of type 2B the conditions look a little bit different.

Proposition 1.5 (Partial Revelation just by Cheap-Talk)**.**
There is a partial revealing equilibrium, where the state space Ω is split up into disjoint subsets if there exists a series of sets $(\Omega_j^{div})_{j=1,\dots,J}$ such that

1. $\bigcup j = 1 J \Omega_j^{div} = \Omega$ *and* $\forall k \neq l : \Omega_k^{div} \cap \Omega_l^{div} = \emptyset$.
2. $\forall \Omega_j^{div}\ \exists \hat{a}_j \in \dot{A}(\Omega_j^{div})$ *such that* $u_S(\hat{a}_k, \omega) > u_S(\hat{a}_l, \omega)\ \forall \omega \in \Omega_k^{div}$ *with* $k \neq l$.

The first condition says that the subsets have to be disjoint and add up to the complete state space. The second condition ensures that the Sender has no incentive to lie if the Receiver chooses the actions that maximize his expected utility for each subset. As before, we can write this as a set, this time consisting of series of tuples of actions and subsets of the state space:

Definition 1.9.

$$\begin{aligned} \mathbf{\Omega}_A^{div}(\Omega) = \Big\{ \big(\hat{a}_j, \Omega_j^{div}\big)_j \ \Big| \ & 1)\ \forall \Omega_k^{div} : a_k \in \dot{A}(\Omega_k^{div}) \\ & 2)\ \bigcup j \Omega_j^{div} = \Omega \text{ and } \forall k \neq l : \Omega_k^{div} \cap \Omega_l^{div} = \emptyset \\ & 3)\ \forall \omega \in \Omega_k^{div} : \forall \hat{a}_k \neq \hat{a}_l : u_S(\hat{a}_k, \omega) > u_S(\hat{a}_l, \omega) \quad \Big\} \end{aligned}$$

This set contains all the different possibilities of series of tuples that split the state space into subsets.

Partial revelation by a combination of verifiable messages and cheap-talk

For the combination of two types of partial revelation it is not sufficient to combine $\mathbf{\Omega}^{vI}$ and $\mathbf{\Omega}^{ct}$, because we need to use the same action $\hat{a}$ for the states, that are not revealed.

Theorem 1.4 (Partial Revelation by type 1 and 2A)**.**
All combinations of revelation by verifiable message and cheap-talk (type 2A) are given by:

$$\mathbf{\Omega}^{vI+ct}(\Omega) = \Big\{ \left(\Omega^{vI}, \Omega^{ct}\right) \Big| \; 1)\ \Omega^{vI} \cap \Omega^{ct} = \emptyset \\ 2)\ \exists \hat{a} \in \dot{A}\left(\Omega \setminus (\Omega^{vI} \cup \Omega^{ct})\right) \textit{ such that} \\ (\hat{a}, \Omega^{vI}) \in \mathbf{\Omega}^{vI}_{A}(\Omega \setminus \Omega^{ct}) \textit{ and} \\ (\hat{a}, \Omega^{ct}) \in \mathbf{\Omega}^{ct}_{A}(\Omega \setminus \Omega^{vI}) \Big\}$$

This means that all states in Ω^{vI} are revealed by verifiable messages and those in Ω^{ct} by cheap-talk. Therefore it is necessary that $\hat{a}$ maximizes the Receiver's utility for the remaining states $\Omega \setminus (\Omega^{vI} \cup \Omega^{ct})$. By the definition of $\mathbf{\Omega}^{vI}$ and $\mathbf{\Omega}^{ct}$ it is ensured that $\Omega^{vI} \cup \Omega^{ct} \subsetneq \Omega$ holds, because otherwise there would be full revelation.
Similar to the combination of type 1 and type 2A, it is also possible to combine type 1 and type 2B. This means that there are some states revealed by verifiable information (type 1) and the remaining states are divided into subsets of the state space (type 2B).

Theorem 1.5 (Partial Revelation by type 1 and 2B)**.**
All combinations of revelation by verifiable message and cheap-talk (type 2B) are given by:

$$\mathbf{\Omega}^{vI+div}(\Omega) = \Bigg\{ \left(\left(\hat{a}_j, \Omega^{div}_j\right)_j, \Omega^{vI}\right) \Big| \; 1)\ \left(\bigcup j \Omega^{div}_j\right) \cup \Omega^{vI} = \Omega \\ 2)\ \forall \Omega^{div}_k :\ \Omega^{div}_k \cap \Omega^{vI} = \emptyset \\ 3)\ (\hat{a}_j, \Omega^{div}_j)_j \in \mathbf{\Omega}^{div}_{A}(\Omega \setminus \Omega^{vI}) \\ 4)\ \forall \hat{a}_k : (\hat{a}_k, \Omega^{vI}) \in \mathbf{\Omega}^{vI}_{A}\left(\Omega \setminus \left(\bigcup l \neq k \Omega^{div}_l\right)\right) \Bigg\}$$

Condition 1) and 2) ensure that the subsets of states are disjoint, but united are equal to the entire state space. Condition 3) makes sure that the states are split up, if there is no revelation by a verifiable message. The Receiver plays different actions on different subsets of states, with Condition 4) the Sender will send the verifiable message in all states in Ω^{vI} and will not deviate to an action $\hat{a}_k$ from the series $(\hat{a}_j)$.

1.2.2.3 Increasing and Decreasing Differences

For increasing (or decreasing) differences, we can state the existence of partial revealing equilibria with verifiable messages in a way similar to Proposition 1.1. The most important change is that the answer to cheap-talk is no longer a^*_R, but $\hat{a}$ such that this action maximizes the Receiver's utility on the non-revealed states.

Proposition 1.6 (Partial Revelation by verifiable messages under increasing differences)**.**
Let $\Omega = \{\omega_1, \dots, \omega_L\}$ and sort A such that $A = \{a_R^*(\omega_1), \dots, a_R^*(\omega_L)\}$. We can ignore all actions, which are never the best reply for the Receiver in a single state.
There is a partial revealing equilibrium that reveals the states just in $[\underline{\omega}, \overline{\omega}]$ by verifiable messages if

$$\exists \hat{a} \in \dot{A}(\Omega \setminus [\underline{\omega}, \overline{\omega}]) \text{ with } a_R^*(\underline{\omega}) > \hat{a} \text{ such that}$$
$$1)\ u_S \text{ has increasing differences on } \Omega' = [\underline{\omega}, \overline{\omega}] \text{ and } A' = [a_R^*(\underline{\omega}), a_R^*(\overline{\omega})]$$
$$2)\ u_S(a_R^*(\underline{\omega}), \underline{\omega}) - c > u_S(\hat{a}, \underline{\omega})$$
$$3)\ \forall \omega_i \in [\underline{\omega}, \overline{\omega}] : u_S(a_R^*(\omega_i), \omega_i) \geq u_S(a_R^*(\omega_{i-1}), \omega_i)$$
$$4)\ \forall \omega_j \in \Omega \setminus [\underline{\omega}, \overline{\omega}] : u_S(\hat{a}, \omega_j) > u_S(a_R^*(\omega_j), \omega_j) - c$$

Proposition 1.7 (Partial Revelation by verifiable messages under increasing differences)**.**
Let $\Omega = \{\omega_1, \dots, \omega_L\}$ and sort A such that $A = \{a_R^*(\omega_1), \dots, a_R^*(\omega_L)\}$. We can ignore all actions, which are never the best reply for the Receiver in a single state.
There is a partial revealing equilibrium that reveals the states just in $[\underline{\omega}, \overline{\omega}]$ by verifiable messages if

$$\exists \hat{a} \in \dot{A}(\Omega \setminus [\underline{\omega}, \overline{\omega}]) \text{ with } a_R^*(\overline{\omega}) < \hat{a} \text{ such that}$$
$$1)\ u_S \text{ has increasing differences on } \Omega' = [\underline{\omega}, \overline{\omega}] \text{ and } A' = [a_R^*(\underline{\omega}), a_R^*(\overline{\omega})]$$
$$2)\ u_S(a_R^*(\overline{\omega}), \overline{\omega}) - c > u_S(\hat{a}, \overline{\omega})$$
$$3)\ \forall \omega_i \in [\underline{\omega}, \overline{\omega}] : u_S(a_R^*(\omega_i), \omega_i) \geq u_S(a_R^*(\omega_{i+1}), \omega_i)$$
$$4)\ \forall \omega_j \in \Omega \setminus [\underline{\omega}, \overline{\omega}] : u_S(\hat{a}, \omega_j) > u_S(a_R^*(\omega_j), \omega_j) - c$$

We can do a similar change to Condition 3) as before and also get the same results for decreasing differences by the same changes as done between Propositions 1.1 and 1.2.
In addition it is possible to rewrite these conditions that they hold for more than just a single interval $[\underline{\omega}, \overline{\omega}]$, but for a disjoint series of intervals $\left([\underline{\omega_k}, \overline{\omega_k}]\right)_k$.

1.3 Continuous model

In many settings it is not enough to focus on a finite action or state space, but assume that both of them are continuous. For example at wage negotiations or any discussions concerning prices, we have to deal with a continuous interval. In this section we do not limit our attention any more to the discrete setting, but switch to a continuous model. So in general we can assume $A = \Omega = [0, 1]$. We state different conditions under which there is no possibility for a fully revealing equilibrium. Afterwards we use the example of the quadratic loss function to visualize our results. Theorem 1.1 and Theorem 1.2, which give the conditions for fully revealing equilibria with only a single message type, still hold. The conditions in these theorems still have to hold for every state, which is more strict in the continuous model. The following Theorems 1.6 to 1.8 give us necessary conditions for the existence of different fully revealing equilibria, where the continuity of u_S and a_R^* are the most important factors. Combined with the results from the discrete model we also get the sufficient conditions.

Theorem 1.6 (Full Revelation under continuous u_S and a_R^*).
Assume that $a_R^*(\omega) : \Omega \to A$ is continuous and that $u_S(a, \omega) : A \times \Omega \to \mathbb{R}$ is continuous in both arguments. Then full revelation can only be achieved either by cheap-talk messages in every state or by verifiable messages only.

Theorem 1.7 (Full Revelation under continuous $u_S(a, \omega)$).
Assume that $u_S(a, \omega)$ is continuous. There can be a fully revealing equilibrium with both message types used if there exists $[\underline{\omega}, \overline{\omega}] \subseteq [0, 1]$ such that for all $\omega \in [0, 1]$

1) $\lim_{\omega \nearrow \underline{\omega}} a_R^*(\omega) \neq a_R^*(\underline{\omega})$

2) $\lim_{\omega \searrow \overline{\omega}} a_R^*(\omega) \neq a_R^*(\overline{\omega})$

3) If $\underline{\omega} \neq \overline{\omega} : \forall \omega_i \in [\underline{\omega}, \overline{\omega}] : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i) \ \forall \omega_j \in [\underline{\omega}, \overline{\omega}], \omega_j \neq \omega_i$

holds.

Remarks.

- *If $\underline{\omega} = 0$, then the first condition is always satisfied.*
- *If $\overline{\omega} = 1$, then the second condition is always satisfied.*
- *There may exist more than one interval satisfying the conditions of Theorem 1.7.*

Theorem 1.7 states that if u_S is continuous in both arguments, the function a_R^* has to be discontinuous. The interval $[\underline{\omega}, \overline{\omega}]$ gives the interval of states in which the Receiver believes the Sender's cheap-talk message. For that a_R^* has to be neither right-continuous nor left-continuous at a single $\hat{\omega}$ or not right-continuous at $\underline{\omega}$ and not left-continuous at $\overline{\omega} > \underline{\omega}$. In addition, the Sender also is not allowed to have any incentive to deviate to a different cheap-talk message for states in $[\underline{\omega}, \overline{\omega}]$. In a continuous state and action space with continuous utilities for Sender and Receiver, there exists no action $\hat{a}$ that the Receiver can use as a threat point such that the Sender uses verifiable messages in some states, but cheap-talk in those states in which the Receiver's most preferred action is $\hat{a}$. The reasoning is as follows: Let $\hat{\omega}$ denote the state of the world in which the Receiver wants the action $\hat{a}$, i.e. $a_R^*(\hat{\omega}) = \hat{a}$. At a state $\hat{\omega} + \epsilon_1$ close to $\hat{\omega}$ the Receiver prefers another action, because of the continuity it is close to $\hat{a}$, i.e. $\hat{a} + \epsilon_2$. If the Receiver uses $\hat{a}$ to reply to cheap-talk, the Sender uses cheap-talk not only in $\hat{\omega}$, but also for states close to $\hat{\omega}$. In that case his utility is $u_S(\hat{a}, \hat{\omega} + \epsilon_1)$ which is larger than $u_S(\hat{a} + \epsilon_2, \hat{\omega} + \epsilon_1)$ because of the continuity in the utility functions of Sender and Receiver. Only if there is some discontinuity the Sender can have an incentive to use the verifiable message close for states close to $\hat{\omega}$ as stated in Theorem 1.7.

Corollary 1.4.
There is a fully revealing equilibrium with both message types used if there exists $[\underline{\omega}, \overline{\omega}] \subseteq [0,1]$ *such that:*

- $[\underline{\omega}, \overline{\omega}]$ *satisfies the conditions of Theorem 1.7.*
- *Theorem 1.3 is satisfied with* $\hat{\Omega} = [\underline{\omega}, \overline{\omega}]$.

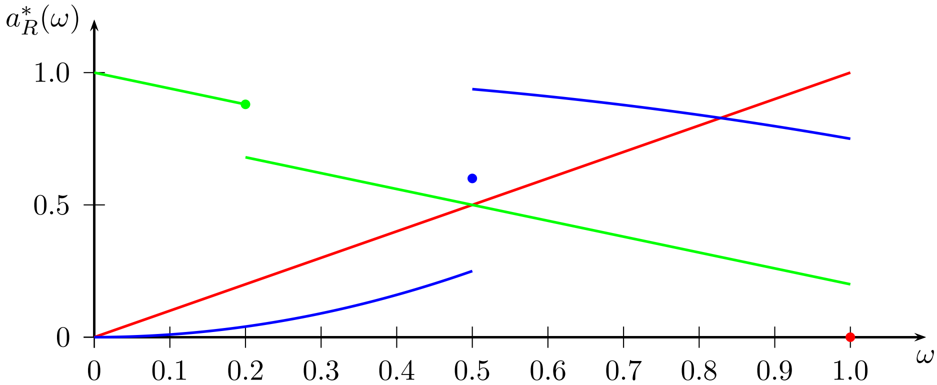


Figure 1.1: Different examples for $a_R^*(\omega)$.

Figure 1.1 shows three different discontinuous functions $a_R^*(\omega)$. For the blue graph there can be a fully revealing equilibrium with both message types, where the threat point is at $a_R^*(\frac{1}{2})$. The red graph shows a situation where the possible threat point is at the border of the interval, here at $a_R^*(1)$. So Condition 2) of Theorem 1.7 is always satisfied. As the function is discontinuous for $\omega = 1$, Condition 1) also holds. An example where Theorem 1.7 implies that there can be no full revelation is given by the green graph. The function is continuous coming from below and so does not satisfy Condition 1).
If a_R^* is continuous the previous theorem does not hold, but we need that u_S is discontinuous to achieve full revelation under the usage of both message types.

Theorem 1.8 (Full Revelation under continuous $a_R^*(\omega)$)**.**
Assume that $a_R^*(\omega)$ *is continuous. Only if* $u_S(a, \omega)$ *is not continuous in at least one argument, there can only be a fully revealing equilibrium with both message types used.*

Remark 1.1. *Theorems 1.7 and 1.8 just state necessary, but not sufficient conditions for the different types of fully revealing equilibria.*

Theorem 1.6 states another possibility for discontinuity that allows for full revelation in which the Sender uses different message types in different states of the world. The reasoning is the same as for Theorem 1.7.

1.3.1 Quadratic loss function

For this second part we like to focus on the quadratic loss utility for the Receiver and a biased quadratic loss utility for the Sender. We show how our general results from the continuous model work and what the intuition behind the missing of the fully revealing equilibria is. The utility functions are $u_R = -(a-\omega)^2$ and $u_S = -(a-\omega-b(\omega))^2$, where $b(\omega) \in \mathbb{R}$ is the state dependent bias function of the Sender. We assume that this bias function is continuous. For positive values of b, the Sender wants to have a higher action than state, while for negative values she wants to have a lower action than state. This is similar to the example Crawford and Sobel (1982) use, but we allow that the bias function is state-dependent.

Clearly we have the problem that $a_R^*(\omega)$ and $u_S(a,\omega)$ are continuous and therefore all fully revealing equilibria just include the use of one message type. As for $A = \Omega = [0,1]$ the function $a_R^*(\omega)$ is bijective and so every action is the best reply for one state, we can focus on pure strategies. It will happen that we misuse notation a little and denote actions by ω as well. Then we simply mean the action $a = \omega$.

As an immediate conclusion from Theorem 1.6 we see that there can be no fully revealing equilibrium with both message types used. As long as the bias function $b(\omega)$ is not constant equal to 0, the Sender will not always tell the truth by cheap-talk. In addition it is also impossible to have a fully revealing equilibrium where the Sender just uses the verifiable messages, because every possible threat point $\hat{a}$ is the Receiver's best reply to one state. This means that in that state the Receiver will never use the verifiable message, but prefers to save the costs and goes for cheap-talk.

Corollary 1.5.
For $A = \Omega = [0,1]$ and quadratic loss utility functions for the players, there are no fully revealing equilibria.

This follows immediately from the continuity of u_S and a_R^* and Theorem 1.6. We can see it in more detail with the help of the following lemma:

Lemma 1.1.
There is a fully revealing equilibrium if

$$\exists\hat{\omega} : 1)\ \forall\omega > \hat{\omega} : b(\omega) > \frac{\hat{\omega}-\omega}{2} - \frac{c}{2(\hat{\omega}-\omega)}$$
$$2)\ \forall\omega < \hat{\omega} : b(\omega) < \frac{\hat{\omega}-\omega}{2} - \frac{c}{2(\hat{\omega}-\omega)}$$

Lemma 1.1 states the condition for a fully revealing equilibrium, where the Sender uses a cheap-talk message in $\hat{\omega}$ and the verified messages in all other states. We can state the same result for a set of states with cheap-talk messages, but use this case to illustrate the problem of the continuous model.

Figure 1.2 shows Lemma 1.1 for three different values of $\hat{\omega}$. For $\hat{\omega} = 0$, the function $b(\omega)$ has to be above the blue curve (in the blue area). If the Receiver answers every cheap-talk message by the action $\hat{\omega} = 0$, the Sender should not prefer this action over the one responding to the true state. This can be achieved by a positive bias function, or for some values also by a slightly negative one. For $\hat{\omega} = 1$, the function $b(\omega)$ has to be below the red curve (in the red area). For $\hat{\omega} = 0.5$, the function $b(\omega)$ has to be below the green curve for $\omega < 0.5$ and above for $\omega > 0.5$ (in the green shaded area).

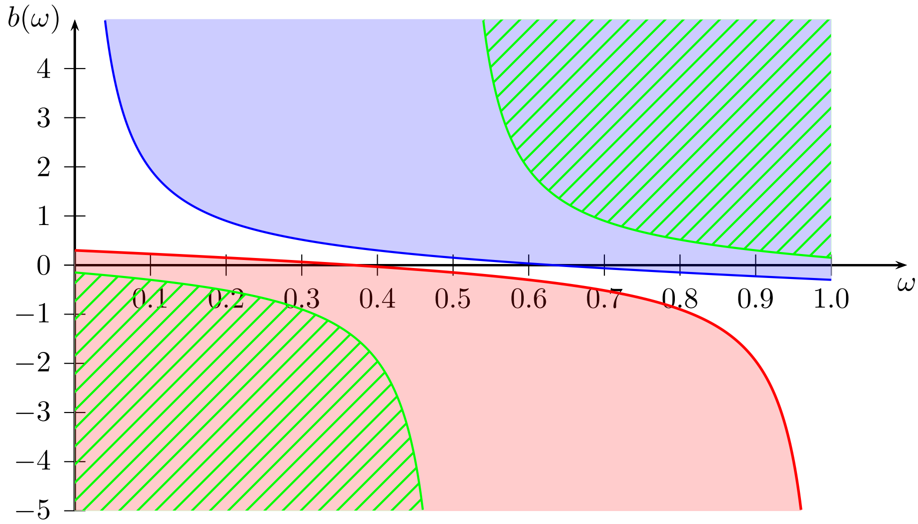


Figure 1.2: Regions of fully revealing equilibria, for $\hat{\omega} = 0, 0.5$ and 1 with $c = 0.4$

This figure already reveals a problem with this setting: No matter the value of $\hat{\omega}$, it is necessary that the bias function $b(\omega)$ gets either really high or low values. The problem here is that the bias function has values in the real numbers, but Conditions 1) or 2) require $|b(\omega)| = \infty$, for some ω. This means if the Receiver answers every cheap-talk message with $\hat{\omega}$ there is always a neighborhood around $\hat{\omega}$ where the Sender prefers sending the costless cheap-talk message over sending the costly verifiable message. The Sender's utility loss by the quadratic loss function (difference between action and state) is less than the utility loss resulting from the costs c.

1.3.1.1 Discretization

One way to achieve full revelation, while keeping the quadratic loss functions, is to discretize the type space.

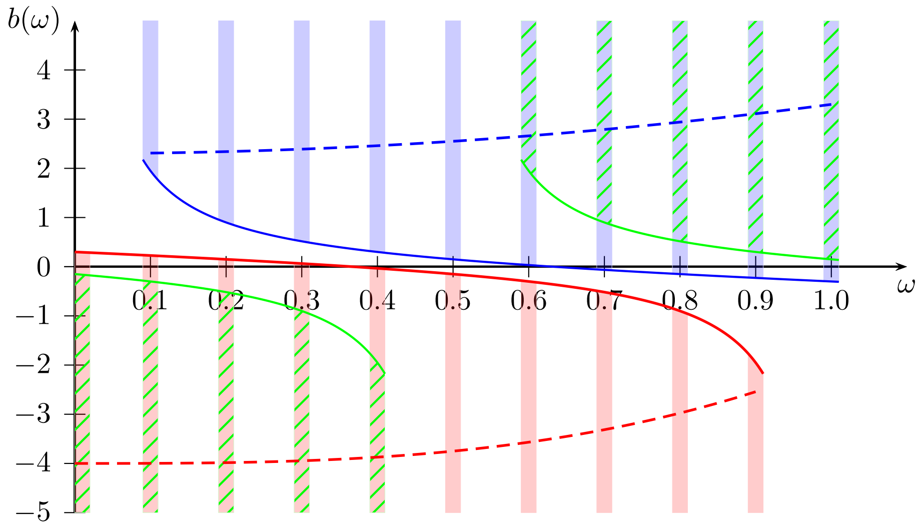


Figure 1.3: Regions of fully revealing equilibria, for $\hat{\omega} = 0, 0.5$ and 1 with $c = 0.4$

Figure 1.3 shows a discretization for example for $\Omega = \{0, 0.1, 0.2, \dots, 1\}$. There can be a fully revealing equilibria, even with costly verification, quadratic loss functions and a continuous action space. Again for $\hat{\omega} = 0$, the function $b(\omega)$ has to be above the blue curve (in the blue area) for $\omega \in \{0.1, 0.2, \dots, 1\}$. As we do not need this condition for values close to $\hat{\omega}$, but just starting with 0.1 the area is cut off at 0.1. This avoids that the bias function needs too high values. Analogue for $\hat{\omega} = 1$, the area is cut off at $\omega = 0.9$ and the function $b(\omega)$ has to be below the red curve (in the red area) for $\omega \in \{0, 0.1, \dots, 0.9\}$. There are two cuts for $\hat{\omega} = 0.5$. One at the state lower than 0.5, which is 0.4 and the other one at the next higher state, 0.6. The function $b(\omega)$ has to be below the green curve for $\omega \in \{0, 0.1, \dots, 0.4\}$ and above for $\omega \in \{0.6, 0.7, \dots, 1\}$ (in the green shaded area).

Lemma 1.2.
Assume that the Sender's utility is modeled by a quadratic loss function $u_S(a, \omega) = -(a - \omega - b(\omega))^2$. *If* $b(\omega)$ *is non-decreasing,* u_S *satisfies increasing differences.*

The application of Proposition 1.1 and the following corollaries can be seen in Figure 1.3 as well. For $\hat{a} = 0$ and $b(\omega)$ increasing we have as first condition that $b(\omega)$ should be above the blue curve. An example is given by the dashed blue curve. The maximal cost c have to be lower than the utility difference is 0.1, which is illustrated as the difference between the blue curve and the dashed blue curve. Similar for $\hat{a} = 1$ and the red dashed curve, here the critical condition is that $b(\omega)$ stays below the red curve even for $\omega = 0.9$.

1.4 Extensions

In this section we want to give some ideas of extension possibilities to fit our model into different situations. We do not go much into detail, but just state our ideas and possible implications.

State dependent costs

One simple extension of our model is to allow that the costs for the verifiable message c are state dependent, i.e. $c(\omega) : \Omega \to \mathbb{R}$. As long as the costs are strictly positive for all states, there are no mayor changes. Of course all conditions of the previous results are slightly different for each state, but the ideas stay the same. A more dramatic change would happen if we allow that the costs c are equal to zero for some states. Then there is no possibility to guarantee that there exists a fully revealing equilibrium, where the Sender uses just cheap-talk. The simple reason is that, in the states where the verifiable message is for free, she is indifferent as both messages yield to the same utility. So Theorem 1.1 does not hold any longer.

The most dramatic changes happen to our results in the continuous model: Even with continuous utility u_S and continuous a^*_R, there can be a fully revealing equilibrium if $c(\omega) = 0$ holds for some ω. This might be an interesting point for future research.

Sender mixing

As long as there is full revelation, the Sender will never mix messages, so the only part where the mixing plays a role is for the cases of partial revelation. Only under special circumstances the Sender has an incentive to mix, for example if she is indifferent between several actions resulting from different messages. If the Sender does not mix, the Receiver will use the probabilities of each state to maximize his utility, as stated in section 1.2.2. Knowing that the Sender might mix his actions, the Receiver will use Bayes' rule to update his beliefs and maximize according to them. This means we have to replace the probabilities $\mathbb{P}$ in the maximization problems with the Receiver's beliefs μ. We state three examples for this:

Example 1.5.
Assume that $\Omega = \{\omega_1, \ldots, \omega_l\}$ *and for simplicity* $\mathbb{P}[\omega_i] = \frac{1}{L} \forall \omega_i \in \Omega$. *Furthermore we assume that the Sender sends* m_1 *in* ω_1 *and* ω_2, *but nowhere else. After reading the message* m_1 *the Receiver updates his beliefs to* $\frac{1}{2}\omega_1, \frac{1}{2}\omega_2$.

Example 1.6.
Assume that $\Omega = \{\omega_1, \ldots, \omega_l\}$, $\mathbb{P}[\omega_i] = \frac{1}{L} \forall \omega_i \in \Omega$ *and that the Sender mixes such that:*

$$\begin{array}{lcccc} \omega_1 \to & \frac{1}{3}m_1 & + & \frac{2}{3}m_2 \\ \omega_2 \to & \frac{1}{2}m_1 & + & \frac{1}{2}m_2 \end{array}$$

In all other states she does not send m_1 *or* m_2. *Then if the Receiver gets the message* m_1, *he beliefs that the true state is:* ω_1 *with probability* $\frac{1/3}{1/3+1/2} = \frac{2}{5}$ *and* ω_2 *with probability* $\frac{1/2}{1/3+1/2} = \frac{3}{5}$. *Analogue for message* m_2.

Example 1.7.
Assume that $\Omega = \{\omega_1, \ldots, \omega_l\}$, $\mathbb{P}[\omega_i] = \frac{1}{L} \forall \omega_i \in \Omega$ *and that the Sender mixes as follows:*

$$\begin{array}{llllll} \omega_1 \to & \frac{1}{3}m_1 & + & \frac{2}{3}m_2 & & \\ \omega_2 \to & & & \frac{1}{2}m_2 & + & \frac{1}{2}m_3 \\ \omega_3 \to & & & & & m_3 \end{array}$$

Then the Receiver's beliefs are:

$$\begin{array}{llll} m_1 \to & \omega_1 & & \\ m_2 \to & \frac{2}{5}\omega_1 & \frac{3}{5}\omega_2 & \\ m_3 \to & & \frac{1}{3}\omega_2 & \frac{2}{3}\omega_3 \end{array}$$

States as Sender's types

Many economic examples do not consider different states of the world, but different Sender types. We can easily see our states as types. For the typical notation we replace Ω by Θ with elements θ_1 to θ_L instead of $\omega_1, \ldots, \omega_L$.

A standard example is to see the Sender as an agent looking for a job, with skills θ, while the Receiver is the employer who wants to hire a well skilled worker. He can either choose to hire the Sender or not. Of course his action depends on his belief of the Sender's skills. The Sender either just mentions her skill sets, which is the cheap-talk message, or she can verify it by some certificates. If we assume that the Sender's utility just depends on his employment and not on his skills, the Receiver's threat point here is not to hire the Sender after a cheap-talk message, but just if the verifiable message shows that the Sender has the necessary skill level.

Receiver's utility

To avoid multiple equilibria and the waste of money for verifiable messages when they are not necessary, we can modify the utility function of the Receiver: $\hat{u}_R(a, \omega, \tilde{u}_S) = u_R(a, \omega) + \epsilon_R \cdot \tilde{u}_S$, with u_R and $\tilde{u}_S$ as before and $\epsilon_R > 0$, but small. We assume ϵ to be such small that

$$|\epsilon_R| \cdot \max_{a_x} \hat{u}_S(a_x, m, \omega) < |u_R(a_i, \omega) - u_R(a_j, \omega)| \; \forall \omega \; \forall a_i, a_j, i \neq j \; \forall m \in M \cup \{v\}. \quad (1.2)$$

Equation (1.2) implies that the Receiver aims to maximize his utility directly by u_R. He will not choose an action that gives him not the highest u_R, but a high utility through $\tilde{u}_S$. With this assumption we assure that the solutions of our maximization problems stay the same, but if there are multiple solutions, the Receiver will choose the solution giving the highest utility to the Sender. Especially for the case, where the preferences of the Sender and Receiver are the same, this is helpful. In this setting there can be several fully revealing equilibria, but just the one where the Receiver beliefs every cheap-talk message is Pareto-efficient.

Sender's utility

In the previous sections we ignored the possibility that the Sender might be indifferent between sending a cheap-talk message or the verifiable message. We can change the utility function to $\hat{u}_S(a, \omega, m, u_R) = \tilde{u}_S(a, \omega, m) + \epsilon_S \cdot u_R$ with $\tilde{u}_S$ and u_R as before. Under some assumptions it is reasonable that the Sender prefers cheap-talk, under other assumptions she should prefer the verifiable messages. The best reason for cheap-talk is the one stated in the previous extension: The Sender wants to maximize his utility, but at the same time she prefers a higher utility for the Receiver over a lower one, in that case ϵ_S should be positive. On the other hand one can argue that sending a verifiable message gives her certainty, what she might prefer. Then ϵ_S should be negative. In both cases $|\epsilon_S|$ should be small enough not to interfere with the Sender's maximization problem, as we stated in Equation (1.2) for the Receiver.

1.5 Conclusion

In this paper we have combined the cheap-talk model of Crawford and Sobel (1982) with the models dealing with verifiable messages. In our Sender-Receiver game the informed Sender can choose between verifiable and non-verifiable messages. While the Receiver only learns the true state for sure after reading a verifiable message, a cheap-talk message will not reveal the true state to him, but just let him update his belief system. We stated conditions for a discrete setting under which the Sender reveals the true state to the Receiver. The main idea behind is known from other models as well: The Receiver punishes the Sender for not using the verifiable message by answering every cheap-talk message with an action the Sender dislikes. As we limit our attention to non-dominated action, there always exists a belief system which makes this action best reply and so it makes the threat credible.

If such action does not exist, full revelation can only be achieved by common interests. In that case the Sender has no reason to lie and the Receiver can trust every cheap-talk message. Otherwise there can be only partial revelation or no revelation at all. In the first case we differ between different ways of revelation, for each of them we state conditions. We have not only analyzed different examples for partial revelation and have shown that there exist several ways for the Receiver to maximize his utility, but also stated general results. For the case that the utility functions have increasing or decreasing differences, all conditions simplify.

In a continuous model the enforcement of full revelation is more difficult. Under continuous utility functions for the Sender and the Receiver there is no fully revealing equilibrium where the Sender uses both message types. We have illustrated that with the standard example of the quadratic loss function and we also have shown a way to counteract it: By discretization of the state space. All in all we stated results that allow to check whether there are fully revealing equilibria or not. Therefore we differ between three different types of fully revealing equilibria: The one where both message types are used and those where the Sender always sticks to one kind of message.

For future research we would like to further characterize the group of utility functions which allow for full revelation, either by using specific properties as single-crossing or by discretization of specific utility functions. There are several ways in which we can push these ideas, but with this model we created a suitable foundation.

Appendix of Chapter 1

Proof. Theorem 1.1

Only if: Clearly there is a fully revealing equilibrium just with cheap-talk messages if Condition (1.1) holds: The Receiver will trust every cheap-talk message and the Sender has no incentive to deviate.

If: Proof by contradiction. Let us assume that $\exists \omega_k$ such that $u_S(a_R^*(\omega_k), \omega_k) \ngtr u_S(a_R^*(\omega_j), \omega_k)$ $\forall \omega_j \neq \omega_k$. This implies that there exists ω_j such that $u_S(a_R^*(\omega_k), \omega_k) < u_S(a_R^*(\omega_j), \omega_k)$ holds. Then the Receiver has an incentive to lie in ω_k and send the cheap-talk message ω_j, so there is no full revelation. □

Proof. Theorem 1.2

Only if: Follows directly.

If: Proof by contradiction.

Step 1) Let us assume that Condition 2) does not hold. Then there exists a ω_j such that $u_S(a_R^*(\omega_j), \omega_j) - c < u_S(\hat{a}, \omega_j)$ holds. This implies that the Sender prefers sending a cheap-talk message and getting action $\hat{a}$ over sending the verifiable message and action $a_R^*(\omega_j)$. So she will deviate in ω_j and there will be no full revelation.

Step 2) Let us assume that Condition 1) does not hold, then Condition 2) does not hold and we can follow Step 1). □

Proof. Theorem 1.3

Only if: The equilibrium is as follows: For $\omega \in \hat{\Omega}$ the Receiver trusts the cheap-talk and in all other states the Sender uses the verifiable message.

If: Proof by contradiction.

Step 1) Let us assume that Condition 1) does not hold. This implies that there exist $\omega_i \notin \hat{\Omega}$ and $\omega_j \in \hat{\Omega}$ such that $u_S(a_R^*(\omega_j), \omega_i) > u_S(a_R^*(\omega_i), \omega_i) - c$ holds. So the Sender prefers cheap-talk (and action $a_R^*(\omega_j)$) over the verifiable message (and action $a_R^*(\omega_i)$) and there will be no full revelation, because $a_R^*(\omega_i) \neq a_R^*(\omega_j)$.

Step 2) We assume that Condition 2) does not hold and follow the same steps as in the proof of Theorem 1.1. □

Proof. Proposition 1.1

We split the proof in two parts. First we show that for states higher than ω_j, the Sender prefers the verifiable message, then we do the same for lower states. Assume $\omega_i > \omega_j$ holds. For $\omega_i = \omega_{j+1}$ we have Condition 2.1), which states that the Sender prefers the costly verifiable message (yielding $a_R^*(\omega_{j+1})$), over the cheap-talk message (yielding $\hat{a}$).

It remains to show that for all $\omega_i > \omega_{j+1} : c < u_S(a_R^*(\omega_i), \omega_i) - u_S(a_R^*(\omega_j, \omega_i)$ hold.

$$\begin{aligned} u_S(a_R^*(\omega_i), \omega_i) - u_S(a_R^*(\omega_1), \omega_i) &\overset{3.1)}{\geq} u_S(a_R^*(\omega_{i-1}), \omega_i) - u_S(a_R^*(\omega_j), \omega_i) \\ &\overset{1.)}{\geq} u_S(a_R^*(\omega_{i-1}), \omega_{i-1}) - u_S(a_R^*(\omega_j), \omega_{i-1}) \end{aligned}$$

We repeat these two steps until

$$\begin{aligned} &\geq u_S(a_R^*(\omega_{j+1}), \omega_{j+1}) - u_S(a_R^*(\omega_j), \omega_{j+1}) \\ &\overset{2.1)}{>} c \end{aligned}$$

Analogue steps yield the proof for $\omega_i < \omega_j$. □

Proof. Proposition 1.2
The proof follows the same ideas as the proof of Proposition 1.1. The only difference is that we go step by step from the boundary states and actions to the threat point, while in the previous proof we moved from the threat point towards the boundaries.
For example for $\omega_i > \omega_j$:

$$\begin{aligned} u_S(a_R^*(\omega_i), \omega_i) - u_S(a_R^*(\omega_j), \omega_i) &\overset{3.1)}{\geq} u_S(a_R^*(\omega_{i+1}), \omega_i) - u_S(a_R^*(\omega_j), \omega_i) \\ &\overset{1.)}{\geq} u_S(a_R^*(\omega_{i+1}), \omega_{i+1}) - u_S(a_R^*(\omega_j), \omega_{i+1}) \end{aligned}$$

We repeat these two steps until

$$\begin{aligned} &\geq u_S(a_R^*(\omega_L), \omega_L) - u_S(a_R^*(\omega_j), \omega_L) \\ &\overset{2.1)}{>} c \end{aligned}$$

□

Proof. Proposition 1.3
If Condition 1) holds, the Sender has an incentive to use the verifiable message in all states in Ω^{vI}. She also sends the cheap-talk message in all other states, because of Condition 2). For the Receiver $\hat{a}$ is by definition an action that maximizes his expected utility on $\Omega \setminus \Omega^{vI}$. As the states in Ω^{vI} are revealed, he will play the action he likes the most there. So both players have no incentive to deviate and we have a partial revealing equilibrium. □

Proof. Proposition 1.4
Analogue to the proof of Proposition 1.3. □

Proof. Proposition 1.5
Condition 1) ensures that Ω is split up in disjoint subsets. If Condition 2) holds, there is at least one action $\hat{a}$ for each subset that maximizes the Receiver's expected utility that is such that the Sender does not want to deviate to another action. This means that no player wants to deviate. □

Proof. Theorem 1.4
The first condition is necessary to have disjoint sets for partial revelation by cheap-talk and verifiable messages. The action $\hat{a}$ that satisfies Condition 2) maximizes the Receiver's utility on the remaining states and by definition of $\mathbf{\Omega}^{vI}$ enforces the Sender to use the verifiable message in the states in Ω^{vI}. In the states in Ω^{ct} the Sender has no incentive to deviate to another cheap-talk message by the definition of $\mathbf{\Omega}^{vI}$. □

Proof. Theorem 1.5
Conditions 1) and 2) make sure that Ω is completely split up into disjoint subsets. In each subset Ω_j^{div} the Sender sends a different cheap-talk message, so that the Receiver knows in which subset he is. The Receiver maximizes his expected utility in each of these subsets by $\hat{a}_j$. If Condition 4) holds, the Sender sends a verifiable message in all states in Ω^{vI} and has no incentive to deviate to any $\hat{a}_j$. □

Proof. Proposition 1.6
By Condition 4) the Sender prefers sending cheap-talk over sending a verifiable message outside the interval $[\underline{\omega}, \overline{\omega}]$. Therefore it is correct that $\hat{a}$ is maximizing the Receiver's utility outside that interval.
Condition 1) ensures that for $\underline{\omega}$ the Sender sends the verifiable message. It remains to show that she does so for the rest of the interval as well. For $\omega_i \in [\underline{\omega}, \overline{\omega}]$ we need $u_S(a_R^*(\omega_i), \omega_i) - u_S(\hat{a}, \omega_i) > c$. As in the proof of Proposition 1.1 we show that $u_S(a_R^*(\omega_i), \omega_i) - u_S(\hat{a}, \omega_i) > u_S(a_R^*(\underline{\omega}), \underline{\omega}) - u_S(\hat{a}, \underline{\omega})$. By the first condition the result then follows. Starting from the left side:

$$\begin{aligned} u_S(a_R^*(\omega_i), \omega_i) - u_S(\hat{a}, \omega_i) &\overset{3)}{\geq} u_S(a_R^*(\omega_{i-1}), \omega_i) - u_S(\hat{a}, \omega_i) \\ &\overset{1)}{\geq} u_S(a_R^*(\omega_{i-1}), \omega_{i-1}) - u_S(\hat{a}, \omega_{i-1}) \end{aligned}$$

Repeating these steps until we reach $\underline{\omega}$ yields the result.
We can use the increasing difference property, because by assumption $a_R^*(\underline{\omega}) > \hat{a}$ holds.

□

Proof. Proposition 1.7
The proof follows the same steps as the previous one, but it might help to rewrite the definition of increasing differences: $\forall a' \geq a, \omega' \geq \omega$:

$$\begin{aligned} u_S(a', \omega') - u_S(a, \omega') &\geq u_S(a', \omega) - u_S(a, \omega) \\ \Leftrightarrow u_S(a, \omega) - u_S(a', \omega) &\geq u_S(a, \omega') - u_S(a', \omega') \end{aligned}$$

Compared to the proof of the previous proposition, this time the steps go up from ω_i to $\overline{\omega}$, using $\hat{a} > a^*_R(\overline{\omega})$. □

Proof. Theorem 1.6
The possible existence of fully revealing equilibria with just one type of message sent follows from the conditions imposed in Theorem 1.1 and Theorem 1.2. Assume that the Sender sends a cheap-talk message just in $\hat{w}$ and uses the verifiable message in all other states. The argumentation for sending cheap-talk in several states or intervals will be the same. The Sender has an incentive to use the verifiable message if $u_S(a^*_R(\omega), \omega) - c > u_S(a^*_R(\hat{\omega}), \omega)$. So for the states close to $\hat{\omega}$ we get:

$$u_S(a^*_R(\hat{\omega} \pm \epsilon), \hat{\omega} \pm \epsilon) - c > u_S(a^*_R(\hat{\omega}), \hat{\omega} \pm \epsilon) \tag{1.3}$$

For $\epsilon \to 0$ and the continuity of u_S and a^*_R this is equivalent to:

$$u_S(a^*_R(\hat{\omega}), \hat{\omega}) - c > u_S(a^*_R(\hat{\omega}), \hat{\omega})$$

This then leads to $c < 0$, which is clearly a contradiction. So under this assumptions it is not possible that there is a fully revealing equilibrium where the Sender uses both message types. □

Proof. Theorem 1.7
Assume that 1) or 2) do not hold, the problem is the same as described in Equation (1.3), which requires negative costs c.
Let us assume that 3) does not hold, then the Sender deviates when the real state is in the interval $[\underline{\omega}, \overline{\omega}]$ and so there cannot be full revelation. □

Proof. Theorem 1.8
The proof is analogue to the proof of Theorem 1.7, using the discontinuity of u_S instead of a^*_R. □

Proof. Lemma 1.1

Assume that the Receiver answers every cheap-talk message with $\hat{\omega}$.

The utility of the Sender for any state ω is given by:

u_S("verifiable message") $= -(-b(\omega))^2 - c$

u_S("cheap-talk message") $= -(\hat{\omega} - \omega - b(\omega))^2 = -\left[(\hat{\omega} - \omega)^2 - 2(\hat{\omega} - \omega) \cdot b(\omega) + (b(\omega))^2\right]$

So the Sender will use the verifiable message if and only if:

$$\begin{aligned} & -(-b(\omega))^2 - c && > -\left[(\hat{\omega}) - \omega)^2 - 2(\hat{\omega} - \omega) \cdot b(\omega) + (b(\omega))^2\right] \\ \Leftrightarrow \quad & -2b(\hat{\omega} - \omega) && > -(\hat{\omega} - \omega)^2 + c \end{aligned}$$

Case 1: $\omega > \hat{\omega}$

$$\begin{aligned} \Leftrightarrow \quad & -2b && < -(\hat{\omega} - \omega) + \tfrac{c}{\hat{\omega} - \omega} \\ \Leftrightarrow \quad & b && > \tfrac{\hat{\omega} - \omega}{2} - \tfrac{c}{2(\hat{\omega} - \omega)} \end{aligned}$$

Case 2: $\hat{\omega} > \omega$

$$\begin{aligned} \Leftrightarrow \quad & -2b && > -(\hat{\omega} - \omega) + \tfrac{c}{\hat{\omega} - \omega} \\ \Leftrightarrow \quad & b && < \tfrac{\hat{\omega} - \omega}{2} - \tfrac{c}{2(\hat{\omega} - \omega)} \end{aligned}$$

□

Proof. Lemma 1.2

Increasing Differences mean that the following condition have to hold $\forall a' \geq a, \omega' \geq \omega$:

$$\begin{aligned} & u_S(a', \omega') - u_S(a, \omega') && \geq u_S(a', \omega) - u_S(a, \omega) \\ \Leftrightarrow \quad & -(a' - \omega' - b(\omega'))^2 + (a - \omega' - b(\omega'))^2 && \geq -(a' - \omega - b(\omega))^2 + (a - \omega - b(\omega))^2 \\ \Leftrightarrow \quad & a^2 - (a')^2 + 2(b(\omega') + \omega')(a' - a) && \geq a^2 - (a')^2 + 2(b(\omega) + \omega)(a' - a) \\ \Leftrightarrow \quad & b(\omega') + \omega' && \geq b(\omega) + \omega \end{aligned}$$

This condition is clearly satisfied if $b(\omega)$ is increasing, because $\omega' \geq \omega$ holds. □

CHAPTER 2

Information Transmission in Hierarchies

This chapter is based on the paper "Information Transmission in Hierarchies".
All notations and definitions are independent of those from the previous chapter.

Abstract: We study a game in which players with a unique piece of information are arranged in a hierarchy. Over a finite number of rounds the players can either hold their information or pass it to their successor. Players discount over time and passing generates an immediate payoff so the players have an incentive to pass early. On the other hand, the players have an incentive to pass the information later, because the last player that passes the information to a successor gets an additional reward. Once a successor has collected all information from her workers she can pass the information herself and faces the same problem. We state conditions for the existence of different Subgame Perfect Nash Equilibria and analyze the time it takes the principal in each hierarchy to centralize all information. This allows us to compare different structures and state in which hierarchical structure information centralization is the fastest depending on the information distribution and other parameters.

Keywords : communication network, dynamic network game, hierarchical structure, information transmission.
JEL Classification : D83, C72, C73.

2.1 Introduction

The aim of this paper is to develop a model of information transmission in hierarchies where the principal wants to centralize all information of her agents. There are n players arranged in a hierarchy below the principal and each player has a unique piece of information. All players face the same deadline until they have to submit their information to their successor in the hierarchy. In each period, each player decides either to hold her information or to pass the information on to her successor. A player gets a payoff only when she passes the information and we assume that different players may get different payoffs. A successor can only pass the information to her own successor once she has centralized the information from all her predecessors. An additional payoff is given to the last player who passes information to a successor, i.e. to the player who completes the information the successor has to collect. This yields to an incentive for the players to hold back their information. Each player has an incentive to pass the information as early as possible as well, because players discount over time and players receive their payoff as soon as they pass. All players make their decisions simultaneously.

We start our analysis with a flat hierarchy, i.e. in a model in which all players report directly to the principal and in which no intermediaries exist. We derive all the Subgame Perfect Nash Equilbria of the game. We find that the time it takes the principal to collect all information in equilibrium is either one or two periods or equal to the given deadline. Furthermore, we show that the parameters that have an impact on this duration are the discount factor, the reward, the deadline and the lowest two payoffs of all players. In addition, we analyze the effects of all these parameters and illustrate them.

Then, we extend our model to allow for hierarchies with several layers. The main difference that arises is that an intermediary, i.e. a player who has to collect information from her agents before she can pass all information to her successor, may collect the information in a different period than another intermediary. Hence, the SPNE also depends on these collection times. We find that, in comparison to the model with a flat hierarchy, the time it takes the principal to collect all information is not necessarily increasing in the discount factor. As an example, we give a detailed analysis of all possible ways to arrange four players in a hierarchy and compare all those hierarchical structures. We show that for different values of parameters, the hierarchy with the shortest duration varies and we illustrate this result. Furthermore, we give a set of hierarchical structures that are, respective to the duration, (weakly) dominated by other structures.

One motivation for our model is the situation when a company finds a new investor. The boss of the company will require information about different ways to use the new money for improvements in each department. The head of the departments collect information from their seniors who themselves have to get information from other employees. The agents can make preparations for the improvements once they submitted their information and obviously those improvements may give different payoffs to different agents. In our model we assume that the last player to pass the information gets an additional reward. One motivation for this reward is that the head of the department first may not know how much money she is allowed to spend so the payoffs she gives to her seniors are only pessimistic estimates because the head of the department does not want to

make promises she can not hold. When the last senior passes her information to the head of the department, the head of the department can pass on the entire information and learns how much money the department will get and has to spend. There will be a difference between the real payment and the pessimistic estimates that were paid out and the department has to spend it. So, the head of the department gives this difference as an additional payment to the last senior.
Another example is the writing of a proof. The principal is working on a proof and relies on other researches. Those researches have some information that are helpful for the proof. Obviously, only information that has been proofed by itself can be submitted and this may require information from other people as well. Hence, there is an entire hierarchy of scientists and results. The payoff a researcher gets from passing the information can be of different nature, it can be for example a mentioning in a footnote, a citation, co-authorship or the invitation to a workshop. It seems reasonable to assume that the principal enjoys completing the proof and that she gives a high reward to the scientist who delivers the final piece of the puzzle.
For most of this paper the good transmitted in the hierarchy does not have to be information, instead it could be a machine that is assembled by the principal who gets different parts from different players. We focus on information transmission because information is one of the few goods that can be sent without any costs and without any delay. Furthermore, in one extension we analyze the situation that arises when an agent can pass the information to two intermediaries who then face not only competition for the reward, but also for the information they share. We argue that only information can be copied so easily that it can be passed to two (or more) people at the same time, while creating copies of machine parts seem to come with effort and costs.
A model of information transmission in networks was first introduced by Hagenbach (2011). In her model, players are connected in a network and compete for information. Each player has an incentive to pass the information and an incentive to hold back the information, because the player that centralizes all information gets a higher payoff than all other players. Hence, the players face a similar problem as in our model. While we focus on (directed) tree networks, Hagenbach gives general results depending on the (undirected) network structure. In undirected networks, for the same parameters, there can be several equilibria at the same time and in different equilibria different players will centralize the information. We argue that in many applications it is more reasonable to assume that there is one fixed player whose task it is to centralize all information, while all other players pass the information towards her.
Hierarchical structures, as we use them, can be found in many different articles. There are many papers on hierarchies which focus on different solution concepts and which compare these concepts, e.g. van den Brink and Steffen (2012) and Álvarez-Mozos et al. (2017). van den Brink and Steffen (2008) analyze the power that comes with positions and the arrangement of positions in hierarchies. They take into account the role of the decision making mechanism and focus on the dominance relation between different players in a hierarchy. Closer to our model is the work of Garicano (2000). He deals with knowledge production and transmission in networks. While players in a low level of the hierarchy solve simple problems, more knowledge is held by the specialists in upper levels and these specialists solve more complicated problems. The author shows that this split of tasks is optimal, but the firm has to give additional incentives if the complexity of

a problem is not observable. An empirical work on production in networks was done by Garicano and Hubbard (2016).

In the networks in our paper some players act as intermediaries. The literature on the role of intermediaries in networks has been growing recently. Manea (2015) models the reselling of a good, which also might be information, in a network until the good reaches the final buyer. The utility generated by the players comes from bargaining over the price. The author studies differences between intermediaries who bargain with players on the same layer of the network and those who interact with players from different levels. Siedlarek (2015) focuses more on the competition between different routes that a given good can take through the network from the source to the final buyer. Other papers dealing with trade and intermediaries in networks are for example L. E. Blume et al. (2009) and Choi et al. (2014). Manea (2016) gives an overview over different models of bilateral trade, while the survey of Galeotti and Condorelli (2016) focuses on the role of intermediaries in networks.

The first models of organizational economics go back to Sah and Stiglitz (1986) and Radner (1993). Sah and Stiglitz (1986) argue that the structure in which players are arranged in a firm has an impact on the quality of decision making. They show how the probabilities of accepting good project proposals and declining bad project proposals differ between different hierarchical and polyarchical structures. In Radner (1993) the managers of a firm are the information processors. The author studies the efficiency of different structures under specific circumstances. We do similar comparisons of structures in our model.

Jehiel (1999) also deals with different hierarchical structures. He models a communication structure in which a decision maker needs to centralize information to make a decision about a project. The utility of all players is given by a share of the surplus generated by that project, which incentivizes the players to work as a team. After the decision is made, the decision maker is fired if she made a bad decision. The author gives conditions under which a communication structure is optimal for players who want to communicate their private information. Other models on information transmission, but without the networks aspect, can be found in Lewis and Sappington (1997) and Levitt and Snyder (1997). Lewis and Sappington (1997) derives a way for an agent to acquire information optimally, while Levitt and Snyder (1997) focuses on information transmission in a principal-agent model.

A connection between communication and networks is done in Ambrus et al. (2013). The authors create a model in which communication in a network takes place by cheap-talk between different intermediaries.

This paper is organized as follows. In Section 2.2 we introduce a model with just two layers of hierarchy, give examples and derive all equilibria. We focus on the time it takes the principal to collect all information and analyze the impact of all parameters on this duration. In Section 2.3 we generalize. In a model with more than two layers we state several examples and compare different hierarchical structures. Furthermore, we state some general results and we give some more results depending on additional conditions. The results from the two-layer model help with the analysis here. We discuss two different extension possibilities in Section 2.4. Section 2.5 concludes. All proofs are relegated to the appendix.

2.2 Two layer model

In this part we focus our attention on networks that just consist of two different layers of hierarchy. There is one successor/principal and a set of predecessors/agents $N = \{1, \ldots, n\}$. We assume that there are at least two players with information. Each of the agents $i \in N$ has an information item with value $x_i > 0$. Without loss of generality we say that $x_1 \geq x_2 \geq \ldots \geq x_n > 0$ holds.
The game has a finite deadline of $T > 1$ periods. Starting from period $t = 1$ each player can decide to pass her information in that period or hold it. We denote the action player i took at time t by $a_i^t \in \{P, H\}$ and limit our attention to pure strategies.
Passing the items generates a utility according to the value of information x_i. Once a player passed at time τ_i, she is eliminated from the game, i.e. $a_i^t = H \; \forall t > \tau_i$. All players make their decision simultaneously. The player who passes her information last gets an additional reward $R \in (0, n \cdot (x_{n-1} - x_n) + (n-2) \cdot x_{n-1})$. The upper bound for the reward depends not only on the number of players, but also on the difference between the value of information of the two least informed players. The larger the difference is, or the larger the number of players is, the larger the reward R can be. It is not the value of information of the least informed players, but her difference to the value of information of the second least informed players that limits the reward. Additionally, in larger organizations with more players, the reward can be higher. In an extension in Section 2.4 we relax this assumption.
One can argue that also the first player who passes the information and starts the whole centralization should get a reward R'. In Proposition 2.3 we show that each player either passes in the first period or gets a share of the reward R. Introducing the additional reward R' therefore has the same effect as decreasing the reward R which is discussed in Section 2.2.4.
If several players pass the last information in the same period the reward is split equally between them. All players discount over time by the same discount factor $\delta \in (0, 1)$. Holding the information generates no utility for the players, i.e. $u_i = 0$. The present value of player i's utility is as follows:

$$u_i(x_i) = \delta^{\tau_i - 1} \cdot \begin{cases} x_i & \text{, if } \exists j \in N, j \neq i \text{ such that } \tau_j > \tau_i \\ x_i + \frac{R}{\ell} & \text{, if } \forall j \in N : \tau_j \leq \tau_i \text{ with } \ell = \{j \in N \mid \tau_j = \tau_i\} \end{cases}$$

Our aim is to find the Subgame Perfect Nash Equilibrium depending on δ, R, T and $(x_i)_{i \in N}$. We assume that in a situation where several players have an incentive to pass the information, the players with more valuable information pass first. This assumption only changes which players get a share of the reward, but not the time it takes the principal to collect all information. A more detailed explanation can be found in the following example with two players and two periods.

Remark 2.1. *All players will pass their information at one point. In particular in $t = T$ all the players who have not passed their information will pass.*

Even in the very last period the players generate a utility strictly larger than 0 from passing, because $\delta > 0$ holds.

Remark 2.2. *As soon as there is only one player left who has not passed, this player will pass immediately.*

In the situation where there is just one player left, that player does not face any competition, but gets the reward R for herself for sure. Since the value of her information decreases over time, she will pass as soon as possible.

2.2.1 Two players, two period example

Let us start with a simple example of just two players and two periods, i.e. $T = 2$. We have already noted that in the last period $t = T$ all remaining players will pass, so it remains to analyze the players in $t = 1$.

	P	H
P	$x_1 + \frac{R}{2}$ $x_2 + \frac{R}{2}$	x_1 $\delta \cdot (x_2 + R)$
H	$\delta \cdot (x_1 + R)$ x_2	$\delta \cdot (x_1 + \frac{R}{2})$ $\delta \cdot (x_2 + \frac{R}{2})$

In this game player 1 chooses the row, while player 2 selects the column. We can simplify the conditions, because we have sorted the players such that $x_1 \geq x_2$ holds and we get the following Nash equilibria:

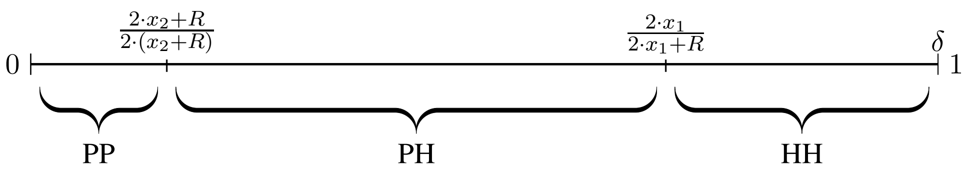


Figure 2.1: Subgame Perfect Nash Equilibria

We can see that for a low value of δ both players prefer to pass in the first period, for a medium value player 2 prefers to wait to get the reward for herself, while for a high value of δ both players will wait until the second period and share the reward. This matches the fact that players get more patient when the discount factor is larger. There is no area in which player 1 holds her information, but player 2 passes, because this interval would be included in the second interval and we assume that in that case always the players with the more valuable information passes. This assumption influences which player is holding the information and by that is getting a share of the reward. It is important to point out that this assumption has no impact on the duration: In the entire interval one player holds while the the other player passes, which yields to a duration of two periods. Even if there are more players, the assumption just determines which players pass their information, but for the duration it is not important which players hold or pass. The time it takes the principal to get all information is only determined by the number of players that keep their information private in the first period. We argue that this is a reasonable assumption to make, because the larger intervals (here where player 1 passes and player 2 holds) always exist, while the existence of smaller sub-intervals depends on the values of the parameters.

2.2.2 n players, two period example

Similarly to the previous example we can search for the SPNE in a game with n players and two periods. In a later part we show the relation to games with any finite number of periods.
Again we know that in $t = T = 2$ all the remaining players are going to pass their information, which gives all players the choice between the following utilities:

$$\begin{aligned} u_i(\text{P} \mid \text{all other players pass}) &= x_i + \frac{R}{n} \\ u_i(\text{H} \mid \text{all other players pass}) &= \delta \cdot (x_i + R) \\ u_i(\text{P} \mid j \leq n-1 \text{ players hold}) &= x_i \\ u_i(\text{H} \mid j \leq n-1 \text{ players hold}) &= \delta \cdot (x_i + \frac{R}{j+1}) \end{aligned}$$

With these utilities we can write down the game in matrix form and get the first proposition to state all the SPNE:

Proposition 2.1 (SPNE for $T = 2$)**.**

1. *All players pass their information in* $t = 1$ *if and only if* $\delta \in \left(0, \frac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$.
2. *One player holds her information, while all other players pass in* $t = 1$*, if and only if* $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)$.
3. $i \in \{2, \ldots, n-1\}$ *players hold their information, while all other players pass in* $t = 1$*, if and only if* $\delta \in \left(\frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}, \frac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R}\right)$.
4. *All players hold their information in* $t = 1$ *if and only if* $\delta \in \left(\frac{n \cdot x_1}{n \cdot x_1 + R}, 1\right)$.

The complete SPNE profile, if $i \in \{1, \ldots, n\}$ players pass in $t = 1$, is that

- the players 1 to $(n - i)$ player pass in $t = 1$ and by definition hold in $t = 2$ and
- the players $(n + 1 - i)$ to n hold in $t = 1$ and pass in $t = T$.

With this proposition we can sort all SPNE depending on δ. With a higher δ more players will hold their information in the first period, while for a low δ they prefer to pass sooner.
Subgame Perfect Nash Equilibria depending on regions similar to those can be found in many later parts of this paper. Again we can illustrate the $(0, 1)$ interval. In Figure 2.2 we see that the number of players who hold their information in $t = 1$ is increasing in δ. By the assumption we made on the reward R we get that the equilibrium is unique, because $\frac{n \cdot x_n + R}{n \cdot (x_n + R)} < \frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R} < \frac{3 \cdot x_{n-2}}{3 \cdot x_{n-2} + R} < \ldots < \frac{n \cdot x_1}{n \cdot x_1 + R}$ holds.

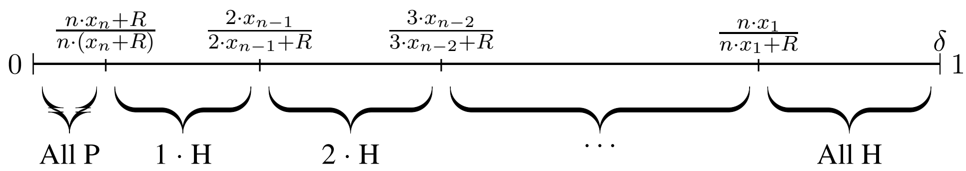


Figure 2.2: Different equilibria depending on δ

2.2.3 n players, T periods

In this general setting with T periods we first state some more results about the behavior of the players. These results then help us to state the SPNE.

Remark 2.3. *In equilibrium no player will hold her information first and then pass it later without getting a share of the reward.*

The reason for this remark is simply given by the discounting. Passing in the first period generates an utility of x_i, while passing at time t' without getting any part of the reward gives only $\delta^{t'-1} \cdot x_i$. The same holds for all other periods. If the player passes in period t, she gets $\delta^{t-1} \cdot x_i$, for any period $t' > t$ she only gets $\delta^{t'-1} \cdot x_i < \delta^{t-1} \cdot x_i$.

Proposition 2.2.
There exists no SPNE such that $j > 1$ players hold in $t = 1$ and then all pass in a period $t' \neq T$.

This proposition states that there is never an equilibrium in which at least two players hold their items in the first period and then collect the reward together in the following period, but instead they will wait until the last period to pass. Players cannot coordinate on actions and so the players that have hold in the first period cannot agree to pass together at the second period. If it would be optimal for one player to pass in the second period all other players will hold in the second period. So the player who has passed in the second period would get no share of the reward, but only a payoff according to her value of information. The only possibility for all players to ensure that they get a part of the reward is to wait until the last period. In that period all remaining players will pass their information, because otherwise they would not get any utility at all.
With the help of this proposition we can list all the possible SPNE:

Corollary 2.1.
There can exist only the following SPNE:

1. *All players pass in $t = 1$.*
2. *Players 1 to $n-1$ pass in $t = 1$, just player n holds in $t = 1$ and passes in $t = 2$.*
3. *Players 1 to $n-i$ ($i \in (1, n-1)$) pass in $t = 1$, players $n+1-i$ to n hold in $t \in \{1, \ldots T-1\}$ and pass in $t = T$.*
4. *All players hold in $t \in \{1, \ldots T-1\}$ and pass in $t = T$.*

By backward induction we can state also conditions for each of the SPNE as in Proposition 2.1.

Proposition 2.3 (SPNE).

1. *All players pass in* $t = 1$ *if and only if* $\delta \in \left(0, \dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$.
2. *One player holds her information, while all other players pass in* $t = 1$*, if and only if* $\delta \in \left(\dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\dfrac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$*. This one player then passes in* $t = 2$.
3. $i \in \{2, \ldots n - 1\}$ *players hold their information in* $t = 1$*, while all other players pass in* $t = 1$*, if and only if* $\delta \in \left(\left(\dfrac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}\right)^{\frac{1}{T-1}}, \left(\dfrac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R}\right)^{\frac{1}{T-1}}\right)$*. The players who have hold in* $t = 1$*, also hold in all* $t \in \{2, \ldots T - 1\}$ *and pass in* $t = T$.
4. *All players hold their information in* $t = 1$ *if and only if* $\delta \in \left(\left(\dfrac{n \cdot x_1}{n \cdot x_1 + R}\right)^{\frac{1}{T-1}}, 1\right)$*. All players will also hold in* $t \in \{2, \ldots T - 1\}$*. They pass in* $t = T$.

The complete SPNE profile is:

- If all players pass in $t = 1$, they hold by definition for all $t > 1$.
- If one player (player n) holds in $t = 1$, the remaining players hold by definition for all $t > 1$, while this player n passes in $t = 2$ and then also holds by definition for $t > 2$.
- if $i \in \{3, \ldots, n\}$ players pass in $t = 1$, is that
  - the players 1 to $(n - i)$ player pass in $t = 1$ and by definition hold in $t > 1$ and
  - the players $(n + 1 - i)$ to n hold in $t \in \{1, \ldots, T - 1\}$ and pass in $t = 2$.

We can see that these results are similar to Proposition 2.1. Only the upper bound of the second case and the boundaries of the third and fourth cases got the exponent $\frac{1}{T-1}$. This is caused by the fact that if more than one player holds in $t = 1$, the players wait for the last period to pass and so they have to discount $T - 1$ times. It is obvious that the equilibrium is still unique, because the first boundary is not changed and all others are increased in the same way.

Duration

Our aim is to compare different hierarchies and see in which hierarchy the information is centralized fastest or slowest. We have seen in Proposition 2.2 that all the games have either a duration of 1, 2 or T periods. We can modify Proposition 2.3 easily to focus on the duration of the game:

Corollary 2.2.

1. *The game ends after 1 period if and only if* $\delta \in \left(0, \dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$.
2. *The game ends after 2 periods if and only if* $\delta \in \left(\dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\dfrac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$.
3. *The game ends after* T *periods if and only if* $\delta \in \left(\left(\dfrac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$.

It is to notice that the duration only depends on the parameters R, T and δ and the value of information of the two least informed players x_n and x_{n-1}. The intuition behind is as follows: In $t = 1$ all players will only pass if no one has an incentive to deviate. Because of the order of players $(x_1 \geq \ldots \geq x_n)$ and our assumption that the more informed players pass first, the least informed player may deviate first and so her value of information influences the boundary between a duration of one and two periods.

In the case that player n holds, the remaining players have to pass to achieve a duration of two periods. Again, because of the order of players, the boundary is defined by the value of information of the least informed of the remaining players, player $n-1$. If she decides to hold in $t = 1$ as well, the players who hold in the first period will wait until the last period to pass, as stated in Proposition 2.2. In that case the duration is T.

2.2.4 Impact of the parameters

The previous corollary gives us the boundaries for the duration of the game. We can see that the reward R, the deadline T, the number of players n and the information distribution $(x_i)_{i\in N}$ have an impact on the duration. In this part of the paper we show the impact of the different parameters on the duration. This allows us to compare two hierarchies in the later part. We will do the comparison for hierarchies with the same number of players and for hierarchies with a different number of players. Some results are illustrated in the last part of this section.

Effect of the information distribution

In this part we want to give a short analysis on how the information distribution affects the duration. We fix δ, R, T and n and compare different information distributions, to see under which distribution the information is centralized fastest.

Let X be the value of all information in the hierarchy with n players, i.e. $X = \sum_{i=1}^{n} x_i$. The interval of δ for a duration of one period becomes maximal when $\frac{n\cdot x_n+R}{n\cdot(x_n+R)}$ is maximal, so for given n and R an increase in x_n increases the interval. By that we get that the interval is maximal if the information is distributed equally between all players, i.e. $x_i = \frac{X}{n}$ for all players i.

Even for this information distribution the discount factor can be such that the duration is longer than one period. To maximize the interval which yields a duration of two periods we have to maximize $\left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}$ for a given n, R and T, while we no longer care about the lower bound $\frac{n\cdot x_n+R}{n\cdot(x_n+R)}$. As already described the lower boundary decreases with an decrease of x_n, so to minimize this boundary, x_n should be minimal. Therefore it is $x_n = \epsilon$ with $\epsilon > 0$. To maximize the upper bound of the interval we have to maximize x_{n-1}. As before we see that an equal information distribution yields the best result: $x_i = \frac{X-\epsilon}{n-1}$ for $i \neq n$.

We see that for equally distributed information the game ends even for higher δ after one period. If δ is too high then a duration of one period cannot be achieved by any distribution. In that case the information distribution in which one player gets the smallest possible piece of information (ϵ) and the remaining players have information with the same value is optimal. Under this distribution the interval with a duration of two periods is maximized.

We should notice that also a long deadline T can be beneficial for the principal. With increasing T also $\left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}$ increases, so the interval in which the game ends after 2 periods increases, while the interval where the game ends after T periods decreases.

Effect of the deadline

We have already seen that with an increase of the deadline the interval $\delta \in \left(\frac{n\cdot x_n+R}{n\cdot(x_n+R)}, \left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}\right)$ becomes larger. It is also interesting to analyze which intervals decrease by an increase of T.

Proposition 2.4 (Effect of an increase in T)**.**
An increase in the deadline T has the following effects:

1. *The interval $\delta \in \left(0, \frac{n\cdot x_n+R}{n\cdot(x_n+R)}\right)$ is unchanged.*
2. *The upper bound of the interval $\delta \in \left(\frac{n\cdot x_n+R}{n\cdot(x_n+R)}, \left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}\right)$ increases and so the size of the interval increases as well.*
3. *Both boundaries of the interval $\delta \in \left(\left(\frac{i\cdot x_{n+1-i}}{i\cdot x_{n+1-i}+R}\right)^{\frac{1}{T-1}}, \left(\frac{(i+1)\cdot x_{n-i}}{(i+1)\cdot x_{n-i}+R}\right)^{\frac{1}{T-1}}\right)$ increase, while the interval becomes smaller.*
4. *The lower bound of the interval $\delta \in \left(\left(\frac{n\cdot x_1}{n\cdot x_1+R}\right)^{\frac{1}{T-1}}, 1\right)$ increases, so the whole interval decreases in size.*

This result states that for an increasing deadline the size of the interval for δ in which the game ends in two periods increases, while all the intervals in which the game ends after T periods become smaller. This means that, if we increase the deadline T to T', there is an interval in which under the old deadline T the duration is T periods, while under the new deadline T', the duration is only 2 periods. So an increase in T can decrease the duration from T periods to 2 periods.

Effect of the reward

A decrease of the reward R has a different effect than the change of the previous two parameters. The interval $\left(0, \frac{n\cdot x_n+R}{n\cdot(x_n+R)}\right)$ increases, so the interval in which the duration is one period becomes larger. Also the upper bound of $\left(\frac{n\cdot x_n+R}{n\cdot(x_n+R)}, \left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}\right)$ increases, but the overall effect depends on the remaining parameters. For some combination of the information distribution and T the interval grows, while for other the size decreases.
The interval $\left(\left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}, 1\right)$ diminishes. So all in all a decrease of R speeds up the information transmission. This fits our intuition, because with a lower reward R the players have less incentive to hold. Of course, an increase of the reward R has the complete opposite effect.
An effect similar to the decrease of the reward R can be achieved if there is another reward R' that is given to the first player who passes.

Effect of the number of players

To analyze the impact of a change of the number of players on the duration, we also have to take the information distribution into account. We do a comparison of two different hierarchies with a different number of players in the next section. To analyze the pure effect of the number of players, we assume that the value of information of the two least informed players, i.e. x_n and x_{n-1} do not change. So this effect can be seen as the adding of well informed players:
The derivative of $\frac{n\cdot x_n+R}{n\cdot(x_n+R)}$ with respect to n is $-\frac{R}{n^2(x_n+R)}$, which is negative. This means that an increase of n decreases the size of the interval in which the duration is one period and increases the interval with a duration of two periods. The number of players also has an effect on the different SPNE, but it has no further effect on the duration.

Summary

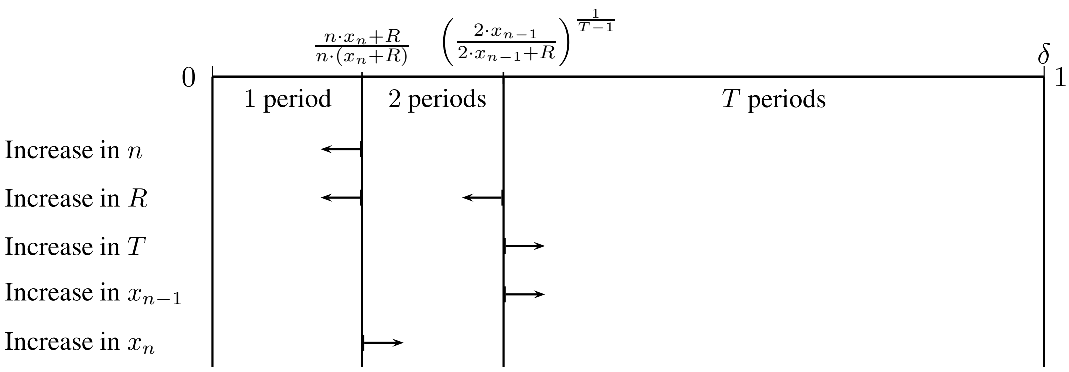


Figure 2.3: Summary of the impact of all parameters on the duration

Figure 2.3 sums up all the changes that come with an increase of a single parameter. We can see that only a change in the reward R influences both boundaries, while increases in all other parameters just change a single boundary. We can use this result to compare two different hierarchies.

2.2.4.1 Comparison of two hierarchies

Even with just two layers of hierarchies we have seen that there are several effects on the duration. To prepare the comparison of larger hierarchies we first have to start with the comparison of two-layer hierarchies with the same deadline T and reward R. In a second step we compare two hierarchies with a different number of players.

Comparison of two hierarchies with the same number of players

We will start with the comparison of two hierarchies with the same number of players n. Let us denote the values of information in the first hierarchy by $x_1, \ldots, x_n$ and in the second by $y_1, \ldots, y_n$. Without loss of generality we can assume that $y_n \geq x_n$, i.e. that the least informed player of the second hierarchy is not less informed than the one of the first hierarchy. For different values of δ we get different durations for both structures, some of the following cases are shown in the Figures 2.4, 2.5 and 2.6. In each figure, above the axis we see the duration of the first hierarchy, while below the duration of the second hierarchy is shown.

1. For $\delta \in \left(0, \dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}\right)$ the game ends after 1 period for both structures.

2. For $\delta \in \left(\dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}, \dfrac{n \cdot y_n + R}{n \cdot (y_n + R)}\right)$ the second structure still just takes one period, while in the other hierarchy the information is centralized slower. We have to separate between two cases:

   - If $\left(\dfrac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}} > \dfrac{n \cdot y_n + R}{n \cdot (y_n + R)}$ holds the first hierarchy takes two periods. This is shown in Figures 2.4 and 2.5.
   - Otherwise there exists an interval $\left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the first structure needs two periods and the interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, \frac{n \cdot y_n + R}{n \cdot (y_n + R)}\right)$ in which it takes T periods. This combination is displayed in Figure 2.6.

3. For $\delta \in \left(\dfrac{n \cdot y_n + R}{n \cdot (y_n + R)}, \left(\dfrac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ the second structure needs two periods for the centralization.

   - If $y_{n-1} > x_{n-1}$ holds there exists an interval $\left(\frac{n \cdot y_n + R}{n \cdot (y_n + R)}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the first structure also takes two periods and one interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, \left(\frac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the duration of the first structure is T (see Figures 2.4 and 2.6).
   - Otherwise the first structure also needs just two periods (see Figure 2.5).

4. For $\delta \in \left(\left(\dfrac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$ the game with the second hierarchy ends after T periods.

   - If $y_{n-1} > x_{n-1}$ holds the first structure has the same duration (see Figure 2.4 and Figure 2.6).
   - Otherwise there is an interval $\left(\left(\frac{2 \cdot y_{n-1}}{2 \cdot y_{n-1} + R}\right)^{\frac{1}{T-1}}, \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}\right)$ in which the duration of the first structure is just two periods, while on the remaining interval $\left(\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}, 1\right)$ both hierarchies need T periods (see Figure 2.5).

It is clear that all combinations are possible and only the comparison of the two least informed players in both hierarchies determines in which the information is centralized faster. Three combinations are illustrated below.

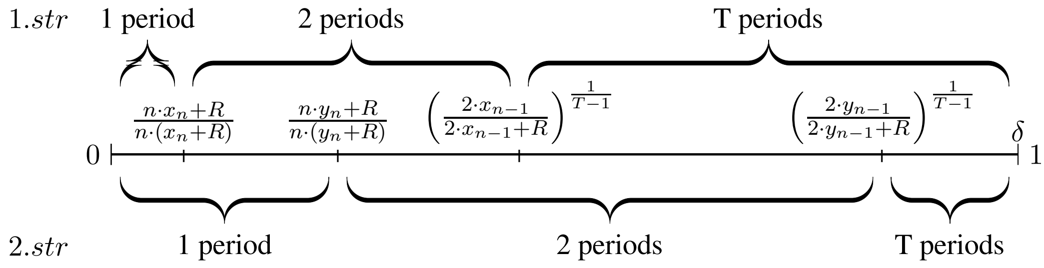


Figure 2.4: First combination

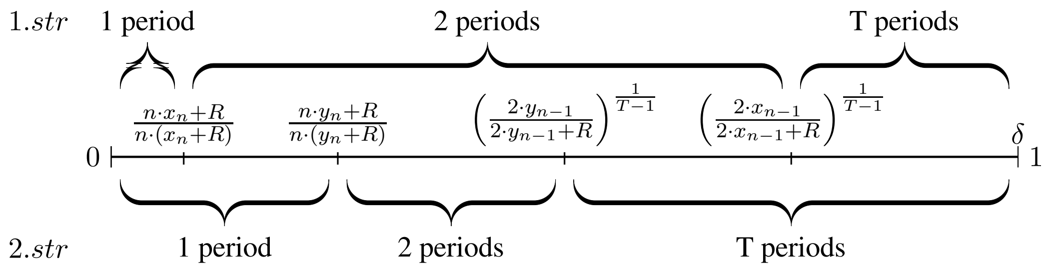


Figure 2.5: Second combination

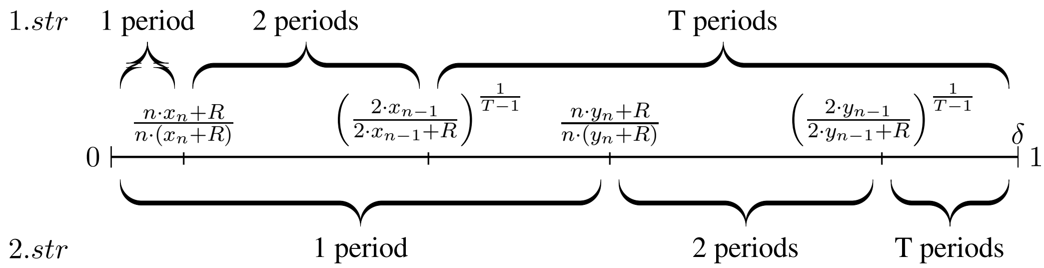


Figure 2.6: Third combination

Comparison of two hierarchies with a different number of players

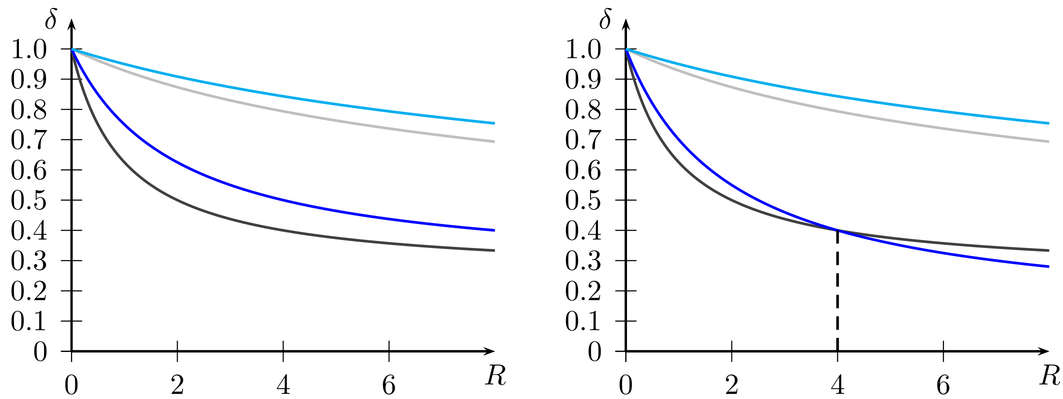
If we compare two hierarchies with a different number of players, the analysis gets more complex, but basically we still have to compare the lower two bounds.

Let there be n players in the first hierarchy with information $x_1, \ldots, x_n$ and m players in the second structure with information $y_1, \ldots, y_m$. Without loss of generality we can assume $m > n$. If in addition also $x_n > y_m$ holds we get $\frac{m\cdot y_m+R}{m\cdot(y_m+R)} < \frac{n\cdot x_n+R}{n\cdot(x_n+R)}$. In that case, if δ is in the interval $\left(0, \frac{m\cdot y_m+R}{m\cdot(y_m+R)}\right)$ both structures need just one period, while for the interval $\left(\frac{m\cdot y_m+R}{m\cdot(y_m+R)}, \frac{n\cdot x_n+R}{n\cdot(x_n+R)}\right)$ just the first hierarchy needs a single period, while the other structure needs two or T periods, depending on the other boundary. If this additional assumption does not hold, i.e. $x_n < y_m$ is true, we cannot make any general statement. The second lowest boundary is independent of x_n and y_m. For $x_{n-1} > y_{m-1}$ we get that $\left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}} > \left(\frac{2\cdot y_{m-1}}{2\cdot y_{m-1}+R}\right)^{\frac{1}{T-1}}$ holds. Therefore there exists an area $\left(\left(\frac{2\cdot y_{m-1}}{2\cdot y_{m-1}+R}\right)^{\frac{1}{T-1}}, \left(\frac{2\cdot x_{n-1}}{2\cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}\right)$, where the first structure takes two periods, while the duration in the second structure is T periods. Again all combinations are possible.

Different deadlines or rewards

As soon as we compare two hierarchies which have not the same deadlines T and which also may not have the same rewards R, we have to look at the effect of T and R on the boundaries of the SPNE. In Proposition 2.4 we have already seen that an increase of T increases the boundary at which the duration changes from 2 to T periods. It remains to analyze how an increase of R shifts the intervals for δ:
$\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$ has a negative first derivative with respect to R, so this boundary is strictly decreasing in R. Obviously also $\left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}\right)^{\frac{1}{T-1}}$ is decreasing in R, so both upper boundaries decrease and create a larger interval in which the game takes T periods to end. We see that an increase in R has a different effect than an increase in T.

Graphics



(a) Same number of players (b) Different number of players

Figure 2.7: Duration of two different hierarchies depending on R and δ

The figure above shows the differences between the comparison of hierarchies with the same number of players and with different number of players. The gray curves characterizes the boundaries for the first hierarchy (with 4 players), while the blue graphs display those of the second hierarchy. The lighter curves are the boundaries between two and T periods, while the darker curves are the first boundary. This means that below the dark gray (blue) curve the first (second) hierarchy needs one period to centralize the information, between the dark and the light curve it takes two periods and above the light one, T periods.
We have fixed the deadline as $T = 4$. In the first hierarchy we have 4 players with $x_4 = 1$ and $x_3 = 2$. On the left hand the second hierarchy also has four players, but they have more valuable information with $y_4 = 2$ and $y_3 = 3$. We see that in the left figure the lines just intersect for $R = 0$. For all values of R the areas stay the same: Below the dark gray line both hierarchies need one period, between the dark gray and dark blue the first hierarchy takes two periods, while the second still centralizes the information in one period. In the area between the dark blue and the light gray curves both take two periods, between the light gray and light blue the first hierarchy needs T periods, while the second just takes two periods and in the area above the light blue graph

none of the two hierarchies centralizes the information before T.
So even with a change in R, as long as the information distribution and the deadline stay constant these areas just shift, but none vanishes and no new area is created.
This is different if we change the number of players in the second hierarchy to 10. For values of R less than 4 we still have the same areas as before, but then the dark gray and dark blue curves cut and create a new area: In the area that is bound by the dark blue curve from below and by the dark gray function from above the first hierarchy finishes in one period, while the second needs two periods. In this example we see that the comparison of two structures with a different number of players depends more on the parameters, in this case on R.
If we instead would change the number of players in the first hierarchy from 4 to 10, the dark gray curve would shift downwards, but no other changes will occur. The remaining analysis of Figure 2.7 (b) is the same as for (a), because the second boundary does not depend on the number of players.

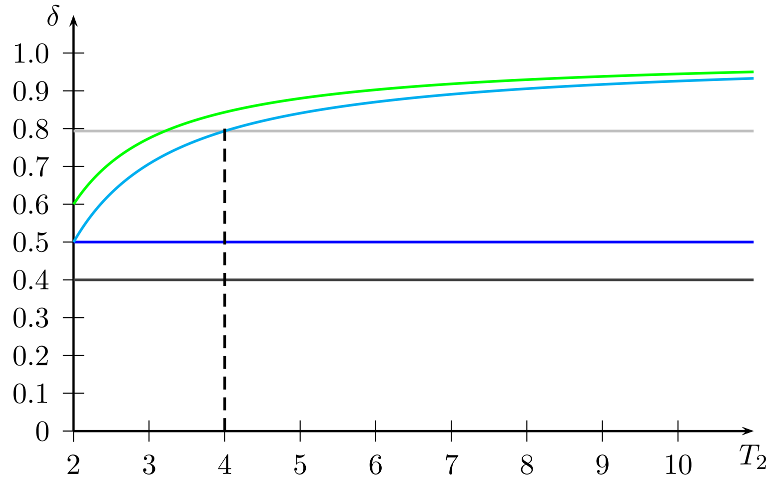


Figure 2.8: Duration of two different hierarchies depending on T_2 and δ

Figure 2.8 displays the impact of the deadline on the intervals for the SPNE: Again we compare two hierarchies. We have fixed $R = 4$. The boundaries for the SPNE of the first hierarchy are shown by the gray lines. In this setting the hierarchy contains 4 players with $x_n = 1$ and $x_{n-1} = 2$ and has a deadline of $T_1 = 4$. Below the dark gray curve all players pass in $t = 1$, so the duration is one period. Between the dark and light gray lines the duration is two periods and above the light gray it is $T_1 = 4$.
The second structure also has four players and also the same reward, but the least informed player has information of value $y_n = 2$. The deadline is T_2 which is displayed on the x-Axis. Similar to the first hierarchy below the dark (blue) line the information centralization takes only a single period, above it takes two periods at least. The light blue curve shows the boundary between two and T_2 periods for $y_{n-1} = 2$, below the duration of the game in the second hierarchy is two periods, above it takes T_2 rounds. We see that for $T_2 = 4 = T_1$ the values of both hierarchies for the upper bound are the same. For lower values of T the boundary for the second hierarchy is lower, for larger values it is larger.

For a small deadline, e.g. $T_2 = 3$ there is an area where the second structure centralizes the information slower than the first one, while for large values the centralization works faster. The green curve is a simple shift from $y_{n-1} = 2$ to $y_{n-1} = 3$. We observe that the curve shifts upwards as we have already noticed in the figures before.

2.3 Multi layer model

In this Section we extend our model to allow for more than just two layers of hierarchy. There is one principal that is linked to some players, who themselves might be the principal for other players. The players on the lowest level of the hierarchy (the leafs of the tree) are those who are only connected to their successor.

Coming from the two layer model, we can use the results to analyze larger hierarchies with several layers. The main difficulty we face in this part is that those players who are the principals for some other players, also have to pass the information. We assume that they can only do it as soon as they have collected all information from their predecessors. Let the set of total players in the hierarchy be N, with $|N| = n \geq 2$. Each player $i \in N$ has some first period when she can pass, we denote this entry time by t_i. For the players who are leafs it is obviously that $t_i = 1$ holds and they can pass the information from the first period on. If player j collects all information in period τ she can pass them in the next period, i.e. $t_j = \tau + 1$. We assume that the entry times are public knowledge. If the players would not know if there is a competitor already in the game, or if one will enter later, there is more uncertainty. This type of uncertainty is analyzed in Bobtcheff et al. (2016).

A player who is not a leaf can either have some information from the beginning or not. In both cases we simply add the value of information she gets from her agents to the value of information she has initially. By that the players on a higher level of the hierarchy have higher value of information than the lower players.

Each hierarchy with more than two layers, consists of different sub-hierarchies. By $L = \{1, \ldots, \ell\}$ we denote the set of two-layer sub-hierarchies. We start the numbering with the lowest levels and then move upwards. So sub-hierarchy 1 always consists of some of the players who are leaves in the complete structure, while sub-hierarchy ℓ is always the one at the top.

Each sub-hierarchy $h \in L$ has some deadline T_h and reward R_h. If one sub-hierarchy has the deadline T_h, we assume that the structure that includes this branch has a deadline $T' > T_h + 1$. This assumption holds for all branches and ensure that each player has some time left to pass the information after receiving them. The more interesting cases arise if we assume that for each layer of hierarchy we add at least three more periods.

Let N_h be the set of players in one sub-hierarchy, by $N_h^t \subseteq N_h$ we denote the set of players in this branch who have already collected all their information, i.e. $N_h^t = \{i \in N_h \mid t_i \leq t\}$. For the competition only the players who have not passed their information yet are important. Let $M_h^t \subseteq N_h^t$ be the set of players who have centralized all information from their predecessors and have not passed to their successor yet. If $a_i^t \in \{\text{P, H}\}$ is the strategy of player i, then we can write M_h^t as $M_h^t = \{i \in N_h^t \mid a_i^\tau = \text{H} \; \forall \tau \in \{t_i, t-1\}\}$.

In contrast to the two-layer model we do not sort player primary based on their value of information, but on their entry time. Player 1 of a sub-hierarchy is the player who has the lowest entry time, while player n enters last. If two or more players have the same entry time we sort those players by value of information as before. By this we still get the same order for the two-layer model. In the situation where several players have an incentive to pass the information, we still assume that the players with the more valuable information pass the information. If there are more players with the same value, we select those to pass the information, with a smaller entry time.
We can repeat some Remarks from the two layer model:

Remarks.

- *All players will pass their information at one point.*
- *In $t = T_h$ all the players from sub-hierarchy h, who have not passed their information will pass.*
- *As soon as there is only one player left in a sub-hierarchy, who has not passed and no other player will enter afterwards, this player will pass immediately.*
- *In equilibrium no player will hold her information first and then pass it later without getting a share of the reward.*

For the hierarchies where all the players enter in with the beginning of the game, i.e. $t_n = 1$, the boundaries of the SPNE are the same as before. Only for the higher layer hierarchies we have to take into account those players who enter later.

2.3.1 SPNE in higher layers

The last player who enters into a sub-hierarchy is player n, she enters at time t_n. With just some minor adjustments of Proposition 2.3, we get the SPNE in period $t = t_n$. One of these adjustments comes with the different entry times.

Corollary 2.3 (SPNE in a sub-hierarchy at $t = t_n$)**.**
Let denote the value of information of player $i \in N_h$ by x_i.
The Subgame Perfect Nash Equilibria at $t = t_n$ depend on δ.

1. *All players pass in t if and only if $\forall i \in M_h^t : \delta < \dfrac{|M_h^t| \cdot x_i + R_h}{|M_h^t| \cdot (x_i + R_h)}$ holds.*
2. *A single player holds in t if and only $\exists j : \delta > \dfrac{|M_h^t| \cdot x_j + R_h}{|M_h^t| \cdot (x_j + R_h)}$ and*
   $\forall i \neq j : \delta < \left(\dfrac{2 \cdot x_i}{2 \cdot x_i + R_h}\right)^{\frac{1}{T_h - t}}$.
3. *$k \in \{2, |M_h^t| - 1\}$ players hold in t and pass in T_h if and only if $\exists J \subset M_h^t$ with $|J| = k$ and $\forall j \in J : \delta > \left(\dfrac{k \cdot x_j}{k \cdot x_j + R_h}\right)^{\frac{1}{T_h - t}}$ and $\forall i \in M_h^t \setminus J : \delta < \left(\dfrac{(k+1) \cdot x_i}{(k+1) \cdot x_i + R_h}\right)^{\frac{1}{T_h - t}}$.*
4. *All players hold in t and pass in T_h if and only if $\forall i \in M_h^t : \delta > \left(\dfrac{|M_h^t| \cdot x_i}{|M_h^t| \cdot x_i + R_h}\right)^{\frac{1}{T_h - t}}$ holds.*

These changes in the conditions are necessary to adjust to the fact that the players may enter at different times and we do not sort the players by their value of information any longer. In case 2 of the corollary it is the least informed player that holds the information, i.e. j is such that $x_j \leq x_i \; \forall i \neq j$ and $\forall i \neq j$ with $x_j = x_i$, $t_j \leq t_i$ holds. Similar in case 3: $\forall i \in M^t \setminus J$ and $\forall j \in J : x_i \leq x_j$ holds and $\forall i \in M^t \setminus J$ with $\exists j \in J$ such that $x_i = x_j$: $t_j \leq t_i$ holds.
This corollary just defines the remaining players' behavior starting from period t_n, but we also want to characterize the players' behavior before. The problem that arises for the higher layers is the following: There exist two different behaviors of players: There are players who enter in one period and pass as soon as possible without getting a share of the reward and those who wait to get the reward. The game tree in Figure 2.9 shows the choices of player 1, where d, e, f and g stand for different duration and i, j, k and l for the different number of players who share the reward.
As we can see in Figure 2.9, only if player 1 decides to pass in the period she enters, her payoff

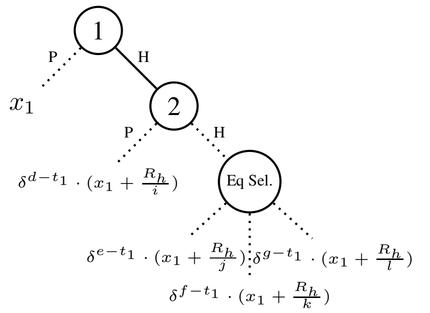


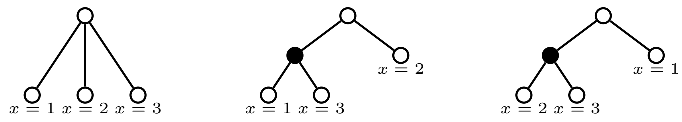
Figure 2.9: Example of a decision tree

is not affected by the decision of the other players. If she decides to hold, she will hold until the last period. Depending on the decision of the other players (just player 2 in the figure) and the equilibrium, player 1 will get a different utility. Still the game is solvable by backward induction, where the equilibrium depends on δ and follows the assumptions we made.

2.3.2 Comparison examples

In this part we compare different hierarchical structures. The main question we want to address is "which hierarchy centralizes the information fastest?". We will show that the answer to this questions depends on the parameters. Even the change of a single parameter will change in which hierarchical structure the information is centralized fastest. To show that we make the following simplifications: We assume that for all sub-hierarchies the reward R and deadline T are the same, furthermore the value of information of the least informed player is set to 1, for the second-least informed player to 2 and is increasing in steps of 1 for the remaining players.
The smallest example we can start with, is a model with only three players. There are only three different ways how to arrange three informed players, while satisfying the assumptions we made so far. With the introduction of a fourth informed player, there are already 25 possible hierarchical-structures, which we compare in the second part.

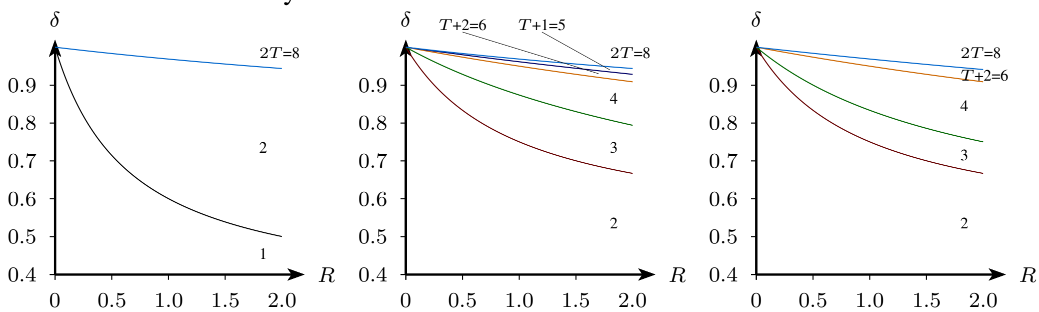
3 Players



(a) Hierarchy A (b) Hierarchy B (c) Hierarchy C

Figure 2.10: All possible multi-layer hierarchies for 3 players

Figure 2.10 shows the three different ways how to arrange the three informed players in a hierarchy. While in the flat hierarchy the positions of all players are the same, it is different if we install one uninformed intermediary (filled node). Then the only feasible possibilities are that the players with value of information 1 and 3 or those with information 2 and 3 pass their items to that intermediary. It is not possible to let the players 1 and 2 be in that part of the hierarchy, because this would contradict our assumptions we made on R in Section 2.2.
We assume that the total deadline for the hierarchy is $2 \cdot T$ and in the hierarchies B and C the deadline for the sub-hierarchy is T.



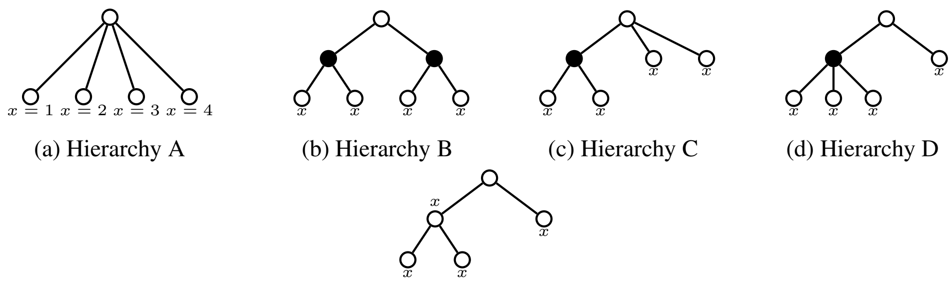
(a) Hierarchy A (b) Hierarchy B (c) Hierarchy C

Figure 2.11: $T = 4$

In the figure above we can see the duration of the different hierarchies, depending on δ and R. For the flat hierarchy the only possible values are one period, two periods and $2T$ periods. As soon as there are three layers the duration can be either two, three, four, $T+1$, $T+2$ or $2T$ periods. While the first three values are just possible if the sub-hierarchy works fast, $T+1$ and $T+2$ just occur if the the sub-hierarchy has a duration of T periods.
In Figure 2.11(b) we can observe something that has never happened in the two-layer model: An increase of δ or R yields to a decrease of the duration. Above the orange line the lower hierarchy needs T periods to centralize the information. Below the dark-blue curve the player with $x = 2$ will wait and then get the reward for herself in period $T + 2$, but above the dark-blue curve the intermediary, which then has a value of information equal to 4, will also hold. This would lead to a duration of $2T$ periods. If δ and R are not high enough, so in this example below the light-blue curve, the player with $x = 2$ decides not to wait for her opponent to centralize the information, but to pass in the very first period. Then the intermediary faces no competition when she finally has centralized the information and passes immediately, yielding to a duration of only $T + 1$ periods.
In other words between the two blue curves the incentive to wait for the player $x = 2$ is not large

enough.
Comparing the different structures leads to an interesting insight: There are no values for δ and R for which Hierarchy B is faster than Hierarchy A. This is quite obvious for the parameters, where Hierarchy A needs less than $2T$ periods. For both, Hierarchy A and B, the change to $2T$ periods (the light-blue curve) is defined by $\left(\frac{4.0}{4.0+R}\right)^{\frac{1}{7}}$.
Still Hierarchy A is not always faster than Hierarchy C, because for Hierarchy C the light-blue curve is defined by $\left(\frac{10}{10+R}\right)^{\frac{1}{3}}$, which is larger than $\left(\frac{4.0}{4.0+R}\right)^{\frac{1}{7}}$. This means in the interval $\left(\left(\frac{4.0}{4.0+R}\right)^{\frac{1}{7}}, \left(\frac{10}{10+R}\right)^{\frac{1}{3}}\right)$, the duration of Hierarchy C is less than the duration of the flat hierarchy.

4 Players



(a) Hierarchy A (b) Hierarchy B (c) Hierarchy C (d) Hierarchy D

(e) Hierarchy E

Figure 2.12: All possible multi-layer hierarchies for 4 players

In Figure 2.12 we see the different hierarchical structures that are possible for four players. Starting from Hierarchy B it makes a difference which player has which value of information. If we add all the different possibilities we end up with 25 different possibilities. In Hierarchy E all four players have information from the first period, this means that the intermediary is informed. In the Hierarchies B, C and D only the leafs have information, but the intermediaries (filled nodes) are uninformed.
With the value of information of the players as $1, 2, 3$ and 4, there are two feasible possibilities to arrange the players in Hierarchy B, six in Hierarchy C, four in Hierarchy D and 12 in Hierarchy E. Note that in Hierarchy B it is not possible that the players with $x = 1$ and $x = 4$ report to the same intermediate, because of the assumption we made about R. For some of these possibilities there is an easy way to exchange two players, while keeping the structure, to speed up the centralization. By that we can already delete 10 cases which are never faster than others. The following 15 cases remain:

1) Hierarchy A

- Hierarchy B with:

  2) $x = 1$ and $x = 2$ reporting to the same intermediary
  3) $x = 1$ and $x = 3$ reporting to the same intermediary

- Hierarchy C with:

  4) $x = 1$ and $x = 2$ reporting to the intermediary
  5) $x = 1$ and $x = 3$ reporting to the intermediary
  6) $x = 1$ and $x = 4$ reporting to the intermediary
  7) $x = 2$ and $x = 4$ reporting to the intermediary
  8) $x = 3$ and $x = 4$ reporting to the intermediary

- Hierarchy D with:

  9) $x = 4$ not reporting to the intermediary
  10) $x = 2$ not reporting to the intermediary
  11) $x = 1$ not reporting to the intermediary

- Hierarchy E with:

  12) $x = 2$ and $x = 3$ reporting to intermediary $x = 1$
  13) $x = 2$ and $x = 4$ reporting to intermediary $x = 1$
  14) $x = 3$ and $x = 4$ reporting to intermediary $x = 1$
  15) $x = 3$ and $x = 4$ reporting to intermediary $x = 2$

Figure 2.13 shows the duration from all those 15 cases, depending on δ. The jumps from one duration to another differ between the cases. We show only the interval $(0.4, 1)$, because the duration for lower values of δ is the same as for 0.4 for all hierarchies.

For low values of δ the duration of each hierarchy is low, until it increases at certain boundaries. For the non-flat structures the lowest duration is 2 periods, then it increases to 3 and then to 4. While for some hierarchies there is a jump directly from 4 to $T + 2$ periods, in others there is a region with a duration of $T + 1$ periods in between. We can also see that in the asymmetric hierarchies the same effect occurs as in the three player example: With an increase in δ the duration decreases to $T+1$. This effect happens in case 4), 5), 9) and 12) and again is caused by the fact that one of the players without intermediary has no incentive to wait and to get into the competition with the intermediary. In Hierarchy C, where there are two players directly connected to the principal these jumps can even happen twice, as we can see in 4).

If we compare all these 15 cases we see that some cases are weakly dominated: 2), 3), 6), 7) and 8) are never faster than 13). The structures 4), 9), 10) and 11) are weakly dominated by 12). As we may argue that a flat hierarchy should not be feasible, we ignore the dominance from case 1). By that the remaining cases are 1), 5), 12), 13), 14) and 15).

In most applications it seems unreasonable to assume that all players can pass directly to the principal. For example in the political case, there will be no system where each city reports directly to the administration of the whole country. We can observe this for example in Germany, where most cities or rural districts are under the administration of a state. The exceptions are the areas of Hamburg, Bremen and Berlin, which are not only cities, but also states. We can find such asymmetries fitting for our Hierarchies C, D and E.

Also in the work of Radner (1993) and Jehiel (1999), the flat hierarchy is avoided. While Radner models it with a maximal capacity for each player, Jehiel introduces a probability of information

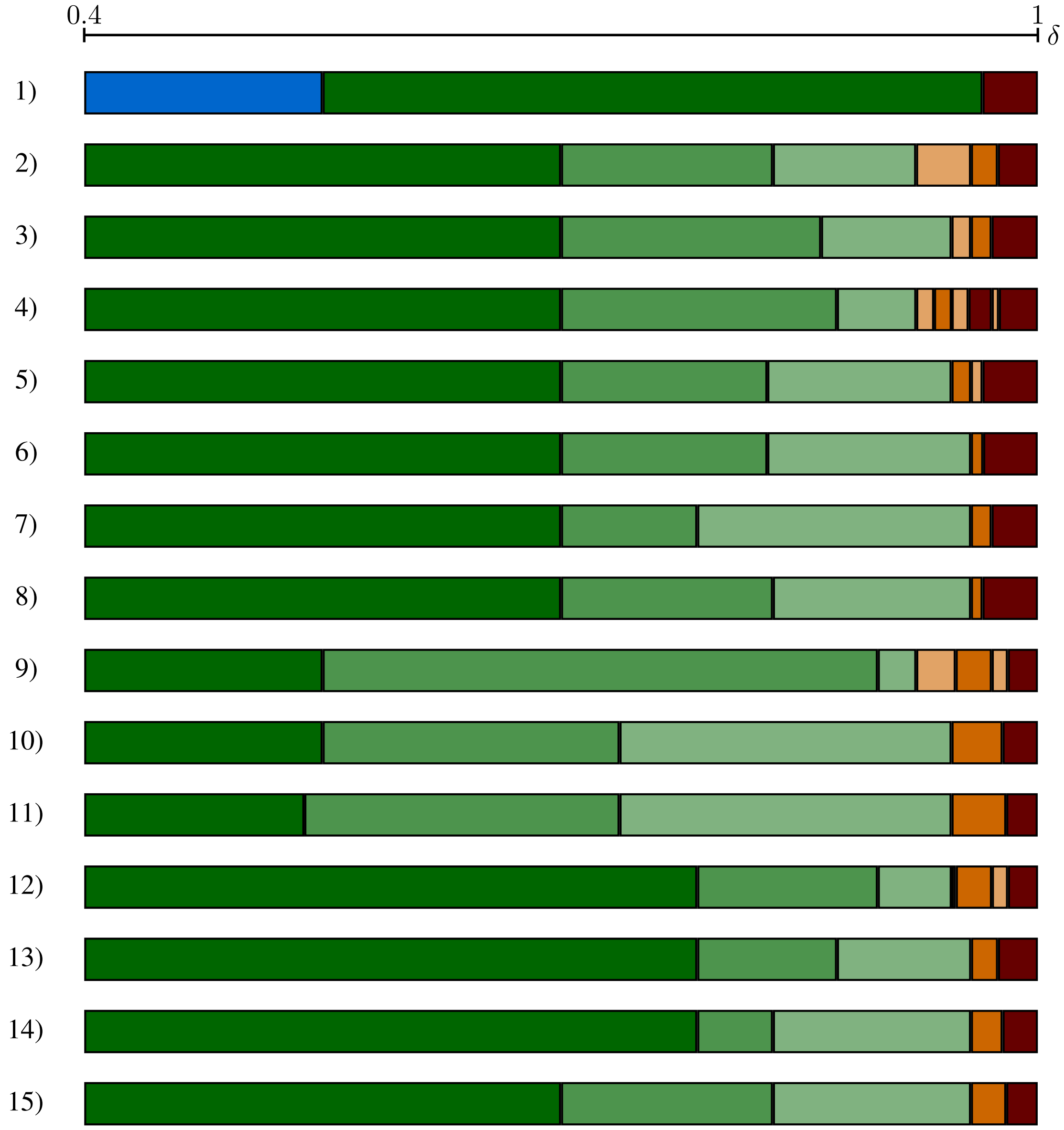


Figure 2.13: Duration of all 15 cases with $T = 5$ and $R = \frac{3}{2}$.
Colors: 1 period, 2 periods, 3 periods, 4 periods, T+1 periods, T+2 periods, 2T periods

loss for this case. Our way is similar to Radner, as we do not consider hierarchies in which many agents report to one player. In the example with only three players, we already mentioned that there exists a region of δ where the flat hierarchy is not the one with the smallest duration. The same holds with four players as we can see in the figure above.

For the remainder of this example, we will focus on the hierarchies with three layers. Figure 2.14 shows the intervals in which the remaining five hierarchical structures centralize the information fastest. Below it is written which is the shortest duration possible for that interval.

While a dark-green bar means the hierarchy is the unique fastest one, lighter shades show the intervals in which other hierarchies are also the fastest. We want to point out that in Figure 2.14 we modified the δ-Axis such that all intervals are clearly visible. The real sizes of the intervals do not correspond to the size shown in the figure.

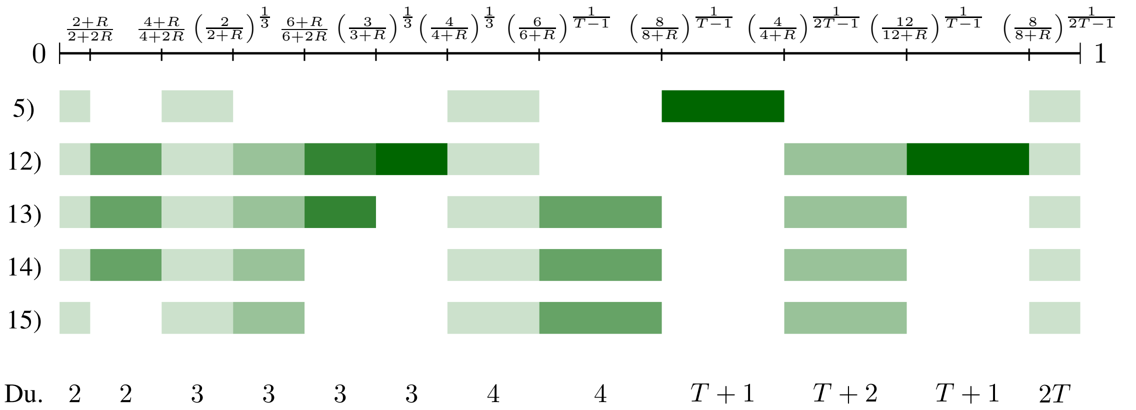
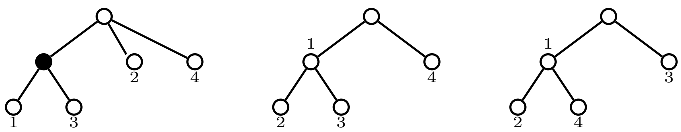


Figure 2.14: Duration of the remaining five cases with $T = 5$ and $R = \frac{3}{2}$

It is important to point out that we do not make any statement about the duration of a hierarchy in an interval where it is not the fastest. For example in the second last interval the duration of the hierarchies 5), 13), 14) and 15) differs between $T + 2$ and $2T$ periods.

From this figure we can clearly see that only the hierarchies 5) and 12) have some intervals in which this hierarchy is the unique fastest to centralize. Furthermore whenever the hierarchies 15) is the fastest, so is 14) and the same holds for 14) compared to 13). This means that for all intervals in which 14) or 15) have the shortest duration, so does the hierarchy from case 13). The reason why we did not delete 14) and 15) before is that they dominate 13) for some values, but for these values 5) or 12) are even faster. Since we just focus on the fastest hierarchies, we can neglect 14) and 15) for the rest of the analysis. The remaining three structures are the following:



(a) Case 5) (b) Case 12) (c) Case 13)

Figure 2.15: Fastest three-layer hierarchies for 4 players

For most values of δ the hierarchy displayed in case 12) (b) has either the shortest duration or shares the shortest duration with another hierarchy. Just for the interval $\left(\left(\frac{6}{6+R}\right)^{\frac{1}{T-1}}, \left(\frac{8}{8+R}\right)^{\frac{1}{T-1}}\right)$ hierarchy 13) is faster, and in $\left(\left(\frac{8}{8+R}\right)^{\frac{1}{T-1}}, \left(\frac{4}{4+R}\right)^{\frac{1}{2T-1}}\right)$ hierarchy 5) is the fastest.

The reasons for this are as follows: Hierarchy 13) is faster than 12), because in that specific interval the sub-hierarchy of case 13) still needs just two periods, while the sub-hierarchy in case 12) already needs T periods. This is simply caused by the fact that in case 13) the second player in the sub-hierarchy has a value of information $x = 4$, while it is only $x = 3$ in case 12).

The dominance of case 5) for the interval defined above is due to the different structure. In this interval the sub-hierarchy of case 5) needs T periods, but the players connected directly to the principal ($x = 2$ and $x = 4$) pass their information already in the first period, so that the intermediary faces no competition once she received the information. So the duration is only $T + 1$ periods. In the other two remaining hierarchies for those values of δ the one player who reports directly to the principal still would wait, which leads to a duration of $T + 2$ periods.

From this example we can learn that there is always an interval in which the flat hierarchy is not optimal and if we ban the flat hierarchy in different asymmetric hierarchies the information is centralized fastest, depending on δ.

2.3.3 General results

The previous examples have shown that the hierarchical structure plays an important role in the speed of centralization. In this part, we will give some more general results. We will show that some hierarchies can never be optimal and give more detailed analysis for a special case.

Proposition 2.5 (Uninformed Intermediary)**.**
Any hierarchy that contains a sub-hierarchy with an uninformed intermediary and at least three players directly connected to this intermediary is weakly dominated.

Proposition 2.6 (Informed Intermediary)**.**
In a sub-hierarchy with one intermediary and $n-1 \geq 2$ players who pass to this intermediary, the only non-dominated sub-hierarchies are those where either the least informed player or the second-least informed player is the intermediary.

These two propositions already permit to exclude several structures for our further analysis. We can neglect all hierarchies which have a sub-hierarchy with an uninformed intermediary who has at least three agents reporting to her. The simple reason for this is that we can get the same or a smaller duration by installing one of these agents as the (informed) intermediary. We have already seen in the four player example that Hierarchy D, which contains these kind of sub-hierarchy, is always dominated.
On the other hand we know that the centralization of information works fastest when the intermediary has only little information. Let us compare a sub-hierarchy with an informed intermediary, which has more information that two of her agents to those sub-hierarchies where one of the least informed players is the intermediary. Taking the least informed player as an intermediary yields to the result that the boundaries (for δ) for a duration of one and two periods both shift upwards and by that make the process faster, because both x_{n-1} and x_n of the sub-hierarchy increase. If we take the second-least informed player instead, it shifts only the boundary between 2 and T periods upwards, but by that also improves the speed. The arguments for those are the same as in Section 2.2.4. A more detailed analysis can be found in the proofs of Proposition 2.5 and 2.6. It is important to note that the structures described in those propositions still can be faster than other, completely different hierarchies, but they are never the unique fastest.

Impact of the parameters
Figure 2.14 shows two main differences between the two-layer model and the multi-layer model. Even though it is just the example with four players, we see that the duration is no longer increasing in δ. Furthermore the boundaries of the different intervals are more complex than before. This makes it impossible to state general results on the impact of an increase of δ and R on the duration. We still can make statements about the impact of the number of players and the information

distribution.
If we add a well informed player to a sub-hierarchy, the duration of this sub-hierarchy does not increase, but may decrease. The reason for this is the same as before: The addition of a well informed player, i.e. $x_i > x_{n-1} > x_n$ is equivalent to an increase in the number of players n. By that the boundary for the duration between one and two periods gets shifted upwards, which decreases the duration for certain values of δ. Similarly, an increase in x_{n-1} or x_n shifts the boundaries such that the duration decreases for a specific interval of δ, while it stays the same for the remaining values.

Simplification
Our model allows for more results if we simplify the reward R and the deadline T. For this part of our analysis we do the same as in the examples: We focus on the same reward R for all hierarchies, i.e. $R_h = R$ and on the same deadline for all sub-hierarchies. This means that the lowest hierarchies have a deadline of T periods, the ones that include this sub-hierarchy have $2T$ periods and so on. By this the entire hierarchy has a deadline equal to $(\#\text{layers}-1) \cdot T$.

Proposition 2.7.
Assume that the least informed player in the highest-level hierarchy has more valuable information than the second-least informed player of any other hierarchy.
If the highest-level hierarchy needs the entire deadline when all agents of this hierarchy arrive at the same time, then each sub-hierarchy with at least two players needs their entire sub-deadline.

Proposition 2.8.
The total duration is minimal (equal to $(\#\text{layers}-1)$) if the following three conditions are true:

1. *The lowest level hierarchies have a duration of just a single period*
2. *The number of players is non-increasing from bottom to top*
3. *Each asymmetric linked player has at least the value of information as the least informed player in the lowest level of the hierarchy.*

These two results give us a possibility to check for the entire duration of a hierarchy just by checking certain sub-hierarchies. If the highest part needs the entire deadline, we know that each sub-hierarchy needs the entire time available. If, on the other hand, the lowest levels just have a duration of one period, we get that the duration of the entire structure is minimal.

2.4 Extensions

In the previous sections we have created a basic model for information transmission in hierarchies. While there may be several ways to extend this model, we want to discuss two important extensions in this section. The first extension relaxes the assumption we made on the reward R. In the second extension we introduce shared information.

2.4.1 Extension 1: Unbound reward

For the entire paper we were assuming that the reward R is not larger than $n \cdot (x_{n-1} - x_n) + (n - 2) \cdot x_{n-1}$. This assumption ensures that even in a setting with just two periods, all different types of SPNE exist, i.e. $\frac{n \cdot x_n + R}{n \cdot (x_n + R)} < \frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1} + R}$ holds. If we relax this assumption and select a larger reward, this is no longer true. The equilibrium in each period and the SPNE will no longer be unique, unless we specify an equilibrium selection rule.

The only equilibrium that can coexist with another equilibrium for the same values of the parameters is the equilibrium in which all players pass in the first period. At the same time there is no more interval in which just one player holds her information in the first period. For higher values of δ, but which are less than $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$, there will be two equilibria: On the one hand the same as before, where some players hold their information in the first period, and in addition the equilibrium in which all players pass in $t = 1$. If R is very large and if the discount factor is close to 1, there can even be a region in which there is an equilibrium with all players holding in the first period and at the same time an equilibrium with all players passing in the first period.

Naturally two different ways of equilibrium selection exist. We can either always select the equilibrium where more players hold or where more players pass. In the first case the analysis stays quite similar to the one we did with the assumption on R. The only difference is that in a two-layer model the duration changes directly from a single period to T periods, but there is no interval for δ with a duration of 2 periods.

This is not the case if we always select the equilibrium in which all players pass. For values of δ larger than $\frac{n \cdot x_n + R}{n \cdot (x_n + R)}$, some players will hold their information in the first period. In this new setting Proposition 2.2 no longer holds. This means that even if two or more players hold their information in $t = 1$, they will not necessarily wait until T to pass. In this case we get additional conditions for the SPNE, which describe the players behavior starting from the second period. Furthermore, the duration not necessarily will be strictly increasing in δ.

We also need similar conditions for other equilibrium selection methods, as long as not always the equilibrium where most players hold gets selected.

2.4.2 Extension 2: Shared information

While in some examples the uniqueness of the information of the different players comes naturally, one can also argue that in some cases there should be shared information. A researcher might propose the same idea for different projects, or a mathematical proof gives a stronger result than necessary.

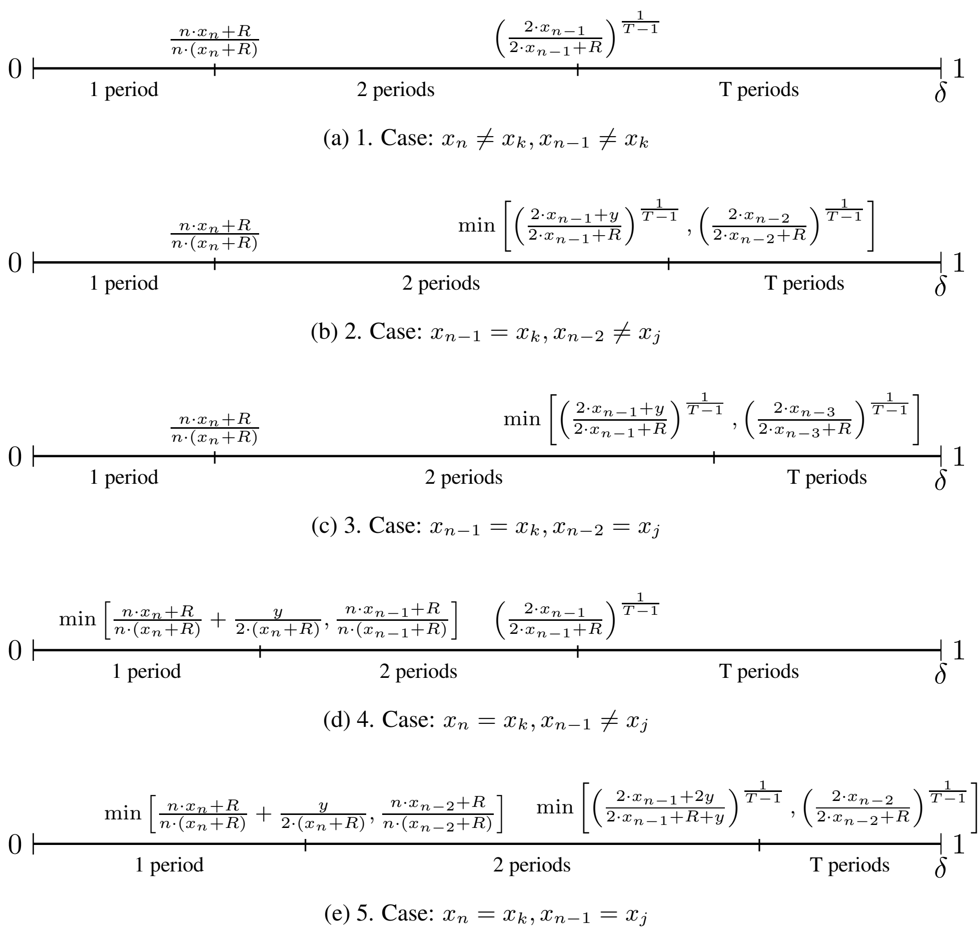
For simplification we will focus on a two-layer model again. We include the possibility that two players have some shared information in addition to their unique information. Only the player who passes the information first gets the payoff according to the value of the shared information. We assume that if both players pass at the same time, they split the payoff equally. In Section 2.2.4 we have described the impact of an increase in the value of information on the duration. We have seen that only if the value of one of the two least informed players is changed, the duration is affected. In this situation the result will be similar. If at least one of the players $n - 1$ and n

has shared information, the duration may decrease. Still the effect is different than an increase in x_n (or x_{n-1}), because the players do not get this additional payoff for sure. We still assume that if several players have an incentive to pass the information, the players with the more valuable unique information pass first.
Let player j and k have the shared information with value $y > 0$. Without loss of generality we can assume that $x_j \geq x_k$ holds. To have a duration of a single period all players need to prefer passing over holding in the first period, i.e.

$$\begin{aligned} \forall i \in N \setminus \{j,k\}: \qquad & x_i + \tfrac{R}{n} && > \delta(x_i + R) \\ \Leftrightarrow \qquad & \delta && < \tfrac{n \cdot x_i + R}{n \cdot x_i + R} \end{aligned}$$

$$\begin{aligned} \forall i \in \{j,k\}: \qquad & x_i + \tfrac{R}{n} + \tfrac{y}{2} && > \delta(x_i + R) \\ \Leftrightarrow \qquad & \delta && < \tfrac{n \cdot x_i + R}{n \cdot x_i + R} + \tfrac{y}{2 \cdot (x_i + R)} \end{aligned}$$

We can see that this boundary is not changed if it is not the least informed player n, who has the additional shared information. In a similar way we can also get the other boundaries, which result in five cases. In Figure 2.16 we can see how the shared information changes the duration. If the



(a) 1. Case: $x_n \neq x_k, x_{n-1} \neq x_k$

(b) 2. Case: $x_{n-1} = x_k, x_{n-2} \neq x_j$

(c) 3. Case: $x_{n-1} = x_k, x_{n-2} = x_j$

(d) 4. Case: $x_n = x_k, x_{n-1} \neq x_j$

(e) 5. Case: $x_n = x_k, x_{n-1} = x_j$

Figure 2.16: Duration in case of shared information

shared information is split between two players that are both not the least or second least informed player, there is no change at all. This benchmark is shown in the first case.
In the second case the second least informed player has the shared information. This increases the upper boundary of the interval with a duration of two periods. So there exists an interval in which the information is centralized in two periods, while in the benchmark it takes T periods. The boundary does not only depend on x_{n-1}, but also on x_{n-2}. Even in the benchmark $\delta < \left(\frac{2 \cdot x_{n-2}}{2 \cdot x_{n-2}+R}\right)^{\frac{1}{T-1}}$ has to hold, but as we described in Section 2.2, this is always satisfied for $\delta < \left(\frac{2 \cdot x_{n-1}}{2 \cdot x_{n-1}+R}\right)^{\frac{1}{T-1}}$.
In the third case the third least informed player also has the shared information, so the boundary between a duration of 2 and T periods depends on x_{n-3}. This condition ensures that the players 1 to $n-3$ prefer to hold and will not deviate. In some cases the boundary between 2 and T periods will be the same for the second and third case, there is also the possibility that the boundary is higher in the third case.
The change in the fourth case is different. In that case the least informed player has shared information, while the second least informed player does not. This shifts up the upper bound for the interval in which the duration is one period. Both boundaries increase if the least and the second least informed player share the information. In that case the increase of the boundary between 2 and T periods is stronger than in the second case. Whether the increase in the third or fifth case is stronger depends on several variables. The first boundary of the fifth case can also be larger than that of the fourth case.
We can see that as soon as at least one of the players n and $n-1$ has shared information, the boundaries change. We still need the conditions to hold for all other players, so we use the minimum of both conditions.
These changes are different than those we did in Section 2.2.4. If we just increase x_n or x_{n-1}, we observe a different change. A comparison of these changes shows that for some values of the parameters the duration decreases more if the players have shared information, while for other values the duration decreases stronger if just one of the players has additional information.

2.5 Conclusion

In this paper we have analyzed a model of hierarchies in which information flows from the leafs to the root. Each player has unique information and faces the problem to transmit this information to her predecessor at the best time. While late passing comes with the disadvantage of the discounted value of information the last player to pass gets an additional payoff.
For the two-layer model we have shown that the information in a hierarchy with two layers is always centralized in either one, two or T periods. The parameters such as the number of players, the additional reward, the deadline and the information distribution all have different effects on the duration. The most surprising result may be that an increase of the deadline T can lower the duration from T to two periods. Furthermore, we have compared different hierarchies in combination with a different information distribution and we have visualized the importance of the number of

players and different deadlines for the duration of two hierarchies.

In a hierarchy with several layers we have proposed a model in which the players can only pass their information once they have collected all information from their agents. The players' problem of timing is more complex, because they have to take into account that other players may centralize their information later and then enter the game. For the period when all players have entered, we have given conditions for all SPNE, for the players who have entered the game before, we have stated the problem they face.

We have compared all different hierarchical structures for three and four players and have shown that not only the parameters have an impact on the duration, but also that the structure and the information distribution play an important role. Even in a general multi-layer model, some hierarchies and some arrangements of players can never be optimal, while the dominance of the remaining possibilities depends highly on the discount factor and the reward.

In the entire paper we make one crucial assumption, which is the upper limit on the reward R. In one of the extensions we have seen that weakening this assumption creates intervals for δ in which the equilibrium is not unique. Defining an equilibrium selection rule or a mechanism and then studying the model again, can be a nice first step to a more general model. A similar change would arise if we allow the players to have different discount factors.

We have already shown the impact of shared information between two players on the duration in a two-layer model. This can be extended even more, either by letting different pairs of players share information, or even by shared information between more than two players.

Other extension possibilities can be found easily by slight modifications of the utility function: Can we replicate the results if the reward is not split equally, but according to the value of information? How do the results change if the reward is depending on the value of information? These questions should be answered to analyze the behavior of players who have some kind of fixed wage and a variable wage, depending on their work.

Appendix of Chapter 2

Proof of Proposition 2.1.
Player j's utility is given by:

$$\begin{aligned}
&u_j(\text{ Pass} \mid \text{all other players pass }) &&= x_j + \tfrac{R}{n}\\
&u_j(\text{ Hold} \mid \text{all other players pass }) &&= \delta \cdot (x_j + R)\\
&u_j(\text{ Pass} \mid \text{i players hold }) &&= x_j\\
&u_j(\text{ Hold} \mid \text{i players hold }) &&= \delta \cdot \left(x_j + \tfrac{R}{i+1}\right)\\
&u_j(\text{ Pass} \mid \text{all other players hold }) &&= x_j\\
&u_j(\text{ Hold} \mid \text{all other players hold }) &&= \delta \cdot \left(x_j + \tfrac{R}{n}\right)
\end{aligned}$$

1) This implies that all players prefer to pass over hold if and only if
$\forall j \in \{1, \ldots n\}$:

$$\begin{aligned}
&x_j + \tfrac{R}{n} > \delta \cdot (x_j + R)\\
\Leftrightarrow\quad &\delta < \tfrac{n \cdot x_j + R}{n \cdot (x_j + R)}
\end{aligned}$$

With $x_1 \geq x_2 \geq \ldots \geq x_n$ we get $\delta < \dfrac{n \cdot x_n + R}{n \cdot (x_n + R)}$

2) The players 1 to $n-1$ prefer to pass and player n prefers to hold if and only if
$\forall j \in \{1, \ldots, n-1\}$:

$$\begin{aligned}
&x_j > \delta \cdot \left(x_j + \tfrac{R}{2}\right)\\
\Leftrightarrow\quad &\delta < \tfrac{2 \cdot x_j}{2 \cdot x_j + R}
\end{aligned}$$

and for player n we get the opposite result for 1). Then with $x_1 \geq x_2 \geq \ldots \geq x_n$ we get the interval of δ.

3) The players 1 to $n-i$ prefer to pass, while the players $n+1-i$ to n hold if and only if
$\forall j \in \{1, \ldots, n-i\}$:

$$\begin{aligned}
&x_j > \delta \cdot \left(x_j + \tfrac{R}{i+1}\right)\\
\Leftrightarrow\quad &\delta < \tfrac{(i+1) \cdot x_j}{(i+1) \cdot x_j + R}
\end{aligned}$$

and $\forall j \in \{n+1-i, \ldots n\}$:

$$\begin{aligned}
&\delta \cdot \left(x_j + \tfrac{R}{i}\right) > x_j\\
\Leftrightarrow\quad &\delta > \tfrac{i \cdot x_j}{i \cdot x_j + R}
\end{aligned}$$

With $x_1 \geq x_2 \geq \ldots \geq x_n$ we get $\delta \in \left(\dfrac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}, \dfrac{(i+1) \cdot x_{n-i}}{(i+1) \cdot x_{n-i} + R}\right)$

4) All players prefer to hold if and only if $\forall j \in \{1, \ldots n\}$:

$$\begin{aligned}
&\delta \cdot \left(x_j + \tfrac{R}{n}\right) > x_j\\
\Leftrightarrow\quad &\delta > \tfrac{n \cdot x_j}{n \cdot x_j + R}
\end{aligned}$$

With $x_1 \geq x_2 \geq \ldots \geq x_n$ we get $\delta > \dfrac{n \cdot x_1}{n \cdot x_1 + R}$ □

Proof of Proposition 2.2.

Let us first assume that there are only 3 periods, i.e. $T = 3$.

If in $t = 2$ there are i players left, the discount factor has to be such that all pass. As there is only one period left the SPNE are as in Proposition 2.1. To not have the equilibrium where all players hold their information we need $\delta < \frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}$.

In $t = 1$ we need that i players hold and $(n - i)$ players pass. In that situation the utility of player $n + 1 - i$ is $u_{n+1-i}(\text{H}) = \delta \cdot (x_{n+1-i} + \frac{R}{i})$. If this player passes she gets $u_{n+1-i}(\text{P}) = x_{n+1-i}$. So the player prefers to hold if and only if $\delta \cdot (x_{n+1-i} + \frac{R}{i}) > x_{n+1-i}$ which is equivalent to $\delta > \frac{i \cdot x_{n+1-i}}{i \cdot x_{n+1-i} + R}$ and contradicts the behavior in $t = 2$.

So it is impossible that the remaining players all pass before the deadline and by Remark 2.3 also no subset of players will pass, therefore all remaining players will wait until the last period to pass their information.

This also does not change for any other $T > 3$, because if we require that all the remaining i players pass in $t = 2$, they will do so in $t = 3$ as well. This generates the same utility for all i players if they all hold as in the setting with $T = 3$. □

Proof of Proposition 2.3.

The proof follows the same steps as the proof of Proposition 2.1, but we have to discount the utility for holding $T - 1$ times, because of Proposition 2.2. □

Proof of Proposition 2.4.

1) Obviously the interval is not affected by T.

2) The lower boundary stays unchanged. The exponent of the higher boundary decreases (between 0 and 1) and since the base is smaller than one, the whole term increases.

3) By the same idea of 2) both values increase. It remains to show that the difference between the upper and lower boundaries decreases: The size of the interval is

$$\frac{[(i+1) \cdot x_{n-i}]^{\frac{1}{T-1}} \cdot [i \cdot x_{n+1-i} + R]^{\frac{1}{T-1}} - [i \cdot x_{n+1-i}]^{\frac{1}{T-1}} \cdot [(i+1) \cdot x_{n-i} + R]^{\frac{1}{T-1}}}{[(i+1) \cdot x_{n-i} + R]^{\frac{1}{T-1}} \cdot [i \cdot x_{n+1-i} + R]^{\frac{1}{T-1}}}$$

We can rewrite that as $\frac{a^{\frac{1}{x}} - b^{\frac{1}{x}}}{c^{\frac{1}{x}}}$ with $c > a > b > 0$. Derivation yields

$$\frac{1}{c^{\frac{1}{x}} * x^2} \left[b^{\frac{1}{x}} \cdot (\ln(b) - \ln(c)) - a^{\frac{1}{x}} \cdot (\ln(a) - \ln(c)) \right]$$

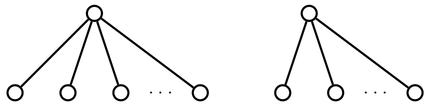
It remains to show that this term is negative. Obviously $\frac{1}{c^{\frac{1}{x}} * x^2}$ is positive, so we need to show that

$$b^{\frac{1}{x}} \cdot (\ln(b) - \ln(c)) < a^{\frac{1}{x}} \cdot (\ln(a) - \ln(c))$$

holds. As $b < a$ implies $b^{\frac{1}{x}} < a^{\frac{1}{x}}$ and also $\ln(b) < \ln(a)$ we see that the inequality holds and the size of the interval is decreasing with T.

4) Obviously the lower boundary is increasing, which decreases the interval. □

Proof of Proposition 2.5.
We compare the following two structures:



(a) Uninformed intermediary

(b) Informed intermediary

In hierarchy (a) with the uninformed intermediary there are n agents connected to the intermediary, while in hierarchy (b) there are only $n-1$ players, because we have selected one of the players to become the intermediary.
For simplification we sort the players by value of information, i.e. $x_1 \geq x_2 \geq \ldots x_n > 0$. The duration for the left hierarchy is:

- 1 period if and only if $\delta < \frac{n \cdot x_n + R}{n \cdot (x_n + R)}$
- 2 periods if and only if $\delta \in \left(\frac{n \cdot x_n + R}{n \cdot (x_n + R)}, \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)}^{\frac{1}{T-1}} \right)$
- T periods if and only if $\delta > \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)}^{\frac{1}{T-1}}$

If we take player $i \neq n, i \neq n-1$ as the intermediary in hierarchy (b) we get:

- 1 period if and only if $\delta < \frac{(n-1) \cdot x_n + R}{(n-1) \cdot (x_n + R)}$
- 2 periods if and only if $\delta \in \left(\frac{(n-1) \cdot x_n + R}{(n-1) \cdot (x_n + R)}, \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)}^{\frac{1}{T-1}} \right)$
- T periods if and only if $\delta > \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1} + R)}^{\frac{1}{T-1}}$

Clearly the boundary between 1 and 2 periods is higher for hierarchy (b).
If we take player n or $n-1$ as the intermediary also the other boundaries shift:

- 1 period if and only if $\delta < \frac{(n-1) \cdot x'_n + R}{(n-1) \cdot (x'_n + R)}$
- 2 periods if and only if $\delta \in \left(\frac{(n-1) \cdot x'_n + R}{(n-1) \cdot (x'_n + R)}, \frac{2 \cdot x'_{n-1}}{2 \cdot (x'_{n-1} + R)}^{\frac{1}{T-1}} \right)$
- T periods if and only if $\delta > \frac{2 \cdot x'_{n-1}}{2 \cdot (x'_{n-1} + R)}^{\frac{1}{T-1}}$

By x'_n and x'_{n-1} we denote the value of information of the two least informed players after removing the former player n or $n-1$. If we take player n as the intermediary we have $x'_n = x_{n-1}$ and $x'_{n-1} = x_{n-2}$ and if we take player $n-1$ we get $x'_n = x_n$ and $x'_{n-1} = x_{n-2}$. Obviously the two least informed players in hierarchy (b) have at least the same value of information as those in hierarchy (a). We have described this effect already in the first part of Section 2.2.4. With this reasoning we get that the boundaries shift upwards and so hierarchy (b) weakly dominates hierarchy (a). □

Proof of Proposition 2.6.
As we have seen several times (e.g. in the previous proof) the duration of a hierarchy depends on the value of information of the two least informed players. If we remove one of these players, because we install her as the intermediary and add the previous intermediary who has more valuable information, it is beneficial for the duration. This step is equivalent to giving one of the two least informed players more information. □

Proof of Proposition 2.7.
We assume there are k-levels of the hierarchy. Assume that the players of the highest level of the hierarchy all enter at time $\tau \leq (k-2) \cdot T$. Then the total duration is $(k-1) \cdot T$ if and only if $\delta > \frac{2 \cdot x_{n-1}}{2 \cdot (x_{n-1}+R)}^{\frac{1}{(k-1) \cdot T - \tau}}$, where x_{n-1} denotes the value of information of the player who has the second least valuable information.
For all other sub-hierarchies the respective x_{n-1} has to be lower than that of the highest level hierarchy, because there are also other players who have information. By that we get that a value of δ that implies the longest duration in the highest part of the hierarchy, also implies the longest duration in all sub-hierarchies. □

Proof of Proposition 2.8.
The lowest hierarchies all have a duration of one period if $\delta < \frac{n \cdot x_n + R}{n \cdot (x_n + R)}$ holds for all these hierarchies. With Condition 2 we get that n is increasing if we move to higher-levels, which makes this boundary higher. Since the intermediaries add the value of information of their agents, also x_n increases going from the bottom to the top. Together with the third condition we then get that for all hierarchies δ is such that the duration is only one period. □

CHAPTER 3

Centralizing Information in Endogenous Networks

This chapter is based on the paper "Centralizing Information in Endogenous Networks" with Ana Mauleon[1] and Vincent Vannetelbosch[2].
All notations and definitions are independent of those from previous chapters.

Abstract: We analyze a model of information centralization in teams where players can only exchange information through an endogenous network. Over several periods each player can either pass or not pass her information to her neighbors. Once one player has centralized all the information, all players receive some payoff. The winner who collects all the information gets an additional reward. Since each player discounts payoffs over time, she faces the dilemma of either letting another player centralizing all the information fast, or trying to collect all the information by herself and overtaking the leadership. We find that there is always a single winner who centralizes the information at equilibrium and that only minimally connected networks can be pairwise stable. We also characterize the winner and the duration for any network and for any discount factor.

Keywords : dynamic network game, competition for leadership, information centralization, network formation, pairwise stability.
JEL Classification : C72, C73, D83, D85.

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3.1 Introduction

The objective of the paper is to develop a model of information centralization in teams where team members can only exchange information through a communication network and to predict which communication networks could emerge in the long run. There are n players arranged in a connected network and each player belonging to the team has a unique piece of information. The task is to centralize all the information. The team must collect all the information facing some exogenously given deadline. In each period, each player decides either to hold on her information item or to pass on to her neighbors in the communication network. Players take their decisions sequentially following some decision order or hierarchical ranking. The timing in each period is as follows. Player 1 takes first her decision. Next, each player chooses his action knowing the choices made by the players preceding him in the ranking. After all players have selected an action, pass on or hold on, the information is passed according to their decisions. The decision order or hierarchical ranking can be interpreted as the leadership structure within the team. Hence, player 1 is the current leader, and so players, beside collaborating for collecting all the information, compete for becoming the winner or the new team leader and getting the reward. If there is no winner after the items have been transmitted, the next period starts. As soon as at least one player has centralized all the information the game ends and all players receive some positive payoff. The player who collects all the information gets an additional reward. If several players centralize all the information at the same time they split the reward equally. The players have time preferences and so the utility of each player depends on the time it takes until one player centralizes all the information and whether she gets the reward or not.

We find that there is always a single winner who centralizes the information at equilibrium. When the common discount factor is low, the player with the lowest eccentricity is the winner, where the eccentricity of a player is the geodesic distance from this player to the player furthest away from him in the communication network. All players prefer to centralize the information as fast as possible. The player with the lowest eccentricity can centralize the information fastest and so she wins. The duration is given by the number of periods it takes the winner to centralize the information and is then equal to the minimal eccentricity (i.e. the radius) of the network. When the discount factor becomes large, players care less about the duration and focus more on becoming the new leader and getting the additional reward. Obviously, the leader has an advantage over the other players since she takes her decision first. She can then enforce the other players to pass on their information since otherwise they will not be able to centralize the information at all. However, the leader can decide not to centralize all the information if it is beneficial for her that another player collects faster all items. Another reason, why the leader may not end up being the player who centralizes all the information is that some player blocks her from winning. Some players may have incentives to hold on their items long enough making it impossible for the leader to win. For instance, some player may hold on his information first, to avoid that the leader who has a much higher eccentricity than him wins. As soon as the leader cannot win anymore, he passes on his information. This is the only type of situations where, at equilibrium, the duration is not equal to the eccentricity of the winner.

Beside characterizing the winner and the duration for any network and for any discount factor, we also predict which communication networks could emerge in the long run. We assume there is an infinitesimally small cost to form a link. Under this assumption only minimally connected networks can be pairwise stable and players will add a link to each other if the duration for centralizing all the information becomes shorter. We also find conditions that help us to exclude some networks for being pairwise stable without having to check all possible additional links. In addition, we provide more information about which network structures are always (star or star-like networks) or never (line networks) pairwise stable. Finally, we show that all our results are robust if we instead require that players only need to collect at least $n-1$ items. We do further robustness checks and analyze closely related settings. We show that we can replicate most of our results even if two players have the same information item or if a single player is more patient or impatient than all other players.

One motivation for our model is the process of finding a spokesman or a promoter or a coordinator for a research project. All players contribute a part to the success of the project, but in the end only one player coordinates and promotes the project to the outside world. The additional reward can be either some additional funding the spokesman of a project gets or some benefits in terms of scientific reputation. Other examples include R&D joint ventures, political agreements, international agreements for fighting terrorism or climate change negotiations. Another example are the Panama Papers[3] investigations in 2015 where many different journalists and newspapers worked together, but the project is now promoted by only a small subset of the people who worked on it. The German journalist Bastian Obermayer was first to obtain all the raw data. He and his newspaper (Süddeutsche Zeitung) realized they could not analyze all the data by themselves. They contacted the International Consortium of Investigative Journalists and started the team project to retrieve information from the data. So, in our model Bastian Obermayer would be designated as the first player or leader, while the process of working through all the documents represents the centralization of (useful) information. In this example we can find all our assumptions satisfied. Obviously, the entire team had a strong incentive to finish the project early as they wanted to stop the damage that was dealt by the firms and people they investigated. This corresponds to our assumption that the players discount over time. In this application, all newspapers and journalists were rewarded with the Pulitzer Prize and so they all got the same payoff. At the same time Bastian Obermayer was the one that represented the project in many interviews and on television, which gave him additional publicity. Clearly, the International Consortium of Investigative Journalists can be represented as a network. Even though, all newspapers might be able to share information with each other, this does not hold for all journalists in the newspapers and especially not for the external specialists that were hired directly from individual newspapers. These specialists worked on certain tasks and sent their results to the newspaper who hired them. Then, the information was shared by this newspaper with other journalists. Even the assumption that all the players have unique information is closely approximated in this example. Everyone had access to the approximately 2.6 Terabyte of data, but the tasks were split between the players. The people working on this project completed their aim to go through all the information, because they did not want

[3] All information available at https://panamapapers.icij.org/

anyone to get away with what they had done.

We now turn to the related literature. A similar setting was first introduced in Hagenbach (2011) but with an exogenously given communication network. The players are arranged in a network and want to centralize the information, but they decide simultaneously whether to pass or to hold on their information. This game of transmission information has multiple equilibrium outcomes. We rather adopt a sequential decision order yielding a unique equilibrium outcome. This decision order or ranking allows us not only to make more precise statements about the winner and the duration, but also to endogenize the communication network. Radner (1993) studies the efficiency of hierarchies in a model where players process information. He compares different structures and shows which hierarchical structures are efficient for a given set of variables. Another approach of information centralization is done by Jehiel (1999). In his setting a decision maker needs to gather some information to decide about a project. The decision maker's future employment depends on the outcome of the project. The decision maker gets fired if he selects a bad project. All other players get a certain share of the surplus of the project. The author states conditions for optimal communication structures from the players and from the decision maker perspective. Closer to Hagenbach (2011) is the work of Schopohl (2017), who starts with a similar setting, but focuses on exogenous networks in which a given player wants to centralize the information. All other players are arranged in a hierarchy and compete for a reward. Schopohl (2017) compares different network structures with the focus on the time it takes until all the information is centralized.For strategic information transmission networks and its related literature we refer to Galeotti et al. (2013). They study a model of cheap-talk on networks and show how the players' welfare increases with more truthful messages. In addition, they find that in larger communities the communication decreases. Bonacich (1990) conducted two experiments on communication in networks where the participants had to find a quotation. Each player receives a different subset of letters and she could pass on the information to her neighbors or hold on the information private. Similar to our setting, the first player who completes the quotation receives a reward. One difference between his experiments and our model comes up with the predictability of a missing piece of information. While in the experiments the players could make guesses about information they could not centralize, this cannot happen in our setting. However, our results are still valid if we instead require that players only need to collect at least $n-1$ items.

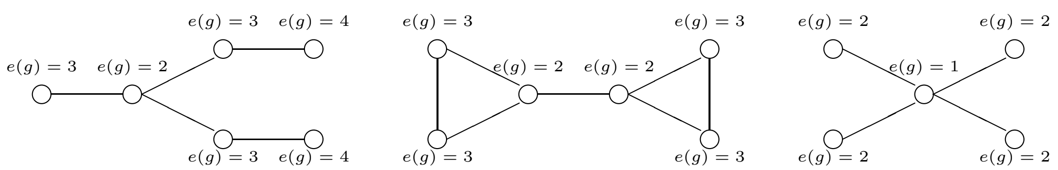
In economics the idea of leadership has been neglected for a long time. Hermalin (1998) builds a model and asks 'why players should follow a leader?'. Hermalin (1998) shows that the leader has two ways to convince her fellow agents. Either by leading by example or by making a sacrifice. The first is associated with long working times of the leader to motivate the agents, while the second represents the idea that the leader gives small gifts to show that the work of her agents is valuable. Komai et al. (2007) focus on a different aspect of leadership. They compare the collective decisions an organization makes in two settings. If all players have full information players can free ride on the decision making. On the other hand, if only the leader has full information and reveals only a part of it to the other players, the players have to invest into effort and cannot free ride. This improves the efficiency of the decision making compare to the first situation. By this they give an answer to the question 'why there should be a leader?'. Even if there has to be a leader it remains open who should become the leader. Dewan and Squintani (2017) propose a model where a leader is selected and then this leader receives cheap-talk messages from the agents. To trust the messages the leader must first build a network of trustworthy associates. Dewan and Squintani (2017)) show that the quality of the leadership depends on the judgment and wisdom of the people surrounding the leader. Dewan and Myatt (2008) model the influence of a leader on a mass. For a survey on leadership, see Ahlquist and Levi (2011).

The paper is organized as follows. In Section 3.2 we introduce the model and some definitions. In Section 3.3 we show that there is always a single winner and that only minimally connected networks can be pairwise stable. In Section 3.4 we characterize the winner and the duration for any network and for any discount factor, and we also predict which communication networks could emerge in the long run. In Section 3.5 we discuss the robustness of our main results by relaxing some main conditions. Finally, in Section 3.6 we conclude.

3.2 Model

Let $N = \{1, \dots, n\}$ be a set of finite players, which are arranged in an undirected network g. A link in g between player i and player j is denoted by $ij \in g$. Let the set of players who have at least one link in g be denoted by $N(g) = \{i \in N \mid \exists\, j \in N \text{ such that } ij \in g\}$ and let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbors of player i in g. A player i who has only one neighbor, $|N_j(g)| = 1$, is called a loose end. A path in g from i to j is a sequence of distinct links, which connect player i and player j. The length of a path is equal to the number of links on the path. A network g is said to be connected if there exists at least one path between all players $i, j \in N(g)$ with $i \neq j$. A network g is minimally connected if for all players $i, j \in N(g)$ with $i \neq j$, there exists exactly one path that connects player i and player j. Given a connected network g, the geodesic or shortest distance between player i and player j is the length of the shortest path between them and is denoted by $d_{ij}(g)$. Throughout the paper if we do not mention $N(g)$ for a connected network g, we implicitly presume that $N(g) = N$.

Definition 3.1 (eccentricity, radius and diameter)**.**
Let g be connected. The eccentricity of player i is $e_i(g) = \max_{j \in N} d_{ij}(g)$. The radius of the graph g is $r(g) = \min_{i \in N} e_i(g)$ and the diameter is $\text{dia}(g) = \max_{i \in N} e_i(g)$.



(a) min. connected, $r(g) = 2, \text{dia}(g) = 4$

(b) not min. connected, $r(g) = 2, \text{dia}(g) = 3$

(c) star, min. connected, $r(g) = 1, \text{dia}(g) = 2$

Figure 3.1: Eccentricity, radius and diameter.

The eccentricity of player i is the geodesic distance from player i to the player furthest away from her. Given a connected network g, the radius is the minimal eccentricity while the diameter is the maximal eccentricity. Figure 3.1 illustrates the notions of eccentricity, radius and diameter for three different networks.

At the beginning of the game each player belonging to the team is in the possession of a unique piece of information. We label the information items such that player i has item i in her possession, while player j has item j in his possession. There is a finite number of periods T. In each period the players can either pass on (P) all their information to their neighbors or hold on (H) the information private. When player i passes on her information, she passes copies of all the information items in her possession to all her neighbors $j \in N_i(g)$. The information set of each player is non-decreasing over time t. Players only pass on copies of their items to their neighbors, but they also keep the information in their possession. Each player either passes on all her information to her neighbors or holds on. That is, she is not able to split her information into different information items. This is consistent with R&D collaborations in chemistry or biology where after adding item A to item B it is only possible to give information about the resulting item AB, but not about both components. Once a player has collected all the information, the game ends and the payoff of each player equals 1, except for the player who centralizes all the information. This player receives an additional reward R. If there are several players who have collected all information in the same period, those players share the reward equally. We call the player(s) who get a share of the reward the winner(s). For every network g, the set of winners of the game with deadline T is denoted by $W(g,T)$. The duration of the game is given by the number of periods it takes the winner to centralize the information, and is denoted by $\tau(g,T)$. If no player centralizes all the information, all players get a payoff of 0. Players have time preferences with constant discount factor, $\delta \in (0,1)$, and the utility function of player i is given by

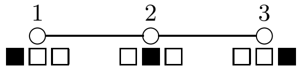
$$u_i(g,T) = \begin{cases} 0 & \text{if } W(g,T) = \emptyset \\ \delta^{\tau(g,T)-1} \cdot 1 & \text{if } W(g,T) \neq \emptyset \text{ and } i \notin W(g,T) \\ \delta^{\tau(g,T)-1} \cdot \left(1 + R \cdot \frac{1}{|W(g,T)|}\right) & \text{if } i \in W(g,T). \end{cases}$$

The effects of the reward R and the discount factor δ are similar in terms of players' incentives to pass or hold on their information. If the discount factor or the reward is large, then each player focuses on getting the reward for herself and cares less about the duration it takes for centralizing all the information. On the contrary, if the discount factor or the reward is small, then each player tends to prefer that all information is centralized as soon as possible. For the rest of the paper, we fix the reward $R = 1$.

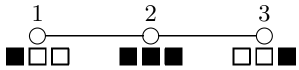
In each period t, the players choose sequentially their action $a_i^t \in \{P, H\}$, $i \in N$. There is a fixed decision order or current leadership ranking $\{1, 2, ..., n\}$ that determines the order of who is making a choice along the sequence. The timing in each period or period t is as follows. The current leader, player 1, starts and takes her decision, which is then observed by all other players. Afterwards, player 2 decides, already knowing what player 1 did. Then, player 3 chooses his action knowing the choices made by both players preceding him in the ranking. When player n has to pick an action, he takes into account the decisions of all other players. After all players have selected an action, pass on (P) or hold on (H), the information is passed according to their decisions. If there is no winner after the items have been forwarded, the next period starts.

Assumption 3.1. *If a player is indifferent between passing and holding on, we assume that she passes on the information.*

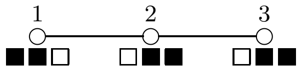
The game is solved backwards looking for subgame perfect Nash equilibria.



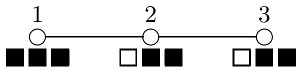
(a) Players and their information at $t = 1$



(b) Players and their information after $t = 1$, with $a_1^1 = a_3^1 = P$ and $a_2^1 = H$



(c) Players and their information after $t = 1$, with $a_1^1 = H$ and $a_2^1 = a_3^1 = P$



(d) Players and their information after $t = 2$, with $a_1^1 = a_1^2 = H$ and $a_2^1 = a_2^2 = a_3^1 = P$

Figure 3.2: A three-player line network

In Figure 3.2 we look at one possible way to arrange three players in a line. Each box represents a piece of information. A box is filled if the player has that piece of information in her possession. If we face a short deadline of $T = 1$, the only equilibrium outcome is that player 1 and player 3 pass on their information and player 2 is the unique winner as displayed in (b). If player 1 or player 3 would deviate, all players would obtain an utility of 0, so they prefer to pass on all their information. The action of player 2 makes no difference for the utility, because the other players cannot get all items within one period. As soon as we extend the deadline to $T = 2$, the analysis becomes more complex and the equilibrium now depends on δ. If players are sufficiently impatient, $\delta \leq \frac{1}{2}$, the equilibrium is as before: player 1 and player 3 pass on their information in the first period and player 2 wins. But, if players are patient, $\delta > \frac{1}{2}$, player 1 has incentives to hold on her information during both periods forcing the other two players to pass on their information to her (see (c) and (d)). The intuition works as follows. For low values of δ, the players rather focus more on centralizing all the information, while for higher values of δ, the reward becomes more important than the duration for centralizing all the information. If player 1 holds on her information in the first period, player 3 is indifferent and so he passes on his information to player 2. In the second period, player 1 can enforce player 2 to pass on his information by holding on her information once again.

If the network g is not connected, it is impossible for any player to centralize all the information. Otherwise, the shortest time for player i to centralize all the information in case every other player passes on his information in every period is equal to player i's eccentricity, $e_j(g)$. Only if the deadline T is larger or equal to the eccentricity of player i, player i can win the game. From now on, we set $T = |N| - 1$ and we write $W(g)$ for $W(g, |N| - 1)$ and $u_i(g)$ for $u_i(g, |N| - 1)$.

It implies that each player can potentially be the winner of the game. If we would set a shorter deadline, some players could not exante win the game. For any deadline greater than $|N| - 1$, the current leader, namely player 1, would even have a stronger advantage over the other players. Player 1 takes her decision first and can enforce the rest of the players to pass on their information, if she can threat that there would be no winner in case they hold on. In fact, we will show later on that player 1 does not win either if she prefers that another player wins or if she is blocked from winning. In the later case, at least one other player holds on his information long enough so that player 1 cannot become the winner anymore. With a deadline greater than $T = |N| - 1$, the incentives for the other players to block player 1 decrease.

3.3 One winner and minimally connected networks

Our objective is to characterize the winner(s), the duration and the networks that emerge in the long run. We first provide some general results that turn to be helpful for the characterization carried out further on. We show that, given any connected network, only a single player will get the reward at equilibrium. This result does not depend on the discount factor δ nor on the network g as long as g is connected. In addition, we show that only minimally connected networks can emerge in the long run. Since no player can centralize all the information if the network g is not connected, we mostly consider connected networks and we set $\tau(g) = \infty$ if g is not connected.

Proposition 3.1. *There is a unique winner at equilibrium, i.e.* $|W(g)| = 1$.

Proof. Since it is a game with perfect information, it cannot be optimal for the players that no one centralizes all the information. The players prefer to pass on without getting the reward over failing to centralize all the information. Hence, there is at least one winner.

Suppose that player i and j are both winning and share the reward after τ periods. Without loss of generality let $i < j$. To end up in a situation where both i and j win, both players need to pass on their information at least once, so that i can obtain the item of j and vice versa. The decision order in each period is such that player i always decides before player j. Hence, player i can decide to hold on her information in any period and be the unique winner, because she enforces player j to still pass on his information. If player j would also hold on his information, his utility would be 0 and so player j prefers passing over holding on his information in any period. Solving the game backwards, we know that player j passes on his information at the latest in the last period, so that player i wins. If player i holds on in the period before, player j anticipates that he will pass on his information in the last period and make player i the winner. Since players have time preferences, player j prefers to pass on in the second to last period so that the game ends earlier. Repeating this argument yields to the conclusion that if player i holds on her information in all periods, player j passes on his information so that player i is the unique winner. □

The easiest way to understand the intuition behind Proposition 3.1 is to consider the game with just two connected players. No matter the deadline $T > 1$, player 1 always wins after a single period. If both players hold on their information before the last period T, player 1 still holds on her information in T forcing player 2 to pass on his information. Otherwise, both players would be worst off (ending with a payoff of 0). In the second to last period $T-1$, player 2 has incentives to pass on his information since he anticipates that if he holds on, he will pass on in period T, and he ends up worse off because of time preferences. Repeating the above argument until we reach period $t = 1$, we get that player 2 already passes on his information during the first period, and player 1 is the single winner. We denote by w the player who is the single winner.

Beside looking for who is going to be the winner, we are also interested in understanding which communication networks are likely to arise in teams when players need to centralize all the information. To do so, we need to define a notion which captures the stability of a network. We restrict our analysis to pairwise deviations and we suppose that there are positive but infinitesimally small costs to forming links. In other words, the cost of any link is always less important than any payoff from centralizing all the information. Hence, as in Goyal and Joshi (2003) or Mauleon et al. (2014), we use a strict version of the notion of pairwise stability of Jackson and Wolinsky (1996). See Vannetelbosch and Mauleon (2016) for an overview on network formation games. A network is pairwise stable if no player has an incentive to delete one of her links and no other two players strictly benefit from adding a link between them.

Definition 3.2 (pairwise stability)**.**
A network g is pairwise stable if (i) for all $ij \in g$, $u_i(g) > u_i(g - ij)$ and $u_j(g) > u_j(g - ij)$, and (ii) for all $ij \notin g$, if $u_i(g) < u_i(g + ij)$ then $u_j(g) \geq u_j(g + ij)$.

Corollary 3.1. *Take any g such that $ij \notin g$. Player i and player j will add the link ij if and only if the network $g + ij$ has a shorter duration than g, i.e. $\tau(g + ij) < \tau(g)$.*

Corollary 3.1 follows directly from Proposition 3.1. Both players have to pay for the additional link ij and they only do so, if they both benefit from the link. Since at equilibrium only one player is the winner, the only possibility for both players to benefit from an additional link, is that this link decreases the time it takes until the information is centralized.

Proposition 3.2. *Only minimally connected networks can be pairwise stable.*

All the proofs not in the main text can be found in the appendix. The intuition behind Proposition 3.2 is as follows. As soon as a network is not minimally connected, it cannot be pairwise stable, because there is always at least one link that can be deleted while the duration remains constant. Even if players are connected by two or more paths, the information just flows through one path. Since links are costly, the players prefer to delete links until the network is minimally connected. Proposition 3.2 restricts our search for pairwise stable networks to minimally connected networks. Given any minimally connected network, we only have to consider additional links that players may want to form. Players will never delete a link since it would be impossible to centralize all the information, and all players would end up with zero utility. It then follows directly that a network that differs from a pairwise stable network by just one link cannot have a shorter duration than the duration of the pairwise stable network.

3.4 Winner, duration and stability

In this section we characterize the winner, the time it takes for the winner to centralize all the information and the stable networks. From the preceding section we already know that the winner and the duration for collecting all the information are affected by the value of the discount factor δ. For low values of δ, the players care less about the reward and more about the time. We show that, for $\delta \leq \frac{1}{2}$, the duration is the shortest possible and the eccentricity determines who is going to be the winner. For high values of δ, player 1 has an advantage over all other players thanks to the decision order. Nevertheless, we show that player 1 is not guaranteed to be always the winner. We first consider low discount factors. This analysis conveys the main intuition behind the results. We next provide general results for any value of δ. Moreover, we provide corollaries that simplify the search for the winner for high discount factors.

3.4.1 Low discount factor

For $\delta \leq \frac{1}{2}$ the players prefer to decrease the duration for centralizing all the information by one period over becoming the winner at a later time.

Proposition 3.3. *Assume $\delta \leq \frac{1}{2}$ holds. The winner is $w = \min_{e_i = r(g)} i$ and the duration is $\tau(g) = r(g) = e_w(g)$.*

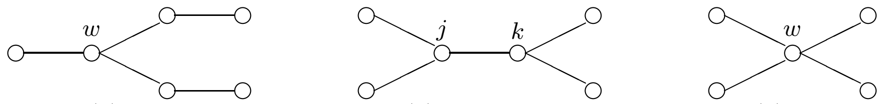
Proof. For $\delta < \frac{1}{2}$ all players prefer centralizing all the information in a given period and not winning over winning one period later. If player i can end up the centralization in period τ by passing on, but then another player wins, player i's utility is equal to $\delta^{\tau-1}$. If she holds on her information and then wins in period $\tau+1$ her utility is equal to $\delta^{\tau} \cdot 2$ but less than $\delta^{\tau-1}$. For $\delta = \frac{1}{2}$ she is indifferent and passes on by assumption. Since this reasoning holds for all players it follows that the player who can centralize all the information fastest wins and the duration is minimal and equal to the radius of g. If there are several players with the same minimal eccentricity, then the one who is first in the decision order wins. If she holds on her information in all periods, she becomes the unique winner and gets as utility $\delta^{e_i(g)-1} \cdot 2$, while if she passes on at least once her utility is at most $\delta^{e_i(g)-1}$. The other players with the same eccentricity, anticipating the behavior of the player ranked before them in the decision order, have no incentives to hold on their information as shown in the first part of the proof. □

Proposition 3.3 tells us that, for low discount factors ($\delta \leq \frac{1}{2}$), the player with the lowest eccentricity is the winner. All players prefer to centralize the information as fast as possible. The player with the lowest eccentricity can centralize the information fastest and so she wins. If there are several players with the lowest eccentricity, then the one who is first in the decision order wins. This player can hold on her information to avoid that other players win. Since all the other players want to centralize the information as fast as possible, they will pass on their information to her. The duration is then equal to the radius of the network.

Proposition 3.4. *Assume $\delta \leq \frac{1}{2}$ holds. A minimally connected network g is pairwise stable if and only if there is no link $ij \notin g$ that decreases the radius, i.e. $\forall ij \notin g : r(g + ij) = r(g)$.*

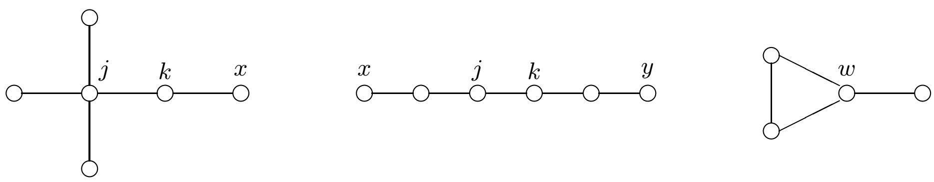
Proof. From Proposition 3.3 we know that the duration equals the radius. Proposition 3.1 states that there is always a unique winner at equilibrium. Combining these two results with Proposition 3.2, it follows that two players can only have incentives to form a link if this link decreases the radius and by that decreases the duration. □

The duration for centralizing all the information is equal to the radius. Two players will only form an additional link, if they both benefit from this new link. But, only one of them could be the winner. Hence, this new link must decrease the duration in order to offset the infinitesimally small cost for forming links. Figure 3.3 shows us three different network structures that are pairwise stable. In (a) and (c) there is a single player with the lowest eccentricity, while in (b) two players have the lowest eccentricity. In none of the networks it is possible to add a single link that decreases the radius. Hence, these three networks are pairwise stable. On the contrary, in network (a) of Figure 3.4, players have incentives to form links. In this network, depending on the decision order, either player j or k wins. Player j has an incentive to link with player x, because it decreases the duration by one period and player j would become the winner even if he makes his decision after player k. In network (b) the two players (j and k) who are most central have incentives to form links with the players who are the most far away (j to y or k to x). With such additional link the radius decreases to two and so the duration decreases by one period. The network (c) is not minimally connected and so it cannot be pairwise stable.



(a) $r(g) = 2$, Winner: w (b) $r(g) = 2$, Winner: $\min j, k$ (c) $r(g) = 1$, Winner: w

Figure 3.3: Pairwise stable networks for $\delta \leq \frac{1}{2}$.



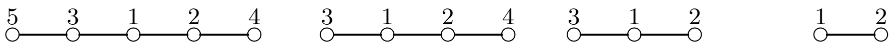
(a) $r(g) = 2$, Winner: $\min j, k$ (b) $r(g) = 3$, Winner: $\min j, k$ (c) $r(g) = 1$, Winner: w

Figure 3.4: Not pairwise stable networks for $\delta \leq \frac{1}{2}$.

3.4.2 Low and high discount factor

When the discount factor becomes large, players care less about the duration and focus more on becoming the new leader and getting the additional reward. Obviously, player 1 who is the current leader has an advantage over the other players since she takes her decision first. She can then enforce the other players to pass on their information since otherwise they will not be able to centralize the information at all. However, player 1 can decide not to centralize all the information if it is beneficial for her that another player i collects faster all items. This can happen only if the discount factor δ is not too high and there is a significant difference in the eccentricity of the players 1 and i. The other reason, why player 1 may not end up being the player who centralizes all the information is that some player(s) block(s) her from winning. Some player(s) may have incentives to hold on their items long enough making it impossible for player 1 to win. For instance, player i may hold on his information first, to avoid that player 1 who has a much higher eccentricity than him wins. As soon as player 1 cannot win anymore, player i passes on his information. This is the only type of situations where, at equilibrium, the duration is not equal to the eccentricity of the winner.

Given the network g, the deadline T and the actions in the period $s = 1, \dots, t-1$ are according to $(a_j^s)_{j\in N}$, we denote by $\Delta_i\left(g, t, T, (a_j^s)_{j\in N}^{s=1,\dots,t-1}\right)$ the remaining time needed from period t so that player i can centralize all the information. At the beginning of the game, the minimum time needed for player i to centralize all the information is equal to her eccentricity, that is $\Delta_i(g, 1, T, \emptyset) = e_i(g)$. In any minimally connected network, a player who is a loose-end node plays no role once she has passed on her information. We can remove such player from the network and look at the reduced network instead. For a minimally connected g, the remaining time needed from period t so that player i can centralize all the information, $\Delta_i(g, t, T, (a_j))$, is equal to the eccentricity of player i on the reduced network $\widehat{g}$ with deadline $T - (t+1)$. It means that $\Delta_i(g, t, T, (a_j)^{s=1,\dots t-1}) = \Delta_i(\widehat{g}, 1, T-(t+1), \emptyset) = e_i(\widehat{g})$, and $\Delta_i(g, 1, T, (a_j)^{s=1,\dots t-1}) = t - 1 + \Delta_i(\widehat{g}, 1, T-(t+1), \emptyset) = t - 1 + e_i(\widehat{g})$.



(a) g (b) g_1 (c) g_2 (d) g_3

Figure 3.5: A line-network g and its reduced networks g_1, g_2, g_3.

In Figure 3.5 we illustrate the notion of remaining time and reduced network, and how we can remove players who are loose-end nodes from the network once they have passed on their information. Starting from network g we obtain that the remaining time so that player 1 can centralize all the information is equal to $\Delta_1(g, 1, 4, \emptyset) = e_1(g) = 2$. Suppose first that $a_5^1 = P$ and $a_4^1 = H$. We can remove player 5 from g and we get the reduced network g_1. Then, $\Delta_1(g, 2, 4, a_5^1 = P, a_4^1 = H) = \Delta_1(g_1, 1, 3, \emptyset) = e_1(g_1) = 2$. Suppose instead that $a_4^1 = a_5^1 = P$, then we can remove both players 4 and 5 from g and we get the reduced network g_2 with $\Delta_1(g, 2, 4, a_4^1 = a_5^1 = P) = \Delta_1(g_2, 1, 3, \emptyset) = e_1(g_2) = 1$. Suppose next that $a_4^1 = a_5^1 = P$ and $a_3^2 = P$. Then, we can first remove both players 4 and 5 and next player 3 from g and we get the reduced network g_3 with $\Delta_1(g, 3, 4, a_4^1 = a_5^1 = P, a_3^2 = P) = \Delta_1(g_3, 1, 2, \emptyset) = e_1(g_3) = 1$.

Definition 3.3 (blocking)**.**
We say player k has the power and incentive to block player j in favor of player i at time t_k^j if there exists $\tau_k^j \in \mathbb{N}_{++}$ such that

(i) $\Delta_j(g, t_k^j, T, (a_l)) + \tau_k^j > T - t_k^j$ and

(ii) for $i \neq k$ we have $\Delta_i(g, t_k^j, T, (a_l)) + \tau_k^j < \Delta_j(g, t_k^j, T, (\widehat{a}_l))$

with $a_k^t = H \;\forall t < t_k^j$ and $\exists t < t_k^j$ such that $\widehat{a}_k^t \neq a_k^t$.

Definition 3.3 tells us that player k blocks player j if this decreases the duration it takes to centralize all the information. The first condition (i) ensures that if player k holds on his information for τ_k^j periods, player j cannot win. The second condition (ii) gives the incentives for player k to block player j in favor of player i. The time it takes for player j to collect all the information once player k holds on his information during τ_b^k periods is less than the time it would take for player j in case that k passes on.

Definition 3.4 (mutual blocking)**.**
Let player k have the power and incentive to block player j and let player j have the power and incentive to block player k. There is mutual blocking if the periods in which player j and player k block each other are identical, i.e. $t_k^j = t_j^k$ and $\tau_k^j = \tau_j^k$.

Figure 3.6: Blocking and mutual blocking.

Figure 3.6 illustrates the notions of blocking and mutual blocking. Suppose that $\delta > \left(\frac{1}{2}\right)^{\frac{1}{2}}$. In the left-hand network (a) player 2 wins after two periods at equilibrium. In the first period player 1 and player 4 pass on their information, while in the second period player 3 and player 5 pass on. The other actions have no impact on the equilibrium outcome. The reason that player 1 passes on in the first period is that, if she would deviate and hold on her information, player 4 would have an incentive to block her. Indeed, player 4 would hold on in the first period and by that he could achieve that player 1 is unable to win and so player 2 wins after three periods. The first period of blocking would increase the duration by one, so player 1 does not deviate and no blocking happens at equilibrium. In the right-hand network (b), player 2 has incentives to block player 1 from winning and player 1 has incentives to block player 2 from winning at equilibrium. This leads to a duration of three periods with player 3 as the winner. In the first period player 1 and player 2 hold on their information and hence prevent that one of them wins. If player 1 or player 2 would win, the duration has to be four periods. After that period of mutual blocking, both player 1 and player 2 pass on their information and player 3 wins after three periods. Both players who block each other have incentives to do so since they will end up with an utility of δ^2, while if they deviate their utility would be only of δ^3. However, if we increase the deadline T by an additional period, player 2 would have to hold on two periods to avoid that player 1 wins. Then, player 2's utility would be the same whether player 1 wins after four periods or whether player 3 wins after $2 + 2$ periods. By assumption he would pass on, making player 1 the winner. This last example emphasizes that increasing the deadline gives even more advantage to the current leader, namely player 1.
The next two propositions characterize the winner and the duration for any value of the discount factor, $\delta \in (0, 1)$.

Proposition 3.5. *Player $i \in N$ wins if and only if*

1. *$\forall j < i$ we have $e_j(g) > e_i(g)$, and either*
   - 1.1. *$\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{e_j(g)-e_i(g)}}$ or*
   - 1.2. *there is some $k \in N$ who has the power and incentive to block j in favor of i,*
2. *$\forall j > i$ either*
   - 2.1. *$e_j(g) \geq e_i(g)$ or*
   - 2.2. *$e_j(g) < e_i(g)$, $\delta > \left(\frac{1}{2}\right)^{\frac{1}{e_i(g)-e_j(g)}}$ and there is no $k \in N$ who has the power and incentive to block i in favor of j.*

Proposition 3.5 tells us that several conditions have to be satisfied for making player i the winner. First, it must take more time for all players who decide before i to centralize all the information (part 1) and these players must either prefer that i wins, because the discount factor is small (part 1.1) or they must be blocked from winning (part 1.2). Second, all players who decide after player i either cannot centralize all the information faster (part 2.1) or can centralize all the information faster but the discount factor is such that i prefers to win and i is not blocked from winning (part 2.2).

Proposition 3.6. *Let player $i \in N$ be the winner. The duration $\tau(g,T)$ is given by $e_i(g)+\tau_{mutual}$ where τ_{mutual} is the number of mutual blocking periods, i.e.*

$$\tau_{mutual} = \left| \bigcup_{x=1}^{|N|} \left(\bigcup_{y=1}^{|N|} \left(\{t_x^y, \ldots, t_x^y + \tau_x^y - 1\} \cap \{t_y^x, \ldots, t_y^x + \tau_y^x - 1\} \right) \right) \right|$$

with t_x^y and τ_x^y as in Definition 3.3.

Proposition 3.6 tells us that the duration is equal to the eccentricity of the winner plus the number of mutual blocking periods, where the number of mutual blocking periods is the union of all mutual blocking periods of player x with other players, which is given by the intersection of blocking periods of player x and player y. In the left-hand network (a) of Figure 3.6, player 1 is blocked from winning, but there is no mutual blocking period, so that player 2 wins after two periods. In the right-hand network (b) there is a mutual blocking period, which stops player 1 and player 2 from winning and leaves player 3 as the winner after three periods.
For small discount factors, Proposition 3.3 already helps us to find the winner easily. For larger discount factors, Proposition 3.5 enables us to get some corollaries that simplify searching for the winner.

Corollary 3.2. *Assume $\delta > \frac{1}{2}$ holds. If player 1's eccentricity is lower or at most $(i-1)$ higher than player i's eccentricity, i.e. $\forall i \neq 1 : e_i(g) + i - 1 \geq e_1(g)$, then player 1 is the unique winner at equilibrium. The duration is equal to the eccentricity of player 1, $\tau(g) = e_1(g)$.*

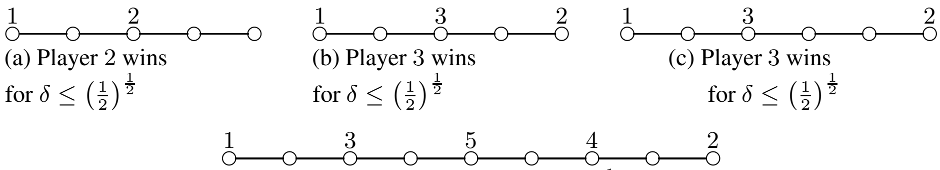
(a) (b)

Figure 3.7: Two networks with player 1 as winner.

When the discount factor is high, the advantage of taking the decision first is great. As long as player 1's eccentricity is just slightly larger than the eccentricities of the other players, player 1 centralizes all the information. For each player ranked further away in the decision order, the difference between the eccentricity of this player and the eccentricity of player 1 can be larger, while player 1 still remains the winner. Figure 3.7 looks at two networks where player 1 is the winner. In both networks, the eccentricity of player 2 is $e_1(g) - 1$ and the eccentricity of player 3 is $e_1(g) - 2$, but it is still player 1 who wins at equilibrium because the difference between the eccentricities is relatively small.

Corollary 3.3. *Assume $\delta > \frac{1}{2}$ holds. Player i wins after e_i periods if*

1. $\forall j < i : e_i(g) < e_j(g) - 1$ *and* $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{e_j(g) - e_i(g)}}$, *and*
2. $\forall j > i$ *either*
   2.1. $e_j(g) \geq e_i(g) - 1$ *or*
   2.2. $e_j(g) < e_i(g) - 1$ *and* $\delta > \left(\frac{1}{2}\right)^{\frac{1}{e_i(g) - e_j(g)}}$.



(a) Player 2 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$
(b) Player 3 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$
(c) Player 3 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$
(d) Player 5 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$

Figure 3.8: Four networks where player 1 is not the winner.

For intermediate values of the discount factor δ, results are similar. If δ is low enough and $e_i(g)+1$ is smaller than the eccentricity of all the players that take their decision before player i and the same does not hold for a player who makes his decision after player i, then player i is the winner who centralizes all the information. In Figure 3.8 we look at four networks where Corollary 3.3 holds. In the network (a) player 1 prefers that player 2 wins since the discount factor is too low. Even though the neighbors of player 2 would prefer to win, they cannot win, because player 2 makes his decision before them. The analysis from networks (b) and (c) is similar. Player 1 and player 2 prefer that player 3 wins, since player 3 can win in two periods while player 1 or player 2 cannot. In network (d), even though player 3 can win faster than player 1 and player 2, there is another player, namely player 5, who can centralize all the information even faster. Indeed, Condition 2.1 holds for player 3 but Condition 2.2 does not hold because of player 5.
Beside characterizing the winner and the duration for any network, our objective is to predict which networks could emerge in the long run. We already know that two players will add a link

to each other if the duration for centralizing all the information becomes shorter. Moreover, only minimally connected networks can be pairwise stable. We now provide some propositions that help us to exclude some networks for being pairwise stable without having to check all possible additional links.

Lemma 3.1. *There is no network in which three or more players want to block each other.*

There are many different network structures where three (or more) players have a long geodesic distance to each other. It cannot occur that those players can block each other from winning. The main reason behind this result is the modeling of the deadline $T = |N| - 1$.

Proposition 3.7. *No network, in which there occurs a mutual blocking period at equilibrium, is pairwise stable.*

Using Proposition 3.7 we get directly that the right-hand network (b) in Figure 3.6 cannot be pairwise stable for $\delta > \left(\frac{1}{2}\right)^{\frac{1}{2}}$. On the other hand, the left-hand network (a) is pairwise stable for all values of δ. There exists no way to decrease the duration from two to one period, because no additional single link can change the radius to one. Even though player 1 would like to form an additional link to become the winner, the other players have no incentive to do so.

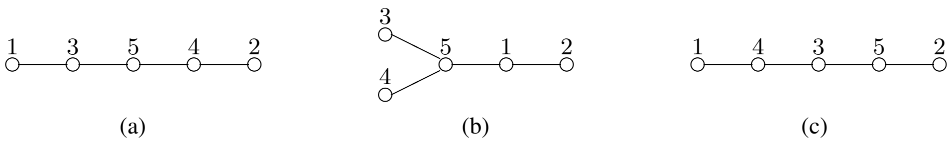


Figure 3.9: Pairwise stable networks depending on the discount factor δ.

Remark 3.1. The set of pairwise stable networks is neither increasing nor decreasing in δ.

The stability of a network depends on the discount factor. Some networks are pairwise stable for all values of δ, some are only pairwise stable for low values of δ, while others are only pairwise stable for high values of δ. The left-hand network (a) of Figure 3.9 is pairwise stable for $\delta \leq \frac{1}{2}$ since no additional link can decrease the duration (and the radius) from two periods to one period. For $\delta > \frac{1}{2}$, player 1 wins after four periods. In that case all players except player 3 have an incentive to link with player 1 and by that decrease the time it takes for player 1 to centralize all the information. Thus, there are networks that are pairwise stable only for low values of δ. In the center network (b), player 1 wins after two periods when $\delta > \frac{1}{2}$. The players have no incentive to add links, because even if player 2 and player 5 add the link 25, player 1 would remains the winner after two periods. In addition, it is not possible for player 1 to win after one period by just adding a single link. For $\delta \leq \frac{1}{2}$, it is player 5 who wins after 2 periods. In that case player 2 and player 5 have incentives to form the link 25 so that the duration decreases by one period. Thus, there are networks that are pairwise stable only for high values of δ. The right-hand network (c) is pairwise stable for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$. In that case player 3 wins after two periods. For $\delta > \left(\frac{1}{2}\right)^{\frac{1}{2}}$, player 1 and player 2 hold on in the first period, but block each other from winning. After that mutual blocking period, player 3 wins, but it takes him three periods to centralize all the information. Player 1 and

player 2 or player 1 and player 5 have an incentive to link since then player 1 would win after two periods. Thus, even for $\delta > \frac{1}{2}$, there are networks which are pairwise stable only for some δ, namely $\delta \in \left(\frac{1}{2}, \left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$.
Even with the results above it is hard to find the set of pairwise stable networks. We know that we can limit our attention to minimally connected networks, but even then there are many different network structures and for each structure there are several ways how to arrange the players. The next proposition give us additional information about which network structures are always or never pairwise stable.

Proposition 3.8.

1. *If $n > 3$ then any star network is pairwise stable;*

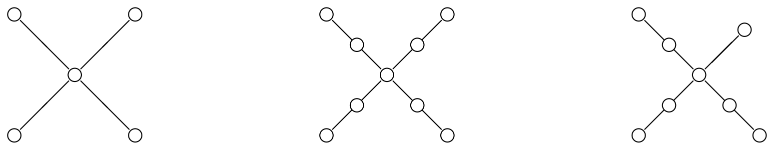
2. *Any symmetric star-like network with at least three arms*[4] *is pairwise stable;*

3. *If $n > 2$ is even then no line network is pairwise stable.*

Proof. Part 1. Take $n > 3$. In any star network, the player who wins is either the center or player 1. If $\delta \leq \frac{1}{2}$, the center is the unique winner since all players want to centralize all the information as fast as possible. There is no incentive for an additional (costly) link since the players already centralize all the information in a single period. If $\delta > \frac{1}{2}$, then player 1 is the single winner. Suppose that player 1 is not the center of the star. Her optimal strategy is to hold on during all periods and forcing the other players to pass on their information to her. The duration is 2 periods. Player 1 cannot add just a single link to decrease her eccentricity from 2 to 1 since there are more than three players, $n > 3$. Thus, player 1 has no incentive to form a single additional link. The other players who are not the center cannot decrease the duration since an additional link would not change their eccentricity and player 1 (who is the first player to take her decision in each period) can stick to her optimal strategy. Using Corollary 3.1 yields the result. Suppose now that player 1 is the center of the star. Then, we can use the same arguments as for $\delta \leq \frac{1}{2}$.
Part 2. The proof is analogue to the one of part 1. The center cannot form any link and the remaining players cannot reduce their eccentricity by adding a link. In any symmetric star-like network with at least three arms, there always exist at least two players for any given player to whom she has her maximal geodesic distance. Thus, a single additional link cannot decrease the duration. From Lemma 3.1 it follows that at equilibrium there is no blocking in any symmetric star-like network with at least three arms.
Part 3. Take $n > 2$ even. For each player i in any line network g there exists exactly one player j with the same eccentricity, $e_i(g) = e_j(g)$. When player i is the winner, it implies that $i < j$ for j such that $e_i(g) = e_j(g)$. Suppose that i and j are such that $e_i(g) = e_j(g)$, $i < j$ and $ij \notin g$. Both i and j have incentives to add the link ij since it does not change the winner but it decreases the duration for centralizing all the information. Suppose now i and j are such that $e_i(g) = e_j(g)$, $i < j$ and $ij \in g$. Then, there exists a single player k for the winner w to whom she has her

[4] An arm of a star-like network is simply a line network.

maximal geodesic distance. Both w and k want to form a link since it decreases the duration, and moreover, this new link does not change who is the winner. □



(a) symmetric (b) symmetric (c) asymmetric

Figure 3.10: Star-like networks.

In any star network where one central player is connected to all other players, only the players who are not the center of the star can form a link, but no link can decrease the duration for centralizing all the information. Thus, the star network is pairwise stable. A similar argument holds for symmetric star-like networks with radius $r \geq 2$. In a star-like network the length of the arms can be longer. A star-like network is symmetric if all arms of the star have the same length. Figure 3.10 provides some examples of symmetric and asymmetric star-like networks. While stars and symmetric star-like networks are pairwise stable, most line networks are not pairwise stable. The reason is that, if the number of players is even ($n > 2$), there always exists an additional link that decreases the duration. Thus, a line network can only be pairwise stable if the number of players is two or odd. But, if the discount factor is low ($\delta \leq \frac{1}{2}$), using Proposition 3.4, we conclude that all line networks with more than five players are not pairwise stable. For $n > 2$ odd and $\delta > \frac{1}{2}$, line networks are pairwise stable since the unique centered player cannot decrease her eccentricity by forming a single link.

Many minimally connected network structures can be pairwise stable. One could wonder whether some of them are more likely to emerge in the long run than others. If we start from some randomly selected connected network and we assign the same probability to all links the players want to delete or to form, we will follow improving paths leading towards some pairwise stable network. For instance, start from a connected network that is not pairwise stable and suppose that there are two links that players want to cut and three links that the players want to form. We select each of these five changes with equal probability, but depending on which link gets selected the improving paths may lead to completely different pairwise stable networks. The probability for ending up in some pairwise stable network varies widely and this probability does not only depend on the network structure but also on the position of the players within the network. Take $\delta > \frac{1}{2}$ and $n = 5$. The star network with player 1 as the center arises with a probability of 27.9%, while the star networks with a different center have a probability of approximately 4.3% each. In addition, the probability for ending up in any other pairwise stable network lies between 0.8% and 2.5%. When $\delta \leq \frac{1}{2}$, only the network structure influences the probability.

As already mentioned earlier, the competition for the reward can be interpreted as a competition for the team leadership when the task of the team is to centralize all the information. The decision order represents the current hierarchical ranking within the team with player 1 being the current leader. One can argue that the fight for the leadership not only happens once, but gets repeated over and over again. Once a player has centralized all the information, she becomes the new leader of the team and moves to the first spot in the ranking, and she heads the team to execute a new task of centralization. Then, she has a better position in the decision order, but the same position in the network. From Proposition 3.5 it follows directly that this player will stay the winner as long as the network structure does not change. The pairwise stable network of the previous round is still pairwise stable under the new decision order. For instance, if the star network with the current leader as the center emerges, the current leader is the one who centralizes the information. She will keep the leadership and in the next rounds this star network with her as the center will remain pairwise stable. However, in some situations, the players may start from a different network in the next round. For instance, if one player leaves the team and is replaced by another player, the new player may not have the same links as the old one. Then, another pairwise stable network can emerge and this might change the leader.

3.5 Robustness

What happens if players only need to collect at least $n-1$ items?
In some situations it may be reasonable to assume that the players do not need to centralize all information items. For instance, it is often the case that if we collect $n-1$ pieces of the puzzle we are able to guess the missing piece. We now show that most of our results are robust to such change.
Assume that players whose information get centralized obtain the same utility as before: 1. The player who first centralizes at least $n-1$ items gets an additional reward $R=1$. If a single winner centralizes more than $n-1$ items, she also obtains $1+R$. It remains to point out two important features. First, a player whose information is not centralized by some winner gets 0 since he does not contribute to the team task. Second, if there are two (or more) players who first centralize at least $n-1$ items, they share the reward equally, and all players whose information is centralized receive a payoff of 1. For instance, consider a team consisting of five players and let the reward R be equal to 1. If after two periods player 1 has collected the items $1, 2, 3, 4$ and player 2 has collected the items $2, 3, 4, 5$, player 1 and player 2 obtain an utility of $\delta \cdot 3/2$, while player 3, player 4 and player 5 obtain an utility of δ.

Proposition 3.9. *Only minimally connected networks g such that $|N(g)| = n-1$ can be pairwise stable.*

Proposition 3.9 tells us that, if players need only to centralize at least $n-1$ items, then all pairwise stable networks consist of one isolated player and one minimally connected network connecting the remaining $n-1$ players. Obviously, for such network g connecting $n-1$ players, the results about the winner and the duration from the previous sections hold. However, even though the network g could be pairwise stable if we would restrict the analysis to $n-1$ players, the network g may not be pairwise stable since some players may have incentives to link with the isolated player. On the other hand, if the network g is not pairwise stable when we restrict the analysis to $n-1$ players, then the network g is not pairwise stable in general.

(a) non-stable (b) stable (c) non-stable (d) non-stable

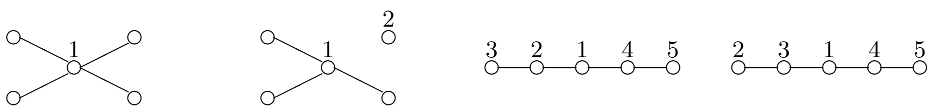
Figure 3.11: Centralizing $n-1$ items with $\delta > \left(\frac{1}{2}\right)^{\frac{1}{2}}$.

In Figure 3.11 we look at the stability of networks when only at least $n-1$ items need to be centralized. Proposition 3.9 is not satisfied in network (a). The center has no incentive to be linked to all other players, because she only needs the information of $n-2$ other players, so she cuts a link and we end up in (b). In network (b), one player is isolated (or disconnected) from all other players, while they form a star. In both (a) and (b) player 1 is the only winner and the duration is one period. The network (c) illustrates the case in which a star would be pairwise stable if the team would be composed of only four players, but it not pairwise stable once the team consists of five players. Player 1 wins after two periods but player 2 has an incentive to link with the isolated player, because he then can centralize the information in a one period. In network (d) the line network is even not pairwise stable if the team would be composed of only four players. Player 1 is the winner after three periods. The duration can be decreased with an additional link between player 1 and player 3 or between player 1 and player 5 or between player 4 and player 5. Player 1 has an incentive to link with player 2, because it would decrease the duration as well. Moreover, player 3 wants to form a link with player 2, so that he can centralize all information within one period.

What happens if two players have the same information?

In other situations players may have the same information in the beginning. While in some cases this setting is similar to the one previously described, there are other cases in which this changed setting yields to new results. Assume that two players have the same information and that the players only need this information once for a successful project. Furthermore, assume that all players connected in the network get a payoff of 1 if the information is centralized by another player.

Then, the pairwise stable networks consist either of one minimally connected network of n players or of one isolated player and one minimally connected network connecting the remaining $n-1$ players. In the second case the isolated player is one of the players with a copy.



(a) non-stable (b) stable (c) stable (d) non-stable

Figure 3.12: Examples with $\delta > \left(\frac{1}{2}\right)^{\frac{1}{2}}$, where players 1 and 2 have the same items.

Figure 3.12 shows some examples of stable and non-stable networks under the assumption that players 1 and 2 have the same information. In network (a) player 1 would cut the link to player 2, because she already has the same information as player 2. Therefore, she has no incentive to keep the link with player 2. In general, no star network of n players is pairwise stable. If two players

on the periphery have the same information the center cuts the link with one of them. On the other hand, if the center shares the information with another player, as in network (a), the center cuts the link to that player. Removing the link between player 1 and player 2 yields to the network displayed in example (b), which is pairwise stable. Player 1 still centralizes the information of the other players within one period.
In this setting, networks that consist of one minimally connected component of all players can be pairwise stable as well. One example is shown in network (c). Even though player 1 and player 2 have the same information, they have to keep their connection, because otherwise no player will be able to centralize all information. In this example player 1 centralizes the information after two periods. For that she does not need the information of player 2, but she needs player 2 to pass on the information from player 3 to her. One additional link cannot decrease the duration and so the network is pairwise stable. This is not the case in example (d). Player 2 is a loose end and therefore he does not spread the information of other players. So, player 3 has an incentive to cut the link with player 2.

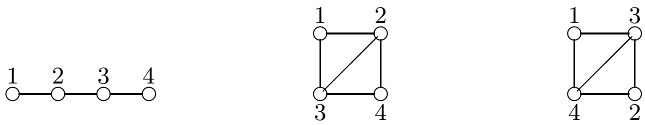
What happens if one player is more patient or impatient than the others?
In all previous results we rely on the assumption that the players have the same discount factor. Especially in the situation when firms cooperate with each other and work together on a joint project this might not hold. As soon as one player has a different discount factor we cannot make statements like Proposition 3.3 or Proposition 3.4. Still those results hold if $\delta_i \leq \frac{1}{2}$ holds for all players $i \in N$, where δ_i denotes the discount factor of player i. Furthermore, we can generalize Proposition 3.5:

Proposition 3.5*. *Player $i \in N$ wins if and only if*

1. *$\forall j < i$ we have $e_j(g) > e_i(g)$, and either*
   - 1.1. *$\delta_j \leq \left(\frac{1}{2}\right)^{\frac{1}{e_j(g)-e_i(g)}}$ or*
   - 1.2. *there is some $k \in N$ who has the power and incentive to block j in favor of i,*
2. *$\forall j > i$ either*
   - 2.1. *$e_j(g) \geq e_i(g)$ or*
   - 2.2. *$e_j(g) < e_i(g)$ and either*
     - 2.2.1 *$\delta_i > \left(\frac{1}{2}\right)^{\frac{1}{e_i(g)-e_j(g)}}$ and there is no $k \in N$ who has the power and incentive to block i in favor of j or*
     - 2.2.2 *$\delta_i \leq \left(\frac{1}{2}\right)^{\frac{1}{e_i(g)-e_j(g)}}$, but $\exists k \in \{i+1, \ldots j-1\}$ such that $e_k(g) > e_j(g)$, $\delta_k > \left(\frac{1}{2}\right)^{\frac{1}{e_k(g)-e_j(g)}}$ and there is no $\ell \in N$ who has the power and incentive to block k in favor of j.*

As described in Section 3.4.2 Condition 1.1. ensures that player $j < i$ does not want to win. Of course, this condition depends on the discount factor of player j. On the other hand we need to

modify Condition 2.2. If player j can centralize the information faster than player i (Condition 2.2), then either player i wants to win herself because her discount factor δ_i is large enough (2.2.1) or player i wants player j to win, but there exists another player k who decides in between those players that centralizes the information slower than j (i.e. $e_k(g) > e_j(g)$) and this player does not want player j to win (2.2.2).
With this changed proposition we can find the winner even if different players have different discount factors.



(a) Winner: 2
Duration: 2

(b) Winner: 2
Duration: 1

(c) Winner: 1
Duration: 2

Figure 3.13: Examples with $\delta_1 \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$ and $\delta_{2,3,4} > \left(\frac{1}{2}\right)^{\frac{1}{2}}$

In Figure 3.13 we look at the winner and the duration of networks when the discount factor of player 1 is lower than that of the other players. In network (a) player 2 wins after 2 rounds. If all players had the same discount factor $\delta > (\frac{1}{2})^{\frac{1}{2}}$, player 1 would win after 3 rounds, but because of her low discount factor player 1 prefers to pass on her information in the first round. The network displayed in example (b) has player 2 as the winner and a duration of 1 period. If $\delta_1 > (\frac{1}{2})^{\frac{1}{2}}$ holds, player 1 wins after 2 rounds. As in example (a), player 1 prefers to let player 2 win, because he has a lower eccentricity. This corresponds to the new Condition 1.1 of Proposition 3.5*. On the other hand, in example (c) we see a network that has the same winner and the same duration independent of the discount factor of player 1. If player 1 would pass on her information item in the first round player 2 becomes the winner after 2 rounds and so she prefers to hold on which makes her the winner after 2 rounds. In this example the new Condition 2.2.2 is crucial, because only player 1 prefers that player 3 wins after a single period, but this does not hold for player 2 because he has a different discount factor.

3.6 Conclusion

We have proposed a model of information centralization in teams where players can only exchange information through an endogenous communication network. Players face a trade-off between cooperating for centralizing all the information fast and competing for being the one who collects all the information and becomes the team leader. Over several periods each player can either pass or not pass her information to her neighbors. Once one player has centralized all the information, all players receive some payoff. The winner who collects all the information gets an additional reward. Since each player discounts payoffs over time, she faces the dilemma of either letting another player centralizing all the information fast, or trying to collect all the information by herself and overtaking the leadership.
We have found that there is always a single winner who centralizes the information at equilibrium and that only minimally connected networks can be pairwise stable. We have characterized the winner and the duration for any network and for any discount factor, and we have predicted which

networks emerge in the long run. Whether some player becomes the winner or not depends on her position in the network, her rank in the decision order and the discount factor. If the discount factor is low, the player with the lowest eccentricity wins. On the other hand, if the discount factor is high, a player first ranked in the decision order can outweigh a player with a low eccentricity. In addition, at equilibrium some players might hold on their information just to ensure that no player with an excessive eccentricity wins. Only minimally connected networks can be pairwise stable. It follows that additional channels of communications between two players either do not benefit both players or lead to a breakdown of other links afterwards. The information only flows through a single path between the players. Again, whether some minimally connected network is pairwise stable or not depends on the decision order and the discount factor. For instance, switching two players in a pairwise stable network may destabilize the network, because of the crucial role played by the decision order or current leadership ranking.

Finally, we have shown that our results are robust. For instance, if players only need to collect at least $n-1$ items, then all pairwise stable networks consist of 1 isolated player and one minimally connected network connecting the remaining $n-1$ players. Similarly, most our results can be modified easily if several players have the same information or if players have different discount factors.

Appendix of Chapter 3

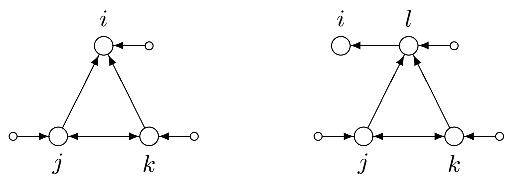
Proof. Proposition 3.2

From Proposition 3.1 we know that there is a single winner at equilibrium. Hence, two players (neighbors or not) will never share the reward at equilibrium. We now show that networks which are not minimally connected networks cannot be pairwise stable.

Suppose that g is not minimally connected and player i is the single winner. Hence, there is a cycle in g and player i receives all information items from the other players, but she just needs to receive them once.

1. Player i is part of the cycle. Suppose that player j and player k are also part of the cycle. The equilibrium outcome is independent of the number of items that player j or k has. The information flows either from j over k to i, or from k over j to i, or from j to i and from k to i. In the first (second) case, the links between j (k) and i are useless and will be deleted to save costs. In the third case, player j and player k have no incentive to be linked, except than being connected through player i. If the information reaches player i from two different paths, the slower path has no purpose and can be deleted.

2. Player i is not part of the cycle. Suppose that player j, player k and player l are part of the cycle. At equilibrium the winner i just needs to receive all the information once. Since player i is not part of the cycle, she can receive the information from players j, k and l only through one of them. If (without loss of generality) player j collects the items of players k and l, and then passes on the information items to player i, at least one link between them can be deleted to save costs.



(a) Winner i is part of the cycle

(b) Winner i is not part of the cycle

Figure 3.14: Only minimally connected networks can be pairwise stable

Figure 3.14 shows the simplified cycles without the players in between j, k and i (respectively l). In both the left-hand and right-hand networks, the link between j and k does not decrease the duration. It means that the duration $\tau(g - jk)$ is equal to the duration $\tau(g)$ and so player j and player k have incentives to cut the link since links are costly. □

Proof. Proposition 3.5

Suppose that player i is the winner. Then: (i) all players j who decide before i (i.e. $j < i$) must need more time than player i for centralizing all the information and either want that player i wins or they are blocked from winning; all players j who decide after i (i.e. $j > i$) either must need more time than player i to centralize all the information or if they can centralize all the information faster than player i, they want i to win and nobody can block her from winning. □

Proof. Proposition 3.6
We show by contradiction that the duration can be neither larger nor smaller than the sum of the eccentricity of the winner and the number of mutual blocking periods.

$\not>$ Given that the strategies of the players are optimal, the duration cannot be longer.

$\not<$ Let τ_k be the number of mutual blocking periods in total, starting from period t_k. The same argument holds if several mutual blocking periods take place not successively. By the definition of mutual blocking, we get

$$\forall i \in N : \Delta_i(g, t_k, T, (a_l)) = \Delta_i(g, t_k + \tau_k, T, (a_l)).$$

This implies that the duration cannot be lower than the eccentricity of the winner plus the number of mutual blocking periods.

□

Proof. Lemma 3.1
We show that three players have no incentive to mutually block each other. Without loss of generality, take player 1, player 2 and player 3. We first show that in a star-like network g with three arms, where player 1, player 2 and player 3 are the loose-end nodes (i.e. the node at the end of each arm), they will not mutually block each other. A star-like network is one type of network architecture, but if the players do not mutually block each other in such type of network, they still do not block once we add additional links to the network.
Suppose that player 1, player 2 and player 3 block each other and that they all have the same eccentricity $e_1(g) = e_2(g) = e_3(g)$. Let β be the number of blocking periods and let player c be the center of the star-like network g. It follows that the duration $\tau(g)$ is equal to the sum of $e_c(g)$ and β. In addition, $\beta + e_1(g) = T + 1 = |N|$ has to hold since the number of blocking periods has to be large enough to prevent player 1, player 2 and player 3 from winning. Replacing β by $|N| - e_1(g)$ in the expression for the duration yields: $\tau(g) = e_c(g) + |N| - e_1(g)$. Players only block each other if the duration including blocking is lower than their eccentricity: $\tau(g) < e_1(g)$. Merging both equations gives us $|N| + e_c(g) < 2 \cdot e_1(g)$. In a symmetric star-like network, the eccentricity of the loose-end nodes is twice the eccentricity of the center, and so we get $|N| + e_c(g) < 4 \cdot e_c(g)$ which reverts to $|N| < 3 \cdot e_c(g)$. Since the eccentricity of the center of a symmetric star-like network with x arms is $e_c(g) = (|N| - 1)/x$, we get $|N| < (3/x) \cdot (|N| - 1)$. This inequation never holds if there are at least three arms, $x \geq 3$. Thus, in a symmetric star-like network with at least three arms, the loose-end nodes will never block each other. One can consider all other networks as symmetric star-like networks with additional players and links and then the same arguments hold. □

Proof. Proposition 3.7
From Lemma 3.1 we know that only two players j, k can block each other. Without loss of generality, assume $j < k$. The shortest number of blocking periods is one, which happens only on the line network. The loose-end nodes block each other for one period and hence they achieve that a player located in the middle of the network wins. In case of a line the eccentricity of player j and player k is $e_j(g) = e_k(g) = |N| - 1$ and the eccentricity of the winner is $e_w(g) = |N|/2$ if $|N|$ is even or $e_w(g) = (|N| - 1)/2$ if $|N|$ is odd. The time it takes the winner to centralize all the information is equal to the sum of her eccentricity plus the number of blocking periods, $e_w(g) + 1$. It follows that the network cannot be pairwise stable, because player j and player k would prefer to add the link jk. If g is a line, we get $e_j(g + jk) = e_k(g + jk) = |N|/2$ if $|N|$ is even or $e_j(g+jk) = e_k(g+jk) = (|N|-1)/2$ if $|N|$ is odd. Then player j is the winner after $e_j(g+jk)$ periods which is strictly less than $e_w(g) + 1$. All other networks can be considered as adding players and links to the line network. For each player we add to the line, player j and player k have to block each other for an additional period. Let g' be the network where we have added x players to the line with $|N'|$ players, and so we have a tree with $|N'| + x$ players in total. The smallest eccentricity is $e_w(g') = |N'|/2$ if $|N'|$ is even and $(|N'| - 1)/2$ if $|N'|$ is odd. Therefore, we get that the duration of g' is $\tau(g') = e_w(g') + 1 + x$. Again player j and player k will link. The furthest the additional players can be away is if they are all linked to the center of the line. By this we get $e_j(g' + jk) \leq |N'|/2 + x$ if $|N'|$ is even and $e_j(g' + jk) \leq (|N'| - 1)/2 + x$ if $|N'|$ is odd. It follows that $e_j(g' + jk) \leq e_w(g') + 1 + x$ and so we have shown that the network g' cannot be pairwise stable. Notice that these arguments hold since it is assumed that in case of indifference the players pass on their information. □

Proof. Proposition 3.9
Notice that any connected network that is not minimally connected cannot be pairwise stable following the same arguments as in the proof of Proposition 3.2. We have to show that only minimally connected networks g such that $|N(g)| = n - 1$ can be pairwise stable. We first show that a minimally connected network g such that $|N(g)| = n$ cannot be pairwise stable. There always exists at least one player k who has the highest geodesic distance to the winner. Obviously, player k is a loose-end node. Let player j be the player who is linked to k ($jk \in g$). Player j has an incentive to cut the link jk, because even without that link the winner can centralize the information of $n - 1$ players in the same time, and player j saves the infinitesimally small cost for having link jk. Therefore, any minimally connected network g with $|N(g)| = n$ is never pairwise stable. □

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