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Smart Products: Liability, Timing of Market Introduction, and Investments in Product Safety*

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Abstract

This paper addresses the role of product liability for the emergence and development of smart products such as autonomous vehicles (AVs). We analyze how the liability regime affects innovative activities, as well as the timing of market introduction and market penetration of such smart products. We develop a dynamic model in which at each point in time, a potential (monopolistic) innovator decides on how much to invest in the safety stock of the smart product and on the product price, once it has been launched. Calibrating the model to the U.S. car market, our analysis reveals policy-relevant trade-offs when shifting more liability on the producers of AVs. First, while this improves the safety of AVs in the long run, the safety stock is accumulated more slowly. Second, it delays the market introduction of AVs, and also slows down market penetration, which hampers the innovator's incentives for safety investments in the short- and intermediate term. As a result, the safety level of AVs at a given point in time decreases as the liability regime becomes more stringent. Furthermore, there is a threshold for the innovator's burden of liability beyond which she forgoes to develop the AV altogether. Finally, we find that direct AV safety regulation is welfare-superior compared to a stringent liability regime, as it induces higher levels of AV safety in the short and intermediate term.

JEL-Codes: O31, K13, L11, L62

Keywords: Product Innovation, Liability, Digital Economy, Autonomous Vehicles, Smart Products, Optimal Investment Dynamics

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1 Introduction

Motivation The advent of digitization has already changed our everyday lives in many fundamental ways, and it is widely expected that this development will continue in the future at an even accelerated speed. As a consequence, we will move more and more towards a *digital economy*, with far-reaching consequences for many areas such as workplace relationships, product markets, international trade or mobility. These developments also constitute major policy challenges, for example with respect to infrastructure investments, standardization, regulation, and legislation.[1]

From a technological point of view, the key drivers for these developments are recent advances in areas such as robotics, artificial intelligence (AI), cloud computing, or big data analytics (see e.g., UN Conference on Trade and Development, 2017), which enable the development of so-called *smart products*. According to Porter and Heppelmann (2015), the capabilities of smart products can be grouped in four areas: monitoring, control, optimization, and autonomy. This paper focusses on autonomy. Products exhibiting autonomy rely less and less on human decision-making, and major decisions are instead being taken by the product's operation system, based on algorithms. One example of particular topicality are Autonomous Vehicles (AVs), which is the prime application in this paper, but similar developments are also observed for other smart products ranging from household appliances to heavy military weapons.[2]

One alleged major benefit of AVs is that in the long run, when fully matured, they are potentially much safer than conventional vehicles in terms of their accident rate.[3] The main reason is that computerized decision-making based on AI algorithms is potentially less error-prone than decision-making by human drivers, who are often impaired by poor sight and slow reactions times, for example due to fatigue, distraction, alcohol or drug consumption. While (conventional) cars have become considerably safer over the last decades, car crashes are still a significant problem and create a huge social cost, so that any significant reduction in accident risks would generate potentially large social benefits.[4]

However, these long-run benefits of AVs cannot be expected to be reaped immediately upon their market introduction. In the meantime, as long as the technology is not yet fully matured, AVs are likely to be involved in crashes in the same way as conventional cars, potentially at even a higher rate. This is highlighted by the recent fatal crashes caused by AVs of Uber and Tesla during test drives, triggering considerable public attention.[5]

[1] For an overview over some of these policy challenges, see e.g., OECD (2017); UN Conference on Trade and Development (2017)

[2] See Schellekens (2015, p.507) for a more detailed classification of different degrees of automation for vehicles. Sassone et al. (2016) provide a survey of different areas where smart products are expected to be available in the near and intermediate future.

[3] Apart from a lower accident rate, AVs are expected to give rise to further benefits such as reducing traffic congestion or improving mobility of the elderly (see e.g., Douma and Palodichuk, 2012).

[4] For example, according to a recent report by the U.S. National Highway Traffic Safety Adminstration (Blincoe et al., 2015), in 2010 there were 32,999 people killed in car crashes in the U.S., and 3.9 million were injured. Moreover, for that year the total social costs due to car accidents is estimated to be $ 836 billion.

[5] See e.g., `https://www.nytimes.com/2018/03/31/business/tesla-crash-autopilot-musk.html`

Under the assumption that the safety of AVs increases over time as the technology improves, this raises the question of *when* and with which safety level AVs should be brought to the market. For example, it is argued that upon introduction, AVs should be at least as safe as conventional cars, if not much safer (see e.g., Schellekens, 2015, p. 510). In a similar vein, in a recent panel discussion Dieter Zetsche, CEO of Daimler (one of the world's major car manufacturers), argued that even if AVs caused only a tenth of the number of fatalities as conventional cars, this would still not be tolerated by the public. As a result, "they will have to be a hundred times better, but then it gets really difficult from a technological point of view".[6]

The possibility of accidents caused by AVs which are controlled by non-human drivers raises interesting and novel aspects with respect to the issue of liability. In particular, unlike other and less significant product innovations, in the case AVs a new type of injurer emerges: the AV's operating system.[7] Since computer systems cannot be held legally responsible, however, the question arises who should be liable for the harm arising from accidents caused by AVs? As one main feature (and alleged benefit) of AVs is that its driver retains less control and hence becomes more like a passenger rather than the operator of the car, many argue that there should be less scope for legal responsibility of drivers, and more scope for car manufacturers and automotive software developers.[8] Despite the current considerable legal uncertainty to which extent producers of AVs and their operating systems will ultimately be held liable, they can be expected to face substantially higher liability costs than they currently do with conventional cars.[9]

One crucial question is to which degree these higher liability risks affect firms' innovation activities and other business policies, such as R&D investments (e.g., to improve the safety of AVs), the timing of market introduction of AVs and pricing. Together these

and `https://www.nytimes.com/2018/03/19/technology/uber-driverless-fatality.html`.

[6]See `https://www.faz.net/-ijt-9gpve`.

[7]Marchant and Lindor (2012, p.1325) discuss autopilots in airplanes as one example with similar features.

[8]Galasso and Luo (2018a) discuss some general legal challenges for tort law in the age of AI. Moreover, Douma and Palodichuk (2012), Colonna (2012), Marchant and Lindor (2012), Schellekens (2015) and Gless et al. (2016) contain legal discussions in the context of AVs in the United States. Moreover, the European Parliament has adopted a resolution containing suggestions how to establish legal rules for tort cases with AI tortfeasors (European Parliament, 2017). Finally, Germany's "Ethics Commission for Autonomous and Connected Driving" recommends that apart from car owners, also the producers and operators of autonomous cars and its supportive technological systems should be considered as potentially liable parties (Federal Ministry of Transport and Digital Infrastructure, 2017).

[9]Currently (i.e., without AVs on the streets), in the vast majority of accidents the damage is apportioned between the involved drivers and/or their insurance companies. For example, according to the 2008 *National Motor Vehicle Crash Causation Survey* (NHTSA), more than 93 percent of the analyzed crashes were classified as being caused by erroneous behavior of the respective drivers, and only the remaining 7 percent were due to vehicle problems or adverse weather conditions. Car manufacturers currently face a risk of product liability, which might apply when a car model exhibits systematic technical defects. Moreover, Viscusi (2012) argues that there is a tendency of courts and juries to be tougher on injurers in liability cases where "novel risks" are involved. Given the high degree of product novelty of AVs compared to conventional cars, this could further contribute to the expected liability cost of firms engaged in the development and production of AVs.

decisions determine (i) when AVs are eventually launched (if at all) and with which safety level, (ii) how quickly they penetrate the market, and (iii) how long it takes until the long-term benefit of a higher safety of AVs compared to conventional cars can be expected to materialize. For example, legal scholars such as Marchant and Lindor (2012, pp. 1336), Colonna (2012, p.84) and Schellekens (2015) argue that a stringent liability regime for producers of AVs might considerably delay these processes.

More generally, from a theoretical perspective, the effect of a more stringent liability regime on the incentives to innovate seems ambiguous: one the one hand, it might increase the incentive to undertake product innovations to improve product safety. But on the other hand, it might have a "chilling effect" on innovative activities, for example in the form of choosing to not develop new products (Viscusi, 1991; Parchomovsky and Stein, 2008; Viscusi, 2012), even if they are potentially superior than existing ones.[10] From an empirical point of view, both effects seem relevant (see e.g., McGuire, 1988; Viscusi and Moore, 1993; Galasso and Luo, 2017, 2018b). Hence, in the light of the potentially large benefits from AVs in the long-term, the stringency of liability for firms involved in the development, production and operation of AVs and other smart products seems a highly relevant policy issue.

Framework and results We consider a dynamic model of product innovation, where there already exists a market for an established product (the *old* product), and where a monopolistic innovator can introduce a new one (the *smart* product). Consumers differ with respect to their valuation for the smart product. Both products are prone to accidents, which are socially harmful. In each period the innovator decides on (i) how much to invest into the safety level of the smart product which reduces its accident rate, and (ii) the price at which it is sold, once it has been launched.

We analyze how the dynamically optimal decisions and the resulting state dynamics depend on the liability regime, which apportions the damage accruing from accidents with the smart product between the innovator and the consumers. More precisely, we are interested in how the liability regime affects (i) the safety level of the smart product in the long run (steady state), (ii) the optimal dynamic paths for safety investments as well as output and pricing choices, thereby also determining when and with which safety level the smart product is launched, (iii) the development of the smart product's safety level after market introduction and (iv) overall welfare.

We first derive an analytical characterization of the effect of the liability regime on the steady states of the model. The dynamic properties of the optimal paths are then examined using numerical methods, where we calibrate the model using data from the US automobile market. Considering the impact of the liability regime on the optimal behavior of the innovator, our analysis reveals two basic trade-offs which policy makers and legislators are facing: The first trade-off concerns the existence and optimality of an *active* steady state, in which the innovator indeed accumulates a safety stock for the smart

[10] This was also a crucial argument in the decision of the U.S. Supreme Court in *Riegel vs. Meditronic* in 2008.

product, and eventually introduces it to the market: As the liability regime becomes more stringent (i.e., increasing the share of the damage accruing to the innovator), this leads to an increase of the safety level of the smart product in an active steady state, given that it exists and is optimal for the innovator. However, a too stringent liability regime might forestall the development of the smart product altogether. In particular, the innovator might prefer to induce a *passive* steady state, in which there is no investment into the safety stock of the smart product, and where it is never launched. Overall, these results are consistent with empirical findings that the effect of liability on innovation activities is ex ante unclear and might go in either direction (Viscusi and Moore, 1991, 1993; Galasso and Luo, 2017, 2018b).

The second trade-off concerns the short- and long term effects of the liability regime in an active steady state: As above, a more stringent liability regime has the long-run benefit of inducing a higher steady state safety level of the smart product. However, the analysis of the dynamic patterns over time reveals that it also leads to a delay in the accumulation of the safety stock in early periods. Intuitively, the more stringent the liability regime, the longer it takes until the smart product is launched, and the smaller the quantities produced and hence the market penetration (thereby also leading to deadweight loss). In turn, this reduces the innovator's incentives for safety investments. As a result, the units brought to the market in the short- and intermediate term exhibit a lower safety level compared to those that would have been produced under a less stringent liability regime.

For the context of AVs, our analysis suggests that these short-term costs outweigh the long-benefit from more stringent liability. As a result, it would be socially optimal to *not* impose additional liability costs on the producers of AVs, compared to the status quo with conventional cars only. Moreover, we find that direct safety regulation of AVs (in the sense that they can only be launched once they satisfy a given minimum safety standard) might be a better policy than liability rules, as this would ameliorate the issue of delay in safety stock accumulation, and would lead to smaller distortions with respect to quantity choices.

Relation to literature Our paper is related to the following strands of literature: First, we contribute to a current academic and non-academic legal debate to which extent existing laws and other legal procedures need to be adapted in response to the new legal challenges arising in the digital economy, in particular in the context of AVs (see e.g., Marchant and Lindor, 2012; Colonna, 2012; Douma and Palodichuk, 2012; Schellekens, 2015; Smith, 2017). To the best of our knowledge, our paper is the first one to analyze these issues from a theoretical perspective a dynamic model framework.

Second, we relate to the literature in economics on how (product) liability affects the incentive of firms to improve the safety of their products, either by making existing products safer or by developing new products which exhibit higher safety levels. For example, in the survey by McGuire (1988) conducted among firms, about a third of respondents answered that the threat of liability has led them to improve the safety of their products, but an equal number of respondents said that it has lead them to not introduce new products. In their rather critical general discussion of product liability, Polinsky and Shavell (2010) argue that incentives to invest into product safety can also be stipulated through other (and arguably less expensive) channels, such as market forces (as consumers anticipate that they will have to bear the consequences of the accident risk, which reduces their willingness to pay for the product) or direct safety regulation. These arguments are supported by our analysis in the sense that the consumer side is indeed an important driver of investments into the safety of AVs, even when liability is not more stringent than for producers of conventional cars. In fact, we find that social welfare is highest under such a lenient liability regime. However, it can be further increased when combining it with an ex ante safety regulation in the form of requiring a minimum safety standard (i.e., a maximum accident rate) for AVs upon market introduction, as this speeds up the accumulation of the AV safety stock in the time interval before the minimum safety standard is reached.[11]

As for the impact of liability on innovation incentives, the evidence is mixed. For example, in their seminal empirical study in the U.S. manufacturing sector Viscusi and Moore (1993) find that the relationship between liability and the intensity of firms' innovative activities is positive (negative) when the stringency of liability is high (low), with the average effect being positive.[12] More recently, Galasso and Luo (2017) empirically analyze the impact of tort reforms in the health sector (which have reduced the stringency of liability), and they find that this has led to a decrease of innovative activities of upstream suppliers. By contrast, Galasso and Luo (2018b) consider vertical relationships in the medical implant industry. They find a strong negative effect of higher liability of upstream suppliers on their innovative activities in that sector. Overall, the existing empirical evidence suggests that whether or not liability has a chilling effect on innovation activities crucially depends on the industry under consideration and how stringent the liability regime actually is. This view is supported also in our calibration exercise for the U.S. automobile industry. Viscusi and Moore (1993), Galasso and Luo (2017) and Galasso and Luo (2018b) also develop static models in which they capture the main effects of the empirical analysis. By contrast, our dynamic framework allows for a more detailed analysis of the interplay of the various effects in the short- und long-term, and the timing of innovations.

Daughety and Reinganum (1995) consider a setting of incomplete information, where a monopolistic firm may use the product price as a signaling device for product safety (which is unobserved by consumers). They analyze the impact of liability on the equilibrium properties. Their model is static, but the timing of events is similar to the stage game in our dynamic setup under complete information. In addition, like them we show that direct safety regulation might outperform liability rules in terms of social welfare, although the underlying mechanism in their paper differs from ours.

Third, since the safety standard of an AV is a key quality parameter, the innovator's

[11] In this respect, we also contribute to the literature which analyzes (in static models, without R&D investments and innovation activities) under which circumstances a policy which combines ex ante instruments (e.g., safety regulation) with ex-post instruments (e.g., liability rules) can be welfare-superior compared to the use of single instrument (see e.g., Shavell, 1984a,b; Kolstad et al., 1990; Schmitz, 2000).

[12] See also Viscusi (1991), Viscusi and Moore (1991), and Viscusi (2012).

investment in building up the safety stock is closely related to (quality enhancing) product innovation investments. Although dynamic models of optimal innovation investments have mainly focused on process innovations, several contributions (e.g. Lambertini and Mantovani (2009); Dawid et al. (2015)) have characterized optimal product innovation investments of monopolists in different market settings. With respect to the explicit consideration of the interplay between innovation investments and the timing of market introduction our paper is related to Hinloopen et al. (2013), where the focus is, however, on process innovation. Moreover, none of these papers considers the impact of liability.

The remainder of the paper is organized as follows: Section 2 lays out the model framework. Section 3 analyzes the different steady states of the model, and how the steady state optimally induced by the innovator depends on the liability regime. Section 4 analyzes in more detail how the liability regime affects the dynamic paths leading into the respective steady state. In doing so, we proceed numerically and calibrate the model to the US automobile market. In Section 5 we consider safety regulation as a further policy variable in addition to liability. Section 6 explores the policy implications of our findings and discusses further model extensions. The Appendix contains all proofs, details about our numerical method and additional simulation results.

2 Model

We consider a continuous-time framework in which a monopolistic firm (the "innovator") can introduce a smart product to the market. At each point in time t, there is a unit mass of consumers. Each consumer i has unit demand and is characterized by her gross valuation v_i obtained from the smart product in every period in which it is functional. The valuations v_i are distributed uniformly on $[\underline{v}, \bar{v}]$ with $\underline{v}, \bar{v} > 0$, so that the resulting distribution is $F(v_i) = \frac{v_i - \underline{v}}{\bar{v} - \underline{v}}$. Alternatively, consumers can purchase a standard or "old" product which yields the same value v_o to each consumer in every period in which it is functional, where $\underline{v} < v_o < \bar{v}$. The unit cost of production of both the old and the smart product is $c \geq 0$ (there are no fixed costs of production).[13] The old product is sold at a competitive market at price $p_o = c$ in all periods, whereas the price of the smart product in period t is denoted by $p(t)$.

The lifetime of both the old and the smart product is distributed exponentially with a constant failure rate $\rho > 0$. In the course of a product's lifetime, accidents may occur, each causing a damage $D > 0$. For the old product, an accident occurs at a given constant rate $\alpha_o > 0$. For the smart product, the accident rate is $\alpha(S)$, where S is the innovator's knowledge stock determining the safety of the smart product at the time when it is sold (explained in more detail below). We assume that a higher safety stock reduces the accident rate $\alpha(S)$ of the smart product, where $\alpha'(S) < 0, \alpha''(S) > 0$ and $\lim_{S \to \infty} \alpha(S)$ is denoted by $\alpha^{min} \in [0, \alpha_o)$. The last property reflects that AVs can become safer than conventional cars if the innovator builds up a sufficiently high knowledge stock,

[13] Since the focus of our analysis is on the (relative) accident rate of the smart product and the induced liability risk, we abstract from differences in production costs between the two products.

but accidents might not be fully avoided even under enormous safety investments.

As discussed in the introduction, the scope of liability for producers is expected to increase upon the introduction of smart products. Consequently, in our model, producers are not liable for accidents with the old product, and all liability risk is borne by the consumers. By contrast, for accidents with a smart product, a fraction $\beta \cdot D$ accrues to the innovator, so that the parameter $\beta \in [0,1]$ measures the strength of the liability risk for producers of smart products.[14]

The share of liability which is not borne by producers (that is, D and $(1-\beta) \cdot D$ for accidents with the old and the smart product, respectively) remains with the consumers, and they evaluate it with a function $k(\cdot)$ which is increasing and concave (i.e., $k' > 0$ and $k'' < 0$) and satisfies $k(0) = 0$, and $k'(0) = 1$.[15] Intuitively, while consumers strongly (or even fully) internalize small damages, the degree of internalization decreases as the damage becomes larger. In this way, we aim to capture the empirical feature that, at the margin, consumers often do not fully internalize their liability share which, for example, could be due to insurance policies with deductibles, or under-insurance (or even no insurance at all) in combination with wealth constraints.[16] As a result, the expected liability of each consumer is $\alpha_o \cdot k(D)$ and $\alpha(S) \cdot k((1-\beta)D)$ for the old and smart product, respectively. Finally, all market participants have a common discount rate $r > 0$.

For a given price p for the smart product in a given period, consumer i's expected discounted value from purchasing the old respectively the smart product is

[14] Of course, also the producers of conventional cars are currently facing product liability (at least in the U.S.). Since our focus is on the effect of additional liability risks due to accidents being caused by the operation system of AVs, we implicitly subsume all other liability risks (e.g., design defects of the brake system or the gas tank) in the production costs of firms. Accordingly, we associate a legal regime where the innovator faces the same liability risk as the producers of the conventional car with $\beta = 0$. All qualitative findings of our analysis would remain unchanged when considering instead a strictly positive (but not too large) lower bound on β.

[15] As will become clear below, when there is no wedge between the liability cost of producers and consumers, then under linear demand, any shift of the division of a damage D between liability between the innovator ($\beta \cdot D$) and the consumers ($(1-\beta) \cdot D$) would be offset one-to-one by a respective price change. This feature also arises, for example, in the setting of Daughety and Reinganum (1995) with complete information, where the firm's profit depends only on the total social damage from an accident, but not on how it is divided between the firm and the consumers.

[16] Uninsured driving is an empirically relevant phenomenon. For example, while there is mandatory insurance in the virtually all states of the U.S., a recent study of the Insurance Research Institute (IRC) estimates that in 2015, 13 percent of all motorists were uninsured (`https://www.insurance-research.org/sites/default/files/downloads/UMNR1005.pdf`). The high empirical relevance of *judgement-proofness* in tort cases is extensively discussed by Gilles (2006). As a result, the personal liability of many individuals is often much smaller than the damage caused in the course of their negligent behavior. For example, when the consumer has an insurance contract with a deductible, her marginal liability effect is one for damages below the amount of the deductible, and zero above it. While larger damages might result in higher insurance premiums, all we need is that the expected liability cost faced by consumers increases under-proportionally in the underlying (remaining) damage amount.

$$
\begin{aligned}
U_o &= \int_0^\infty e^{-(r+\rho)t}\left[v_o - \alpha_o k(D)\right] dt - c \\
&= \frac{v_o - \alpha_o k(D) - (r+\rho)c}{r+\rho} && (1) \\
U(p,S;v_i) &= \int_0^\infty e^{-(r+\rho)t}\left[v_i - \alpha(S)k((1-\beta)D)\right] dt - p \\
&= \frac{v_i - \alpha(S)k((1-\beta)D) - (r+\rho)p}{r+\rho} && (2)
\end{aligned}
$$

We assume $v_o > \alpha_o k(D) + (r+\rho)c$ and hence $U_o > 0$, so that all consumers purchase either one of the two products. Consumer i with valuation v_i prefers the smart product if and only if $U(p,S;v_i) \geq U_o$. The innovator has incentives to sell positive quantities of AVs if and only if there are some consumers who prefer the AV if it is priced at marginal costs. This corresponds to the condition $U\left(c + \frac{\beta\alpha(S)D}{r+\rho}, S; \bar{v}\right) \geq U_o$. We denote the lowest safety stock at which this condition holds by $\underline{S}$, which we assume to be finite and strictly positive, and the corresponding accident rate by $\underline{\alpha} = \alpha(\underline{S})$.[17] If $S > \underline{S}$ demand is determined by the mass of consumers whose valuation for AVs is above that of the indifferent consumer with valuation v^*. The (linear) demand function $q(p)$ for the smart product can be derived from $q(p) = 1 - F(v^*)$ as

$$
q(p) = \min\left[1, \frac{1}{\bar{v} - \underline{v}}\Big(A - (r+\rho)(p-c) - \alpha(S)k((1-\beta)D)\Big)\right], \tag{3}
$$

where $A = \bar{v} - v_o + \alpha_o k(D)$ denotes the difference between the maximal valuation of AVs and the valuation of conventional cars, taking into account their expected liability costs for consumers. The second term gives the (flow utility equivalent of the) price difference between AVs and conventional cars, and the last term captures the effect of consumers' liability risk when purchasing an AV.

At each point in time t, the innovator chooses an investment level $x(t) \geq 0$ which improves the safety stock of the smart product. This safety stock evolves according to

$$
\dot{S}(t) = x(t) - \delta S(t) \tag{4}
$$

where $\delta > 0$ measures the stock depreciation.[18] Since we are interested in the innovator's ex ante decision whether or not to invest in a wholly new technology (the smart product), we assume $S(0) = 0$. As is standard in dynamic investment models, the cost of investment is given by $h(x) = \theta x\left(1 + \frac{\eta}{2}x\right)$, where $\theta > 0$ measures the marginal costs at $x = 0$, and $\eta > 0$ is a parameter capturing convexity.

[17] Formally, $\underline{S}$ is implicitly determined by $\alpha(\underline{S}) = \frac{\bar{v} - v_o + \alpha_o k(D)}{\beta D + k((1-\beta)D)} > 0$. We assume that $\alpha(0) > \frac{\bar{v} - v_o + \alpha_o k(D)}{\beta D + k((1-\beta)D)} > \alpha_{min}$. Due to the strict monotonicity of $\alpha(S)$ this implies that the equation above has a unique strictly positive solution.

[18] That a positive investment is needed to maintain the stock can be interpreted as a reduced-form modeling of the fact that the complexity of the smart product increases over time.

The innovator chooses the trajectories $p(t)$ and $x(t)$ to maximize its expected discounted profit, which is given by

$$\widetilde{\Pi} = \int_0^\infty e^{-rt} \left[q(p(t), S(t)) \cdot \left(p(t) - c - \int_t^\infty e^{-(r+\rho)(\tau - t)} \alpha(S(t)) \beta D \, d\tau \right) - h(x(t)) \right] dt \quad (5)$$

subject to the state dynamics of the safety stock as described in (4). Thereby, the second integral term in (5) denotes the expected (discounted) liability cost generated by each unit of the AV sold at date t. Since the price choice has no intertemporal effect, in each point in time t, it can be determined by standard monopoly pricing.[19] This leads to the following expressions for the monopoly price and quantity for $S > \underline{S}$:

$$p^*(S) = c + \frac{1}{2} \left[\frac{A + \alpha(S) \cdot (\beta D - k((1-\beta)D)}{r + \rho} \right], \quad (6)$$

$$q^*(S) = \frac{A - \alpha(S)(\beta D + k((1-\beta)D))}{2(\bar{v} - \underline{v})}. \quad (7)$$

Note that whereas the impact of the safety stock S on the monopoly quantity is clearly positive (i.e., q^* is increasing and concave in S), the effect on the price depends on the sign of the expression $\beta D - k((1-\beta)D)$ and is hence ambiguous: Intuitively, a safer product increases the consumers' willingness to pay (demand effect), and it also lowers the innovator's cost for each unit of the smart product sold (liability effect). If the innovator's share of liability is high (i.e., for β large), the liability effects dominates, so that $p^*(S)$ is decreasing in S. By contrast, for β sufficiently low, the demand effect is stronger than the demand effect and $p^*(S)$ is increasing in S. [20]

Substituting the optimal price $p^*(S)$ and the optimal quantity $q^*(S)$ into (5) yields the reduced-form objective function

$$\Pi = \int_0^\infty e^{-rt} \left[\pi^*(S) - h(x) \right] dt \quad (8)$$

where $\pi^*(S) = q^*(S) \cdot \left(p^*(S) - c - \frac{\alpha(S)\beta D}{r+\rho} \right)$ denotes the expected instantaneous profit which is generated by the (optimal) sales in a given period for a given safety stock S. The intertemporal optimization problem for the innovator is then to maximize (8) w.r.t $x(\cdot)$ under the state dynamics (4) and the constraint $x(t) \geq 0$ for all t.

[19]Note, however, that the optimal (monopoly) price is not uniform, but will differ across periods, as the safety stock S also varies over time. We abstract from learning effects of the form that the marginal cost of production in a given period is decreasing in the cumulative output in earlier periods, see the discussion in Section 6.

[20]Similarly, such an ambiguous relationship between the optimal price and accident risk also arises in Daughety and Reinganum (1995) for the case of complete information.

3 Optimal safety stock accumulation: active and passive steady states

In this section we characterize the intertemporally optimal investment path of the innovator, combining Optimal Control and Dynamic Programming methods. Using the Maximum Principle (see e.g. Grass et al., 2008) we directly obtain the following necessary optimality conditions:

Lemma 1. *Let $x^*(t)$ be an optimal solution for the innovator's intertemporal optimization problem and S^* the corresponding state trajectory. Then there exists a piece-wise differentiable costate trajectory $\lambda(t)$ such that*

$$x^*(t) = \max\left[\frac{\lambda(t) - \theta}{\theta\eta}, 0\right], \tag{9}$$

S^* evolves according to the state equation given in (4), and $\lambda(t)$ satisfies the costate equation

$$\dot{\lambda} = (r + \delta)\lambda + \alpha'(S) \cdot q^*(S)\frac{\beta D + k((1-\beta)D)}{r + \rho}. \tag{10}$$

Furthermore, the transversality condition

$$\lim_{t\to\infty} e^{-rt}\left[\pi^*(S^*(t)) - h(x^*(t)) + \lambda(t)(x^*(t) - \delta S^*(t))\right] = 0$$

holds.

The optimal investment schedule x^* is the typical one for a dynamic investment problem with quadratic investment costs. The innovator only invests if the value of the co-state (which corresponds to the derivative of the innovator's value function with respect to the safety stock S) is larger than the marginal investment costs at $x = 0$, where $h'(0) = \theta$. Above that level, investment grows linearly with the co-state value λ. Furthermore, the level of investment decreases as the cost function becomes more convex (i.e., as $\theta\eta$ increases).

In a next step, we characterize potential steady states of the system under optimal investment. Denoting by $\phi^*(S)$ the inter-temporally optimal investment strategy, we use the following terminology throughout.

Definition 1.

i) A safety stock S^* is denoted as a steady state, if it is a fixed point of the state dynamics (4) under the investment strategy $x = \phi^*(S)$.[21]

[21] It should be noted that not all fixed points of the canonical system (4) and (10) correspond to steady states in that sense. However, for every steady state there exists a suitable co-state such that the state/co-state tuple is a fixed point of the canonical system. Hence, the set of fixed points of the canonical system provides candidates for steady states.

*ii) A steady state S^* is called **active** if it exhibits strictly positive levels of both safety stock and production of the smart product ($S^* > 0$ and $q^*(S^*) > 0$).*

*iii) A steady state S^* is called **passive** if these two levels are zero ($S^* = q^*(S^*) = 0$).*

We only consider these two types of potential steady states, because the two other cases $S^* > 0, q^*(S^*) = 0$ or $S^* = 0, q^*(S^*) > 0$ are irrelevant: The former case can never be optimal because the firm does not gain from a positive safety stock if it does not produce the smart product at all. The latter case is ruled out by the strictly positive minimum safety stock $\underline{S} > 0$ required for production to take place, i.e., $q^*(S^*) = 0$ must hold whenever $S^* = 0$.

The candidates for a steady state of the problem can be identified by considering the intersection of the two isoclines $g_\lambda(S)$ and $g_S(S)$ of the canonical system (4) and (10), i.e., setting $\dot{\lambda} = 0$ and $\dot{S} = 0$:

$$g_\lambda(S) \;:=\; -K_1\alpha'(S)q^*(S), \tag{11}$$

$$g_S(S) \;:=\; \begin{cases} \theta\eta\delta S + \theta & \text{for } S > 0 \\ [0,\theta] & \text{for } S = 0, \end{cases} \tag{12}$$

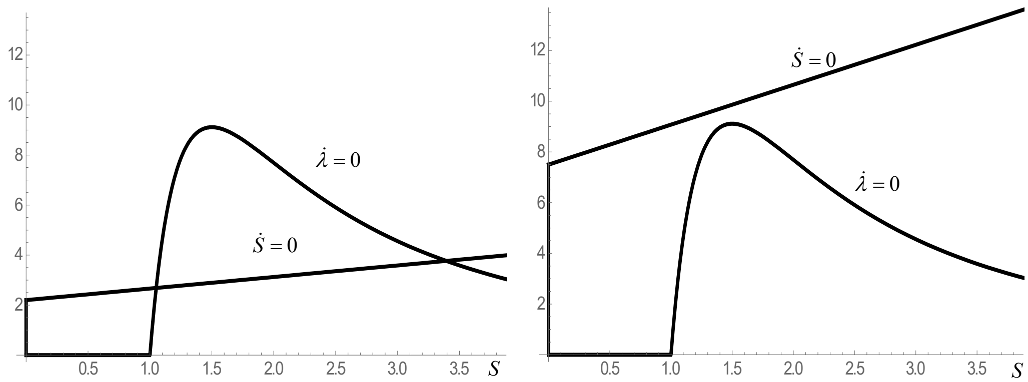
where $K_1 = \frac{\beta D + k((1-\beta)D)}{(r+\rho)(r+\delta)}$.

First, since a strictly positive safety stock $\underline{S} > 0$ is required before the smart product is launched, we have $q^*(0) = 0$, and hence $g_\lambda(0) = 0$. This implies that the two isoclines intersect at $S = 0$. Hence, the point $S = 0$ is always a candidate for a passive steady state in the sense that under the expectation that $S = 0$ prevails for all future it is indeed optimal for the innovator to invest $x = 0$ (and to produce $q = 0$) in all periods. Clearly, this does not necessarily imply that staying at $S = 0$ is the globally optimal trajectory for the innovator. Furthermore, for $S > \underline{S}$ we have $g_\lambda(S) > 0$, since $q^*(S) > 0$, where $\lim_{S\to\infty} g_\lambda(S) = 0$.[22] Taking into account that $g_\lambda(S)$ is independent from θ and that $\lim_{\theta\to 0} g_S(S) = 0$ for all $S > 0$, it follows that for sufficiently small values of θ there also exist (at least two) candidates for a steady state with $S > 0$. In Figure 1 we illustrate this observation by showing the isoclines of the canonical system for two values of θ. Whereas for a large θ the only candidate for a steady state is the one with $S = 0$, for small θ two additional fixed points of the canonical system with positive safety stock exist. Whereas, the one with the smaller value of S is an unstable node, the one with the larger safety stock is saddle point stable in the state/co-state space and hence indeed a candidate for an (active) steady state.

The following lemma makes this observation more rigorous. It shows that the safety stock evolves in a monotonous way. Moreover, regardless of the liability regime, there always exists a threshold for the cost parameter θ such that, whenever θ is above that threshold, the innovator's optimal investment leads to a safety stock of zero in the long run.

[22] The last equality follows because due to $\alpha'(S) < 0$ and $\alpha(S) \geq \alpha^{min}$ for all $S > \underline{S}$ we must have $\lim_{S\to\infty} \alpha'(S) = 0$.

Figure 1: Isoclines of the canonical system for different values of θ.



(a) θ low (b) θ high

Lemma 2.

(i) Every optimal path of the innovator's intertemporal optimization problem converges in a (weakly) monotone way to a finite steady state. There exists a finite upper bound $\bar{S}(\theta)$ with $\bar{S}'(\theta) \leq 0$ such that any steady state is in $[0, \bar{S}(\theta)]$ for all $\beta \in [0, 1]$.

(ii) For any $\beta \in [0, 1]$ there exists a threshold $\theta^{max}(\beta)$ such that for $\theta > \theta^{max}(\beta)$ any optimal path with $S(0) \in [0, \bar{S}(\theta)]$ converges monotonously to $S = 0$.

In light of our assumption that the initial safety stock is given by $S(0) = 0$, in what follows we focus on the description of the innovatorś optimal behavior under these initial conditions. The proposition then characterizes in more detail the conditions for the existence of active and passive steady states and their properties:

Proposition 1. *Assume that $S(0) = 0$. There exist thresholds $\underline{\theta} < \bar{\theta}$ with the following properties.*

(i) For $\theta \leq \underline{\theta}$ the optimal trajectory converges to an active steady state with $S^*(\theta, \beta) > \underline{S}$ where the innovator produces $q^*(S^*) > 0$ in the long run for all $\beta \in [0, 1]$.

(ii) For $\theta \geq \bar{\theta}$ the innovator implements a passive steady state by optimally choosing $x(t) = 0$ and $q^*(S(t)) = 0$ for all t and for all $\beta \in [0, 1]$.

(iii) For $\theta \in (\underline{\theta}, \bar{\theta})$ there exists a threshold $\hat{\beta}(\theta) \in (0, 1)$ such that for $\beta > \hat{\beta}$ the innovator implements the passive steady state by optimally choosing $x(t) = 0,\ \forall t$. For $\beta < \hat{\beta}$ the optimal trajectory converges to an active steady state with $S^*(\theta, \beta) > 0$ where the innovator produces $q^*(S^*) > 0$ in the long run. For $\beta = \hat{\beta}$ the innovator is indifferent between the optimal path leading to the active steady state with $S^*(\theta, \hat{\beta}(\theta))$, and staying at the passive one.

The first two parts of the proposition show that for sufficiently high or low values of the cost parameter θ, whether an active or a passive steady state is reached is independent of the liability regime (β). For S sufficiently large, the innovator's profit from the smart product is always positive. Hence, in the absence of any investment costs, the firm would always invest in build up a safety stock, independent of the liability regime. By continuity, there exists a threshold $\underline{\theta}$ such that this also holds for sufficiently small values of $\theta \leq \underline{\theta}$. Furthermore, as discussed above, a sufficiently large value of the cost parameter θ implies that $S = 0$ is the only candidate for a steady state for all $\beta \in [0, 1]$. Hence, for $\theta \geq \bar{\theta}$ it is not optimal for the firm to invest in the smart product.

The most interesting scenario occurs for intermediate values of θ. For any such value of θ, in the absence of liability (i.e., $\beta = 0$), the active steady state is stable and the path leading from $S = 0$ to this stable steady state generates a strictly positive discounted payoff stream. Since the firm generates a payoff stream of zero if it stays at the passive steady state $S = 0$, it is optimal for the innovator to invest in the safety stock of the smart product and to eventually launch it. As β increase, the value of the path leading to the active steady state becomes smaller. Hence, for $\theta \in (\underline{\theta}, \bar{\theta})$ there exists a threshold $\hat{\beta} \in (0, 1)$ such that for this value the optimal path leading from $S = 0$ to the active steady state yields a discounted payoff stream of zero. Hence, only for $\beta \leq \hat{\beta}$ is it optimal for the innovator to pursue the path to the active steady state.[23] By contrast, for all $\beta > \hat{\beta}$, the expected liability cost is sufficiently high such that the innovator optimally forgoes the development of the smart product. This is in line with empirical findings of McGuire (1988), Viscusi and Moore (1993), and Galasso and Luo (2018b) that the impact of liability on innovative activities depends crucially on how stringent the liability regime actually is and characteristics of the industry under consideration.

Proposition 1 characterizes the steady state properties of our model (i.e., the long run attractor from $S = 0$), and the impact of the liability regime. In a next step, we analyze in more detail how the liability regime affects the dynamic patterns of product safety and market evolution.

4 Dynamic patterns of product safety and market evolution: The countervailing effects of liability

In this section we investigate in more detail the economic mechanisms underlying the dynamic patterns of the optimal investment and production decisions (x and q), the accumulation of the safety stock (S), the accident rate ($\alpha(S)$), the innovator's profit ($\pi^*(S)$), and social welfare. We then explore how a change in the liability regime affects the dynamics of these variables. Due to the non-linear nature of the firm's investment problem,

[23] Considering the problem from the perspective of the state space, for any $\theta \in (\underline{\theta}, \bar{\theta})$ and $\beta > \hat{\beta}$ there is a positive threshold $S^S > 0$ such that the optimal trajectory for any $S(0) < S^S$ leads to the passive steady state $S = 0$, whereas for $S(0) > S^S$ the optimal path leads to the active steady state S^*. Such a threshold S^S is referred to as a Skiba point (see e.g., Skiba, 1978; Hinloopen et al., 2013). In this interpretation the value $\hat{\beta}$ is defined as the point where the Skiba point coincides with $S = 0$.

it is infeasible to obtain a closed form solution for the optimal investment strategy. Consequently, we perform a numerical investigation of a calibrated version of the model.

4.1 Calibration for US automobile market

To obtain an empirically meaningful parametrization of the model, we focus on the US automobile market and consider the development of Autonomous Vehicles (AV, the smart product) as an alternative to conventional cars (the old product).[24] In particular, we choose our parameters with the following empirical targets in mind (reference year 2017): As for the discount rate, we follow Blincoe et al. (2015) and use $r = .03$. Moreover, Bento, Roth, and Zuo (2018) find a current average lifetime of cars in the US of 15.6 years, leading to a failure rate $\rho = .065$, which we assume to apply for both types of car.

As for accident rates, for conventional cars we set $\alpha_o = .074$. This is obtained from US data from the Insurance Information Institute, reporting a total of 7.4 claims per year per 100 insured cars.[25] For the AV, we use the functional form $\alpha(S) = \bar{\alpha}/(m + S)$, which satisfies all properties for $\alpha(S)$ as assumed in Section 2, and where we set $\bar{\alpha} = 0.12, m = 0.05$ to attain a reasonable timing for the introduction of the AV.

For the damage in the course of an accident, we use $D = 39.2$ which is calculated as follows. According to a report by the U.S. National Highway Traffic Safety Adminstration (Blincoe et al., 2015), the total social costs due to car accidents in 2010 were \$836 billion with a total of 24 million damaged vehicles. Taking also into account price inflation between 2010 and 2017 (12.4 % in total), we estimate the average damage per damaged vehicle per accident in 2017 as $(836.000 \cdot 1.124)/24 = 39.152$. As for the share of the expected damage borne by the consumers, we use $k(z) = \frac{\tau \cdot z}{\tau + z}$ which satisfies all the properties as assumed in Section 2, where we set $\tau = 50.000$.

According to data from Kelley Blue Book, the average transaction price for cars in 2017 was 34.782\$ and, hence, $p_o = 34.8 = c$.[26]

As for consumer valuations, we could not find any direct evidence for the US market. Instead, we use data reported by Brenkers and Verboven (2006) in their empirical study of several EU automobile markets, and we find that for the case of Germany, on average the consumer surplus equals the price of the car.[27] In the numerical analysis, we assume

[24] Empirical studies of the automobile markets in the U.S. and in Europe have been carried out for example by Goldberg and Verboven (2001), Brenkers and Verboven (2006) and Kagawa et al. (2006).

[25] See `https://www.iii.org/fact-statistic/facts-statistics-auto-insurance`.

[26] Kelley Blue Book is a leading source of price information for the US car market. They issue monthly press releases with information on transaction prices for the different manufacturers and models. Our value of 34.782\$ is obtained from taking the average of the prices reported for the 12 months of the year 2017. That also $c = 34.8$ holds follows from our model assumption that conventional cars are sold at marginal costs.

[27] Brenkers and Verboven (2006) report that a loss in consumer surplus of 444 Mio Euro corresponds to 1.7% of the average annual consumer surplus generated in the market for new cars in Germany between 1970-1999 (see their Table 6). Hence, the average annual consumer surplus during that time period can be calculated as 26.1 Billion Euro. Dividing this number by the average annual car sales in Germany of 2.48 Mio during that period (obtained from their Table 1), we obtain a consumer surplus per sold car of 10.531 Euro. This number almost coincides with the average sales price of 10.520 Euro in Germany

that the same property also holds for the US. Based on this, the consumer surplus can be derived as the difference between the willingness to pay (per period of usage) net of expected liability costs and the sales price (i.e. $\frac{v_o-\alpha_o k(D)}{r+\rho} - p_o$), which leads to $v_o = 8.2$. With respect to consumer valuations for the AV, we do not consider those consumers, whose expected discounted value derived from an AV is negative even if the AV had an accident rate of zero and would be sold at the price of the conventional car. This implies $\underline{v} = p_o(r+\rho) = 3.3$. To approximate the market size for AVs, we follow the seminal work on the diffusion of innovations by Rogers (2003), who argues that 16% of the consumers can be classified as either *innovators* or *early adopters*, which are the most eager ones to switch to a new product. Hence, in our numerical analysis, we assume that 16% of the mass of the (uniform) distribution of valuations on the interval $[\underline{v}, \bar{v}]$ is above $v_o(= 8.2)$. This leads to $\bar{v} = 9.2$.

Finally, with respect to the safety stock S of the AV, we assume that it depreciates with a rate $\delta = 0.07$. Moreover, as for the cost of investment into that stock, $h(x) = \theta x(1+\frac{\eta}{2}x)$, we use $\theta = 3$ and $\eta = 2.2$. This parameter choice ensures that there indeed exist a liability regime $\hat{\beta} \in (0,1)$, such that the innovator prefers the optimal path leading to the active steady state for $\beta \leq \hat{\beta}$, and staying at the passive steady state for $\beta > \hat{\beta}$.[28] For our calibration, this leads to $\hat{\beta} = 0.29$.

In order to determine the optimal paths for our calibrated model we numerically determine the value generated by the optimal path form $S(0) = 0$ leading to the candidate for an active steady state (if it exists) and compare it to the value of staying at the passive steady state, which is zero. The optimal path leading to the candidate for the active steady state and the associated value are calculated by solving the Hamilton-Jacobi-Bellman (HJB) equation for the innovator's dynamic optimization problem, associated with a path converging to a positive steady state, on the state space $S \in [0, \bar{S}(\theta)]$. The function $\tilde{V}(S)$ solving this HJB equation is evaluated at the initial state $S(0) = 0$ and we obtain the value $V(0) = \max[0, \tilde{V}(0)]$ at this state. The scenarios for which positive investment and convergence to an active steady state are optimal for the innovator are therefore characterized by the condition $\tilde{V}(0) \geq 0$. Since an analytical solution for the HJB equation is not available, we obtain an (approximate) solution to the equation using a collocation method. The formulation of the HJB equation and a short description of our method is given in Appendix B.

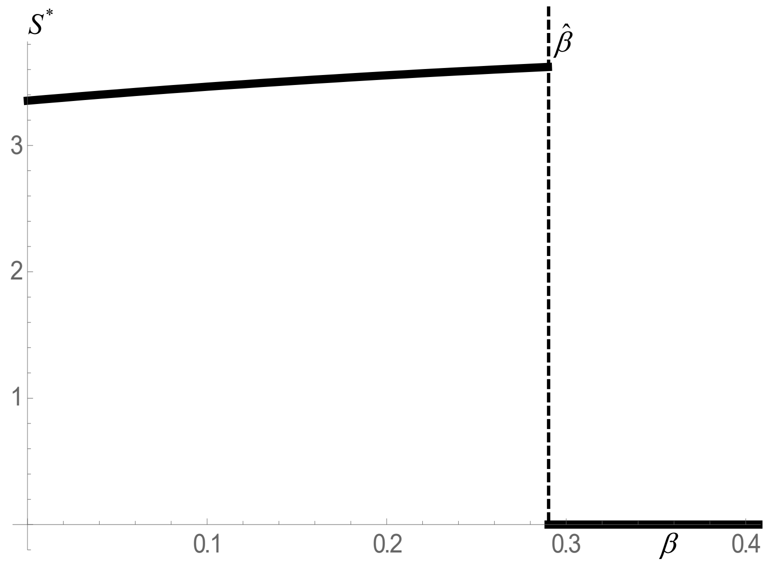
4.2 Active and passive steady states of safety stock accumulation

We start with analyzing the impact of the liability regime (β) on the existence and properties of active and passive steady states as characterized in Proposition 1 above. As illustrated in Figure 2, when the innovator's liability share (β) is sufficiently low (i.e.,

during the same period (also obtained from their Table 1).

[28] Put differently, our specification ensures that the isoclines (11) and (12) intersect twice as in panel (a) of Figure 1, and that we are in part (iii) of Proposition 1.

Figure 2: Safety stock in *active* and *passive* steady states for different liability regimes

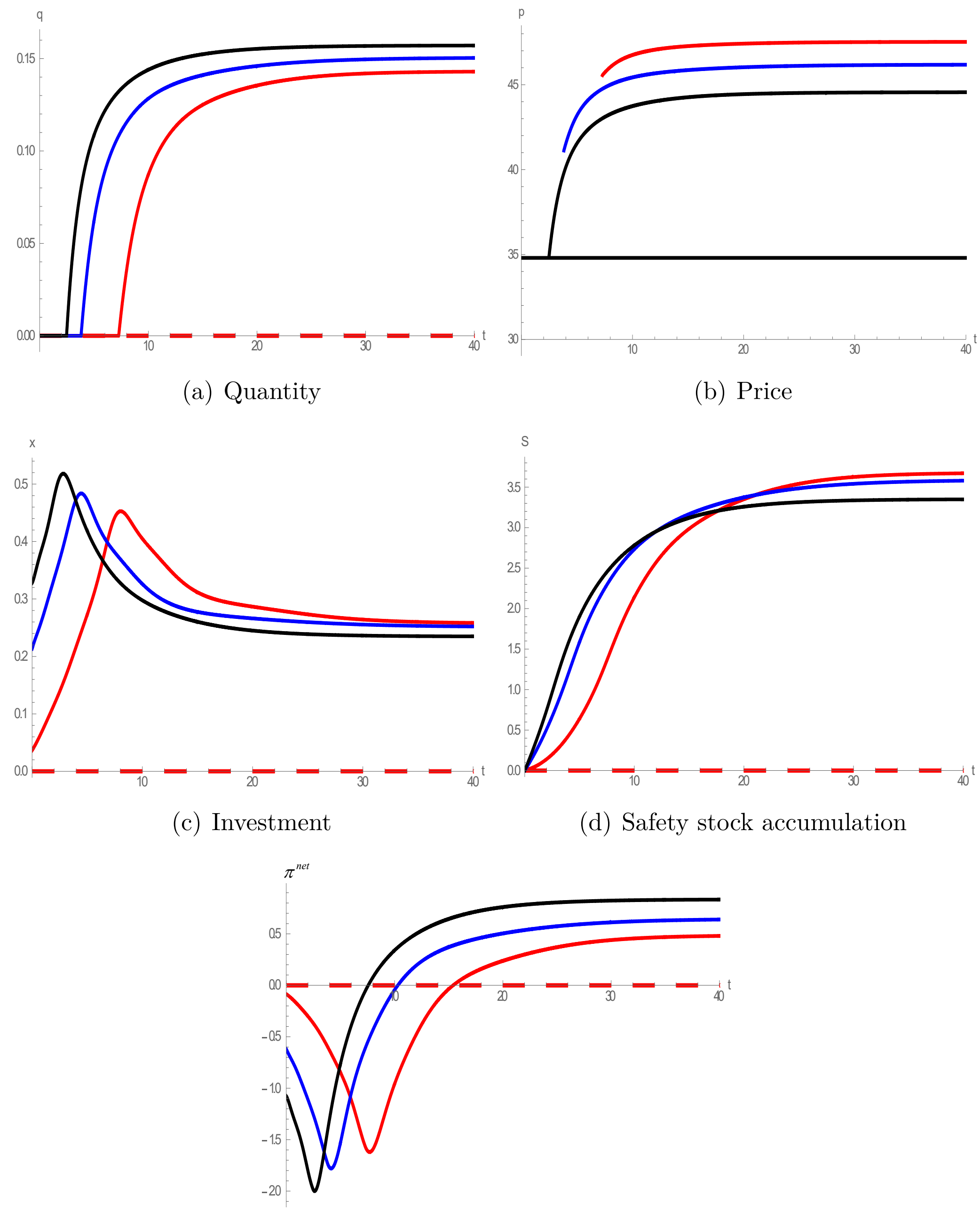


when $\beta \leq \hat{\beta}$), an *active* steady state emerges in which the innovator accumulates a positive safety stock $S^*(\beta) > 0$. Moreover, the steady state level of the safety stock is increasing in β. By contrast, for high values of $\beta > \hat{\beta}$, the *passive* steady states emerges, where the innovator (with a starting point of $S = 0$) will optimally not accumulate any safety stock over time (i.e., $S^*(\beta) \equiv 0$ for all $\beta > \hat{\beta}$). For our calibration, the critical value above which the active steady state ceases to be optimal is $\hat{\beta} = 0.29$. Hence, this property illustrates one trade-off when shifting more liability on the producers of smart products: Given that these products will eventually be launched at all, more liability indeed increases the level of product safety in the long-run, but it might also discourage the activity altogether. Overall, these results show that the relationship between liability and product innovation can in principle be positive or negative, and that it depends (among others) on the characteristics of industry under consideration and the level of liability actually chosen. This is in line with the empirical findings of McGuire (1988); Viscusi and Moore (1993); Galasso and Luo (2017, 2018b) as discussed above.

4.3 Optimal dynamics

From now on, we focus on liability regimes $\beta \leq \hat{\beta}$, such that optimal investment by the innovator leads to convergence to the active steady state. In the following, we analyze in more detail the dynamic properties of the key variables of interest, and how these trajectories are affected by the liability regime. The results are illustrated in Figure 3, where we depict trajectories for $\beta = 0, \beta = 0.15$ and $\beta = \hat{\beta} = 0.29$. Lemma 2 directly implies that the safety stock grows monotonously under any optimal investment path leading to an active steady state. Hence, understanding how the key variables depend on

Figure 3: Dynamic patterns leading to the active steady state



(a) Quantity

(b) Price

(c) Investment

(d) Safety stock accumulation

(e) Profit

The solid lines of different colors show the optimal paths for different values of firm liability: $\beta = 0$ (black), $\beta = 0.15$ (blue), $\beta = 0.29$ (red). For the threshold value $\hat{\beta} = 0.29$ both the path converging to the active steady state (solid red line) and the one staying in the passive steady state (dashed red line) are optimal. In panel (b), the three depicted paths show the optimal price for the AV starting from the respective period for which the optimal quantity $q^*(S)$ becomes strictly positive. The horizontal black line indicates the price of the conventional car ($p_o = 34.8$).

S also allows to understand their behavior over time.

Production quantities As can be seen from panel (a), the AV is not immediately launched. Rather there is a time interval beforehand, during which the innovator does not produce and only invests to improve the AVs' safety (this issue will be discussed on more detail in Section 4.4 below). Once production does take place, the (instantaneous) quantity is indeed increasing and concave over time for all three depicted liability regimes (see the discussion of Eqn. (7) above). Intuitively, as the safety stock increases over time (see panel (d)), the AV becomes safer which, ceteris paribus, decreases the innovator's unit costs and increases consumers' net benefit from it. Both effects lead to higher sales. Moreover, while a more stringent liability regime does not affect the dynamics of output decisions qualitatively, it does, however, induce a later date of market introduction of the AV and lower production quantities afterwards, and hence a slower market penetration.

Prices Panel (b) of Figure 3 shows that the price for the AV optimally chosen by the innovator increases over time as the safety stock increases. Recall from the discussion of Eqn. (6) above, that the effect of S on the optimal price of the AV $p^*(S)$ is a priori ambiguous, and that it is increasing in S for β sufficiently small, in which case the demand effect dominates the liability effect. In our calibration, it turns out that this is indeed the case in the relevant parameter range $\beta \in [0, \hat{\beta}]$ where the innovator optimally induces the active steady state. Moreover, also more stringent liability leads to higher prices.

Investment Panel (c) of Figure 3 reveals a hump-shaped investment pattern over time. Again, this process is affected by the liability regime: For low values of β, initial investments are high, with a peak being reached already in early periods, and then exhibiting a relatively strong decline. By contrast, for larger values of β, the initial investment outlays are lower, the peak is reached at a later date, and the decline is less severe. As a result, in the short-run (intermediate and long run), the investment are the lower (higher), the more stringent the liability regime.

Several channels are at work which jointly determine the dynamic investment pattern. To gain an intuition, recall from Lemma 1 that the optimal investment is linearly increasing with the co-state of $\lambda(t)$ (see (9)), which corresponds to the state derivative of the value function. For the purpose of illustrating the qualitative effects, in what follows, we focus on the effects of a marginal increases of the safety stock on (discounted future) market profits. In doing so we abstract from the fact that an increase of S might also have an impact on the future investment paths and the associated discounted costs. Hence, we approximate the state derivative of the value function by the derivative of the innovator's discounted future market profits with respect to S. Formally, we define $\tilde{\Pi}^*(\tilde{S}) = \int_0^\infty e^{-rt}\pi^*(S(t))dt$ with $S(t)$ following the path induced by optimal investment from $S(0) = \tilde{S}$.[29] Using this approximation, the optimal investment level is linearly

[29]To clarify our approximation it should be noted that the actual value function is defined as $V(\tilde{S}) = \int_0^\infty e^{-rt}(\pi^*(S(t) - h(x^*(t))dt$ with $S(t)$ following the path induced by optimal investment $x^*(t))$ from

increasing in the term $\frac{d\tilde{\Pi}^*}{d\tilde{S}}$ and we consider this term to gain an understanding of the factors determining optimal investment. Taking into account the envelope theorem, it can be written as follows:

$$\begin{aligned}\frac{d\tilde{\Pi}^*(\tilde{S})}{d\tilde{S}} &= \int_0^\infty e^{-rt}\left[\frac{\beta D}{r+\rho}q^*(S(t)) + \left(p^*(S(t)) - c - \frac{\alpha\beta D}{r+\rho}\right)\frac{k((1-\beta)D)}{\bar{v}-\underline{v}}\right]\cdot\left|\frac{\partial\alpha(S(t))}{\partial\tilde{S}}\right| dt \\ &\approx \Bigg[\underbrace{\frac{\beta D}{r+\rho}Q^*(\tilde{S})}_{\substack{\text{Profit Margin Effect}\\ \text{(PME)}}} + \underbrace{\left(P^*(\tilde{S}) - \frac{c}{r+\delta}\right)\frac{k((1-\beta)D)}{\bar{v}-\underline{v}}}_{\substack{\text{Quantity Effect}\\ \text{(QE)}}}\Bigg]\cdot\underbrace{\left|\frac{\partial\alpha(\tilde{S})}{\partial\tilde{S}}\right|}_{\substack{\text{Safety Effect}\\ \text{(SE)}}}, \end{aligned} \quad (13)$$

where $Q^*(\tilde{S}) = \int_0^\infty e^{-(r+\delta)t}q^*(S(t))dt$ and $P^*(\tilde{S}) = \int_0^\infty e^{-(r+\delta)t}\left(p^*(S(t)) - \frac{\alpha(S(t))\beta D}{r+\rho}\right)dt$ denote aggregated discounted future sales quantities and net prices (i.e. prices minus liability costs) for an initial safety stock $\tilde{S}$. Note that to obtain the second line we have used the approximation $\frac{\partial\alpha(S(t))}{\partial\tilde{S}} \approx e^{-\delta t}\frac{\partial\alpha(\tilde{S})}{\partial\tilde{S}}$ based on the fact that the safety stock depreciates at a rate of δ.

Intuitively, the multiplicative third term captures direct effect of an increase S, namely a reduction in the accident rate of the AV (SE). The first two terms capture the effect of a decrease in the accident rate on the profit margin (PME) and the quantity of AV sold (QE), respectively: As for the PME, for a given aggregate quantity Q^*, a lower $\alpha(S)$ increases the profit margin of the innovator for each unit sold now and in the future, as it leads to a decrease of liability costs, which is constant over time. As for the QE, the demand function (3) shifts upwards by $\frac{k((1-\beta)D)}{\bar{v}-\underline{v}}$ as α decreases. To obtain the total marginal profit of this quantity increase, it is multiplied with the discounted sum of the marginal profits over time.[30]

To understand the dynamics of the optimal investment over time we analyze how the different effects in Eq. (13) vary in S (formally, we analyze $\frac{\partial^2\tilde{\Pi}^*}{\partial\tilde{S}^2}$). The following three (countervailing) channels drive the optimal investment pattern for the time *after* the AV has been launched (i.e., when $q^* > 0$). First, the PME becomes stronger because the discounted sum of future sales, $Q^*(\tilde{S})$, increases with $\tilde{S}$. Hence, this channel leads to an increasing investment incentive over time. Second, also QE becomes stronger, because an increase in S leads to both a higher price (see panel (b)) and a lower (expected) liability cost per unit sold (since α decreases with S) in all future periods. Hence the term $P^*(\tilde{S})$ increases over time and also this channel generates growing investment incentives. Third, the SE generates a decreasing investment incentive over time, because the marginal effect of a higher safety stock on the accident rate decreases over time (recall that $\alpha'(S) < 0$ and $\alpha''(S) > 0$). This term is multiplicative such that it scales the size of the two previous channels. For the time period before the market introduction of the AV, a fourth channel

$S(0) = \tilde{S}$. Hence, under our simplifying assumption that $\frac{\partial}{\partial\tilde{S}}\int_0^\infty e^{-rt}h(x^*(t))dt \approx 0$ we obtain that $\frac{\partial V(\tilde{S})}{\partial\tilde{S}} = \frac{\partial\Pi^*(\tilde{S})}{\partial\tilde{S}}$.

[30] The change of the optimal price induced by an decrease in α and the resulting quantity change do not affect the maximized profit because of the envelope theorem.

is at work. During this time, the innovator seeks to accumulate, at minimum cost, the level of safety stock $\underline{S}$ with which the AV is launched. Note that the discounted profit generated after the introduction of the AV does not explicitly depend on the time of the market introduction. For a given market introduction date, in the absence of discounting, due to the convexity of the investment cost function $h(x)$, it would be optimal to spread the investment outlays evenly over the time period before the AV is launched. However, with discounting, the value of this fixed future profit becomes larger as the date of market introduction comes closer. This explains the initially increasing part of the investment trajectories before the AV is launched. The interplay of these four effects generates the observed hump-shaped pattern.

Equation (13) is also useful to develop an intuition of how the optimal investment trajectories are affected the liability regime (β). Note first that, ceteris paribus, an increase in β does not affect the SE. With respect to the PME and QE, we obtain the following derivative with respect to β:

$$\begin{aligned}\frac{\partial[PME+QE])}{\partial\beta} = & \underbrace{\frac{D}{r+\rho}Q^*(\tilde{S})}_{>0} + \underbrace{\frac{\beta D}{r+\rho}\frac{\partial Q^*(\tilde{S})}{\partial\beta}}_{<0} + \underbrace{\frac{\partial P^*(\tilde{S})}{\partial\beta}\frac{k((1-\beta)D)}{\bar{v}-\underline{v}}}_{<0} \\ & + \underbrace{\left(P^*(\tilde{S}) - \frac{c}{r+\delta}\right)\frac{-Dk'((1-\beta)D)}{\bar{v}-\underline{v}}}_{<0}. \end{aligned} \tag{14}$$

Intuitively, the first of the four terms captures that a higher β fortifies the impact of changes in α on the profit margin (i.e., the coefficient of Q^* increases with β). This leads the innovator to respond with higher investment, and the effect is proportional to quantity. As for the second term, in light of (7), there is a direct negative effect of β on the accumulated discounted future quantities ($\frac{\partial Q^*(\tilde{S})}{\partial\beta} < 0$). This reduces the size of the MPE and hence dampens investment incentives. Note that this argument abstracts from the indirect effect that a change in β also affects the optimal path of $S(t)$, which also influences the accumulated discounted quantities and prices. Similarly, with respect to the third term, abstracting from the indirect effect and using (6) we obtain

$$\frac{\partial P^*(\tilde{S})}{\partial\beta} = \frac{D(k'((1-\beta)D)-1)}{2(r+\rho)}\int_0^\infty e^{-(r+\delta)t}\alpha(S(t))dt < 0. \tag{15}$$

This expression is negative, since a higher β leads to higher unit costs, which is not fully offset by the increase in demand resulting from the lower remaining liability share $(1-\beta)D$ borne by consumers (this is due to $k'(\cdot) < 1$). As a result, an increase in β induces a lower profit margin (price minus liability costs) for the innovator, which lowers investment incentives. Finally, as for the last term in (14), shifting liability from the consumers to the innovator, makes demand react less strongly to an increase of the safety stock, thereby reducing investment incentives.

The interplay of these four effects as captured in (14) explains why the qualitative implications of an increase in product liability differs between the short-term and the long-term (see panel (c) of Figure 3). Note that only the first term in (14) generates a positive effect of β on investment incentives, which is proportional to the discounted stream of future sales (Q^*). In early periods, Q^* is small and therefore this positive effect is dominated by the three negative effects. This explains why initially investments are lower for larger values of β. Eventually, in later periods Q^* becomes sufficiently large such that the first positive effect dominates, and we obtain a positive relationship between β and safety stock investment.

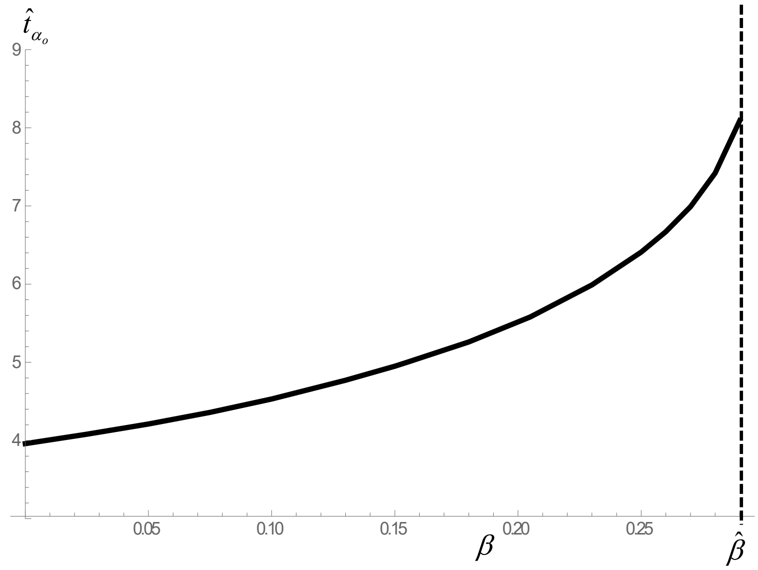
Finally, it is noteworthy that in the absence of liability, the innovator's investment incentives are not zero, even in the short-run. The reason is that in this case, all liability risk would be borne by the consumers. This would lower their willingness to pay for the AV, hence shrinking the market for these vehicles. To countervail this effect, the innovator does optimally invest in product safety, even when she does not face a direct liability risk. These are exactly the "market forces" stressed by Polinsky and Shavell (2010) in their critical discussion of product liability.

Accumulation of safety stock The dynamic patterns of investment incentives determine the impact of the liability regime on the process of safety stock accumulation. In line with our observation that in the long run an increase of β induces higher investment, panel (d) of Figure 3 shows that the safety stock ultimately reached in the active state steady is increasing in the innovator's liability share β. However, panel (d) also conveys that, as β increases, the accumulation of the safety stock occurs more slowly in early periods, due to the lower initial investment incentives. For example, during the first 8 years, S is largest for $\beta = 0$ followed by $\beta = 0.15$ and $\beta = 0.29$ and it takes 20 years until the safety stock is highest under the most stringent of the three liability regimes. This effect is illustrated in more detail in Figure 4. It shows that the time elapsing until the AV eventually reaches the same safety standard (accident rate) as the conventional car, is increasing and convex in the stringency of liability.[31] That is, as long as β is relatively small, making liability more stringent only causes a moderate delay in the accumulation process compared to the benchmark without liability ($\beta = 0$). But delay becomes considerable as β moves closer towards the threshold level ($\beta = 0.29$), for which the active steady state ceases to be optimal for the innovator.

In summary, even when confining attention to liability regimes which are not too stringent in the sense of preventing the introduction of the smart product altogether, our analysis points to a second trade-off associated with shifting more liability to the producers of smart products: One the one hand, this leads to a higher safety level of these products in the long run. But on the other hand, it delays the process of safety stock accumulation, so that it takes longer until a given safety standard is reached. As a result, under a more stringent liability regime, the units being sold in the short- and intermediate run are less safe than they would have otherwise been under a less stringent regime.

[31] Formally, we define $\hat{t}_{\alpha_o}$ as the point in time t at which $\alpha(S(t)) = \alpha_o$ holds.

Figure 4: Delay in safety stock accumulation depending on the liability regime



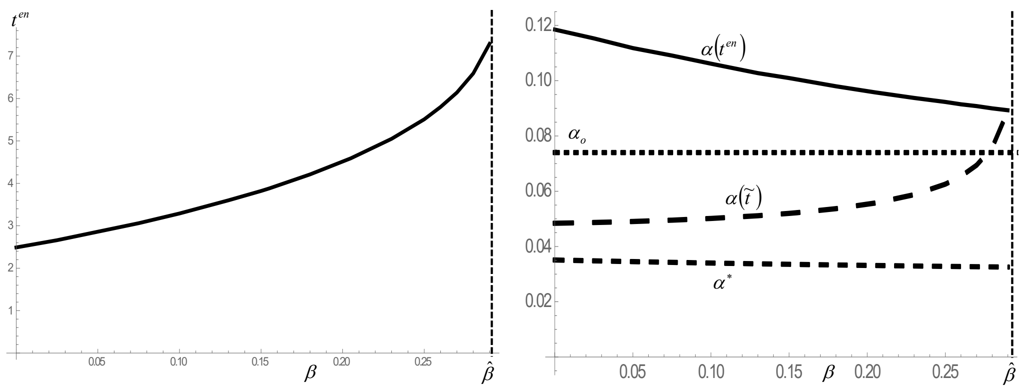
The vertical axis measures the time ($\hat{t}_{\alpha_o}$) until the accident rate of the AV equals that of the conventional car, i.e., $\alpha(S(\hat{t}_{\alpha_o})) = \alpha_o = 0.074$.

Instantaneous profits Panel (e) of Figure 3 illustrates that the resulting instantaneous innovator's net profit ($\pi^{net} = \pi^*(S(t)) - h(x(t))$) from the AV exhibits a U-shaped pattern over time. In particular, in early periods large losses incur, and they are even increasing during this time, which is due to the increasing investment profile in the safety stock of the AV before market production as discussed above. After the AV is launched, however, profits eventually become positive, and are increasing and concave over time. This latter feature is driven by the concavity of the optimal production quantities over time. As for the impact of the liability regime, the U-shaped pattern is more balanced as β increases, since initial investments (and hence losses) are smaller, but the break-even point is reached later (due to the later market introduction of the AV), and also the profits achieved in later periods are lower.

4.4 Timing of market introduction of AV, and safety standard of first generation

In this section we investigate in more detail the impact of liability on the date of market introduction of the AV (denoted by t^{en}) and the corresponding safety level $\alpha(t^{en})$ of this first generation AV. As shown in panel (a) of Figure 5, a more stringent liability regime leads to a delay with respect to the market introduction of the AV, which is increasing in the stringency of liability. This feature is due to two effects: First, as discussed above, in early periods the negative effects of a higher β on investment incentives dominate, so that it takes longer until any given level of safety stock S is reached. Second, a higher β

Figure 5: Date of market introduction of AV and corresponding safety standard



(a) Date of AV market introduction (b) Accident rate of AV

Panel (a): $t^{en}(\beta)$ indicates the point in time at which the AV is launched (i.e., where the optimal instantaneous quantity becomes strictly positive) for a given β. Panel (b): accident rate of AV i) in the first generation ($\alpha(t^{en})$), ii) at a fixed point in time $\tilde{t}$ ($\alpha(\tilde{t})$) and iii) in the active steady state (α^*). The dotted line denotes the accident rate of the conventional car ($\alpha_o = 0.074$).

increases the safety level $\underline{S}$ with which the AV is launched (see panel (b)).

Panel (b) of Figure 5 also shows that, whereas β has a beneficial effect on the accident rate of the AV at the time of market introduction and in the long run (steady-state), there is also a negative effect in the short run. If we consider a fixed point in time $t = \tilde{t}$ not too far in the future, then the accident rate at $\tilde{t}$ is increasing in β.[32] Intuitively, when β is low, the AV is launched earlier and the innovator's investment incentives increase once it sells positive quantities. In particular, the difference between $\alpha(t^{en})$ and $\alpha(\tilde{t})$ shows by how much the accident rate is reduced due to the earlier market introduction for a low β compared to the case of $\beta = \hat{\beta}$. Put differently, if a policy maker aims at maximizing the safety level of AVs at a given point in time not too far in the future, then having a lenient product liability regime ($\beta = 0$) is optimal. However, the policy maker should be aware that under such a regime AVs are introduced to the market early and the first product generations exhibit a safety level that is substantially lower than first generation AVs under more stringent liability. Panel (b) of Figure 5 also reveals that for a large range of β the accident rate at $t = \tilde{t}$, is actually already below that of the conventional car. Last, but not least, the figure also illustrates that for our calibration, the long run accident rate of the AV is below that of the conventional car (α_o) for all β. This is consistent with respective claims concerning the long-term benefit of AVs compared to conventional cars.

4.5 Welfare

Having studied the implications of an increase in β on the timing of market introduction of AVs and the evolution of the safety level, we now consider the total welfare effects of

[32]For the purpose of illustrating this effect, in the figure we have set $\tilde{t} = 7.29$, which is the date of market introduction of the AV for $\beta = \hat{\beta}$.

the liability regime. As usual we define welfare as the sum of firm profits and consumer surplus. Since we are interested in the welfare implications of AVs, in what follows we consider the additional welfare generated by their introduction relative to the status quo where they are not available.[33] This leads the following measure ΔW for the (instantaneous) welfare effect of AVs:

$$\Delta W = \int_{v^*}^{\bar{v}} \left(\frac{v - \alpha(S)D - (v_o - \alpha_o D)}{r + \rho} \right) \frac{1}{\bar{v} - \underline{v}} dv - h(x), \tag{16}$$

where the first term measures the additional welfare generated by those consumers who prefer to purchase the AV instead of the conventional car.[34] Recall that the threshold $v^* = \bar{v} - q^*(S)(\bar{v} - \underline{v})$ is determined by the valuation of the consumer who is just indifferent between purchasing either type of car.

We proceed in two steps: In a first step, we compare the optimal investment path of the innovator with the welfare-maximizing path for a *given* level of β.[35] As is illustrated in Figure 9 in Appendix C, the socially optimal path exhibits substantially larger safety stock investments compared to the innovator's optimal path. Since the accident rate upon market introduction does not differ between the two paths, this implies that AVs are launched earlier under the socially optimal path. Furthermore, at each point in time after market introduction, the quantity is higher under the socially optimal investment level. Overall, this comparison suggests that from a welfare perspective, it is desirable to speed up investments into the safety stock of AVs and to increase their sales volume.

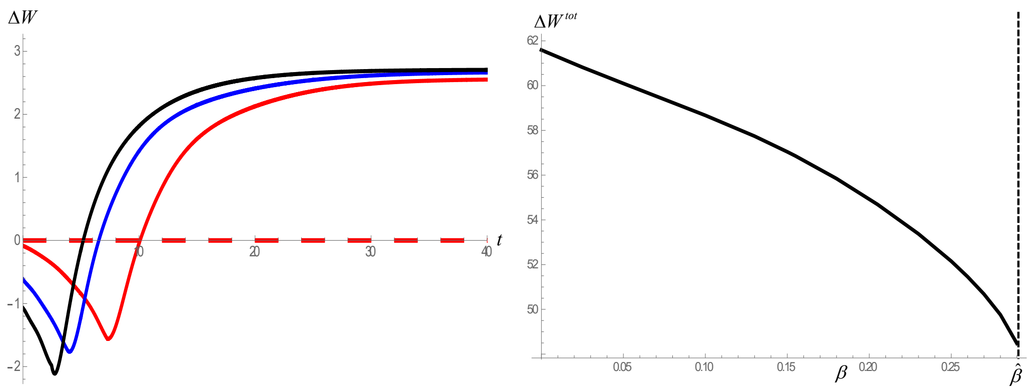
In a second step, we consider the welfare effect of a change of β under the innovator's optimal path, and the results are shown in Figure 6. As can be seen from panel (a), instantaneous welfare exhibits a U-shaped pattern over time. Intuitively, it is initially negative due to the investment outlays before the AV is launched, and then eventually becomes positive afterwards. Moreover, as the liability regime becomes more stringent, this leads to a more balanced pattern (i.e., lower initial welfare losses, but for a longer period of time, and also lower gains in later periods). In particular, the long-run (i.e., steady-state) welfare level decreases with β, although a higher value of β induces a lower long-run accident rate. Intuitively, an increase of β leads to lower AV sales in the long-run, and the resulting welfare loss dominates the positive welfare effect of a better safety.

[33] Hence, we consider the difference in welfare between the cases with and without AVs, respectively. Since in the latter case, the resulting welfare is independent of S and β, this does not have any qualitative effect on the results.

[34] As for the damage resulting from accidents, note that simply adding up the liability shares of the respective consumer and the innovator (i.e., $k((1-\beta)D)+\beta D)$ would leave unaccounted for the remaining share, which has to be borne by someone else in the economy such as, for example, another driver (consumer) involved in the accident or an insurance company. To avoid this, our welfare measure captures the total social damage D.

[35] Formally, we consider the objective function $\int_0^\infty e^{-rt} \Delta W(S(t), x(t)) dt$ subject to the state dynamics (4), where $\Delta W(S(t), x(t))$ is given by (16). In order to allow for a clean comparison with the investment incentives of the innovator, in determining the socially optimal investment path we consider the same pricing behavior as in the main analysis.

Figure 6: The welfare effect of liability in the active steady state



(a) Welfare dynamics (b) Total welfare effect

Panel (a): Dynamics of instantaneous welfare effect for $\beta = 0$ (black), $\beta = 0.15$ (blue) and $\beta = \hat{\beta} = 0.29$ (red). Panel (b): Total discounted welfare effect ΔW^{tot} for $\beta < \hat{\beta}$.

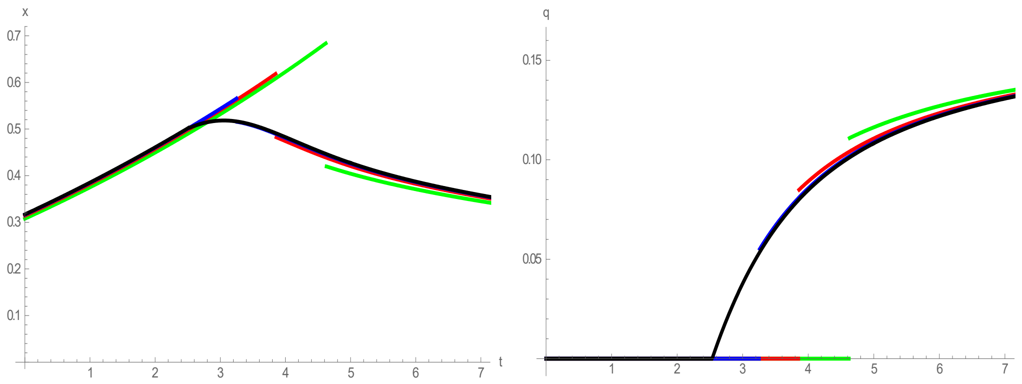
Panel (b) of Figure 6 depicts the difference in total discounted welfare ΔW^{tot} as a function of the liability regime. As long as the active steady state is induced at all (i.e., for $\beta \leq \hat{\beta} = 0.29$), the resulting welfare difference is positive, and decreasing in β. Intuitively, the initially high welfare losses induced by higher investment under a low value of β are outweighed by the intermediate and long term gains due to earlier market introduction and a larger sales volume of AVs. Our finding of an overall negative net effect of more stringent liability on welfare suggests that liability for producers of AVs should not be more stringent than it is for conventional cars ($\beta = 0$). Hence, one main conclusion that can be drawn from our analysis is that, even if the innovator is not directly liable for accidental harm caused by its AVs on the streets, substantial investment incentives are generated by the fact that a large accident rate reduces demand. In this sense our findings corroborate the argument put forward in Polinsky and Shavell (2010), that safety investments can be stipulated by market forces even in the absence of liability. Moreover, they argue that incentives to improve product safety can also be generated through direct regulation ex-ante. This issue is considered next.

5 Extension: Ex-ante safety regulation

In this section, we consider a model variant where the safety standard for AVs is directly regulated ex ante, rather than indirectly affected through the liability regime. In particular, the regulator imposes a minimum safety standard which a new product must fulfill before it can be launched. In our context, this corresponds to a maximum accident rate of the AV, denoted by α^{max}, where a high (low) value of α^{max} indicates a weak (strict) regulation.

We first illustrate the impact of α^{max} on the dynamic patterns of the two optimal

Figure 7: Impact of ex-ante safety regulation on optimal dynamic investment and output patterns



(a) Investment (b) Quantity

The lines of different colors show the optimal paths for different values of the maximum accident rate α^{max} for the AV: $\alpha^{max} = 0.12$ (black), $\alpha^{max} = 0.09$ (blue), $\alpha^{max} = \alpha_0 = 0.074$ (red), $\alpha^{max} = 0.06$ (green).

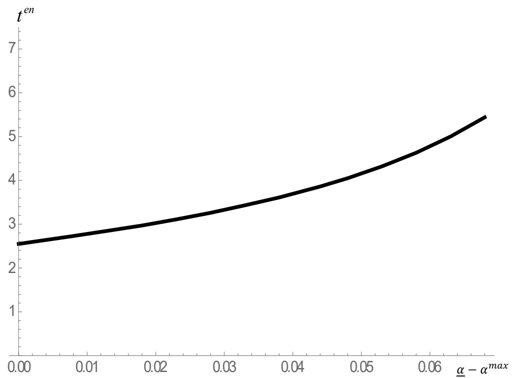
controls (i.e., investment in safety stock and output) under a liability regime with $\beta = 0$, for which the social welfare was maximized in the calibrated model without regulation (see panel (b) of Figure 6 above). The results are illustrated in Figure 7. With respect to optimal investment, consider first the black line in panel (a) which corresponds to the case of weak regulation ($\alpha^{max} = 0.12$). For this value of α^{max} regulation is not binding in the sense that, even without regulation, upon introduction the accident rate of the AV would already be below α^{max}.[36] As a consequence, the investment pattern is the same as for the case without regulation, as depicted in panel (c) of Figure 3 above.

As the regulation becomes stricter (i.e., as α^{max} decreases), the optimal dynamic investment pattern changes and exhibits a jump.[37] Intuitively, this jump occurs at the point in time at which the imposed minimum safety standard α^{max} is reached. As soon as this is the case, the investment pattern is the same as in the case without regulation considered above. By contrast, before the threshold α^{max} is reached, the regulation gives rise to a prolonged period of increasing investment compared to the case without regulation, and this period lasts the longer, the stricter the regulation. The reason is that the instantaneous profit from selling the AV would already be positive in this extended segment, but the innovator cannot reap it, because the safety level does not yet meet the standard α^{max}. This gives the innovator an incentive to intensify her safety investments to reach the threshold faster. Hence, one advantage of stricter safety regulation is that it accelerates the process of safety stock accumulation, rather than slowing it down as under more stringent liability.

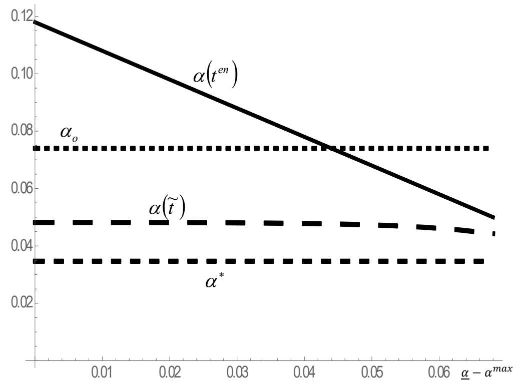
[36]This can be seen from panel (b) of Figure 5 above, where there was no regulation in place: For the case $\beta = 0$ considered here, the accident rate of the AV upon introduction is $\alpha = 0.118$, and hence below $\alpha^{max} = 0.12$.

[37]As detailed at the end of Appendix B, the possibility of such jumps require slight modifications of the numerical methods used.

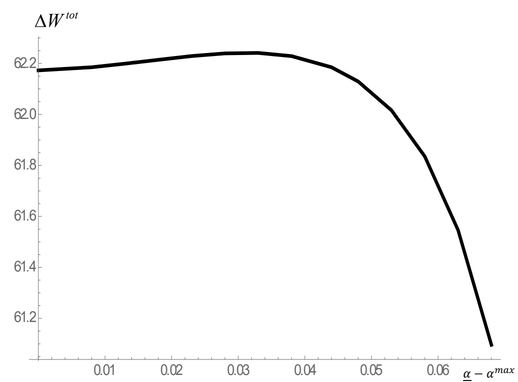
Figure 8: The impact of safety regulation on timing of market introduction, speed of safety stock accumulation and welfare



(a) Date of AV market introduction of AV



(b) Accident rate of AV



(c) Welfare

Panel (a): $t^{en}(\beta)$ indicates the point in time at which the AV is launched (i.e., where the required accident rate α^{max} is reached. Panel (b): accident rate of AV i) in the first generation ($\alpha(t^{en})$), ii) at a fixed point in time $\tilde{t}$ ($\alpha(\tilde{t})$) and iii) in the active steady state (α^*). The dotted line denotes the accident rate of the conventional car ($\alpha_o = 0.074$). Panel (c): the total discounted welfare effect of safety regulation. All panels are drawn for $\beta = 0$ and the horizontal axis measures how far the regulated accident rate α^{max} is below the accident rate at market introduction without regulation ($\underline{\alpha} = 0.118$).

With respect to optimal output choices illustrated in Figure 7(b), by the same argument as above, there is no effect when the regulation is sufficiently weak such that it is not binding. Furthermore, after the safety threshold has been reached the output pattern is the same as without regulation. However, the regulation leads to a delay of the market introduction of the AV, and this delay is the larger, the stricter the regulation. A qualitative difference compared to the scenario without safety regulations is that the innovator sells a strictly positive quantity already at the time of market introduction leading to a jump also of the quantity trajectory.

Next, consider the effect of safety regulation on the date of market introduction of AVs and accident rates as depicted in panels (a) and (b) of Figure 8, respectively. Comparing these two panels with their counterparts for the liability regime in Figure 5 above reveals that, as under more stringent liability, a stricter safety standard leads to a delay in the market introduction of the AV. Moreover, we observe that the level of regulation has no effect on the steady state accident rate α^*. This is due to the fact that the regulation no longer has any impact once the accident rate $\alpha(S)$ is below the critical threshold α^{max}.[38] Hence, differently from the case of more stringent liability, safety regulation has no positive long-run effect on AV safety. However, the safety level in the short- and intermediate term (as captured by the accident rate $\alpha(\tilde{t})$ at a given date time $\tilde{t}$) becomes *lower* as the regulation becomes stricter. Again, this is qualitatively different compared to more stringent liability, which induces higher accident rates in the short and medium run. This can be seen by comparing the function $\alpha(\tilde{t})$, which is increasing in Figure 5(b), while it is decreasing in Figure 8(b). Intuitively, whereas making the liability regime more stringent reduces investments before market introduction, tightening the safety regulation has exactly the opposite effect by inducing the innovator to invest more in order to reach the required safety threshold faster. Note that this also implies that under a given safety requirement for the first generation of AVs on the market, under safety regulation the AV is launched much earlier compared to liability. For example, in order to achieve an accident rate of $\alpha = 0.1$ at market introduction, under liability a value of $\beta = 0.16$ is required which leads to a market introduction of AVs after 4 years (see Figure 5). By contrast, under safety regulation with $\alpha^{max} = 0.1$, the AV is launched already after 3 years (see Figure 8(a)).

From a welfare perspective safety regulation has two countervailing effects. On the positive side, the induced increase of investments leads to a faster accumulation of the safety stock. On the negative side, regulation prevents firms from selling AVs in situations where this would be beneficial for both the innovator and consumers. In our calibrated model it turns out that under an appropriate choice of α^{max} the positive effect prevails. As can be seen in Figure 8(c), the difference between the welfare arising with and without the development of the AV exhibits a hump-shaped pattern as the safety standard becomes stricter, and is maximized for $\alpha^{max} \approx 0.085$. Hence, combining the instrument of safety regulation with a lenient liability regime ($\beta = 0$) outperforms the sole use of the liability regime.[39]

[38] Of course, this observation no longer applies if the regulation is excessively strong, i.e., $\alpha^{max} < \alpha^*$.

[39] Daughety and Reinganum (1995) highlight a different channel through which safety regulation might be welfare improving compared to the sole use of liability. In their signaling framework the price set by the firm serves as a signaling device for unobserved product safety. They argue that, by credibly excluding low realizations from the set of relevant safety levels, such regulation can improve overall efficiency attained in equilibrium. Our results also complement the findings from previous studies analyzing the joint use of safety regulation and liability rules (see e.g. Shavell, 1984a,b; Kolstad et al., 1990; Schmitz, 2000). This literature shows that combining the two instruments in an optimal way can lead to strictly higher welfare compared to relying on one instrument only.

6 Discussion

In this paper, we set up a dynamic model of product innovation and product safety to analyze the short- and long-term effects of product liability on the market evolution and safety features of smart products, in particular autonomous vehicles (AVs). Because of non-human tortfeasors (i.e., the product's operating system, which is based on algorithms), this issue constitutes novel challenges for tort law.

Our analysis informs legislators and policy makers about crucial trade-offs associated with imposing liability in the context of such innovations. In particular, we highlight the tension between the long-term benefits of stricter liability (in the form a higher steady-state product safety), and the possibility of foregoing the innovation altogether, or the short- and intermediate term costs (in the form of a slower accumulation of product safety due to later market introduction and slower market penetration). In our model calibration for the U.S. automobile market we find that the latter costs are so significant that extending the scope of producer liability compared to the status quo with conventional cars only, leads to welfare losses. Our results hence suggest that one should be careful with such policies.

Our model exhibits a number of simplifying features, which were helpful in fleshing out the main trade-offs. Hence, in future work our framework could also be extended in number of further directions. First, in our calibration direct safety regulation leads to higher welfare than a pure liability regime by strengthening early stage safety investments, but the effect is rather small. This suggests that an alternative policy which fosters investments more directly, e.g. through subsidies, could be even more efficient. Such subsidies would reduce the innovator's marginal investment cost (θ), and would ceteris paribus lead to higher levels of investment in each period. However, since such subsidies will have to be financed by the consumers (e.g., through taxation), this might lower their demand for consumption goods. It would be interesting to explore the overall effects of such a subsidy scheme in our model framework.

Second, our model does not yet capture a potential externalities which seems particularly relevant in the context of smart products such as AVs. It arises from the fact that the accident rate of an AV might not only depend on its own safety stock S, but also on the number of AVs on the streets. Since AVs can communicate with other AVs but not with conventional cars, a higher share of AVs improves the overall "connectedness" among vehicles, which should lead to fewer accidents.

Third, there might be learning curve effects on the production side implying that the marginal cost of production in a given period decrease in the total output produced prior to this period. This would give the innovator an incentive to depart from monopoly pricing in each period. From a technical point of view, it would make the analysis of the model more challenging as this would add a second state variable to the dynamic framework.

Appendix

A Proofs

A.1 Proof of Lemma 1

We define the current-value Hamiltonian function

$$H(x,S,\lambda) = [\pi^*(S) - h(x)] + \lambda(x - \delta S), \tag{17}$$

where λ is the costate for the safety stock S. Applying Pontryagin's maximum principle (see e.g., Grass et al., 2008), we derive the optimal control from the first order condition $\frac{dH}{dx} = 0$. Taking into account the non-negativity constraint of the control, this yields (9). The costate equation follows from

$$\begin{aligned}\dot{\lambda} &= r\lambda - \frac{\partial H}{\partial S}\\ &= r\lambda - \frac{\partial \pi}{\partial q^*}\cdot\frac{\partial q^*}{\partial S} - \frac{\partial \pi}{\partial S} + \delta\lambda\\ &= (r+\delta)\lambda - \frac{\partial \pi}{\partial S}\end{aligned}$$

where $\pi = \left[q^*(S)\cdot\left(p(q^*(S),S) - c - \frac{\alpha(S)\beta D}{r+\rho}\right) - h(x)\right]$ is the integrand of (8). With respect to the term $\frac{\partial \pi}{\partial S}$, it follows from the envelope theorem that $\frac{\partial \pi}{\partial q^*} = 0$, so that only the direct effect of S on π needs to be considered. That is, we have $\frac{\partial \pi}{\partial S} = q^*(S)\cdot\left(\frac{\partial p(q,S)}{\partial S} - \frac{\alpha'(S)\cdot\beta D}{r+\rho}\right)$, where $p(q,S)$ is the demand function derived from (3) and given by

$$p(q,S) = c + \frac{A - (\bar{v} - \underline{v})\cdot q - \alpha(S)\cdot k((1-\beta)D)}{r+\rho}.$$

Taking this into account when calculating the derivate of $\pi(s)$ with respect to S yields the expression in the lemma. The transversality condition follows from Michel (1982). ∎

A.2 Proof of Lemma 2

(i): Since this is a time-autonomous infinite horizon optimization problem with a one-dimensional state space, standard results (see e.g., Hartl, 1987) imply that every optimal trajectory has to be (weakly) monotone. To see that no optimal path can induce divergence of S to infinity, note that $\pi^*(S)$ is bounded from above. For any monotonously increasing path we must have $x > \delta S$. Furthermore there exists a lower bound $\tilde{S}(\beta,\theta)$ such that $h(\delta S) > \pi^*(S)$ for all $S > \tilde{S}$. Since $h(\delta S)$ is shifted upwards for increasing θ, whereas $\pi^*(S)$ is independent of θ, if follows that $\frac{\partial \tilde{S}}{\partial \theta} \leq 0$ for all β. Defining $\bar{S}(\theta) = \max_{\beta\in[0,1]} \tilde{S}(\beta,\theta)$, we obtain that $\bar{S}' \leq 0$. Therefore for any path diverging to infinity there is a $T > 0$ such that $S(t) > \bar{S}(\theta)$ for $t > T$. Hence, $\pi^*(S(t)) - h(x(t)) < 0$

for all $t > T$, which implies that such a path can never be optimal. Together these insights imply that every optimal path monotonously converges to some steady state in $[0, \bar{S}(\theta)]$.

(ii): Considering the canonical system derived from the Maximum Principle, any steady state $S^* > 0$ has to satisfy $g_\lambda(S^*) = g_S(S^*)$, where g_λ and g_S are given by (11) and (12). The function g_λ is finite, continuous in S and does not depend on θ. Fix some small but positive θ_{min}. The function $g_S(S)$ increases monotonously in θ for all $S \in [0, \bar{S}(\theta^{min})]$ with $\lim_{\theta\to\infty} g_S(S) = \infty$. Therefore for any given $\beta \in [0,1]$ there exists a finite $\theta^{max}(\beta) = \inf\left\{\theta \geq \theta^{min} \mid g_S(S) > g_\lambda(S)\ \forall S \in [0, \bar{S}(\theta_{min})]\right\}$. By definition for each $\theta > \theta^{max}(\beta) \geq \theta^{min}$ we have $g_S(S) > g_\lambda(S)$ and $S \in [0, \bar{S}(\theta)]$. Hence, there is no candidate for a steady state in $(0, \bar{S}(\theta)]$. Since any optimal path entering $(0, \bar{S}(\theta)]$ must monotonously converge to a steady state in this interval, this implies that any optimal path starting in $[0, \bar{S}(\theta)]$ must converge monotonously to $S = 0$. ■

A.3 Proof of Proposition 1

(i): Consider some $\beta \in [0,1]$ and denote by $\epsilon < \delta\underline{S}$ a small positive number. Then consider the intertemporal optimization problem of the innovator with the additional constraint $x(t) \geq \epsilon\ \forall t \geq 0$. We denote this as the $\epsilon-$constrained problem of the innovator. Standard arguments show that the problem has an optimal solution and we denote by $V_\epsilon(0;\beta,\theta)$ the value function of this problem for the initial state $S(0) = 0$. A decrease in θ shifts the function $h(x)$ downwards, and since investment is positive along the optimal path in the $\epsilon-$constrained problem, it follows that $V_\epsilon(0;\beta,\theta)$ is a strictly decreasing function of θ. Furthermore, we show that $V_\epsilon(0;\beta,\theta) > 0$ for sufficiently small θ. To see this, note that $\lim_{S\to\infty} \pi^*(S) > 0$ and hence $\pi^*(\hat{S}) > 0$ for sufficiently large $\hat{S}$. Now consider a path $\{\hat{x}(t)\}$ with the properties that the induced state dynamics $S(t)$ reaches $\hat{S}$ for $t = T$ and $\hat{x}(t) = \delta\hat{S}\ \forall t \geq T$. For the discounted net present value of this path, $\hat{V}$, we obtain

$$\begin{aligned}
\hat{V} &\geq -\int_0^T e^{-rt} h(\hat{x}(t))dt + \frac{e^{-rT}}{r}\left(\pi^*(\hat{S}) - h(\delta\hat{S})\right) \\
&= -\theta\left[\int_0^T e^{-rt}\left(\hat{x}(t) + \frac{\eta}{2}\hat{x}(t)^2\right)dt + \frac{e^{-rT}}{r}\left(\delta\hat{S} + \frac{\eta}{2}(\delta\hat{S})^2\right)\right] + \frac{e^{-rT}}{r}\pi^*(\hat{S})
\end{aligned}$$

The square bracket does not depend on θ and therefore the negative term goes to zero for $\theta \to 0$. Moreover, $\pi^*(\hat{S}) > 0$, which implies that $\hat{V} > 0$ for sufficiently small θ. Since $V_\epsilon(0;\beta,\theta) \geq \hat{V}$, we obtain $\lim_{\theta\to 0} V_\epsilon(0;\beta,\theta) > 0$. From part (ii) of Lemma 2 it follows that $\lim_{\theta\to\infty} V_\epsilon(0;\beta,\theta) < \lim_{\theta\to\infty} V(0;\beta,\theta) = 0$ and therefore taking into account the monotonicity and using the intermediate value theorem we obtain that there exists a unique $\hat{\theta}_\epsilon(\beta)$ such that $V_\epsilon(0;\beta,\hat{\theta}_\epsilon(\beta)) = 0$. Clearly any optimal path giving rise to this value must converge monotonously to some steady state and since $\pi^*(S) = 0$ for all $S \leq \underline{S}$ and $h(x(t)) > 0$ along any path in the $\epsilon-$constrained problem, this steady-state, which we denote by $S^*_\epsilon(\beta)$, must satisfy $S^*_\epsilon(\beta) > \underline{S}$ for all $\epsilon > 0$. Taking the limit $\epsilon \to 0$ it follows that for $\theta = \hat{\theta}(\beta) := \lim_{\epsilon\to 0} \hat{\theta}_\epsilon(\beta)$ the value of the unconstrained problem of the innovator is zero and that this value can be obtained both by the path $x(t) = 0\ \forall t$ and by a path with positive investment, which induces convergence of the state S to the steady state $S^*(\beta) = \lim_{\epsilon\to\infty} S^*_\epsilon(\beta) > \underline{S}$.[40]

Furthermore, we show that $\hat{\theta}(\beta)$ is strictly decreasing with respect to β. Consider an arbitrary $\beta \in (0,1)$. The value function of the problem for $\theta = \hat{\theta}(\beta)$ by definition is $V(0;\beta,\hat{\theta}(\beta)) = 0$. As shown above, this value is generated by an optimal path for which we have $q(t) > 0,\ \forall t > T$ for some large T. Using the envelope theorem, we obtain from (8)

$$\frac{\partial \pi^*(S(t))}{\partial \beta} = -q^*(S(t))\frac{\alpha(S(t))D}{r+\rho} < 0$$

for all t where $q(S(t)) > 0$ and zero otherwise. Hence, $V(0;\tilde{\beta},\hat{\theta}(\beta)) > 0$ for all $\tilde{\beta} < \beta$. Similar arguments as used above show that $V(0;\beta,\theta)$ is a strictly decreasing function of θ if the optimal path induces $q(S(t)) > 0$ in a time interval with positive measure. It follows that $\hat{\theta}(\tilde{\beta}) > \hat{\theta}(\beta)$, which establishes the strict monotonicity of $\hat{\theta}(\beta)$ with respect to β. Defining $\underline{\theta} = \min_{\beta\in[0,1]} \hat{\theta}(\beta)$ we immediately obtain the claim of (i) with $\underline{\theta} = \hat{\theta}(1)$.

(ii): Defining $\bar{\theta} = \hat{\theta}(0)$, it follows from the arguments given in part (i) that for $\theta = \bar{\theta}$ and $\beta = 0$ the innovator has two optimal paths giving the value $V(0;0,\bar{\theta}) = 0$, one of them converging to a positive steady state and one staying at $S = 0$. For any positive β only the path $S(t) = 0,\ \forall t$ is optimal. This implies directly that for any $\theta > \bar{\theta}$ any path from $S(0) = 0$ which converges to a positive steady state generates a negative value regardless of $\beta \in [0,1]$.

(iii): Follows directly from the strict monotonicity of $\hat{\theta}(\beta)$ with respect to β and the definitions $\underline{\theta} = \hat{\theta}(1)$ and $\bar{\theta} = \hat{\theta}(0)$. ■

B Details of the Numerical Method

In this section, we briefly outline the numerical method used in the analysis.

Main analysis To determine the innovator's optimal investment path from the initial condition $S(0) = 0$ we first check whether for a given parameter setting there is a candidate for an active steady state. If this is the case, we then rely on the HJB equation associated to the innovator's dynamic optimization problem in order to determine the optimal path leading to this candidate and the associated value of the innovator's objective function (8).[41]

To identify candidates for an active steady state we numerically solve the equation $g_\lambda(S) = g_S(S)$ with g_λ and g_S given by (11) and (12). As discussed in Section 3, this

[40]It should be noted that by the same arguments as used above, the value of any path monotonously converging from $S(0) = 0$ to a steady state at $\underline{S}$ has to be negative. Therefore $S^*(\beta) > \underline{S}$ must hold.

[41]See also Miranda and Fackler (2002) or Dawid et al. (2015) for more details of the application of collocation for the solution of HJB equations.

equation (apart from the non-generic case of tangency between the two isoclines) has at least two solutions if it has any. The smallest solution is always repelling in the state/co-state space and therefore we always consider the second smallest solution of this equation as the location of the candidate for an active steady state.[42] We denote this candidate by $\tilde{S}^*$.

The value of an optimal path to $\tilde{S}^*$ is determined using the HJB equation. Since the problem is time-autonomous and has an infinite horizon, we can consider stationary investment and value functions, which do not explicitly depend on time. Denoting by $\tilde{V}(S)$ the value function associated to the optimal path leading to $\tilde{S}^*$, the HJB equation is given by

$$r\tilde{V}(S) = \begin{cases} \max_{x\geq 0}\left[-h(x) + \frac{\partial \tilde{V}(S)}{\partial S}(x-\delta S)\right] & S \in [0, \underline{S}), \\ \max_{x\geq 0}\left[q^*(S)\cdot\left(p^*(S) - c - \frac{\alpha(S)\beta D}{r+\rho}\right) - h(x) + \frac{\partial \tilde{V}(S)}{\partial S}(x-\delta S)\right] & S \geq \underline{S}. \end{cases} \tag{18}$$

Maximization of the right hand side of the HJB equation yields (see also (9))

$$x^*(S) = \max\left[\frac{1}{\theta\eta}\frac{\partial \tilde{V}(S)}{\partial S} - \frac{1}{\eta}, 0\right], \tag{19}$$

and inserting this expression into (18) yields a non-linear first order differential equation in $\tilde{V}$. The fact the we consider paths (at least locally) leading to $\tilde{S}^*$ is incorporated by the condition

$$\tilde{V}(\tilde{S}^*) = \frac{1}{r}\left(q^*(\tilde{S}^*)\cdot\left(p^*(\tilde{S}^*) - c - \frac{\alpha(\tilde{S}^*)\beta D}{r+\rho}\right) - h\left(\delta\tilde{S}^*\right)\right). \tag{20}$$

We numerically solve the HJB equation on the state space $[0, \bar{S}]$ with $\bar{S} > \tilde{S}^*$ relying on a collocation method using Chebychev polynomials. To this end we generate a set of n Chebychev nodes $\mathcal{N}$ in $[0, \bar{S}]$ (see e.g. Judd (1998) for the definition of Chebychev nodes and Chebychev polynomials).

Our goal is to calculate a polynomial approximation of $\tilde{V}(S)$ which (approximately) satisfies (18) on the set of interpolation nodes $\mathcal{N}$. The set of basis functions for the polynomial approximation is determined as $\mathcal{B} = \{B_j(S), j = 1, .., n\}$ with

$$B_j(S) = T_{j-1}\left(-1 + \frac{2S}{\bar{S}}\right),$$

where $T_j(x)$ denotes the j-the Chebychev polynomial. Since Chebychev polynomials are defined on $[-1, 1]$, the state variables have to be transformed accordingly.
For a given value of β the function $\tilde{V}$ is then approximated by

$$\tilde{V}(S) \approx \hat{V}(S;\beta) = \sum_{j=1}^{n} C_j(\beta)B_j(S),\ S \in [0, \bar{S}], \tag{21}$$

[42]In all scenarios covered in this paper the equation $g_\lambda(S) = g_S(S)$ has at most two solutions.

where $C(\beta) = \{C_j(\beta)\}$ with $j = 1, \cdots, n$ is the set of n coefficients to be determined. To calculate these coefficients we set up a system of non-linear equations derived from the condition that $\hat{V}$ satisfies the HJB equation (18) on the set of interpolation nodes $\mathcal{N}$. This system consists of n equations with n unknowns (i.e. the coefficients $C_j(\beta)$) and is solved by a standard numerical procedure for solving systems of non-linear equations.

Once a solution vector $C(\beta) \in \mathcal{R}^n$ is obtained the accuracy of the solution is checked in two ways. First, the absolute difference between the left hand side and right hand side of (18) for $\tilde{V}(S) = \hat{V}(S;\beta)$ relative to the value of $\hat{V}(S)$ is calculated on the state space $[0, \bar{S}]$. In all numerical results presented in the paper this difference value is below $2 \cdot 10^{-3}$, which shows that $\hat{V}(S;\beta)$ almost exactly solves the HJB equation, not only on $\mathcal{N}$ but on the entire state space. Second, the value $\hat{V}(0;\beta)$ has been verified by calculating a discrete time approximation (for small time steps) of the optimal investment path and state dynamics derived from $\hat{V}$ and explicitly calculating the corresponding value of (8). For our numerical results also this test confirms that the obtained value functions and optimal investments obtained from inserting $\hat{V}(S;\beta)$ into (19) are very close approximations of the actual optimal values.

The condition (20) is enforced by initially determining the coefficient vector $C(\beta)$ for $\beta = 0$ and verifying that with this coefficient vector $\tilde{S}^*$ is indeed the fixed point of the state dynamics induced by the feedback function (19) if $\tilde{V} = \hat{V}$, which implies by the HJB equation that (20) holds. Furthermore for $\beta = 0$ we have $\hat{V}(0;0) > 0$ and therefore the path leading to $\tilde{S}^*$ is optimal from the initial condition $S = 0$. The parameter β is then increased stepwise with a small step-size ϵ and the coefficient vector $C(\beta - \epsilon)$ is used as the initial guess in the iterative algorithm solving (18) on $\mathcal{N}$ for β. This continuous adjustment of the value function implies that there is always a positive steady state of the dynamics induced by $\hat{V}$ and the function $\hat{V}$ captures the value of the optimal paths converging to this positive steady state. As β increases the value of $\hat{V}(0;\beta)$ declines and the value of the threshold $\hat{\beta}$ is determined by the condition that it is the smallest value of β such that $\hat{V}(0;\beta + \epsilon) < 0$.

Adjustments for case of ex ante safety regulation In order to determine the value function and the optimal feedback function for the extension with the ex-ante safety regulation discussed in Section 5, we have to slightly adjust the numerical algorithm. The reason is that under this regulation the quantity of the innovator jumps from zero to a positive amount once the accident rate $\alpha(S)$ crosses the regulatory requirement α^{max}. Since such a jump in the quantity implies a kink in the value function and such a kink cannot be captured by a polynomial approximation, we have to solve the HJB equation for each value of α^{max} separately on the intervals $[0, \alpha^{-1}(\alpha^{\max})]$ and $[\alpha^{-1}(\alpha^{\max}), \bar{S}]$. Since we assume a value of $\beta = 0$ for our safety regulation analysis and only consider scenarios where $\alpha(S^*) < \alpha^{max}$, i.e. where the safety regulation is satisfied in the steady state without regulation, the value function does not depend on α^{max} on the interval $[\alpha^{-1}(\alpha^{\max}), \bar{S}]$. Actually, $\hat{V}$ on this interval coincides with the value function we have calculated in Section 4 for $\beta = 0$, i.e. $\hat{V}(S;0)$. To determine the value function on $[0, \alpha^{-1}(\alpha^{\max})]$ for a given

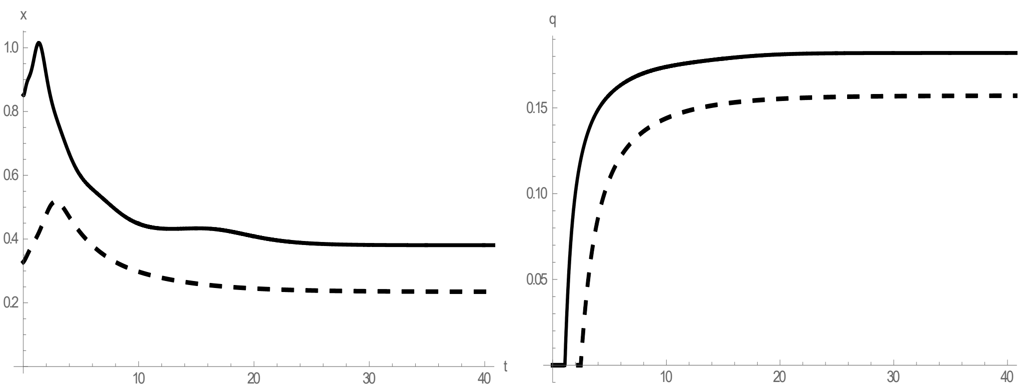
α^{max} we use the collocation method for this reduced state space to solve

$$r\tilde{V}(S) = -h(x^*(S)) + \frac{\partial \tilde{V}(S)}{\partial S}(x^*(S) - \delta S)$$

with $x^*(S)$ given by (19) and the boundary condition $\tilde{V}(\alpha^{-1}(\alpha^{\max})) = \hat{V}(\alpha^{-1}(\alpha^{\max}); 0)$.

C Comparison with the socially optimal investment path

Figure 9: Socially optimal investment and quantity paths



(a) Investment (b) Quantity

Dynamics of investment (panel (a)) and quantity (panel (b)) under the welfare maximizing path (solid line) and the optimal path of the innovator (dashed line) for $\beta = 0$.

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