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T. Hellmann and F. Panebianco

Center for Mathematical Economics (IMW)
Bielefeld University
Universitätsstraße 25
D-33615 Bielefeld · Germany

e-mail: imw@uni-bielefeld.de
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The Transmission of Continuous Cultural Traits in Endogenous Social Networks

T. Hellmann[b,*], F. Panebianco[a]

[a] Department of Economics and Finance, Cattolica University, Milan
[b] Center for Mathematical Economics (IMW), Bielefeld University, P.O. Box 100131, D-33501 Bielefeld

Abstract

We study a model of transmission of continuous cultural traits across generations where children learn their cultural trait from their parents and their social environment modeled by a network. Parents can engage in the socialization process of their children by biasing links in the network in order for their children to adopt a cultural trait similar to their own. In this endogenous network, we study the emergence of positive and negative role models, the existence of a steady state cultural trait, its characterization in terms long-run influence of each dynasty, and the speed of convergence.

Keywords: Cultural transmission; Social networks; Persistence of cultural traits; Network Formation
JEL-Classification: A14; C72; D83; D85; Z13

1. Introduction

Culture plays a major role for economic outcomes (see, e.g. Guiso et al., 2008; Tabellini, 2010). How cultural traits form and evolve is hence of central interest to economists. There is large empirical evidence that cultural traits are shaped by parents' cultural traits and the cultural traits in the social environment.[1] Beyond that, social networks also play an important role for the evolution of cultural traits.[2]

The seminal paper of Bisin and Verdier (2001) analyzes the purposeful intergenerational transmission of culture and provides a theoretical explanation of the empirically observed phenomenon of persistent cultural traits.[3] Only recently some research, e.g. Buechel et al.

*Corresponding author
Email addresses: tim.hellmann@uni-bielefeld.de (T. Hellmann), fabrizio.panebianco@unicatt.it (F. Panebianco)
URL: https://sites.google.com/site/thellmannimw/ (T. Hellmann)

[1] For instance, Dohmen et al. (2012) show that the degree of risk aversion and trust can be explained by risk preferences and trust attitudes of parents and the social environment.

[2] It is documented that cultural traits often differ across geographic regions, e.g. cities (Guiso et al., 2008; Voigtländer and Voth, 2012), or countries (Algan and Cahuc, 2010) and that these differences are persistent.

[3] For instance, Guiso et al. (2008) report persistence of trust levels in Italian cities, Nunn and Wantchekon (2011) show that (mis-)trust attitudes in African families are prevailing throughout many generations, and Voigtländer and Voth (2012) find that attitudes towards the Jewish population persisted over many centuries in German cities.

(2014), Panebianco (2014), and Prummer and Siedlarek (2017), extend this approach to analyze the role of social network structure for the transmission of continuous traits, while Panebianco and Verdier (2017) study random networks and discrete traits transmission. These models, however, consider the network as exogenously given. Bisin and Verdier (2010) note that "the cultural composition of society is at least partly under the control of parents: they in fact choose schools, neighborhood, peers, and so on." [p.352] and therefore discuss in favor of a research program that studies the role of endogenous networks. This issue is also acknowledged in Buechel et al. (2014) who note that "a time-varying interaction structure [shall be introduced] to allow parents to control the whole network of their children." [p.294]

In this paper, we introduce an endogenous network for the transmission of continuous cultural traits. While we assume that dynasties are located on an exogenous social network, we allow parents of each generation to bias positively or negatively each link of their children's network, identifying positive or negative role models for their children, and reacting by increasing or decreasing the weights of the initial network. We assume that parents exert costly efforts to change their children's network because they feel imperfect empathy towards their children expressing a desire to have their children's cultural trait as close as possible to their own cultural trait (see also Bisin and Verdier, 2001; Buechel et al., 2014, for analogous preference). We study the consequences of this differential bias on the endogenous network and on the long run effects in terms of cultural traits.

2. The Model

We employ the model by Buechel et al. (2014) and Panebianco (2014) for the transmission of continuous cultural traits on social networks. Consider an overlapping generations society populated by the adults of a finite set of dynasties $N = \{1, \ldots, n\}$. At the beginning of any period $t \in \mathbb{N}$, each adult has one offspring. Adults of period $t \in \mathbb{N}$ are characterized by a *cultural trait* $V_i(t) \in I$. As we consider continuous traits, $I \subseteq \mathbb{R}$ is assumed to be compact and convex. Following empirical evidence, children learn their cultural trait from their parents and their social environment which is determined by a social network which stays fixed over time, for instance a city. The network is represented by a $n \times n$ row stochastic matrix $\mathbf{G} = (g_{ij})_{ij \in N}$ which is primitive.[4] The weights g_{ij} may represent time-independent relative influences between dynasties, given for instance by the inverse of geographic distance.

The key novel contribution of this paper is that we allow parents $i \in N$ to control the relative influences of their children's social environment given by the i-th row $\mathbf{g}_i$ of $\mathbf{G}$ with network biases $\mathbf{d}_i(t) \in \mathbb{R}^n$. The influence of each family $j \in N$ is then altered from g_{ij} to $g_{ij}(t) := g_{ij} - d_{ij}(t)$. Since these are relative influences, we impose that $\sum_{j \in N} d_{ij}(t) = 0$ to retain $\sum_{j \in N} g_{ij}(t) = 1 \ \forall i \in N$. The resulting network of relative influences is then denoted by $\mathbf{G}(t)$ such that the cultural trait of a child of family $i \in N$ is given by $\sum_{j \in N} g_{ij}(t) V_j(t)$.

[4]The assumption is satisfied if the network is aperiodic and strongly connected. Note that the same assumption is also made in Buechel et al. (2014).

At the end of any period $t \in \mathbb{N}$, the adults die, the children become adults in period $t+1$, and carry over the adopted trait into their adult period. The dynamics of cultural traits is then defined by

$$\mathbf{V}(t+1) = \mathbf{G}(t)\mathbf{V}(t) \tag{1}$$

We assume imperfect empathy: parents want their children to adopt the same cultural trait as they carry themselves (see for a motivation e.g. Bisin and Verdier, 2010). Moreover, parental socialization is costly, resulting in the following functional form of utility,

$$U_i(t) = -[V_i(t+1) - V_i(t)]^2 - \tfrac{1}{2}c_i(t) \sum_{j \in N} [d_{ij}(t)]^2 \tag{2}$$

where $c_i(t) > 0$ for all $i \in N$, $t \in \mathbb{N}$. The first term of the RHS of (2) is the intergenerational utility component which is decreasing in the distance between parent's and child's trait and therefore reflects imperfect empathy. The second term is composed of the cost which depends on the euclidean distance between the altered influence vector $\mathbf{g}_i(t)$ to the original influence vector $\mathbf{g}_i$ weighted by the generation and dynasty dependent cost factor $c_i(t)/2$. While the intergenerational utility component follows along the lines of Buechel et al. (2014) and Panebianco (2014), the cost component simply reflects the idea that cost is increasing in the magnitude of the network change.

2.1. Optimal Network Choices and Role Models

In each period $t \in N$, the adults choose optimal biases determined by

$$\underset{\mathbf{d}_i \in \mathbb{R}^n : \sum_{j \in N} d_{ij} = 0}{\arg\max} \; -(V_i(t+1) - V_i(t))^2 - \tfrac{1}{2}c_i(t)||\mathbf{d}_i||^2 \quad \text{s.t. } V_i(t+1) = (\mathbf{g}_i' - \mathbf{d}_i')\mathbf{V}(t) \tag{3}$$

where $||\cdot||$ denotes the euclidean norm, and $\mathbf{x}'$ denotes the transpose of a vector $\mathbf{x}$. Optimal choices are characterized by Proposition 1.

Proposition 1. *For each generation $t \in \mathbb{N}$ and for each dynasty $i \in N$, the entries of the optimal vector of network biases as a solution to* (3) *are given by*

$$d^*_{ij}(t) = \frac{[V_i(t) - V_j(t)]\,[V_i(t) - \sum_{k \in N} g_{ik} V_k(t)]}{c_i(t) + \sum_{k \neq i} [V_i(t) - V_k(t)]^2}, \;\; \forall j \in N. \tag{4}$$

Defining by $\bar{V}_i(t+1) := \sum_{k \in N} g_{ik} V_k(t)$ the unbiased trait obtained if the adult of generation t in dynasty i does not engage in the socialization process of its child, we receive:

$$sgn(d^*_{ij}(t)) = sgn[V_i(t) - V_j(t)][V_i(t) - \bar{V}_i(t+1)] \tag{5}$$

$sgn(d^*_{ij}(t))$ determines whether dynasty j is a positive or negative role model for dynasty i in period $t \in \mathbb{N}$. Indeed, a positive role model has her final share increased with respect to g_{ij}, so that $d^*_{ij}(t) < 0$. A negative role model has $d^*_{ij}(t) > 0$. Since it is generically true that $\bar{V}_i(t+1) \neq V_i(t)$, parents exert effort to have $|V_i(t+1) - V_i(t)| < |\bar{V}_i(t+1) - V_i(t)|$. If $V_j(t)$ and $\bar{V}_i(t+1)$ are both smaller or both larger than $V_i(t)$, then dynasty j contributes to influencing child i to move away from its parent's trait $V_i(t)$. Consequently parent i chooses to reduce the influence of dynasty j i.e. $d^*_{ij} > 0$. The reduction of influences could even lead to negative final influence weights $g_{ij}(t)$ such that j becomes an anti-role model with the child adopting the opposite of those traits. If on the other hand $V_k(t)$ is smaller (larger) than $V_i(t)$ and $\bar{V}_i(t+1)$ is larger (smaller) than $V_i(t)$ then k has a positive influence on the child's trait as this influence prevents the child's trait $V_i(t+1)$ to depart even further from the parent's trait $V_i(t)$. In this case i will encourage the child to be more influenced by this positive role model k, i.e. $d^*_{ij} > 0$. For the very same reason, it is immediate to see that if i has the smallest or the largest cultural trait, then she only has negative role models. This is the only case where agents behave as in the Bisin and Verdier framework, as only the own influence is increased. At last, notice that being a positive or negative role model is not necessarily reciprocal, as we can have $d^*_{ij}(t) > 0$ and $d^*_{ji}(t) < 0$.

3. The Dynamics of Cultural Traits

Given these optimal biases, defining $\tilde{c}_i(t) := c_i(t)\frac{1}{\sum_{j\neq i}(V_i^t - V_j^t)^2}$ for all $i \in N$ and plugging (4) into (1), we get

$$V_i(t+1) = \frac{\tilde{c}_i(t)}{\tilde{c}_i(t)+1}\sum_{k\in N} g_{ik}V_k(t) - (1 - \frac{\tilde{c}_i(t)}{\tilde{c}_i(t)+1})V_i(t). \tag{6}$$

The cultural traits of the next generation are a convex combination of the projected cultural trait $\bar{V}_i(t+1)$ and the current cultural trait $V_i(t)$, with weights given by the cost term $c_i(t)$ and the average cultural distance to other dynasties $\sum_{j\neq i}(V_i^t - V_j^t)^2$. In the long-run we then get convergence to a single cultural trait.

Proposition 2. *Let $c_i(t) > 0$ for all $i \in N$, $t \in \mathbb{N}$. Then, $\lim_{t\to\infty} V(t)$ exists and is such that all dynasties' traits coincide, $\lim_{t\to\infty} V_i(t) = \lim_{t\to\infty} V_j(t)$ for all $i, j \in N$*

Although parents bias the children's network in each period to achieve a cultural trait close to their own, it is unavoidable in the long-run that a melting pot society emerges where all dynasties share the same cultural trait.

As the dynamical system is time inhomogeneous since $c_i(t) \in \mathbb{R}_+$ may vary arbitrarily with time, we will use the following assumption on costs in order to obtain further results.

Assumption 1. *For all $i \in N$, $c_i(t) = c_i\sum_{j\neq i}(V_i(t) - V_j(t))^2$ for all $t \in \mathbb{N}$ such that $c_i > 0$.*

By Assumption 1, each dynasty $i \in N$ experiences costs of changing the network given by a dynasty specific base cost term $c_i > 0$ weighted with the mean cultural distance to other dynasties $\sum_{j\neq i}(V_i(t) - V_j(t))^2$. In other words, being able to change the network is more difficult for outsiders, i.e. families who are far away from the mean cultural trait, as school choice and other network changes require more effort.[5] With Assumption 1, we are able to characterize the long run outcome.

Proposition 3. *Suppose Assumption 1 holds. Then, the long-run steady state trait of all dynasties $i \in N$ is given by $\lim_{t\to\infty} V_i(t) = \mathbf{x}'\mathbf{V}(0)$ such that $\mathbf{x} = \frac{1}{\sum_{j=1}^{n} \frac{c_j+1}{c_j} y_j} \mathbf{C}^{-1}\mathbf{y}$ where $\mathbf{C} = diag(c_i/c_i + 1)$ and $\mathbf{y}$ is the unique left unit eigenvector of $\mathbf{G}$ satisfying $\sum_{j\in N} y_j = 1$.*

By Proposition 3, long-run influence $\mathbf{x}$ only depends on cost term $\mathbf{c} = (c_1, \ldots, c_n)$ and the normalized left unit eigenvector $\mathbf{y}$ of $\mathbf{G}$. Long-run influence of dynasty $i \in N$ is hence decreasing in c_i and increasing in y_i. Small base costs c_i allow each generation of a given dynasty to choose larger biases implying that cultural traits stay close to the parents' traits leading to a long-run persistence. A large entry in the normalized left unit eigenvector means that a lot of other dynasties are influenced (directly or indirectly) in the underlying network. I.e., when a dynasty is very central in the underlying network, then although this network is altered in each period, the cultural trait propagates through the network more easily.

Propositions 2 and 3 may seem to contradict results of the classical literature on dynamics of cultural traits where long-run heterogeneity of cultural traits is predicted when there are only a finite number of possible traits (one prominent example is Bisin and Verdier, 2001, while for a literature survey the reader may be referred to Bisin and Verdier, 2010). Instead, in continuous cultural traits models, long-run convergence to a single melting pot trait is frequently observed.[6] The empirical phenomenon of long-run persistence is explained in these models by low convergence rates (see Buechel et al., 2014). Similarly, in our model, biasing children's interactions to achieve a next period's cultural trait close to parents' cultural traits slows down convergence which is shown via comparison of the second eigenvalue of the law of motions associated with different cost terms.

Proposition 4. *Suppose Assumption 1 holds and $\mathbf{G}$ is symmetric. Then, speed of convergence of the dynamics $\mathbf{V}(t)$ is increasing in the cost vector $\mathbf{c}$.*

Note that if $c_i \to \infty$ for all $i \in N$, then no changes to the network are made and convergence is fasted. Thus, allowing parents to change their childrens' network slows down convergence. What is more, for lower costs convergence becomes slower and slower. So while we get long-run convergence to a melting pot society by Proposition 2, we may explain the empirically observed phenomenon of persistence of cultural traits by very slow convergence.

4. Conclusion

This paper is the first attempt to build a cultural transmission framework in which the socialization network is endogenous and shaped by parents. While standard models just

[5]Note that the assumption entails that when the cultural traits of all dynasties are aligned such that $\sum_{j\neq i}(V_i(t) - V_j(t))^2 = 0$, then there are no costs of changing the network. This however, only happens in the long-run, if for the initial traits $\mathbf{V}(0) \neq a\mathbf{1}$ for all $a \in \mathbb{R}$. In the latter case, the traits of all dynasties equal initially such that there is also no incentive for any dynasty to change the network.

[6]One notable exception is the recent work by Cheung and Wu (2018) who employ a continuous imitative dynamic to achieve long run heterogeneity.

allow parents to bias in favor of own type, leaving unchanged the relative weights allocated to the other groups, we allow parents to direct socialization to different targets. This enables us to micro-found the emergence of positive and negative role models as functions of original network structure and distribution of cultural traits in the population. In this way an endogenous network of oblique socialization is built.

Studying the resulting dynamics of cultural traits, we have shown that, given any initial network and cultural traits distribution, a steady state is always reached. Under some simplifying assumption on the socialization costs, we characterize the level of cultural traits shown in long run. We find that families with low costs of changing the network and a central position in the underlying network, have strong influences on the long-run cultural trait. Finally, we demonstrate that the changes made to the social network by each generation slow down convergence. Moreover, the smaller the cost term, the larger is this effect, providing a theoretical explanation for the observed persistence of cultural trait. While this is only possible to be shown for the natural but quite specific assumption made on the costs term, we expect these findings to hold more generally when parents engage in the socialization process of children.

While this is the first attempt to have such a targeted vertical socialization, further steps are needed in the analysis. First of all, the role of children is assumed passive, as it is the case in all of this strand of literature. Allowing children to form their own network and engage in the identity formation process shall be addressed in the future. At the same time, in the dynamics, we consider the underlying social network to be fixed which we interpret as inverse distances in e.g. a city. However changes to the network made by parents (or children) may carry over to the next generation. In that case, future models of cultural transmission may include inherited changes to the network.

A. Appendix

Proof of Proposition 1. Since the relative influences after biasing must sum up to one, we have $\sum_{j \in N} d_{ij} = 0$ implying that $d_{ii} = -\sum_{j \neq i} d_{ij}$. Thus, we can rewrite (3), to obtain

$$(3) \Leftrightarrow \arg\max_{(d_{ij})_{j \neq i}} -\,(\sum_{j \neq i}(g_{ij} - d_{ij})V_j(t) + (g_{ii} + \sum_{j \neq i} d_{ij} V_i(t) - V_i(t))^2 - c_i(t)(\sum_{j \neq i} d_{ij})^2$$

Solving the system of $n-1$ first order conditions $j \neq i$,

$$-\left(\sum_{k \in N} g_{ik} V_k(t) - V_i(t) - \sum_{k \neq i} \delta_{ik}(t)(V_k(t) - V_i(t))\right)(V_i(t) - V_j(t))^2 = c_i(t)(V_i(t) - V_j(t))\delta_{ij}(t)$$

then obtains

$$\delta^*_{ij}(t) = (V_i(t) - V_j(t)) \frac{V_i(t) - \sum_{k \in N} g_{ik} V_k(t)}{c_i(t) + \sum_{k \neq i}(V_i(t) - V_k(t))^2}.$$

□

Proof of Proposition 2. Since (6) holds for all $i \in N$, we can write the dynamics of all cultural traits as

$$\mathbf{V}(t+1) = [\tilde{\mathbf{C}}(t)\mathbf{G} + (\mathbf{I} - \tilde{\mathbf{C}}(t))]\mathbf{V}(t) \tag{A.1}$$

where $\tilde{\mathbf{C}}(t) = diag\left(\frac{\tilde{c}_i(t)}{\tilde{c}_i(t)+1}\right)$ and $\mathbf{I}$ denotes the $n \times n$ identity matrix. The law of motion of the dynamic system is hence defined by the $n \times n$ time inhomogeneous matrix

$$\mathbf{M}(t) := \tilde{\mathbf{C}}(t)\mathbf{G} + [\mathbf{I} - \tilde{\mathbf{C}}(t)] \tag{A.2}$$

We then get, $\mathbf{V}(t+1) = \mathbf{M}(t) \cdot \mathbf{M}(t-1) \cdot \ldots \cdot \mathbf{M}(0)\mathbf{V}^0$. Note that $\mathbf{M}(t)$ must be row stochastic for all $t \in \mathbb{N}$. By substituting $\tilde{\mathbf{S}}(t) := \mathbf{I} - \tilde{\mathbf{C}}(t)$ into (A.2), we get $\mathbf{M}(t) = \left(\mathbf{I} - \tilde{\mathbf{S}}(t)\mathbf{G} + \tilde{\mathbf{S}}(t)\right)$ such that $\tilde{\mathbf{S}}(t)$ is diagonal with entries $\tilde{s}_{ii}(t) \in (0,1)$ for all $i \in N$, $t \in \mathbb{N}$. Thus, the result is implied by Panebianco (2014, Proposition 2). □

Proof of Proposition 3. By Assumption 1, we have that $\tilde{c}_i(t) = c_i$ and hence by (A.2)

$$\mathbf{M}(t) = \mathbf{M} := \mathbf{C}\mathbf{G} + [\mathbf{I} - \mathbf{C}] \tag{A.3}$$

for all $t \in \mathbb{N}$ with $\mathbf{C} := diag(c_i/(c_i+1))$. Since convergence of cultural traits is guaranteed by Proposition 2, we have that $\mathbf{M}(\infty) := \lim_{t\to\infty} \mathbf{M}(t) = \lim_{t\to\infty} \mathbf{M}^t$ exists. Standard results on power series of matrices imply $\mathbf{M}(\infty) = \mathbf{1}\mathbf{x}'$ where $\mathbf{1} = (1, ..., 1)$ and $\mathbf{x}$ is unique normalized left unit eigenvector of $\mathbf{M}$ with $\sum_{j=1}^n x_j = 1$. Using (A.3), we get $\mathbf{0} = \mathbf{x}'(\mathbf{M}-\mathbf{I}) = \mathbf{x}'\mathbf{C}(\mathbf{G}-\mathbf{I})$ implying that $\mathbf{x}'\mathbf{C}$ must be a left unit eigenvector of $\mathbf{G}$ which is unique since $\text{rk}(\mathbf{G}-\mathbf{I}) = n-1$. In other words, we get $\mathbf{x}' = r\mathbf{y}'\mathbf{C}^{-1}$ with $r = \frac{1}{\sum_{j=1}^n \frac{c_j+1}{c_j} y_j}$ to have $\sum_{j=1}^n x_j = 1$. □

Proof of Proposition 4. Since $\mathbf{G}$ is symmetric and primitive, all eigenvalues are real and we can order according to size such that $1 = \lambda_1(\mathbf{G}) > \lambda_2(\mathbf{G}) \geq \ldots \geq \lambda_n(\mathbf{G})$.

Now, consider base cost vectors $\mathbf{c}, \hat{\mathbf{c}} \in \mathbb{R}^n$ with $\hat{c}_i \leq (<) c_i$ for all $i \in \{1, \ldots, n\}$ and denote the diagonal matrices $\mathbf{C} = diag(c_i/(c_i+1))$ and $\hat{\mathbf{C}} = diag(\hat{c}_i/(\hat{c}_i+1))$, as before, and the law of motions associated with these cost vectors by $\mathbf{M}$ and $\hat{\mathbf{M}}$, respectively. From (A.3), we then get $\mathbf{M} - \mathbf{I} = \mathbf{C}(\mathbf{G} - \mathbf{I})$. Note that $\mathbf{C}(\mathbf{G} - \mathbf{I})$ and $\mathbf{C}^{1/2}(\mathbf{G} - \mathbf{I})\mathbf{C}^{1/2}$ have the same eigenvalues, since $\mathbf{G}$ is symmetric and $\mathbf{C}$ is diagonal. Letting $\hat{\mathbf{D}} = diag((c_i\hat{c}_i + \hat{c}_i)/(c_i\hat{c}_i + c_i))$, we get $\hat{\mathbf{C}} = \hat{\mathbf{D}}\mathbf{C}$ and thus, $\hat{\mathbf{M}} - \mathbf{I} = \hat{\mathbf{C}}(\mathbf{G} - \mathbf{I}) = \hat{\mathbf{D}}\mathbf{C}(\mathbf{G} - \mathbf{I}) = \hat{\mathbf{D}}(\mathbf{M} - \mathbf{I})$. Analogously, $(\hat{\mathbf{M}} - \mathbf{I})$ and $\hat{\mathbf{D}}^{1/2}(\mathbf{M} - \mathbf{I})\hat{\mathbf{D}}^{1/2}$ have the same (real) eigenvalues.

For the eigenvalues of both law of motions, we can apply Theorem 1 in Ostrowski (1959), implying that $\lambda_k(\mathbf{C}^{1/2}(\mathbf{G} - \mathbf{I})\mathbf{C}^{1/2}) = \theta_k \lambda_k(\mathbf{G} - \mathbf{I})$, where θ_k are real numbers with $\lambda_n(\mathbf{C}^{1/2}\mathbf{C}^{1/2}) \leq \theta_k \leq \lambda_1(\mathbf{C}^{1/2}\mathbf{C}^{1/2})$. Since $\mathbf{C}^{1/2}\mathbf{C}^{1/2} = \mathbf{C}$ is diagonal with strictly positive diagonal entries bounded above by 1, it holds that $\lambda_k(\mathbf{C}^{1/2}(\mathbf{G} - \mathbf{I})\mathbf{C}^{1/2}) > \lambda_k(\mathbf{G} - \mathbf{I})$ for all k such that $\lambda_k(\mathbf{G} - \mathbf{I}) < 0$. The latter is satisfied for $\lambda_k(\mathbf{G}) < 1$, and thus for all $\lambda_k(\mathbf{G})$ such that $2 \leq k \leq n$. We get, $\lambda_k(\mathbf{M}) = \lambda_k(\mathbf{M} - \mathbf{I}) + 1 > \lambda_k(\mathbf{G} - \mathbf{I}) + 1 = \lambda_k(\mathbf{G})$ for all $2 \leq k \leq n$.

Analogously, $\lambda_k(\hat{\mathbf{M}}) = \lambda_k(\hat{\mathbf{M}} - \mathbf{I}) + 1 \geq (>) \lambda_k(\mathbf{M} - \mathbf{I}) + 1 = \lambda_k(\mathbf{M})$ since the eigenvalues (i.e. the entries) of $\hat{\mathbf{D}}$ satisfy $0 < (c_i\hat{c}_i + \hat{c}_i)/(c_i\hat{c}_i + c_i) < 1$ as $\hat{c}_i \leq (<) c_i$ for all $i \in N$. Thus, speed of convergence of $\mathbf{M}^t$ is higher than $\hat{\mathbf{M}}^t$ since $\lambda_2(\mathbf{M}) \geq (>) \lambda_2(\hat{\mathbf{M}})$.

□

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