Dynamic and Strategic Analysis of Innovations

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Meinen Eltern, Sevil und Çağan

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Chapter 1

Introduction

Innovations have played a major role since the industrial revolution and nowadays, they play an even more important role. Innovations, in particular product innovations are accompanied with changing industrial structures and hence, they affect economic decisions. An important question then is how product innovations affect decisions made by firms. For instance, how are capacities of products adjusted if in the future a new substitute product is expected to be developed which will enlarge the market but at the same time cannibalize existing demand? In other words, how do firms react to an expected lower value of its installed capacity in the future? Such questions have been addressed in multi-stage models where the number of products offered by firms depends on the stage of the game (cf. Dawid et al. (2010b), Dawid et al. (2013a)). Unfortunately, in such models, neglecting the dynamic structure of industries might lead to wrong conclusions (cf. Cabral (2012) and Dawid et al. (2010a)). Hence, in this thesis, time is modeled continuously and optimal control and differential game models are used to analyze rigorously dynamic variables such as capacities, quantities, prices, investments etc.

Typically, dynamic programming, i.e. the Hamilton-Jacobi-Bellman Equation is employed in order to derive Markov-perfect Nash equilibria of differential games. There, smoothness assumptions are made, in particular continuously differentiable value functions are assumed in order to employ the HJB-equation and its smoothness is verified ex-post. However, sufficiency conditions might not be fulfilled e.g. if the value functions have kinks which might lead to multiple equilibria. In particular, the decision maker might be indifferent between approaching different steady states. Such indifference points are called *Skiba* or *DNSS* points, respectively, giving credit to the researchers Dechert, Nishimura, Skiba and Sethi who have started studying such points in the 70's. A standard example is the shallow lake model which involves multiple equilibria with 'clean' and 'dirty' steady states (see e.g. Wagener (2003)). We encounter Skiba points as well leading either to staying with an established product or to a new product's introduction.

By the introduction of new products the market structure changes substantially. In dynamic models, such a change is incorporated by considering different modes, e.g. a mode with one product and another mode with two products. Transition between different modes has been considered in games as well. Such games are called 'piecewise deterministic' or 'multi-mode' differential games in the literature (see e.g. Dockner et al. (2000)). There the transition between modes is stochastic but all other components are deterministic. In Chapters 2 and 3 of this thesis, there is no stochastic component but firms, to be more precise an innovator can decide on the time when to switch to another mode, here by introducing a new product. Hence, we focus on optimal timing of product introductions. In Chapter 2, we find a situation where initial capacities have only an indirect effect on the timing decision. More precisely, we find an optimal threshold for the firm's established capacity where the new product is introduced, hence timing is affected indirectly by initial capacities since it just takes longer for the state to arrive at the threshold if the state is further away or vice versa. However in Chapter 3, we consider a different situation. There, optimal timing is affected directly by initial conditions, and the switch to other modes might take place at different values instead of at a certain threshold point or curve in a two-dimensional state space, respectively.

While in Chapters 2 and 3, an implicit utility function of a representative consumer leads to an inverse demand function which is stationary, demand is considered explicitly in a durable goods model in Chapter 4. In Chapters 2 and 3, the inverse demand function is considered to be stationary even though examples of products given there are also rather durable than non-durable. In Chapter 4, modeling demand explicitly by considering consumers with different valuations for quality allows to analyze waiting effects. Here, again a similar setting is considered where a monopolist introduces a new product, but here at an exogenously given instant of time. Here, we focus on optimal pricing of the established and the new product in the presence of rational consumers. Rational consumers might delay their purchases. In general, there are two drivers of delay. First, in the standard durable goods literature, waiting occurs due to consumers' expectation of lower prices in the future, for the same product. Second, consumers might delay their purchase in order to buy the new product when it is introduced. We could have assumed a flexible price for the established product and hence have considered both drivers of delay but foregoing this option allows us to analyze the waiting effect only due to innovation and makes the problem analytically more tractable. However, not modeling the selling period of the established product dynamically and assuming a static setting would not allow to characterize changing willingness to pay of the consumers and its effect on pricing decisions. In other words, consumers' preferences are dynamic which adds pressure on firms price setting. What makes this Chapter special is that the firm cannot commit to its future price and hence the firm's price setting is restricted to be *credible*. More precisely, consumers restrict the firm's action by forming a rational expectation for the future product's price which affects prices in a rational expectations equilibrium crucially.

Chapter 2

Delaying Product Introduction: A Dynamic Analysis with Endogenous Time Horizon

2.1 Introduction

For many firms, especially those operating in the high-tech sector, whenever a new technology is available, they have to decide whether to adjust the product range by incorporating the new technology and if yes, when to do so.

Wang and Hui (2012) provide examples of firms hesitating to incorporate new available technologies and choosing to stay with the old technology for a while. Examples include the technology of DVD that has been developed much earlier than vendors started promoting DVDs. Another example is the MP3 standard.

In an empirical investigation, Chandy and Tellis (2000) have found that a large fraction of product innovations has been achieved by incumbents. Indeed, we face such a situation described above often in real-world markets and in many industries, submarkets evolve and coexist with the established product. An example is the TV Industry where CRT televisions and flatscreens were sold simultaneously for a long time (cf. Dawid et al. (2015)).

We consider an incumbent firm which has the option to introduce a horizontally and vertically differentiated substitute product which has a higher quality than the established one. For realizing this option, it incurs one-time adoption costs. Thus, the firm has to determine if the product introduction is profitable and if yes, when the optimal time of product introduction is. After introduction, we assume that the firm sells both products.

The firm faces the following trade-off: At the one hand, by launching the new product it cannibalizes demand for the established product and at the other hand, it benefits from the new product with higher quality by exploiting higher willingness to pay of the consumers. We find that the cannibalization effect alone cannot cause a delay. Delay is optimal if and only if there are adoption costs as well e.g. coming from adjustment costs of the plant, advertisement activities or fees paid to developers for using their technologies.

In particular, we find that if the firm is strong at the established market, i.e. its capacities are at a high level, then the firm decides to wait and hence to introduce the improved product later. By delaying, the firm benefits from discounting adoption costs while it decreases the capacity of the established product before the new product is introduced. This reduction of capacity increases the marginal values of the capacities of the established and the new product at the time of product introduction. Amongst others, this enables the incumbent to build-up capacities for the new product faster when it is introduced, compared to immediate introduction.

There is a large literature on capital accumulating firms which has been extended by Dawid et al. (2015) who analyzed the optimal R&D effort for product innovation and capital accumulation of established and new products, where the breakthrough probability of developing a new product depends on both, a knowledge stock and current R&D efforts via a hazard rate. Hence, in that paper innovation time is stochastic and it is assumed that the new product is introduced immediately once it is available. We focus on the optimal timing of product introduction and optimal investment in capacities and differ from Dawid et al. (2015) in not considering R&D efforts to develop a new product and not linking successful development to market introduction but considering the time of market introduction as a choice variable. The classical literature on optimal timing of technology adoption (see. e.g. Kamien and Schwartz (1972) for a single firm and Reinganum (1981) and Fudenberg and Tirole (1985) for a duopoly) assumes that quality increases due to technological progress and the only decision variable is the time of technology adoption. Farzin et al. (1998) and Doraszelski (2004) extend this stream of literature by considering the quality improvement as a stochastic process. In contrast, in our model, the quality of the new product is fixed and the firm cannot gain additional quality by delaying. Thus, our analysis focuses on the dependence on initial characteristics whose importance has been addressed a lot, e.g. in Hinloopen et al. (2013) where initial marginal costs determine if a technology is developed further or not. Real options models (see e.g. Dixit and Pindyck (1994)) have focussed on optimal timing in continuous time where demand is stochastic e.g. evolving according to a Brownian motion. A simultaneous analysis of optimal timing and optimal investment in capacities in the real options literature has been provided by Huisman and Kort (2015) where the price of the good is stochastic. We differ from that stream of literature by considering a deterministic environment and continuous adjustments of capacities.

The problem of an incumbent delaying product introduction has been addressed in Wang and Hui (2012). They apply a discrete three-period time framework where they do not take into account capacity adjustments. In contrast to Wang and Hui (2012), in our model, delaying cannot be optimal if there are no adoption costs.

From a technical perspective, we employ Pontryagin's Maximum Principle for free end time (see. e.g. Grass et al. (2008)) to obtain analytical results concerning the optimal investments and the optimal time of market introduction.

Moreover, in this optimal control problem, due to the non-concave structure of the value function, the Arrow-Mangasarian sufficiency conditions are not met which for certain states lead to the presence of multiple optimal investment paths. In particular, we characterize situations in which the firm is indifferent between approaching different steady states (see Skiba (1978)). In such models, qualitative properties of solutions depend very much on parameters (cf. Hinloopen et al. (2013)). Therefore, we use a bifurcation analysis to assess industry dynamics for different values of adoption costs where we encounter a deformed pitchfork bifurcation.

The analysis in this paper is carried out for a monopoly setting. Even though the real-world examples we have raised stem from competitive environments, we believe that it is important to consider the monopoly as it is interesting in its own right. Indeed, timing of product introduction is not only influenced by competing firms but from competing substitute products as well even if there is only a single firm. As the established and new product are substitutes, there is 'internal' competition between those two products. In order to disentangle rivalry between products and between firms, it is reasonable to analyze the monopoly case before proceeding to the competition case.

The paper is organized as follows. We introduce the model in Sect. 2.2. Sect. 2.3 is devoted to the technical analysis. In Sect. 2.4, we provide an economic interpretation, conduct a bifurcation analysis and present optimal timing curves. Sect. 2.5 analyzes welfare effects of delaying product introduction. Model assumptions are discussed in Sect. 2.6 and Sect. 2.7 concludes.

2.2 Model

We consider an incumbent firm which has initial capacity K_1^{ini} to produce an established product. A new substitute product with higher quality has been developed and is ready for market introduction. Product introduction comes with lump-sum adoption costs F. An important assumption is that the incumbent cannot invest in capacities of the new product before introducing it, i.e. there are no capacities at the time of introduction for the new product.

We follow the literature on optimal capital accumulation by relying on a standard linear model (see e.g. Dockner et al. (2000)). Thus, the firm faces a linear inverse demand function which is given by

$$p_1(t) = 1 - K_1(t). \tag{2.1}$$

After product introduction, the inverse linear demand system[1] is given by

$$p_1(t) = 1 - K_1(t) - \eta K_2(t), \tag{2.2}$$

and

$$p_2(t) = 1 + \theta - \eta K_1(t) - K_2(t), \tag{2.3}$$

where η with $0 < \eta < 1$ measures the degree of horizontal and $\theta > 0$ the degree of vertical differentiation of the substitutes.

The firm wants to determine the optimal time of product introduction T and the optimal investment strategies before and after product introduction. There is no inventory, i.e. capacities equal sales[2]. The capacity dynamics are

$$\dot{K}_i(t) = I_i(t) - \delta K_i(t), \quad i = 1, 2, \tag{2.4}$$

$$K_1(0) = K_1^{ini}, \quad K_2(t) = K_2^{ini} = 0 \ \forall\, t \leq T, \tag{2.5}$$

where $\delta > 0$ measures the depreciation rate. As has been done in Dawid et al. (2015), we allow the firm to intentionally scrap capacities, i.e. $I_i \in \mathbb{R}$ while capacities have to remain non-negative:

$$K_i(t) \geq 0 \ \ \forall\, t \geq 0, i = 1, 2. \tag{2.6}$$

Adjusting capacities is costly, in particular it comes with quadratic costs

$$C\Big(I_i(t)\Big) = \frac{\gamma}{2} I_i^2(t), \quad i = 1, 2. \tag{2.7}$$

Normalizing production costs to zero, the objective function of the firm is given by the following expression:

$$\begin{aligned} \max_{T, I_1(t), I_2(t)} J = &\int_0^T e^{-rt}\Big(p_1(t)K_1(t) - C(I_1)\Big)dt \\ &+ \int_T^\infty e^{-rt}\Big(p_1(t)K_1(t) + p_2(t)K_2(t) - C(I_1) - C(I_2)\Big)dt - e^{-rT}F. \end{aligned} \tag{2.8}$$

We refer to this problem as $\mathcal{P}(K_1^{ini})$.

[1]This demand system is motivated by the fact that the two products are substitutes and competing with each other. According to the seminal result of Kreps and Scheinkman (1983), setting prices optimally subject to ex-ante capacity commitments reduces to a Cournot setting which we adopt here.

[2]This assumption has been used in large parts of the literature on dynamic capacity investment, see e.g. Goyal and Netessine (2007). See Section 2.6 for a discussion of this assumption.

2.3 Analysis

In case that the firm wants to introduce the improved product at some finite time T, there will be a structural change of the model. Therefore, we denote by mode 1 (m_1) the optimal control problem up to time T and by mode 2 (m_2) the problem after T. Denote by $V^{m_1}(K_1)$ and $V^{m_2}(K_1, K_2)$ the corresponding value functions of the infinite horizon control problems where the mode is fixed and hence does not change[3]. The optimal control problem at hand where the mode m might change is denoted by $V(K_1, K_2, t, m)$ and we refer to this problem as the *optimal control problem with introduction option*.

The subproblem in m_2 is linear-quadratic with infinite time horizon which can be solved easily, as has been done in Dawid et al. (2015). The optimal strategy and the value function are stationary for this problem, i.e. $V(K_1, K_2, t, m_2) = V^{m_2}(K_1, K_2) - F$. There is a unique globally asymptotically stable steady state under the optimal strategy and the value function is given by[4]

$$V^{m_2}(K_1, K_2) = a + bK_1 + cK_1^2 + dK_2 + eK_2^2 + fK_1K_2. \tag{2.9}$$

The typical shape of the value function of m_2 is depicted in Figure 2.1.

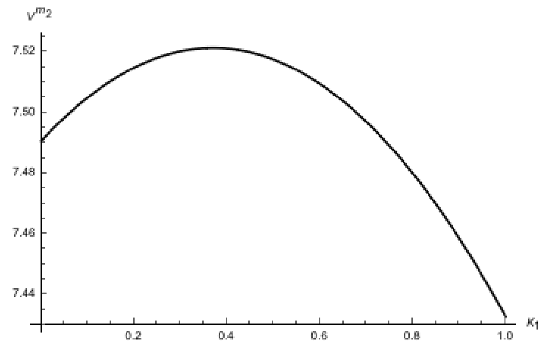


Figure 2.1: Value function of m_2 at T, i.e. for $K_2 = 0$. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15$.

By regarding the value function of the subproblem as the salvage value of the

[3] We suppress the argument t wherever it is possible and does not cause confusion.

[4] Equations for coefficients are given in Dawid et al. (2015).

optimal control problem with introduction option, we can rewrite (2.8) by

$$\max_{T, I_1(t)} J = \int_0^T e^{-rt}\Big(p_1(t)K_1(t) - C(I_1(t))\Big)dt + e^{-rT}S\Big(K_1(T)\Big), \tag{2.10}$$

where $S\Big(K_1(T)\Big) = V^{m_2}(K_1(T), 0) - F$.[5] This problem can be solved analytically by Pontryagin's Maximum Principle for variable terminal time. The Hamiltonian is

$$H(K_1, I_1, \lambda, t) = (1 - K_1)K_1 - \frac{\gamma}{2}I_1^2 + \lambda(I_1 - \delta K_1), \tag{2.11}$$

where λ is the co-state variable and the optimal investment is given by

$$I_1 = \frac{\lambda}{\gamma}. \tag{2.12}$$

The co-state equation reads

$$\dot{\lambda} = (r + \delta)\lambda - (1 - 2K_1), \tag{2.13}$$

and the transversality condition is given by[6]

$$\lambda(T) = S_{K_1} = V_{K_1}^{m_2}(K_1, 0). \tag{2.14}$$

For nonzero finite T^*, let $\Big(K_1^*(\cdot), I_1^*(\cdot)\Big)$ be an optimal solution to (2.10) on the optimal time interval $[0, T^*]$. Pontryagin's Maximum Principle for variable end time implies an additional constraint for the terminal time, which is given by

$$H(K_1^*(T^*), I_1^*(T^*), \lambda(T^*), T^*) = rS\Big(K_1^*(T^*)\Big) - S_T\Big(K_1^*(T^*)\Big). \tag{2.15}$$

Note that the salvage value does not depend explicitly on T^* and hence,

$$S_T\Big(K_1^*(T^*)\Big) = 0. \tag{2.16}$$

So, equation (2.15) requires that at the optimal time T^*, the instantaneous revenue from staying in m_1 plus the assessment of the change of the state variable on the one hand (which is given by the current-value Hamiltonian, abbr. by H) and the interest on the salvage value (abbr. by rS) on the other hand are equal. This is

[5] $K_2(T) = 0$ since there are no capacities for the new product at T, yet.

[6] The canonical system, isoclines, the steady state for staying in m_1 and its stability properties are given in Appendix 2.A.1.

quite intuitive since otherwise it would be optimal to stay longer in m_1 if H is higher than rS or to have introduced earlier if rS is higher than H.

In Lemma 2.2 in Appendix 2.A.2, we state that there are two solutions for equation (2.15). By that lemma and Proposition 2.1 below, we show that for $F = 0$, both solutions coincide and $H \leq rS$ for all values of established capacity, i.e. immediate introduction is optimal and hence $T^* = 0$. For $F > 0$, there are two distinct points satisfying the terminal condition. In the corresponding interval, where the boundaries are given by the two points satisfying (2.15), there is $H \geq rS$ (cf. Figure 2.12 in Appendix 2.A.2), i.e. for initial capacities in the interval, it is optimal to reduce capacities down to the lower bound and to introduce the new product, we say to jump to m_2. We denote the two solutions of (2.15) by K_1^{lb} and K_1^{ub}, respectively for lower and upper bound of the interval with $K_1^{lb} \leq K_1^{ub}$. As mentioned above, for $F = 0$, both solutions coincide[7], i.e. $K_1^{lb} = K_1^{ub}$ (see Appendix 2.A.2), which we denote by $K_1^{F=0}$.

So, for higher capacities than K_1^{ub}, the unique solution is to introduce the new product immediately again. In particular at K_1^{ub}, the firm is indifferent between both options. However, higher capacities than K_1^{ub} will not be analyzed further as there the firm switches immediately to m_2 which has been analyzed in Dawid et al. (2015).

As the optimal introduction time depends on the size of capacity, we consider it as a correspondence depending on K_1^{ini} and denote it by $T^*(K_1^{ini})$[8]. It is a correspondence since there are situations with multiple optimal values as we will discuss in the following. We start by characterizing finite solutions.

Proposition 2.1. *If $T^*(K_1)$ is finite for all K_1, then for all $K_1 \leq K_1^{lb}$, it is optimal to innovate immediately. For all $K_1^{lb} < K_1 \leq K_1^{ub}$, it is optimal to reduce capacities and to innovate when the capacity reaches K_1^{lb}, i.e. $T^*(K_1) > 0$.*

[7]Technically, in case of no adoption costs, H and rS are tangential at $K_1^{F=0}$:

$$\frac{\partial}{\partial K_1} H(K_1^{F=0}, I_1^*(T^*), \lambda(T^*), T^*) = \frac{\partial}{\partial K_1} rV^{m_2}(K_1^{F=0}, 0). \tag{2.17}$$

[8]An alternative would have been to define a function which gives the remaining time in m_1 not depending on the initial but current capacity (cf. Long et al. (2017)).

Proof. See Appendix 2.A.3 □

Proposition 2.1 states that immediate introduction is optimal if capacity for the established product is lower than a certain threshold (given by K_1^{lb}) whereas for capacities above, it is optimal to wait and to decrease capacities on the established market before product introduction. Note, that there are infinite solutions where it is not optimal to innovate immediately even though $K_1^{ini} \leq K_1^{lb}$ as we will discuss at the end of this section.

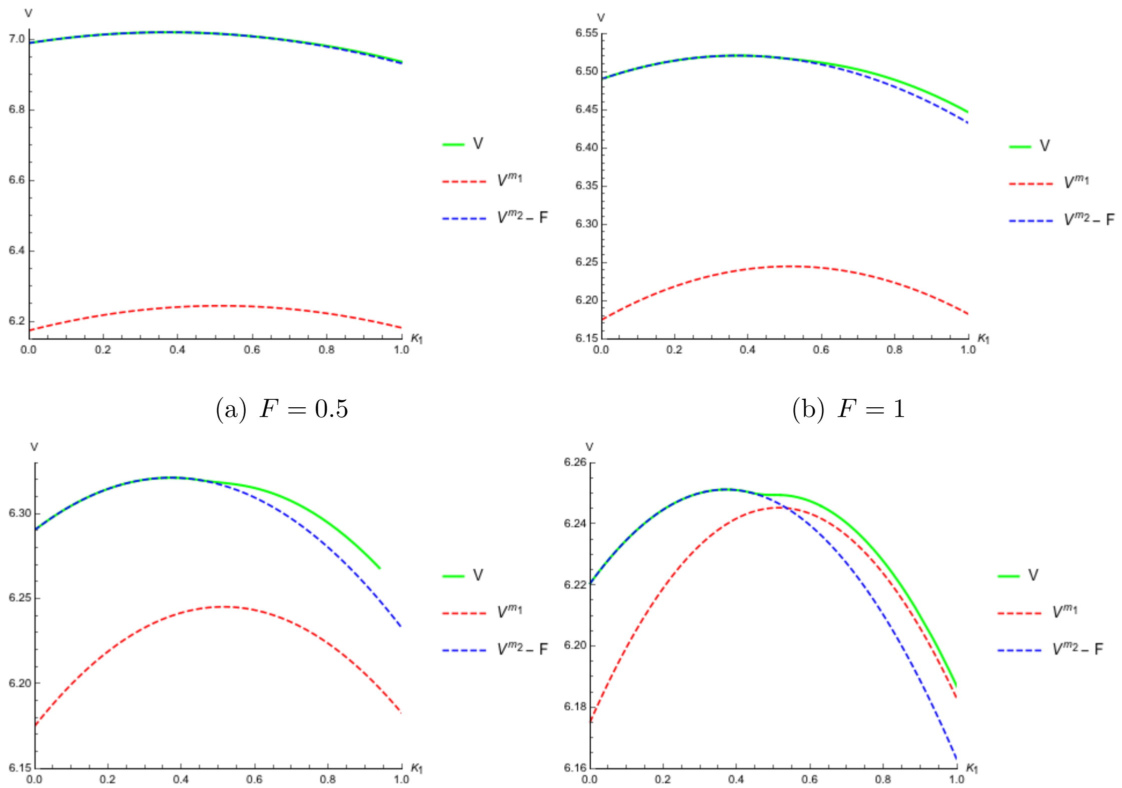
In the next lemma we focus on the dependence of K_1^{lb} on F and find that K_1^{lb} is decreasing in F, i.e. as adoption costs increase, it takes longer to arrive at K_1^{lb} for a fixed starting point $K_1^{ini} > K_1^{lb}$.

Lemma 2.1. *K_1^{lb} is decreasing in F.*[9]

Proof. See Appendix 2.A.4. □

In Figure 2.2, we illustrate how the value function evolves as F increases. For $K_1^{lb} < K_1 < K_1^{ub}$, the value function of the problem with introduction option is higher than the value function of m_2. As F increases and discounting adoption costs become more important, the difference of the value function with introduction option and the scrap value function gets larger. Furthermore, as the products are vertically differentiated, the value of the problem of m_2 is higher than of m_1 for no adoption costs. Thus, the value of the problem with introduction option is higher than the value of the infinite problem of m_1. Obviously, for large enough F, the value function of the problem with introduction option will hit the value function of the infinite horizon problem of m_1 and infinite solutions will occur, i.e. product introduction will not be sufficiently attractive anymore. We show in Appendix 2.A.5 in Lemma 2.3 that there exists a unique value of adoption costs $\tilde{F}$ where this happens for the first time (see Figure 2.3). Thus, $\tilde{F}$ is the lowest value of adoption costs for which it exists some initial value of capacity where the firm abstains from product introduction. This result leads to the following corollary.

[9] Moreover, K_1^{ub} is increasing in F. Thus, for increasing F, the interval $[K_1^{lb}, K_1^{ub}]$ expands around $K_1^{F=0}$.



(a) $F = 0.5$ (b) $F = 1$

(c) $F = 1.2$ (d) $F = 1.27$

Figure 2.2: Value functions for different values of F. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15$.

Corollary 2.1. *For $F < \tilde{F}$, $T^*(K_1)$ is finite for all initial capacities and Proposition 2.1 applies.*

Proof. Follows directly from Lemma 2.3 in Appendix 2.A.5. □

To sum up the results so far, for $F = 0$, the firm wants to launch the new product immediately. For increasing F, there arises an interval given by $[K_1^{lb}, K_1^{ub}]$ wherein the higher K_1^{ini} the longer it takes to arrive at K_1^{lb} where the firm wants to launch the new product, i.e. the stronger the firm on the established market, the more the firm delays. Moreover, due to Lemma 2.1, the higher the adoption costs, the lower is the switching capacity, i.e. the firm wants to reduce capacities more in advance before switching to m_2.

Denote by $\tilde{K_1}$ the lowest value of initial capacity where an infinite solution exists for $\mathcal{P}(\tilde{K_1})$:

$$\tilde{K_1} = \min\{K_1 \mid T^*(K_1) = \infty\}. \tag{2.18}$$

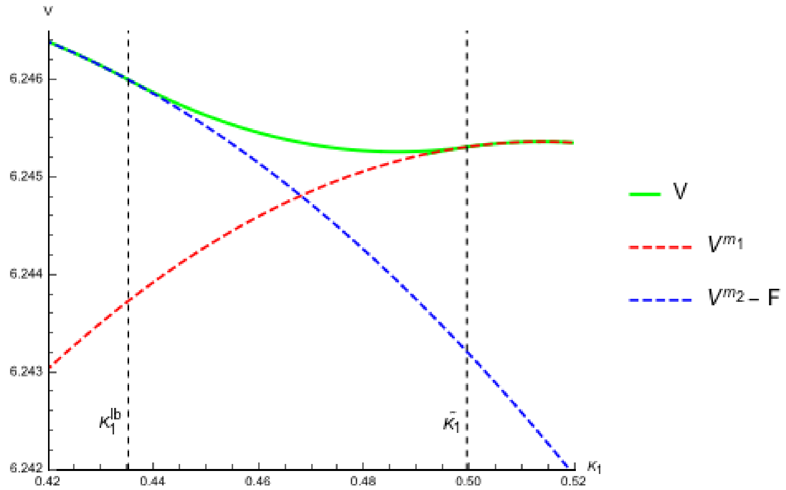


Figure 2.3: Value function for $F = \tilde{F} = 1.27437$. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15$.

Note that $\tilde{K}_1$ exists for $F \geq \tilde{F}$. The following proposition and corollary characterize the situation at $\tilde{F}$.

Proposition 2.2. *At* $F = \tilde{F}$*,*

$$\tilde{K}_1 = K_1^{ss,m_1} \text{ [10]} \tag{2.19}$$

and the free end-time problem $\mathcal{P}(\tilde{K}_1)$ *has a unique solution with* $T^* = \infty$*.*

Proof. See Appendix 2.A.7. □

Corollary 2.2. *At* $\tilde{F}$*, for* $K_1 < \tilde{K}_1$*,*

$$T^*(K_1) < \infty, \tag{2.20}$$

and for $\tilde{K}_1 \leq K_1 < \bar{K}_1$*,*

$$T^*(K_1) = \infty. \tag{2.21}$$

Proof. Due to the definition of $\tilde{K}_1$, for $K_1 < \tilde{K}_1$ only finite solutions are optimal. According to the proof of Proposition 2.2, for $\tilde{K}_1 \leq K_1 < K_1^{ub}$, only infinite solutions are optimal. □

[10] K_1^{ss,m_1} is the unique steady state for staying infinitely in m_1 given in Appendix 2.A.1.

Proposition 2.2 and Corollary 2.2 state that at $\tilde{F}$, K_1^{ss,m_1} is a threshold separating finite and infinite solutions. That is, for $K_1 \geq K_1^{ss,m_1}$ the firm prefers not innovating and stays in m_1, whereas for $K < K_1^{ss,m_1}$ the firm decreases[11] capacities to K_1^{lb} and hence introduces the new product eventually.

For characterizing the evolution of $\tilde{K_1}$, we denote by $\bar{F}$ the value of adoption costs for which

$$V^{m_1}(K^{lb}) = V^{m_2}(K^{lb}) - F \ (= S(K^{lb})) \tag{2.22}$$

holds, i.e. where the firm is indifferent between introducing immediately and delaying infinitely at K_1^{lb}.

Proposition 2.3. *$\tilde{K_1}$ is decreasing in F and for all $\tilde{F} < F < \bar{F}$, the free end-time problem $\mathcal{P}(\tilde{K_1})$ has two different solutions with optimal terminal times $0 < T^f < \infty$ and $T^\infty = \infty$, i.e. $\tilde{K_1}$ is a Skiba point where the firm is indifferent between introducing the product after some delay and not at all.*

Proof. See Appendix 2.A.8. □

A consequence of Proposition 2.3 is that as F increases, the range of capacities where the firm stays with only one product enlarges as $\tilde{K_1}$ decreases. Moreover, there is a finite and infinite solution at $\tilde{K_1}$[12]. As before, the timing for capacities lower than $\tilde{K_1}$ is finite. So there exist three different ranges of capacities where optimal time of product introduction is either 0, infinite or in-between. We refer to $[\tilde{F}, \bar{F})$ as the *intermediate range* of F and for $F \in [\tilde{F}, \bar{F})$ we refer to $(K_1^{lb}, \tilde{K_1})$ as the *waiting region*.

Denote by $\bar{\bar{F}}$ the value of adoption costs where thereafter finite solutions disappear for the first time[13], i.e.

$$T^*(0) = \infty \ . \tag{2.23}$$

Now, we show that at $\bar{F}$ the waiting region vanishes and only immediate or infinite solutions for T remain.

[11]In Appendix 2.A.6 in Lemma 2.4, we show that at $\tilde{F}$, $K_1^{lb} \leq K^{ss,m_1}$ holds.

[12]There is no other value of capacity where both solutions are optimal.

[13]As $\tilde{K_1}$ is decreasing in F, at $\bar{\bar{F}}$, $K_1 = 0$ is the only remaining value for capacity such that the firm is indifferent between immediate and no product introduction.

Corollary 2.3. *For $\bar{F} \leq F < \bar{\bar{F}}$, there exists a $\tilde{K}_1 > 0$ such that for all $K_1 < \tilde{K}_1$ the firm introduces the new product immediately whereas for all $K_1 > \tilde{K}_1$ the firm never introduces the new product. At $\tilde{K}_1$, the incumbent is indifferent, in particular the free end-time problem $\mathcal{P}(\tilde{K}_1)$ has two different solutions with $0 = T^f < T^\infty = \infty$. Moreover, at $\bar{F}$, $\tilde{K}_1 = K_1^{lb}$.*

Proof. By definition of $\bar{F}$, the firm is indifferent between immediate and infinite product introduction. By Proposition 2.3, $\tilde{K}_1$ is decreasing and hits K_1^{lb} at $\bar{F}$ where solutions with $0 < T < \infty$ vanish. □

Thus, for all F, $\tilde{K}_1$ is separating finite and infinite solutions for T. Note that for $F < \bar{F}$, the value function of m_2 and the value function of the problem with introduction option paste smoothly at K_1^{lb}, i.e.[14]

$$\frac{\partial V(K_1^{lb}, 0, m_1)}{\partial K_1} = \frac{\partial V^{m_2}(K_1^{lb})}{\partial K_1} . \tag{2.24}$$

Furthermore, at $\tilde{F}$, the value function of the problem with introduction option and the value function of m_1 paste smoothly at $\tilde{K}_1$ (see Figure 2.3) whereas for $F > \tilde{F}$ the value function has a kink at $\tilde{K}_1$ (cf. Figure 2.4).

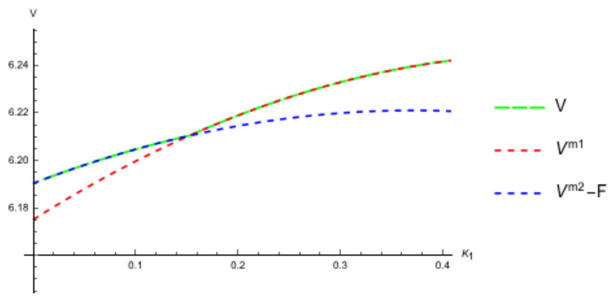


Figure 2.4: Value function for high F. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15, F = 1.3$.

2.3.1 Summary of Results

In total, as long as F is intermediate (i.e. $\tilde{F} \leq F < \bar{F}$), we can split the state space in three parts:

[14]Note that the value function is time-invariant and hence the time argument can be omitted, i.e. $V(K_1^{lb}, K_1^{lb}, t, m) = V(K_1^{lb}, K_1^{lb}, m)$.

i) 'Immediate introduction': $K_1 \leq K_1^{lb}$: Firm innovates immediately, $T^* = 0$.

ii) 'Delayed product introduction': $K_1^{lb} < K_1 \leq \tilde{K_1}$: Firm delays introduction and introduces product later at $0 < T^* < \infty$.

iii) 'No introduction': $K_1 \geq \tilde{K_1}$: Firm delays introduction infinitely, i.e. there is no product introduction.

For increasing F the indifference point $\tilde{K_1}$ shifts to the left and eventually the waiting region vanishes where $\tilde{K_1}$ and K_1^{lb} coincide and only two possibilities remain: Either the firm innovates immediately (for low capacities) or never (for high capacities). Hence, for $F \geq \bar{F}$, the value function is given by the upper curve of the value functions V^{m_1} and V^{m_2} (see Figure 2.4).

We call F *low* if $0 < F < \tilde{F}$, *intermediate* if $\tilde{F} \leq F < \bar{F}$, *high* if $\bar{F} \leq F \leq \bar{\bar{F}}$ and *very high* if $F > \bar{\bar{F}}$.

- If there are no adoption costs, only scenario i) is prevalent.
- For low adoption costs, scenarios i) and ii) are possible depending on the initial capacity level.
- If F is intermediate, all three scenarios are possible.
- For high adoption costs, only scenarios i) and iii) are possible.
- For very high adoption costs, only scenario iii) is prevalent.

2.4 Dynamics

In Section 2.4.1, we give an economic interpretation of the optimal capacity investments and the timing decision. A bifurcation analysis is presented in Section 2.4.2. Optimal timing curves and its dependence on parameters of horizontal and vertical differentiation are given in Section 2.4.3.

In order to derive dynamics, we consider the following default parameter setting taken from Dawid et al. (2015):

$$r = 0.04,\ \delta = 0.1,\ \eta = 0.9,\ \theta = 0.1,\ \gamma = 0.15. \tag{2.25}$$

2.4.1 Economic Interpretation

The intuition for the 'Immediate Introduction' and 'No Introduction' scenario is straight forward. The benefit from the new product is either so high that the firm does not want to wait or the benefit is too low such that the firm stays with the established product. Thus, we focus on the interpretation of the interesting case of delay. Note that for finite T^*, before T^*, the Hamiltonian H is greater than the interest on the salvage value rS and at T^*, they are equal[15]. In a sense the firm exploits profits in m_1 before moving to m_2. By choosing $T^* > 0$, the Hamiltonian is affected[16] via the co-state $\lambda(t)$. In economic terms, the following mechanisms can be identified.

First, the delay in time leads to stronger discounting of the scrap value $V^{m_2} - F$. The firm saves adoption costs as F is paid as a lump-sum, but gets V^{m_2} later as well. The latter is smoothed by the concave structure of the value function of m_2 as the firm reduces capacities of the established product and hence V^{m_2} increases[17].

Second, in the proof of Lemma 2.2 in Appendix 2.A.2, we find that

$$\frac{\partial V^{m_2}}{\partial K_2}(K_1^{F=0}, 0) = 0, \tag{2.26}$$

which has an interesting economic intuition. In contrast to m_1, in m_2, the firm is able to invest in K_2. For $F = 0$ at $K_1^{F=0}$ and elsewhere, there is no reason for waiting. But for higher $F > 0$, waiting yields discounting of adoption costs while at $K_1^{F=0}$, (2.26) still holds and thus there is no gain from immediate switching to m_2 and investing in K_2. Thus, by postponing the product introduction, the incumbent can decrease the capacity of K_1 before switching such that $\frac{\partial V^{m_2}}{\partial K_2}(K_1^{lb}, 0) > 0$, i.e. when switching, the marginal value of the new product's capacity is higher and

[15] Note that this is not necessarily true for the infinite case since if T^* is infinite, the transversality condition for the co-state variable and hence the Hamiltonian would be altered.

[16] Note that the investment in established capacity depends on the co-state as well.

[17] This holds as long as the switching capacity K_1^{lb} is greater than the maximal argument of V^{m_2} which is true for the considered parameter setting.

hence there is an immediate gain from investment in K_2. Hence, the investment pattern in m_2 is affected, where due to the reduced capacity of the established product, the firm has stronger incentives to build-up capacities for the new product and the disinvestment in the established product is weaker[18] than it would be without delay. Hence, in m_2, profits drop and are initially lower than in m_1 as there is a strong investment in capacities of the new product but sales increase only gradually for the new product. By delaying, the firm can postpone this drop in profits and enjoy 'high' profits in m_1. However, the drop in profits is stronger compared to immediate introduction.

2.4.2 Bifurcation Analysis

We have a situation in mind where a new improved version of a product is launched which is a close substitute to the established product. This is reflected by a relatively high η and low θ. We do robustness checks with respect to those parameters in Section 2.4.3. The other parameter choices are very standard.

From Figure 2.3, it is clear that the value function is not concave in K_1 and hence does not satisfy the Arrow-Mangasarian sufficiency conditions. Thus, as mentioned earlier, in this section we examine the qualitative properties of the steady states of the control problem with introduction option with respect to the parameter F. We start by drawing a bifurcation diagram of m_1 (Figure 2.5).

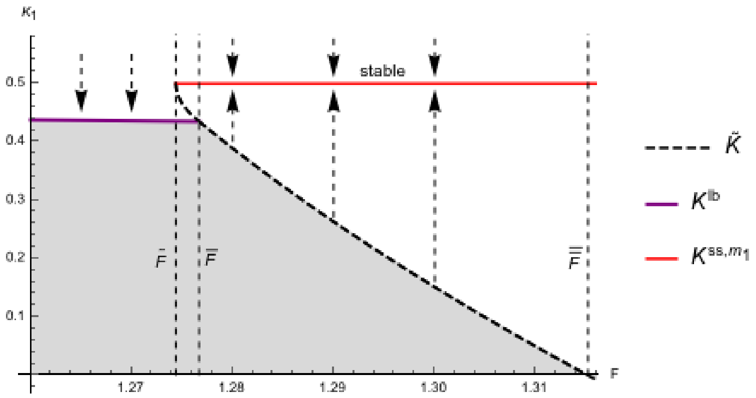


Figure 2.5: Bifurcation diagram of m_1.

[18]This is due to the increased marginal value of the established capacity.

The gray area is not present in m_1 since if the firm starts in that area or arrives there, it introduces the new product and hence is no more in m_1 but in m_2. As we are interested in characterizing dynamics in m_1 and in m_2 together, we draw a superimposed bifurcation diagram of both modes (cf. Hinloopen et al. (2017)) in Figure 2.6. For $F < \tilde{F}$, we have a unique stable steady state. No matter if

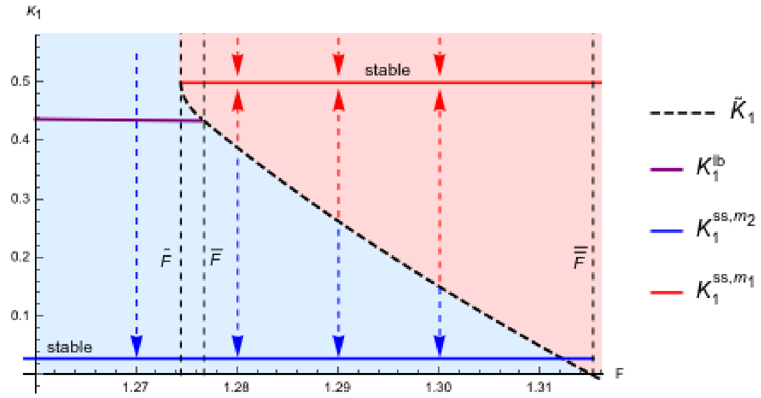


Figure 2.6: Superimposed diagram.

the firm delays product introduction or not, it will eventually arrive at the steady state level of K_1 in m_2 denoted by K_1^{ss,m_2}. As analyzed before, at $\tilde{F}$ there arises a second steady state where for initial capacities $\tilde{K}_1 \leq K_1 \leq K_1^{ub}$ (which are in the red area in Figure 2.6) the firm stays in m_1 and eventually arrives at K_1^{ss,m_1}.

At $\bar{\bar{F}}$ the equilibrium point K_1^{ss,m_2} vanishes and it remains only K_1^{ss,m_1} for $F > \bar{\bar{F}}$ (see Figure 2.7).

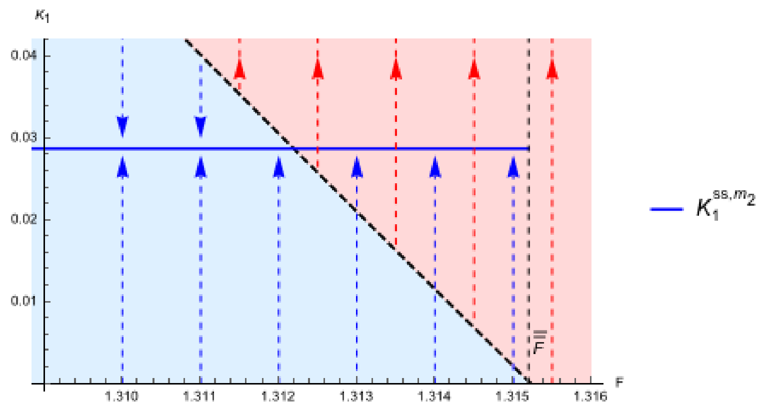


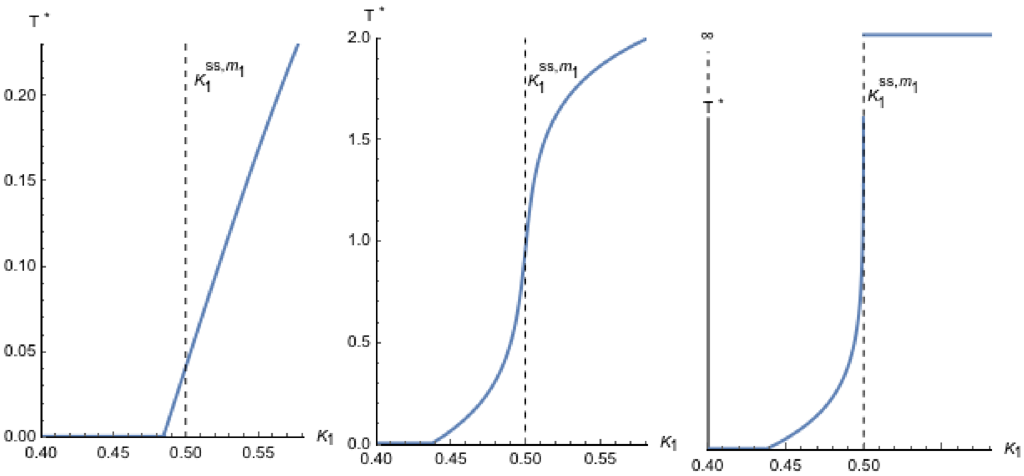
Figure 2.7: Dynamics around $\bar{\bar{F}}$.

Besides, we have a deformed pitchfork bifurcation which exhibits a hysteresis phenomenon where initially only one stable steady state exists and for higher F a second equilibrium arises 'out of the blue sky', where a repelling curve separates the two basins of attraction (red and blue area) where for very high F only the second equilibrium remains. The black dashed curve is the Skiba curve (which is repelling except at the two steady states where it is semi-stable). Note that for capacities on the Skiba curve in between the two steady states, optimal paths are moving in opposite directions but for capacities on the Skiba curve below K_1^{ss,m_2} both optimal paths move in same direction (see Figure 2.7). Note that this is a superimposed diagram and not a bifurcation diagram in the classical sense and the latter is possible since there the firm either jumps immediately to m_2 or never, which means that we actually consider two disjoint optimal control problems where the mode can be interpreted as a further state variable.

2.4.3 Characterization of Optimal Timing Curves

As discussed in Section 2.3, for $F \geq \tilde{F}$, $\tilde{K}_1$ separates finite and infinite solutions for the optimal introduction time. Thus, it jumps at $\tilde{K}_1$ to infinity. Hence, for $\tilde{K}_1 \leq K_1 \leq K_1^{ub}$, the value function of the problem with introduction option is equal to the value function of the problem without introduction option.

We now investigate in detail what happens when F approaches $\tilde{F}$. The graphs of the optimal introduction time are depicted in Figure 2.8. For low adoption costs,



$F = 1$ $F = 1.274$ $F = \tilde{F} \approx 1.27437$

Figure 2.8: Optimal time of switching for increasing F.

the correspondence is concave for $K_1 \geq K_1^{lb}$. As analyzed in Section 2.3, it is finite for low adoption costs whereas it becomes infinite at $\tilde{F}$ for $K_1 \geq \tilde{K}_1 = K_1^{ss,m1}$. For F approaching $\tilde{F}$, $T^*(K_1)$ becomes convex-concave and very steep at K_1^{ss,m_1}, i.e K_1^{ss,m_1} becomes an inflection point (see Figure 2.8) which means that the firm decreases higher capacities and "stays around" K_1^{ss,m_1} for a while until it starts decreasing again down to K_1^{lb}. Note that for $F < \tilde{F}$, $T^*(K_1)$ is finite everywhere, whereas at $\tilde{F}$, $T^*(K_1)$ is infinite for $K_1 \geq K_1^{ss,m1}$.

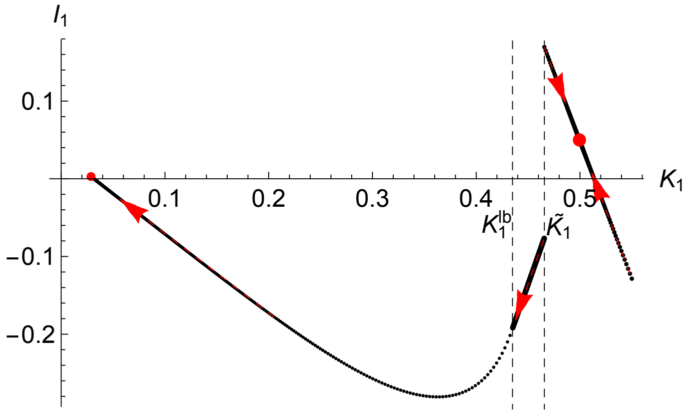


Figure 2.9: Capacity-investment dynamics for $F = 1.275$.

Figure 2.9 depicts optimal curves in the (K_1, I_1) space for the interesting case of intermediate adoption costs where $\tilde{K}_1$ separates the two basins of attraction. For $K_1^{lb} < K_1 < \tilde{K}_1$, the firm decreases capacities down to K^{lb} and introduces the new product. In m_2, it continues decreasing capacities of K_1 down to K_1^{ss,m_2} while it builds up capacities for the new product up to K_2^{ss,m_2}.

Effect of Horizontal and Vertical Differentiation

For decreasing degree of horizontal differentiation η, the products become more differentiated and thus the firm is expected to benefit from this. As both markets get more independent we expect that the firm is willing to introduce the new product earlier. Numerical experiments are in line with this intuition (see Figure 2.10). Analogously, for increasing θ we get similar results.

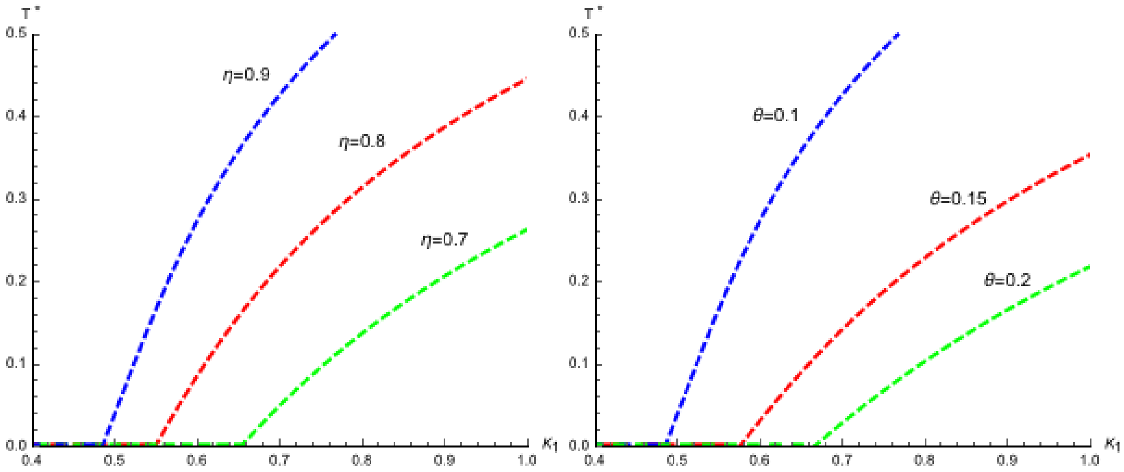


Figure 2.10: Optimal time of switching for different parameterizations of η and θ.

2.5 Welfare Implications

For analyzing welfare implications, note that the inverse demand functions stem from the following utility function of the consumers where M is the initial endowment:

$$CS(t) = u(K_1, K_2) = K_1 + (1+\theta)K_2 - \frac{1}{2}(K_1^2 + K_2^2) - \eta K_1 K_2 + (M - p_1 K_1 - p_2 K_2). \tag{2.27}$$

The welfare depends on the interpretation of adoption costs. If it is paid to the developer of the technology, then it is considered as a transfer and it is always profitable to introduce the new product immediately. But if it is considered as 'real' costs, then it has to be taken into account. In that case, the social planner maximizes the difference of consumer surplus and costs of investment and adoption:

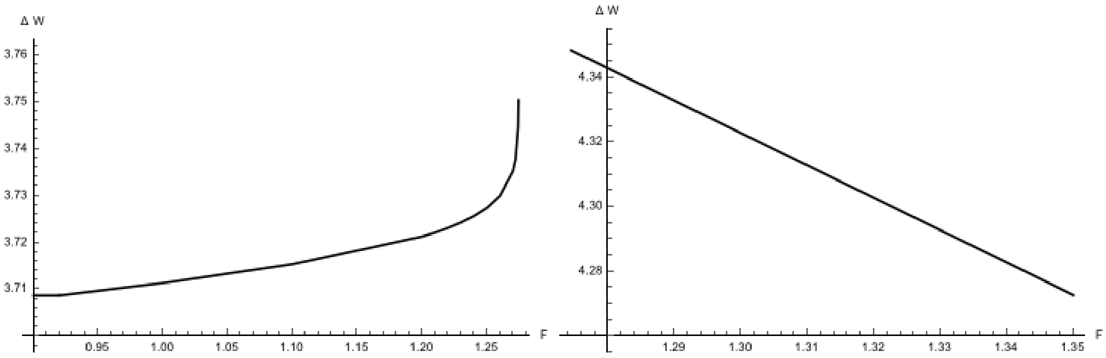
$$\max_{T, I_1(t), I_2(t)} J = \int_0^T e^{-rt}\Big(u(K_1, 0) - \frac{\gamma}{2}I_1^2\Big)dt + \int_T^\infty e^{-rt}\Big(u(K_1, K_2) - \frac{\gamma}{2}(I_1^2 + I_2^2)\Big)dt - e^{-rT}F. \tag{2.28}$$

We expect that product introduction is favorable from a social point of view as in m_2, there is a new product of higher quality which affects the consumer only positively. For the given parameter setting, we find that delaying product introduction occurs only for very large F, in particular for $F > 2.4492$ [19]. So, as expected, from

[19] Note that for the profit maximizing firm delay occurs even for $F = \epsilon$, $\epsilon > 0$, which is substantially lower than 2.4492.

the perspective of a social planner, it is optimal to introduce immediately for a wide range of F.

For the case of 'real' costs, the welfare difference of the situation of a profit maximizing firm and the situation where the firm is controlled by a social planner is depicted in Figure 2.11 for $K_1^{ini} = K_1^{ss,m_1}$. The welfare loss is initially constant



(a) (b)

Figure 2.11: Welfare gain for $K_1 = K_1^{ss,m_1}$. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15$.

as in both situations, immediate introduction is optimal (as long as $K_1^{ss,m_1} < K_1^{lb}$) but at some critical F (where $K_1^{lb} < K_1^{ss,m_1}$), the firm starts delaying the product introduction which increases the welfare loss. However, for $F \geq \tilde{F}$, the welfare loss decreases (see Figure 2.11(b)) as the firm stays in m_1 where F does not have an effect whereas the welfare for the social planner decreases as costs of switching to m_2 increase.

2.6 Discussion of Results and Assumptions

Somewhat surprising is that the first appearance of solutions where the firm stays with the established product is accompanied by a threshold point separating finite and infinite solutions for the terminal time. One might think that the rationale behind is that m_1 and m_2 are endogenously linked as in m_2 the number of products increases. But the phase-plane analysis (given in Appendix 2.A.7) shows that this situation might occur even for a switch to an exogenously given mode, in particular whenever the terminal pair is on the unstable manifold.

From an economic perspective, delay was expected in order to discount adoption costs and increase the scrap value at the time of introduction. Our analysis shows that the decrease of established capacities is accompanied by a larger marginal value for the new product in m_2, i.e. investing in the capacities of the new product is stronger than it would be with immediate introduction.

In our analysis, we abstract from competition. However, a monopoly could turn into a competing environment if entry is possible. Thus, if there is a threat of possible entrants, we expect that this would accelerate product introductions.

Another issue is that we do not consider the phase of development of the new product. For the interpretation that the new product is developed by the incumbent himself, it is clear that the firm is not going to engage in R&D activities if the product is not introduced eventually. In the case where the product is introduced with some delay, we expect that R&D efforts would be less in the development phase which would have a similar impact on the introduction time.

For the interpretation of external developers generating a new technology where adoption costs mainly consist of buying the patent for the new technology, an alternative option to adoption costs which has to be paid once when the product is introduced, would be to consider fees per unit which has to be paid to the owner of the patent. There, as long as the fee per unit is constant and less than θ, introduction would occur immediately since fees are paid continuously, so adoption costs are 'spread over time'.

We made the assumption that capacities are fully used, i.e. production equals sales. We believe that this assumption is of minor consequence to our results since in our model, there are no capacities for the new product in T and investment in capacities has quadratic costs such that capacities are not build up as a 'lump-sum' but slowly while the capacity of the established product is reduced slowly. Moreover, in the case of delay, the incumbent starts reducing capacities even in m_1. A rigorous analysis of the full usage of capacity assumption yields that it is optimal to exploit full capacity if the following conditions hold:

$$2K_1 + \eta K_2 \leq 1, \tag{2.29}$$

$$\eta K_1 + 2K_2 \leq 1 + \theta. \tag{2.30}$$

Numerical experiments suggest that conditions (2.29) and (2.30) seem to be satisfied for reasonable values of K_1 ($\leq K_1^{ss,m_1}$)[20].

Furthermore, e.g. for decreasing demand, it is argued that in practice firms reduce prices in order to maintain production rather than reducing production due to contracts with employees and suppliers, even though such contracts are not modeled here (cf. Goyal and Netessine (2007)). However, counterexamples exist as well where firms have excess capacity e.g. for deterring entry (see Chicu (2012)).

This analysis focuses on the effect of adoption costs. However, for some products, not adoption costs but differences in production costs may be the main reason for firms to abstain from product introduction, in particular if the old and new product's production costs differ a lot. Apple had developed a mouse in 1979 whose production costs were too much such that Apple abstained from further development of this mouse and hence from introducing it (cf. Hinloopen et al. (2013)).

[20]In the case of no horizontal and vertical differentiation, i.e. $\eta = 1$ and $\theta = 0$, conditions (2.29) and (2.30) are satisfied if

$$K_1 \geq \frac{1}{3} \wedge K_2 \leq \frac{1}{3}\ , \tag{2.31}$$

or

$$K_1 \leq \frac{1}{3} \wedge K_2 \geq \frac{1}{3}\ . \tag{2.32}$$

For our default parameter setting with $F = 1.275$, (2.31) and (2.32) are satisfied. In the case of horizontal and vertical differentiation, (2.29) and (2.30) are weakened. For higher θ, the incumbent wants to build up capacities for the new product faster, but also to decrease capacities of the established product faster. For lower η, as products are more differentiated and competition of the established and the new product is weakened, investment in the new product's and disinvestment of the established product's capacities are slower. Thus, in both cases, we expect that (2.29) and (2.30) are not affected much.

2.7 Conclusion

Using a fully dynamic framework we identify different scenarios where the firm's behavior depends crucially on the capacity of the established product and on the level of adoption costs. There is an interesting case where it is not optimal for the firm to introduce the new product immediately but to delay product introduction. By delay in time, adoption costs are discounted while the firm prepares for product introduction by reducing capacities on the established market which increases the marginal value of the established and new products' capacities. Moreover, the incumbent postpones investment in new capacity and hence benefits longer from high profits before product introduction. Noteworthy is the occurrence of Skiba points where the firm is indifferent in approaching different steady states which affects the number of products produced by the firm. We assumed that firms cannot invest in capacities beforehand. Allowing for investment before introduction might have an effect on the time of introduction, in particular we expect that this would accelerate product introduction while we think that qualitative results will be the same. Furthermore, we abstained from competition which is analyzed in Chapter 3.

2.A Appendix

2.A.1

The canonical system is given by

$$\begin{aligned} \dot{K}_1 &= \frac{\lambda}{\gamma} - \delta K_1, \\ \dot{\lambda} &= (r+\delta)\lambda - (1 - 2K_1), \end{aligned} \tag{2.33}$$

and the isoclines are

$$\begin{aligned} \dot{K}_1 = 0 \quad &\Leftrightarrow \quad \lambda = \delta\gamma K_1, \\ \dot{\lambda} = 0 \quad &\Leftrightarrow \quad \lambda = \frac{1-2K_1}{r+\delta}. \end{aligned} \tag{2.34}$$

If the firm does not introduce the new product, i.e. for staying in m_1 infinitely, there is a unique steady state

$$K_1^{ss,m_1} = \frac{1}{\delta\gamma(r+\delta)+2}, \quad \lambda^{ss,m_1} = \frac{\delta\gamma}{\delta\gamma(r+\delta)+2}\ . \tag{2.35}$$

The steady state is a saddle point as the Jacobian is

$$\begin{pmatrix} -\delta & \frac{1}{\gamma} \\ 2 & r+\delta \end{pmatrix} \tag{2.36}$$

with

$$\det J = -\delta(r+\delta) - \frac{2}{\gamma} < 0. \tag{2.37}$$

The eigenvalues are given by

$$\mu_{1,2} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 + \delta(r+\delta)}, \tag{2.38}$$

so eigenvalues have different sign and the steady state is indeed a saddle point.

2.A.2

Lemma 2.2. *Condition (2.15) holds for*

$$\left(K_1^*\right)_{1,2} = -\frac{d}{f} \pm \sqrt{\frac{2\gamma r F}{f^2}}. \tag{2.39}$$

Proof.

Consider the terminal condition[21] (2.15):

$$H(K_1^*, I_1^*, \lambda(T^*), T^*) = rS\left(K_1^*\right) \tag{2.40}$$

$\Leftrightarrow$

$$(1-K_1^*)K_1^* - \frac{\gamma}{2}I_1^{*2} + \lambda(T^*)(I_1^* - \delta K_1^*) = r(V^{m_2}(K_1^*) - F) \tag{2.41}$$

$\Leftrightarrow$

$$(1-K_1^*)K_1^* - \frac{\gamma}{2}I_1^{*2} + \frac{\partial V^{m_2}}{\partial K_1}(I_1^* - \delta K_1^*) = r(V^{m_2}(K_1^*) - F). \tag{2.42}$$

[21] For convenience, we henceforth omit the dependence of state and control variables on T^*.

The HJB-equation in m_2 at T^* is given by[22]

$$(1-K_1^*)K_1^* - \frac{\gamma}{2}(I_1^{*2}+I_2^{*2}) + \frac{\partial V^{m_2}}{\partial K_1}(I_1^* - \delta K_1^*) + \frac{\partial V^{m_2}}{\partial K_2} I_2^* = rV^{m_2}(K_1^*). \quad (2.43)$$

For $I_2^* = \frac{V_{K_2}^{m_2}}{\gamma}$, we have:

$$(1-K_1^*)K_1^* - \frac{\gamma}{2}I_1^{*2} + \frac{\partial V^{m_2}}{\partial K_1}(I_1^* - \delta K_1^*) + \frac{1}{2\gamma}\left(\frac{\partial V^{m_2}}{\partial K_2}\right)^2 = rV^{m_2}(K_1^*). \quad (2.44)$$

Using (2.44) and (2.42) yields

$$rF = \frac{1}{2\gamma}\left(\frac{\partial V^{m_2}}{\partial K_2}\right)^2, \quad (2.45)$$

which under consideration of $K_2 = 0$ yields the two solutions

$$K_1^{lb} := -\frac{d}{f} - \sqrt{\frac{2\gamma r F}{f^2}}, \quad (2.46)$$

and

$$K_1^{ub} := -\frac{d}{f} + \sqrt{\frac{2\gamma r F}{f^2}}. \quad (2.47)$$

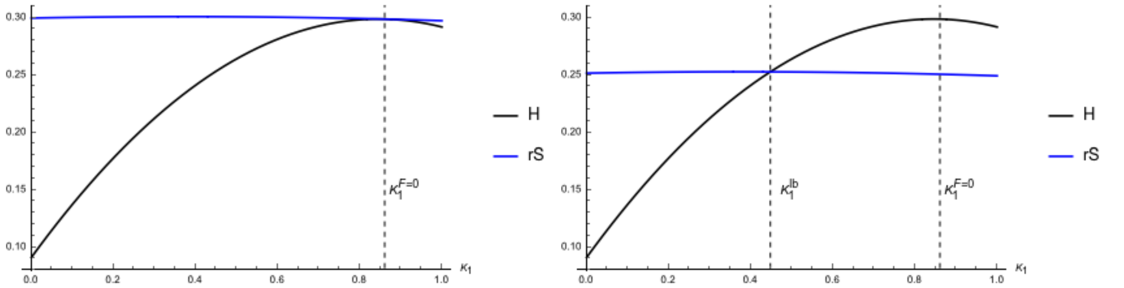
□

2.A.3

Proof of Proposition 2.1. By Lemma 2.2 in Appendix 2.A.2, we know that for $F = 0$ the terminal condition of the Maximum Principle holds for $K_1^{F=0}$ and $H < rS$ for other values of capacity[23]. For $F > 0$, F occurs negatively on the right hand side of the terminal condition and only there. Thus, there arises an interval whose bounds are given by (2.46) and (2.47) wherein $H > rS$ (see Figure 2.12). For K_1^{ini} outside the interval, the opposite holds. Hence, for $K_1^{ini} \leq K_1^{lb}$, the interest on the salvage value is higher than the current value Hamiltonian. Thus, immediate introduction is optimal.

[22] Note that F is paid for switching to m_2 and does not occur in m_2 anymore.

[23] Cf. Appendix 2.A.2. For $F = 0$, the square root in (2.39) vanishes and both solutions coincide. Moreover, note that for $F = 0$, the only extra term in (2.44) in comparison to (2.42) is $\frac{1}{2\gamma}\left(\frac{\partial V^{m_2}}{\partial K_2}\right)^2$ which is non-negative. Hence for all K_1, H is less or equal than rS (it is equal for $K_1^{lb}(= K_1^{ub})$ as $\frac{1}{2\gamma}\left(\frac{\partial V^{m_2}}{\partial K_2}\right)^2 = 0$).



(a) $F = 0$ (b) $F = 1.2$

Figure 2.12: Left hand side (H) and right hand side (rS) of terminal condition.

For $K_1^{lb} < K_1 \leq K_1^{ub}$ the optimal switching capacity K_1^{lb} has to be reached by the transversality condition. Thus the firm reduces capacities down to K_1^{lb} and innovates. □

2.A.4

Proof of Lemma 2.1. Taking the derivative of K_1^{lb} with respect to F yields

$$\frac{\partial K_1^{lb}}{\partial F} = -\frac{2\gamma r}{2f^2\sqrt{\frac{2\gamma r F}{f^2}}} = -\sqrt{\frac{\gamma r}{2Ff^2}} < 0 \ . \tag{2.48}$$

□

2.A.5

Lemma 2.3. *$\exists!\ \tilde{F} > 0$ such that $\forall F \geq \tilde{F}$, $\exists\ K_1$ with $T^*(K_1) = \infty$, i.e. $V(K_1) = V^{m_1}(K_1)$ and $\forall F < \tilde{F}, \nexists\ K_1$ with $T^*(K_1) = \infty$.*

Proof. The value function of m_1 without the option to switch to m_2 is independent of F whereas the value function of the control problem with introduction option is decreasing in F due to the decreasing salvage value. Thus, there is some $\tilde{F}$ where the value function of the control problem with introduction option hits the value function of m_1 for the first time which is greater than 0 since for $F = 0$, switching is costless and in m_2, there is the option of producing the new product which has a higher quality ($\theta > 0$)[24]. □

[24] Even for no vertical differentiation, introducing the new product is beneficial as the market

2.A.6

Lemma 2.4. *At $F = \tilde{F}$,*

$$K_1^{lb} \leq \tilde{K}_1 \tag{2.49}$$

holds.

Proof. Let $F = \tilde{F}$. Assume $\tilde{K}_1 < K_1^{lb}$. Then, for $\tilde{K}_1$, $H < rS$, which yields that the unique solution is to switch to m_2 which contradicts $F = \tilde{F}$. □

2.A.7

We first state the following lemma which is necessary for the proof of Proposition 2.2.

Lemma 2.5. *The dynamics at the terminal pair $\left(K_1^{lb}, \lambda(T)\right)$ are not $\dot{K}_1 > 0$ and $\dot{\lambda} > 0$ simultaneously.*

Proof. The terminal pair is determined by $H = rS$ and $\lambda(T) = S_{K_1}$. The line $\lambda(T) = S_{K_1} = b + cK_1$ has a positive ordinate ($b > 0$) as K_1's marginal value is positive if there are no capacities installed. One might think that this line could pass through the area to the right-upper of the intersection point of $\dot{K}_1 = 0$ and $\dot{\lambda} = 0$ where $\dot{K}_1 > 0$ and $\dot{\lambda} > 0$ hold. This would yield different dynamics than studied so far. However, one can easily show that for terminal pairs in that area, there is no candidate for an optimal solution with $0 < T^* < \infty$. In particular, for $K_1^{ini} > K_1^{lb}$, there are either no candidate paths or only non-monotone paths arriving at the terminal pair which cannot be optimal[25]. Converging to the steady state of m_1 along the stable manifold is not optimal as well as time consistency is violated since for $K_1 < K_1^{lb}$, $H < rS$ holds. Thus, there are no optimal paths for $K_1^{ini} > K_1^{lb}$ which yields a contradiction and proves that this situation cannot

is expanded and the firm is able to split the total quantity among the two products which yields a higher price (cf. Dawid et al. (2015)).

[25]Non-monotone paths imply a set of Skiba points which generates fluctuating paths for $T^* = \infty$, which contradicts to the uniqueness property of the steady state of the infinite horizon problem.

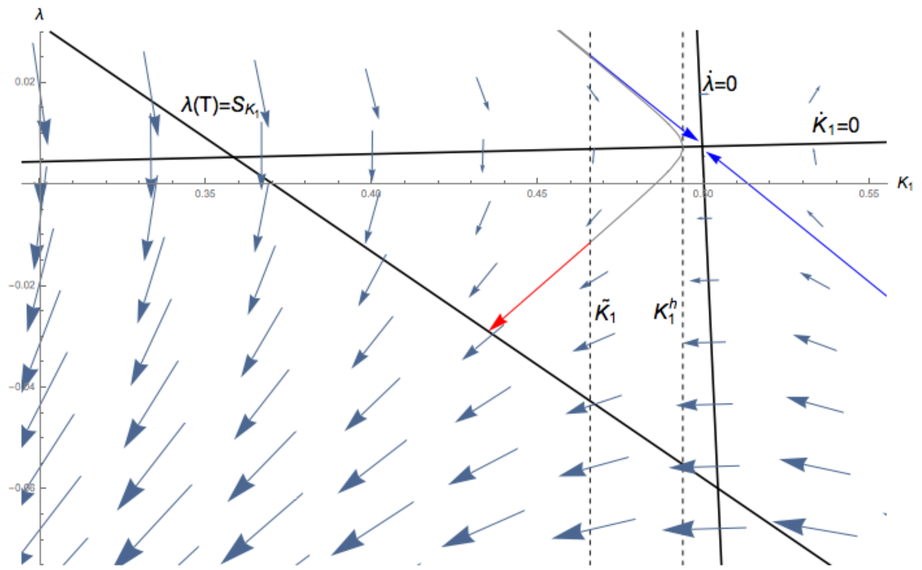


Figure 2.13: Vector plot for $F = 1.275$ $(> \tilde{F})$.

occur. Moreover, the slope of the $\lambda(T)$ line is necessarily negative ($c < 0$), i.e. the marginal value of K_1 is decreasing as $\frac{\partial^2 V^{m_2}}{\partial K_1^2} = c$.

□

Proof of Proposition 2.2. As the steady state of m_1 is a saddle-point, there is a stable and unstable manifold. If T^* is finite but not zero, then the switching pair $\big(K_1(T), \lambda(T)\big)$ in the (K_1, λ) space is derived from the condition $H = rS$ and the transversality condition $\lambda(T) = S_{K_1}$. As F increases and K_1^{lb} decreases, there is an F, where $\big(K_1^{lb}, \lambda(T)\big)$ is on the unstable manifold with $\dot{K}_1 < 0$ and $\dot{\lambda} < 0$[26]. Denote that F by F^{uns}. For arriving at that pair, the initial pair has to be on the unstable manifold. Thus, for all $K_1 \geq K_1^{ss,m_1}$, there is no optimal path which leads to $(K_1^{lb}, \lambda(T))$, i.e. for all $K_1 \geq K_1^{ss,m_1}$, $T^*(K_1) = \infty$.

Next, we prove that $\tilde{F} = F^{uns}$. Obviously, $\tilde{F} \leq F^{uns}$ [27]. Assume $\tilde{F} < F^{uns}$. Then, by Lemma 2.1, at $\tilde{F}$, the terminal pair is right to the unstable manifold. Denote for all possible terminal values $K_1(T)$ the value of the path which leads

[26]As shown in Lemma 2.5, the dynamics at the terminal pair are not $\dot{K}_1 > 0$ and $\dot{\lambda} > 0$ simultaneously. Hence, the line passes through the area where $\dot{K}_1 < 0$ and $\dot{\lambda} < 0$ holds as it has a positive ordinate and negative slope.

[27]Note that for F^{uns} infinite solutions for T exist. As $\tilde{F}$ is the minimal value of adoption costs for which infinite solutions exist, $\tilde{F} \leq F^{uns}$ holds.

to the terminal pair by $V^{term}(K_1(t), K_1(T), F)$ which in this case exists for all $K_1 \geq K_1(T)$ and for all $F < F^{uns}$ and is continuous in F.

In order to avoid confusion, for an F, we denote the corresponding K_1^{lb} by $K_1^{lb}(F)$. For $K_1^{ini} > \tilde{K}_1$,

$$V^{term}(K_1^{ini}, K_1^{lb}(\tilde{F}), \tilde{F}) < V^{m_1}(K_1^{ini}), \tag{2.50}$$

holds[28]. Hence, $\exists\ F^l < \tilde{F}$ with

$$V^{term}(K_1^{ini}, K_1^{lb}(F^l), F^l) = V^{m_1}(K_1^{ini}), \tag{2.51}$$

which contradicts the minimality of $\tilde{F}$. Hence, the assumption $\tilde{F} < F^{uns}$ was wrong and $\tilde{F} = F^{uns}$ holds.

Now, we prove that $\tilde{K}_1$ is not less than K_1^{ss,m_1} again by contradiction. Assume that $\tilde{K}_1 < K_1^{ss,m_1}$. Then, consider K_1^{int} for which $\tilde{K}_1 < K_1^{int} < K_1^{ss,m_1}$ holds. For $F = \tilde{F}$, we have[29]

$$V^{term}(K_1^{int}, K_1^{lb}(\tilde{F}), \tilde{F}) < V^{m_1}(K_1^{int}). \tag{2.52}$$

Again, by continuity of V^{term} in F, there exists an $F^l < \tilde{F}$ with

$$V^{term}(K_1^{int}, K_1^{lb}(F^l), F^l) = V^{m_1}(K_1^{int}), \tag{2.53}$$

which contradicts the minimality of $\tilde{F}$. Thus, $\tilde{K}_1 = K_1^{ss,m_1}$ and it is a threshold point[30] where the firm is not indifferent.

[28]It can not be $V^{term}(K_1^{ini}, K_1^{lb}(F), F) = V^{m_1}(K_1^{ini})$ since for $K_1^{ini} \geq \tilde{K}_1$, trajectories of the finite and infinite solution move in the same direction (as due to Lemma 2.4, $K_1^{lb} \leq \tilde{K}_1$) and according to Proposition 1 in Caulkins et al. (2015), in that case, the trajectories have to coincide for all $t \in [0, T^*(K_1^{ini})]$ which is apparently not true. Moreover, $V^{term}(K_1^{ini}, K_1^{lb}(F), F) > V^{m_1}(K_1^{ini})$ cannot hold either since this leads to another solution for the problem without introduction option via moving to $\tilde{K}_1$ along the path corresponding to the finite solution of T and switching at $\tilde{K}_1$ to the solution of the problem without introduction option.

[29]Note that in this case, V^{term} exists for $K_1 < \tilde{K}_1$. Moreover, as this problem is time invariant and trajectories of the finite and infinite solution move in opposite directions and due to the monotonicity of the trajectory of the infinite solution (see Hartl (1987)), the trajectory of the finite solution is monotone as well and there can not be an overlap region, i.e. there is no interval of Skiba points (cf. Caulkins et al. (2015)). Thus, at $\tilde{F}$ for K_1^{int}, the infinite solution is the unique optimal solution.

[30]Here, a threshold point is characterized by having finite and infinite solutions for T in every neighborhood (cf. Caulkins et al. (2015)).

□

2.A.8

Proof of Proposition 2.3. As K_1^{lb} decreases with F, for $\tilde{F} < F < \bar{F}$, the terminal pair $\Big(K_1(T), \lambda(T)\Big) = \Big(K_1^{lb}, \lambda(T)\Big)$ is left to the unstable manifold (cf. proof of Proposition 2.2 in 2.A.7). There, the dynamics are given by $\dot{K}_1 < 0$ and $\dot{\lambda} < 0$. Starting at the terminal pair $\Big(K_1^{lb}, \lambda(T)\Big)$ and moving backwards along the arc leading to it, i.e. considering V^{term} introduced in 2.A.7 (cf. Figure 2.13), we can identify candidates for the optimal starting point for different K_1^{ini}'s. This arc hits the $\dot{K}_1 = 0$ line at some K_1^h. This is the highest K_1 for which a finite candidate T exists since following the arc further gives further candidates for $K_1^{lb} \leq K_1 < K_1^h$ as there is $\dot{K}_1 > 0$, which implies non-monotone paths for K_1 which can not be optimal (cf. Appendix 2.A.7). Hence, V^{term} is well defined. For any $K_1 < K_1^{ss,m_1}$, it is also possible to converge to the steady state of m_1 by following the stable arc of the steady state. Comparing values of both candidates by taking the upper curve of the value functions corresponding to both options we obtain the value function and the optimal strategies of the control problem with introduction option. Hence, there is an indifference point $0 < \tilde{K}_1 \leq K_1^h$ where the firm is indifferent moving to the steady state along the stable manifold and moving to K_1^{lb}. Thus, $\tilde{K}_1$ is a Skiba point. As F increases, K_1^{lb} and K_1^h decreases. Next, we prove that $\tilde{K}_1$ decreases as well by contradiction. For $F^a, F^b \in (\tilde{F}, \bar{F})$, with $F^a < F^b$, denote the corresponding indifference points by $\tilde{K_1}^a$ and $\tilde{K_1}^b$ and assume that $\tilde{K_1}^a \leq \tilde{K_1}^b$, i.e. $\tilde{K}_1$ is nondecreasing in F. Then,

$$V^{m_1}(\tilde{K_1}^b) = V^{term}(\tilde{K_1}^b, K_1^{lb}(F^b), F^b) < V^{term}(\tilde{K_1}^b, K_1^{lb}(F^a), F^a) \leq V^{m_1}(\tilde{K_1}^b) \tag{2.54}$$

which yields a contradiction[31] . Hence, $\tilde{K}_1$ is decreasing in F. □

[31]The last inequality is due to the following: $\tilde{K_1}^a \leq \tilde{K_1}^b$ and for $K_1 \geq \tilde{K_1}^a$, infinite solutions are optimal.

Chapter 3

Delaying Product Introduction in a Duopoly: A Strategic Dynamic Analysis

3.1 Introduction

Technological change is a crucial driver of industrial dynamics. Improved versions of products appear regularly. Furthermore, product innovations lead to differentiated products and new submarkets arise. According to an empirical investigation by Chandy and Tellis (2000), most of the product innovations has been achieved by established incumbents. Typical examples include Asus which has been active on the notebook market and has introduced netbooks in 2007 or Apple's introduction of the iPad in 2010 which generated a huge submarket for tablet computers. For a firm competing with others on a homogeneous market, a product innovation can be very valuable. Given that a product innovation has been made, we examine whether there are incentives for an innovator not to introduce a new product immediately but to delay the product introduction strategically or not to introduce at all[1]. Wang and Hui (2012) provide examples where the market introduction of products has been delayed, e.g. DVD players and MP3-related products which

[1]Several studies (Mansfield (1977), Åstebro (2003) and Åstebro and Simons (2003)) have found out that a large fraction of product innovations is not brought to the market.

could have been introduced earlier.

Given that two firms are competing on an established homogeneous market, we assume that one of the firms has the option to introduce a new product whereas his rival sticks with producing the established product. Moreover, we assume that the new product is horizontally and vertically differentiated, in particular that it has a higher quality than the established product. Both firms are restricted by production capacities which they adjust over time. The setting after product introduction has been analyzed in Dawid et al. (2010a). They find that not only the innovator benefits but the non-innovator is better off as well in most cases, in particular if the products are not too differentiated. The innovator strongly reduces capacities on the established market in order to increase demand for the established product.

Adjustments of capacities of established products *prior* to a product innovation has been studied in a stochastic setting in Dawid et al. (2017) who consider a duopoly where both firms can also invest in R&D in order to increase the probability of product innovation (see Dawid et al. (2013b) for an exogenous hazard rate). In contrast to those approaches, we assume that the innovation has been made already and the time of product introduction is an additional choice variable and hence is not directly linked to the time of the successful completion of an R&D project. The separation of innovation and introduction has been employed by Dawid et al. (2009), however only in a three-stage model where continuous capacity adjustments are not taken into account and the timing of product introduction could not be addressed.

The game we are considering is a multi-mode differential game where one of the firms can induce a regime switch (in our context adding a second differentiated product to its product range) at any time in contrast to models where a regime switch occurs when the state variable hits some critical threshold (see e.g. Reddy et al. (2015) and Masoudi and Zaccour (2013)).

Optimal timing of innovation has been analyzed extensively in the optimal stopping and real options literature (see e.g. Dutta et al. (1995), Hoppe and Lehmann-Grube (2005) and Dixit and Pindyck (1994)). Recent contributions consider for stochastic demand, both, optimal timing and capacity choice simultaneously (see e.g. Huberts et al. (2015) and Huisman and Kort (2015)). The latter finds in a setting with two firms who have the option to enter a new market that firms invest earlier compared to the monopoly setting. In particular, the first investor overinvests in order to delay market entry of the second investor. The innovation of the present paper relative to this literature is that it considers the dynamic adjustment of capacities before and after the innovation, whereas mostly one-time investments have been treated in the real options literature.

The monopoly version of this paper has been analyzed in Chapter 2 where a deterministic setting is considered where a monopolist has the option to introduce a substitute product. Even in a monopoly, where competition effects are excluded, the firm might delay product introduction if it incurs adoption costs. By delaying the product introduction, the monopolist benefits from discounted adoption costs, which has to be paid as a lump sum at the time of product introduction. Furthermore, the monopolist can increase the marginal value of the new product by decreasing established capacities. Similar effects are also present in the duopoly here, however strategic interaction adds substantial new effects.

Optimal timing has not been considered a lot in differential game models. Yeung (2000) derives feedback Nash equilibria for games with endogenous time horizon by restricting terminal values for state variables. Recently, Gromov and Gromova (2017) formalize the class of hybrid differential games and characterize a switching manifold in the time-state space which is determined by a switching condition. They argue that deriving feedback Nash equilibria for state-dependent switching is complicated and resort to open-loop Nash equilibria, which in certain games, parametrized by initial conditions yields feedback Nash equilibria.

In terms of timing, the most related contribution is Long et al. (2017) where in a differential game model with multiple regimes, the concept of piecewise-closed loop Nash equilibria (PCNE) is introduced. They derive necessary conditions for the optimal switching time in a two player setting, where both players can induce a change of the regime of the game. The timing decision is given implicitly by the state variable arriving at a certain state which is derived by optimality conditions.

However, in their setting, it is assumed that firms commit to their switching time in the sense, that they would not alter that time even if the other firm would deviate from its equilibrium control path. Hence, the considered equilibrium is not fully Markov perfect with respect to the timing decision.

In our approach, we consider a case where the innovator can fully commit to its product introduction time. Hence, the competitor cannot influence the timing of the product introduction. An equilibrium is given if the choice of the product introduction time, T, maximizes the value of the game for the innovator while given this T, the investment strategies played by both players constitute a Markov-perfect Nash equilibrium in the classical sense. Note that the timing decision is made in the beginning of the game for given initial capacities and hence it is an open-loop strategy whereas the continuous control variables constitute a Markov perfect equilibrium using closed-loop strategies. Characterizing a fully closed-loop equilibrium in which the introduction of the new product is triggered if the state variable hits a switching manifold (to be optimally determined by the innovator) is technically challenging and might lead to non-existence of equilibria (see Long et al. (2017) for details).

From an economic perspective, the commitment to the product introduction time might be due to a preannouncement. There is a huge literature on preannouncements considering its effects on various interest groups such as consumers, competitors and others. Preannouncements are made for various purposes (cf. Lilly and Walters (1997)). They are used e.g. for building interest for the new product before the market launch (Bao et al. (2005)), in order to stimulate consumers to delay purchases, in particular to wait for a better product (Su and Rao (2010) or to deter entry of potential entrants or to induce a competitor to adjust capacities or to reposition (see Farrell and Saloner (1986) and Heil and Robertson (1991)).

We use dynamic programming for solving for the optimal capacity investment strategies and derive an optimality condition for the optimal timing which depends on the time-derivative of the corresponding value function at the outset of the game. This game might be interpreted as a two stage game where in the first stage only the innovator decides on the introduction time and in the second stage both firms play simultaneously either starting with only the established product or with both products in case that the innovator introduces immediately.

We find that whenever it is optimal to delay the product introduction, the optimal introduction time is increasing in adoption costs. Furthermore, we find that the optimal introduction time increases in both initial capacities, i.e. the stronger the innovator or the non-innovator on the established market, the later the product introduction. The latter is in accordance with results of Dawid et al. (2017) where R&D investments are negatively affected by both firms' capacities.

Additionally, we find that in a duopoly, the innovator introduces the product less often compared to a monopoly scenario. In case of product introduction, he introduces earlier compared to the monopoly. Thus, this paper contributes to the debate initiated by Schumpeter and Arrow where we see a connection between both views where market concentration facilitates innovation but slows down its arrival.

In section 3.2, we provide the model and in section 3.3, we derive a general sufficient condition for delaying the product introduction. Furthermore, we derive general necessary conditions for optimal timing which has to hold at the outset of the game. A particular parameter setting is discussed in Section 3.4. Welfare implications are characterized in Section 3.5. A discussion is given in Section 3.6. Section 3.7 concludes.

3.2 Model

We consider a duopoly where both firms, denoted by firm A and B, produce a homogeneous established product, denoted as product 1. Due to product innovation, firm A has the option to introduce a horizontally and vertically differentiated substitute product with higher quality, denoted as product 2. We call this firm the innovator whereas the other firm, firm B is called the non-innovator. The innovator incurs a lumpy cost F at the time of introduction. For simplicity, we assume that the innovator can only start to invest in the capacity of the new product after introduction, i.e. there are no capacities at the time of introduction for the new product, yet.

Before product introduction, i.e. for all $t \leq T$, the linear inverse demand function for the established product is given by

$$p_1^{m_1}(K_{1A}(t), K_{1B}(t)) = 1 - K_{1A}(t) - K_{1B}(t), \tag{3.1}$$

whereas after product introduction, i.e. for all $t \geq T$, the inverse demand system is given by

$$p_1^{m_2}(K_{1A}(t), K_{1B}(t), K_{2A}(t)) = 1 - \Big(K_{1A}(t) + K_{1B}(t)\Big) - \eta K_{2A}(t), \tag{3.2}$$

and

$$p_2^{m_2}(K_{1A}(t), K_{1B}(t), K_{2A}(t)) = 1 + \theta - K_{2A}(t) - \eta\Big(K_{1A}(t) + K_{1B}(t)\Big), \tag{3.3}$$

where η with $0 < \eta < 1$ measures the degree of horizontal and $\theta > 0$, the degree of vertical differentiation of the strategic substitutes.

The innovator wants to determine the optimal time of product introduction T and the optimal strategies for investment in capacities before and after product introduction whereas the non-innovator only determines the optimal investment strategies in its capacity of the established product. For simplicity, it is assumed that capacities are always fully used (see Section 2.6 for a discussion of this assumption). Adjustment of capacities is costly but production costs for given capacities are normalized to zero. There is no inventory, i.e. production equals sales.

In total, there are 2 modes in this capital accumulation game:

- mode 1 (m_1): New product is developed by the innovator and is ready for market introduction which is common knowledge. Only the established product is sold.

- mode 2 (m_2): New product is introduced to the market. Both products are sold.

Investment in capacities is costly, in particular comes with quadratic costs

$$C_1(I_{1f}(t)) = \frac{\gamma_1}{2} I_{1f}^2(t), \quad f = A, B, \tag{3.4}$$

and

$$C_2(I_{2A}(t)) = \frac{\gamma_2}{2} I_{2A}^2(t). \tag{3.5}$$

The capacity dynamics in m_1 are

$$\dot{K}_{1f} = I_{1f} - \delta K_{1f}, \quad f = A, B, \tag{3.6}$$

for initial capacities

$$K_{1f}(0) = K_{1f}^{ini}, \; f = A, B, \tag{3.7}$$

where $\delta > 0$ measures the depreciation rate of the capacities. In m_2, there is an additional state for the capacity of the new product which evolves in the same way according to

$$\dot{K}_{2A} = I_{2A} - \delta K_{2A}, \tag{3.8}$$

$$K_{2A}(t) = 0 \quad \forall t \leq T. \tag{3.9}$$

As in Dawid et al. (2010a), we allow the firms to intentionally scrap capacities (i.e. investments might be negative) while capacities have to remain non-negative, i.e. $K_{1f} \geq 0 \; \forall \; t$, $f = A, B$, and $K_{2A} \geq 0 \; \forall \; t$.

The discounted stream of profits of the innovator is given by

$$\begin{aligned} J_A = &\int_0^T e^{-rt}\Big(p_1^{m_1}(\cdot)K_{1A} - C_1(I_{1A})\Big)dt \\ &+ \int_T^\infty e^{-rt}\Big(p_1^{m_2}(\cdot)K_{1A} + p_2 K_{2A} - C_1(I_{1A}) - C_2(I_{2A})\Big)dt - e^{-rT}F, \end{aligned} \tag{3.10}$$

which is maximized with respect to T, I_{1A} and I_{2A}. For the non-innovator, it is given by

$$J_B = \int_0^T e^{-rt}\Big(p_1^{m_1}(\cdot)K_{1B} - C_1(I_{1B})\Big)dt + \int_T^\infty e^{-rt}\Big(p_1^{m_2}(\cdot)K_{1B} - C_1(I_{1B})\Big)dt, \tag{3.11}$$

where the control variable of firm B is I_{1B}.

3.3 Optimality Conditions

In this section, we will derive some sufficient and necessary conditions for the optimal timing of the product introduction. It should be noted that those conditions hold generally for models where two firms' controls affect the dynamics of a continuously evolving state variable and one of the firms can induce a regime switch.

For the sake of brevity, denote the capacity pair (K_{1A}, K_{1B}) by K. Let

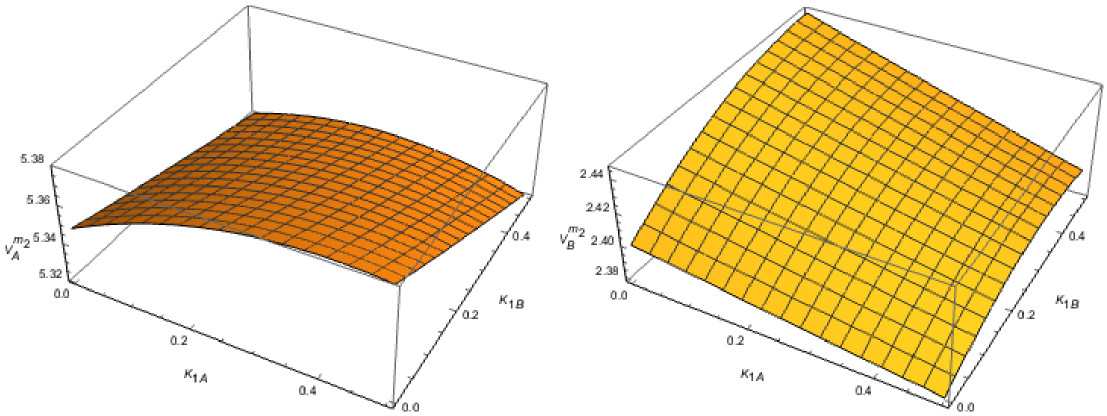
$$\phi_{1,f}(K, K_{2A}, t, m), \ f = A, B$$

be the Markovian investment strategies[2] of both firms in mode m and $T = \tau(K)$ the timing strategy of the innovator. Then, a strategy vector of the innovator is a pair $\psi_A = (\phi_{1A}, \tau)$ whereas the strategy of the non-innovator is given by $\psi_B = \phi_B$. A strategy profile (ψ_A, ψ_B) is called an equilibrium if given τ, (ϕ_{1A}, ϕ_{1B}) constitutes a Markov perfect equilibrium and τ maximizes the objective functional of the innovator.

In the case that the innovator introduces the improved product at some finite time T, there will be a structural change of the model. Denote by $V_f^{opt}(K_{1A}, K_{1B}, K_{2A}, t, m)$ the value function of firm f in mode m where the switching time from m_1 to m_2 is selected *optimally* by the innovator. Furthermore, denote by $V_f^{m_1}(K_{1A}, K_{1B})$ and $V_f^{m_2}(K_{1A}, K_{1B}, K_{2A}),\ f = A, B$, the value functions of the corresponding infinite horizon games where the mode is fixed and hence does not change. This immediately gives $V_f^{opt}(K_{1A}(t), K_{1B}(t), K_{2A}(t), t, m_2) = V_f^{m_2}(K_{1A}(t), K_{1B}(t), K_{2A}(t)) - F$, $f = A, B$ since in m_2, the mode does not change anymore. Since the infinite horizon games are time-autonomous, we consider stationary strategies and hence the value functions of those infinite horizon games do not depend on time, explicitly. The subproblem of m_2 is of linear-quadratic type which can be solved easily by the dynamic programming approach. Due to the linear quadratic structure of the game, the value functions have the following form

$$\begin{aligned} V_f^{m_2} =& C_f^{m_2} + D_f^{m_2} K_{1A} + E_f^{m_2} K_{1A}^2 + F_f^{m_2} K_{1B} + G_f^{m_2} K_{1B}^2 + H_f^{m_2} K_{2A} + J_f^{m_2} K_{2A}^2 \\ &+ L_f^{m_2} K_{1A} K_{1B} + M_f^{m_2} K_{1A} K_{2A} + N_f^{m_2} K_{1B} K_{2A}, \ f = A, B. \end{aligned} \tag{3.12}$$

[2]Note that we do not have to consider the investment strategy of firm A for product 2's capacity, explicitly, since below, we will use the value function of m_2 as a salvage value for the game.



(a) (b)

Figure 3.1: Value functions of m_2 for $K_{2A} = 0$. Parameters: $r = 0.04,\ \delta = 0.2,\ \eta = 0.5,\ \theta = 0.1,\ \gamma = 0.1$.

Using this functional form, the HJB-equations can be reduced to a set of algebraic equations which has to be satisfied by the coefficients of the quadratic value functions. Coefficients can be found by standard numerical methods for a given parameter setting (cf. Dawid et al. (2010a) for a similar model with slightly different inverse demand functions). Figure 3.1 illustrates the shape of the value functions in m_2. By regarding the value of the subproblem (minus adoption costs) as the salvage value of the finite time horizon problem in mode m_1, i.e.

$$S(K_{1A}(T), K_{1B}(T)) = V_A^{m_2}(K_{1A}(T), K_{1B}(T), 0) - F \ , \tag{3.13}$$

we can write the optimization problems of both firms in m_1 as

$$\begin{aligned} \max_{T, I_{1A}} \int_0^T & e^{-rt}\Big(p_1 K_{1A} - C_1(I_{1A})\Big)dt \\ & + e^{-rT}\Big(V_A^{m_2}\Big(K_{1A}(T), K_{1B}(T), 0\Big) - F\Big), \end{aligned} \tag{3.14}$$

and

$$\max_{I_{1B}} \int_0^T e^{-rt}\Big(p_1 K_{1B} - C_1(I_{1B})\Big)dt + e^{-rT} V_B^{m_2}\Big(K_{1A}(T), K_{1B}(T), 0\Big). \tag{3.15}$$

If an infinite time horizon is optimal, then the salvage value disappears and the value of the game is simply given by $V_f^{m_1}(\cdot)$ for $f = A, B$ and there is a unique stable steady state (see Reynolds (1987) and Jun and Vives (2004)).

As discussed above, we assume that the innovator announces the date of product introduction and has commitment power such that he cannot deviate from the announced date even though ex post it would be better to do so. Thus, the non-innovator takes T as given by the preannouncement and chooses his investment strategy in order to maximize the value of the game. Technically speaking, we employ Markov (feedback) strategies for the investment in capacities and open-loop strategies for the introduction time T.

Note that for any fixed T, the game in m_1 is still of linear quadratic structure. Since the problem in m_1 has a finite time horizon the coefficients in the value function depend on time and from the HJB-equations a set of Riccati equations for those coefficients is obtained. We solve this system using standard numerical solvers. The corresponding HJB-equations to be fulfilled are given in Appendix 3.A.2. Denote the value function of the game starting in m_1 and switching to m_2 at a fixed T by $V_f(K, t; T)$, $f = A, B$, and the corresponding profile of Markovian strategies in equilibrium by $\phi_f(K, t; T)$, $f = A, B$.

Note that the solution is time-invariant since the game is time-autonomous, i.e. t appears explicitly only in the discounting term e^{-rt}. Hence,

$$V_f(K, t; T) = V_f(K, 0; T - t), \;\; f = A, B, \tag{3.16}$$

$\forall\ K$ and $t \leq T$ (cf. Caulkins et al. (2015)). In order to endogenize the time horizon of the game, we proceed as follows. We consider a sufficiently large fixed time horizon and compute the optimal distance to the terminal time where the firm wants the game to start. For this, we use a *large* T, which is defined below.

Standard turnpike arguments (see Grüne et al. (2015) and McKenzie (1986)) yield that for $T \to \infty$, the change in the value function becomes small since it is converging to the (time-independent) value function of the infinite horizon game in mode m_1, $V_f^{m_1}$. For an ϵ with $0 < \epsilon \ll |V_A^{m_2}(K^{ini}, 0) - V_A^{m_1}(K^{ini})|$[3] and an initial capacity K^{ini}, a large T satisfies

$$\left| V_f(K^{ini}, 0; T) - V_f^{m_1}(K^{ini}) \right| \leq \epsilon. \tag{3.17}$$

[3]Note that for higher choices of ϵ, inequality (3.17) might be satisfied for all T and hence would not yield a *large* T.

We denote by $T^l(\epsilon, K^{ini})$ the minimal T for which inequality (3.17) holds for all $T \geq T^l$. Among all capacities which yield positive prices, we select the maximal T^l which we denote by $T^L(\epsilon)$, i.e. $T^L(\epsilon) := T^l(\epsilon, K_{max})$ where $K_{max} = \arg\max_K(T^l(\epsilon, K))$.

For finite T, we denote the right hand side of the HJB-equation of firm A (equation (3.44) in Appendix 3.A.2) by[4]

$$\begin{aligned} H(K) = p_1^{m_1}(\cdot)K_{1A} - C(\phi_{1A}(K,t;T)) + V^{m_2}_{A,K_{1A}}(\cdot)(\phi_{1A}(K,t;T) - \delta K_{1A}) \\ + V^{m_2}_{A,K_{1B}}(\cdot)(\phi_{1B}(K,t;T) - \delta K_{1B}). \end{aligned} \tag{3.18}$$

Note that the optimal strategies ϕ_{1A} and ϕ_{1B} stem from m_1 whereas derivatives of the value function of m_2 are considered. We assume that $V(K, t; T)$ is sufficiently smooth, i.e. let $V(K, t; T)$ be continuously differentiable in K and t for all T. Then, the following lemma gives a sufficient condition for delaying the product introduction.

Lemma 3.1. *For a K^{ini}, if*

$$H(K^{ini}) > r(V_A^{m_2} - F) \tag{3.19}$$

holds, then for K^{ini}, the optimal time of product introduction T^* is positive, possibly infinite.

Proof. Consider the value for the innovator to stay for the duration of ϵ in m_1 and afterwards to switch to m_2 under the equilibrium strategy $\phi = (\phi_{1A}, \phi_{1B})$:

$$V_A(K(0), 0; \epsilon) = \int_0^\epsilon e^{-rs} F_A^{m_1}(K(s), \phi(K(s), s; \epsilon)ds + e^{-r\epsilon}(V_A^{m_2}(K(\epsilon)) - F). \tag{3.20}$$

where $F_A^{m_1}(\cdot)$ is the instantaneous profit function of the innovator in m_1. For a finite time horizon, since we consider non-stationary strategies, altering the terminal time would yield different investments in m_1 and hence different values for the terminal capacities. Thus, for the sake of clarity, here we denote the capacity at t

[4] Actually, $H(K)$ is the Hamiltonian where the co-state variable is replaced by the state derivatives of the scrap value (cf. Pontryagin's maximum principle with finite time horizon e.g. in Dockner et al. (2000)).

for terminal time T by $K_{1f}(t,T)$, $f = A, B$. $K_{1A}(\epsilon,\epsilon)$ can then be derived via the initial value $K_{1A}(0,\epsilon)$ and the investments from 0 until ϵ:

$$K_{1A}(\epsilon,\epsilon) = K_{1A}(0,\epsilon) + \int_0^{\epsilon} (\phi_{1A}(K(\tau,\epsilon),\tau;\epsilon) - \delta K_{1A}(\tau,\epsilon))d\tau. \tag{3.21}$$

Its derivative with respect to ϵ is then given by

$$\frac{\partial K_{1A}(\epsilon,\epsilon)}{\partial t} + \frac{\partial K_{1A}(\epsilon,\epsilon)}{\partial T} \tag{3.22}$$

$$= \phi_{1A}(K(\cdot),\tau;\epsilon) - \delta K_{1A}(\cdot) + \int_0^{\epsilon} \frac{\partial \phi_{1A}(K(\tau,\epsilon),\tau,\epsilon) - \delta K_{1A}(\tau,\epsilon)}{\partial T} d\tau. \tag{3.23}$$

In equation (3.20), subtracting $V_A(K(0),0;0)$ on both sides, dividing by ϵ and considering the limit $\epsilon \to 0$ yields

$$\begin{aligned}\frac{\partial V(K,0,0)}{\partial T} &= p_1^{m_1}(\cdot)K_{1A}(\cdot) - C(\phi_{1A}(K(\cdot),0,0)) \\ &+ V^{m_2}_{A,K_{1A}}(\cdot)\left(\dot{K}_{1A}(0,0) + \frac{\partial K_{1A}(0,0)}{\partial T}\right) \\ &+ V^{m_2}_{A,K_{1B}}(\cdot)\left(\dot{K}_{1B}(0,0) + \frac{\partial K_{1B}(0,0)}{\partial T}\right) + V^{m_2}_{A,t}(\cdot) \\ &- r\left(V^{m_2}_A(K_{1A}(0,0),K_{1B}(0,0)) - F\right)\end{aligned} \tag{3.24}$$

However,

$$\frac{\partial K_{1f}(0,0)}{\partial T} = 0, \;\; f = A, B. \tag{3.25}$$

Moreover, as we consider stationary strategies in m_2, $V_{A,t}(\cdot, m_2) = 0$. Then, due to inequality (3.19),

$$\frac{\partial V_A(K,0,0)}{\partial T} > 0, \tag{3.26}$$

which proves that delaying the product introduction marginally is better than introducing immediately. □

From optimal control theory, it is known that for $H(K^{ini}) > r(V^{m_2}_A(K^{ini}) - F)$, the innovator prefers not introducing the product immediately but introducing whenever $H = r(V^{m_2}_A - F)$ holds. Here, $H = r(V^{m_2}_A - F)$ is satisfied on a *switching line* (see Appendix 3.A.1). In an optimal control setting, the firm exerts control such that the state arrives at the switching line and the switch occurs. But in a game, due to the other player who influences the dynamics of its own and its competitors capacity, this might not be possible in an equilibrium, i.e. there might not exist a terminal time T, where the state arrives at that line. In mathematical terms, the existence of such a terminal time requires that

$$H(K(T)) = rS(K(T) \quad \text{and} \quad H(K(t)) > rS(K(t) \; \forall \; t < T, \tag{3.27}$$

where $K(t)$ is the induced trajectory by the announcement of T. For the parameter setting considered in section 3.4, a terminal time satisfying conditions (3.27) could not be found.

Note that it is not possible to derive a (local) sufficient condition for immediate introduction since marginally being worse-off does not imply necessarily that immediate introduction is optimal. For some $T > 0$, the corresponding value might still outweigh immediate introduction's value.

Our main result is given in the following proposition.

Proposition 3.1. *Let $V_f(K, t; T^L)$ be the value function of the game for a fixed large end time $T^L(\epsilon)$ for $f = A, B$. Let t^* be the time argument maximizing V_A for an initial pair $K^{ini} = (K^{ini}_{1A}, K^{ini}_{1B})$, i.e.*

$$t^*(K^{ini}) = \arg \max_{t \in [0, T^L]} V_A(K^{ini}, t; T^L). \tag{3.28}$$

If $t^*(K^{ini}) > 0$, then

$$T^*(K^{ini}) = T^L - t^*(K^{ini}), \tag{3.29}$$

is the optimal time of product introduction for $K(0) = K^{ini}$ and the value function in m_1 for $f = A, B$ and for initial capacities K^{ini} is given by

$$V_f^{opt}(K, 0, t, m_1) = V_f(K, t; T^*(K^{ini})). \tag{3.30}$$

Furthermore, if $t^*(K^{ini}) = 0$ for all $T \geq T^L(\epsilon)$ (i.e. for all $T^L(\tilde{\epsilon})$ with $\tilde{\epsilon} \leq \epsilon$), then

$$T^*(K^{ini}) = \infty, \tag{3.31}$$

is the optimal time of product introduction for $K(0) = K^{ini}$ and the value function is given by

$$V_f^{opt}(K, 0, t, m_1) = V_f^{m_1}(K), \;\; f = A, B. \tag{3.32}$$

Proof. Due to time invariance, the current value of the initial game defined on the time interval $[0, T^L]$ at t^* is equal to the current value at 0 of the game defined over $[0, T^*]$ where $T^* = T^L - t^*$. Hence, it is sufficient to derive the optimal distance to a fixed terminal time where the innovator wants the game to start.

If $t^*(K^{ini}) > 0$, i.e. $t^*(K^{ini})$ is interior in $[0, T^L]$, then for all $T \geq T^L$, according to inequality (3.17), $t^*(K^{ini})$ (shifted by $T - T^L$) is still an interior maximum. Hence, $T^L - t^*(K^{ini})$ is the optimal distance to the terminal time T^L.

If $t^*(K^{ini}) = 0$ for all $T \geq T^L(\epsilon)$, then the maximizing argument is at the left boundary. More precisely, for reducing ϵ and thereby increasing T^L, $t^* = 0$ remains optimal. Thus, $V_A(K^{ini}, t, T)$ is monotonously increasing in T. Hence, $T^* = \infty$ is optimal.

□

Note that for a finite T^*, the choice of ϵ is not unique. More precisely, ϵ can be any number from the interval $(0, \bar{\epsilon})$ where $\bar{\epsilon}$ is $V(K^{ini}, 0, T^*(K^{ini}), m_1) - V^{m_1}(K^{ini})$.

Essentially, from a family of value functions of the game for different T's, i.e. for varying terminal times, the innovator has to select that one which maximizes his profits for the initial capacity. So, the optimal time of product introduction can be found via considering the value function for a fixed initial pair K^{ini} and a fixed sufficiently large terminal time and determining the optimal distance to the terminal time[5]. In the next corollary, we provide necessary conditions for the slope of the time derivative of the value function at the outset of the game.

Corollary 3.1. *i) If immediate product introduction, i.e. a corner solution $T^* = 0$ is optimal, then*

$$\lim_{T \to 0} \left(\lim_{t \to T^-} V_{A,t}(K^{ini}, t; T) \right) \geq 0, \tag{3.33}$$

and

$$H \leq rS. \tag{3.34}$$

[5]The idea of considering large values for the terminal time has been employed by several works, e.g. in Grass (2012).

ii) If no product introduction, i.e. $T^* = \infty$ *is optimal, then*

$$\lim_{T\to\infty} V_t(K^{ini}, 0; T) \leq 0, \tag{3.35}$$

iii) For an interior solution, i.e. $0 < T^* < \infty$ *to be optimal we must have*

$$V_t(K^{ini}, 0; T^*) = 0. \tag{3.36}$$

Proof. i) For a corner solution $T^* = 0$, the maximizing argument of (3.10) is on the right boundary, i.e. $t^* = T^L$. Thus,

$$\lim_{T\to 0}\left(\lim_{t\to T^-} V_{A,t}(K^{ini}, t; T)\right) \geq 0,$$

for $T^* = 0$. The HJB-equation for $T^* = 0$ yields

$$rS - V_t = H. \tag{3.37}$$

As the limit of V_t stays positive,

$$rS \geq H. \tag{3.38}$$

ii) For a corner solution $T^* = \infty$, the maximizing argument is on the left boundary, i.e. $t^* = 0$ which corresponds to $T^* = T^L$ [6]. Thus, $\lim_{T\to\infty} V_t(K^{ini}, 0; T) \leq 0$ for $T^* = \infty$.

iii) For an interior solution $0 < T^* < \infty$, the first-order condition for a maximum is given by

$$V_t(K^{ini}, 0; T^*) = 0. \tag{3.39}$$

□

Note that Corollary 3.1 yields necessary conditions only. In particular, condition (3.39) might be satisfied for local maximums which are not globally maximal.

In the derivation of the HJB-equation (see e.g. Dockner et al. (2000)), when time proceeds from t to $t + \Delta$, the value of the game is altered due to the change in the state variable and due to the change of the time which affects investment

[6]Note that T^L is selected such that it can reproduce the infinite solution.

patterns[7]. The effect on the state, i.e. the transition from $K(t)$ to $K(t+\Delta)$ is evaluated via V_K and $\dot{K}$ while the pure effect of time is taken into account via the derivative with respect to the second argument of the value function, i.e. V_t. Consider the difference of the value of the game for a fixed state variable vector when time moves from t to $t+\Delta$, $\Delta > 0$:

$$V(K^{ini}, t+\Delta; T^L) - V(K^{ini}, t; T^L). \tag{3.40}$$

As we are free to choose between $t+\Delta$ and t, (3.40) measures the change in the value function in current-value terms. If (3.40) is positive, it is (locally) optimal for the firm to choose a later starting point than t, and an earlier starting point, else. As K^{ini} is not affected by the choice of T^*, maximizing with respect to the second argument of the value function yields for fixed T^L the (globally) optimal time of product introduction of the free end time game.

3.4 Dynamics

In this section, we first examine the behavior of the firms for an exogenously given product introduction time T. We then explore optimal timing and its dependence on adoption costs and initial capacities. In case of delay, we analyze how capacities evolve before introduction.

3.4.1 Exogenous Time Horizon

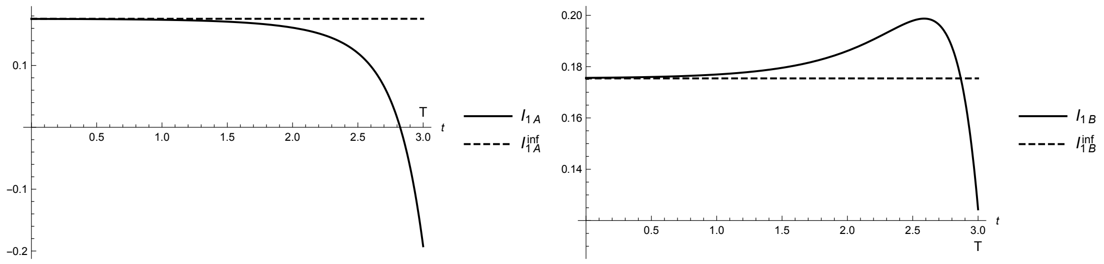
In order to depict optimal time and investment paths, we use the following fixed parameter setting (similar to the parameter setting of Dawid et al. (2010a)):

$$r = 0.04,\ \delta = 0.2,\ \eta = 0.5,\ \theta = 0.1,\ \gamma_A = \gamma_B = 0.1 \tag{3.41}$$

We start by analyzing the equilibrium investment strategies $\phi_f(K, t; T)$, $f = A, B$, for a large fixed time horizon $T^L = 3$, and fixed initial capacity $K^{ini} =$

[7] Note that investment strategies in m_1 are non-stationary.

(0.35, 0.35), which is depicted in Figure 3.2 [8]. The dashed line corresponds to the infinite horizon case in m_1. Obviously, T^L is large enough to resemble the infinite horizon investment strategy at $t = 0$. In panel (a), we see that the innovator



(a) (b)

Figure 3.2: Optimal Investments of both firms at a fixed pair of capacity $K^{ini} = (0.35, 0.35)$ for $F = 1$.

reduces his investments as time approaches T^L which is due to the decreased marginal value of the established capacity when the innovator introduces the new product. For the non-innovator, we have an interesting investment strategy which is non-monotone in t. Note that the marginal value of its capacity is decreased in m_2 as well. Hence, eventually investments decline. The initial increase is due to the innovator's decreasing willingness to invest. Moreover, there is an intertemporal strategic effect, i.e. by increasing investment, via a higher capacity and lower price in the future, a firm can even further reduce the future investment of its competitor. As the innovator is affected on both markets by the established capacity while the non-innovator is affected only at the established market (since it is not producing product 2), the non-innovator has more influence on its competitor than the other way around.

Figure 3.2 is also suitable to assess the changes in investment incentives if there is an unexpected product innovation and immediate preannouncement by the innovator, given that capacities are at (0.35, 0.35). For the innovator, this yields a downward jump of its investment in established capacities. For the non-innovator, it depends on the length of T. For $T \lessapprox 0.15$, there is a downward jump

[8]Note that in Figure 3.2, the investment strategy for a fixed capacity pair is depicted. Hence, it is not an investment trajectory.

whereas for higher T, there is an upward jump.

3.4.2 Endogenous Time Horizon

As described in Proposition 3.1, for each K^{ini}, we are able to derive the optimal T to be preannounced by the innovator. Note that due to time invariance, instead of calculating value functions for different terminal times, it suffices to calculate the value functions for a single T^L and then to determine the optimal distance to the terminal time (see section 3.6). An example is depicted in Figure 3.3.

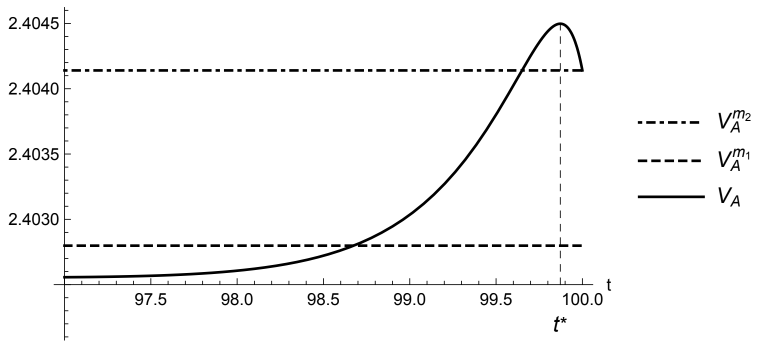
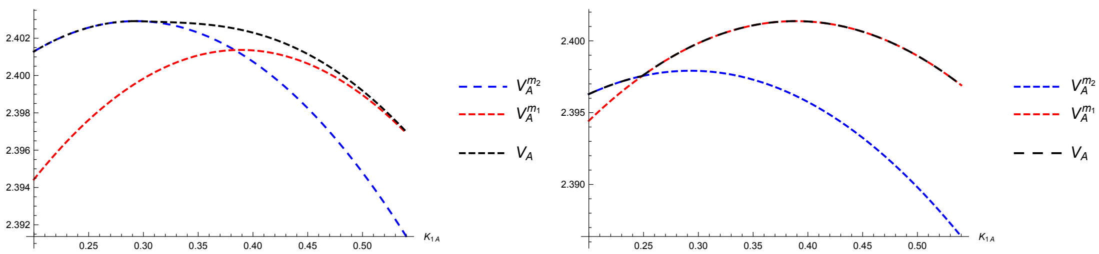


Figure 3.3: Value function for the innovator for $F = 2.94$, $K^{ini} = (0.35, 0.35)$ and for $T^L = 100$.



(a) $F = 2.94$. (b) $F = 2.945$.

Figure 3.4: Value function for $K_{1B} = K_{1B}^{m_1,ss} \approx 0.3697$.

Hence, we can obtain the value for each pair of initial states. For a fixed K_{1B}, more precisely for the steady state value of K_{1B} for the infinite horizon game in m_1 which we denote by $K_{1B}^{m_1,ss}$, the value for the innovator for different initial states K_{1A} at $t = 0$ is depicted in Figure 3.4(a) where for low initial K_{1A}^{ini}, the innovator introduces immediately whereas for higher initial capacity, there is a gain by delaying the product introduction[9].

For higher values of F, not introducing becomes optimal for high capacities and hence infinite solutions for T occur. There arises an indifference point[10], where introducing after some delay and not introducing at all yield the same value for the innovator. Moreover, in general, the value function has a kink at that point since strategies depend on the derivative of the value function w.r.t. its own capacity and strategies in the two equilibria are very different. Note that at the point where the innovator is indifferent between introducing immediately and delaying marginally (around 0.3 in Figure 3.4(a)) , the value function is smooth unless F becomes too high such that either the firm introduces immediately or never (see Figure 3.4(b)) and cf. Chapter 2 for a rigorous treatment of these issues via bifurcation diagrams in a monopoly).

For the non-innovator, in general, the value function is not smooth at that point where the innovator is indifferent between marginally delaying and introducing. The reason is that generically, if the non-innovator were the one who could decide on when to switch to m_2, then he might want to introduce earlier or later compared to the innovator's decision. Hence, for $T > 0$, at $t = 0$, the derivative of the value function for the non-innovator left and right to the switching line might be different. However, due to the transversality condition requiring value matching at $t = T$, the investment path is smooth even though it depends on the derivative of the value function. Hence, there will be no jump in the investment of the non-innovator when the innovator introduces the new product. Intuitively, in a setting with fixed switching time T firm B anticipates the marginal effect of investment on profits in m_2 even before T and therefore investment incentives do not change at $t = T$.

[9] The value functions of immediate and no switching intersect at a point where the slopes of the value functions are very different and hence there is a kink. We see that the option of delaying 'smoothes' the value functions.

[10] In the literature, indifference points are called Skiba points.

3.4.3 Optimal Timing and Investment

Here we consider the effect of adoption costs on the timing choice of the innovator. The optimal timing for the same fixed pair $K^{ini} = (K^{ini}_{1A}, K^{ini}_{1B}) = (0.35, 0.35)$ is given in Figure 3.5 where we see that for low adoption costs the firm wants to introduce the new product immediately. Above some threshold $\bar{F}_{(K^{ini}_{1A}, K^{ini}_{1B})}$, the

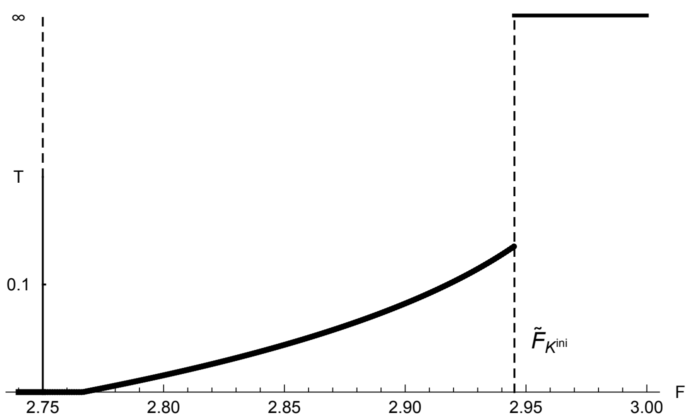


Figure 3.5: Optimal time to switch to m_2.

firm does not want to introduce the new product immediately but after some delay. This delay is higher the higher F is. There is another threshold $\tilde{F}_{(K^{ini}_{1A}, K^{ini}_{1B})}$ where the innovator abstains totally from product introduction and stays with its established product. Thus, there is a jump from some finite T to infinity at this threshold. Note that the thresholds depend on initial capacities.

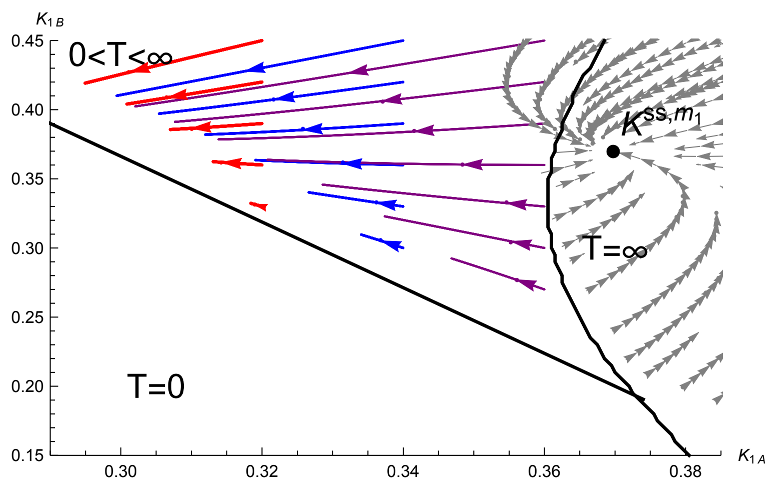


Figure 3.6: Optimal trajectories for different initial capacities.

A qualitative description of optimal timing for different levels of capacities of both firms is given in Figure 3.6. Here, the steady state of m_1 lies in the interior of the $T = \infty$ area, but still a trajectory might leave that area in the meantime and return eventually which is clearly a feature of the open-loop strategy for the timing choice. Moreover, there are parameter settings where the steady state of m_1 does not lie in the corresponding area such that every trajectory starting in the $T = \infty$ area would end up in another $[0 < T < \infty]$ area where ex-post, the firm would like to introduce the product (possibly after some delay) if there were no commitment.

Furthermore, we are interested in how the optimal time of product introduction is influenced by the capacities of both firms. Regarding the capacity of the non-innovator, one might expect that if the non-innovator is stronger on the established market, the innovator has higher incentives to introduce the new product earlier in order to escape competition. But there is another effect as well, namely higher capacity of the non-innovator leads not only to a lower price of the established product but also to a lower price of the new product in m_2. In order to compensate for that, the innovator has incentives to decrease its own capacity on the established market in m_1 in order to be 'more prepared' when switching to m_2. Figure 3.6 suggests that the latter effect is stronger such that the stronger the competitor, the later the product introduction, i.e. T is increasing in K_{1B}. Moreover, the duration in m_1 is increasing in the innovator's capacity as well. Note that for the parameters considered here, the switching line is never reached. No matter how close to that line, the innovator cannot force the state in an equilibrium with Markov-perfect investment strategies to hit that line.

Another interesting observation is that for the innovator, for every initial capacity in the delaying region, it is optimal to reduce capacity whereas for the non-innovator, the dynamics of its capacity depends on initial capacities, in particular on K_{1B}. If K_{1B} is relatively low, then its capacity increases, otherwise it decreases as well. Note that the steady state value of the non-innovators capacity in m_2 is higher than in m_1. Thus, it is very natural, that the non-innovator increases its capaticity already in m_1.

In comparison to the monopoly case where the non-innovator does not exist which has been analyzed in Chapter 2 we find the following interesting pattern: The innovator introduces earlier, i.e. the delay in product innovation is shorter but at the same time innovation occurs for a smaller range of costs of product introduction, i.e. for some F the innovator would innovate in monopoly but not in presence of a competitor even though the competitor is only active on the established market. Thus, we see a connection between the Schumpeterian and Arrowian perspective where market concentration facilitates innovation but decreases its speed.

3.5 Welfare Implications

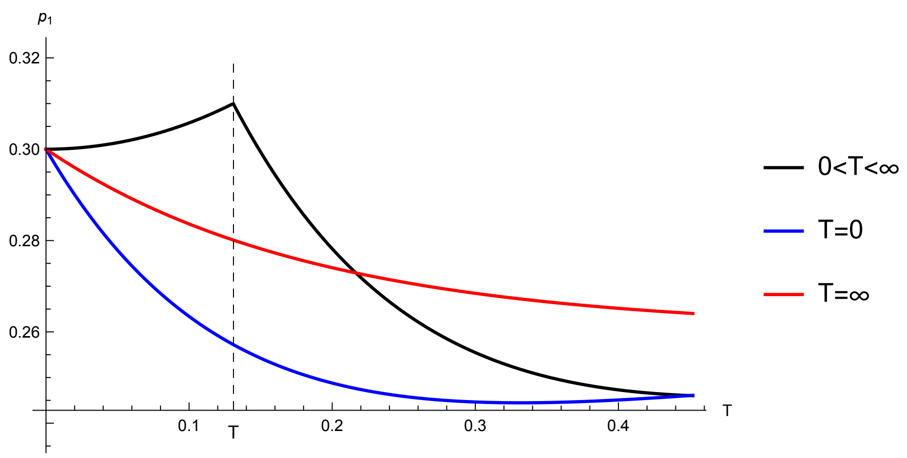


Figure 3.7: Price trajectories for $F = 2.9413$ and $K^{ini} = (0.35, 0.35)$.

Here, we aim at describing welfare implications qualitatively only[11]. As in Section 2.5, welfare depends on the interpretation of F. Interpreting it as a transfer, obviously, due to the higher quality of the new product and the monopolist's freedom to introduce the product, welfare increases, compared to no product introduction, whenever product introduction is profitable. However, for the case of delay, in m_1 the price for the established product evolves differently and hence affects consumer surplus as well. In Figure 3.7, price trajectories belonging to

[11]See Section 2.5 for a rigorous treatment.

immediate, optimal delayed and no product introduction are compared. We see that for those parameters and initial capacities, in case of delayed product introduction, the price for the established product is substantially higher compared to immediate introduction. Thus, consumer surplus if affected negatively. However, due to the reduced capacity of the innovator's established product, investment in capacities of the new product are typically slightly higher leading temporarily to a slightly lower price of the new product, compared to the immediate introduction case. Thus, the total effect of delay on consumer surplus depends on parameters and initial capacities and hence might be ambiguous.

3.6 Discussion

Due to the time invariance property of the considered problem, the optimal strategies in m_1 depend only on the distance of the current time and the terminal time. Hence, it is sufficient to calculate the optimal strategies for a large T^L and then to look for the optimal starting point, i.e. to go backwards in time. This facilitates the analysis in view of the fact that we have to compute optimal investment strategies only once instead for all possible T.

As we assumed that the innovator's timing choice is an open loop strategy and hence he is committed to his decision of T, the optimal investment strategies are subgame perfect given that T is fixed. But if the firm were allowed to make a new choice of the time of introduction, then it might choose a different terminal time.

Technically speaking, we employ open-loop strategies for the decision of optimal innovation time and closed-loop strategies for the decision of optimal investment in capacities. At first sight, this might look like an apparent drawback of the solution method since the innovator might not want to commit to its timing decision. However, in this asymmetric setting, whenever the innovator fears to be worse off in an equilibrium with closed-loop timing strategies, he could accomplish playing a different equilibrium by preannouncing the time of the new product introduction, thereby generating such strong commitment as considered in our setup.

3.7 Conclusion

We identified different scenarios depending on the initial capacities and the value of adoption costs. In the interesting case of delay of product introduction, the innovator reduces capacities of the established product before the new product is introduced whereas the dynamics of the non-innovator's capacity depends on initial capacities. Compared to the monopoly setting, the innovator abstains from product introduction more often. We derived sufficient conditions for delaying the product introduction and necessary conditions for the time derivative of the value function which has to hold at the outset of the game. An interesting topic for future research seems to be the investigation of the existence of a fully closed loop equilibrium.

3.A Appendix

3.A.1

As derived in Lemma 3.1, the innovator is indifferent between waiting marginally and introducing the new product if and only if $H = rS$, which reduces to

$$\frac{1}{2\gamma_2}\left(\frac{\partial V_A^{m_2}}{K_{2A}}\right)^2 = rF. \tag{3.42}$$

Rearranging equation (3.42) yields the switching line

$$K_{1B} = \frac{\sqrt{2r\gamma_2 F} - H_A^{m_2} - M_A^{m_2} K_{1A}}{N_A^{m_2}}. \tag{3.43}$$

3.A.2

Given the terminal time T, the HJB-equations for non-stationary Markovian investment strategies are given by

$$\begin{aligned} rV_A(K_{1A}, K_{1B}, t) - \frac{\partial V_A(K_{1A}, K_{1B}, t)}{\partial t} = \max_{I_{1A}} \Big[& p_1 K_{1A} - C_1(I_{1A}) + \frac{\partial V_A}{\partial K_{1A}}(I_{1A} - \delta K_{1A}) \\ & + \frac{\partial V_A}{\partial K_{1B}}(I_{1B}^* - \delta K_{1B}) \Big] \end{aligned} \tag{3.44}$$

and

$$rV_B(K_{1A}, K_{1B}, t) - \frac{\partial V_B(K_{1A}, K_{1B}, t)}{\partial t} = \max_{I_{1B}} \Big[p_1 K_{1B} - C_1(I_{1B}) + \frac{\partial V_B}{\partial K_{1A}} (I^*_{1A} - \delta K_{1A}) + \frac{\partial V_B}{\partial K_{1B}} (I_{1B} - \delta K_{1B}) \Big] \tag{3.45}$$

with the transversality conditions

$$V_f(K_{1A}(T), K_{1B}(T), T) = V_f^{m_2}(K_{1A}(T), K_{1B}(T), T), f = A, B. \tag{3.46}$$

Maximizing the right hand side of the HJB-equations yields

$$I_{1f} = \frac{1}{\gamma} \frac{\partial V_f}{\partial K_{1f}}, \quad f = A, B. \tag{3.47}$$

Additionally, firm A has to select the optimal value for T maximizing its discounted stream of profits. Due to the linear-quadratic structure of the game, we impose the following form for the value function:

$$V_f = C_f(t) + D_f(t)K_{1A} + E_f(t)K_{1A}^2 + F_f(t)K_{1B} + G_f(t)K_{1B}^2 + L_f(t)K_{1A}K_{1B}, \ f = A, B. \tag{3.48}$$

Due to the finite time horizon, we consider non-stationary Markovian strategies and hence coefficients depend on time. Comparison of coefficients yields the following system of 12 riccati differential equations which are solved by standard numerical methods:

$$
\begin{aligned}
rC_A(t) &= \frac{D_A(t)^2 + 2F_A(t)F_B(t) + 2\gamma_1 C'_A(t)}{2\gamma_1} \\
rD_A(t) &= \frac{\gamma_1 + D_A(t)(-\gamma_1\delta_1 + 2E_A(t)) + F_B(t)L_A(t) + F_A(t)L_B(t) + \gamma_1 D'_A(t)}{\gamma_1} \\
rE_A(t) &= \frac{2E_A(t)(-\gamma_1\delta_1 + E_A(t)) + L_A(t)L_B(t))}{\gamma_1} - 1 + E'_A(t) \\
rF_A(t) &= \frac{2F_B(t)G_A(t) + F_A(t)(-\gamma_1\delta_1 + 2G_B(t)) + D_A(t)L_A(t) + \gamma_1 F'_A(t))}{\gamma_1} \\
rG_A(t) &= \frac{G_A(t)(-4\gamma_1\delta_1 + 8G_B(t)) + L_A(t)^2 + 2\gamma_1 G'_A(t))}{2\gamma_1} \\
rL_A(t) &= \frac{2(-\gamma_1\delta_1 + E_A(t) + G_B(t))L_A(t) + 2G_A(t)L_B(t) + \gamma_1(-1 + L'_A(t))}{\gamma_1} \\
rC_B(t) &= \frac{2D_A(t)D_B(t) + F_B(t)^2 + 2\gamma_1 C'_B(t)}{2\gamma_1} \\
rD_B(t) &= \frac{D_B(t)(-\gamma_1\delta_1 + 2E_A(t)) + 2D_A(t)E_B(t) + F_B(t)L_B(t) + \gamma_1 D'_B(t)}{\gamma_1} \\
rE_B(t) &= \frac{(-4\gamma_1\delta_1 + 8E_A(t))E_B(t) + L_B(t)^2 + 2\gamma_1 E'_B(t))}{2\gamma_1} \\
rF_B(t) &= \frac{\gamma_1 + F_B(t)(-\gamma_1\delta_1 + 2G_B(t)) + D_B(t)L_A(t) + D_A(t)L_B(t) + \gamma_1 F'_B(t)}{\gamma_1} \\
rG_B(t) &= \frac{2G_B(t)(-\gamma_1\delta_1 + G_B(t)) + L_A(t)L_B(t))}{\gamma_1} - 1 + G'_B(t) \\
rL_B(t) &= \frac{2E_B(t)L_A(t) + 2(-\gamma_1\delta_1 + E_A(t) + G_B(t))L_B(t) + \gamma_1(-1 + L'_B(t))}{\gamma_1}
\end{aligned}
\tag{3.49}
$$

with transversality conditions $C_f(T) = C_f^{m_2}, D_f(T) = D_f^{m_2}, E_f(T) = E_f^{m_2}, F_f(T) = F_f^{m_2}, G_f(T) = G_f^{m_2}, L_f(T) = L_f^{m_2}, f = A, B.$

Chapter 4

Optimal Pricing of an Improving Durable Good in the Presence of Rational Consumers

4.1 Introduction

Nowadays, consumers care a lot about improvements of products, in particular technological products. There, the question arises how those improvements, e.g. built-in into new versions of products, affect the market. Potential buyers might react by postponing the purchase and waiting for the market launch of the improved product. Indeed, many products are refreshed in regular time intervals. Examples include smartphones, processors and cars. Lobel et al. (2016) find that there is a substantial difference in sales before and after new smartphone introductions by Apple. Another example is the announcement of Airbus in 2001 to introduce the A380 in 2006 which has been assessed to have a negative impact on sales of the predecessor aircraft (see Kristiansen (2006)).

Consider a consumer who has some valuation about some current product but is aware of an improved version of that product which will be introduced at a certain level of quality and at a certain point in time in the future. If he buys now, he starts consuming immediately. Alternatively, he could wait for the improved version of that product and derive some higher utility per period with the drawback of starting consuming later. The intuition is that if the market launch is far away, only few care about future products but as time evolves and the introduction of the improved product approaches, some consumers might decide to postpone the purchase and to wait for the better product. For heterogeneous consumers, we are interested in which consumer types are going to wait for the new product and which are going to buy the established product and whether there are consumers who do not care at all about product innovations.

By setting a relatively high price for the current product, the firm would decrease its sales but this would cause more accumulation of consumers and more sales of the new product. In the extreme case, the monopolist could set a high price which is above the willingness-to-pay of the consumer with highest valuation such that no consumer buys the established product in order to sell only the improved product when it is introduced. Contrary, setting a low price for the established product would result in less consumer accumulation and hence a decrease of sales of the new product.

One critical issue is that the price for the new product has to be optimal at the time of product introduction, i.e. optimal ex-post. This complicates the analysis as the price expectation of the new product influences demand of the established product and hence the final distribution of consumers at the time of product introduction. Hence, we aim at finding a rational expectations equilibrium (cf. Stokey (1981)) which in this setting reduces to finding a solution to a fixed-point problem. If the firm had commitment power, he could announce the price of the new product at the beginning, thereby influencing sales and hence the distribution of the consumers at the introduction time of the new product. Here, the classical problem of intertemporal pricing arises that the firm announces a price which maximizes overall profits but is not optimal ex-post since there might be an incentive to deviate from that announced price.

We identify different cases, which depend on the pricing of the established and new product, where the evolution of demand differs substantially. For the equilibrium pricing, we find that in most cases, the pricing is such that the price of the new product is not commensurate with its quality, in particular it is cheaper in per quality units. Consumers start earlier to wait and this leads to a relatively high accumulation of consumers which enables the firm to charge a relatively low price for the second product which increases sales of the new product. In particular, sales stop before the introduction time of the new product. A real-world example of a firm introducing regularly new versions and stopping sales early is OnePlus which has announced in March 2018 that the current version of its smartphone (OnePlus 5T) will no more be available even though the new version (OnePlus 6) was not introduced, yet[1].

The paper is organized as follows. In Sect. 4.1.1 we discuss related literature. In Sect. 4.2, we introduce the model. The analysis of consumers' and the monopolist's behavior is given in Sect. 4.3. Firm Objectives and terminal consumer distributions are presented in Sect. 4.4. A numerical example in provided in Sect. 4.5. Sect. 4.6 concludes.

4.1.1 Related Literature

In a durable goods setting, the Coase conjecture (Coase (1972)) states that due to the lack of commitment of a monopolist to future prices, consumer expectation of decreasing prices leads to delay of purchases and as price adjustments become more frequent the monopolist profit converges to zero. This work has inspired works in the intertemporal pricing literature by Stokey (1981), Bulow (1982), Gul et al. (1986) and Besanko and Winston (1990) where the Coase conjecture has been considered for particular settings.

In that literature, delay of purchase of a product is driven by the expectation of declining prices. In our model, we would like to focus on the delay of purchase caused by consumers awareness of a new improved product in the future. For analytical tractability and in order to analyze only the effect of the product innovation, we assume that the price of the current product is fixed and hence constant over time.

[1]See e.g. https://www.androidcentral.com/oneplus-5t-no-longer-sale-north-america.

Most of the durable goods literature assumed a simultaneous arrival of consumers until Conlisk et al. (1984) and Sobel (1991) allowed for sequential arrival of consumers which weakens the monopolist's propensity to lower prices as time passes. In the management literature, sequential consumer arrival is typically modeled via a Poisson process (see e.g. Elmaghraby et al. (2009) and Yin et al. (2009) and the survey by Gönsch et al. (2013)). Here, we assume a constant arrival rate as has been done in Su (2007) in order to keep the model simple.

Following the literature on vertical differentiation (see e.g. the seminal work by Mussa and Rosen (1978)), we assume that consumers' taste for quality differs. For two types of consumers, Moorthy and Png (1992) analyze differences between simultaneous and sequential selling of different qualities. Assuming homogeneity of consumers, Anton and Biglaiser (2013) consider optimal pricing of subsequent products of different quality if consumers remain in the market and have the opportunity to upgrade to a newer product. In Fudenberg and Tirole (1998), upgrades are also considered. Additionally, secondhand markets are taken into account in a two-period framework to analyze the impact of improving products for heterogeneous consumers under different informational assumptions, in particular anonymous, semianonymous and identified consumers (see Zhao and Jagpal (2006) for a related work with entry of new consumers). Fishman and Rob (2000) consider accumulated R&D costs for improving a product's quality and implementation costs of introducing new products to the market. However, they consider only a homogeneous set of consumers.

Lobel et al. (2016) consider the optimal launch policy of a monopolist who faces strategic consumers. At any time, the firm has the option to implement the current technology, which is driven by a Brownian motion, to its product. They consider a stock of consumers which do not leave the market after purchase but can improve their utility by switching to the new product. However, the price for consecutive versions of a product is assumed to be the same. This can be interpreted as an announcement of prices. By relaxing this assumption and allowing for different prices for consecutive versions of a product, we can take into account the effect of different prices on consumer accumulation and whether it is consistent with consumers' expectations. In particular, the prices has to be selected such that given the resulting terminal distribution of consumers, the price for the new product is optimal ex-post. Kristiansen (2006) is another paper which considers in a three-stage model the effect of expected product innovations on R&D and hence the timing of product introductions under competition.

Another important issue is how demand evolves over time. In diffusion models (see e.g. the seminal work by Bass (1969)), consumer categories such as early adopters and laggards are distinguished (see Krankel et al. (2006) for a recent work). Here, demand varies as well, but not because of diffusion effects (we assume that the product is well known directly from the scratch) but because of an expectation of a future product.

Our paper is most related to Dhebar (1994) (see Kornish (2001) for some related work) who has analyzed the impact of improving products on consumers and firms optimal strategies within a two-period model. There, in the first period consumers can buy the established product or wait for the second period in order to buy the improved product. Banerjee and Soberman (2013) consider a similar model with two types, i.e. high and low type consumers with different size and derive differences between myopic and forward-looking buyers and differences when quality is observable and not. Unfortunately, those frameworks are not able to investigate dynamic issues such as consumers changing willingness-to pay during the time interval where the improved product has not been introduced yet. Thus, we employ a dynamic framework in continuous time and investigate changing consumer behavior and optimal pricing strategies even before the new product is introduced.

4.2 Model

We consider a monopolist who sells a durable product, denoted by 1, with quality q_1 in $t = [0, T)$ which is replaced by a new version, denoted by 2 with higher quality q_2 in $t = T$, i.e. $q_2 > q_1$. Hence, in every instant of time, there is only one product sold, i.e. the older product is no more sold as soon as the new product is introduced[2]. For simplicity, production is costless and the problem ends at T. Here, the time of product introduction is exogenously given by T and is common knowledge. Resales are forbidden.

Consumers want to buy exactly one unit and are uniformly distributed in the unit interval. Consumer's valuation is denoted by θ, i.e. $\theta \in [0, 1]$. $\theta = 0$ is the consumer with lowest and $\theta = 1$ is the consumer with highest valuation. At the outset of the problem, there are no consumers. Consumers are infinitesimally small and arrive according to a deterministic flow of constant rate. Consumer arrival rate is normalized to one (cf. Su 2007).

The durable good is infinitely durable and consumers who have bought leave the market forever. Consumers who haven't bought yet remain in the market. The firm is free to choose the price of its products, but we assume that the price is kept fixed over the selling period. The quality of the new product is common knowledge and based on this and the price of the established product, consumers form a price expectation for the new product[3]. Given the current price of the established product p_1 and the expected price of the improved product p_2^e in $t = T$, consumers in the market decide whether to buy the established or the new product.

The monopolist maximizes his discounted stream of profits by setting prices optimally taking into account that consumers build rational expectations for the price of the new product and can infer in an equilibrium the optimal price at T for the new product from the distribution of consumers at T.

Either a consumer prefers buying the existing product immediately upon arrival[4] or prefers waiting for the new product which will be introduced at T. Either way, consumers do not buy necessarily. They buy only if buying yields also posi-

[2]An implicit assumption is that the new product has been developed but not introduced, yet. We assume that its quality is known.

[3]In Dhebar (1994), the quality of the new product is endogenous, determined via R&D efforts of the firm. Hence, there consumers build an expectation for the quality as well.

[4]Note that as we have assumed that the price of the established product is fixed over $[0, T)$, there is no gain of buying the established product later. In particular, for a positive discount rate, there is a loss of buying product 1 later. This claim is formally proved below in Lemma 4.1.

tive consumer surplus. If their valuation is below the price charged, they do not buy at all even though they prefer buying the one product over the other.

How consumers behave depends on their individual valuation, i.e. its type (speaking figuratively its location in the unit interval), the remaining time up to the product introduction, the extent of quality improvement of the new product and on prices, in particular the current price of the established product and the expected future price of the new product.

4.3 Analysis

We start by analyzing the behavior of consumers and proceed then to the analysis of the monopolist.

4.3.1 The Consumers' Problem

Let $u_1^{CV}(\theta)$ represent the current-value consumer utility if she purchases the established product and $u_2^{CV}(\theta)$ if she purchases the improved product in period T. For a consumer θ, buying the established product with quality q_1 for the fixed price p_1 yields utility

$$u_1^{CV}(\theta) = q_1\theta - p_1. \tag{4.1}$$

Buying the new product yields utility[5]

$$u_2^{CV}(\theta) = q_2\theta - p_2^e. \tag{4.2}$$

Note that for consumers in the unit interval, $p_2 \leq q_2$ holds necessarily since otherwise, no consumer would buy at T which is not optimal, ex-post.

[5]We have assumed a linear correlation between the valuation of consumers and the importance of technological improvement, i.e. the higher the valuation for the product, the higher is the benefit of waiting. This seems to be reasonable since on the one hand a consumer who has very low value for the product do not care much about its quality improvements. On the other hand, if some consumer derives high utility of using a product, the new version has a higher influence on her.

Let us denote the present-value of utility for a consumer arriving at s and buying at t by

$$u_1(\theta, t; s) = e^{-r(t-s)} u_1^{CV}(\theta) \tag{4.3}$$

where $r > 0$ is the discount rate. Analogously, waiting for the new product yields in present-value terms

$$u_2(\theta, T, s) = e^{-r(T-s)} u_2^{CV}(\theta). \tag{4.4}$$

As the price of the established product is assumed to be constant before the introduction of the new product, there is no gain from waiting and buying the established product later.

Lemma 4.1. *If a consumer buys the established product, then he buys immediately (upon arrival).*

Proof. As the price is constant and does not change in $[s, T)$, buying immediately upon arrival at s strictly dominates every purchasing time in (s, T). □

Thus, for buying the established product, we consider $u_1(\theta, s; s)$ which does not depend on s, however. Thus, we simply write $u_1(\theta)$ and we henceforth omit the time argument in $u_2(\theta, T; s)$ and simply write $u_2(\theta; s)$.

A consumer θ prefers buying the established product immediately upon arrival in s than in T if

$$u_1(\theta) \geq u_2(\theta; s) \tag{4.5}$$

$\Leftrightarrow$

$$q_1\theta - p_1 \geq (q_2\theta - p_2^e)e^{-r(T-s)} \tag{4.6}$$

(self-selection constraint) and if

$$u_1(\theta) \geq 0 \tag{4.7}$$

(market-participation constraint for the established product).

Let us rearrange the self-selection constraint in order to get more insight to the relation of prices and qualities:

$$e^{-r(T-s)} p_2^e - p_1 \geq \theta(e^{-r(T-s)} q^2 - q^1). \tag{4.8}$$

From this inequality, we can infer that a consumer prefers buying the established product if the increase in price in present-value terms exceeds the increase in quality, again in present-value terms[6]. As we will see below, it is important to distinguish the cases where the product is improving in present-value terms and not. Hence, we denote by *case A* the situation where the product is not improving, i.e.

$$e^{-r(T-s)}q^2 - q^1 \leq 0, \tag{4.9}$$

and by *case B*, where it is improving, i.e.

$$e^{-r(T-s)}q^2 - q^1 > 0. \tag{4.10}$$

By the assumption of $q_2 > q^1$, we will certainly end up in case B before T. In particular, for relatively high T, the problem starts in case A and switches to case B as the time evolves or for relatively low T, the problem starts immediately in case B.

Dhebar (1994) argues that in case B, a necessary condition for consumers buying the established product is that the price is expected to be increasing as well, in particular the expected increase in price has to overweigh the increase in quality since otherwise, waiting and buying the new product in T would be better whenever the market-participation constraint is satisfied.

In the next lemma, we characterize the situation for consumers with relatively low valuation for the product.

Lemma 4.2. *For all consumers $\theta < \frac{p_2^e}{q_2}$, the market-participation constraint for the new product is not fulfilled, i.e. the value of the improved product is negative.*

Proof. $u_2(\theta; s)$ is negative for all $\theta < \frac{p_2^e}{q_2}$ and for all s. □

Lemma 4.2 leads directly to the following conclusion.

Conclusion 4.1. *For all consumers $\theta < \frac{p_T^e}{q^2}$, if the market-participation constraint for the established product is satisfied, consumers buy the established product.*

[6]Note that the increase in quality is scaled by θ, i.e. the individual valuation.

Proof. Due to Lemma 4.2, for those consumers the value of the improved product and hence the right-hand side of inequality (4.6) is negative. If the market-participation constraint holds, then the left-hand side of inequality (4.6) is positive such that the inequality holds, i.e. buying the established product is optimal. □

Thus, for all consumers $\theta < \frac{p_T^e}{q^{2,e}}$, the market participation constraint is the binding constraint only. Hence, setting $u_1(\theta) = 0$ and rearranging yields the willingness-to-pay (henceforth wtp) function for consumers $\theta < \frac{p_2^e}{q_2}$,

$$wtp_1(\theta, t) = \theta q_1, \quad \theta < \frac{p_2^e}{q_2}, \tag{4.11}$$

which actually does not depend on time.

According to inequality (4.6) , for all consumers $\theta \geq \frac{p_2^e}{q_2}$, if the self-selection constraint is fulfilled, then the market participation constraint holds necessarily, i.e. $u_1(\theta) \geq 0$. Thus, we can neglect the latter constraint and consider the self-selection constraint for purchase timing. By rearranging inequality (4.6), we can derive the wtp of a consumer, i.e. the highest price the firm could charge from consumer θ such that she still buys:

$$wtp_1(\theta, t) = \theta q_1 + e^{-r(T-t)}(p_2^e - \theta q_2), \quad \theta \geq \frac{p_2^e}{q_2}, \tag{4.12}$$

where the second term is always negative, i.e. compared to the no innovation setting, the awareness of the introduction of a better product in the future reduces the wtp of consumers with relatively high valuation[7]. Note that the wtp functions coincide for p_T^e/q^2, i.e. the wtp function is continuous in θ and has a kink at p_2^e/q_2. In contrast to (4.11), for $\theta > p_2^e/q_2$ the wtp depends explicitly on time. Let us rearrange (4.12) in order to gain more insight:

$$wtp_1(\theta, t) = \theta(q_1 - q_2 e^{-r(T-t)}) + p_2^e e^{-r(T-t)}. \tag{4.13}$$

Neglecting the option of buying the established product, the price of the new product discounted to the present time t is given by $p_2^e e^{-r(T-t)}$. Taking this option into account, the comparison of the established and the new product is measured

[7]Note that in the no innovation case, only the market-participation constraint has to be fulfilled.

via the first term which is positive in case A and negative in case B. In case A where the product is not improving in present-value terms, it measures the mark-up a consumer is willing to pay in order to consume now and avoid waiting. In Case B where the product is expected to improve in present-value terms, it measures the mark-down the consumer wants to save in order to consume the established 'worse' product now instead of waiting and consuming the better product.

For starting in case A, the switching time to case B can be easily derived and is given by

$$\tau := T - \frac{\ln q_2 - \ln q_1}{r}, \tag{4.14}$$

which is less than T as we have assumed that $q_2 > q_1$. If the problem starts in case B, then we set $\tau := 0$. The situation at τ for starting in case A is characterized in the following Lemma.

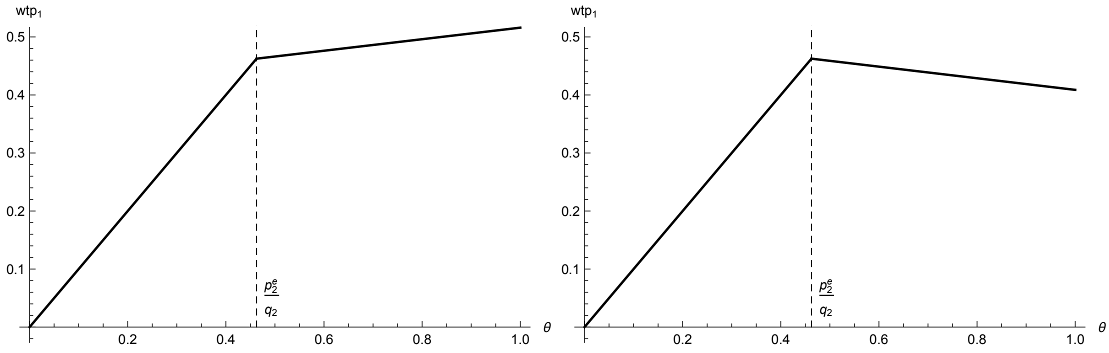
Lemma 4.3. *If $\tau > 0$, i.e. for starting in case A, at the switching time τ, the wtp of all consumers $\theta \geq \frac{p_2^e}{q_2}$ is the same, namely $e^{-r(T-\tau)}p_2^e$.*

Proof. According to the definition of cases A and B, at τ, $q_1 - q_2e^{-r(T-t)} = 0$ such that the first term of the wtp function cancels out and it remains $wtp_1(\theta, \tau) = e^{-r(T-\tau)}p_2^e$ which is independent of θ. □

Lemma 4.3 states that at τ there exists a threshold, namely $\frac{p_T^e}{q^{2,e}}$, above which every consumer has exactly the same wtp.

In case B, for $\theta \geq \frac{p_T^e}{q^{2,e}}$, the higher the valuation, the lower is the willingness to pay for the established product since there the product is improving. It shows that high-valuation consumers' attention is devoted to the new improved product whereas low-valuation consumers do not care at all about product innovation. Note that the wtp function for $\theta > \frac{p_T^e}{q^{2,e}}$ decreases in t whereas for $\theta \leq \frac{p_T^e}{q^{2,e}}$, the wtp is constant.

In Figure 4.1a), we illustrate that initially the new product is not improving in present-value terms. The wtp has a kink and follows intuition that the higher the valuation for a product, the higher the wtp, i.e. the wtp is monotonously increasing. In Figure 4.1b) however, the new product is improving in present-value terms. Hence, there is a peak at $\theta = p_2^e/q_2$. A consumer located in the



(a) case A (b) case B

Figure 4.1: Willingness-to-pay functions in case A and B.

center has a higher wtp than a consumer located at the ends. For a price below the kink, it can be directly observed that central consumers buy whereas low types do not buy since it does not yield positive consumer surplus and high types do not buy as well but for a different reason,they prefer to wait for the improved product.

4.3.2 The Monopolist's Problem

Up to now, we have only characterized the demand structure. This section is devoted to the analysis of the monopolist's behavior. As mentioned above, we consider a firm who wants to set prices p_1 and p_2 such that the discounted stream of profits is maximized. A durable goods monopolist creates his own competition: by selling today he decreases demand tomorrow. There are the following strategic issues to take into account. Demand varies due to two reasons, first because of the varying willingness to pay of the consumers and second due to the influx of new consumers. In the language of dynamic optimal control problems, one may interpret the prices as the control variables and the stock of consumers or more specifically the distribution of consumers as the (infinitely dimensional) state variable.

Let $g(\theta, t; p_1, p_2^e)$ be the density function of consumers at time t describing the stock of consumers and $G(\theta, t; p_1, p_2^e)$ its cumulative distribution function. For simplicity, the arrival rate is given by 1 and at the outset there are no consumers. For instance, if prices are set such that consumers do not buy, then $G(\theta, t; p_1, p_2^e)$

is given by $t\theta$. Let $I(\theta, p_1, p_2^e)$ be the time interval[8] where consumer θ buys the established product. Let $\pi_t(p_1, p_2^e)$ be the profit of the firm for $t < T$. For $t = T$, we have $\pi_T(p_2, G(\theta, T; p_1, p_2^e))$, which depends on the final distribution of consumers which is generated by the expected price of the new product and on the price p_2 which is indeed set by the firm at T.

We are interested in finding prices p_1 and p_2 where consumers' expectations reveal to be true and no consumer regrets his purchase decision.

Definition 4.1. *Prices* (p_1, p_2) *constitute a credible price pair, if given* p_1*, consumers expect* p_2*, i.e.* p_2 *is maximizing the terminal profit given the terminal distribution of consumers* $G(\theta, T; p_1, p_2)$*.*

Hence, for a credible price pair, the monopolist's choice of price at T is consistent with the consumers' expectations and no consumer regrets his purchase or waiting decision. Then, the objective function of the firm is given by

$$\max_{p_1, p_2} \int_0^T e^{-rt} \pi_t(p_1, p_2^e) dt + e^{-rT} \pi_T\Big(p_2, G(\theta, t, p_1, p_2^e)\Big) dt \tag{4.15}$$

subject to

$$p_2^e = \arg\max_{p_2} \pi_T(p_2, G(\theta, T, p_1, p_2^e)) \tag{4.16}$$

and

$$\begin{aligned} &\dot{g}(\theta, t; p_1, p_2^e) = 0 \quad \forall\, \theta \in [0,1] \quad \text{and} \quad \forall\, t \in I(\theta, p_1, p_2^e), \\ &\dot{g}(\theta, t; p_1, p_2^e) = 1 \quad \forall\, \theta \in [0,1] \quad \text{and} \quad \forall\, t \notin I(\theta, p_1, p_2^e), \\ &g(\theta, 0; p_1, p_2^e) = 0 \quad \forall\, \theta \in [0,1], \end{aligned} \tag{4.17}$$

i.e. the firm maximizes among all credible price pairs which yields the rational expectations equilibrium. Technically, this is a fixed-point problem whose existence is not guaranteed per se as we will discuss below.

4.4 Firm's Objective and Consumer Distribution

We start at the terminal time T where the new product is introduced and the established product is taken from the market. At the terminal date, we deal with

[8]The wtp is monotonously decreasing with respect to t, hence $I(\theta, p_1, p_2^e)$ is indeed a connected set, i.e. an interval.

a particular demand structure. Since the problem ends at this point, this is a static optimization problem and the wtp of the consumers is no more conditional on the next improved product but depends merely on the valuation, i.e. the type of the consumer and the product's current quality which has increased at T. Thus, the wtp of a consumer with valuation θ reads

$$wtp_2(\theta) = q_2\theta. \tag{4.18}$$

Hence, the indifferent consumer is given by p_2/q_2. Thus, the objective functional at T reads

$$\max_{p_2} \pi_T(p_2, G(\theta, T; p_1, p_2^e)) = \max_{p_2} \int_{\frac{p^2}{q^2}}^{1} p_2 dg(\theta, \cdot) = \max_{p_2} \; p_2\Big(G(1, T; p_1, p_2^e) - G(\frac{p_2}{q_2}, T; p_1, p_2^e)\Big),$$

s.t. (4.16). For $t < T$, we start with characterizing the indifferent consumer who is indifferent between buying the established product upon arrival and the new product at T (cf. Dhebar (1994)). In case A, she is characterized by

$$\begin{aligned} \theta_{1A} := \min\Big\{\theta : 0 \leq \theta \leq 1,\; q^1\theta - p_1 \geq 0, \\ e^{-r(T-t)}p_2^e - p_1 \geq \theta(e^{-r(T-t)}q_2 - q_1)\Big\}. \end{aligned} \tag{4.19}$$

In case B, she is given by

$$\begin{aligned} \theta_{1B} := \max\Big\{\theta : 0 \leq \theta \leq 1,\; q^1\theta - p^1 \geq 0, \\ e^{-r(T-t)}p_2^e - p_1 \geq \theta(e^{-r(T-t)}q_2 - q_1)\Big\}. \end{aligned} \tag{4.20}$$

Hence, in case A, all $\theta > \theta_{1A}$ prefer buying the established product whereas in case B, it is the other way around, i.e. all $\theta < \theta_{1B}$ prefer buying the established product over the new product. From

$$e^{-r(T-t)}p_2 - p_1 = \theta(e^{-r(T-t)}q_2 - q_1) \tag{4.21}$$

the consumer which is indifferent between buying the established and the new product can be derived:

$$\tilde{\theta}(t) := \theta_{1A} = \theta_{1B} = \max[0, \min[1, \frac{e^{-r(T-t)}p_2^e - p_1}{e^{-r(T-t)}q_2 - q_1}]].$$

If there were no new product, the market participation constraint would have to be satisfied only, i.e. the consumer who is indifferent between buying and not buying would be given by

$$\theta_1 := \min\Big\{\theta : 0 \leq \theta \leq 1,\ q^1\theta - p^1 \geq 0\Big\}.$$

The optimal price for the new product at T depends on the distribution of consumers who have not bought the established product before. The distribution, however, depends on both, p_1 and p_2^e . There are three cases of prices (p_1, p_2^e) which lead to structurally different distributions. More precisely, it depends on whether p_1 is above, equal to or below the price at the kink of the wtp, i.e. whether

$$p_1 \lesseqqgtr p_2^e \cdot q^1/q^2, \tag{4.22}$$

which is the price at the kink point $\theta = p_2^e/q_2$. As discussed earlier, note that the kink is in the unit interval since $p_2^e \leq q_2$.

Denote the time where the willingness to pay of a consumer θ equals p_1 by $\hat{t}(\theta)$, if it exists (see Appendix 4.A.1 for its formula.). If not, set $\hat{t}(\theta) = 0$. The three cases are characterized by

- *Case* I: $p^1 > p_2^e\frac{q^1}{q^2}$ $(\Leftrightarrow \hat{t}(1) \leq \tau)$: For $t \leq \hat{t}(1)$ (in case A), the set of buyers is given by $[\theta_{1A}, 1]$ and the indifferent consumer θ_{1A} increases such that sales stop eventually in case A and there are no sales anymore. If $\hat{t}(1)$ =0, then there are no sales at all before T. The objective functional is then given by

  $$\max_{p_1,p_2} \int_0^{\hat{t}(1)} e^{-rt}p_1(1-\theta_{1A})dt + e^{-rT}\pi_T(p_2, G(\theta, T; p_1, p_2^e)),$$

  s.t. (4.16).

- *Case* II: $p^1 < p_2^e\frac{q^1}{q^2}$ $(\Leftrightarrow \hat{t}(1) \geq \tau)$: For $t \leq \hat{t}(1)$, the set of buyers is given by $[\theta_1, 1]$ and for $t \geq \hat{t}(1)$, the demand is given by $[\theta_1, \theta_{1B}]$. [9] The objective functional is then given by

  $$\max_{p_1,p_2} \int_0^{\hat{t}(1)} e^{-rt}p_1(1-\theta_1)dt + \int_{\hat{t}(1)}^{T} e^{-rt}p_1(\theta_{1B}-\theta_1)dt + e^{-rT}\pi_T(p_2, G(\theta, T; p_1, p_2^e)),$$

  s.t. (4.16).

[9] Note that here, $\hat{t}(1)$ might not exist for two reasons, either if wtp of $\theta = 1$ stays above p_1 until T or is below p_1 from the beginning. In the latter case $\hat{t}(1)$ must be set 0. In the first case, both terms of the integral yield the same, i.e. $\hat{t}(1) = T$ is possible as well. For simplicity, we have assumed that $\hat{t}(1) = 0$, once it does not exist.

- *Case* III: $p^1 = p_2^e \frac{q^1}{q^2}$ ($\Leftrightarrow \hat{t}(1) = \tau$): In case A, demand is given by $[\theta_1, 1]$ and in case B, there are no sales at all[10], i.e. demand vanishes abruptly when the case switches form A to B. Thus, the objective functional is given by

$$\max_{p_1,p_2} \int_0^{\tau} e^{-rt} p_1(1-\theta_1)dt + e^{-rT}\pi_T(p_2, G(\theta, T; p_1, p_2^e)).$$

  s.t. (4.16).

Intuitively, in case I, the price of the established product is relatively high such that only consumers with high valuation buy but as the introduction of the new product gets closer, demand for the established product vanishes. In case II, the price of the established good is relatively low such that until $\hat{t}(1)$, all consumers for which the market participation constraint is fulfilled buy. After $\hat{t}(1)$, demand decreases as the consumers at the 'high-end' start waiting for the new product. Case III is the case in between where demand jumps to 0 at τ.

In an equilibrium, consumers form an expectation for p_2 and the firm sets at T the price which has been expected by consumers[11]. Here the problem arises that the firm cannot commit to future prices, i.e. there is no commitment device which guarantees that the firm is not going to set a different price. Hence, p_2 has to maximize the profit at T, ex-post. Thus, we are looking for price pairs (p_1, p_2) which maximize the monopolist's discounted stream of profits among all credible price pairs.

We find that there are no credible price pairs in case II, i.e. there is no expectation p_2^e for p_2 which is actually set by the firm at T.

Lemma 4.4. *In case II, there is no credible price pair* (p_1, p_2).

Proof. In case II, demand is given by $[\theta_1, 1]$ for $t < \hat{t}(1)$ and by $[\theta_1, \theta_{1B}]$ for $t \geq \hat{t}(1)$. At T, θ_{1B} is given by $\frac{p_1 - p_2^e}{q_1 - q_2}$. Hence, there are no consumers in the interval $[\theta_1, \theta_{1B}]$=$[\frac{p_1}{q_1}, \frac{p_1 - p_2^e}{q_1 - q_2}]$. Consider the indifferent consumer at T which is given by p_2/q_2. Rearranging the definition of case II yields

$$\frac{p_1}{q_1} < \frac{p_2}{q_2}. \tag{4.23}$$

[10]Note that in that case, $\theta_1 = \theta_{1A}$.

[11]Note that actually observing p_1 and forming an expectation for p_2 is much less costly for the consumers than deriving the rational expectations equilibrium under all credible price pairs.

Furthermore, again from the definition of case II, we can derive

$$
\begin{aligned}
& p_2^e \frac{q_1}{q_2} > p_1 \\
\Leftrightarrow \quad & p_2 q_1 > q_2 p_1 \\
\Leftrightarrow \quad & p_2(q_1 - q_2) > (p_1 - p_T^e) q_2 \\
{}^{12}\Leftrightarrow \quad & \frac{p_2}{q_2} < \frac{p_1 - p_T^e}{q_1 - q_2},
\end{aligned}
$$

i.e. p_2/q_2 is in that consumerless interval. Hence, this price cannot be optimal ex-post since an optimal price has to be outside this interval. □

Intuitively, this price cannot be optimal ex-post since the firm could increase its price without loosing consumers or would prefer to decrease its price in order to increase sales.

We show in Lemma 4.5 that if the problem starts in case A, then all candidates for credible price pairs in case III are inferior. In other words, it is necessary that the problem starts in case B ($\tau = 0$) for having a credible price pair in case III.

Lemma 4.5. *In case III, if $\tau > 0$, there is no credible price pair.*

Proof. At T, for a credible price pair, the price for the new product must be set to $p_2 = p_1 \frac{q_2}{q_1}$ and the corresponding indifferent consumer is given by $\tilde{\theta} = p_2/q_2$. In case III, the final distribution is given by

$$
G(\theta, T; p_1, p_2^e) = \begin{cases} T\theta & \text{for } \theta \le p_2^e/q_2 \\ (T-\tau)\theta + \tau \; p_2^e/q_2 & \text{for } p_2^e/q_2 \le \theta \le 1 \\ (T-\tau) + \tau \; p_2^e/q_2 & \text{for } \theta \ge 1 \end{cases} \tag{4.24}
$$

Maximizing the terminal profit with respect to p_2 and requiring ex-post optimality yields two candidates, in particular $p_2 = q_2/2$ and

$$
p_2 = \frac{T - \tau}{2T - \tau} q_2. \tag{4.25}
$$

However, whenever $\tau > 0$, there exists another p_2 which yields a higher profit, i.e. the candidate for a credible price is inferior and is not selected (details are given in Appendix 4.A.3). □

[12]Note that $q_1 - q_2 < 0$ holds by assumption.

So rational expectations equilibria might be found in case I and additionally in case III but only if the product is improving right from the beginning. However, note that the latter scenario corresponds to a flat distribution[13] at T and all other choices of p_1 which are higher and thus belong to case I yield the same outcome. Hence, additionally, there will be infinitely many credible price pairs in case I. Thus, for $\tau > 0$, case III can be considered as a hairline case. The necessary derivations for case I in order to obtain profits for a credible price pair are given in Appendix 4.A.2.

4.5 Numerical Results

As the distribution structure changes for varying prices, the optimal prices are calculated numerically. For the default parameter setting

$$q_1 = 1, q_2 = 1.1, T = 2, r = 0.1 \tag{4.26}$$

the problem starts in case A where the product is not improving but switches to case B where the product becomes improving at $\tau = 1.0469$. We define a grid for the price of the new product by $P_2^{e,ini} = [0.4, 0.56]$ with nodes of equal distance 0.001 [14]. Note that there is a natural threshold for the price of the established product since if it becomes too expensive such that no consumer buys anymore the established product, there will be a uniform distribution of consumers at T such that the ex-post optimal price will be $q_2/2$ (cf. proof of Lemma 4.5 and Appendix 4.A.3). Hence, for credible price pairs, p_2 becomes constant as soon as p_1 is too high that no sales occur anymore in mode 1 and hence the distribution of consumers does not change anymore. For finding the credible price pairs, we proceed as follows. For a fixed p_2^e from the grid, we maximize the profit at T with respect to p_2 which yields p_2^* which depends on p_1. Then, we solve for p_1 such that $p_2^* = p_2^e$ holds. For the considered parameter setting, this yields a unique p_1

[13]In this scenario, the optimal price is given by $p_2 = q_2/2$ and the indifferent consumer is at the center, i.e. at $\theta = 1/2$.

[14]The grid does not need to be widened since the distribution of consumers and hence the profit does not change for higher or lower prices, respectively.

for each p_2^e from the grid. After having calculated all candidates for optimal price pairs, i.e. all credible price pairs, by simple comparison of its corresponding total profits, we determine the optimal choice of p_1 and p_2 among the credible price pairs.

For the credible price pairs, we find that initially p_2 is increasing in p_1 but becomes constant once p_1 is so high such that no consumer buys before T and hence the optimal p_2 does not change anymore. We find that the optimal price in equilibrium for p_2 is around 0.509. Thus, we define a new more dense grid (nodes of lower distance 0.0001) $P_2^{e,fine} = [0.508, 0.51]$ and derive credible price pairs which are depicted in Figure 4.2.

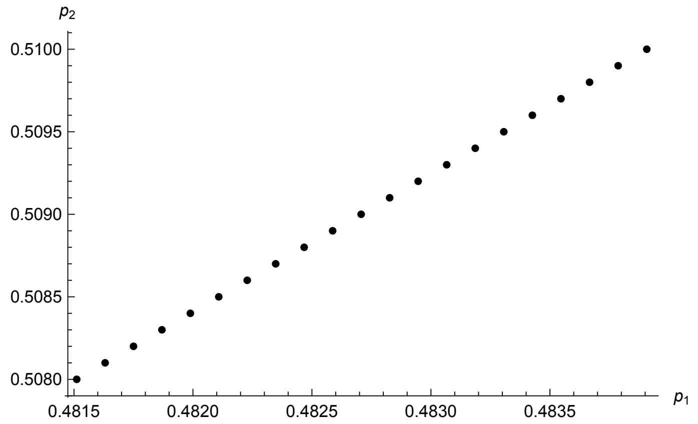


Figure 4.2: Credible price pairs.

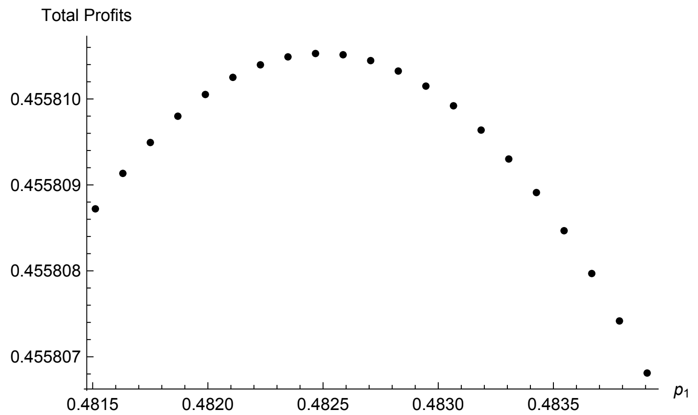
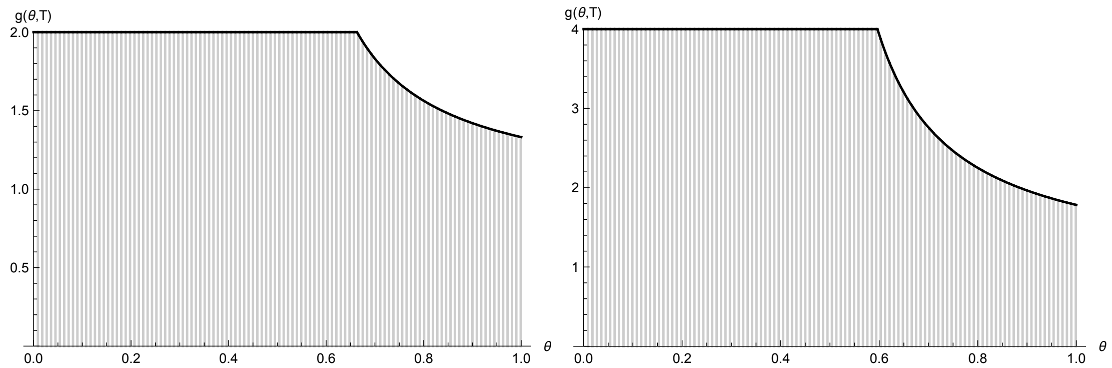


Figure 4.3: Profits of credible price pairs.

Among those pairs, by comparing profits (see Figure 4.3) we find that it attains its maximum at the price pair $(p_1, p_2) = (0.4825, 0.5088)$ which corresponds to case

I, i.e. the firm initially sells in mode 1 but as the time of innovation comes closer demand decreases and sales stop in case A at $\hat{t}(1)$, even before τ. More specifically, initially, consumers in $[0.6633, 1]$ buy the established product but as time passes, demand decreases and sales stop at $\hat{t}(1) = 0.6686$. In particular, the indifferent consumers shifts to the right. Figuratively speaking, the price of the established product is too high and is perceived higher as time goes by such that more and more consumers stop buying and start waiting for the new product.

The density of consumers at T is shown in Figure 4.4(a). At T, the indifferent



(a) $T = 2$ (b) $T = 4$

Figure 4.4: Consumers' density at T.

consumer is at 0.4625.

For an alternative parameter setting with $T = 4$, τ increases by 2 points to $\tau = 3.0469$ and qualitative results remain unaffected[15]. However, prices are much lower compared to the shorter time horizon case, in particular the optimal price pair is given by $(0.4671, 0.4631)$. Here, initially, consumers in $[0.5965, 1]$ buy and sales stop at $\hat{t}(1) = 2.2172$. The corresponding density function is depicted in Figure 4.4(b). Here, the indifferent consumer at T is at $\theta = 0.421$. Compared to the default parameter setting with $T = 2$, consumer surplus is higher since prices are lower and in addition, consumers in $[0.421, 0.4625)$ who were neither buying the established nor the new product are now buying the new product. In a setting with a finite time horizon, increasing the time interval where the established product is sold increases consumers' wtp. Neglecting new consumer arrival, a natural guess

[15]Figures are given in Appendix 4.A.4.

would be that the price for the established product in equilibrium increases as well. However, increasing the length of the problem while keeping the inflow of consumers constant at the rate 1 leads to a higher total accumulation of consumers which increases the firm's propensity to lower the price for the new product. Hence, the latter is dominating and the equilibrium prices for $T = 4$ are lower.

4.6 Conclusion

We have developed a simple and tractable model and have characterized optimal price pairs by modeling demand endogenously in a durable-goods setting. The assumption of rational consumers led to the framework of rational expectations equilibria where consumers perfectly predict the price of the new product. Employing a continuous time framework before product introduction was crucial to characterize consumers' changing willingness to pay for the established product. While consumers' and the monopolist's behavior could be described analytically, rational expectations equilibria have been found numerically proceeding along a grid. More precisely, in our examples, an optimal credible price pair has to be selected among a continuum of credible price pairs which can not be accomplished analytically.

Accounting for consumer's potential lack of computing ability, considering bounded rational consumers might be an interesting topic for future research.

4.A Appendix

4.A.1

For

$$\frac{p_1 - \theta q_1}{p_2 - \theta q_2} > 0, \tag{4.27}$$

$\hat{t}(\theta)$ is easily derived from

$$wtp_1(\theta, \hat{t}(\theta)) = p_1 \tag{4.28}$$

$\Leftrightarrow$

$$\hat{t}(\theta) = T + \frac{1}{r}\ln\left(\frac{p_1 - \theta q_1}{p_2 - \theta q_2}\right). \tag{4.29}$$

Note that for the existence of $\hat{t}(\theta)$, i.e. to have $0 \leq \hat{t}(\theta) \leq T$, $\frac{p_1-\theta q_1}{p_2-\theta q_2}$ is required to be in the interval $[e^{-rT}, 1]$.

4.A.2

In case I, for a credible price pair (p_1, p_2), total profits are given by

$$\int_0^{\hat{t}(1)} e^{-rt} p_1(1-\theta_{1A})dt + e^{-rT}\pi_T(p_2, G(\theta, T; p_1, p_2)).$$

For $\theta \in [0,1]$, the final density function is given by

$$g(\theta, T; p_1, p_2) = \begin{cases} T & \text{for } \theta \leq \tilde{\theta}(0) \\ T - \hat{t}(\theta) & \text{for } \theta > \tilde{\theta}(0). \end{cases} \tag{4.30}$$

$G(\cdot)$ is obtained by integrating $g(\cdot)$ with respect to θ

$$G(\theta, T; p_1, p_2) = \begin{cases} T\theta & \text{for } \theta \leq \tilde{\theta}(0) \\ T\tilde{\theta}(0) - \frac{1}{r}\int \hat{t}(\theta)d\theta & \text{for } \theta > \tilde{\theta}(0). \end{cases} \tag{4.31}$$

where

$$\int \hat{t}(\theta)d\theta = -\frac{p_1}{q_1}\ln(p_1 - q_1\theta) + \theta\ln\left(\frac{p_1 - q_1\theta}{p_2 - q_2\theta}\right) + \frac{p_2}{q_2}\ln(p_2 - q_2\theta), \tag{4.32}$$

and the primitive of the integrand of the first integral is given by

$$\frac{e^{-r(T+t)p_1\left(e^{rT}(p_1-q_1)q_1 - e^{rt}(p_2q_1 - p_1q_2)\ln(e^{r(T-t)}q_1 - q_2)\right)}}{rq_1^2}, \tag{4.33}$$

such that total profits can be calculated.

4.A.3

Note that for flat distributions, the optimal price at T is given by $p_2 = q_2/2$. For non-flat distributions which emerge in case III whenever $\tau > 0$, there might be at most two candidates for the optimal price of the new product since the profit function consists of two concave functions which tied together is not quasiconcave. For credibility, the indifferent consumer at T has to be at the kink of the distribution function. Assuming $p_2 \geq p_2^e$, the terminal profit is given by

$$\begin{aligned}\pi_T(G(\theta,T;p_1,p_2^e)) &= p_2\Big(G(1,T;p_1,p_2^e) - G(\frac{p_2^e}{q_2},T;p_1,p_2^e)\Big)\\ &= p_2\Big(T-\tau+\frac{p_2^e}{q_2}\tau-(T-\tau)\frac{p_2}{q_2}-\frac{p_2^e}{q_2}\tau\Big)\\ &= p_2\Big((T-\tau)(1-\frac{p_2}{q_2})\Big).\end{aligned}$$

The first-order condition yields[16]

$$p_2 = \frac{q_2}{2}, \tag{4.34}$$

whereas the profit for assuming $p_2 \leq p_2^e$ is given by

$$\begin{aligned}\pi_T(G(\theta,T;p_1,p_2^e)) &= p_2\Big(G(1,T;p_1,p_2^e) - G(\frac{p_2^e}{q_2},T;p_1,p_2^e)\Big)\\ &= p_2\Big(T-\tau+\frac{p_2^e}{q_2}\tau-T\frac{p_2}{q_2}\Big)\\ &= p_2\Big(T(1-\frac{p_2}{q_2})-\tau(1-\frac{p_2^e}{q_2})\Big)\end{aligned}$$

Requiring the first order condition to be met and requiring credibility leads to

$$p_2 = \frac{T-\tau}{2T-\tau}q_2. \tag{4.35}$$

In total, we have two candidates which satisfy local first order conditions and are credible, i.e. $q_2/2$ and $q_2(T-\tau)/(2T-\tau)$. However, we find that for $p_2^e = q_2/2$ there exists another candidate p_2 (in the other part, i.e. in $p_2 < p_2^e$), in particular

$$\frac{q_2}{2}\left(1-\frac{\tau}{2T}\right) \tag{4.36}$$

which yields a higher payoff whenever $\tau^2/4 > 0$, which is equivalent to $\tau > 0$. Analogously, for $p_2^e = q_2(T-\tau)/(2T-\tau)$, there is another candidate (also from the other part, i.e. in $p_2 > p_2^e$), given by $p_2 = q_2/2$ which yields a higher payoff whenever $\tau^2 > 0$. Hence whenever $\tau > 0$ holds, the credible price pairs are inferior.

4.A.4

For $T = 4$, figures of credible price pairs and its corresponding profits are given in Figures 4.5 and 4.6.

[16] Note that in the case of $q_2/2 < p_2^e$, credibility is not fullfilled and hence $q_2/2$ is no candidate for the optimal price.

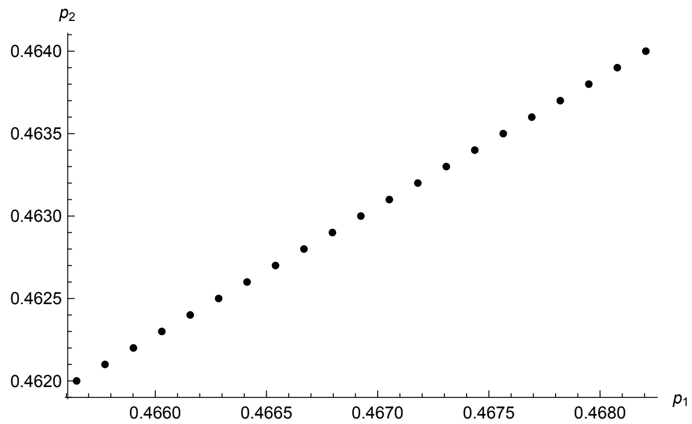


Figure 4.5: Credible price pairs for $T = 4$.

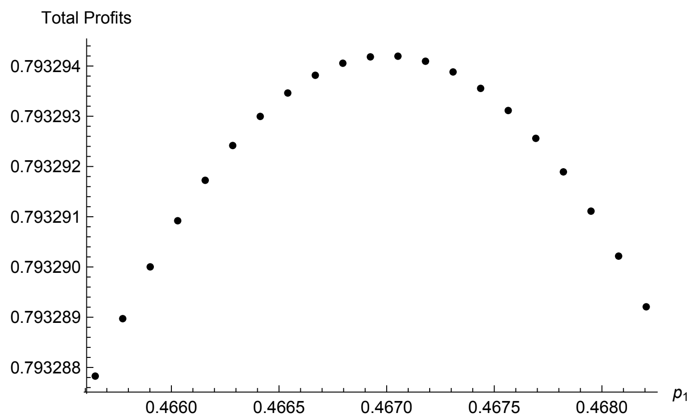


Figure 4.6: Profits of credible price pairs for $T = 4$.

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