# Essays on Energy in Macroeconomic Models

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# Preface

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# Chapter 1

# Introduction

This thesis deals with the economic implications of price volatility in energy markets, in particular its transmission channels at the macroeconomic level. Broadly speaking, the aim to contribute by focusing on the qualitative and quantitative analysis of the consequences of volatility in energy prices. In Chapter 2, I consider the effect of oil price volatility on GDP growth, while accounting for the change in a country's dependency on oil, which is done with the help of several different approaches to depict price fluctuations. Chapters 3 and 4 seek to complement theoretical models of real business cycle frameworks, by integrating the consumption of energy by both, households and firms, int he model. In a further extension, another dimension is added, by allowing for some goods and input factors to be energy-depend, while others are not. In the model I describe in Chapter 3, energy is endogenously generated from finite and renewable sources. However in chapter 4, I assume exogenous energy price shocks for my model analysis, and instead I incorporate heterogeneous agents.

Today, except for very few large oil-producers, the energy sector occupies a rather small share of a country's economic output. At the same time, energy is an indispensable input in the production of almost any product or service, all over the modern world. The availability and affordability of energy has essentially transformed every single industry, since the days of the industrial evolution. The magnitude of the economic impact of the energy sector is therefore a lot larger than its share in GDP. Looking back into history, and in particular the Industrial Revolution, it remains without doubt that energy plays a decisive role in the development process of a country's economy. In the $18^{\text{th}}$ century, four factors drove economic welfare and growth: the availability of labor force, the availability of capital, the advances in technology, and energy. Instead of defining energy as part of technological advancement, I distinguish between them, which enables me to describe how the one influences the other. The increase in productivity by pioneering innovations and discoveries, such as the steam engine or the light bulb, have for example triggered an enormous increase in the demand for primary energy. Hence, energy can be seen as the fuel of technological development, as well as an essential input for most products. Particularly with the start of mass industrialization, economic development started to fully depend on the availability of energy and its price. As a consequence, the ability of a country to access energy has ever since defined its economic future.

The importance of oil is underlined by real life examples of unstable energy supply due to a weak energy infrastructure and energy sector, as known from many developing countries. Such struggling economies production sectors may be unable to produce sufficient final goods, either to be consumed or exported, and consequently they don't grow. In other cases, such as Venezuela, power outages turn a political crisis into humanitarian crisis. This makes energy a crucial element among the driving forces of economic growth to promote welfare and high standards of living.

The term 'energy' is afflicted with various meanings and products, like primary energy products such as oil, gas, coal, or other renewable energy sources, but also final generated energy such as electricity or gasoline. An outstanding role is often ascribed to electricity as the universal carrier of energy. In this thesis, when not further specified, the term energy refers to *primary energy* and its various sources.

Over time, different sources of energy have been dominating. While initially, wood has been used as a multi-functional raw material for everything, its predominant position has been replaced by coal, in the Industrial Revolution, followed by oil, with the rapid growth in automobile production in the beginning of the $20^{\text{th}}$ century. Today, oil still covers the largest proportion of all primary energy sources, at least for most industrial countries, such as Germany and the USA, as is shown in Chapter 2.

The first models of economic growth have neglected resources or energy. But when the oil crisis of the mid 1970s hit in, this triggered economists to developed theoretical models that incorporate the role of resources, including energy, in the growth process. Despite stable supply of labor and capital, non-declining output could no longer be guaranteed, as instead it was shown to depend on both the nature of technology and institutional arrangements (Solow, 1974; Stiglitz, 1974). By considering energy as a further input for economic growth, multiple new paths and outcomes became plausible, especially when recognizing finite resources (Dasgupta and Heal, 1974). In order to find an optimal depletion rate of non-renewable resources, an aim of much of this literature is to determine, whether, and under what circumstances, technical progress is effective in ensuring sustained growth and consumption in an economy (Bretschger, 2005). Furthermore, the finite nature of non-renewable resources, but also the externality problem in terms of pollution, which affects current utility as well as future economic development, have increased awareness for transition to alternatives, such as renewable resources.

Research on the role of energy has not been limited to growth economics alone. Because of the particular link of energy to other factors in the economy, changes in the quantity and price of energy impacts the macroeconomic conditions, and a country's development and welfare. These effects can even spread globally, given the fact that energy resources are not uniformly distributed among countries (e.g. oil), which means that effects spread with trade. With the emergence of excessive oil price shocks along with the oil crises in the mid and end 1970s, price fluctuations in the global oil market and resulting sensitivities of economies have led to a large interest in research on the role of oil, at least with respect to the macroeconomy and in the short term.

Numerous reasons may underlie the fluctuations of oil price, which may be of economic, political, or other nature. But forecasts from microeconomic theory, the general principal of pricing, according to which prices are a result of the equilibrium of supply and demand, cannot always be applied to this commodity. Since a number of recessions have happened in the aftermath of extraordinary oil price peaks, the relationship between prices and economic output is a hot topic.

There are several approaches to investigate the relationship between energy prices, in particular oil prices, and the macroeconomy. Although different methods of analysis have yielded different results, economists have acknowledged changes in the price of energy, as a considerable source of economic fluctuations. According to Blanchard and Gali (2007), such changes have the potential to cause global shocks, as many economies are affected simultaneously, due to their dependency and lack of alternative resources in the short run.

Empirical studies have assessed the effects of oil price shocks on economic activity, in particular since the oil crises. Initially, linear models have shown a significant negative relationship between oil price changes and GDP growth, but this link has lost in significance since the mid-1980s. As a consequence, literature has shifted to several non-linear and asymmetric transformations by using new econometric tools (Hamilton, 1996; Lee et al., 1995; Mork, 1989). Recent studies confirm non-linearity for most industrial countries including Germany (Jiménez-Rodríguez and Sánchez, 2005) but simultaneously question asymmetry (Kilian and Vigfusson, 2013).

Chapter 2 has been published as a paper in 'Energy Economics' in 2019. In this chapter, I re-estimate a vector-autoregressive (VAR) model similar to those of Jiménez-Rodríguez and Sánchez (2005), by extending the analysis to a time span including the years of the financial crisis and up to 2016. Furthermore, I consider the suggestion by Hooker (1996), claiming that the degree of correlation between oil price changes and economic output is not constant but rather weakening over time. I hypothesize that a reduction of the oil-to-energy share, which results in less dependency on oil, leads to this weakens relationship between oil price changes. In order to incorporate this in a formal model, I examine the existence of significant moderator effects, by making use of a moderated regression analysis in form of an interacted VAR (IVAR). But other than the interacted panel VAR by Towbin and Weber (2013), in which all variables interact with the moderator variable, the IVAR model presented in Chapter 2 is further developed and restricts this interaction for price variables only. For the purpose of testing this model, I have constructed a balanced panel dataset of twelve industrialized countries (three oil-exporting and nine oil-importing countries), covering a time horizon of 45 years. Of course, next to the novel modeling approach, the construction of this dataset and the empirical results are rather minor contributions of this chapter. For the analysis, I consider four different approaches of price determination.

The results of this chapter confirm the assumption that changes in oil prices Granger cause GDP growth for most observed countries. Although I cannot confirm rejection of asymmetry such as proposed by Kilian and Vigfusson (2013), I indicate that effects of negative oil price movements are more important than what has so far been assumed in previous studies. Furthermore, I can examine a high significance in the existence of moderator effects. As a result, oil price increases have a lower effect on GDP growth the lower the oil-to-energy ratio, which is in line with the theoretical assumption that a higher dependency on imports of fossil energy resources makes a country more vulnerable to price fluctuations. As a side outcome of this chapter, I have presented an enhanced version of an IVAR model to test for moderator effects for selected variables only.

Studies that analyze the theoretical relationship between macroeconomic variables often include RBC models. In principle, these models investigate the external influence through shocks on the modeled economy, and decompose the effects on the different variables. RBC models are popular because the methodology attempts to explain aggregate economic phenomena on the basis of macroeconomic models derived from microeconomic principles. However, there are some major drawbacks. In particular the role of technological shocks is often not reflected properly. These shocks take a dominant role in affecting the models' dynamics, that are not always confirmed by microeconomic evidences. In this context, McCallum (1988) has identified energy as an essential factor on the supply side, which helps explaining those fluctuation, to which too little attention has been paid thus far. As a consequence, several RBC models that include oil price shocks have been developed, mostly treating energy prices as determined exogenously, and exclusively affecting the production side (Finn, 1995; Kim and Loungani, 1992; Rotemberg and Woodford, 1996). Most recently, there are efforts to allow for full-endogeneity of energy generation in RBC models, and revealing that endogenizing energy prices improves the prediction of business cycles (Argentiero et al., 2018; Huynh, 2016). However, these existing frameworks do not distinguish between different energy resources and lack in quality regarding their predictions of business cycles. Consequently, further research is necessary to cope with these problems, which is exactly what I am concerned with in Chapter 3 and 4 of this thesis.

In Chapter 3, I contribute to energy literature by constructing a multi-sector RBC model, whereby endogenizing energy generation and distinguishing between finite and renewable energy resources. Despite this more complex way of modeling the energy sector, this model is close to Dhawan and Jeske (2008), and considers two types of consumption goods. First, more consumption goods mean an expanded investment portfolio for households. Second, it includes additional transmission channels of energy shocks because energy is consumed by both households and firms. It is a complementary component with respect to durable goods and capital. In an extended version of the model, I allow for constrained replenishment of the finite resource stock. In the real world, this reflects the possibility to transform resources to available and mineable reserves by doing costly R&D e.g. of new mining devices. The model is calibrated to fit the German economy and estimates most of the parameters using Bayesian estimation techniques.

The Bayesian estimation in Chapter 3 confirms a complementary relationship between durable goods and energy in the households sector, as well as between capital and energy in the final production sector. Furthermore, the model confirms the dominant role of volatility in total factor productivity, which is widely accepted as the main force behind business cycles. Nevertheless, this study provides essential improvements in explaining theoretical moments, by distinguishing between durable and non-durable goods, taking energy consumption into account, and also by endogenizing energy generation from two different resources.

A further criticism of traditional neoclassical RBC models concerns the assumption of a representative agent always behaving perfectly rational, and operating in perfectly competitive good, factor, and asset markets. These homogeneous frameworks ignore the existence of heterogeneity in human beings and human decision making, which adds to biased outcomes regarding inequality. However, even when allowing for idiosyncratic behavior of agents, inequality is still eliminated because of the self-regulating (complete) markets. Only in case of incomplete markets, where agents cannot fully insure against idiosyncratic risk, inequality may arise.

In Chapter 3, I intend to complement theoretical RBC literature by including energy in a model, using a heterogeneous approach that combines properties of 'incomplete market models' and 'limited asset market participation models'. The model I construct is close to those of Chapter 4, but considers energy prices as determined exogenously. This simplification is based on the assumption that from a global perspective, Germany is a small country in terms of energy consumption, and has little market power in affecting the world price of energy. Heterogeneity arises through distinguishing between two types of agents (Ricardian households and rule-of-thumb households) and idiosyncratic shocks in labor supply. As agents face an occasionally-binding budget constraint, there is an incomplete capital market with limited risk insurance. In order to solve cross-section capital distribution, I apply the *explicit aggregation* approach, based on Den Haan and Rendahl (2010), assuming that agents' decisions on capital accumulation only depend on first moments of wealth distribution. In addition to the analysis of the variables' responses to aggregate technology shocks and energy price volatility, I also study how inequalities in income and wealth compare to observations from Germany.

With respect to energy price shocks, inequalities in income and wealth decrease, which can mainly be traced back to the complementary relationship of energy with durable goods and capital. I conclude that it is not the low-income agent who benefits from volatility in energy prices, but instead it is the high-income agent who looses both in terms of income and wealth, due to higher absolute sunk costs. Subsequently, I consider policy implication with respect to income redistribution from Ricardian to rule-of-thumb agents. The results are in line with empirical findings, showing a significant reduction of income inequality at the cost of a slight increase in wealth inequality.

The simulation of the models in Chapter 3 and 4 were performed with the help of Dynare (Adjemian et al., 2011). Dynare is a software platform based on MATLAB routines, which can handle a wide class of economic models, such as dynamic stochastic general equilibrium models and overlapping generations models. For the purpose of my research, I have written several MATLAB scripts in order to cope with the explicit aggregation algorithm in Chapter 4. Additionally, I have also developed a toolbox that can numerically simulate the model for heterogeneous agents with idiosyncratic shocks, in order to analyze the evolution of their income and wealth distribution. In this toolbox I also integrated routines for robustness checks and for detailed analyses of various inequality metrics and graphical illustrations. For the purpose of future non-commercial research, I am willing to share the code upon request.

# Chapter 2

# Oil Price Shocks and GDP Growth: Do Energy Shares Amplify Causal Effects?*

## 2.1 Introduction

This paper investigates the effect of oil price fluctuations on GDP growth using several linear and nonlinear VAR models. In particular, an IVAR approach is applied to consider moderator effects in the relationship between oil price changes and GDP growth. It has long been assumed that events in the monetary or in the oil markets contribute to the outset of economic recessions. The monetary market is often characterized by interventions in the credit market that are meant to influence investment behavior and may cause financial frictions. These interventions hamper or accelerate economic growth and magnify business cycles. The oil market, or the fluctuation in the oil price, influences economic development through several channels. On the one side, oil prices have a direct negative effect on the output of an economy by increasing production costs. On the other side, oil price fluctuations generate uncertainty which influences investment behavior in future projects (Bernanke, 1980). Expectations regarding the price evolution impact business outlooks and often lead to a deferral of new investments, which, in the medium and long run, dampens future business development. On an aggregate level, this affects economic growth.

In the literature, along with theoretical explanations, historical data have been analyzed to identify specific properties regarding oil prices, and alongside related behavior of other macroeconomic variables. When based on data from the 1980s or before, linear models have shown a significant negative relationship between oil price changes and GDP growth (Gisser and Goodwin, 1986; Hamilton, 1983; Mork, 1989). However, starting with the mid-1980s, oil price decreases have not had the

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predicted influence on macroeconomic performance, as economic models of the time were outdated. Following a drop in oil prices, GDP growth does not longer increase by the same amount as it would decrease after an equivalent rise in oil price. This new type of relationship has been modeled by changing the analysis in favor of an asymmetric relationship between oil price fluctuation and GDP growth. Starting with Mork (1989), several economists have considered this by adopting the theory of the asymmetric relationship to non-linear models. These new models allow for distinguishing between impacts of positive and negative oil price changes, introducing separate coefficients for both of them (Hamilton, 1996; Lee et al., 1995).

Other insights have become possible by investigating data of a growing time span. For example, it was suggested that the degree of correlation between oil price changes and economic output is not constant but rather weakened over time (Hamilton, 1996; Hooker, 1996). Depending on the respective degree of correlation, the economy has been vulnerable, to a decreasing extend, to fluctuations in the oil market over time. There is an ongoing debate on why the impact of oil is diminishing. The change may either be caused by a higher flexibility in absorbing price shocks through other macroeconomic channels, such as monetary policy. Alternatively, the dependency on oil may have decreased in favor of a dependency on other sources of energy, resulting in a loss of the importance of oil for the respective economy. Indeed, since the 1970s, oil shares have decreased in many economies, in particular in oil demanding countries without own meaningful oil production, as will be investigated in this study.

The present paper contributes to existing literature in two ways. First, we investigate the weakening relationship between oil price changes and GDP growth. We hypothesize that a lowering of the oil-to-energy share, which comes with a decreasing dependency on oil, leads to this weakening relationship between oil price changes and GDP growth. Thereby, we look at the effect of a change in the oil-to-energy share as a moderator effect. Second, we extend the time horizon to 2016, enabling the re-estimation of previous studies. In particular, the data include the transition into the 21st century, with strong increases in oil prices up to levels beyond those of the 1970s.

These objectives have gained importance as extreme fluctuations of oil prices, as well as major reforms regarding new ways of energy production have taken place in many countries in recent years. Hence, our study contributes to existing literature by extending the analysis to a time span including the years of the financial crisis and up to 2016, thereby addressing the latter of the two previously described objectives. In turn, the first of the two objectives has partly been dealt with by Jiménez-Rodríguez and Sánchez (2005). Nevertheless, our study adds further contribution to this objective. It is, to the best of our knowledge, the first paper to describe and quantify moderator effects on the relationship between oil price changes and GDP growth. In doing so, we consider the possibility of the existence of asymmetric effects of oil price changes on GDP growth and thus do not limit the analysis to linear models.

The results of this paper confirm the assumption that changes in oil prices Granger cause GDP growth for most countries. This holds for both, net-oil-consuming and net-oil-producing economies. By allowing for asymmetry in the effect of positive and negative price movements on economic growth, we further indicate that effects of negative oil price movements are more important for some countries than assumed before. In spite of that, magnitudes of positive or negative responses are not equal which supports the original assumption of asymmetric effects by oil price changes. However most important, we find evidence for the existence of a non-linear moderator effect, with the oil-to-energy share acting as the moderator variable. Thereby, our paper makes an important contribution to existing literature, as this significant moderator effect explains that a decline in the oil-to-energy share weakens the causal effect of oil price changes on economic growth.

The paper is organized as follows: Section 2 gives an overview of existing literature examining the relationship between oil prices and GDP growth. It summarizes models which assume asymmetric effects of oil prices, explaining the focus on non-linear instead of linear models. Section 3 reflects the historical development of oil shares in different countries to determine whether their paths have been changing. Section 4 describes the present dataset and introduces the functional form of the model including the moderator effect. Section 5 presents the empirical results and analyzes them. Section 6 concludes.

## 2.2 Effects of Oil Price Changes on Economic Growth

Price fluctuations in the global oil market and resulting sensitivities of economies have led to a large interest in research on the role of oil with respect to the macroeconomy. This large body of literature has a particular focus on questions related to the two oil crises in the mid and end 1970s. Reasons for oil price fluctuations may be economic, political, or other. But unlike forecasted by microeconomic theory, the general principal of pricing, saying that prices are a result from the equilibrium of supply and demand, cannot always be applied to this commodity. Since a number of recessions have been preceded by extraordinary peaks in the oil price market, it is a topical issue covered in research of economic development and growth dealing with the relationship between prices and economic output.

Side by side, theoretical and empirical studies have been evolved to analyze the role of exhaustible resources such as oil and coal over the business cycle. On the theoretical side, noticeable work has been published by Stiglitz (1974) who implements a general non-renewable resource to a basic Cobb-Douglas economy solving for the optimal growth path. Noteworthy, the analysis by Dasgupta and Heal (1974) examines how depletion of a finite product should optimally set when allowing for substitution between exhaustible resources and other reproducible inputs. Bernanke (1980) and Bernanke et al. (1997) analyze the effects of fluctuations of resource prices on investment behavior and related responses by monetary authorities. Davis and Haltiwanger (2001) pick up new empirical findings to analyze job creation and destruction with respect to oil price changes in the US manufacturing sector between 1972 and 1988. They find that oil price fluctuation causes twice as much variability in employment growth as monetary shocks. Summing up, the theoretical results explain the effects of changes in oil prices either by influencing the production and consumption of an economy directly or by intensifying uncertainty such as on the investment behavior.

### 2.2.1 Symmetric and linear effects of oil prices

On the empirical side, an influential study has been published by Hamilton (1983) based on the six-variable system by Sims (1980). He extends the model observing the relationship of several main macroeconomic variables and movements in the oil price. Hamilton finds a strongly significant negative correlation between rising oil prices and seven out of eight post-war recessions in the USA between 1948 and 1972. Accordingly, he concludes that the main oil price shocks have had a significant impact on aggregated economic levels. However, there is no significant evidence that it is oil price shocks alone, that Granger cause economic downturns. Instead, he names other macroeconomic channels, such as monetary interventions, who may have played a role as well. Nevertheless, due to its simplicity but also its explanatory power, Hamilton's linear model has a strong influence on business cycles theory and its way to simulate models such as Eichenbaum and Singleton (1986), Gisser and Goodwin (1986), and McCallum (1988).

### 2.2.2 Asymmetric and nonlinear effects of oil prices

Until the early 1980s, when oil prices have pushed mainly in an upwards direction, linear models have performed reasonably well. But with frequent ups and downs as well as considerable drops in prices in the 1980s, the theory of linearity between oil price changes and economic growth has been revised. Despite decreasing oil prices, economic growth has not reflected the prediction. In fact, it turned out that there is a non-symmetric relationship between both variables. Price declines have been followed by only weak enhances or even negative economic developments. Meeting that, Mork (1989) finds strong significance for asymmetric impacts on economic growth in the USA between 1949 and 1988. On the one side, he identifies large negative effects of oil price increases, but on the other side oil price decreases do not show any significant effect. His study (hereafter called: asymmetric approach) distinguishes between positive and negative linear changes in the oil price with no further modification. The results have been confirmed for the majority of other industrial countries (Mork et al., 1994). However, it has been sporadically criticized that the usage of asymmetric linear approaches is not consistent to explain the role of oil in the macroeconomic business cycle (Hooker, 1996). In fact, the main criticism has been to weight the pre-1980 period too much in disfavor of the 1980s and 1990s, leading to an underrepresentation of observations for the latter. Subsequently, other economists proposed alternative methods to match asymmetric behavior. Similar to Mork, two leading contributions by Lee et al. (1995) and Hamilton (1996) handle asymmetry by exploiting nonlinearities. They construct nonlinear transformations of oil prices while at the same time maintaining Granger causality to other macroeconomic variables. It is commonly argued in the literature that these approaches do not replace the symmetric methodology but are also valid for the pre-1980 period. However, this period lacked of information by facing only price increases and considerably less fluctuation wherefore both, linear and non-linear asymmetric instruments lead to significant results.

To be more specific, Lee et al. (1995) incorporate changes in oil prices by normalizing these with regard to price variability. This transformation, called scaled specification, is obtained by a GARCH model. The measurement allows to distinguish between oil price movements which appear sharply but frequently and movements which are small but sporadic. Hence, the degree to which an oil shock affects the economy is measured according to its appearance with respect to time and amplitude. The degree of impact from an equal oil price shock is higher in a stable environment with unexpected movements than in a noisy one. The authors argue that the failure of linear relationship stems from the price volatility since the 1980s which has not been observed before. Much better forecasts of GDP growth are obtained by using transformed oil prices considering recent price volatility.

Hamilton (1996) replies to the criticism from Hooker (1996) by comparing the actual oil price with the maximum value from the previous four quarters. If the current value is higher, then the percentage change over previous year maximum is plotted, otherwise it is zero. Hence, this transformation, called net oil price increase, does not deal with quarterly price changes generally. This allows to consider many price changes as a correction to earlier price adjustments without directly affecting economic growth.

Both transformations have in common that they aim to modify the determination of price changes rather than just precluding negative or positive price changes by their sign. In the following years, these three methods have been established in various studies extended by further economies and time periods. Despite criticism, recent literature has repeatedly confirmed the nonlinear relationship between oil price changes and economic growth (Ferderer, 1997; Herrera and Pesavento, 2009; Jiménez-Rodríguez and Sánchez, 2005). Ferderer's focus is on price volatility of oil measured on a daily variance with respect to monthly averages. Additionally, he focuses on the extend of reaction of monetary policy due to oil price volatility. Ferderer confirms asymmetric results which have been found in previous studies.

Moreover, his study confirms the theory stating that monetary policy is sensitive to oil price changes between 1970 and 1990 but contradicts that these reactions are more restrictive following an oil price increase. Consequently, monetary policy does not explain the asymmetry puzzle. Jiménez-Rodríguez and Sánchez confirm asymmetry by focusing on European countries. Using a variance decomposition analysis, they argue that oil price shocks are a considerable source of volatility for many macroeconomic variables. Their analysis is close to our study by looking at a similar selection of countries as well as covering some common methods. Herrera and Pesavento (2009) investigate, among others, in how far changes in the dynamic response of GDP growth by oil price shocks can explain the decline in volatility of the US economy. Herrera and Pesavento (2009) find that magnitude but also duration of the response of GDP growth by oil price shocks have diminished during 1980s and 1990s.

We will revert to the three main transformation methods by Mork (1989), Lee et al. (1995), and Hamilton (1996) in our study. An evaluation of different modification methods has been done by Hamilton (2003). He investigates some existing asymmetric solutions to identify which specification is the best. To do this, he applies several tests for stability of coefficients on oil prices. He concludes that the scaled specification by Lee et al. works out the best with regard to historical US data, performing slightly better than the net oil price increase covering three years. This paper finds similar results.

A problem to deciphering causal effects of oil price movements to economic growth lies in the wide acceptance of oil price formation being endogenous with respect to other macroeconomic forces. To consider this, Hamilton (2003) isolates the exogenous components of the oil price with respect to its effect on growth by identifying and controlling for a number of military conflicts in the observed time horizon. These events are assumed to be exogenous with regard to the US economy and resulted to a shortage of oil affecting the supply side of the economy. However, a weak assumption says that the lack of exogeneity should not be overvalued due to the lagged response of oil prices with respect to changes in macroeconomic conditions. Kilian (2008) remarks that recursively identified VAR is a well-selected approach to deal with the relationship of oil-prices and economic growth, independent of the degree of transformed prices.

The concept of asymmetry is still ambiguous. Whereas non-linearity has never been questioned after its implementation in specifications like Lee et al. (1995) and Hamilton (1996), the support of asymmetry has decreased. Recently, occasional empirical studies have reconsidered the concept of combined non-linearity and asymmetry in the relationship between oil prices and other macroeconomic variables. Kilian and Vigfusson (2013) re-estimate US real GDP from oil prices, using an asymmetric approach and net oil price increases. By applying a modification of these methods, they confirm non-linearity but contradict asymmetry. They conclude that the empirical success of the Hamilton approach is due to nonlinearity features rather than to asymmetry. Alternatively, they find significant support for non-linear symmetry by focusing on the question whether oil prices deviate from their most recent extreme values instead of distinguishing between positive or negative oil price changes, called net oil price change.[1]

## 2.3 Historical Development of Oil Shares

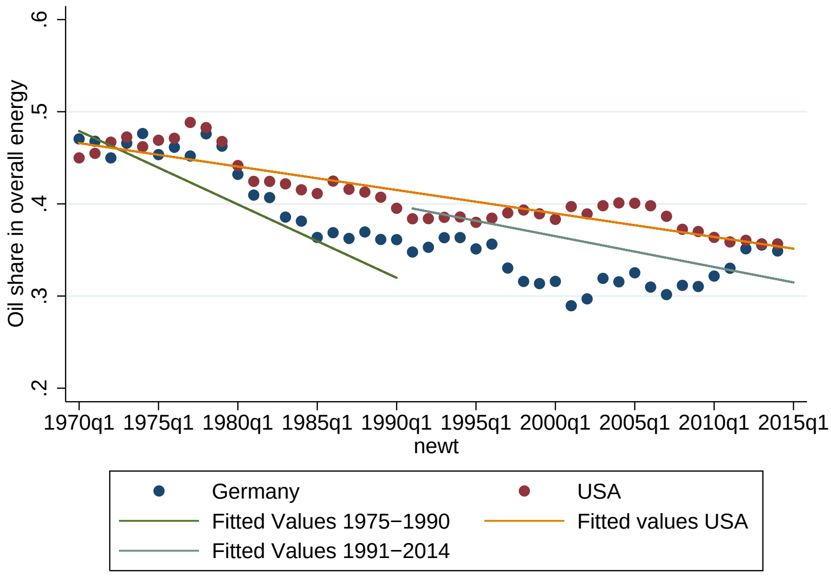
The literature covering the relationship between oil price changes and economic growth with respect to a dynamic energy mix is rare. By considering energy ratios, the literature mainly refers to the proportion of energy relative to other production factors such as labor or capital, hence, energy intensity. Kilian (2008) points out that the energy share, defined as the nominal valued added in oil and gas extraction divided by nominal GDP, is irrelevant in regression estimates because they do not fluctuate sufficiently on a quarterly basis. Hooker (2002) concludes that the sharp decline in the pass-through to core inflation caused by oil price changes results from the declining energy intensity. However, the oil-to-energy ratio has not been considered, and consequently the possibility of substitution of oil with respect to alternative energy sources.

In contrast, the relationship between energy consumption and economic growth has been dealt with in a wide range of literature. On the one side, the substitution or complementation between both variables is considered in several studies (Acaravci and Ozturk, 2010; Belke et al., 2011; Griffin and Gregory, 1976). The findings show mixed evidence on the causal relations of both variables depending on the econometric methodology or specific conditions concerning the selection of the observation sample. Among others, these include manifold consumption patterns or variations in the structure and stage of economic development. According to Payne (2010) this disunity does not allow for a classification of individual groups of countries to be energy dependent or energy-neutral. Stern (2011) provides an overview over several studies which analyze the causality between energy and GDP by applying cointegration methods with differing results according to time frames, methodologies, regions and measures. Despite inconsistent results, he concludes that both, energy use and output are tightly coupled, especially when putting more weight on the most recent studies.

On the other side, it is indisputable that sustained growth over a longer period goes along with a growing demand for energy. From the theoretical viewpoint, a production process is usually described by consisting of input factors such as capital and labor. In mainstream economic growth theory it is often underestimated that energy also accounts for part of the production. However, considering recent development it is hard to deny that the intensity of energy relative to GDP has decreased

[1] Hamilton (1996) combines extreme deviation from most recent extreme values but does not exclude to distinguish between positive and negative changes.

Figure 2.1: Oil-to-energy share of Germany and the USA




Data consists of ratio of oil supply and total primary energy supply from IEA.

over time. Hence, it accounts for a lower proportion in the production function today due to technological progress and more efficient usage of energy. Consequently, an increase in aggregated output does not automatically mean a proportional rise in the usage of energy.

While overall energy demand has shown a long-term increasing trend, especially since its appreciation and usage in the industrial revolution through coal and oil, its composition with regard to non-renewable and renewable raw resources has varied over time. Numerous factors have had influence on this shifting such as availability of resources, technological progress, innovations, or market- and political influence. The ecological economists Tahvonen and Salo (2001) have investigated the development of energy transition of finite and renewable energy resources in an economic growth model. They find that, at an early stage, an economy gathers its energy from renewable energy sources. Later, with an increasing economic growth, it changes to a balanced demand for both renewable and non-renewable resources whereas at the most developed stage, it decreases its share of depletable resources. The whole process mirrors an inverted U-curve of the share of fossil energy resources, similar to the Environmental Kuznets Curve (Grossman and Krueger, 1995) which represents the environmental degradation with increasing per capita income.

In this paper, we concentrate on oil as a specific representative for fossil energy resources. Firstly, this resource covers the largest proportion of all non-renewable energies, at least for most industrial countries such as Germany and the USA (see Figure 2.B.1). Secondly, its general price setting is easily ascertainable by taking the world reference prices into account. The limited geographic availability of oil and oligopoly formations like OPEC have led to consistent prices by all oil-exporters. A historical investigation of the development of oil shares shows some common properties between groups of countries. Concerning economies which are categorized as industrial countries and hence countries at a highly developed economic stage, these face a downward sloping oil-to-energy share for the past forty years.

Figure 1 shows oil shares relative to the overall energy use for two of the major industrial countries, namely the USA and Germany. It reflects a persistent decreasing trend in the importance of oil within the economy. Concerning the observed period of 40 years, the US economy has had an average annual decline of about 0.26 percentage points. Other industrial countries face similar trends (see Figure 2.B.2). However, some countries underwent apparent structural breaks such as slow-downs in the speed of decline. As in the example Germany's decline of oil-to-energy ratio has been temporary interrupted by the Germany reunification at the beginning of the 1990s but went back on track again after a few years. Nevertheless, all countries have experienced a significant decline in their oil-to-energy shares, ranging from around 15% for the USA to 60% for Sweden in the long run. However, we will look at moderator effects of static oil-to-energy shares in the analysis later on. Hence, we do not consider possible structural changes.

Overall, the development of oil shares confirms theoretical considerations on the composition of the overall energy mix as indicated by Tahvonen and Salo (2001). Further, the negative trend has been stable over a longer period which can be seen to be less affected by significant and unexpected events happening in a short time horizon such as price pressures due to economic or political events, or natural disasters. Substituting oil in favor for other alternative energy resources is not feasible instantaneously, but it is rather subject to long-term orientations due to restructuring of large investments in e.g. infrastructure.

## 2.4 Methodology and Data

Before analyzing the relationship between oil price fluctuation and GDP growth and the influence of the oil-to-energy share, we give an overview of how to proceed. At the beginning, we set up a linear vector autoregression (VAR) model similar to Mork (1989) as a general basement for the comparative analysis.[2]

Next, we set up asymmetric VAR models by distinguishing between positive and negative oil price changes to analyze but also to compare the behavior of asymmetric effects of oil prices changes. Here, we follow three approaches provided in existing literature. Firstly, we differentiate only between positive and negative price changes without making any adjustments similar to Mork et al. (1994). Secondly, we use the Scaled Specification Scheme by Lee et al. (1995). Thirdly, we pick up the Net

[2]In fact, Mork (1989) uses a seemingly unrelated regression framework which is a special form of a VAR-model with the restriction to allow for correlation between the error terms of each time series. Due to the more complex structure, we use general VAR.

Price Increase method by Hamilton (1996). After investigating the general baseline model and the three further approaches, the moderator effects are introduced and model extensions are explained and tested.

### 2.4.1 Symmetric linear model

Similar to Hamilton (1983) and Mork (1989), the variables of the first baseline model are based on the version of the six-variable system which has been set up by Sims (1980). Despite its simplicity, it provides a good approximation of macroeconomic activities. The VAR is a seven variable model which includes economic growth in form of real GDP growth. Robustness checks indicate a better outcome by taking four lagged exploratory variables. Next, changes in the oil price are taken into consideration. Further variables are added to control for macroeconomic effects not caused by changes in the oil price. These are the CPI (Consumer Price Index) to measure inflation, interest rate, unemployment rate, the IPI (Industry Producer Index), and expenses for oil relative to GDP.[3] The latter is considered to take into account the weight of dependency on oil relative to the overall economic outcome. Consequently, a country, whose industry relies strongly on fossil energy sources is more affected by cost fluctuations in oil prices than a country with lower shares.

The general linear baseline model is constructed as a VAR(p) model of order $p = 4$. Respectively, for the asymmetric approach, this model is described by the reduced form

$$y_t = c + \sum_{j=1}^{4} \alpha_j y_{t-j} + \varepsilon_t \tag{2.1}$$

where $c$ is the (7x1) interception vector, $\alpha_j$ is the $j^{th}$ (7x7) matrix of autoregressive coefficients and $y_t$ is a (7x1) vector of endogenous variables described below. $\varepsilon_t$ is the generalization of the uncorrelated white noise process with zero mean.

Different from Hamilton (1983), we use the interest rate representing the financial sector by the monetary channel through adjustments of the interest rate instead of the control of money supply (M1). For most of the countries, we take the short term interest rate. This complies with the current literature. As a proxy for domestic prices and the inflation rate, we add the CPI. We consider the IPI as an approximation for economic development outside the country. Positive effects on the growth rate can result from an increase in the net export rate which might have its origin abroad. Hence, this variable is included to measure exogenous export demand. In this regard, it is the industrial production index for the G7 countries[4] which covers the main trade partners of most countries. The original models by Sims (1980), Hamilton (1983), and Mork (1989) use import prices whereas Mork et al.

[3] See Appendix 2.A for a detailed description.

[4] This measurement includes the G7 countries until 2015: Canada, France, Germany, Italy, Japan, UK, and the USA.

(1994) show that this index represents foreign business cycles more properly and that the coefficients of the two do not differ significantly. Alternatively, this variable can been regarded as an indicator of the general state of the global economy as it covers 1/3 of global economic activities. As all of the observed countries are highly integrated in the global market, the state of the global economy can has an exogenous effect on the domestic economy which is measured by the IPI G7 index. For further definitions and descriptions of the variables see Appendix 2.A.

Using an orthogonalized system aims to avoid that error terms are correlated with each other in the IRFs. By triangularizing the reduced VAR, we get orthogonalization of the residuals which also yields to a recursive structure. This process is also known as using the Cholesky decomposition in the reduced VAR as suggested by Sims (1980). Along with triangularization, the order of the endogenous variables becomes important as it determines the restriction of influence of the variables. The first predicted variable is determined by all lagged regressors, whereas the second variable is furthermore contemporaneously affected by the current first variable, and so on. In this model, we use the order: GDP growth, oil price changes, changes in CPI, interest rate, unemployment rate, IPI, and oil-to-GDP ratio. By setting the order of the first three variables, we assume that oil-prices do not contemporaneously affect GDP but inflation instead. This is in line with the literature, such as Jiménez-Rodríguez and Sánchez (2005), as oil price settings are often ascribed to have a certain degree of exogenous behavior, dissociated from general price developments. Giving that it has a higher ranking also considers its influences in the production process through affecting the price level or interest rate.[5] Ordering interest rate as the forth variable implies that the former values react with a distinctive lag, similar as the IPI and oil-to-GDP ratio.

### 2.4.2 Asymmetric nonlinear model

We extend our analysis by applying a non-linear approach through the estimation of three different methods of price determination. The i) asymmetric approach, the ii) scaled specification approach, and the iii) net oil price increase approach. These three approaches were selected because of their widespread use in existing literature. All three specifications only differ with respect to the determination of the oil price, hence, the overall model structure including the growth rates along with the control variables does not change. This allows for comparing the symmetric with the asymmetric as well as the non-linear models to examine different properties of the behavior of oil price changes on economic growth. The three specific approaches have all been chosen as they are very frequently used in existing literature, and hence enable a direct comparison of our work to the literature.

[5] Alternatively, we have also considered alternative ordering such as allowing oil price changes for contemporaneous impacts on GDP growth which are not reported here. Similar to Jiménez-Rodríguez and Sánchez (2005), it is only the contemporaneous effect that changes. With respect to causality, there are no significant changes in the results.

The asymmetric approach by Mork (1989) distinguishes between positive and negative oil price changes. Consequently, the oil price variable is split up into two parts with no further modification in level values,

$$\begin{aligned}\Delta oil^{+} &= \begin{cases}\Delta\text{oil} & \text{if } \Delta\text{oil} > 0\\ 0 & \text{otherwise}\end{cases}\\ \Delta oil^{-} &= \begin{cases}\Delta\text{oil} & \text{if } \Delta\text{oil} < 0\\ 0 & \text{otherwise}\end{cases}\end{aligned} \tag{2.2}$$

The scaled oil price increase (SOPI) approach by Lee et al. (1995) follows price normalization with regard to its variability using an autoregressive process. The model is based on a GARCH structure which includes a four lagged autoregressive process with a one lagged AR process of its variance.

$$\begin{aligned}\Delta\text{oil}_t = \alpha_0 + \sum_{i=1}^{4}\beta_i\Delta\text{oil}_{t-i} + \epsilon_t, \quad \epsilon_t \mid I_{t-1} \sim N(0, h_t)\\ h_t = \gamma_0 + \gamma_1\epsilon_{t-1}^2 + \gamma_2 h_{t-1}\\ SOPI_t = \max\left(0, \hat{\epsilon}_t/\sqrt{\hat{h}_t}\right)\\ SOPD_t = \min\left(0, \hat{\epsilon}_t/\sqrt{\hat{h}_t}\right)\end{aligned} \tag{2.3}$$

where information about $\epsilon_t$ is contained in information set $I_{t-1}$.

This AR(4)-GARCH(1,1) specification follows Jiménez-Rodríguez and Sánchez (2005) and the approach by Lee et al. (1995), but has also been verified by sensitivity analysis in our case.[6] The final scaled oil price is determined by the expected error of the AR change in oil price formation and the expected standard error of its variance. From intuition, this means that during both a period of stable prices changes as well as a period of high volatility the scaled price change is fluctuating less compared to the case in which a smooth period is followed by a sudden peak in price changes. Hence, the impact of shocks contributes stronger than a continuous trend. In addition to the initial proportion by Hamilton (1996), we also observe the model with scaled oil price decreases (SOPD).

[6] We obtain the lag-order selection of the autoregressive model from the Akaike information criterion. To test for ARCH effects, we perform the Engle Lagrange multiplier test, where ARCH(1) is valid for all countries except Japan and the USA. For these two countries, we perform two analyses: one with ARCH(1) effects and one without. As the results do not vary significantly, we uniformly consider ARCH(1) effects for all countries.

The net oil price increase (NOPI) approach has been proposed by Hamilton (1996), including an AR(4) process of oil prices in levels. It only permits the current oil price to change and to have an impact on the economy if it exceeds the highest price from the previous four periods. Otherwise, the NOPI value is assigned to be zero. Consequently, the change in oil prices reflected by NOPI is not equal to a quarterly oil price change. For $j = 1, 2, 3, 4$ hold

$$NOPI_t = \max(0, 100 * \{\ln(\text{oil}_t) - \ln[\max(\text{oil}_{t-j})]\}). \tag{2.4}$$

### 2.4.3 Oil share as moderator

Next to investigating the effect of oil price changes on GDP growth, we are further interested in whether this effect is different when the economy is faced with various oil-to-energy shares. All four baseline models are extended to allow for the investigation of the role of oil and energy shares within the aggregated economy. To do that, we make use of a moderated regression analysis in form of an Interacted VAR (IVAR) which is an otherwise VAR model but in which an interaction term substitutes the original price predictor. The interaction term is determined by the variable which will be shocked and the conditional variable. In theory, this term measures a moderation effect that affects the strength of the relation between a predictor variable and a criterion variable. If there is significant relationship of the predictor variable on the dependent variable, moderation is supported. In that case, we find evidence that the moderator influences the effect of the independent and dependent variable, either by amplifying or weakening the relationship between both.

IVAR have been recently introduced in several studies to analyze the impact of structural characteristics on the response of other variables to a macroeconomic shock. Towbin and Weber (2013) investigate the transmission of an external shock on output and investments with the influence of varying foreign currency debt, raw materials and exchange rate regimes. Leroy and Lucotte (2019) study the effect of competition in the financial sector on credit procyclicality. Caggiano et al. (2015) use an IVAR to examine the role of uncertainty at the zero lower bound by fully endogenizing the conditioning variables. The current study is based on the Interacted Panel VAR by Towbin and Weber (2013).[7]

For each oil price determination approach respectively, the recursive form of the IVAR is described by

[7] We thank Towbin and Weber for providing their MATLAB codes of the toolbox for Interacted Panel VAR estimations (based on Towbin and Weber, 2011).

$$\begin{pmatrix} 1 & 0 & \dots & 0 \\ \alpha_{0,t}^{2,1} & 1 & \dots & 0 \\ \vdots & \alpha_{0,t}^{3,2} & \ddots & \vdots \\ \alpha_{0,t}^{7,1} & \dots & \alpha_{0,t}^{7,6} & 1 \end{pmatrix} y_t \tag{2.5}$$

$$= c + \sum_{j=1}^{4} \begin{pmatrix} \alpha_{j,t}^{1,1} & \alpha_{j,t}^{1,2} & \dots & \alpha_{j,t}^{1,7} \\ \alpha_{j,t}^{2,1} & \alpha_{j,t}^{2,2} & \dots & \alpha_{j,t}^{2,7} \\ \vdots & \alpha_{j,t}^{3,2} & \ddots & \alpha_{j,t}^{3,7} \\ \alpha_{j,t}^{7,1} & \dots & \alpha_{j,t}^{7,6} & \alpha_{j,t}^{7,7} \end{pmatrix} y_{t-j} + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma)$$

where the impact matrix on the left hand side is a lower triangular matrix. The error terms are, by construction, uncorrelated across equations and orthogonalized to each other with a diagonal covariance matrix $\Sigma$. This has the advantage that the full system can be solved sequentially using OLS. As we use the same identification scheme as before, the variables remain in the same order. c is the intercept and $\varepsilon_t$ describes the error term of the equation.

The baseline VAR-models from the previous section only include endogenous variables which respond to each other respectively. In contrast to that, variables describing a structural condition are assumed to be exogenously given and independent of the remaining variables in the IVAR model. This is reasonable in the short term horizon since a direct response of the oil-to-energy share includes changes of structural infrastructure and other investments whose implications have effects in the long run. Observing the historical development of the oil-to-energy shares whose speed of change has been slow, supports this assumption. Furthermore, the coefficients in this model are allowed to vary with these deterministic structural characteristics. In other words, the autoregressive $\alpha_{j,t}^{w,q}$ coefficients are functions of the cross-time-varying level of oil-to-energy shares:

$$\alpha_{j,t}^{w,q} = \beta_j^{w,q} + \eta_{j,1}^{w,q} \cdot s_t + \eta_{j,2}^{w,q} \cdot s_t^2 \tag{2.6}$$

where $\beta_{j,t}^{w,q}$ and $\eta_{j,1}^{w,q}$ are vectors of coefficients and $s_t$ is the oil-to-energy share. The dynamic responses of the endogenous variables to the oil-price shock are conditionally linear. However, only oil prices are restricted to interact with the oil-to-energy share: For all remaining $\alpha_{j,t}^{w,q}$ coefficients

$$\alpha_{j,t}^{w,q} = \beta_{j,t}^{w,q} \quad \text{for all but} \quad q = 2$$

holds.[8]

After estimating the IVAR, a structural analysis is conducted based on varying structural characteristics to measure the consequences of a high and low oil-to-energy

[8]The baseline model can be obtained by assuming that (2.7) holds for all q.

share. In more detail, we observe the effect of oil-price changes on GDP growth for the individual 30th and 70th percentiles of the oil-to-energy share for each country.

To verify robustness of our results, we analyze the order of integration using a unit root test (see Table 2.B.1). According to the Dickey-Fuller test, stationarity has been confirmed for GDP, CPI, IPI, interest rate, unemployment rate, and oil-prices in their first log-differences. For all further variables (oil-to-GDP share and oil-to-energy share[9]) level-values are used.[10] We choose the number of lags in the VAR based on the Schwarz Information Criterion (SIC) and the Akaike Information Criterion (AIC) according to the sensitivity analysis. Along with that, we use lagged values of four quarters of a year for each variable to be able to consider variations which appear over a year. For GDP and the oil price defining variables including its interaction term with oil shares, we also consider the current values.

From a balanced panel dataset, the sample period of all models covers 184 quarters, a time period from 1971:I to 2016:IV for 12 different countries, namely Australia, Belgium, Canada, Finland, France, Germany, Japan, the Netherlands, Norway, Sweden, the UK, and the USA. As a result, it covers the oil crises in 1979/80 as well as the oil price increase in the 2000s and in part the sharp decrease in 2012 onwards. Results from a seemingly unrelated regression as a model framework similar to Mork (1989) and Mork et al. (1994) do not essentially deviate from our finding in the VAR-model.[11]

## 2.5 Empirical Results and Discussion

In this section, we will analyze the linear model as well as the three asymmetric approaches. Hereby, the study of moderator effects of oil shares will be done separately from the general analysis of oil prices in the macroeconomic context. This will ease the analysis by clearly distinguishing between the general study as it has been done by previous researches e.g. Hamilton (1983) or more recently Jiménez-Rodríguez and Sánchez (2005), and the extended part which focuses on a new feature in the relationship between oil prices and economic growth. Moreover, compared to other

[9] According to Wagner and Hong (2016), there is no definite answer in the econometric literature to deal with the concept of integrated and cointegrated processes to the nonlinear environment as it takes place in the oil-to-GDP ratio. As a minimum requirement for a useful extension of this concept they suggested to exclude cointegration, which is why we use level-values.

[10] For the interest rate and the unemployment rate, the Dickey-Fuller test indicates stationarity only for a few level values. However, in the majority of the existing literature (Hamilton, 1996; Hooker, 1996; Lee et al., 1995; Mork, 1989; Mork et al., 1994) models are estimated with level values. To make our results more comparable to the literature, we have performed two analyses: one with level values and one with first difference values for interest and unemployment rate (as it is done by Jiménez-Rodríguez and Sánchez, 2005). Despite these differences in control variables, the relevant results for the analysis of the moderator effect are largely robust and do not vary significantly. Hence, we stick to the results based on our dataset by using first difference values. The complete specification of the model is available upon request by e-mail.

[11] A SUR is a special form of a VAR-model with the restriction that the error terms of each time series are correlated with each other. This allows us to deal with white noise that can affect all local economies commonly which is assumed to be included in all error terms. As a side effect, the amount of estimates are increased compared to the general VAR-model.

studies, we will put more emphasis on Granger causality between oil prices, energy shares, and aggregate growth, to find evidence for the possible role of oil shares emitting moderator effects.

After classifying countries into groups of oil-consuming and oil-producing countries, we look at the models' relative explanatory power using the information criterion. Subsequently, causal relation as well as quantitative influence of oil prices on economic growth are investigated. In the first instance, this is done for the general models followed by their modified versions. For the sake of simplicity, we identify an oil price shock as a positive change in the oil price. Correspondingly, a negative price movement will be called a negative price shock.

### 2.5.1 Classification of oil-importing vs. oil-exporting country

We distinguish between oil-importing and oil-exporting countries. A country is regarded an oil-exporter when it displays a production-consumption ratio larger than unity (see Figure 2.B.3). According to this definition, three out of twelve countries investigated in this study are categorized as oil producing countries. Norway has constantly been an oil-exporting country, with an oil production exceeding consumption ten times in 2013. Canada made the transformation to a pure oil-exporting country in 1980. Since then, the average ratio has amounted to 1.5. The UK has switched from being an oil-exporting (from 1980 onwards) to being an oil-importing country in 2005, with a peak in the productions-consumption ratio at 1.6. However, as the UK is classified as a net-oil-exporter during half of the observed time series and clearly different from the remaining oil-importing countries, we consider the UK as an oil-exporting country. The USA have faced a different development. Due to new technologies to extract shale oil and gas, the country could increase its own oil production significantly since the mid of 2000s. The oil-to-energy share could be increased from 0.4 to 0.75 between 2008 and 2016 and is still showing a further increasing trend. Nevertheless, the USA is categorized as an oil-importing country as, in contrast to Canada, it has never had an oil-to-energy ratio lager than one in the observed time series. Likewise ,the remaining eight countries are classified as oil-importing countries. However, in total, the dependency on oil imports varies largely, from 0.03 for Japan to 0.75 for the USA in 2016.

### 2.5.2 Model selection

Since the four models are non-nested, we cannot use the likelihood-ratio test to make a statement about the quality of the models in comparison to each other. Therefore, we mainly refer to the Akaike information criterion (AIC) and Bayesian information criterion (BIC) which impose no restrictions on. Both criteria measure the goodness of fit of one model compared to another model. Hence, they do not make any proposal regarding the general quality of an individual model, but rather weight the explanatory power relative to that of other models. According to Burnham and

Anderson (2004), the AIC has theoretical advantages compared to the BIC. Among others, the amount of parameters are penalized less strongly using the AIC than using the BIC. Additionally, and particularly in the case at hand, the results might be altered due to the high number of parameters in our models setup. However, considering differences between the standard and extended setups, the results do not vary strongly.

Table 2.1: Information Criteria

| | Symmetric | | Asymmetric + | | Asymmetric - | | SOPI | | SOPD | | NOPI | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AIC | BIC | AIC | BIC | AIC | BIC | AIC | BIC | AIC | BIC | AIC | BIC |
| AUS | 13.771 | 17.442 | 12.845 | 16.516 | 12.851 | 16.522 | 11.773 | 15.444 | 11.808 | 15.479 | 12.301 | 15.973 |
| BEL | 11.487 | 15.088 | 10.530 | 14.131 | 10.558 | 14.159 | 9.061 | 12.662 | 9.137 | 12.738 | 10.201 | 13.802 |
| CAN | 12.685 | 16.286 | 11.776 | 15.377 | 11.645 | 15.246 | 10.729 | 14.330 | 10.638 | 14.239 | 11.333 | 14.934 |
| FIN | 13.094 | 16.695 | 12.160 | 15.761 | 12.096 | 15.697 | 10.740 | 14.341 | 10.761 | 14.362 | 11.776 | 15.377 |
| FRA | 8.497 | 12.098 | 7.581 | 11.182 | 7.534 | 11.135 | 6.112 | 9.713 | 6.183 | 9.784 | 7.123 | 10.724 |
| GBR | 13.112 | 16.783 | 12.126 | 15.797 | 12.150 | 15.821 | 10.818 | 14.489 | 10.926 | 14.597 | 11.636 | 15.307 |
| GER | 9.930 | 13.531 | 8.927 | 12.528 | 9.116 | 12.717 | 7.502 | 11.103 | 7.714 | 11.315 | 8.540 | 12.141 |
| JPN | 10.608 | 14.209 | 9.765 | 13.366 | 9.812 | 13.413 | 8.113 | 11.714 | 8.138 | 11.739 | 9.311 | 12.912 |
| NLD | 12.273 | 15.874 | 11.306 | 14.907 | 11.418 | 15.019 | 9.985 | 13.586 | 9.977 | 13.578 | 10.655 | 14.256 |
| NOR | 18.124 | 21.795 | 17.242 | 20.913 | 17.065 | 20.737 | 16.251 | 19.922 | 16.240 | 19.911 | 16.859 | 20.530 |
| SWE | 13.204 | 16.805 | 12.362 | 15.963 | 12.165 | 15.766 | 10.910 | 14.511 | 10.940 | 14.541 | 12.066 | 15.667 |
| USA | 10.683 | 14.284 | 9.743 | 13.344 | 9.779 | 13.380 | 8.476 | 12.077 | 8.414 | 12.015 | 9.299 | 12.900 |

AIC: estimator of the relative quality of the statistical model based on the Akaike information criterion
BIC: estimator of the relative quality of the statistical model based on the Bayesian information criterion

Table 2.1 shows the results for the baseline models. We investigate both, the AIC and BIC. The results are consistent for all countries. Concerning the standard setup, the NOPI model and the asymmetric linear model, considering price increases only, provide similar results, whereas the former performs slightly better. They are both preferred over the symmetric linear approach. However, all specifications are strictly dominated by the scaled approach considering price increases only. This is in line with other studies such as Hamilton (2003) and Jiménez-Rodríguez and Sánchez (2005) who tend to prefer the SOPI approach. Nonetheless, the results also reflect that all information criteria of these four approaches are on a similar level within each country in our estimations. However, it is notable that in case of Norway, the information criteria are considerable larger which can lead to an overestimation.

Concerning the relative performance of the models, graphical results of respective impulse response functions confirm similar classification (see Figures 2.2–2.7). The graphs show the impact of oil price changes on GDP growth without moderator effects. Comparing the confidence bands, we can figure out the precision of estimation with respect to each other.

Altogether, it has to be assumed from these results that besides choosing between using a symmetric and a non-symmetric model structure, the environment of a model, together with its advantages and disadvantages, has to be considered as well. This is even more important when dealing with the special structure when including moderator effects of oil-to-energy shares. Furthermore, a modeled environment, which considers country specific properties and additional properties such as asymmetry and non-linearity, are valued higher regarding the consequences for GDP growth than in the simple linear-symmetric approaches.

### 2.5.3 Granger causality and response to price shock

Before analyzing the influence of changes in oil prices on aggregated economic growth and the relationship between the two qualitatively and quantitatively, we investigate the causal relationship between oil price changes and GDP growth. Hereby, we distinguish between direct and indirect causality, resulting from oil price changes. At first, we carry out a conventional F-test for each model separately. To be more precise, we investigate whether Granger-causality of oil prices on economic growth is significant by performing a Wald test. According to the latter, we test the null-hypothesis whether all oil price coefficients of each country are jointly zero. For the asymmetric models, these tests are performed individually for positive and negative changes. The results are shown in Table 2.2. In the following, we assume confirmation of a causal relationship at a 10% significance level. By considering indirect causality from oil price changes to GDP growth by third variables, we are able to identify possible channels which are beyond the direct oil-price to GDP growth relationship. We use a structural VAR model, imposing a few constraints, to test whether all oil price coefficient are jointly zero, but in its own equation. The results are summarized in Table 2.3.

Subsequent to the causality test, we analyze the qualitative and quantitative impact of an oil price shock by considering its effects on GDP growth. Table 2.4 comprises the accumulated price effect for each model in period 4 to 12 after the shock. In addition, the results are prepared graphically as orthogonalized impulse response functions (IRF) in Figures 2.2–2.7, looking at a time horizon of 20 periods. The size of a shock is the same for all models (100% increase/decrease in oil price), with these shocks occurring unexpectedly. Subsequently, we discuss the results separately for oil-importing and oil-exporting countries.

Table 2.2: Direct Causality - base models

| | Symmetric | Asymmetric | | Scaled Prices | | Net Prices |
|---|---|---|---|---|---|---|
| | Price +/- | Price + | Price - | SOPI | SOPD | NOPI |
| AUS | 0.737 | 0.649 | 0.716 | 0.397 | 0.564 | 0.312 |
| BEL | 0.025** | 0.001*** | 0.127 | 0.022** | 0.160 | 0.016** |
| CAN | 0.004*** | 0.199 | 0.001*** | 0.487 | 0.000*** | 0.112 |
| FIN | 0.606 | 0.197 | 0.046** | 0.357 | 0.066* | 0.115 |
| FRA | 0.082* | 0.046** | 0.126 | 0.152 | 0.310 | 0.023** |
| GBR | 0.245 | 0.113 | 0.450 | 0.118 | 0.332 | 0.071* |
| GER | 0.056* | 0.005*** | 0.205 | 0.031** | 0.250 | 0.016** |
| JPN | 0.535 | 0.040** | 0.001*** | 0.041** | 0.031** | 0.018** |
| NLD | 0.066* | 0.026** | 0.585 | 0.357 | 0.547 | 0.001*** |
| NOR | 0.089* | 0.077* | 0.346 | 0.254 | 0.520 | 0.279 |
| SWE | 0.299 | 0.373 | 0.120 | 0.726 | 0.378 | 0.833 |
| USA | 0.410 | 0.163 | 0.658 | 0.028** | 0.490 | 0.317 |

$H_0$: all lagged oil-price change coefficients are jointly equal to zero ($\alpha_1^{\text{oil}} = \alpha_2^{\text{oil}} = \alpha_3^{\text{oil}} = \alpha_4^{\text{oil}}$)
Values present two-sided p-value corresponding to the F-statistic result.

Table 2.3: Indirect Causality - base models

| | Symmetric | Asymmetric | | Scaled Prices | | Net Prices |
|---|---|---|---|---|---|---|
| | Price +/- | Price + | Price - | SOPI | SOPD | NOPI |
| AUS | 0.004*** | 0.093* | 0.000*** | 0.053* | 0.003*** | 0.021** |
| BEL | 0.000*** | 0.001*** | 0.002*** | 0.003*** | 0.006*** | 0.001*** |
| CAN | 0.023** | 0.033** | 0.014** | 0.215 | 0.013** | 0.187 |
| FIN | 0.001*** | 0.016** | 0.001*** | 0.013** | 0.005*** | 0.098* |
| FRA | 0.001*** | 0.000*** | 0.008*** | 0.009*** | 0.041** | 0.000*** |
| GBR | 0.035** | 0.041** | 0.074 | 0.101 | 0.248 | 0.215 |
| GER | 0.000*** | 0.000*** | 0.000*** | 0.000*** | 0.001*** | 0.000*** |
| JPN | 0.000*** | 0.002*** | 0.000*** | 0.000*** | 0.000*** | 0.005*** |
| NLD | 0.000*** | 0.000*** | 0.001*** | 0.001*** | 0.000*** | 0.000*** |
| NOR | 0.001*** | 0.005*** | 0.005*** | 0.184 | 0.014 | 0.000*** |
| SWE | 0.121 | 0.597 | 0.000*** | 0.828 | 0.006*** | 0.138 |
| USA | 0.013** | 0.064* | 0.001*** | 0.152 | 0.021** | 0.232 |

Indirect causality is checked by testing for block exogeneity.

$$Y_{1,t} = C_1 + A_1' X_{1,t} + A_2' X_{2,t} + \epsilon_{1,t}$$
$$Y_{2,t} = C_2 + B_1' X_{1,t} + B_2' X_{2,t} + \epsilon_{2,t}$$

where vector $Y_1$ contains all variables except changes in oil prices and $Y_2$ contains the oil-price changes. $X_1$ is the vector of all lagged variables of $Y_2$ and correspondingly $X_2$ contains all lagged values of $Y_2$

*$H_0$: all lagged oil-price change coefficients are jointly equal to zero in all equation of the system except its own, here $A_2' = 0$*

Consequently, the history of the block $Y_2$ (oil-price changes) does not help in forecasting the variable $Y_2$.

A restricted SVAR tests for over-identifying restrictions according to the model above. The test is computed as:

$$\text{LR} = 2(\text{LL}_{var} - \text{LL}_{svar})$$

where LR is the value of the test statistic against the null hypothesis that the over-identifying restrictions are valid, $\text{LL}_{var}$ is the log likelihood from the underlying VAR(p) model, and $\text{LL}_{svar}$ is the log likelihood from the restricted SVAR model. The results are presented as two-sided p-value corresponding to the asymptotically distributed $\chi^2(q)$ where $q$ corresponds to the number of restrictions.

Table 2.4: Accumulation of price effects

Symmetric model with shock in $\Delta oil$

| quarters | AUS | BEL | CAN | FIN | FRA | GBR | GER | JPN | NLD | NOR | SWE | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | -0.010 | -0.007 | 0.002 | -0.009 | -0.008 | -0.011 | -0.003 | -0.005 | -0.006 | 0.009 | 0.012 | -0.020* |
| 6 | -0.010 | -0.016* | -0.004 | -0.028 | -0.015* | -0.020* | -0.011 | -0.006 | -0.008 | 0.009 | 0.001 | -0.025* |
| 8 | -0.009 | -0.014* | -0.004 | -0.036 | -0.019* | -0.021* | -0.014 | -0.005 | -0.011 | 0.008 | -0.002 | -0.024* |
| 10 | -0.004 | -0.012 | -0.001 | -0.038 | -0.017* | -0.019 | -0.013 | -0.004 | -0.012 | 0.008 | -0.002 | -0.020 |
| 12 | -0.003 | -0.013 | 0.000 | -0.037 | -0.015* | -0.015 | -0.012 | -0.003 | -0.012 | 0.008 | 0.000 | -0.019 |

Asymmetric price model with shock in $\Delta oil^{+}$ and $\Delta oil^{-}$

| quarter | AUS | | BEL | | CAN | | FIN | | FRA | | GBR | | GER | | JPN | | NLD | | NOR | | SWE | | USA | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ |
| 4 | -0.026* | 0.005 | 0.004 | -0.020 | -0.008 | 0.016 | -0.011 | -0.007 | -0.014 | -0.003 | -0.022 | -0.002 | -0.006 | 0.009 | -0.031 | 0.013 | -0.011 | -0.006 | -0.005 | 0.033* | 0.016 | 0.015 | -0.050* | 0.004 |
| 6 | -0.021 | -0.001 | -0.016 | -0.022 | -0.019 | 0.014 | -0.035 | -0.030 | -0.024* | -0.012 | -0.038* | -0.014 | -0.025 | 0.004 | -0.029 | 0.011 | -0.019 | -0.002 | -0.010 | 0.035 | -0.007 | 0.013 | -0.065* | 0.009 |
| 8 | -0.022 | 0.002 | -0.012 | -0.021 | -0.016 | 0.012 | -0.041 | -0.046 | -0.029* | -0.015 | -0.036* | -0.019 | -0.032 | -0.001 | -0.025 | 0.010 | -0.023 | -0.004 | -0.010 | 0.034 | -0.012 | 0.010 | -0.065* | 0.014 |
| 10 | -0.014 | 0.006 | -0.007 | -0.021 | -0.012 | 0.014 | -0.041 | -0.052 | -0.028* | -0.013 | -0.032 | -0.019 | -0.027 | -0.001 | -0.022 | 0.012 | -0.022 | -0.006 | -0.010 | 0.034 | -0.011 | 0.009 | -0.060* | 0.017 |
| 12 | -0.010 | 0.007 | -0.010 | -0.020 | -0.010 | 0.015 | -0.038 | -0.051 | -0.025* | -0.010 | -0.028 | -0.015 | -0.023 | -0.002 | -0.021 | 0.012 | -0.022 | -0.006 | -0.010 | 0.034 | -0.009 | 0.010 | -0.058* | 0.018 |

Scaled price model with shock in $\Delta SOPI$ and $\Delta SOPD$

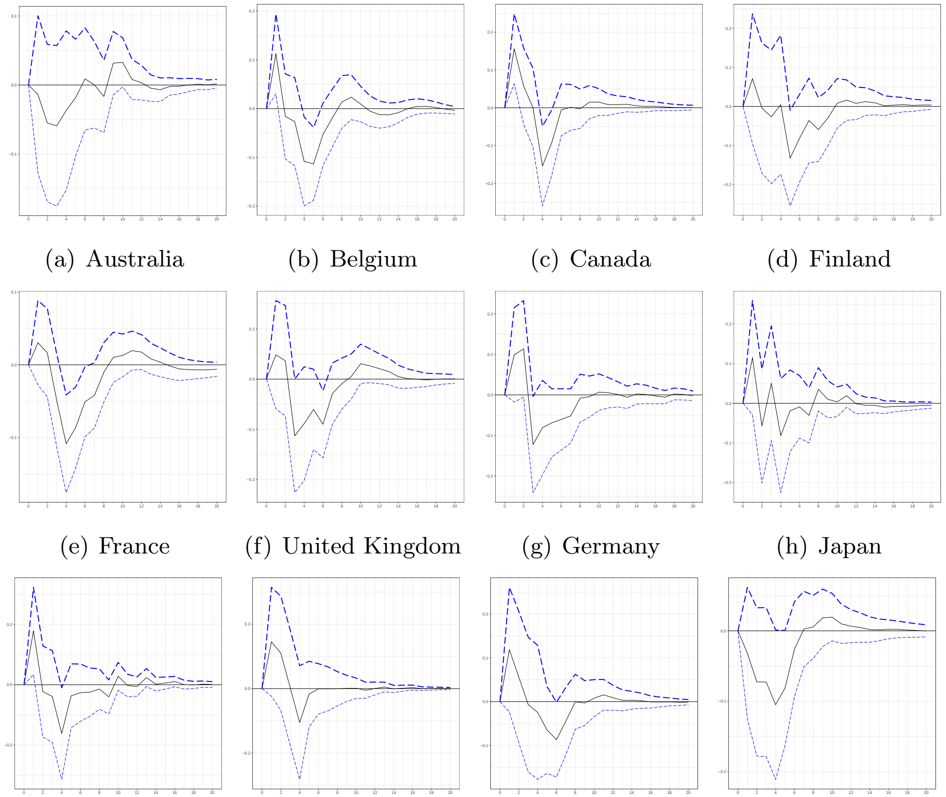
| quarter | AUS | | BEL | | CAN | | FIN | | FRA | | GBR | | GER | | JPN | | NLD | | NOR | | SWE | | USA | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD |
| 4 | -0.052* | 0.003 | 0.012 | -0.036 | -0.034 | 0.020 | -0.071 | -0.013 | -0.035 | -0.004 | -0.080* | -0.022 | -0.042* | 0.018 | -0.099 | 0.055 | -0.052 | 0.005 | 0.007 | 0.050 | 0.005 | 0.015 | -0.114* | 0.009 |
| 6 | -0.048* | -0.004 | -0.021 | -0.039 | -0.046 | 0.007 | -0.140* | -0.060 | -0.056* | -0.019 | -0.094* | -0.043 | -0.056* | 0.007 | -0.098 | 0.056 | -0.044 | 0.018 | 0.006 | 0.049 | -0.035 | 0.005 | -0.144* | 0.014 |
| 8 | -0.047* | 0.000 | -0.019 | -0.034 | -0.047 | 0.001 | -0.165* | -0.089 | -0.071* | -0.025 | -0.090* | -0.053 | -0.071* | 0.001 | -0.093 | 0.057 | -0.059 | 0.015 | 0.007 | 0.052 | -0.042 | -0.004 | -0.156* | 0.023 |
| 10 | -0.035 | 0.007 | -0.013 | -0.033 | -0.044 | 0.003 | -0.173* | -0.103 | -0.072* | -0.023 | -0.082* | -0.050 | -0.061 | 0.000 | -0.086 | 0.060 | -0.057 | 0.010 | 0.007 | 0.052 | -0.041 | -0.005 | -0.154* | 0.028 |
| 12 | -0.029 | 0.008 | -0.018 | -0.033 | -0.042 | 0.006 | -0.168* | -0.101 | -0.066* | -0.017 | -0.077* | -0.043 | -0.061 | -0.002 | -0.085 | 0.061 | -0.055 | 0.009 | 0.007 | 0.052 | -0.038 | -0.003 | -0.151* | 0.032 |

Net price model with shock in $\Delta NOPI$

| quarter | AUS | BEL | CAN | FIN | FRA | GBR | GER | JPN | NLD | NOR | SWE | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | -0.027 | -0.014 | -0.009 | -0.025 | -0.024* | -0.024 | -0.016 | -0.065* | -0.016 | -0.001 | 0.006 | -0.055* |
| 6 | -0.020 | -0.042* | -0.032 | -0.044 | -0.038* | -0.048* | -0.042* | -0.062* | -0.040* | -0.011 | -0.022 | -0.079* |
| 8 | -0.025 | -0.030* | -0.027 | -0.050 | -0.043* | -0.045* | -0.047* | -0.053* | -0.046* | -0.012 | -0.028 | -0.076* |
| 10 | -0.014 | -0.019 | -0.017 | -0.047 | -0.040* | -0.037 | -0.041 | -0.050* | -0.044* | -0.012 | -0.025 | -0.065* |
| 12 | -0.011 | -0.024 | -0.012 | -0.041 | -0.034* | -0.029 | -0.035 | -0.050* | -0.043* | -0.012 | -0.022 | -0.061* |

Table includes accumulated response of GDP growth after a one-standard-deviation in oil prices according to the respected model.

Figure 2.2: Orthogonalized IRF of GDP growth to a one-standard-deviation positive symmetric oil price shock



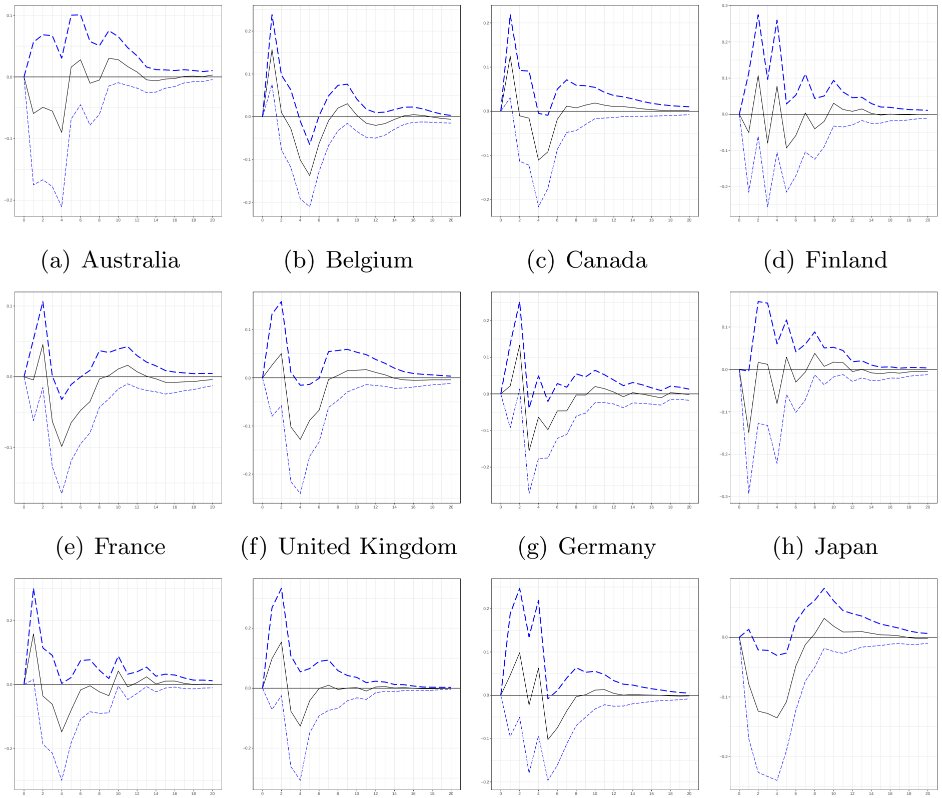
(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Black (solid) line indicates point estimate and blue (sticked) lines indicate 90% confidence band.

Figure 2.3: Orthogonalized IRF of GDP growth to a one-standard-deviation positive asymmetric oil price shock

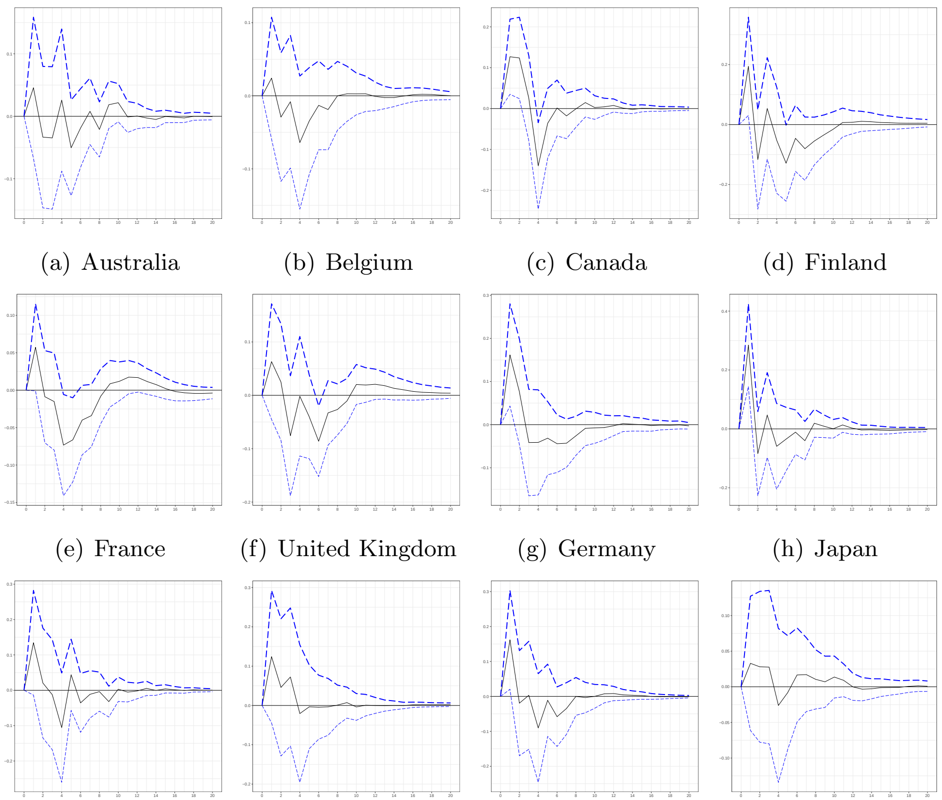


(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Figure 2.4: Orthogonalized IRF of GDP growth to a one-standard-deviation negative asymmetric oil price shock

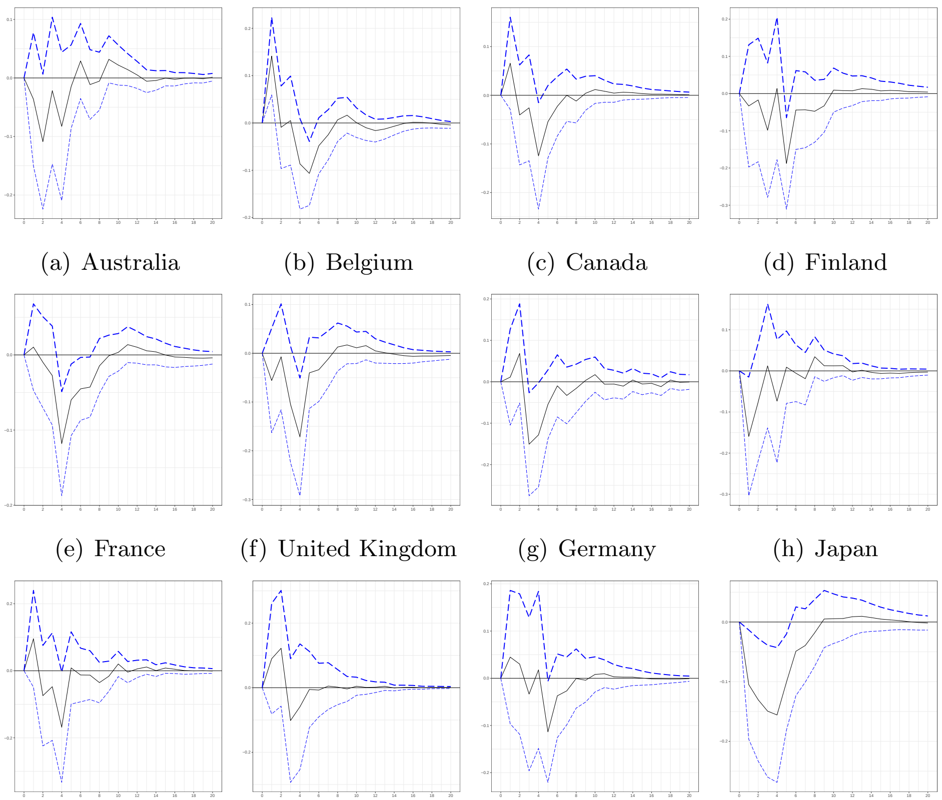


(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Figure 2.5: Orthogonalized IRF of GDP growth to a one-standard-deviation positive scaled oil price shock (SOPI)



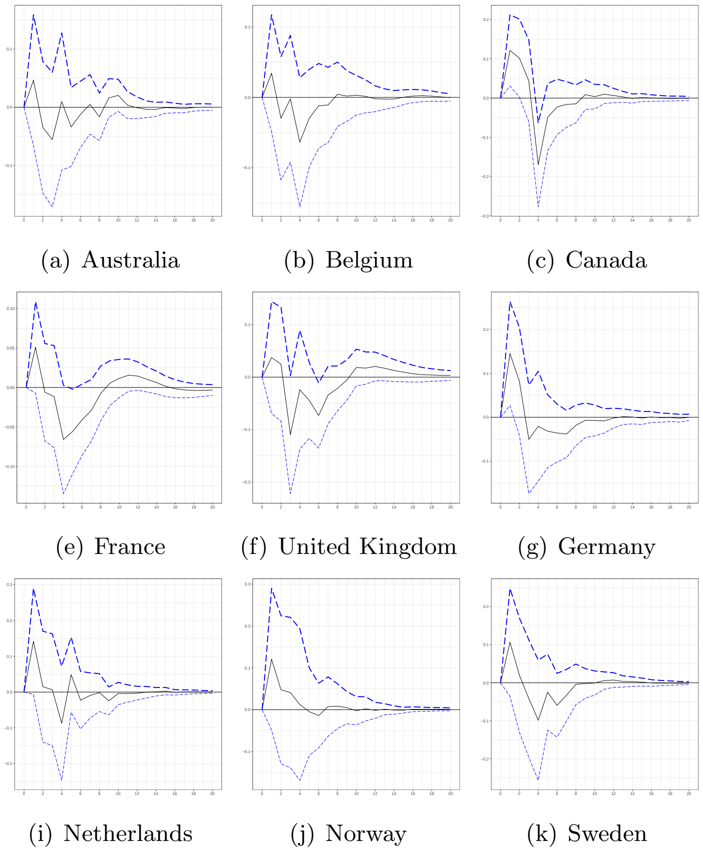
(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Black (solid) line indicates point estimate and blue (sticked) lines indicate 90% confidence band.

Figure 2.6: Orthogonalized IRF of GDP growth to a one-standard-deviation negative scaled oil price shock (SOPD)

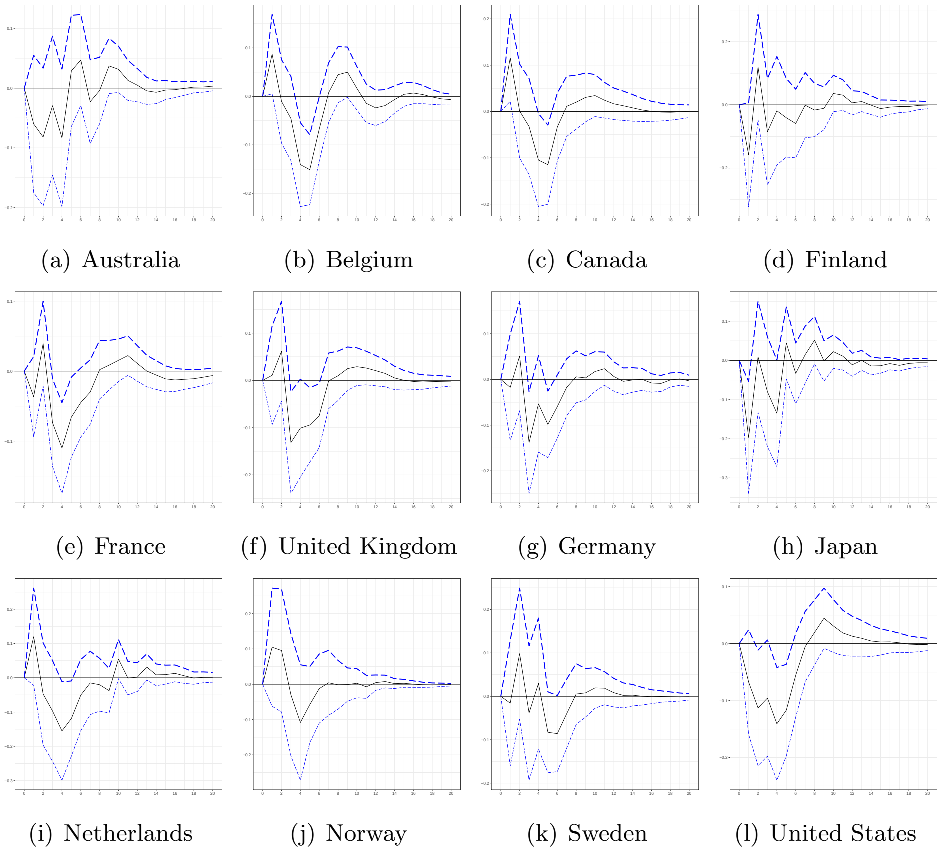


(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Figure 2.7: Orthogonalized IRF of GDP growth to a one-standard-deviation positive net oil price shock (NOPI)



(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Black (solid) line indicates point estimate and blue (sticked) lines indicate 90% confidence band.

#### 2.5.3.1 Oil-importing countries

Considering a positive oil price shock shows an inconsistent economic response among the oil importing countries with respect to Granger causality. Under the assumption of symmetry, a change in oil prices is significantly Granger causing economic growth through the direct channel in only three out of nine oil importing countries, namely Belgium, Germany, and Japan. However, the indirect channel through a third variable takes a more important role, being highly significant for all countries except Canada.[12] Regarding the GDP growth responds after an oil price shock, the majority of countries have a similar development in their responses. A direct increase in GDP growth is followed by a drop in its growth rate which fades out after a few quarters (Figure 2.2). In contrast to that, Japan's economy is faced by frequent fluctuations in GDP growth which neutralize each other. These developments are also visible in the accumulated responses of the price shock. In total, all oil-importing countries face a negative impact whereas it is significantly weaker in Japan, accounting for -0.9% after two years (-3.8% for Finland, -2.9 for the USA, and -2.1% for France; see Table 2.4). Although Sweden's economy is also negatively affected in the second year after the price increase, these effects are offset by its initial positive response. The special case of Japan will be discussed in Section 2.5.4.1.

In the asymmetric framework, the results of the linear-asymmetric, scaled-, and net-price models are qualitatively equal, at least for oil price increases (see Figures 2.3, 2.5, and 2.7). Similar to the symmetric approach, all countries experience negative accumulated impacts on GDP growth after two years. But in contrast to the symmetric approach, also Sweden and Japan experience by clearly negative effects from the second year onwards. The non-European countries Australia, Japan, the USA experience immediate drops in GDP growth which are delayed by a few quarters for the remaining oil-importing countries. Overall, this is consistent with regard to their oil dependency. However, (direct) Granger-causality cannot be confirmed for all countries. Similar to the linear symmetric approach, only a few countries have Granger causality from oil-price changes to GDP growth directly. Alone Belgium and Germany show significant results throughout the three approaches, the Netherlands lacks in significance in the scaled price approach, whereas Japan and the USA have significant results in the scaled price approach only. In contrast to that, indirect influence of changes in oil-prices on GDP growth though at least one of the remaining aggregated macroeconomic indicators are mostly valid for all countries except for Canada.

Regarding the magnitudes, the countries differ partially depending on the model selection. It is noticeable that the USA and Japan face the highest impacts on GDP

[12] In contrast to other literature such as Jiménez-Rodríguez and Sánchez (2005), we could not find a significant indirect causal effect in all five models (for both positive and negative price change) for Canada.

growth. Especially for Japan, these strong consequences are not surprising since it has had by far the highest dependency on oil imports of above 96%. For the USA, it can be explained that the US economy experienced by real exchange rate appreciation which has been pointed out by Jiménez-Rodríguez and Sánchez (2005). In other words, as oil is traded in US dollars on the world market, the USA cannot counteract changes in oil prices by adjusting their currency, in contrast to other countries. Furthermore, it is worth to mention that the most recent development in shale oil extraction have not been explicitly covered in this paper as the time range concerned is only 15% of the whole time series. We leave this to further research.

On average, the linear asymmetric approach generates lower magnitudes while the scaled approach has slightly larger results than the NOPI model. Moreover, the asymmetric models generate higher magnitudes than the symmetric approach. Both are in line with former studies. Although the price shock is qualitatively the same within all models, its effect is enhanced in the net price and above all in the scaled model as both approaches act more sensible to previous price trends. For example, Germany experiences an accumulated GDP growth loss of 1.3% in the symmetric approach, 2.5% in the asymmetric linear approach, 3.7% in the NOPI model, and 4.6% in the scaled price approach.

Observing negative oil price movements, the results contradict with the vast literature which mainly reject any relationship of negative oil price changes with GDP growth. From the F-test, analyzing the direct influence of oil-price changes on GDP growth, we find significant effects for Finland (not SOPD), France, Japan, and Sweden (not SOPD). Indirect channels can be highly identified for all countries. Each of them experiences a positive response in GDP growth rates after price increases turning to become negative in the second year, until it fades out in the subsequent years. The size of the magnitude seems to be inconsistent, leading to clearly positive accumulated results for Australia, Germany, Sweden, and the USA but negative results for Belgium, Finland, and France for both of the negative price approaches. Only Japan and the Netherlands have inconsistent accumulated effects, being negative in the linear-asymmetric approach and positive in the scaled model.

The accumulated negative response for some countries ranging from -0.4% for Japan up to -4.4% for Finland in the linear approach contradicts with the assumption that an economy profits from lower costs on the demand side of economic players. A possible explanation for this unusual effect is that the few sharp oil price decreases have taken place along with low economic growth rates (in the 1980s) up to a recession (2008). Consequently, the countries have been confronted with these shocks at a time period where the domestic economy has been more vulnerable to economic downturns which overall has had stronger negative effects. According to the accumulated IRFs, it is worth to mention that the respective reaction of GDP growth rates after price drops is not as strong as in the case of a positive oil price shock. With the exception of Sweden in the linear model and Belgium in both models, all remaining countries show a lower accumulated magnitude, which is confirmed by the IRF figures (see Figure 2.4 and 2.6).

To conclude, the results after positive shocks on oil prices are in compliance with the literature, saying that overall, countries' GDP growth reacts negatively. However, in contrast to previous studies, decreasing oil prices have significant effects on GDP growth as we have seen for a few countries. Moreover, as the more preferred scaled oil price model (followed by the net price approach) has higher magnitudes in its price coefficients, it seems that the consequences of an oil price shock on economic growth are larger in a volatile economy than in a stable one. This finding prevails throughout this study. It confirms the assumption that economic uncertainty or unexpected sharp price changes which are intensified in the non-linear approaches, induce higher pressure on the economy.

#### 2.5.3.2 Oil-exporting countries

For oil-exporting countries, the F-test mirrors the results of the oil-importing countries, namely that there is almost no support for a direct causal effect from oil-price movements on GDP growth, with the exception of the UK (in NOPI only). Only for Canada, we find some effects but they are limited to oil-price decreases. However, as seen before, the main influence by oil-price changes takes place in indirect channels which also holds for the UK and Norway. For Canada alone, we cannot confirm any significant indirect causal effect which can be traced back to oil price changes only. Qualitatively, the UK and Norway are similarly affected by oil price shocks showing a positive response in GDP growth in the first periods which becomes negative afterwards. Overall, the accumulated price effects are negative after three years as the initial positive development is preponderated by the negative response. Only in the symmetric and scaled price approaches, Norway deviates by experiencing a positive accumulated pressure on GDP growth. At first glance, the adverse effects seem to be unexpected since the oil extracting sector is generally profiting from higher prices. But Jiménez-Rodríguez and Sánchez (2005) find the same results for the UK linking it to the exchange rate appreciation which has been a side effect of oil price hikes.

In our model, the effective exchange rate is not used as a separate variable but integrated in the country's specific oil price. Furthermore, Canada has been an oil-demanding country since the 1970s, and the UK is one since 2008 an oil-demanding country. As such, it is not surprising to find a response resembling that of the oil-importing countries analyzed in the previous section. Considering the magnitude of IRFs, the largest accumulated effects after an oil price shock is in the British economy. In the linear-asymmetric approach, a 100% increase in oil prices leads to an accumulated reduction of economic growth of 2.3% for UK whereas the decrease in growth of Norway values only 1.6%. This trend continues in the other price structures as well.

Table 2.5: Existence of moderator effect - extended models

| Moderator | Symmetric | Asymmetric | | Scaled Prices | | Net Prices |
|---|---|---|---|---|---|---|
| | Price +/- | Price + | Price - | SOPI | SOPD | NOPI |
| AUS | 2.345*** | 2.494*** | 2.220*** | 2.504*** | 2.315*** | 2.450*** |
| BEL | 1.200 | 1.013 | 1.617** | 1.144 | 1.462** | 0.309 |
| CAN | 1.808*** | 1.899*** | 1.582** | 1.887*** | 1.609** | 2.011*** |
| FIN | 1.784*** | 1.856*** | 2.149*** | 1.955*** | 2.005*** | 1.918*** |
| FRA | 1.385* | 1.193 | 1.726*** | 1.499*** | 2.153*** | 1.104 |
| GBR | 3.873*** | 3.949*** | 4.119*** | 3.691*** | 4.045*** | 4.195*** |
| GER | 1.208 | 1.121 | 0.893 | 1.025 | 0.895 | 0.995 |
| JPN | 2.454*** | 2.503*** | 2.639*** | 2.399*** | 3.048*** | 2.270*** |
| NLD | 2.883*** | 2.473*** | 3.317*** | 2.398*** | 3.630*** | 2.561*** |
| NOR | 1.404* | 1.370* | 1.268 | 1.332* | 1.362* | 1.477** |
| SWE | 2.670*** | 3.009*** | 2.543*** | 2.983*** | 2.601*** | 2.628*** |
| USA | 2.641*** | 2.562*** | 2.392*** | 2.628*** | 2.463*** | 2.688*** |

$H_0$: all lagged interaction term coefficients are jointly equal to zero
$(\eta_{1,1}^{w,2} = \eta_{2,1}^{w,2} = \eta_{3,1}^{w,2} = \eta_{4,1}^{w,2} = \eta_{1,2}^{w,2} = \eta_{2,2}^{w,2} = \eta_{3,2}^{w,2} = \eta_{4,2}^{w,2} \quad 1 \leq w < 7, w \in \mathbb{N})$
Coefficient are taken from (2.6): $\alpha_{j,t}^{w,q} = \beta_j^{w,q} + \eta_{j,1}^{w,q} \cdot s_t + \eta_{j,2}^{w,q} \cdot s_t^2$
Values present F-statistics, corresponding significance levels are 1.33 (10%), 1.44 (5%), and 1.66 (1%).

### 2.5.4 Granger causality with moderator effects

The modified version of the model includes interaction effects of changes in oil prices and the quarterly moving average oil-to-energy share to incorporate a possible moderator in the explanation of economic growth. Hence, with the F-test, checking for Granger causality, it is tested whether all moderator coefficients are jointly zero. A significant result indicates that the oil-to-energy share has an impact on how the oil-price affects economic growth. The results are summarized in Table 2.5. As in the baseline model, we consider the accumulated effect of price changes as well as its development over the periods in an IRF graph. The structural characteristic in the interaction term which is represented by the oil-to-energy share, is kept constant and enters the model exogenously. Concerning the accumulated effect, we take the $50^{th}$ percentile of each individual countries' oil-to-energy share. This yields an approximate average of the oil-to-energy development over the investigated time period. For the non-accumulated orthogonalized IRF analysis of GDP growth after and oil price shock, we further calculate the respective results for the $30^{th}$ and $70^{th}$ percentile as depicted in the Figures 2.8–2.13. The black (solid) line indicates the estimated points, while the blue (sticked) line indicates the 70th percentile of oil-to-energy share and the red (dotted) line indicates the 30th percentile of oil-to-energy share. This provides an insight into the relationship between variations in oil prices and economic growth, and how this relationship is affected depending on the shares of different energies it is faced with. Again, we look at oil-importing and oil-exporting countries separately. In doing so, we first discuss the causal relation of oil price changes to GDP growth, followed by investigating the price effects including the moderator variable stemming from the changes in the oil-to-energy share.

#### 2.5.4.1 Oil-importing countries

Using the explanatory power of a F-test, it is investigated whether the inclusion of oil-to-energy shares as moderator variables makes a significant difference and hence, whether including them improves the model. Table 2.5 shows the corresponding results for all countries. We find evidence for Granger causality of the interaction coefficients with GDP growth and according to that the existence of moderator effects. For the majority of oil-importing countries, causality can be confirmed at the 1% significance level. Alternatively, to rule out any misspecification regarding the functional form of our models, we also check the fit of a linear function to determine the oil-price coefficients, allowing for linearity in the moderator effect. However, for most countries, there is a lack of significance, confirming that non-linearity is an essential assumption, as non-linearity reflects the intensification effect of oil-price changes. In sum, the extension of standard oil-price–GDP models by adding the behavior of oil-to-energy share but also its non-linear interaction with oil-prices over time lead to an improvement in estimation results. Considering the linear price approaches, it is striking that all oil-importing countries experience similar responses in GDP growth after a positive oil price shock. With exception of Germany and the USA, the initial reaction is an increase in growth, followed by up-and-down movements which slowly fade out.

The accumulated responses of GDP growth are presented in Table 2.6. In the linear approaches which are less valued according to the IC criteria, all countries are negatively affected by price shocks from the second year onwards with the exception of Australia and the Netherlands in the symmetric model, and Belgium and France in the asymmetric model. Surprisingly, we find a positive response of GDP growth for Japan in all models which contradicts the results from the baseline model. Compared to the other countries, it further seems that the positive pure price effects are persistent at higher levels (1.2% in the symmetric approach and 4.4% in the asymmetric approach). These unusual outcomes for Japan have already been discussed in previous papers (Jiménez-Rodríguez and Sánchez, 2005; Mork et al., 1994). In their studies, economic growth in Japan was positively affected by oil price increases which has been explained with a more resilient Japanese economy. The country overcame the second oil price crisis after 1980 much better than the first crisis (73–74), in particular compared with other oil importing countries. As Japan could not benefit significantly from oil price drops in the 1980s, the resilient effect is even amplified in our symmetric model. But unlike Mork et al. and Jiménez-Rodríguez and Sánchez, this finding can also be obtained from our model. The inclusion of oil-to-energy shares could be the reason why this outcome cannot be found in the baseline model. According to the results, the higher resistance to oil price shocks has existed especially in periods with higher dependency on oil for example the 1970s. At that time, ratios have been far above the shares of remaining oil-importing countries excluding Sweden. Comparing these results to those of low oil-to-energy shares, their accumulated responses of GDP growth have declined, or have even become negative. Hence, as the response of GDP growth has become worse in the subsequent years and the role of oil within the economy has lost in magnitude it can be concluded that resilience could not be maintained by Japan over the time.[13]

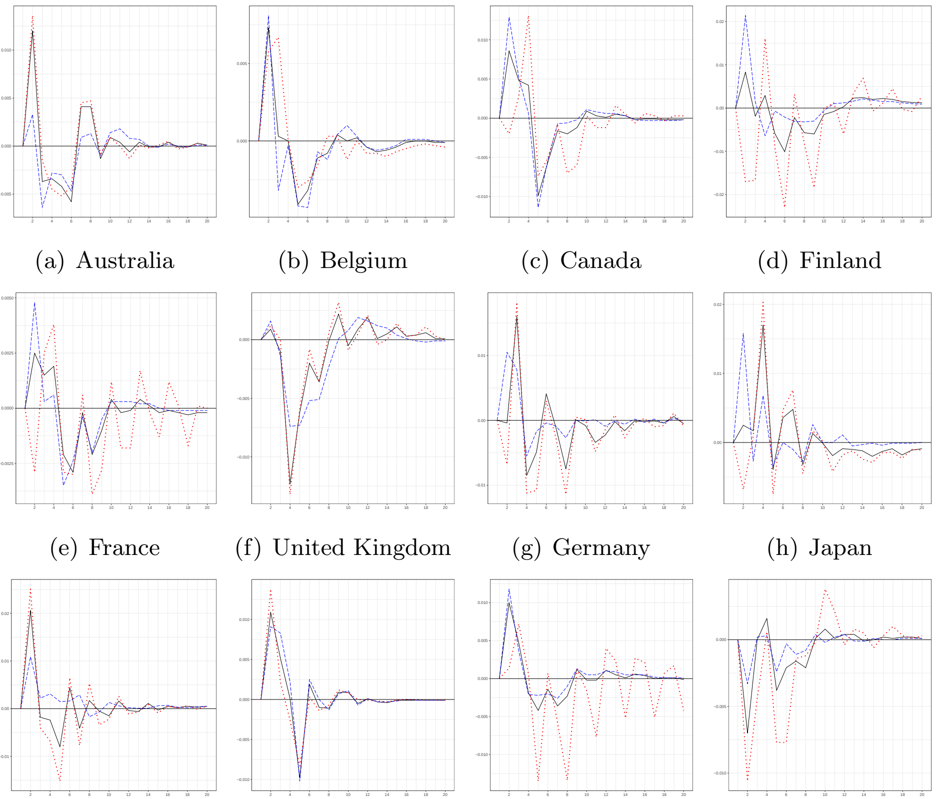
In the non-linear models, the response patterns of GDP growth are very similar. Firstly, the SOPI acts as an amplified version of the NOPI as the qualitative magnitudes of response in the IRF graphs for each country respectively are very similar (compare Figures 2.11 and 2.13). The negative responses are also confirmed by the accumulated output as the qualitative results are mainly equal. Only for Belgium and France, we find positive results in the scaled approach but in both cases, the accumulated effects are negligible. Secondly, there is great variety among countries. As an early reaction, GDP growth undergoes negative pressure in Australia, Finland, the Netherlands and the USA, which turns to become positive with the times. Common features in this group can be constituted in a significant lower dependency in oil-imports, except for Finland. However, the latter does not recover notably as the remaining countries. Contrary to that, Belgium, France, Germany, Japan, and Sweden initially react positively but undergo negative pressure in GDP growth from the second year which offsets the previous gains. Hence, these economies have a delay in facing the concrete consequences which follow from an oil price shock.

The response of GDP growth to a negative oil price shock is similar to the results from the respective baseline models. All oil-importing countries experience positive effects except for Belgium, Finland and France. Again, this is argued with the timing of negative price changes and the state of the economy at that time. Only for the USA, we find diverging qualitative outcomes as their accumulated growth rates become negative (1.3% in the linear approach, 3.1% in the scaled approach). This is in line with Jiménez-Rodríguez and Sánchez (2005) whereas the author cannot confirm it for the scaled approach. It should be noted that when lowering its oil-to-energy share, the US economy's positive and negative responses neutralize each other, leading to results resembling those of Jiménez-Rodríguez and Sánchez.

Additionally, the response of the economy to various oil-to-energy shares are calculated. In particular, the $30^{th}$ and $70^{th}$ percentiles of each individual country's oil-to-energy share are used to show their different effects on GDP growth. The results are enclosed in the IRF graphs and can be gathered from Figure 2.8–2.13. Two trends that go along with high or low oil-to-energy shares can be found. Firstly, an oil-importing country experiences a more negative pressure on its GDP growth when it has a higher oil-to-energy ratio. This makes intuitive sense as in such a case the expenditures for its oil-imports increase. However, this also means that the

[13]In another study, Jiménez-Rodríguez and Sánchez (2012) investigate macroeconomic responses of oil price shocks with respect to structural breaks. Among others, they identified breaks in the mid of 1970s and mid of 1990s in interest rate, wage and exchange rate considering a time series from 1970 to 2008. In models controlling for these breaks, they found that the effects oil price changes are less visible in most recent episodes. Our outcomes does not contradicts with their results as the oil-to-energy share can be seen as another variable to incorporate structural changes.

Figure 2.8: Orthogonalized IRF of GDP growth to a one-standard-deviation positive symmetric oil price shock with moderator

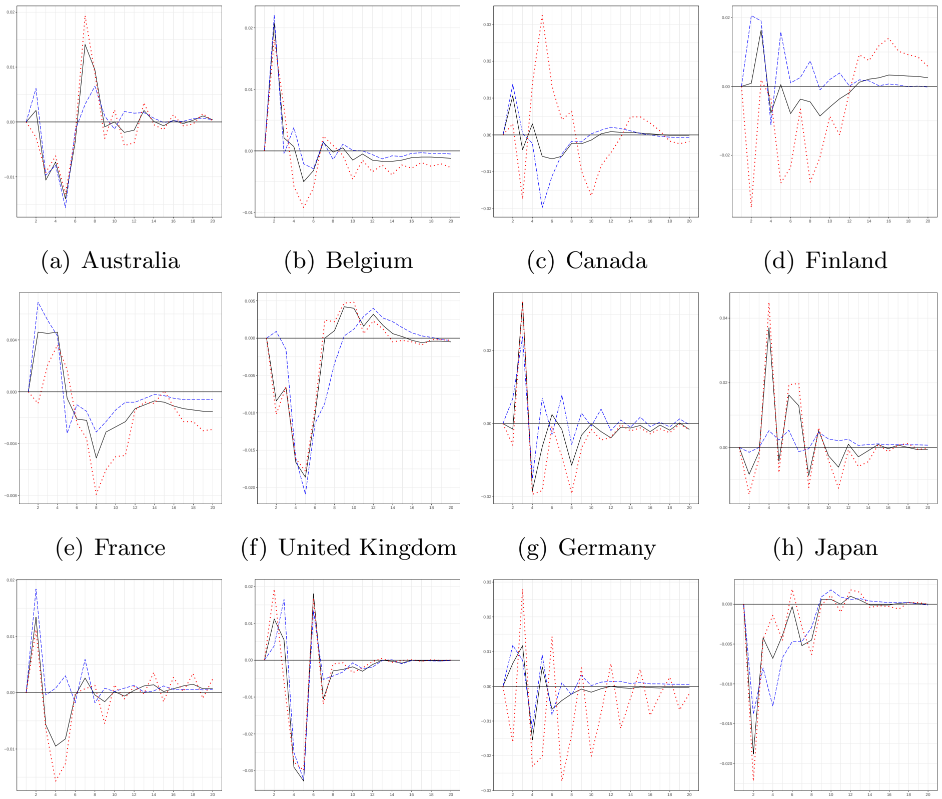


(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Figure 2.9: Orthogonalized IRF of GDP growth to a one-standard-deviation positive asymmetric oil price shock with moderator



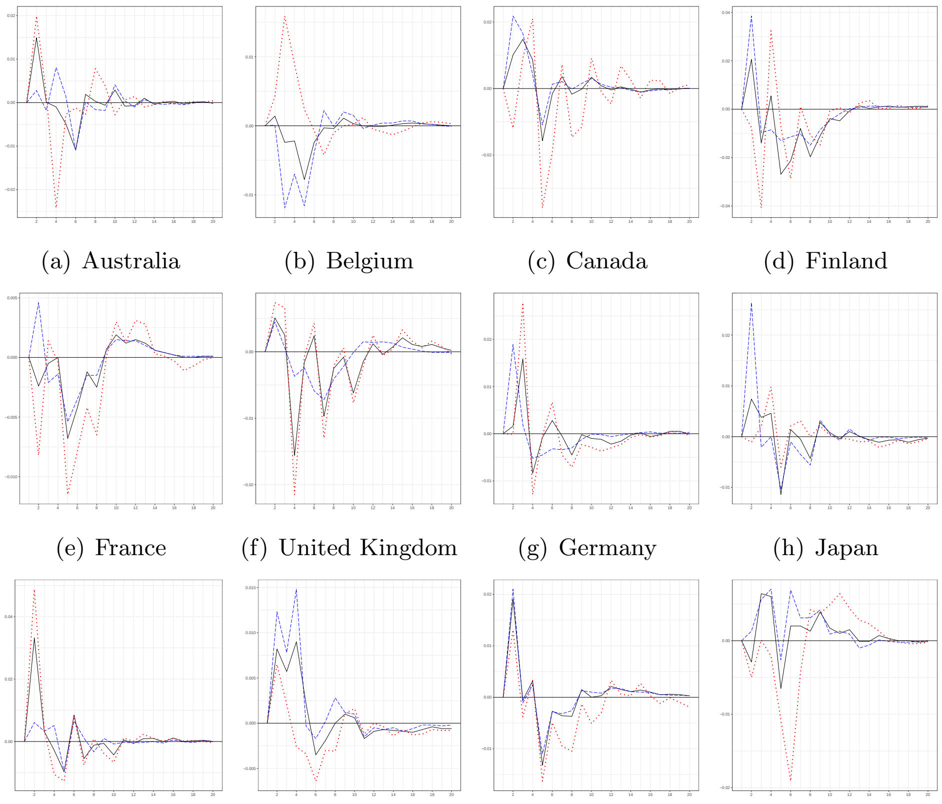
(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Black (solid) line indicates point estimate of GDP growth to a one-standard deviation oil price shock with a consistent oil-to-energy share equal to the country's 50th percentile, blue (sticked) line indicates a consistent 70th percentile of oil-to-energy share, and red (dotted) line indicates a consistent 30th percentile of oil-to-energy share Black (solid) line indicates point estimate, blue (sticked) line indicates 70th percentile of oil-to-energy share, and red (dotted) line indicates 30th percentile of oil-to-energy share.

Figure 2.10: Orthogonalized IRF of GDP growth to a one-standard-deviation negative asymmetric oil price shock with moderator

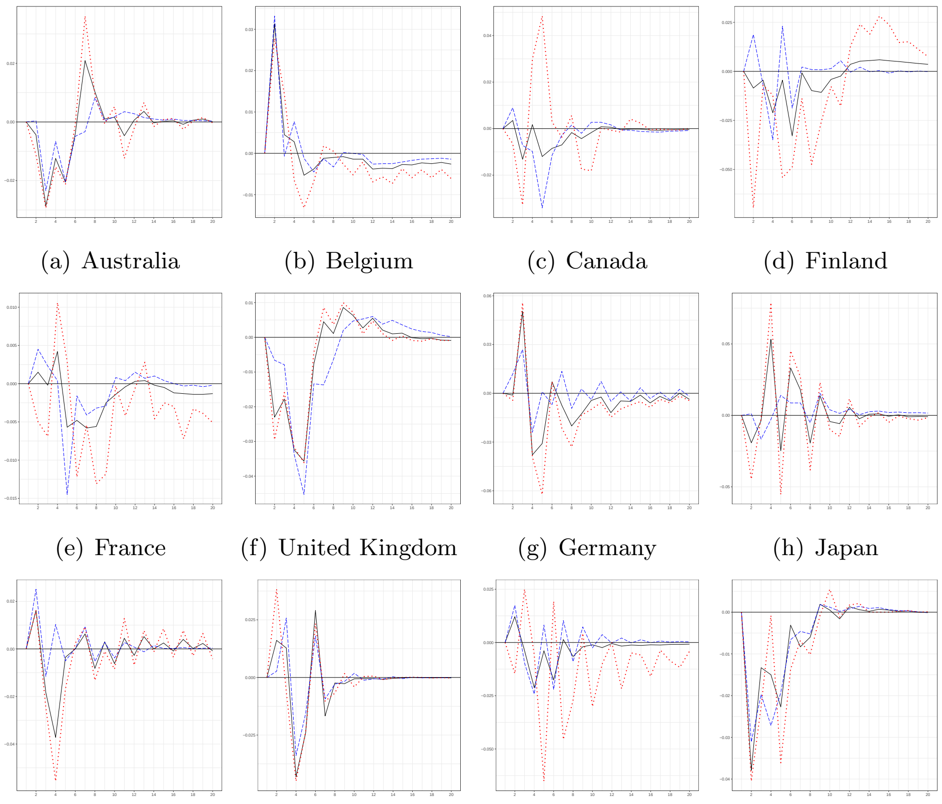


(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Figure 2.11: Orthogonalized IRF of GDP growth to a one-standard-deviation positive scaled oil price shock (SOPI) with moderator



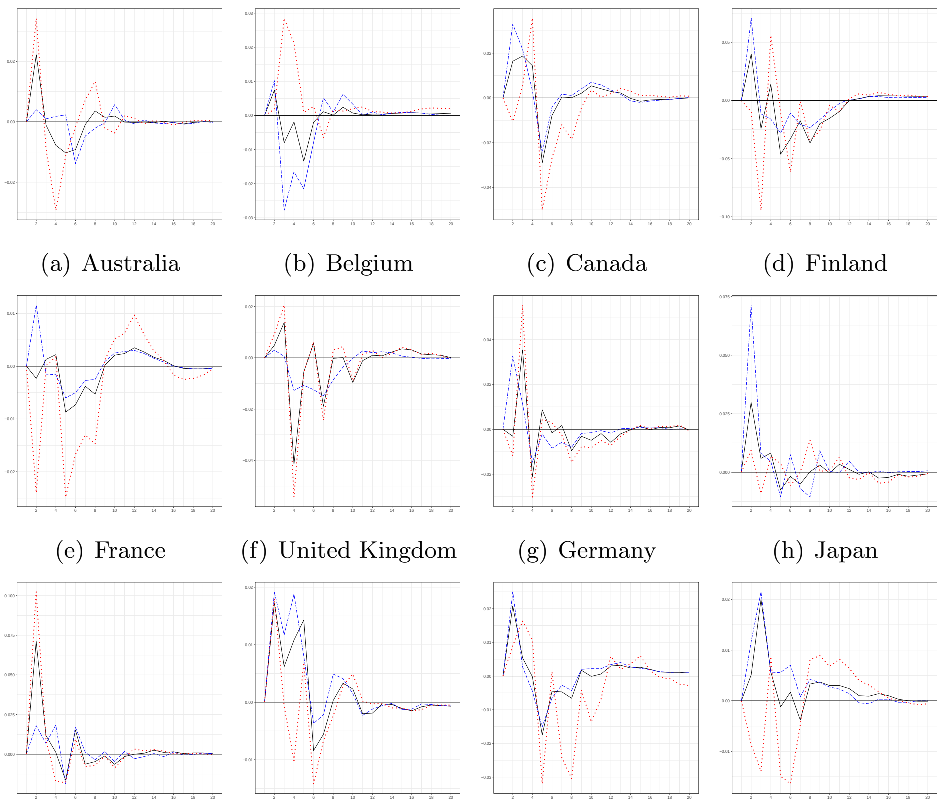
(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Black (solid) line indicates point estimate of GDP growth to a one-standard deviation oil price shock with a consistent oil-to-energy share equal to the country's 50th percentile, blue (sticked) line indicates a consistent 70th percentile of oil-to-energy share, and red (dotted) line indicates a consistent 30th percentile of oil-to-energy share Black (solid) line indicates point estimate, blue (sticked) line indicates 70th percentile of oil-to-energy share, and red (dotted) line indicates 30th percentile of oil-to-energy share.

Figure 2.12: Orthogonalized IRF of GDP growth to a one-standard-deviation negative scaled oil price shock (SOPD) with moderator

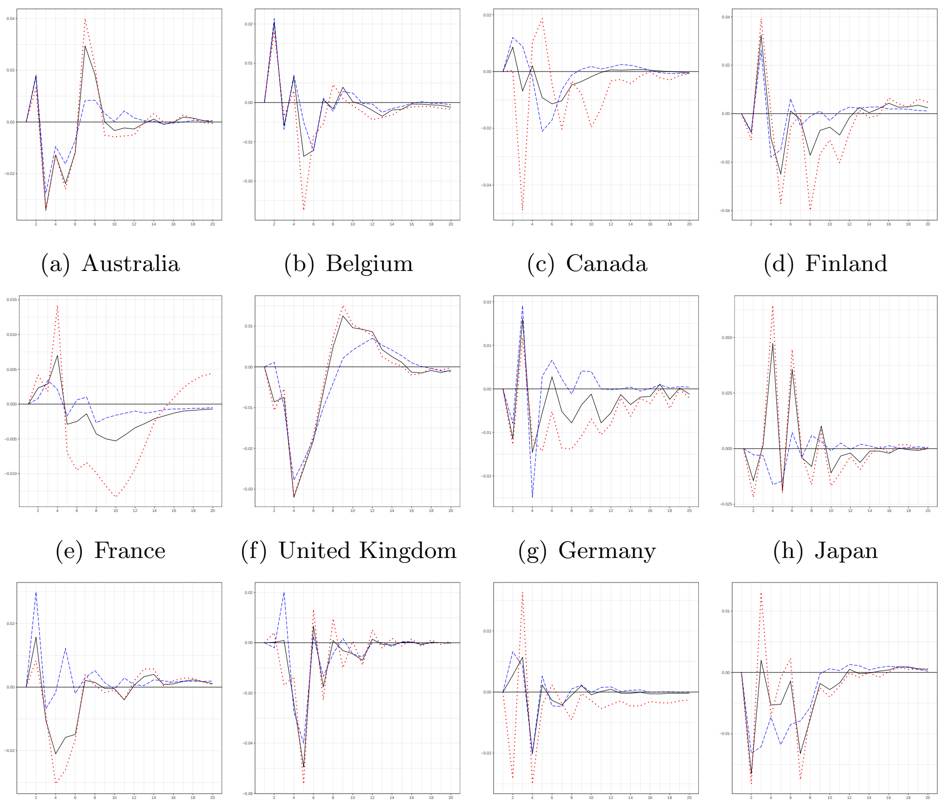


(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Figure 2.13: Orthogonalized IRF of GDP growth to a one-standard-deviation positive net oil price shock (NOPI) with moderator



(a) Australia (b) Belgium (c) Canada (d) Finland

(e) France (f) United Kingdom (g) Germany (h) Japan

(i) Netherlands (j) Norway (k) Sweden (l) United States

Black (solid) line indicates point estimate of GDP growth to a one-standard deviation oil price shock with a consistent oil-to-energy share equal to the country's 50th percentile, blue (sticked) line indicates a consistent 70th percentile of oil-to-energy share, and red (dotted) line indicates a consistent 30th percentile of oil-to-energy share Black (solid) line indicates point estimate, blue (sticked) line indicates 70th percentile of oil-to-energy share, and red (dotted) line indicates 30th percentile of oil-to-energy share.

Table 2.6: Accumulation of price effects with moderators

Symmetric model with shock in $\Delta oil$

| quarters | AUS | BEL | CAN | FIN | FRA | GBR | GER | JPN | NLD | NOR | SWE | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 0.006 | 0.007 | 0.018 | 0.008 | 0.006 | -0.011 | 0.005 | 0.025* | 0.017* | 0.020 | 0.011 | -0.008 |
| 6 | -0.006 | 0.000 | 0.002 | -0.006 | 0.001 | -0.017 | 0.004 | 0.025 | 0.013 | 0.010 | 0.006 | -0.016 |
| 8 | 0.004 | -0.001 | -0.002 | -0.013 | -0.001 | -0.021 | -0.002 | 0.026 | 0.011 | 0.008 | 0.001 | -0.021 |
| 10 | 0.004 | -0.001 | -0.002 | -0.019 | -0.002 | -0.019 | -0.002 | 0.028 | 0.009 | 0.010 | 0.002 | -0.020 |
| 12 | 0.003 | -0.001 | -0.002 | -0.019 | -0.002 | -0.017 | -0.006 | 0.025 | 0.010 | 0.009 | 0.003 | -0.019 |

Asymmetric price model with shock in $\Delta oil^{+}$ and $\Delta oil^{-}$

| quarter | AUS | | BEL | | CAN | | FIN | | FRA | | GBR | | GER | | JPN | | NLD | | NOR | | SWE | | USA | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ | $\Delta oil^{+}$ | $\Delta oil^{-}$ |
| 4 | -0.017 | 0.013 | 0.016 | -0.003 | 0.008 | 0.037*** | 0.008 | 0.010 | 0.018 | -0.002 | -0.035* | -0.006 | 0.009 | 0.008 | 0.025 | 0.016 | -0.002 | 0.033** | -0.011 | 0.024 | 0.002 | 0.023 | -0.026* | 0.011 |
| 6 | -0.036* | -0.002 | 0.010 | -0.010 | -0.002 | 0.018 | 0.002 | -0.031 | 0.015 | -0.011 | -0.067*** | -0.005 | 0.006 | 0.010 | 0.036** | 0.006 | -0.014 | 0.032** | -0.023 | 0.023 | 0.002 | 0.006 | -0.030 | 0.006 |
| 8 | -0.010 | 0.000 | 0.011 | -0.011 | -0.009 | 0.020 | -0.005 | -0.055 | 0.005 | -0.014 | -0.065** | -0.013 | -0.002 | 0.005 | 0.040* | 0.001 | -0.011 | 0.025 | -0.035 | 0.021 | -0.004 | -0.002 | -0.039* | 0.0097 |
| 10 | -0.011 | 0.003 | 0.010 | -0.010 | -0.012 | 0.023 | -0.016 | -0.068* | -0.002 | -0.012 | -0.056** | -0.018 | -0.004 | 0.004 | 0.042 | 0.005 | -0.013 | 0.020 | -0.038 | 0.022 | -0.006 | 0.000 | -0.04 | 0.016 |
| 12 | -0.015 | 0.001 | 0.009 | -0.010 | -0.011 | 0.024 | -0.021 | -0.073* | -0.007 | -0.010 | -0.051** | -0.019 | -0.008 | 0.002 | 0.038 | 0.005 | -0.013 | 0.020 | -0.042 | 0.020 | -0.006 | 0.002 | -0.037* | 0.019 |

Scaled price model with shock in $\Delta SOPI$ and $\Delta SOPD$

| quarter | AUS | | BEL | | CAN | | FIN | | FRA | | GBR | | GER | | JPN | | NLD | | NOR | | SWE | | USA | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD | Δ SOPI | Δ SOPD |
| 4 | -0.061 | 0.014 | 0.033 | -0.002 | -0.008 | 0.064 | -0.045* | 0.024 | 0.007 | 0.001 | -0.099*** | -0.017 | 0.010 | 0.011 | 0.040 | 0.042 | -0.064* | 0.086** | -0.014 | 0.040 | -0.013 | 0.025 | -0.071** | 0.043 |
| 6 | -0.093* | -0.006 | 0.025 | -0.016 | -0.029 | 0.017* | -0.094 | -0.040* | -0.006 | -0.013 | -0.158*** | -0.017 | -0.011 | 0.018 | 0.052 | 0.033 | -0.070* | 0.085** | -0.009 | 0.046 | -0.036 | 0.004 | -0.099** | 0.043 |
| 8 | -0.052 | -0.003 | 0.023 | -0.015 | -0.038 | 0.017* | -0.107 | -0.084* | -0.021 | -0.020 | -0.150*** | -0.031 | -0.034 | 0.010 | 0.050 | 0.028 | -0.073* | 0.074* | -0.028 | 0.040 | -0.041 | -0.007 | -0.114*** | 0.043 |
| 10 | -0.048 | 0.000 | 0.021 | -0.012 | -0.044 | 0.027* | -0.127 | -0.113* | -0.026 | -0.018 | -0.130** | -0.038 | -0.049 | 0.003 | 0.062 | 0.031 | -0.079* | 0.0659 | -0.031 | 0.047 | -0.044 | -0.006 | -0.111*** | 0.052 |
| 12 | -0.054 | 0.000 | 0.017 | -0.011 | -0.042 | 0.036 | -0.125 | -0.120* | -0.026 | -0.013 | -0.119** | -0.038 | -0.061 | -0.005 | 0.062 | 0.035 | -0.076* | 0.064* | -0.032 | 0.042 | -0.047 | -0.002 | -0.112** | 0.059 |

Net price model with shock in $\Delta NOPI$

| quarter | AUS | BEL | CAN | FIN | FRA | GBR | GER | JPN | NLD | NOR | SWE | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | -0.026* | 0.013*** | 0.002 | 0.010 | 0.016 | -0.039* | -0.005 | 0.031* | -0.014 | -0.016 | -0.002 | -0.019 |
| 6 | -0.058*** | -0.003 | -0.011 | -0.006 | 0.009 | -0.073* | -0.006 | 0.046 | -0.042 | -0.045* | -0.002 | -0.025 |
| 8 | -0.015 | -0.004 | -0.020 | -0.019 | 0.001 | -0.074* | -0.013 | 0.035 | -0.039 | -0.056** | -0.005 | -0.045 |
| 10 | -0.018 | -0.001 | -0.023 | -0.027 | -0.012 | -0.056 | -0.015 | 0.034 | -0.040 | -0.062* | -0.005 | -0.049* |
| 12 | -0.023 | -0.003 | -0.023 | -0.034 | -0.022 | -0.042 | -0.022 | 0.030 | -0.043 | -0.065* | -0.004 | -0.050 |

Table includes accumulated response of GDP growth after a one-standard-deviation in oil prices according to the respected model.

higher share amplifies the effect of oil price changes on economic growth. Secondly, there is higher fluctuation in GDP growth the higher the oil-to-energy share. This result is in line with the theoretical assumption that a higher dependency on imports of fossil energy, such as oil, makes a country more vulnerable to price changes. All countries have in common that they experience a prevailingly declining oil-to-energy share over the whole observed time span. In line with our results it can be concluded that the (declining) moderator effect weakens the effect of oil prices on economic growth.

However, the graphical results also show that countries do not respond equally to changes in the oil-to-energy ratio. In case of an oil-price shock, all countries would improve, in terms of GDP, by lowering their oil-dependency. However, while Australia and Belgium would hardly experience any changes, countries such as Finland, France, the Netherlands, and Sweden would face dramatic drops in GDP growth. It is likely that this depends on the respective country's potential to adjust its oil-to-energy ratio, but also on the country's general development so far. For the latter group of countries, the energy shares have varied between 30 (France) and up to 60 percentage points (Sweden). As we set the structural characteristic according to the countries' individual development, this makes it hard to directly compare them quantitatively.

To conclude, the decline of oil-to-energy shares has contributed to a decreased magnitude with which GDP growth reacts to oil price fluctuations. Due to lower (negative) effects, the consequences of uncertainty regarding the short-term development of price changes has also improved. As in the baseline model, oil price increases have a larger magnitude of response to GDP growth compared to the magnitude of response to oil price decreases. In other words, the effects of oil price decreases do not always reflect the mirror image of comparable oil price increases. Consequently, asymmetric frameworks still outperform symmetric ones. We conclude that the change in the energy mix may also be seen as a possible determinant to a changed causal relationship between oil prices and GDP growth.

#### 2.5.4.2 Oil-exporting countries

The results of a F-test to check for the significance of the interaction terms within the functions determining the price coefficients are summarized in Table 2.5. For Canada and the UK, the moderator effect from the oil-to-energy share is consistent and highly significant in all four price models. For Norway, the existence is slightly weaker but still confirmed at the 10% significance level.

Qualitatively, the response of GDP growth to a positive oil price shock does not differ from the baseline model, excluding Norway. The results provide a picture, largely uniform for all countries which are mainly suffering from an increase in prices, even for the case of Norway. Aggregated economic growth responds with instantaneous drops, with Canada's economic response occurring with a short time lag. Overall, the inclusion of the interaction term seems to improve the model by making the results more consistent. However, for an oil-exporting country, the negative response might be surprising, as—theoretically—the terms of trade profit from higher exporting prices. However, Canada and the UK do have something in common concerning their import-export ratio of oil: In 1980, Canada has changed from a former oil importing, into an oil exporting country. Ever since, it has been an importer, even throughout most of the oil crises, making it particularly difficult to classify the effects based on our observations. Moreover, Jiménez-Rodríguez and Sánchez (2005) point out an exchange rate appreciation in the Canadian economy after oil price increases which can justify the negative reaction of GDP growth after 1982 despite exporting crude oil.

The UK has experienced the opposite transformation in the 2000s after having been an oil-producing country since the 1980s. Therefore, it shares structural properties similar to those of oil-importing countries, at least for half of the observed time period, including positive shocks in the 1970s. In contrast to the cases of Canada or the UK, Norway has always been an oil-exporting country, with extraction of oil exceeding domestic consumption multiple times. The unusual response of Norway can be explained with an appreciation of the exchange rate, similar to the case of Canada. Additionally, the Norwegian response can be explained with the development of its oil-to-energy share over the observed time. In contrast to all other countries, Norway has not experienced a steady decrease in its ratio but achieved its bottom of around 30% in early 2000s. Afterwards, the share has increased significantly by more than 13 percentage points. The subsequent phase of oil price increases has taken place simultaneously with low aggregate growth rates due to world recession in 2008. As we calculate the accumulated effects of positive oil price shocks taking the $50^{th}$ percentile, the results might give a distorted picture of the true effects. Therefore, we consider the varying responses by taking different structural characteristics into account.

In case of various oil-to-energy ratios, Canada's and the UK's responses deviate significantly from Norway. Comparing the $50^{th}$ and $70^{th}$ percentiles, higher shares along with oil price shocks do not only impair their economic growth with larger magnitudes but also increase the volatility of GDP growth rates notably. The last reaction particularly applies to Canada. By contrast, Norway's economy is more resilient to changes in its oil-to-energy ratio.

The response of negative oil price shocks on GDP growth is similar as in the baseline model in spite of taking the country's specific oil-to-energy ratio into account. As we can confirm moderation effects, accumulated growth responds positively in Canada and Norway whereas the UK suffers from sharp oil-price declines. However, by analyzing the development of the response, Canada's economy is less robust to price drops compared to Norway which leads to greater negative responses in later periods which even offset the earlier gains in the scaled approach. This contradicts with Jiménez-Rodríguez and Sánchez (2005) who found reverse results for Canada and no significant outcome for Norway.[14] As it can be seen from IRF Figures 2.10 and 2.12, this response is intensified by considering higher oil-to-energy shares. In the case of Canada, larger oil-to-energy shares generate larger magnitudes, especially in the negative range, along with increased variabilities.

To conclude, using the presented models, and thus taking oil-to-energy shares into account when analyzing the response to price changes, does make a considerable difference, especially for the three oil-exporting countries. Overall, accumulated negative effects of oil price increases, as observed in the baseline model, are confirmed for all countries. However, the development and dimension of the same do differ between countries. This is partly due to the historical development in oil-to-energy shares, but also due to the structural alteration of the economies, either from an oil-importing towards an oil-exporting economy (Canada) or due to an economy with two oil-import periods (the UK). Consequently, this group of countries has to be evaluated more sensitively, especially when it comes to cross-country comparisons. Concerning the magnitudes of responses to price increases and decreases, we can confirm the previous findings for oil-importing countries, namely that the price decreases have a smaller effect on aggregated growth.

## 2.6 Conclusion

This paper investigates the role of oil price movements on GDP growth considering four different models of price determination. It shows that oil consuming countries are negatively affected by positive oil price shocks. These results are consistent with the literature. Even by enlarging the sample size by adding new countries to the model and extending the time horizon to 2016, the results remain valid.

Moreover, our results confirm the exclusion of symmetry in the relationship of oil prices and GDP as it has been assumed since the 1980s. However, in contrast to previous studies, the role of decreasing oil prices should not be ignored as we have seen for a few countries. Most strikingly, the paper analyzes the existence of moderator effects caused by a decline in the oil-to-energy share which weakens the causal effect of oil prices on economic growth. In all twelve countries, this moderator is highly significant. We find that oil price increases have a lower effect on GDP growth the lower the oil-to-energy ratio. Hereby, oil-importing countries clearly profit from a decreasing oil-to-energy share whereas oil-exporting countries show a more variable behavior. Furthermore, the response of GDP growth are significantly weakened along with lower oil shares. This result is in line with the theoretical assumption that a higher dependency on imports of fossil energy resources such as oil makes a country more vulnerable to price variations. Since all countries

[14] In their study looking at multivariate correlation between GDP growth and oil-price decreases, Mork et al. (1994) confirm a positive result for Canada and negative outcome for the UK using the same variables.

face declining oil-to-energy ratios, it helps to explain why direct consequences of fluctuations in oil prices on GDP growth have decreased during the past 40 years.

Since this paper only investigates aggregated macroeconomic activities, heterogeneous and detailed changes within the economy such as on the more detailed sector-level are not addressed. Hence, it is probably worthwhile to take a deeper look to sectoral or even firm specific variables to allow for dissimilar developments of energy consumption. A broader analysis helps to control for diverse technical progress on the micro-level which are offset on the macro-level and therefore not visible in our study. Additional insights from more refined models remain on the agenda for further research. Furthermore, the recent development in new technologies to extract shale oil have lowered the dependency on oil imports for some countries, in particular for the USA. Although, this has not been considered due to its short time range, this progress should be recognized in subsequent analysis covering a longer time series. Last, another theory which has not been regarded within this paper is worth to mention. A low oil-to-energy share may also imply a relative advantage within alternative resources and/or technologies. In situations when oil prices are upward moving, this advantage can lead to an increasing demand from countries suffering more from higher prices. This additional stimulus can offset higher costs for fossil energy resources.

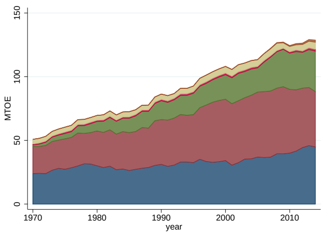
# Appendix

## 2.A Definition of Variables

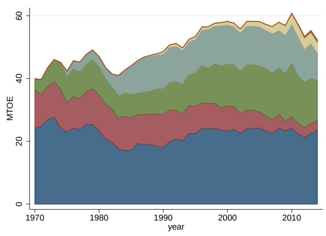
- **Δ GDP growth**: Variable describes the quarterly growth of real Gross Domestic Product (GDP, using expenditure approach) of a country compared to the previous quarter. The data is seasonally adjusted and measured in percentage terms.
  Source: OECD (2012) - Subject B1_GE
- **CPI**: Variable describes quarterly Relative consumer price indices of a country. It is seasonally adjusted and indexed with the base year 2010=100
  Source: OECD (2012) - Subject CCRETT01
- **interest rate**: The variable describes the quarterly short-term interest rates of a country per annum based on 3-months.
  Source: OECD (2012) - Subject IR3TIB
- **unemployment rate**: The variable describes the quarterly Harmonized unemployment rate. It represents the number of unemployed persons as a percentage of the labor force.
  Source: OECD (2012) - Subject LRHUTTTT
- **IPI**: The variable describes the quarterly Industry Producer Index of the G7 countries. It is indexed with the base year 2010=100
  Source: OECD (2012) - Subject INDPROD
- **oil demand relative to GDP**: The variable describes all net oil imports relative to GDP.
  Source: OECD (2012) - Subject OILIMPGDPPPP and TPESGDPPPP
- **PPI**: The variable describes quarterly total producer prices compared to the previous quarter.
  Source: OECD (2012) - Subject PIEAMP01
- **Δ oil price**: The variable describes averaged quarterly growth of oil prices of a country compared to the previous quarter. Nominal costs of OPEC countries crude oil are adjusted by PPI.
  Source: EIA (2015) - FOB Costs of OPEC Countries Crude Oil
- **oil-to-energy share**: The share is measured by the ratio of total oil supply and total primary energy supply which sums up production and imports of energy subtracted by exports and storage changes. It is calculated as the moving average of current the previous three quarters.
  Source: IEA - Subject TPES and OILTPES

## 2.B Additional Figures and Tables

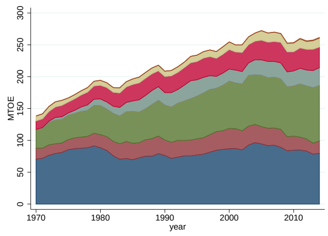
Figure 2.B.1: Country-specific composition of energy products, 1970–2016



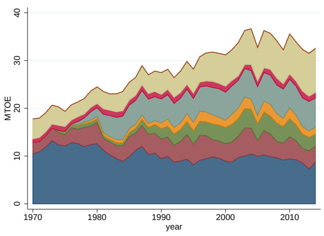
(a) Australia



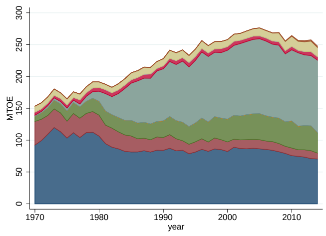
(b) Belgium



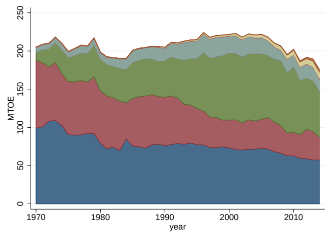
(c) Canada



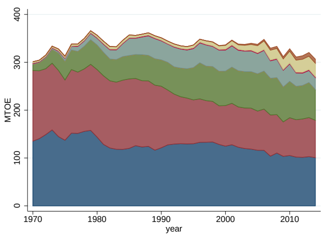
(d) Finland



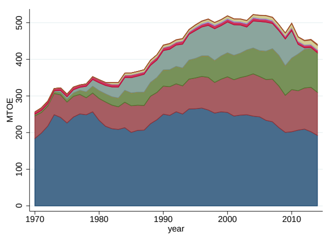
(e) France



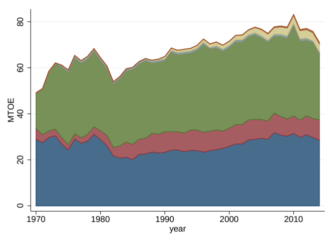
(f) United Kingdom



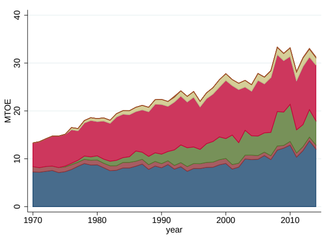
(g) Germany



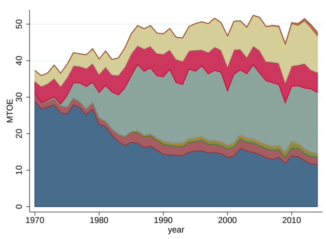
(h) Japan



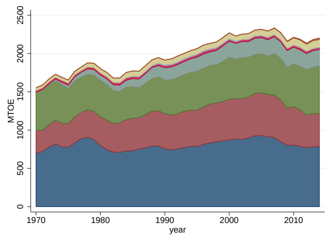
(i) Netherlands



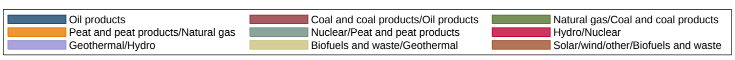
(j) Norway



(k) Sweden

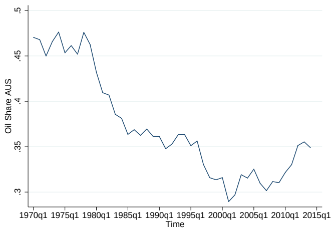


(l) United States

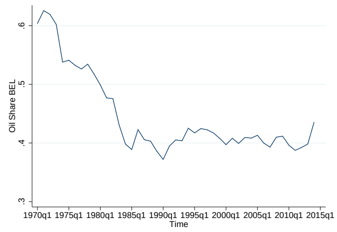



Data describes stacked volumes of energy products of total primary energy supply from IEA (2018). Units are million tonnes of oil equivalent (mtoe)

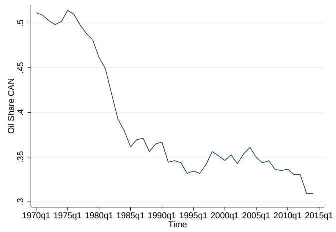
Figure 2.B.2: Country-specific oil-to-energy share, 1970–2016



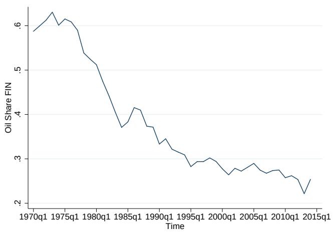

(a) Australia



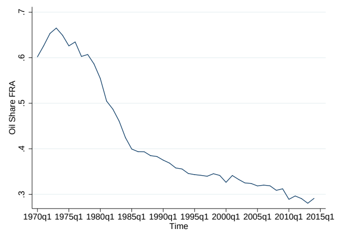

(b) Belgium



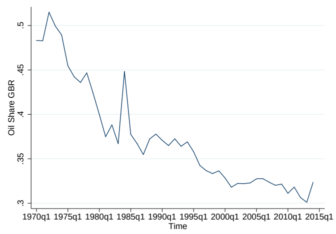

(c) Canada



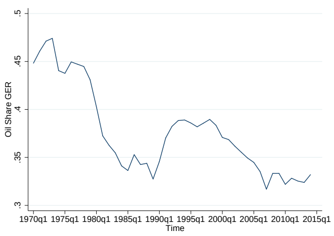

(d) Finland



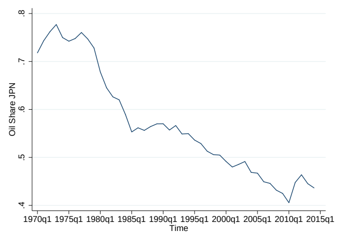

(e) France



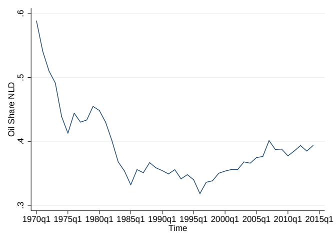

(f) United Kingdom



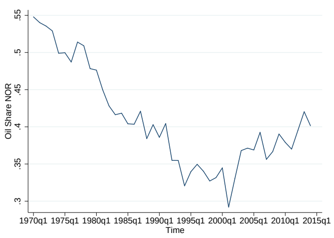

(g) Germany



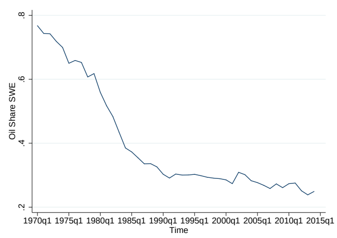

(h) Japan



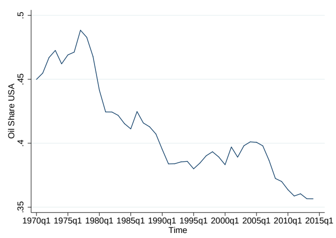

(i) Netherlands




(j) Norway



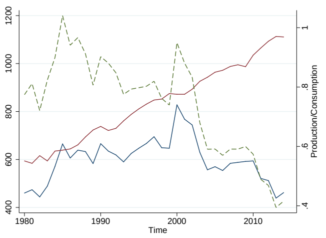

(k) Sweden




(l) United States

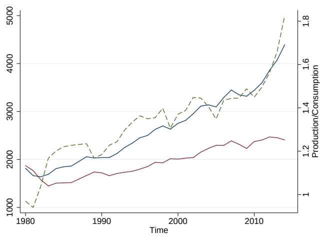
Data consists of ratio of oil supply and total primary energy supply from IEA. In 1984, miner's strike led to a substitution of coal by alternative resources such as oil to ensure security of supply of energy resources. As a consequence, the oil-to-energy share temporary increases from 36% to 44%.

Figure 2.B.3: Oil Production vs. Oil Consumption 1980–2014



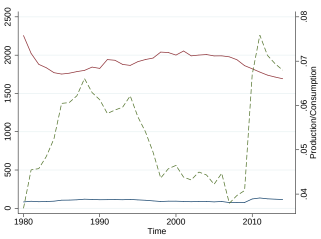

(a) Australia


(b) Belgium

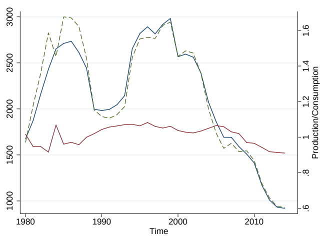



(c) Canada

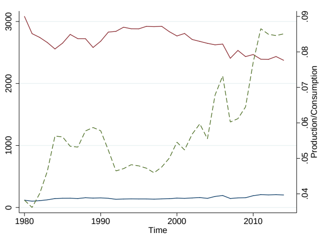

(d) Finland



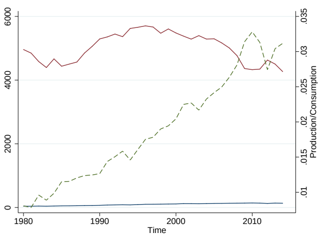

(e) France



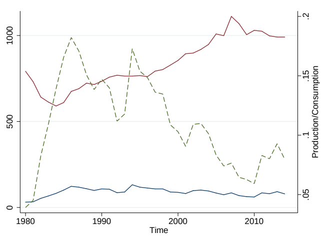

(f) United Kingdom



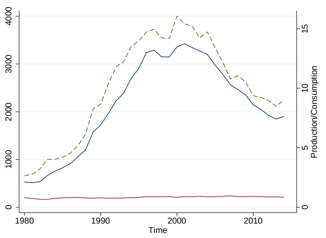

(g) Germany



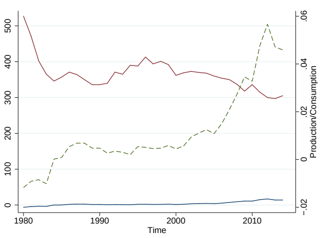

(h) Japan




(i) Netherlands




(j) Norway




(k) Sweden


(l) United States

Data for oil production and oil consumption from IEA (2018).

Table 2.B.1: Unit root test

| | cons&trend | constant | trend | cons&trend | constant | trend |
|---|---|---|---|---|---|---|
| | GDP | | | Δ GDP | | |
| AUS | -1.038 | 4.768*** | 14.748*** | -13.839*** | -13.879*** | -8.909*** |
| BEL | -0.888 | 1.277 | 11.239*** | -8.794*** | -8.559*** | -6.500*** |
| CAN | -1.021 | -0.975 | 4.411*** | -9.225*** | -8.883*** | -6.216*** |
| FIN | -1.021 | -0.975 | 4.411*** | -14.466*** | -14.022*** | -12.007*** |
| FRA | -0.579 | -1.392 | 10.780*** | -8.669*** | -7.964*** | -5.424*** |
| GBR | -1.583 | 1.024 | 8.530*** | -11.050*** | -11.040*** | -8.721*** |
| GER | -2.160 | -0.672 | 6.000*** | -12.274*** | -12.007*** | -9.852*** |
| JPN | -0.262 | -2.504 | 4.713*** | -11.977*** | -10.673*** | -8.827*** |
| NLD | -1.362 | -0.036 | 6.625*** | -15.809*** | -15.608*** | -12.611*** |
| NOR | -2.414 | -0.183 | 6.186*** | -17.291*** | -16.364*** | -11.980*** |
| SWE | -1.443 | 1.330 | 6.156*** | -14.731*** | -14.759*** | -12.198*** |
| USA | -1.870 | 1.399 | 11.203*** | -9.460*** | -9.341*** | -6.633*** |
| | CPI | | | Δ CPI | | |
| AUS | -1.768 | -1.792 | -0.416 | -11.749*** | -11.775*** | -11.808085 |
| BEL | -1.826 | -1.737 | -0.088 | -9.883*** | -9.907*** | -9.933 |
| CAN | -1.598 | -1.585 | -1.054 | -10.143*** | -10.169*** | -10.172 |
| FIN | -1.824 | -0.928 | -0.559 | -9.827*** | -9.824*** | -9.841 |
| FRA | -3.171* | -2.055 | -0.694 | -10.697*** | -10.712*** | -10.724 |
| GBR | -2.204 | -2.221 | -0.263 | -10.583*** | -10.597*** | -10.618 |
| GER | -2.727 | -1.846 | -0.385 | -10.800*** | -10.780*** | -10.808 |
| JPN | -1.279 | -2.036 | -0.059 | -10.504*** | -10.301*** | -10.285 |
| NLD | -3.030 | -2.878* | 0.231 | -10.919*** | -10.878*** | -10.899 |
| NOR | -2.417 | -2.367 | -0.367 | -11.611*** | -11.612*** | -11.645 |
| SWE | -2.905 | -1.439 | -1.532 | -10.653*** | -10.681*** | -10.611 |
| USA | -2.056 | -2.391 | -0.783 | -9.961*** | -9.926*** | -9.954 |
| | interest | | | Δ interest | | |
| AUS | -2.772 | -1.757 | -1.073 | -10.484*** | -10.422*** | -10.411*** |
| BEL | -2.602 | -1.099 | -1.048 | -6.631*** | -6.306*** | -6.203*** |
| CAN | -2.868 | -1.279 | -0.971 | -8.865*** | -8.821*** | -8.824*** |
| FIN | -3.161* | -0.295 | -1.068 | -4.842*** | -4.578*** | -4.378*** |
| FRA | -2.845 | -0.936 | -0.959 | -6.565*** | -6.210*** | -6.097*** |
| GBR | -2.886 | -1.175 | -1.005 | -8.461*** | -8.319*** | -8.265*** |
| GER | -2.390 | -1.401 | -1.356 | -6.448*** | -6.212*** | -6.101*** |
| JPN | -2.136 | -1.171 | -1.469 | -9.188*** | -9.166*** | -9.046*** |
| NLD | -3.669** | -2.277 | -1.493 | -10.561*** | -10.338*** | -10.260*** |
| NOR | -3.632** | -1.990 | -1.170 | -13.923*** | -13.823*** | -13.824*** |
| SWE | -2.061 | -0.848 | -1.067 | -4.406*** | -4.293*** | -4.184*** |
| USA | -3.039 | -1.482 | -1.086 | -8.946*** | -8.943*** | -8.958*** |
| | oil price | | | Δ oil price | | |
| -1.133 | -1.651 | -1.067 | -10.831*** | -10.791*** | -10.710*** | |
| -1.183 | -1.671 | -1.070 | -10.350*** | -10.359*** | -10.336*** | |
| -1.290 | -1.742 | -1.104 | -10.405*** | -10.378*** | -10.312*** | |
| -1.143 | -1.629 | -1.083 | -10.492*** | -10.482*** | -10.427*** | |
| -1.165 | -1.665 | -1.075 | -10.252*** | -10.247*** | -10.206*** | |
| -1.024 | -1.540 | -1.064 | -10.199*** | -10.184*** | -10.103*** | |
| -1.212 | -1.676 | -1.063 | -10.349*** | -10.370*** | -10.363*** | |
| -1.741 | -2.010 | -1.191 | -10.217*** | -10.237*** | -10.249*** | |
| -1.204 | -1.678 | -1.063 | -10.368*** | -10.385*** | -10.373*** | |
| -1.134 | -1.634 | -1.072 | -10.622*** | -10.625*** | -10.567*** | |
| -1.080 | -1.576 | -1.076 | -10.606*** | -10.596*** | -10.517*** | |
| -2.012 | -1.757 | -0.912 | -10.226*** | -10.213*** | -10.178*** | |

| cons&trend | constant | trend | cons&trend | constant | trend |
|---|---|---|---|---|---|
| unemployment | | | Δ unemployment | | |
| -2.089 | -2.244 | 0.194 | -8.196*** | -8.006*** | -7.971*** |
| -2.489 | -2.933** | 0.662 | -11.622*** | -11.170*** | -11.077*** |
| -1.656 | -1.525 | -0.151 | -7.765*** | -7.748*** | -7.769*** |
| -0.695 | -1.358 | 0.176 | -6.986*** | -6.972*** | -6.962*** |
| -1.627 | -1.936 | 1.247 | -7.286*** | -6.973*** | -6.764*** |
| -1.332 | -1.505 | 0.028 | -5.341*** | -5.304*** | -5.332*** |
| -1.458 | -2.193 | -0.345 | -15.400*** | -15.059*** | -15.061*** |
| -0.040 | -1.680 | 0.553 | -11.402*** | -11.055*** | -10.942*** |
| -2.224 | -2.230 | 0.610 | -8.152*** | -8.009*** | -7.969*** |
| -1.391 | -1.380 | 0.218 | -14.365*** | -14.387*** | -14.375*** |
| -1.114 | -0.725 | 0.792 | -9.951*** | -10.006*** | -10.009*** |
| -1.437 | -1.379 | -0.516 | -6.152*** | -6.152*** | -6.1645*** |
| IPI | | | Δ IPI | | |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| -1.321 | -1.541 | 2.886* | -6.383*** | -6.321*** | -6.059*** |
| Oil / GDP | | | Δ Oil / GDP | | |
| -2.064 | -2.311 | -1.736* | -13.159*** | -13.144*** | -13.179*** |
| -2.467 | -2.237 | -2.695*** | -13.725*** | -13.762*** | -13.496*** |
| -2.499 | -1.005 | 0.238 | -5.716*** | -5.831*** | -5.812*** |
| -2.466 | -1.574 | -2.453** | -13.776*** | -13.813*** | -13.526*** |
| -1.745 | -1.507 | -2.786*** | -14.175*** | -14.213*** | -13.493*** |
| -1.841 | -3.107** | -3.297*** | -9.223*** | -8.994*** | -8.944*** |
| -1.897 | -1.325 | -3.206*** | -14.190*** | -14.221*** | -13.494*** |
| -1.595 | -1.083 | -3.267*** | -14.120*** | -14.082*** | -13.520*** |
| -2.085 | -1.735 | -2.185** | -13.813*** | -13.852*** | -13.617*** |
| 0.432 | -1.631 | -0.273 | -5.057*** | -5.210*** | -4.635*** |
| -1.986 | -1.884 | -3.393*** | -13.938*** | -13.938*** | -13.513*** |
| -1.994 | -0.500 | -0.773 | -14.039*** | -13.699*** | -13.644*** |
| oil-to-energy share | | | Δ oil-to-energy share | | |
| -1.476 | -1.597 | -1.804* | -13.641*** | -13.624*** | -13.494*** |
| -2.197 | -3.053** | -2.265** | -13.980*** | -13.774*** | -13.609*** |
| -0.821 | -2.016 | -2.223** | -13.830*** | -13.666*** | -13.495*** |
| -0.933 | -1.687 | -2.890*** | -13.859*** | -13.828*** | -13.510*** |
| -0.662 | -1.671 | -3.622*** | -14.623*** | -14.575*** | -13.685*** |
| -2.421 | -2.014 | -1.311 | -13.570*** | -13.529*** | -13.491*** |
| -1.612 | -1.675 | -1.847* | -13.751*** | -13.753*** | -13.599*** |
| -1.457 | -1.057 | -2.923*** | -14.137*** | -14.174*** | -13.667*** |
| -2.652 | -3.501*** | -1.786* | -14.445*** | -14.143*** | -14.074*** |
| -2.130 | -1.852 | -1.657* | -13.544*** | -13.571*** | -13.498*** |
| -1.147 | -2.782* | -4.302*** | -14.424*** | -14.264*** | -13.541*** |
| -1.736 | -0.936 | -1.571 | -13.633*** | -13.670*** | -13.531*** |

Dickey-Fuller test to check whether a variable of interest follows a unit-root process. The null-hypothesis is that the variable contains a unit-root.

Table shows Dickey-Fuller test statistics. Asterisks ***, **, and * denote significance at 1%, 5%, and 10%.

# Chapter 3

# Durable Goods and Energy in RBC: An Endogenous Multisectoral Model

## 3.1 Introduction

In this paper, we describe a real business cycle (RBC) model, into which we add the endogenous generation of energy from different resources. We integrate energy into both the production process, in the form of a further input factor, and into the household's utility function, in the form of an additional consumption good. This allows us to examine transmission channels of how energy, and also its underlying resources, affect the economy.

A stable supply of energy is essential to ensure a durable and substantial economic development, and to guarantee long-run welfare. Real-life examples where energy supply has been unstable, due to a weak energy infrastructure and energy sector, are known from developing countries, where economies struggle to flourish. Such production sectors of struggling economies are often unable to produce sufficient final goods, either to be consumed or exported, and consequently they are unable to participate in global growth. At the same time, events such as the oil crises in the 1970s and 1980s show that developed countries are not immune to similar problems. Due to the negative consequences that accompanied these crises, many countries started moving towards alternative ways of organizing their energy sector and energy usage, in order to minimize the dependence on singular energy resources, and thus to minimize the risk. A more efficient usage of energy, at the same time beneficial for productivity in general, is just one way to achieve this. Another way is to substitute finite energy resources by alternative and locally produced resources. Macroeconomic models, in particular those investigating business cycles in the short term, have either focused on one single energy resource or have not taken energy into account at all. Those models that do consider energy in the production process propose that shocks in the supply or price formation of energy resources are exogenous.

Apart from this, as will be covered later there is a strong consensus in existing literature, that despite considering energy inputs, total factor productivity is still the major driver of output volatility. However, the former can definitely have effects on the economy through various transmission channels such as the reallocation of resources or disruptive spending on consumption goods (Bernanke et al., 1997; Hamilton, 2008; Herrera, 2018; Kilian, 2008).

The aim of this paper is to deepen existing research by allowing for distinguishing between finite and renewable energy resources in the energy sector. On the one hand, this paper investigates the effects of a stochastic technological progress on the production side, in particular in the energy sector, as the price formation of energy and intermediate energy resources are determined endogenously. On the other hand, this paper considers additional transmission channels, through which macroeconomic variables other than output are affected. In doing so, we assume a complementary link within the bundle of durable goods and energy in the household utility function, as well as within the bundle of capital and energy in the final goods production function. Studying the dynamics within a model calibrated and estimated to match the German economy, we investigate whether this complementary relationship can be confirmed by Bayesian estimation.

To this end, we construct an RBC model of a closed economy with three main sectors: households, final (non-durable) good production, and energy producers. Energy, as a further input good, is consumed by households and used in the production process of final goods. Furthermore, we distinguish between two types of consumable goods: durable goods and non-durable goods, where the former can only be used in combination with energy (the same holds for capital in the production function of final goods). By doing so, we can examine additional transmission channels of energy shocks affecting households' expenditures for different consumption goods. This is motivated by Dhawan and Jeske (2008) who analyze the role of durable consumption goods in a business cycle of the US economy. They model energy as an exogenous variable with a price that is stochastically affected by shocks, and that explicitly enters the model in the utility as well as the production function. In contrast, in the present paper energy is not only endogenized but also generated from a combination of different resources, namely an infinite one and a finite one. Our model extension allows for a transition from non-renewable resources to backstopping resources, caused by a change in relative marginal costs.

In a further extension of the model we allow for a constrained replenishment of the finite resource stock. As shown by Gross et al. (2013), investments in R&D transform resources, which are not accessible with previously available technology, to reserves, which are available as an input factor to produce intermediate energy. But other than done by Gross et al., in our model the capital and R&D are completely supplied by domestic households and the price of intermediate energy is generated by the non-renewable energy sector and determined endogenously. This way, we investigate how dynamics of TFP shocks deviate from the benchmark model in the case of depletion and exploration.

The results of this study show that in our RBC model, Bayesian estimation confirms the complementary relationship between durable goods and energy consumption in the household sector, as well as between physical capital and energy consumption in the final good sector. Furthermore, a TFP shock in the (final and intermediate) energy sectors has a larger effect on durable good purchases than on capital investments in the final good production, which is in line with results from Dhawan and Jeske (2008). Nevertheless, even in the model at hand, with endogenous price determination of energy, TFP in final good production is still the major contributor to the business cycle formation of the national account, which confirms evidence from previous literature (e.g. Kim and Loungani, 1992). Furthermore, this paper provides an essential improvement to explaining theoretical moments by distinguishing between durable and non-durable goods, taking energy consumption into account. Moreover, we regard an extension of our baseline model where we allow for replenishing the constrained fossil stock, but do not find the dynamic responses of the variables to significantly deviate from those observed in our baseline model. Only for goods consumption do we find a slight increase in volatility, resulting from costly R&D, which raises household income.

The paper is organized as follows. In Section 2, we give an overview of existing research dealing with energy and energy-related resources in business cycle models. In Section 3, we present the model and derive the equilibrium in Section 4. In Section 5, we discuss the calibration of parameters, the estimation methods which will be applied, as well as the results of the estimated parameters. In Section 6, we discuss the numerical results, and the accuracy with which they match the data. Section 7 analyzes the dynamic results of the model caused by TFP shocks in the production functions and shocks in the finite reserve/resource stock. In Section 8, we look at the weighting of all individual shocks by performing a variance decomposition. Subsequently, we carry out robustness checks to verify our results. Section 10 concludes.

## 3.2 Energy and Resources in Macroeconomic Models

The amount of literature dealing with the role of energy and similar resources in a theoretical framework is quite extensive. Moreover, the term ”energy” is taken quite vaguely by often specifically describing oil as a finite resource. In general, economists analyze the effects of energy in macroeconomic models through various transmission channels that present the reciprocal relations of energy and other macroeconomic components supported by evidence from literature (Bernanke et al., 1997; Herrera, 2018; Kilian, 2008). In earlier studies, energy has been mainly present on the supply side. However, its degree of importance is differently valued. In the course of time, two strings of theories have been established with contradictory views about the effect and use of energy in the macroeconomic environment.

On the one side, supporters of the "conservation hypothesis" take the view that energy can simply be substituted for alternative input factors. Moreover, technological progress can ease this process and leaves energy as a non-essential component. Hence, energy scarcity would not have negative effects on the economy. This allows economic growth even in the presence of a scarce energy resource where non-finite alternatives act as possible substitutes (Solow, 1974; Stiglitz, 1974; Tobin et al., 1980).

On the other side, the "growth hypothesis" promotes scarce resources as the limiting factor for economic growth due to its binding supply constraint. Considering finite energy as the primary resource in production, this theory is particularly supported by ecological economists (Stern, 2011). Possible substitutes such as capital and labor cannot fully take effect in the production process without energy. Consequently, the latter constitutes a complementary product. In the present paper, when considering the short term, we follow this theory as a possibility of substitution by other components which is constrained by time. To be more precise, investments are needed to enforce these strategy changes. The conservation hypothesis is not completely neglected. The reason is that we allow for different types of energy resources, finite and renewable, and consequently some degree of substitution between them.

Frequent literature that analyzes the theoretical relationship of energy with other macroeconomic variables often include RBC models. In principle, these models investigate the external influence through shocks on the modeled economy and decompose the effects on its variables. But despite the popularity of RBC models that stems from its close to real-life predictions, the role of aggregate technology shocks is controversial. Several researchers such as Plosser (1989) and McCallum (1988) have agreed that some of the facts that characterize economic variations are successfully explained by RBC models. However, it remains a constraint that a number of important issues, such as shocks, that should explain variations in the business cycle have stayed unsolved, or that evidence for them is too fragile to be credible. One criticism is the role of the Solow residual which is often identified as the main source of aggregate fluctuations in the model. On the one side, the nature of technological shocks often remains open. On the other side, the Solow residual includes unexplained behaviors such as energy price shocks that are not necessarily linked to productivity, which leads to overestimation of the productivity factor.

In this context, McCallum (1988) has identified energy as an essential factor on the supply side which contributes to fluctuation to which less attention is paid. In one of the subsequent papers, Kim and Loungani (1992) analyze an RBC model with respect to exogenous energy price changes. By implementing energy in the production function as a further independent input factor, next to the usual inputs such as capital and labor, this allows to extend the source of possible fluctuations affecting total output. In their model, the relative price of energy is modeled as an exogenous stochastic process. All structural parameters are based on the US economy and are chosen in line with microeconomic evidence and certain historical averages. However, the results are in line with those of macro-economists who neglect the impact of shocks by energy factors on an economy. TFP is still the main driver of output volatility while objections such as those by Tobin et al. (1980) who noted that the share of energy in the GNP is too small to generate strong aggregate impacts are confirmed. On the one side, this leads Kim and Loungani to assume prices and wages to be perfectly flexible, which is contrary to empirical studies that derive strong impacts of energy on real variables due to the implementation of some degree of rigidity in prices and wages (Black, 1985; Mork and Hall, 1980). On the other side, energy prices are completely exogenously determined and moreover exclusively affecting the production side. In the present paper, the latter assumption is changed by allowing for energy production determined from within the model and used by the production and consumer side.

To meet the critics, researchers have considered different approaches to implement energy in RBC models. These can be segmented into RBC models following the classical approach and New-Keynesian models with classical market failures. Along with the former, Finn (1995) allowed energy price shocks to affect capital utilization, a method which has been taken over by several subsequent studies (i.e. Leduc and Sill, 2004; Sánchez, 2011). The idea is that, because energy is dependent on capital utilization and necessary for the usage of physical capital, it enters the production function indirectly. Just as Kim and Loungani (1992), Finn's model assumes perfect competition in the production sector. Along with some other modifications, this results in a model which explains 76% to 89% of US output volatility. Both, Kim and Loungani, and Finn conclude that shocks in energy prices account for up to 20% of the aggregate fluctuations in the business cycle.

A further remark made by several economists concerns a possible reallocative effect of energy shocks (Hamilton, 1983; Loungani, 1986; Mork, 1989). Assuming a multi-sector economy, changes in energy prices can induce individual producers to reallocate other input factors across sectors in a costly manner. Consequently, energy price shocks may have an indirect effect on the macroeconomy through other factors, e.g. labor supply. Shocks in energy prices impact substitution of energy with other input components affecting the marginal cost of production. In particular, substitution by capital can influence investment behavior which eventually leads to long-run consequences (Amin and Ferdaus, 2015). In our paper, we consider different sources of energy. Hence, reallocation can even take place within the same input factor that is substituted by an alternative.

Specifically talking about oil, Rotemberg and Woodford (1996) have developed a model that is similar to that analyzed by Kim and Loungani (1992), which uses this resource as an independent input factor. However, in contrast to the present paper, the price determination process is still exogenously determined. In their analysis, Rotemberg and Woodford find that the predicted aggregate effects of a change in oil prices improves significantly by allowing for a modest degree of imperfect competition. Consequently, in Rotemberg and Woodfords' model, they consider an environment with imperfectly competitive elements rather than a perfectly competitive market. These modifications make it possible to introduce mark-ups in prices. Furthermore, the authors argue that an oil price shock could amplify macroeconomic effects by affecting the costs of production. Since the producer faces changes in costs he is likely to adjust his prices by changing the mark-up of what he is selling. Although considering a model with perfect competition only and ignoring mark-ups, we also find some pass-through effects of costs in the energy sector as higher costs are added up to the selling price in the present model.

Researchers following the New-Keynesian approach within dynamic stochastic general equilibrium (DSGE) models generally assume that shocks are independent of each other. However, several economists have questioned the direct influence of energy shocks to the aggregate output. Leduc and Sill (2004) investigate whether recessionary consequences of an oil-price shock are caused by the shock itself or rather by monetary policy responses to the shock, as it has been argued by Bernanke et al. (1997). They find about 40% of the output drop which stems from monetary policy intervention. However, these interventions could not be offset by the negative consequences of an oil shock to the aggregate outcome. Sánchez (2011) is one of the first economists who has introduced oil in a model that was based on the Euro area countries. In doing so he implemented oil into an European economic model following the idea by Finn (1995) of linking the required value of oil to the capital utilization rate. By using a standard DSGE model, he demonstrated that gains in oil usage efficiency lead to an alleviation of inflationary and contractionary consequences when an oil shock affects the economy. In addition, he concluded that a higher degree of flexibility in wages can help ease the impact on output, even though this comes at the expense of larger inflationary pressure. These results are confirmed by Jacquinot et al. (2009) within a open country model.

In the present paper, we take over the approach according to the neoclassical approach looking at real variables rather than distinguishing between nominal and real values as it is done in New-Keynesian model. The aim is to concentrate on the origin of business cycles by allowing for several energy sectors rather than restricting different channels though rigidities or imperfect competition. Hereafter, we concentrate on the classical approach to point out the occurrence of business cycles through the interaction of several input factors and their relationships rather than market failures.

All the models described so far are dealing with energy in a very general context. As either the variable itself or the price determining process are exogenously shaped, next to having only one variable with no more other variables, further detailed properties could be neglected by dealing with the remaining dynamics of the model. However, this goes along with less precise description of what causes these exogenous effects. An input factor such as oil is constrained by being a finite resource and hence, behaving differently to labor, capital, or even a renewable resource. Literature that investigates optimal depletion of finite resources includes Bohn and Deacon (2000) and Gross et al. (2013). By integrating a separate fossil sector within the models, it allows to analyze its influence on the economy. Bohn and Deacon and Gross et al. even go further by endogenizing the stock of the natural resource rather than holding it constant. Firms are allowed to augment existing reserves through exploration which has been previously ignored. However, the price determination process of this resource is still exogenously determined. They find that endogenous reserves have a quite significant effect on the magnitude and persistence of the remaining variables' response to price shocks. In the present paper, we compare both types of stocks but fully endogenize the price-setting.

RBC literature that covers resources with different properties, namely finite ones and renewable ones, is limited. Argentiero et al. (2018) analyze the effects of environmental taxation policy in a model with both resources for China, Europe, and the USA. However, opposite to the present paper, the household sector is much simplified without consumption of energy. Furthermore, the model considers substitution between energy and capital/labor within the production function while we allow for a complementary relationship between energy and capital. While the response of a shock in final good sector's TFP does not distinguish from ours, the results for the dynamics of the remaining shocks do.

Although considering energy as a general and exogenous given variable, Dhawan and Jeske (2008) analyze its role in the household and production sectors. Furthermore, they distinguish between durable and non-durable consumption goods. Assuming a complementary relationship with capital in the production function and durable goods in the household sector they find significant improvements in explaining business cycles. In contrast to Kim and Loungani (1992), disruption in fixed capital investments comes closer to the one observed in the data as the households have an additional channel of adjustments in investments through durable goods. As pointed out by Bernanke et al. (2004), changes in the energy price can induce households to postpone irreversible purchases of durable goods. However, they also find that major impacts causing output fluctuations are still due to productivity shocks. The present paper is based on the work by Dhawan and Jeske by considering multiple margins of investment but endogenizing energy. In doing so, we distinguish between several energy sectors and consequently allow for different properties in energy resources. As a byproduct, this also allows to implement sectoral productivity to consider disaggregated TFP coming from product and process innovation or further fundamental productivity changes (Caliendo et al., 2017). Like Argentiero et al. (2018), Huynh (2016) analyzes a multisectoral model by endogenizing the production process of durable goods and energy, which brings energy volatility closer to its empirical target values. But in contrast to the present paper, Huynh neither allows for energy consumption by households, nor does he distinguish between various resources to generate energy from.

## 3.3 Model

The model consists of three main sectors: Households, final goods-producing firms and energy producing firms. In addition, the latter is divided into three sub-sectors: a general energy sector, a fossil resource sector, and a renewable resource sector. We do not include labor in the production function of the resource sectors as our focus lies on the dynamic change of the capital and reserve stock. In the following, we will describe each sector in more detail. Figure 3.A.1 in the Appendix depicts a graphical description of the model.

### 3.3.1 Households

Households maximize their utility by choosing the optimized demand of non-durable $CN$ and durable consumption goods $CD$, demand of energy $E_{\mathrm{H}}$, supply of labor $L$, and capital which they accumulate through investments $K_{\mathrm{Y}}$,$K_{\mathrm{F}}$,$K_{\mathrm{N}}$.[1] The utility function is assumed to have constant elasticity of substitution (CES) between durable goods and energy which are nested within a Cobb-Douglas function with non-durable goods. Additive-separable disutility is gained from labor supply. The utility aggregation of households follows:

$$U_t = \vartheta \ln \left[ CN_t^{\gamma} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)\,(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}} \right] + (1-\vartheta)\ln\left[1 - L_t\right] \tag{3.1}$$

where $\theta \in (0,1)$ indicates the share of durable consumption good and $\vartheta \in (0,1)$ indicates the share of consumption. As $\gamma \in (0,1)$, non-durable goods and a common basket of durable goods and energy are substitutes while $\zeta < 0$ implies a complementary relationship between durable goods and energy consumed by households.[2] Empirical observations show that the elasticity of substitution between non-durable and durable goods are close to unity (Fernandez-Villaverde and Krueger, 2011; Greenwood et al., 1995; Ogaki and Reinhart, 1998). Hence, we assume of Cobb-Douglas function between non-durables and the complementary composite basket, similar to Dhawan and Jeske (2008). Moreover, by analyzing business cycle fluctuations,

[1] Hereafter, we omit the time index when describing variables.

[2] Note that $\frac{1}{1-\zeta}$ is the elasticity of substitution between capital and energy which determines the degree of substitutability of both.

Bilbiie and Straub (2013) emphasize to check whether overall concavity is fulfilled when using an utility function with non-separable preferences.

**Lemma 1.** *Strict concavity of utility*
*The partial derivatives for the utility function $U_t$ are:*

$$U'_{CN} > 0, U'_{CD} > 0, U'_{E_{\mathrm{H}}} > 0, U'_{L} < 0$$
$$U''_{CN\,CN} < 0, U''_{CD\,CD} < 0, U''_{E_{\mathrm{H}}E_{\mathrm{H}}} < 0$$
$$U''_{LL} < 0, U''_{CN\,CD} = U''_{CD\,CN} > 0, U''_{CD\,E_{\mathrm{H}}} = U''_{E_{\mathrm{H}}\,CD} > 0, U''_{CN\,E_{\mathrm{H}}} = U''_{E_{\mathrm{H}}\,CN} > 0.$$

*Utility function $U_t$ is overall strictly concave in $CN, CD, E_{\mathrm{H}}, L > 0$ iff all the following conditions hold:*

$$0 < \vartheta, \gamma < 1$$
$$\zeta, \theta < 1.$$

*Proof: See Appendix 3.H.1.*

According to Lemma 1, utility increases with consumption of non-durable and durable goods as well as energy, but at a decreasing rate. Energy can be considered to be consumed to enhance the consumption of durable goods in a non-perfect substitutable manner. Alternatively, the presence of energy is required to consume durable goods. On the contrary, the supply of labor diminishes households' utility. Furthermore, overall concavity of utility function $U$ is guaranteed if Lemma 1 holds. Households are restricted by a budget constraint given by:

$$\begin{aligned} CN_t + (p_{\mathrm{H}})_t(E_{\mathrm{H}})_t + (I_{CD})_t + (I_{\mathrm{Y}})_t + (I_{\mathrm{F}})_t + (I_{\mathrm{N}})_t \\ = w_t L_t + (r_{\mathrm{Y}})_t(K_{\mathrm{Y}})_{t-1} + (r_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1} + (r_{\mathrm{N}})_t(K_{\mathrm{N}})_{t-1} \\ + X_t + (\pi_{\mathrm{Y}})_t + (\pi_{\mathrm{E}})_t + (\pi_{\mathrm{N}})_t + (\pi_{\mathrm{F}})_t \end{aligned} \tag{3.2}$$

Income is gained by the supply of labor $L$ in return for wages $w$ and by undertaking investments $I_{CD}$,$I_{\mathrm{Y}}$,$I_{\mathrm{F}}$,$I_{\mathrm{N}}$. Households lend capital to the goods production sector $K_{\mathrm{Y}}$ and each resource sector $K_{\mathrm{F}}$,$K_{\mathrm{N}}$, which they receive back in the next period with a mark-up in the form of interests $r_{\mathrm{Y}}$,$r_{\mathrm{F}}$,$r_{\mathrm{N}}$. We assume that physical investment can only be made to sectors specifically. Hence, once it is invested, it is restricted to the specific sectors' capital stock and distinct from other stocks. Furthermore, households can undertake investments $I_{CD}$ in durable goods $CD$ according to

$$(I_{CD})_t = CD_t - \left(1 - \delta^{CD}\right) CD_{t-1} \tag{3.3}$$

which affects their own utility. $\delta^{CD}$ denotes the depreciation rate of the durable goods stock. Because households own all companies, their income increases by the flow of all profits $\pi_{\mathrm{Y}}$,$\pi_{\mathrm{E}}$,$\pi_{\mathrm{F}}$,$\pi_{\mathrm{N}}$ from these.[3] Expenditures further exist by consuming non-durable goods from final goods production and using energy $E_{\mathrm{H}}$ from the energy sector with the price $p_{\mathrm{H}}$. In an extension, we also allow for R&D in the fossil intermediate energy sector which is provided by households and does not diminish their labor supply. The corresponding profits are equal to $X_t$. The corresponding function is further discussed in Section 3.3.4.

### 3.3.2 Final good production

Non-durable goods $CN$ which are consumed within the household sector are produced by the final good sector, hence $Y = CN$. Here, firms act under perfect competition. Production follows a CES function which is defined by:

$$Y_t = (A_{\mathrm{Y}})_t \left[\eta (K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{1-\alpha} \tag{3.4}$$

$A_Y$ defines Hicks-neutral technological progress which will be later affected by stochastic shocks. $\eta \in (0,1)$ measures the share of capital with respect to energy and $\nu$ the substitution parameter between capital and energy. We assume that $\nu < 0$ which leads to a complementary relationship between both input factors. According to that, the efficient usage of capital $K_{\mathrm{Y}}$ within the production process requires some amount of energy $E_{\mathrm{Y}}$. Moreover, the firm employs labor supplied by households $L$. $\alpha \in (0,1)$ indicates the output elasticity of the capital-energy basket. As the elasticity of substitution between labor and the composite of physical capital and energy is one, final goods are produced with constant returns to scale. This is similar to the aggregated production function used by Kim and Loungani (1992) and Dhawan and Jeske (2008) who also assume a complementary relationship between physical capital and energy. The installation of physical capital takes place with a lag, hence in the period before, which is analogous to having fixed investments.

**Lemma 2.** *Concavity of final production*
*The partial derivatives for the final production function are:*

$$Y'_{K_{\mathrm{Y}}} > 0, Y'_{E_{\mathrm{Y}}} > 0, Y'_L > 0,$$
$$Y''_{K_{\mathrm{Y}}K_{\mathrm{Y}}} < 0, Y''_{E_{\mathrm{Y}}E_{\mathrm{Y}}} < 0, Y''_{LL} < 0,$$
$$Y''_{K_{\mathrm{Y}}E_{\mathrm{Y}}} = Y''_{E_{\mathrm{Y}}K_{\mathrm{Y}}} > 0, Y''_{K_{\mathrm{Y}}L} = Y''_{LK_{\mathrm{Y}}} > 0, Y''_{E_{\mathrm{Y}}L} = Y''_{LE_{\mathrm{Y}}} > 0.$$

*The production function $Y_t$ is overall concave in $A^Y, K_{\mathrm{Y}}, E_{\mathrm{Y}}, L > 0$ iff all the following conditions hold:*

$$\nu, \eta \leq 1 \ \text{or} \ \nu > 1, \eta \geq 1 \ \text{or} \ \nu = 1, \eta > 1$$
$$\alpha < 1.$$

[3] Because all firms act in a perfect competitive market, their profits are equal to zero.

*Proof: See Appendix 3.H.2.*

As Lemma 2 shows, final output increases with installed physical capital, energy and labor but at a decreasing rate. Moreover, overall concavity of the production function is satisfied. The capital stock $(K_{\mathrm{Y}})_t$ is accumulated according to the households investment function:

$$(I_{\mathrm{Y}})_t = (K_{\mathrm{Y}})_t - \left(1 - \delta^{Y}\right)(K_{\mathrm{Y}})_{t-1} \tag{3.5}$$

Final good producing firms face the following profit function:

$$(\pi_{\mathrm{Y}})_t = Y_t - (r_{\mathrm{Y}})_t (K_{\mathrm{Y}})_{t-1} + w_t L_t + (p_{\mathrm{E}})_t (E_{\mathrm{Y}})_t \tag{3.6}$$

By normalizing the price of non-durable goods to one, revenues of firms are equal to $Y$. On the expenditure side, the input factors capital, labor, and energy are payed off with their respective marginal products $w$, $r_{\mathrm{Y}}$, and $p_{\mathrm{E}}$.

### 3.3.3 Energy sector

The energy sector combines both intermediate energy sources (non-renewable and renewable energy) to provide a general energy product to the household sector and the final good sector. As we assume substitution between the input factors, we model the production function as a Cobb-Douglas function.

$$E_t = (A_{\mathrm{E}})_t F_t^{\phi} N_t^{1-\phi} \tag{3.7}$$

where $E$ is the general energy output, $A^E$ is the Hicks-neutral technological progress, $F$ is the non-renewable energy input, and $N$ is the renewable energy input. $\phi$ determines the elasticity of output. The energy sector optimizes its production function with respect to the profit function:

$$\pi_t^E = (p_{\mathrm{E}})_t E_t - (p_{\mathrm{F}})_t F_t - (p_{\mathrm{N}})_t N_t. \tag{3.8}$$

As the energy sector acts under perfect competition, the input factors are payed off with their marginal production, defined by $p_{\mathrm{F}}$ and $p_{\mathrm{N}}$. $p_{\mathrm{E}}$ is the price for the energy output which is the same for both consumers, households and final good firms.

### 3.3.4 Fossil resource sector

In the fossil resource sector, the resources are extracted from a finite resource stock and, when combined with physical capital, transformed to the intermediate energy good. Here, we follow the idea of Gross et al. (2013) with some minor adjustments. In the present model, the economy is completely closed and, consequently, capital is merely supplied by domestic households. Furthermore, the resource stock is owned by the fossil resource sector. Hence, the sector does not face additional occupational costs that have to be paid to another owner. The general production function is defined by:

$$F_t = (A_{\mathrm{F}})_t (K_{\mathrm{F}})^{\varphi}_{t-1} S^{1-\varphi}_{t-1} \tag{3.9}$$

where $F$ is the intermediate energy good, $A^F$ is the Hicks-neutral technological progress, $K_{\mathrm{F}}$ is the physical capital stock, and $S$ is the resource stock. $\varphi \in (0,1)$ measures the output elasticity of capital. As we assume constant returns to scale this Cobb-Douglas function, $1-\varphi$ displays the output elasticity of the reserve stock. Capital is supplied by the household sector. The accumulation of the physical capital stock $K_{\mathrm{F}}$ in the production function is standard, following the investment function:

$$(I_{\mathrm{F}})_t = (K_{\mathrm{F}})_t - \left(1-\delta^F\right)(K_{\mathrm{F}})_{t-1} \tag{3.10}$$

where the capital stock depreciates with the rate $\delta^F$.

The fossil resource sector is further constrained by a stock of finite reserves $S$, that decreases by the amount of intermediate energy, extracted each period $F$. Moreover, $\varepsilon_{\mathrm{S}} \overset{\mathrm{iid}}{\sim} N\left(0, \sigma^2_{\mathrm{S}}\right)$ defines a direct shock to the quantity of available reserves:

$$S_t = S_{t-1} - F_t + \omega D_t - e^{\varepsilon_{\mathrm{S},t}}. \tag{3.11}$$

In an extension of the model, the fossil energy sector is able to increase the reserve stock by investing in R&D which is paid off to the households. By assumption, we distinguish between reserves and resources. Reserves have been already discovered and can be technically extracted at the current point of time with the available technology. However, resources denote the amount of crude resources that are not feasible to be extracted either due to high costs or due to missing technology. Investment into R&D allows the transformation of certain share of resources into reserves. After this definition, $S_t$ is always the amount of reserves available at that moment. $D$ is the amount of reserves which is replenished through R&D whereas $\omega \in (0,1)$ is an efficiency parameter. If $\omega = 0$, there is no R&D in the model and consequently no possibility to replenish the resource stock (marking the baseline model). Expenses in R&D are determined by a non-linear cost function:

$$C\left(D_t, V_t\right) = \left(\frac{D_t}{V_t}\right)^{\upsilon} = X_t \tag{3.12}$$

where $D_t$ is the replenished amount of reserves or amount of transformation from resources to reserves (amount of exploration). $V_t$ is the stock of resources. Although this expenditure function is different, its properties resemble the model by Gross et al. (2013) as we abstract from the assumption of a finite bound in the level of resources as is done by Bohn and Deacon (2000). This is fulfilled by assuming that additional reserves can be discovered but at increasing costs. Therefore, R&D underlies a convex cost function when the second derivative is positive, which is satisfied iff $\upsilon > 1$. To transform the last resource unit to a reserve unit comes with infinite costs. Hence, it will not be mined by the sector. Similar to the reserve stock, the resource stock is finite and bounded by the constraint:

$$V_t = V_{t-1} - D_t - e^{\varepsilon_{\mathrm{V},t}} \tag{3.13}$$

where $\varepsilon_{\mathrm{V}} \overset{\text{iid}}{\sim} N\left(0, \sigma_{\mathrm{V}}^2\right)$ defines a direct shock to the quantity of available resources. As the fossil resource sector performs under the assumption of perfect competition, its corresponding profit function is given by:

$$(\pi_{\mathrm{F}})_t = (p_{\mathrm{F}})_t F_t - (r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1} - C\left(D_t, V_t\right). \tag{3.14}$$

### 3.3.5 Renewable resource sector

The renewable resource sector generates an intermediate energy good that is completely generated from a capital stock. This follows the assumption that access to renewable natural resources require prior investments in capital. In their paper, Mason and Chassé (2018) describe this approach to expand capacities of renewable resources. Households, who own this physical capital, invest into and hence, accumulate this stock for capital returns. The harvesting function of this non-finite product follows:

$$N_t = (A_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1}^{\psi} \tag{3.15}$$

where $N$ indicates the intermediate energy product, $A^N$ the technological progress which is exogenously determined, and $K_{\mathrm{N}}$ the capital stock of the renewable resource sector. $\psi$ measures the output elasticity of the physical capital input. As $\psi < 1$, the harvesting function has decreasing returns to scale. The capital stock is accumulated according to the following function:

$$(I_{\mathrm{N}})_t = (K_{\mathrm{N}})_t - \left(1 - \delta^N\right)(K_{\mathrm{N}})_{t-1}. \tag{3.16}$$

The corresponding profit function

$$(\pi_{\mathrm{N}})_t = (p_{\mathrm{N}})_t N_t - (r_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1} \tag{3.17}$$

satisfies the assumption of perfect competition by paying of input factor capital with the sector's revenue.

### 3.3.6 Market clearing

To complete the model, the markets have to be cleared. According to that, the two remaining equations are:

$$\begin{aligned} GDP_t &= Y_t - (p_{\mathrm{E}})_t(E_{\mathrm{Y}})_t + (r_{\mathrm{N}})_t(K_{\mathrm{N}})_{t-1} + (r_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1} + X_t \\ &= CN_t + (p_{\mathrm{H}})_t(E_{\mathrm{H}})_t + (I_{CD})_t + (I_{\mathrm{Y}})_t + (I_{\mathrm{F}})_t + (I_{\mathrm{N}})_t \end{aligned} \tag{3.18}$$

$$E_t = (E_{\mathrm{H}})_t + (E_{\mathrm{Y}})_t \tag{3.19}$$

where (3.18) determines the general market clearing and (3.19) describes the clearing of energy. For simplicity, we assume that prices per unit of energy is the same for households and firms. The market value is calculated by using the expenditure approach.

We distinguish between three sorts of shocks that affect the economy of this model exogenously. We assume that innovation $A$, which can lead to an increase in productivity, is sector-specific and non-transferable. All sectors with production functions can face impacts by shocks in TFP which are each independent and identically distributed. Their laws of motion are described by the following log-functions:

$$\ln(A_{\mathrm{Y}})_t = \rho_{\mathrm{Y}} \ln(A_{\mathrm{Y}})_{t-1} + \varepsilon_{\mathrm{Y},t} \tag{3.20}$$

$$\ln(A_{\mathrm{E}})_t = \rho_{\mathrm{E}} \ln(A_{\mathrm{E}})_{t-1} + \varepsilon_{\mathrm{E},t} \tag{3.21}$$

$$\ln(A_{\mathrm{F}})_t = \rho_{\mathrm{F}} \ln(A_{\mathrm{F}})_{t-1} + \varepsilon_{\mathrm{F},t} \tag{3.22}$$

$$\ln(A_{\mathrm{N}})_t = \rho_{\mathrm{N}} \ln(A_{\mathrm{N}})_{t-1} + \varepsilon_{\mathrm{N},t} \tag{3.23}$$

They follow an AR(1) process (autoregressive of order one) where $\varepsilon_i \overset{\text{iid}}{\sim} N\left(0, \sigma_i^2\right), i \in (Y, E, F, N)$, hence with a zero mean and uncorrelated variance. The parameter $\rho_i, i \in (Y, E, F, N)$ measures the persistence of TFP.

Furthermore, a quantity shock can affect the size of the stock of finite resource $\sigma_i$. This is comparable with adjustments of the estimation of probable reserves or forced abandonment of reserves by politics and the society. However, contrary to shocks in TFP, it has only an one-time direct effect to the stock constraints. In the model with replenishment of the reserves, we add a similar shock to the resource constraint $\sigma_i$ reflecting the same properties as a reserve shock.

Moreover, we introduce two taste shifters similar to (Bencivenga, 1992) and (Stockman and Tesar, 1995), each for non-durable good consumption $T_{\mathrm{CN}}$ and durable good consumption $T_{\mathrm{CD}}$. Both shocks, which follow an AR(1) process, consider shifts in demand for consumption that are not captured by the dynamics in our model. This is particularly necessary to avoid incorrect identification by estimating some of the remaining parameters taking empirical consumption data into account.[4] The log laws of motion for the taste shifters are:

$$\ln(T_{\mathrm{CN}})_t = \rho_{\mathrm{T,CN}} \ln(T_{\mathrm{CN}})_{t-1} + \varepsilon_{\mathrm{T,CN},t} \tag{3.24}$$

$$\ln(T_{\mathrm{CD}})_t = \rho_{\mathrm{T,CD}} \ln(T_{\mathrm{CD}})_{t-1} + \varepsilon_{\mathrm{T,CD},t} \tag{3.25}$$

where $\varepsilon_{\mathrm{T},i} \overset{\mathrm{iid}}{\sim} N\left(0, \sigma^2_{\mathrm{T},i}\right), i \in (CN, CD)$.

## 3.4 Competitive Equilibrium

After setting up the model, each actor maximizes its functions to optimize its decision making. In the following, we solve the model for each sector successively. The equations are derived in detail in Appendix 3.H. The representative household decides about its demand for consumption of non-durable goods, durable goods, and energy as well as its supply of labor to maximize expected lifetime utility. The household faces the following optimization problem:

$$\begin{aligned}\max_{\substack{CN_t, CD_t, (E_{\mathrm{H}})_t, L_t,\\ (K_{\mathrm{Y}})_t, (K_{\mathrm{F}})_t, (K_{\mathrm{N}})_t}} U_0 = \; & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \vartheta \ln \left[ CN_t^{\gamma} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}} \right] + (1-\vartheta) \ln [1 - L_t] \right. \\ & + \lambda_t^H \{ CN_t + (p_{\mathrm{H}})_t (E_{\mathrm{H}})_t + (I_{CD})_t + (I_{\mathrm{Y}})_t + (I_{\mathrm{F}})_t + (I_{\mathrm{N}})_t - w_t L_t - (r_{\mathrm{Y}})_t (K_{\mathrm{Y}})_{t-1} \\ & - (r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1} - (r_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1} - X_t - (\pi_{\mathrm{Y}})_t - (\pi_{\mathrm{N}})_t - (\pi_{\mathrm{F}})_t \} \} \end{aligned} \tag{3.1}$$

where $\beta$ serves as a time preference parameter to discount future utility streams. The associated FOCs with respect to $CN$, $CD$, $E_{\mathrm{H}}$, $L$, $K_{\mathrm{Y}}$, $K_{\mathrm{F}}$, and $K_{\mathrm{N}}$ are summarized below:

$$1 = \beta \mathbb{E} \left\{ \frac{1-\gamma}{\gamma} \theta \frac{CD_t^{\zeta-1} CN_t}{\theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta}} \right\} + \beta \mathbb{E} \left\{ \frac{CN_t}{CN_{t+1}} \left(1 - \delta^{CD}\right) \right\} \tag{3.2}$$

$$(p_{\mathrm{E}})_t = \frac{(1-\gamma)(1-\theta)}{\gamma} \frac{CN_t (E_{\mathrm{H}})_t^{\zeta-1}}{\left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)} \tag{3.3}$$

$$w_t = \frac{CN_t}{1-L_t} \frac{1-\gamma}{\gamma \vartheta} \tag{3.4}$$

[4] In fact, it holds that $CN = T_{CN} CN_{\mathrm{net}}$ and $CD = T_{CD} CD_{\mathrm{net}}$. According to the law of large numbers, $T_{CN}$ and $T_{CD}$ are constant and equal to one so $CN = CN_{\mathrm{net}}$ and $CD = CD_{\mathrm{net}}$.

$$1 = \beta \mathbb{E} \left\{ \frac{CN_t}{CN_{t+1}} \left(1 + (r_i)_{t+1} - \delta^i\right) \right\} \qquad \text{for: } i = Y, F, N. \tag{3.5}$$

The trade-off between non-durable consumption goods and the composite basket including durables and energy is described in (3.2) while (3.3) determines the demand for energy, given its price. Equation (3.4) shows the intratemporal optimality condition of labor supply in relation with consumption of non-durables, given the wage. Disutility from labor due to an increase in working hours is compensated by a decrease of consumption at constant wages. Equation (3.5) describes the Euler equations which imply that the current marginal utility of consumption on non-durable goods is equal to the discounted utility of future consumption.

The final good sector maximizes current profits with respect to its input factors which are paid off according to their respective marginal productivities:

$$(r_{\mathrm{Y}})_t = \alpha\eta(A_{\mathrm{Y}})_t \left[\eta(K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha}(K_{\mathrm{Y}})_{t-1}^{\nu-1} \tag{3.6}$$

$$(p_{\mathrm{E}})_t = \alpha\eta(A_{\mathrm{Y}})_t \left[\eta(K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha}(E_{\mathrm{Y}})_t^{\nu-1} \tag{3.7}$$

$$w_t = (1-\alpha)\,(A_{\mathrm{Y}})_t \left[\eta(K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{-\alpha} \tag{3.8}$$

The associated demand functions of the input factors of the energy sector, based on the profit function under perfect competition, given $p_{\mathrm{F}}$ and $p_{\mathrm{N}}$ are:

$$(p_{\mathrm{F}})_t = \phi(p_{\mathrm{E}})_t(A_{\mathrm{E}})_t F_t^{\phi-1} N_t^{1-\phi} \tag{3.9}$$

$$(p_{\mathrm{N}})_t = (1-\phi)\,(p_{\mathrm{E}})_t(A_{\mathrm{E}})_t F_t^{\phi} N_t^{-\phi} \tag{3.10}$$

The fossil resource sector faces a finite resource stock constraint and at given conditions also with a finite reserve stock. Thus, the firm's decision problem depends on choosing the optimal demand for raw resources, physical capital, and optimal setting of R&D strategy. The subsequent dynamic problem is given by:

$$\begin{aligned} \max_{(K_{\mathrm{F}})_t, S_t, D_t, V_t} \quad \pi_0^F = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ & (p_{\mathrm{F}})_t F_t - (r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1} - C(D_t, V_t) \\ & + \lambda_t^{FS} \{S_{t-1} - F_t + \omega D_t - S_t\} + \lambda_t^{FV} \{V_{t-1} - D_t - V_t\}\} \end{aligned} \tag{3.11}$$

The corresponding demand functions read as follows:

$$(p_{\mathrm{F}})_t = \frac{(r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1}}{\varphi F_t} + \beta \mathbb{E}\left\{(p_{\mathrm{F}})_{t+1}\right\} + \beta \mathbb{E}\left\{ \frac{(r_{\mathrm{F}})_{t+1}(K_{\mathrm{F}})_t}{\varphi F_{t+1}} \left[(1-\varphi)\,\frac{F_{t+1}}{S_t} - 1\right]\right\} \tag{3.12}$$

$$(p_{\mathrm{F}})_t = \frac{(r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1}}{\varphi F_t} + \upsilon \frac{D_t^{\upsilon}}{D_t V_t^{\upsilon}} - \beta\upsilon\mathbb{E}\left\{\frac{D_{t+1}^{\upsilon}}{D_{t+1}V_{t+1}^{\upsilon}}\right\} + \beta\mathbb{E}\left\{(p_{\mathrm{F}})_{t+1}\right\} - \beta\mathbb{E}\left\{\frac{(r_{\mathrm{F}})_{t+1}(K_{\mathrm{F}})_t}{\varphi F_{t+1}}\right\} + \upsilon\frac{D_t^{\upsilon}}{V_t^{(1+\upsilon)}} \tag{3.13}$$

As $\beta \in (0,1)$, (3.12) shows that the inflation rate of the price for intermediate fossil energy is positive. Note, that this function is similar to the Hotelling rule (Hotelling, 1931) saying that the rate of price increase equals, among others, the social discount rate. Equation (3.13) denotes that the sector equation its marginal costs of R&D for exploration to the marginal revenue it earns from selling the intermediate fossil energy product.

The renewable energy sector maximizes current-period profit under perfect competition. Consequently, the first order condition for the only input factor physical capital is as follows:

$$(r_{\mathrm{N}})_t = \psi (p_{\mathrm{N}})_t (A_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1}^{\psi-1} \tag{3.14}$$

describing the price of physical capital, invested in the renewable energy sector. It increases with a higher scarcity of capital stock and a higher profit of the renewable sector though higher productivity or selling prices.

## 3.5 Calibration and Estimation of Parameters

In a next step, parameters have to be determined to be able to proceed with the numerical as well as the dynamic analysis of the model. We estimate these values in the course of this paper on the basis of calibrated values which have to be determined in the first instance using real long-term data. Subsequently, we define the distributions, hence the kernel and the variance, on which the posterior parameters are estimated. The estimation is based on Bayesian techniques and is carried out with data about the German economy which is discussed more detailed below.

### 3.5.1 Data and estimation methodology

In order to estimate the parameters to apply the model to Germany, we use data for the period of 1991 to 2014 (1991 being the earliest year in which sufficient detailed data about energy market is available). Two macroeconomic variables and three variables describing the development in the energy sectors are considered. In particular, we look at: (i) economic output, (ii) consumption of durable goods, (iii) total energy consumption, (iv) fossil energy consumption, and (v) renewable energy consumption.

For aggregated economic output, we take the output approach of the gross domestic product (GDP) from OECD (2012) at constant prices based on the reference year 2010 (Code: B1_GA). Durable goods consumption is also taken from OECD at constant prices (Code: P311B). The remaining three energy time series are taken from Eurostat (2017, 2018). Total energy consumption is defined as the gross inland consumption of all energy products (Code: nrg_110a_1). Fossil energy includes the consumption of gas, nuclear energy, solid fuels, and total petrol (Code: nrg_100a_1). All remaining consumption of energy is referred to as renewable energy products. All energy products are measures in terajoule to have a common unit, which allows a better comparison and relation. As records for energy consumption and production as well as the consumption of durable goods are not sufficiently recorded in short term units of time, the data is used on annual frequency. To avoid stochastic singularity, the number of time series also determines the amount of exogenous shocks that have to be at least applied to the model. To make the data applicable to our model, the following measurement equation holds:

$$\begin{bmatrix} \Delta \ln GDP_t \\ \Delta \ln CN_t \\ \Delta \ln CD_t \\ \Delta \ln E_t \\ \Delta \ln F_t \\ \Delta \ln N_t \end{bmatrix} = \begin{bmatrix} \overline{GDP} \\ \overline{CN} \\ \overline{CD} \\ \overline{E} \\ \overline{F} \\ \overline{N} \end{bmatrix} + \begin{bmatrix} GDP_t - GDP_{t-1} \\ CN_t - CN_{t-1} \\ CD_t - CD_{t-1} \\ E_t - E_{t-1} \\ F_t - F_{t-1} \\ N_t - N_{t-1} \end{bmatrix}$$

The first vector includes the log difference from the trend path while the second vector describes the trend growth rate for each variable respectively. The trends are identified by applying the HodrickPrescott (HP) filter of each time series respectively (see Appendix 3.D).[5] In the third vector, the variables are included as the first difference from the previous period. Overall, this equation mirrors the relationship between empirical values from the data on the left-hand side and theoretical values from the model on the right-hand side. As the model includes stationary data only, we pursue to calibrate and estimate the model as well as to fit the theoretical values close to their empirical counterparts.

### 3.5.2 Calibration

Independently of whether or not the model should be analyzed with calibrated parameters or estimated values from priors, we have to critically identify both of them on the basis of the given model. Here, the parameters can be split up into two groups. Structural parameters which determine the dynamics of the model and steady state values, such as average ratios, which describe the general state of the economy. We follow three approaches to match an annual time horizon reflecting most of the features of the German economy. Some parameters are (i) calibrated using empirical data to fit the model with real data, some parameters are (ii) taken from existing literature, mainly in the field of RBC models dealing with energy in general or specific sources of energy production while some other parameters are (iii) calculated from

[5]For yearly data we use an HP parameter of $\lambda = 100$.

the steady state of the model. Altogether, there are 24 structural parameters which can be distinguished by 16 structural parameters and 8 shock related parameters. Structural parameters are categorized as numerical factors defining the system of sectors such as the utility function or production function. As prices, in particular those of resources, are completely endogenously determined, shocks affect the technological progress of each production sector only. They define how TFPs behave over time. Tables 3.1 and 3.2 give an overview of the definition of parameters as well as their prior values, sorted according to their respective category.

A number of parameters are initially taken as fixed. We set $\alpha = 0.365$, leading to a labor income share in goods production output of 63.5%. This is close to the value of 0.36 used by Kydland and Prescott (1982) and Hansen (1985) in general literature. Compared to lower values used in other German RBC models such as Schmidt and Zimmermann (2005) and Flor (2014), the reduction of labor income shares considers a more capital-intensive production as discussed by Schmalwasser and Schidlowski (2006). Here, the authors argue that capital stock grows faster than production because labor is increasingly replaced with capital in recent time. The time discount rate $\beta = 0.99$ and the substitution parameter of the durable good/energy consumption bundle and non-durable goods in the utility function amounts for $\zeta = -2.8748$ according to Dhawan and Jeske (2008). To adapt the elasticity of substitution between physical capital and energy in the final good production function to its corresponding value of the German economy, we set $\nu = -0.15$ following Kemfert and Welsch (2000). This is different to Dhawan and Jeske (2008) and Kim and Loungani (1992) who choose $\nu = -0.7$, whereby the former also perform a sensitivity analysis for alternative values. Usually, all parameters have to be carefully chosen as they cannot be altered by remaining variables later on. But opposite to just calibrating the model, these values are solely used as initial priors to later pick the best kind and structure of the model according to our metrics using several estimation methods. Nevertheless, we will perform robustness tests in Section 3.9 to verify the relationship between capital and energy in the production function as we differ from established theoretical literature. By selecting different priors for $\nu$, this allows us to check the strengths of this parameter.

Moreover, the parameters $\zeta$ and $\theta$ in the utility function and $\nu$ and $\eta$ in the final good production function cannot be simultaneously calibrated, because of having an equation system with one degree of freedom. Hence, either of those must be predetermined – in our case the elasticities in these functions. Subsequently, the particular share parameters are calculated to match empirical data. The weight of overall consumption within the utility function is set to $\vartheta = 0.341$ which is determined by the steady state of the wage equation and labor supply equation. The depreciation rate of durable goods is taken from Dhawan and Jeske (2008) with the assumption that the behavior of US households does not distinguish from German consumers significantly. Accordingly, $\delta^{CD}$ is set to 0.0683. Regarding the motion of the capital stock, used in the final good sector, its depreciation rate is calculated from the time preference rate and the steady state interest rate while the latter is calculated from the long-run first-order condition of the production function.

The depreciation rate of fossil capital is determined by the long-run capital/output ratio and investments into the former. Under consideration of its different weights, the combined rates resemble the general depreciation rate of the German economy. The output elasticity of intermediate fossil energy is set to $\phi = 0.88$ following Argentiero et al. (2018). It approximately reflects the average relation between non-renewable energy with respect to renewable energy. This comes close the average proportion in Germany for the observed time period. The output elasticities of physical capital in the intermediate fossil energy generation function and the intermediate renewable energy generation function are set to $\varphi = 0.62$ according to Gross et al. (2013) and $\psi = 0.3$ according to Argentiero et al. (2018) respectively. The remaining structural parameters are determined from the given parameters and empirical findings. A more detailed derivation of the calibrated parameters can be found in Appendix 3.H.6.

Table 3.1: Structural parameter values

| **Parameter** | **Value** | **Description** |
|---|---|---|
| $\beta$ | 0.990 | discount factor |
| $\vartheta$ | 0.341 | share of consumption in utility |
| $\gamma$ | 0.793 | elas. of substitution of consumption |
| $\zeta$ | -2.875 | elas. of substitution between $CD$ and $E^H$ |
| $\theta$ | 0.999 | share of durable consumption good |
| $\alpha$ | 0.365 | final output elas. of VA |
| $\eta$ | 0.949 | share of capital |
| $\nu$ | -0.150 | elas. of substitution between $K^Y$ and $E^Y$ |
| $\phi$ | 0.800 | output elasticity of fossil resources |
| $\varphi$ | 0.490 | fossil resource share |
| $\omega$ | 1.000 | exploration parameter |
| $\upsilon$ | 2.000 | parameter of exploration cost function |
| $\psi$ | 0.310 | renewable asset share |
| $\delta^{CD}$ | 0.068 | depreciation rate of $CD$ |
| $\delta^{Y}$ | 0.017 | depreciation rate of $K^Y$ |
| $\delta^{F}$ | 0.045 | depreciation rate of $K^F$ |
| $\delta^{N}$ | 0.045 | depreciation rate of $K^N$ |

Note that $\theta$ and $\eta$ depend on $\zeta$ and $\nu$ but also $\delta_{CD}$ and $\alpha$ (see (3.H.60) and (3.H.63) in Appendix 3.H.5).

The shock related parameters, in particular the coefficients describing the autoregressive process of total factor productivity in each production function, are assumed to be uniformly equal to 0.85. This reflects a modest reduction of the direct impulse of stochastic shocks and follows business cycle literature Smets and Wouters (2003).

Table 3.2: Shock related parameter values

| Parameter | Value | Description |
|---|---|---|
| $\rho_{A_{\mathrm{Y}}}$ | 0.850 | persistence technology shock of $A_{\mathrm{Y}}$ |
| $\rho_{A_{\mathrm{E}}}$ | 0.850 | persistence technology shock of $A_{\mathrm{E}}$ |
| $\rho_{A_{\mathrm{F}}}$ | 0.850 | persistence technology shock of $A_{\mathrm{F}}$ |
| $\rho_{A_{\mathrm{N}}}$ | 0.850 | persistence technology shock of $A_{\mathrm{N}}$ |
| $\rho_{\mathrm{T,CN}}$ | 0.850 | persistence consumer taste shock of $CN$ |
| $\rho_{\mathrm{T,CD}}$ | 0.850 | persistence consumer taste shock of $CD$ |
| $\sigma_{A_{\mathrm{Y}}}$ | 0.010 | volatility shock in $A_{\mathrm{Y}}$ |
| $\sigma_{A_{\mathrm{E}}}$ | 0.010 | volatility shock in $A_{\mathrm{E}}$ |
| $\sigma_{A_{\mathrm{F}}}$ | 0.010 | volatility shock in $A_{\mathrm{F}}$ |
| $\sigma_{A_{\mathrm{N}}}$ | 0.010 | volatility shock in $A_{\mathrm{N}}$ |
| $\sigma_{\mathrm{T,CN}}$ | 0.010 | volatility shock in non-durable taste |
| $\sigma_{\mathrm{T,CD}}$ | 0.010 | volatility shock in durable taste |
| $\sigma_{\mathrm{S}}$ | 0.010 | volatility shock in $S$ |
| $\sigma_{\mathrm{V}}$ | 0.010 | volatility shock in $V$ [a] |

[a] The constraint for resources and hence, the shock in its quantity only hold for the model allowing for replenishment of the reserves.

For steady state values, we calibrate parameters that are consistent with long-run historical averages from data. Only for labor supply do we set its long-run steady state value to $\overline{L} = 0.3$ as is also standard in the literature. Although this goes along with Dhawan and Jeske (2008), it holds very close to its German equivalent (see Hristov, 2016). However, as there are not good measures available for some data, modification of certain values is requested. As such, the depreciation rate of the stock of physical capital in the renewable energy sector, belonging to the group of structural parameters, is taken over from its appropriate value in the fossil resource sector. The model does not distinguish between different forms of finite resources, and thus we have to combine its various expressions in one term. These are calculated considering their respective heating values (see Appendix 3.H.6 for a detailed discussion). The ratio between extraction of fossil resources and its stock is calculated from data retrieved from the German Federal Institute for Geosciences and Natural Resources (BGR, 2016). For the German economy, the $\bar{F}/\bar{S}$ ratio is set to 0.12875.

### 3.5.3 Estimation methodology

To determine the model by specifying the parameters, we use the concept of Bayesian estimation which gives us a few advantages. It incorporates the derivation of the modes by combining log-likelihood maximization with confronting the model with data through priors. These priors work as weights in the maximization process to avoid strange peak of the log-likelihood function. Otherwise, as pointed out by Griffoli (2007), this can lead to a frequent property of DSGE models that likelihood maximization can lead to illogical or foolish outcomes that contradict with observations in data which is caused by their stylized and misspecified nature. Furthermore, opposed to GMM, Bayesian estimation fits the complete model and not only particular equilibria. However, this also goes along with an adequate definition of the model to avoid misspecification of all estimation results. Moreover, Bayesian techniques can cope with a lack of identification of parameters and is therefore also more robust to outliers in the data. Assuming a peak of likelihood function using false insufficient priors, it will lead to a low probability of the posterior results. A more detailed explanation about the solution technique of the Bayesian methodology can be found in Appendix 3.B. Subsequent to identifying the likelihood function to estimate the modes of the parameters, we perform a Markov Chain Monte Carlo (MCMC) method applying the Metropolis Hansting algorithm to obtain the full posterior distribution of the values. In addition, this also acts as a diagnostic tool to check the robustness of the results to build up confidence in our estimations. The comparison of the prior and posterior distributions is shown in Appendix 3.C.

### 3.5.4 Prior parameters

Subsequently, we determine the probability distributions of all parameters that will be estimated. These densities reflect beliefs about the parameter values and should be carefully chosen. The previously determined calibration results are taken as means to avoid diffuse results as they mainly are based on data. Standard deviations and prior distributions are listed up in the third and forth column of Tables 3.3 and 3.4.

For the capital-energy bundle substitution parameter in the final good production function as well as the major output elasticities in the remaining production functions $\{\alpha, \phi, \varphi, \psi\}$, we assume a variance of $\{0.05, 0.1, 0.1, 0.1\}$ and a beta distribution to constrained the parameter between zero and one. The elasticity parameters within the durable good-energy bundle and capital-energy bundle $\{\zeta, \nu\}$ are distributed according to a normal probability distributed function with a variance of 0.5 and 0.2 respectively as they do not only contain natural numbers but also all real numbers. The share parameters in both CES functions, the utility function and final good production function $\{\theta, \eta\}$, are determined by $\zeta$ and $\nu$ (see Appendix 3.H.6). The depreciation rates $\{\delta_{CD}, \delta_Y, \delta_F, \delta_N\}$ follow a beta distribution with a standard deviation of 0.05 final good productive capital, fossil and renewable capital deposits and 0.1 for durable good stock.

For shock related parameters, determining the development of technological progress in the production functions, we have beta distributions limiting the range to positive values only. Furthermore, this guarantees a stable development to avoid unit roots. The standard deviations of white noise in these autoregressive functions, which acts as the shock components at the same time, follow an inverse-gamma distribution with a mean of 0.01 and an infinite standard deviation.

### 3.5.5 Posterior parameters

All parameters seem to be well identified which is confirmed by identification tests within Dynare.[6] The resulting values from Bayesian estimation performance are summarized in the last four columns of Tables 3.3–3.4, distinguishing between the posterior mode, the posterior standard deviation, and the 90% confidence interval for the model parameters. In addition, a graphical representation of the prior and posterior densities is included in Appendix 3.C.

The substitution parameter within the durable goods–energy bundle is -2.885 which comes close to its prior value. The posterior of the substitution parameter of capital–energy amounts to -0.288 which slightly deviates from its prior. To avoid misspecification of the latter due to a bad fit of prior settings, we perform a robustness test later. The present outcomes verify the assumption of a complementary relationship between energy and durable goods or physical capital in the utility or production function. Although the 90% confidence interval also includes positive values for $\nu$, its posterior mean deviates from the prior values by becoming even more negative.

The posteriors of the remaining structural parameters lie in the range of the prior values which have been originally calibrated from the data. Furthermore, they roughly correspond to the findings of the literature. The output elasticity of the capital-energy bundle $\alpha$ is slightly lower than its prior which corresponds to its equivalent in German RBC models (e.g. Flor, 2014). Argentiero et al. (2018) estimate a mean of 0.395 but assume a substitutable relationship between energy and capital while Dhawan and Jeske (2008) set an output elasticity of 0.36, assuming the same structure as in the present paper. Posterior estimation values of depreciation of physical capital in all sectors including durable goods are almost identical to their prior estimation values. This can be explained as they are not well identified by the data,[7] in particular through the assumption of equal values for fossil and renewable physical capital. Overall, this is negligible due to a lower share in the production function. Alternatively, a close posterior distribution with respect to its prior can also indicate a very accurate reflection of the given information (Pfeifer, 2014). As we set the prior means equal to their steady state values in consideration of the empirical data, this can justify the posterior results for $\varphi$, $\psi$, and $\phi$.

The posterior values of the means of the shock related parameters describing the autoregressive processes are close to the priors for the energy sector and the fossil resource sector. The same holds for the renewable sector as well as consumer taste of non-durable goods. Looking at the means of the shock related parameters

[6] In fact, we perform two independent tests based on the prior parameters. One checks for identification according to Ratto and Iskrev (2011), the other one is a sensitivity test that looks at unique solutions, indeterminacy, and explosive solutions.

[7] We limit the estimation process to the most relevant sector output series only. Hence, we do not include capital data series which might assess the evolution of depreciation more precisely in comparison to sector output.

describing the autoregressive processes, their posterior values are close to the priors for the energy sector, the fossil resource sector, the renewable resource sector and the consumer taste of non-durable goods.

Stochastic technological change in the final output sector and consumer taste of durable goods vanish at a higher speed. The variance describing the stochastic component are close to the prior. Only for the technological process in the finite and renewable resource sector as well as for the consumer taste of durables are they significantly more volatile with a standard deviation of 1.9%, 4.0%, and 3.2% respectively. This can come along with the fact that we do not consider any additional costs in doing investment adjustments. Hence, households overreact to optimally respond to changes in the economy. By construction, fluctuation in the finite resource stock is not subject to an autoregressive process. According the estimation results, its one time effect is slightly lower than the prior mean. In sum, it has the lowest volatility of all exogenous shocks to the model.

In the model allowing for replenishment of the reserve stock, the posterior structural parameters are in accordance with the results from the baseline model (see Appendix 3.E). In particular, the complementary relationship through $\zeta$ and $\nu$ are again confirmed. The estimated value of the additional parameter $\upsilon$, measuring the exponent of the exploration cost function, is negligibly higher than the priory estimated value which slightly increases the cost of R&D. Volatility in the finite resource stock as well as the reserve stock increases only slightly in comparison to the baseline model without extraction. Again, both shocks have only one-time direct effects their respective stock constraints.

## 3.6 Numerical Results

In the following, we compare the percentage standard deviation (2nd moment) of selected variables from the model with the respective values from German economy data, using an HP-filter. Doing so allows us to test how accurately the models with endogenous energy producing sectors fit the actual German business cycle. We simulate both models, without and with extraction, over 1000 periods, taking the estimated posterior parameters to receive the moments of simulated variables. In addition, we present the results of the simulations, in which we allow for only one shock respectively for the baseline model without extraction. The moments are summarized in Table 3.1.

In the baseline model, which considers an economy without resource-extraction that is affected by all shocks (column 1), output volatility comprises 92% of the fluctuation in the data. Comparing that result with those from the models being affected by a single shock only (columns 2–6), we can identify TFP as the main source of generating fluctuation in the national account. TFP in each of the remaining sub-sectors

Table 3.3: Results from Metropolis-Hastings (parameters)

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $\rho_{A_{\mathrm{Y}}}$ | beta | 0.850 | 0.1000 | 0.655 | 0.1340 | 0.4416 | 0.8832 |
| $\rho_{A_{\mathrm{E}}}$ | beta | 0.850 | 0.1000 | 0.772 | 0.0994 | 0.6189 | 0.9391 |
| $\rho_{A_{\mathrm{F}}}$ | beta | 0.850 | 0.1000 | 0.840 | 0.1027 | 0.6926 | 0.9909 |
| $\rho_{A_{\mathrm{N}}}$ | beta | 0.850 | 0.1000 | 0.791 | 0.0937 | 0.6469 | 0.9485 |
| $\rho_{\mathrm{T,CN}}$ | beta | 0.850 | 0.1000 | 0.804 | 0.0985 | 0.6595 | 0.9699 |
| $\rho_{\mathrm{T,CD}}$ | beta | 0.850 | 0.1000 | 0.602 | 0.1170 | 0.4107 | 0.7952 |
| $\zeta$ | norm | -2.875 | 0.5000 | -2.885 | 0.4941 | -3.6868 | -2.0612 |
| $\nu$ | norm | -0.150 | 0.3000 | -0.288 | 0.2632 | -0.7185 | 0.1476 |
| $\alpha$ | beta | 0.365 | 0.0500 | 0.347 | 0.0491 | 0.2663 | 0.4279 |
| $\varphi$ | beta | 0.490 | 0.1000 | 0.487 | 0.0969 | 0.3323 | 0.6500 |
| $\psi$ | beta | 0.310 | 0.1000 | 0.305 | 0.0991 | 0.1473 | 0.4681 |
| $\phi$ | beta | 0.800 | 0.0100 | 0.813 | 0.0100 | 0.7969 | 0.8297 |
| $\delta^{CD}$ | beta | 0.068 | 0.0100 | 0.067 | 0.0099 | 0.0508 | 0.0829 |
| $\delta^{F}$ | beta | 0.045 | 0.0100 | 0.045 | 0.0098 | 0.0292 | 0.0608 |
| $\delta^{Y}$ | beta | 0.017 | 0.0100 | 0.016 | 0.0092 | 0.0024 | 0.0297 |
| $\delta^{N}$ | beta | 0.045 | 0.0100 | 0.045 | 0.0102 | 0.0287 | 0.0612 |

HPD inf (HPD sup) correspond to the lowest (highest) points of the highest posterior density with a 95% confidence interval.

Table 3.4: Results from Metropolis-Hastings (standard deviation of structural shocks)

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $\sigma_{A_{\mathrm{Y}}}$ | invg | 0.010 | Inf | 0.009 | 0.0014 | 0.0062 | 0.0107 |
| $\sigma_{A_{\mathrm{E}}}$ | invg | 0.010 | Inf | 0.009 | 0.0013 | 0.0067 | 0.0109 |
| $\sigma_{A_{\mathrm{F}}}$ | invg | 0.010 | Inf | 0.019 | 0.0027 | 0.0151 | 0.0237 |
| $\sigma_{A_{\mathrm{N}}}$ | invg | 0.010 | Inf | 0.040 | 0.0058 | 0.0315 | 0.0491 |
| $\sigma_{\mathrm{T,CN}}$ | invg | 0.010 | Inf | 0.008 | 0.0013 | 0.0056 | 0.0097 |
| $\sigma_{\mathrm{T,CD}}$ | invg | 0.010 | Inf | 0.032 | 0.0046 | 0.0245 | 0.0392 |
| $\sigma_{\mathrm{S}}$ | invg | 0.010 | Inf | 0.007 | 0.0031 | 0.0025 | 0.0109 |

HPD inf (HPD sup) correspond to the lowest (highest) points of the highest posterior density with a 95% confidence interval.

can only contribute 5% to aggregate output volatility. With respect to consumption, the model can account for about 76% of consumption volatility of non-durable goods while it is only slightly below the empirical target for durable goods. Although Dhawan and Jeske (2008) have calibrated their model for the US economy, whose data partially deviate from the German data, the present endogenized model is able to map business cycles more accurately. Volatility of total energy is well-matched by the model. Most of the fluctuations are generated by TFP in fossil energy production, accounting for more than 82%. Models with shocks solely in the total energy sector or renewable energy sector can explain 45% and 38% respectively, while TFP shocks in the final good sector only generate 17%. The lower share of renewable energy resources in the total energy mix compared to the finite resource is the main

Table 3.1: Percentage standard deviation

| | | baseline | | | | | | extraction |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | Data | all shocks | w/ $A_y$ | w/ $A_e$ | w/ $A_f$ | w/ $A_n$ | w/ $S$ | all shocks |
| GDP | 1.50 | 1.38 | 1.47 | 0.07 | 0.06 | 0.06 | 0.00 | 1.53 |
| Non-durable goods | 0.83 | 0.63 | 0.12 | 0.01 | 0.01 | 0.01 | 0.00 | 0.65 |
| Durable goods | 2.87 | 2.86 | 0.23 | 0.13 | 0.02 | 0.11 | 0.01 | 2.96 |
| Total energy | 1.74 | 1.82 | 0.29 | 0.78 | 1.44 | 0.66 | 0.03 | 1.78 |
| Energy households | | 1.07 | 0.34 | 0.46 | 0.81 | 0.39 | 0.02 | 1.05 |
| Energy firms | | 2.58 | 0.66 | 1.14 | 2.18 | 0.98 | 0.05 | 2.71 |
| Finite resources | 1.85 | 1.90 | 0.35 | 0.25 | 1.78 | 0.22 | 0.04 | 1.90 |
| Renewable resources | 5.58 | 3.90 | 0.05 | 0.09 | 0.00 | 3.85 | 0.01 | 3.69 |
| Labor | 0.98 | 0.91 | 0.95 | 0.05 | 0.11 | 0.05 | 0.00 | 1.00 |
| Total energy prices | 4.98 | 3.27 | 0.95 | 1.35 | 2.54 | 1.15 | 0.05 | 3.20 |

Values denote the simulated results of percentage standard deviation (2nd moment) over 1000 periods using a HP-filter.

reason for its lower share of the explained variation. Moreover, according to the moments, fluctuation in total energy is mainly caused by the supply side as final good production has the opportunity to partly substitute volatility in energy by an alternative input factor. Unsurprisingly, volatility in outputs of finite and renewable resources are primarily caused by TFP shocks in their respective sectors, because these have an immediate impact on the quantity of production. While the model can replicate fluctuation in finite resources quite accurately, it can only explain 70% of volatility in renewable resources. However, its generation process is simplified in this model by including only capital and investments and hence no other input factors such as labor input and innovation (besides TFP). This simplification may be what is behind the unexplained effects. The same also applies for total energy prices where the model can explain 66% of volatility in the data.

In sum, total output fluctuation is mainly driven by TFP shocks in the final good sector in spite of energy being endogenously generated. However, the presence of energy seems to improve the explanation of fluctuation in durable goods. Volatility of this good is by far closer to its empirical target in German data in comparison to Dhawan and Jeske (2008), even without considering additional costs in adjusting its investments. TFP in final production can only explain 14% of volatility in non-durable goods and 8% of volatility in durable goods. In a model with an exogenous energy price process calibrated for Germany, Schmidt and Zimmermann (2005) can only account for 8% of output volatility for the time period 1987-2002. In the present model, volatility is only slightly below the target value by 8%.

Extending the baseline model by allowing to replenish the available stock of fossil resources, we introduce a further source of fluctuation to the model which directly affects the size of reserve stock of the finite resource. The respective simulated percentage standard deviations in column 7 confirm that most fluctuations generated by the model come closer to the target values in the data. The extended model slightly overestimates volatility in GDP, but at the same time it can account for 78% of the target value of fluctuation in non-durable goods. Durable goods and total energy output volatility are well generated. Only regarding the renewable energy output, the standard deviation declines and moves away from the empirical target in the data.

## 3.7 Dynamic Results

In this section, we examine the effects of changes in the productivity processes $A_{\mathrm{Y}}$, $A_{\mathrm{E}}$, $A_{\mathrm{F}}$, and $A_{\mathrm{N}}$ within the production functions, as well as a shock in the stock of fossil reserves $S$ by describing the impulse response functions. We neglect the effects of a shock in consumer taste in durable and non-durable goods as they are of minor interest when discussing the effects of endogenous energy generation.[8] The dynamic results are based on the calibrated and estimated values, where the size of a positive shocks corresponds to their individual standard deviations $\sigma_{\mathrm{Y}}$, $\sigma_{\mathrm{E}}$, $\sigma_{\mathrm{F}}$, $\sigma_{\mathrm{N}}$, and $\sigma_{\mathrm{S}}$ as shown in Table 3.4. The IRFs, depicted in Figures 3.1–3.5, aim at explaining two questions: Firstly, how do the endogenous variables respond to shocks in TFP and the stock of reserves. Secondly, to what extent do the responses differ when allowing for replenishment of the fossil reserve stock (red curve). For better visibility, we include the dynamics of both models in the same graph. Furthermore, we add the dynamics of the baseline model without durable goods (blue curve) to show the dynamic consequences when distinguishing between consumption goods with different properties.

### 3.7.1 Shock to TFP in the final good sector

Figure 3.1 shows the IRFs after a positive shock in total factor productivity in the final good sector. As expected, there is a positive effect on the sector's output because the same unit of all input factors becomes more productive, other things equal. At the same time, this leads to an increase in GDP because it is part of the national account. On the consumption side, there are more final goods to be consumed by households. On the expenditure side, as productivity of each input factor increases, the marginal products, and hence returns to capital and labor increase (consequently, households' incomes from this sector increase). As a result, households not only increase their expenditures for consumption goods to gain higer utility, but also increase the supply of capital due to the higher interest rates. Here, capital demand by the final good sector is growing over the initial 10 periods because optimization of investments is always lagging behind the adjustment of remaining variables due to the restrictions in the capital constraint. In contrast, the peak of supply and demand of labor occurs without a lag, because the input factor 'labor' is

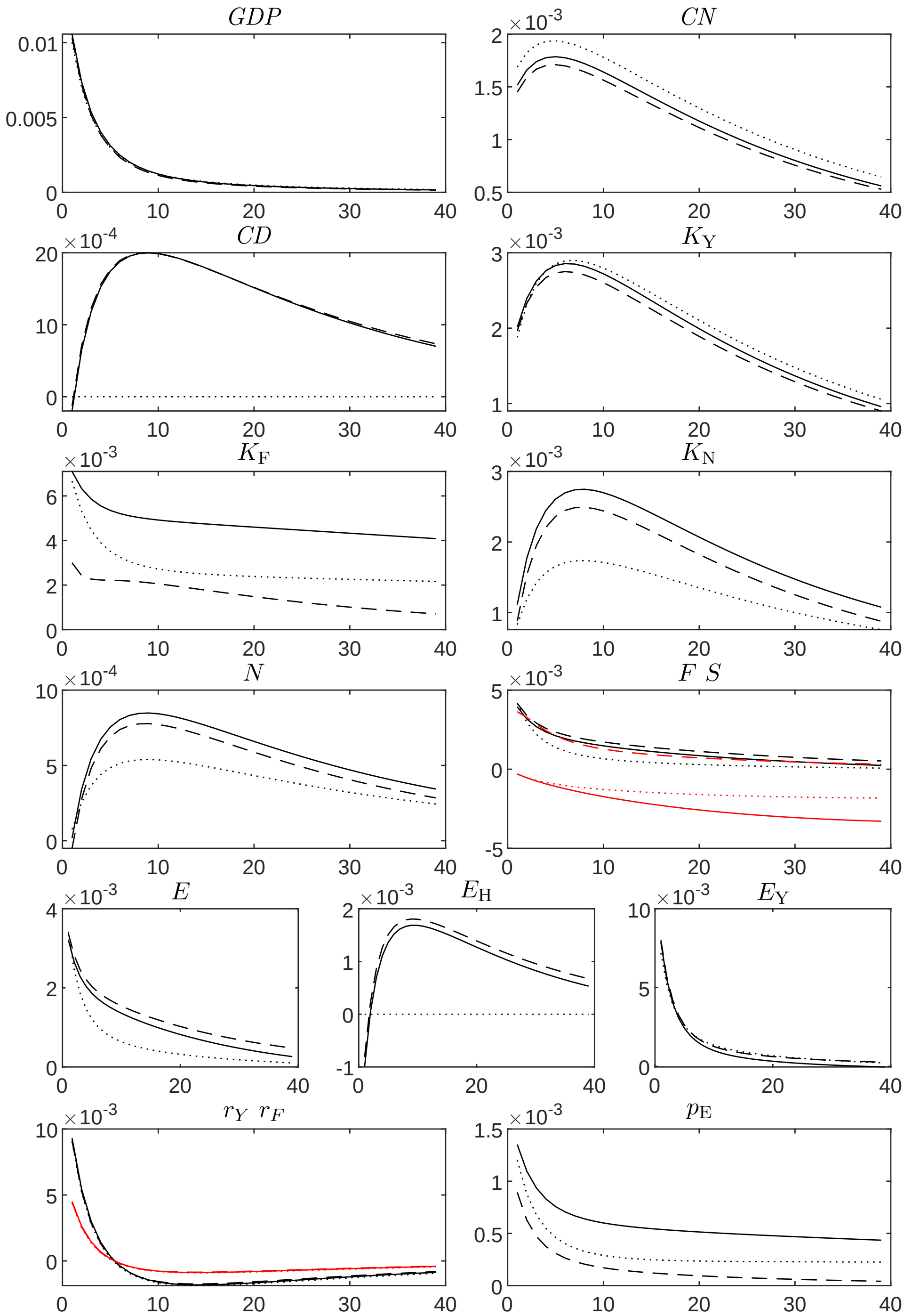
[8] The IRFs of shocks in consumer taste in durable and non-durable goods can be found in Appendix 3.G.

initially used as a substitute for the capital-energy bundle. Similarly, energy demand in final production increases due to the higher demand for non-durable goods by households but also due to its complementary link with capital. The complementary relationship also leads to a boost in the demand for energy by households, because consumption of durable goods increases due to higher income. In sum, the price for energy increases, which has direct impacts the energy and resources sectors.

The energy sector benefits from higher energy prices but also from the increase in demand for energy. In order to fulfill this demand, the sector has to generate more energy. There are two reasons why it is mainly the fossil energy sector that meets this higher demand: First, even though demand for physical capital increases in both intermediate energy sectors (in order to raise production along with higher capital returns), the renewable energy sector can only change its resource generation by adjusting capital investments, which always happens with a lag (just as physical capital in final production). On the contrary, the fossil energy sector is able to respond to these changes almost immediately, by higher depletion of reserves. Second, renewable intermediate energy has a lower output elasticity with constant returns to scale in overall energy production. Hence, higher demand of overall energy leads to an increase of intermediate energy by the same proportional change. But the absolute change of renewable intermediate energy is lower with respect to fossil intermediate energy. Over time, following the AR(1) process, TFP in the final good sector converges to its long-run steady state. Therefore, the amplitudes of the remaining variables diminish as well.

In the model with no durable goods (dotted curves in Figure 3.1), the qualitative effects slightly change in comparison to the baseline model. Households can consume only non-durable goods, with demands for these goods being higher in comparison to the baseline model. Consequently, final good production has a higher demand for labor, capital, and energy. But because households do not consume any energy products, the magnitudes of positive responses within the energy sectors are smaller.

Figure 3.1: Bayesian IRF: orthogonalized shock to $\varepsilon_Y$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

The dashed curves in Figure 3.1 describe the dynamics of the variables in the model, with the possibility to replenish the fossil resource stock. In comparison with the baseline model, the results show a shift towards non-renewable intermediate resources in the final energy sector. The finite intermediate energy sector has the possibility to immediately respond to changes in the demand for intermediate energy by raising the stock of finite resources through an intensification of R&D activity. As a direct consequence of the shock in TFP in the final good sector and the boost in demand for energy, the energy sector substitutes even more renewable intermediate energy with finite intermediate energy in comparison to the baseline model. This is because the usage of the latter is more efficient (less costs and no adjustment lag in all input factors). Positive capital investments are kept lower in comparison to the baseline model, as the reserve stock can be expanded. Although R&D certainly brings along additional costs for the finite intermediate energy sector, total costs can be kept down because less capital has to be used than without R&D. These savings are passed through to a lower price of energy paid by households and final good producers. Furthermore, output by the finite intermediate energy sector is higher, which leads to more energy generation by the energy sector. Hence, as a side effect, both energy consumers slightly increase their demand for energy. In sum, allowing for replenishment of the finite resource stock has no significant impacts on GDP but rather on the energy sectors. In particular the finite intermediate energy sector benefits.

### 3.7.2 Shock to TFP in the energy sector

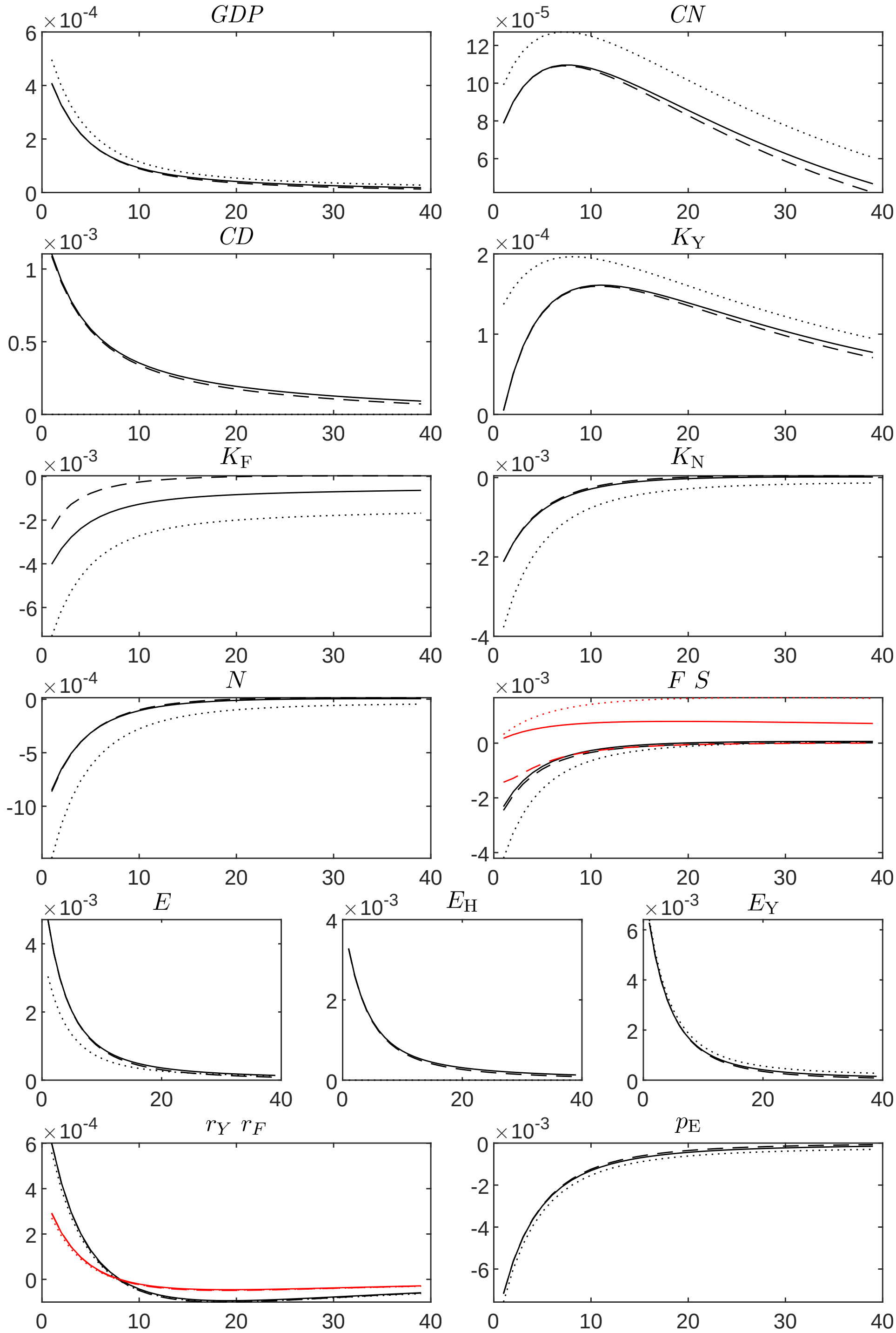
The dynamics of a positive shock in the energy sector are shown in Figure 3.2. As productivity and output in this sector increase, marginal productivities of input factors increase, creating an oversupply of total energy and finally resulting in a drop in energy prices. Consequently, because marginal costs of energy input decrease, the final good sector and households increase their demand of energy, which leads to higher investments in durable goods and increases the demand of the remaining input factors in the production function. Capital utilization increases due to the complementary link with energy. Labor increases, but at a significantly lower extent, to satisfy the demand of non-durable goods through higher production, which cannot be fulfilled by the capital-energy bundle alone. Overall, this has a positive effect on final output and GDP. Despite the increased energy output, the energy sector reduces its demand for intermediate energy due to higher TFP, which puts negative pressure on the price. The intermediate energy sectors are concerned with two impacts. On the one hand, less demand for their outputs leads to a reduction of input factors. On the other hand, the increase in demand for physical capital by the final good sector puts positive pressure on the price. Hence, it becomes more attractive for households to provide capital for final production, which leads to a withdrawal of capital from intermediate energy sectors. As a consequence, capital returns for assets employed in these sectors increase.

The renewable energy sector suffers significantly more than the fossil energy sector and looses shares to the latter. The ability to quickly adjust production by changing the degree of depletion of reserves provides the fossil energy sector with a flexible instrument and comparative advantage over the renewable energy sector. However, these effects diminish over the periods, as the economy converges back to its long-run steady state. The quantity of energy peaks instantaneously, because the shocked variable is present in the energy production sector, whose production function is not constrained by any time lag, opposite to a TFP shock in final production.

In the presence of no durable goods, the dynamics do not essentially change. Notably, the negative effects on the energy sector are higher in absence of a further energy purchaser such as households, in comparison to the baseline model. Consequently, only final production can take advantage of lower energy prices. As households consume one sort of goods only, non-durable consumption is higher, which leads to a more important role of final production. As a result, the reallocation of assets towards the more productive ones, namely physical capital in final production, is intensified. This is done at the expense of the intermediate renewable and finite energy sectors, whose capital stocks decrease significantly.

Allowing for replenishment of the fossil reserve stock does not significantly change the dynamics when comparing them to the baseline model. Solely variables related to the finite intermediate energy sector are affected. The resource stock can be enlarged through an intensification of R&D. As a result, the lack of physical capital due to the redistribution of assets by households can simply be substituted by extending the usage of the resource stock. Renewable energy responses are hardly affected, neither positively nor negatively, which shows that replenishment is directly passed through to the energy production and its purchasers.

Figure 3.2: Bayesian IRF: orthogonalized shock to $\varepsilon_E$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

### 3.7.3 Shock to TFP in the fossil energy sector

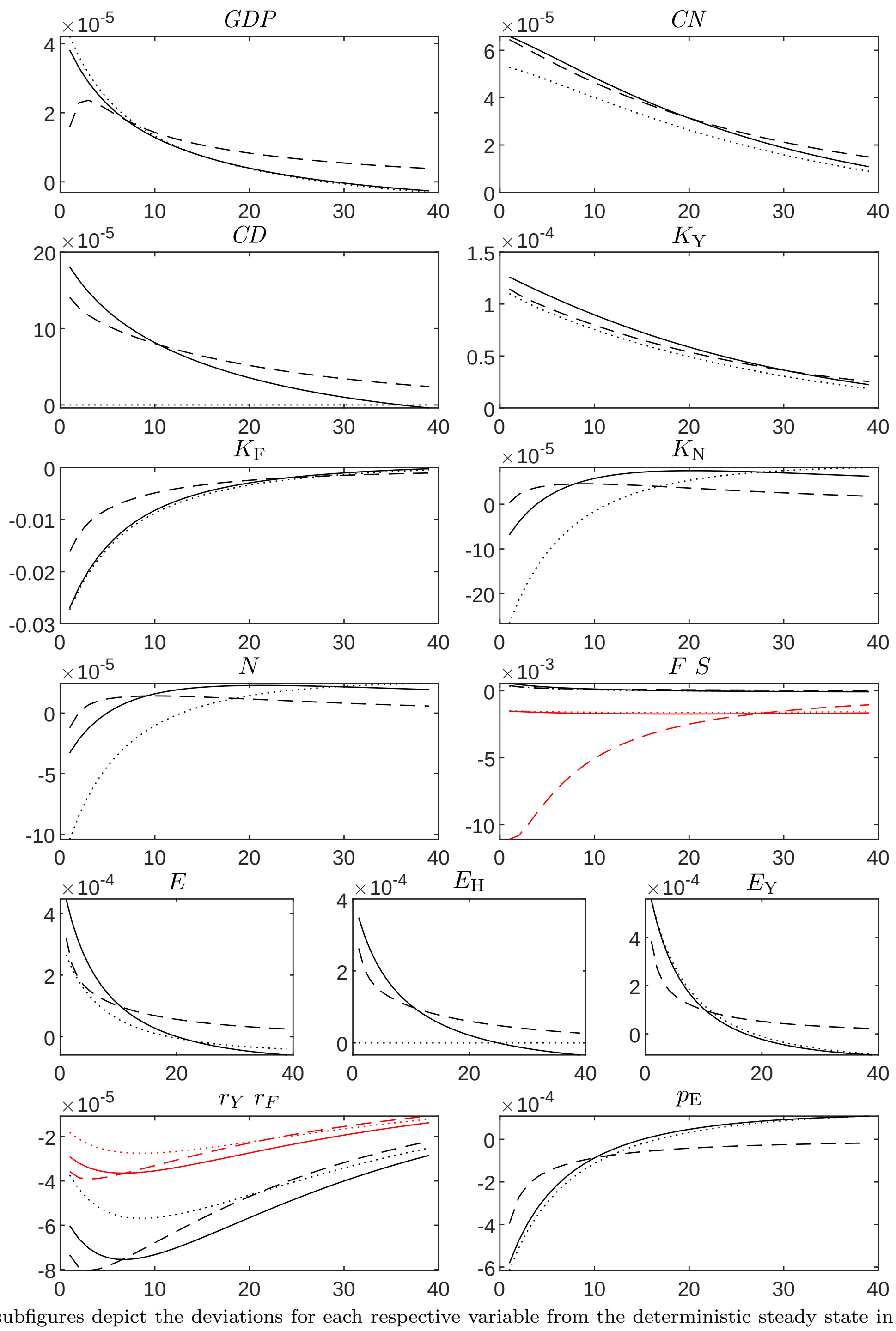
The effects of a positive TFP shock in the fossil energy sector are summarized in Figure 3.3. Initially, the increase in productivity boosts finite intermediate energy output. The response of demand by the energy sector is sedated, so market prices of finite intermediate energy drop to make up for imbalances between demand and supply. As a direct consequence, the depletion rate of reserves drops immediately because a higher TFP leads to an increase in output per unit of input. Since the stock reserves are limited with respect to time, a higher productivity allows the sector to save this valuable input factor. For the same reason, the physical capital is significantly reduced.

Due to lower costs, the final energy sector substitutes renewable energy with fossil energy, which puts downward pressure on the price of renewable intermediate energy and demand for physical capital in this sector. Overall, the final energy sector can pass through the cost saving to energy prices paid by final producers and households, because it acts under perfect competition. This leads to an increased demand in energy as well as consumption. Furthermore, the final good sector substitutes labor by both capital and energy because of cost and efficiency reasons. Households are additionally motivated to reallocate their assets towards the good producing sector, due to the lower demand for capital in the intermediate energy sectors. GDP is positively affected by the TFP shock in the finite intermediate energy sector. This is because of lower energy prices and the resulting higher demand by households for non-durable goods, and in particular for durables, whereas the latter acts as a complementary good with respect to energy.

As we have already seen in the previous cases of TFP shocks (in the final good sector and final energy sector), the consequences of a TFP shock in the finite intermediate energy sector in an economy without durable goods are not significantly different from those in the baseline model. Only for the direct competitor of finite intermediate energy, namely the sector generating renewable intermediate energy, are the negative responses intensified. Due to the missing demand for energy by households, the magnitude of total energy is slightly diminished.

Comparing the baseline model and the model to replenishment of the reserve stock, the dynamic responses do not deviate significantly. However, it is apparent that given the ability to rely on additional reserves, the finite energy sector can reduce the magnitude of negative response in capital and consequently the fluctuation in finite intermediate energy output. As a result, all the effects are smoothed over the observed periods. The initially strong fluctuations diminish and persist for longer time. Furthermore, we notice a light increase in GDP, which can be traced back to R&D activity and its positive contribution to the national account.

Figure 3.3: Bayesian IRF: orthogonalized shock to $\varepsilon_F$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

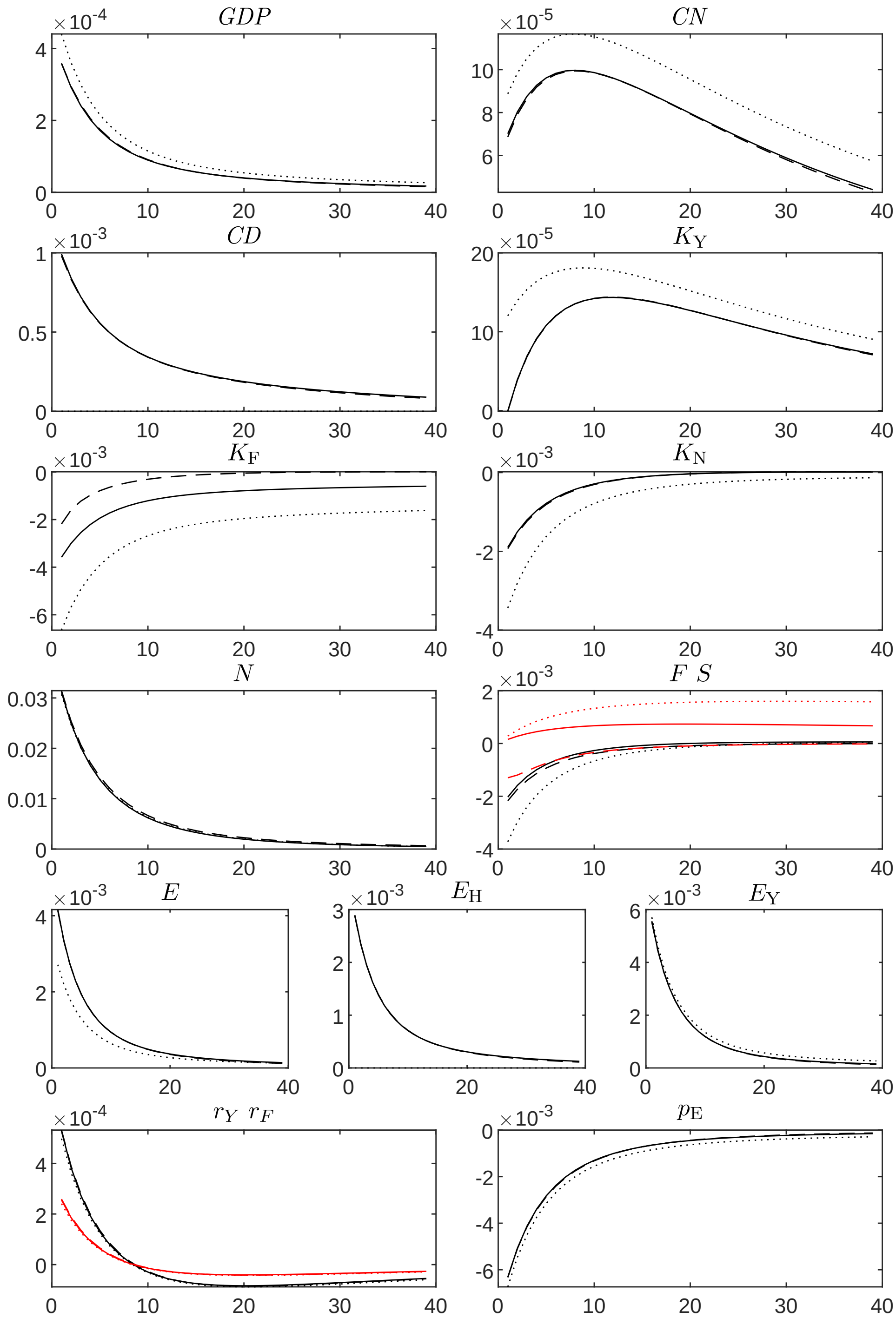
### 3.7.4 Shock to TFP in the renewable energy sector

The responses of a positive TFP shock in the renewable energy sector are similar to those in the previous case, where the finite energy sector was affected by productivity changes. Here, the boost in productivity increases output of the renewable energy sector (see Figure 3.4). Because demand by the energy sector does not respond immediately, the renewable intermediate energy price drops. Even though a single unit of capital becomes more productive, fewer physical capital units have to be demanded to produce the same output, leading to a reduction of capital investments. But other than the TFP shock in the fossil energy sector, the renewable energy sector cannot substitute the usage of capital. Due to this dependency, the marginal product of capital increases, and with it the capital unit costs, but to a lower extent in comparison to the returns to capital in the final good sector.

Again, low factor prices in the final energy sector are passed through to the lower price and higher demand of total energy. Households increase their consumption while final good producers increase production. Due to the demand for capital and its attractive interest returns, households reallocate their assets to final production, putting downward pressure on the interest rate. After around 10 periods, the marginal product of capital in the final good sector falls below that of the renewable and that of the finite energy sector. Consequently, households re-optimize their asset allocation by deducting capital from the final production sector, leading to a hump-shaped capital response curve. In an economy without durable goods, the effects are only intensified. However, there are no qualitative differences in comparison to the baseline model.

Increasing the stock of finite resources through R&D does not affect the output of the renewable energy sector at all. The same holds for the remaining variables, with the exception of assets employed in the finite intermediate energy sector. As the size of the available resource stock can be optimized by lowering the efforts in R&D, transforming resources to usable reserves, the sector can substitute capital losses much better in comparison to in the baseline model. At the same time, this means less capital returns for households which affects consumption expenditures in subsequent periods. However, this happens to such a low extent that GDP is not influenced significantly.

Figure 3.4: Bayesian IRF: orthogonalized shock to $\varepsilon_N$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

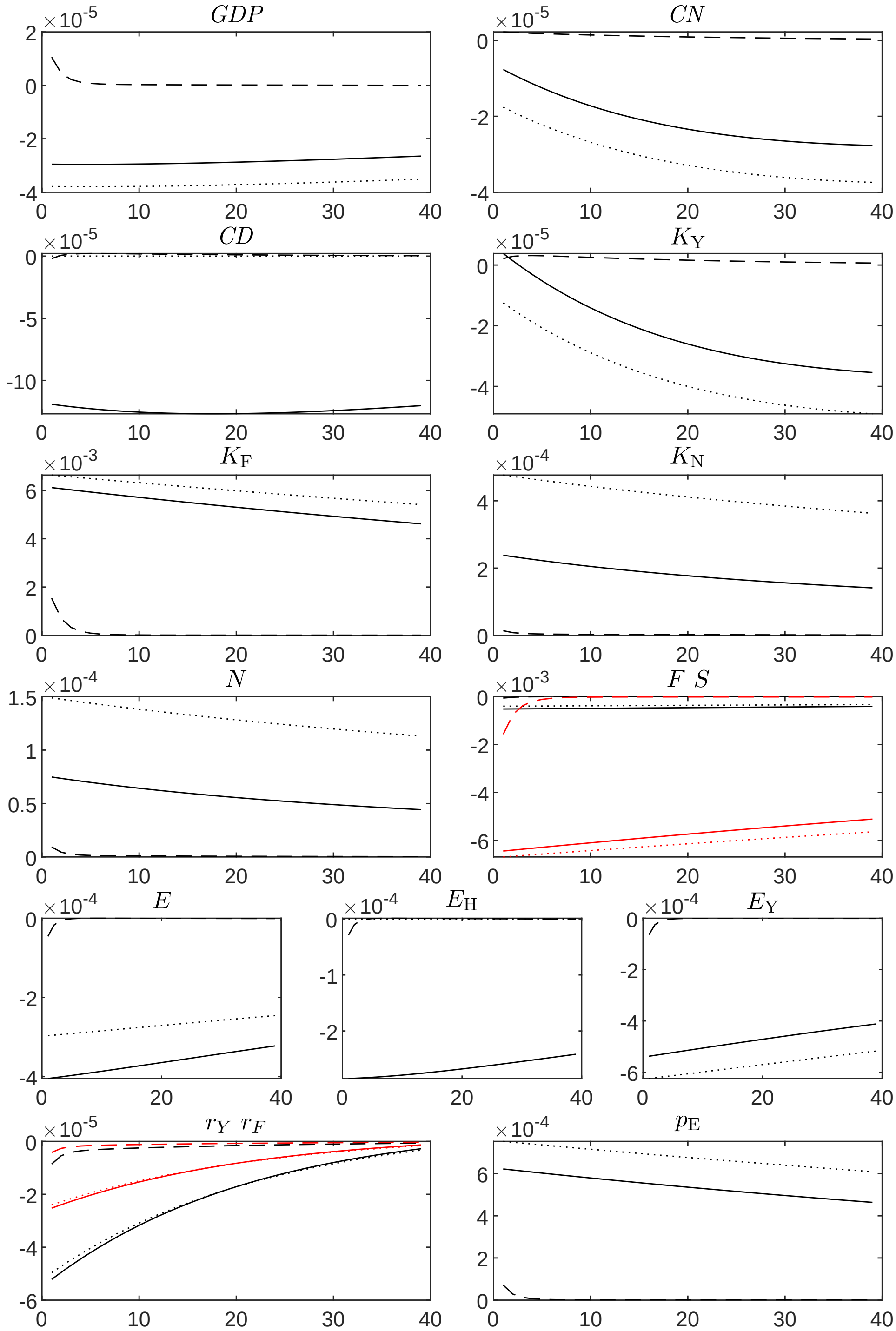
### 3.7.5 Shock to finite reserve stocks

Figure 3.5 shows the IRFs of a shock in the finite reserve constraint. A negative impact leads to a one-time unexpected reduction of reserve stock. The direct consequence is a drop in fossil intermediate energy and an increase in its price. Moreover, the finite energy sector increases its demand for capital as a substitute for the resource stock. At the same time, the energy sector substitutes fossil intermediate energy with its alternative input factor, namely renewable intermediate energy, whereby the renewable energy sector increases its energy generation by employing more capital. However, its output does not offset the loss in fossil energy as the degree of substitution is limited. Consequently, it is not avoidable that the supply of total energy drops, and hence the price of energy increases. Final good producers and households have to reduce their expenditures, leading to less consumption of non-durable and durable goods and a reduction of GDP. In contrast to the preceding shocks, an unexpected change in the reserve constraint has significantly longer persisting impacts on all macroeconomic variables. This can be attributed to the high weight of the fossil sector which is significantly dependent on the reserve stock, and whose reduction has persistent impact on finite intermediate energy generation.

In the model with replenishment of the finite resource stock, an intensification of R&D allows compensation of the loss in the fossil resource stock that comes along with the shock. As a result, the economy can return to its optimal path after some initial periods of adjustments. Moreover, as the high effort in R&D leads to a higher income for households, GDP can even benefit during the first periods.

Overall, the dynamic responses confirm the findings on the behavior of the disruption of fixed capital by Dhawan and Jeske (2008) due to the enlargement of the flexibility to re-balance the household's portfolio. This also explains the low weight of TFP shocks in the energy sectors, which are further discussed below. As there are more channels available to households, they are more flexible in their investment decision. Facing a shock in TFP (in $A_{\mathrm{E}}, A_{\mathrm{F}}, A_{\mathrm{N}}$) leads to adjustments of capital investments in final good production which, in turn, are dominated by adjustments of investments in durable goods by the households. To be more precise, the negative response of capital investments in the final good sector following a reduction in the non-renewable reserve stock are less strong than the reduction of durable goods purchases. Overall, the present paper allows for four channels to re-balance investments, while Dhawan and Jeske consider only two.

Figure 3.5: Bayesian IRF: orthogonalized shock to $\varepsilon_S$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

## 3.8 Shock Decomposition

### 3.8.1 Variance decomposition

Table 3.1 displays the importance of the effects of all seven shocks on the main endogenous variables in the model relative to each other. In other words, Table 3.1 shows the relative contribution of each shock to the variance in the observed variables. In contrast to other papers that determine energy prices exogenously (e.g. Dhawan and Jeske, 2008; Finn, 1995; Kim and Loungani, 1992; Schmidt and Zimmermann, 2005), we distinguish between several productivity shocks, which are allocated to the respective sectors' production or generation functions. As a result, energy and resource prices are endogenously determined as in Jacquinot et al. (2009) and Argentiero et al. (2018). Further sources of fluctuation are taste shocks in consumption goods and quantity shocks in reserved or resources stocks. To be able to observe the degree of influence of each shock over time, we compute the conditional variance decomposition for three different periods. The short-term horizon is defined by the decomposition after four periods, and the mid-term horizon after twelve periods respectively. Additionally, the long-term horizon is computed by an unconditional variance decomposition.

Unsurprisingly, shocks in total factor productivity ($\varepsilon_Y$,$\varepsilon_E$,$\varepsilon_F$,$\varepsilon_N$) have the most influential pressure on output in their respective sectors except for the final energy sector. On the one side, bounded fossil intermediate energy and its high share relative to renewable intermediate energy seems to impact final energy production significantly. We notice a significantly high share of fluctuation in TFP of finite intermediate energy contributing to fluctuations in total energy. On the other side, both the autoregressive coefficient and variance in the total factor productivity shock process of final energy are small and consequently do not boost output notably.

Apparently, productivity in final good production has the most important influence on overall GDP. It remains above 90%, although the share slightly decreases over the different time horizons as the influence of the remaining sectors can evolve with time. This is in line with previous literature such as Kim and Loungani (1992) and Dhawan and Jeske (2008), who detect TFP to be the main driving force behind output fluctuations. It even shows to have a high impact on consumption of non-durable goods, next to the shock in consumer taste ($\varepsilon_{T,CN}$). Its role is obvious, as it has a direct influence on consumption for two reasons. Firstly, the final good sector is the producer of this consumption good, and hence it has a direct impact on its quantity. Secondly, TFP of final production affects the marginal product of input factors, such as capital and labor, which are provided by households. Consequently, it alters households' available income and thus their spending capabilities. The remaining shock processes are negligible with respect to GDP.

Considering durable goods, the influence of each shock depends on the time periods. In the short-term, more than one-third of its variance is explained by the energy sectors. Durable goods respond to changes in quantity and prices of total energy due to its complementary link with energy. Over time, the degree of influence of these changes is reduced, as TFP of final good production becomes more important, because households substitute durable goods with non-durable goods.

Table 3.1: Conditional variance decommposition: baseline model

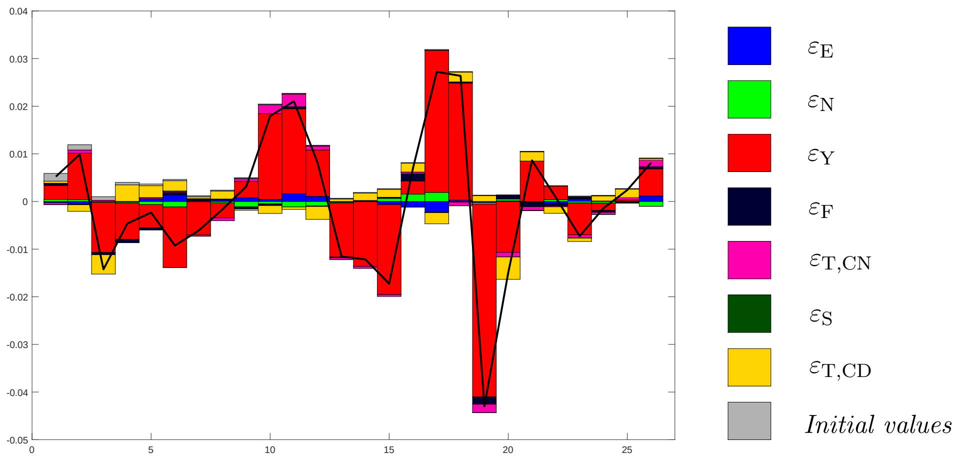
| | $\varepsilon_{\mathrm{E}}$ | $\varepsilon_{\mathrm{N}}$ | $\varepsilon_{\mathrm{Y}}$ | $\varepsilon_{\mathrm{F}}$ | $\varepsilon_{\mathrm{T,CN}}$ | $\varepsilon_{\mathrm{S}}$ | $\varepsilon_{\mathrm{T,CD}}$ |
|---|---|---|---|---|---|---|---|
| | | | short-term horizon | | | | |
| GDP | 0.25 | 0.19 | 96.67 | 0.13 | 1.19 | 0.00 | 1.57 |
| Non-durable goods | 0.25 | 0.19 | 60.45 | 0.14 | 29.58 | 0.00 | 9.38 |
| Durable goods | 20.09 | 16.23 | 21.33 | 0.63 | 26.74 | 0.35 | 14.62 |
| Total energy | 25.27 | 19.09 | 4.96 | 49.69 | 0.07 | 0.14 | 0.78 |
| Finite resources | 2.89 | 2.23 | 8.32 | 85.15 | 0.08 | 0.27 | 1.07 |
| Renewable resources | 0.05 | 99.88 | 0.01 | 0.00 | 0.01 | 0.00 | 0.04 |
| | | | mid-term horizon | | | | |
| GDP | 0.29 | 0.22 | 95.76 | 0.13 | 1.97 | 0.00 | 1.63 |
| Non-durable goods | 0.40 | 0.32 | 66.24 | 0.12 | 26.06 | 0.01 | 6.86 |
| Durable goods | 10.83 | 9.05 | 46.27 | 0.41 | 24.10 | 0.39 | 8.95 |
| Total energy | 25.99 | 20.54 | 8.58 | 43.23 | 0.31 | 0.42 | 0.92 |
| Finite resources | 3.59 | 2.92 | 13.58 | 77.39 | 0.40 | 0.83 | 1.30 |
| Renewable resources | 0.06 | 99.68 | 0.16 | 0.00 | 0.03 | 0.00 | 0.05 |
| | | | long-term horizon | | | | |
| GDP | 0.38 | 0.26 | 94.55 | 0.18 | 2.25 | 0.07 | 2.31 |
| Non-durable goods | 0.76 | 0.61 | 62.33 | 0.18 | 28.22 | 0.48 | 7.41 |
| Durable goods | 8.03 | 6.28 | 47.86 | 0.57 | 24.80 | 4.33 | 8.13 |
| Total energy | 23.64 | 16.53 | 7.26 | 46.04 | 0.66 | 4.92 | 0.95 |
| Finite resources | 2.86 | 2.08 | 10.07 | 74.64 | 0.78 | 8.37 | 1.19 |
| Renewable resources | 0.07 | 99.36 | 0.39 | 0.00 | 0.09 | 0.02 | 0.07 |

The short-term horizon is defined by the decomposition after four periods, the mid-term horizon after twelve periods. The long-term horizon is computed by an unconditional variance decomposition.

The remaining variables describing the outputs of the energy sectors develops similarly. Only renewable intermediate energy seems to be robust to alternative shocks besides its own TFP. Although TFP in the final energy and finite intermediate energy sectors contributes most to volatility in output in the production processes, TFP in final good production gains importance when comparing the short- and mid-term horizon. Final good producers respond to changes in the supply of energy by modifying their allocation of input factors, which in turn also affects the demand of energy. The impact of volatility in the stock of finite resources ($\varepsilon_S$) is negligibly small in the short- and mid-term. However, with respect to long-term development, it has a certain stake in total and finite intermediate energy output as it influences optimization of energy generation in the long-term.

### 3.8.2 Historical decomposition

Figure 3.1: Historical shock decomposition: national account *GDP*.




The black line depicts the business cycle of the corresponding variables (Figure 3.D.1), given the specified parameter set. The colored bars correspond to the contribution of the respective smoothed shocks. Grey colored initial values are part of the business cycles which are not explained by the smooth shocks, but rather by unknown initial values of the state variables.

To investigate how shocks affect deviation from the steady state of the German economy over the sample periods, Figure 3.1 presents the shock decomposition of historical business cycles of Germany between 1991-2016. Overall, a shock in the TFP of final good production ($\varepsilon_{\mathrm{Y}}$) still includes the most influential variance explaining the cycles of the economy. This confirms the variance decomposition analysis from Section 3.8.1. The shock shifting consumer taste ($\varepsilon_{\mathrm{T,CD}}$) is clearly less important for final goods than TFP, followed by shocks in the energy sectors. Finite energy productivity does not play an important role in GDP fluctuation, despite contributing the predominant share to final energy production, which is an input factor in final good production and consumption. On the one side, the final energy sector can substitute either of the forms of intermediate energy for the other, and hence it is flexible in responding to volatility in their supply. On the other side, energy consumers smooth the effects though shifting to substitutes. As a consequence, GDP is relatively robust to fluctuation in energy. The respective historical decomposition of GDP considering the model with resource exploration is equivalent.

In contrast, historically the decomposition of consumption goods is more heterogeneous (see Figures 3.2 and 3.3). Next to exogenous impacts through variation in consumer taste for non-durable and durable goods ($\varepsilon_{T,CN}$,$\varepsilon_{T,CD}$), volatility in the TFP of the energy sectors definitely plays a role in explaining historical fluctuations in consumption of durable goods. In particular, productivity shocks in the total energy ($\varepsilon_E$) and renewable intermediate energy sectors ($\varepsilon_N$) have a high weight, though their effects are offsetting each other. We argue that inconsistency in renewable energy generation is balanced by changing the overall productivity in total energy production. Decomposing historical non-durable goods data shows that energy price shocks or respectively shocks from energy related sectors are negligibly small.

Figure 3.2: Historical shock decomposition: non-durable goods *CN*.

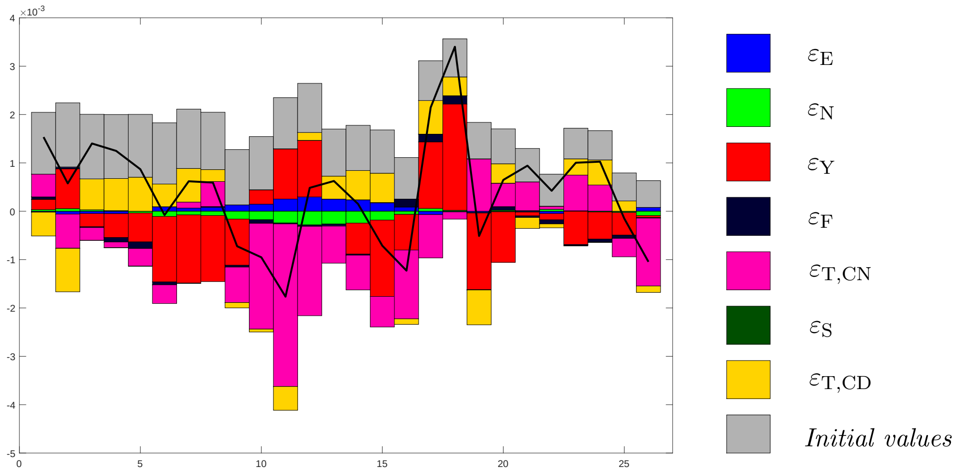
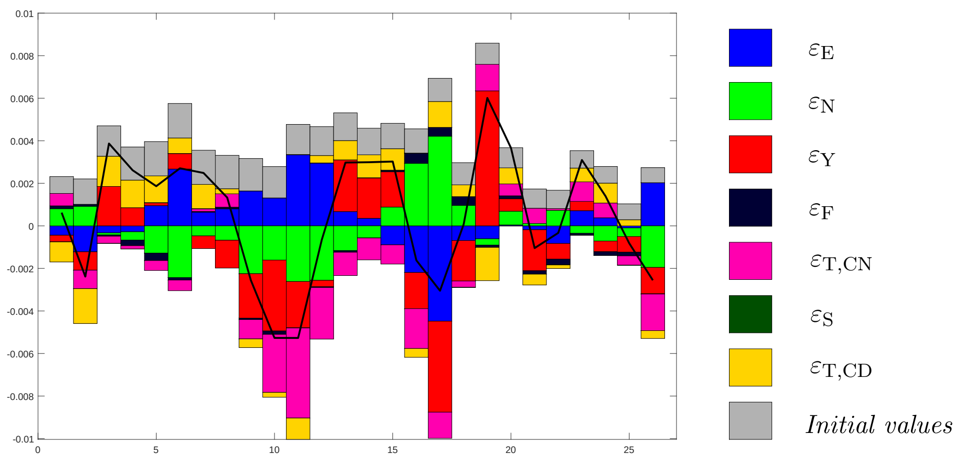


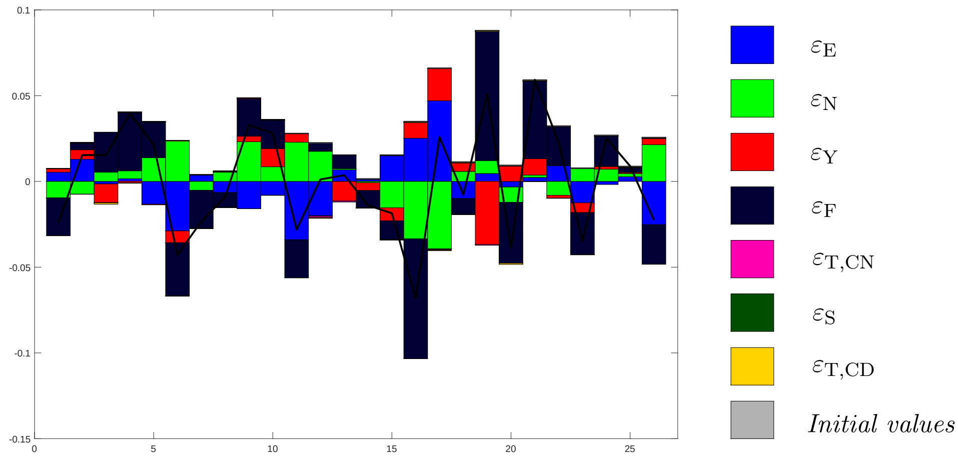

Figure 3.3: Historical shock decomposition: durable goods *CD*.




In addition to the previous decomposition of output variables, we examine what affects the price setting of total energy. Figure 3.4 shows that all energy related sectors have similar impacts on the price. Again, as detected in the decomposition of durable goods, TFP in total energy and renewable intermediate energy are almost neutralizing each other. With the exception of 2007, TFP in the finite intermediate energy sector is decisive for the direction of price development, which can be traced back to its dominant role in the energy mix.

Altogether, the variance decomposition shows that the share of fluctuations resulting from changes in productivity within the energy sectors is negligibly small when considering fluctuation in GDP. The overall share explaining business cycles comes from TFP in the final good sector. However, volatility originating from energy and its underlying sectors affects the behavior of households when optimizing their utility.

Figure 3.4: Historical shock decomposition: energy price $p_{\mathrm{E}}$.




## 3.9 Robustness

When calibrating the distribution and substitution parameters for the energy bundles in the utility function of households and final good production function, there is one degree of freedom each. Consequently, there is an initial value problem shown by:

$$\theta = \frac{1-\beta\left(1-\delta^{CD}\right)}{1-\beta(1-\delta^{CD})+\beta\frac{CD}{E_{\mathrm{H}}}^{\zeta-1}} \quad \text{and} \quad \eta = \frac{\alpha\left(\frac{E_{\mathrm{Y}}}{GDP}\right)^{-1}-1}{\alpha\left(\frac{E_{\mathrm{Y}}}{GDP}\right)^{-1}-1+\left(\frac{K_{\mathrm{Y}}}{E_{\mathrm{Y}}}\right)^{\nu}}.$$

Thus, we examine robustness by checking whether the volatility results are sensitive to alternative parameters. To do so, we choose alternative parameters at the higher and lower range around the benchmark value. We set the elasticity of substitution in the energy-capital bundle of firms $\{\nu\}$ to unity, 0.59, and 0.25 apart from the benchmark of 0.87, or respectively $\zeta \in (0, -0.15, -0.7, -3)$. These values are similar to those of Kim and Loungani (1992) and Dhawan and Jeske (2008) whose benchmark value of 0.59 corresponds to real values of the US economy. For the elasticity of substitution of energy demand and durable goods, we consider values of unity, 0.59, and 0.25, respectively $\zeta \in (0, -0.7, -0.2875)$.

Table 3.1 summarizes the percentage standard deviation of all 12 combinations of substitution parameters for both the baseline and the replenishment model, when all shocks are present. Apart from total energy prices, the moments seem to be robust to variation in both substitution parameters. It is worth to mention that we fully re-estimate all parameters that are not fixed according to the estimation process described in Section 3.5. Hence, also the posterior values of the remaining parameters can deviate from those of the baseline model, to capture the salient features of the data.

Table 3.1: Robustness test: volatility shares

| | baseline model | | | | replenishment model | | | |
|---|---|---|---|---|---|---|---|---|
| | $\nu = 0$ | $\nu = -0.15$ | $\nu = -0.7$ | $\nu = -3$ | $\nu = 0$ | $\nu = -0.15$ | $\nu = -0.7$ | $\nu = -3$ |
| $\zeta = 0$ | | | | | | | | |
| GDP | 1.48 | 1.44 | 1.47 | 1.58 | 1.48 | 1.49 | 1.50 | 1.50 |
| Non-durable goods | 0.63 | 0.65 | 0.65 | 0.64 | 0.66 | 0.66 | 0.68 | 0.65 |
| Durable goods | 2.93 | 2.78 | 2.79 | 2.82 | 2.96 | 2.92 | 2.89 | 2.95 |
| Total energy | 1.90 | 1.89 | 1.84 | 1.83 | 1.87 | 1.86 | 1.84 | 1.77 |
| Finite resources | 1.89 | 1.92 | 1.88 | 1.85 | 1.90 | 1.89 | 1.87 | 1.80 |
| Renewable resources | 3.88 | 3.85 | 3.90 | 3.69 | 3.89 | 3.90 | 3.74 | 3.84 |
| Labor | 0.97 | 0.95 | 0.96 | 1.02 | 0.97 | 0.97 | 0.98 | 0.96 |
| Total energy prices | 2.10 | 2.16 | 2.36 | 2.90 | 2.09 | 2.17 | 2.34 | 2.80 |
| $\zeta = -0.7$ | | | | | | | | |
| GDP | 1.47 | 1.49 | 1.49 | 1.49 | 1.44 | 1.50 | 1.48 | 1.49 |
| Non-durable goods | 0.62 | 0.62 | 0.62 | 0.64 | 0.66 | 0.68 | 0.63 | 0.66 |
| Durable goods | 2.99 | 2.98 | 2.95 | 2.90 | 2.87 | 2.91 | 2.99 | 2.81 |
| Total energy | 1.85 | 1.84 | 1.78 | 1.73 | 1.80 | 1.89 | 1.80 | 1.67 |
| Finite resources | 1.85 | 1.84 | 1.80 | 1.83 | 1.84 | 1.93 | 1.86 | 1.76 |
| Renewable resources | 3.96 | 3.95 | 3.88 | 3.63 | 3.80 | 3.84 | 3.75 | 3.75 |
| Labor | 0.96 | 0.98 | 0.97 | 0.95 | 0.95 | 0.99 | 0.96 | 0.96 |
| Total energy prices | 2.43 | 2.51 | 2.80 | 3.61 | 2.38 | 2.59 | 2.86 | 3.53 |
| $\zeta = -2.875$ | | | | | | | | |
| GDP | 1.46 | 1.38 | 1.55 | 1.45 | 1.44 | 1.53 | 1.49 | 1.52 |
| Non-durable goods | 0.64 | 0.63 | 0.65 | 0.64 | 0.66 | 0.65 | 0.64 | 0.67 |
| Durable goods | 2.96 | 2.86 | 2.71 | 2.83 | 2.85 | 2.96 | 3.08 | 2.87 |
| Total energy | 1.79 | 1.82 | 1.86 | 1.75 | 1.77 | 1.78 | 1.81 | 1.81 |
| Finite resources | 1.81 | 1.90 | 1.88 | 1.88 | 1.83 | 1.90 | 1.88 | 1.94 |
| Renewable resources | 3.97 | 3.90 | 3.54 | 3.81 | 3.74 | 3.69 | 3.75 | 3.72 |
| Labor | 0.95 | 0.91 | 1.01 | 0.93 | 0.95 | 1.00 | 0.96 | 0.98 |
| Total energy prices | 3.06 | 3.27 | 4.05 | 5.83 | 2.99 | 3.20 | 3.94 | 6.01 |

Values denote the simulated results of percentage standard deviation (2nd moment) over 1000 periods using an HP-filter.

Nevertheless, the moments are close to their empirical targets, which conterminously means that the responses of the main variables are robust. For non-durable goods, we can only explain a maximum of 78% (82% with replenishment) of the desired volatility. Hence, households are still very strong with smoothing their consumption expenditures, which is line with Dhawan and Jeske (2008). But contrasting their findings, we cannot find an excess of volatility in (total) energy use. The explanation for this is that in our model, the quantity of energy is endogenously regulated by a set of additional sectors that are, through substitution, more flexible in responding to volatility in factor prices or quantities. In the model by Dhawan and Jeske, the demand of energy responds to energy prices, which follows an autoregressive process with exogenous shocks.

Table 3.1 also shows that increasing the degree of complementarity, volatility in total energy prices increases and approaches its empirical target. This particularly applies to the increase in the substitution parameter of capital and energy. According to this, the more complementary the link between physical capital and energy in final good production, the more volatile total energy prices. A firm's energy use is too volatile as it supplements demand for capital, for instance after a TFP shock in final production. This fluctuation is passed through to create excess volatility in total energy prices. A reduction of the complementary link between energy and capital leads to less volatility in energy prices, while the amplitudes of business cycles in GDP or consumption do not change significantly.

Inspecting the sensitivity analysis of the model with replenishment, we cannot notice any discrepancy to the findings of the respective baseline model. Only for goods consumption do we observe a slight increase in volatility. This is closely linked to additional research costs, which go along with R&D activity when transforming resources into reserves in the finite intermediate energy sector. Households receive an additional source of passive income, which is primarily used for consumption expenditures.

Furthermore, when indicating unity in the elasticity of substitution within the energy bundle as priors, Bayesian estimation leads to the parameter becoming more negative. This indicates support for our initial assumption that there is a complementary relationship between energy and the directly linked input factor, namely capital and durable goods.[9]

## 3.10 Conclusion

We have constructed an RBC model with endogenous energy generation from various different resources. The aim of this paper has been to examine the influence of several shocks and their transmission channels impacting an economy with an extended energy sector. Usable energy is generated from fossil intermediate energy and renewable energy which are each endogenously mined or generated in separate sectors. To avoid exaggerated disruptive investment dynamics, households can invest in a durable good stock next to the usual physical assets for each production sector.

In our estimated RBC model, Bayesian estimation confirms a complementary relationship between durable goods and energy consumption in the household sector as well as between physical capital and energy consumption in the final good sector. We find that a TFP shock in the (final and intermediate) energy sectors has a larger effect on durable good purchases than on capital investments in the final good production. Nevertheless, even in the model at hand with endogenous price determination of energy, TFP in final good production is still the major contributor

[9] The posterior results from the Metropolis-Hasting estimation are available upon request.

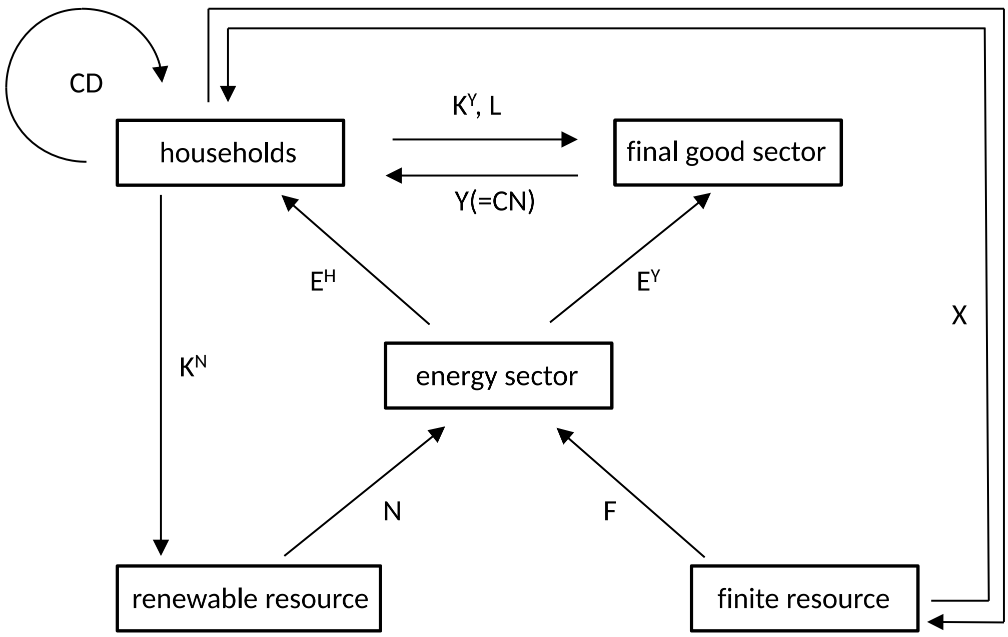
to business cycle formation of the national account, confirming existing literature. However, the explanation of theoretical moments can be essentially improved by distinguishing between durable and non-durable goods, taking energy consumption into account. Moreover, despite allowing the replenishment of constrained fossil stock in an extension, the dynamic response of the variables do not deviate from the baseline model. Solely for goods consumption do we notice a slight increase in volatility resulting from costly R&D, which raises the income of households.

The framework in the present paper can be extended towards several directions. By investigating policy strategies to regulate the usage of different sources of intermediate energy, instruments such as taxes or subsidies can be applied to perform artificial market imperfections. Under this aspect, it is interesting to analyze the inequality of welfare with heterogeneous households which may change on the basis of the corresponding policy instrument. Furthermore, as this model has ignored environmental consequences resulting from different types of energy generation, a consideration of negative externalities and its impact on decision making is left for further research.

# Appendix

## 3.A Model Overview

Figure 3.A.1: Model overview.




## 3.B Bayesian Estimation

To derive the posterior distribution of the parameters, we proceed in two steps. Firstly, we derive the mode of the posterior distribution of the model's parameters using the Bayesian estimation method. Secondly, we apply Markov Chain Monte Carlo (MCMC) simulation methods to obtain the posterior distribution. In the following, we discuss the full procedure:
Employing Bayesian methods allows us to link two approaches to determine the parameters of the model. First, the specification of prior information is obtained from calibration, e.g. though earlier studies of less complex models at the micro and macro level. Next, by using the maximum likelihood approach, the model is confronted with data to estimate the parameters. By combining both approaches, the priors affect the likelihood function in order to weight certain areas of the parameter subspace. This procedure is also known as the Bayes theorem.

The log-linearized model is linked to the data through the following measurement equation:

$$Y_t = Cy_t + \mu_t$$

where $Y_t$ describes the variables from observable data, $y_t$ describes the model variables, $C$ is a matrix mapping the models endogenous variables, and $\mu_t$ characterizes the iid measurement errors. We assume that the log-likelihood function of $Y_t$ is conditional on the vector parameters $\theta \in \Theta$ and thus, the corresponding log-likelihood, using the Kalman-Filter, is expressed by

$$\mathcal{L}\left(\theta|Y_T\right) = -\frac{Tn}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\ln\left|\sum\nolimits_{Y_t,t|t-1}\right| - \frac{1}{2}\sum_{t=1}^{T} e'_{t|t-1}\sum\nolimits_{Y_t,t|t-1}^{-1} e_{t|t-1} \quad (3.B.1)$$

where $Y_T = \{Y_1, Y_2, ..., Y_T\}$ expresses the set of observable variables $Y_t$ from the measurement equation, $n$ is the number of observable variables, $\sum_{Y_t,t|t-1}$ is a predictor of the variance-covariance matrix of the one-step-ahead forecast, and $e_{t|t-1}$ is a vector of the one-step-ahead forecast errors from using parameters $\theta$ to predict sample variables $Y_t$.

Now, we can combine the likelihood function (3.B.1) with the prior density $p(\theta)$ (defined according to prior kernel and values in Section 3.5.4) using the Bayesian theorem in order to obtain the posterior density, given by:

$$p\left(\theta|Y_T\right) = \frac{\mathcal{L}\left(\theta|Y_T\right)p\left(\theta\right)}{\int_{\Theta}\mathcal{L}\left(\theta|Y_T\right)p\left(\theta\right)d\Theta} \quad (3.B.2)$$

where the denominator denotes the marginal density of the data, conditional on the model. The log posterior kernel can be expressed as:

$$\ln\mathcal{K}\left(\theta|Y_T\right) = \ln\mathcal{L}\left(\theta|Y_T\right) + \ln p\left(\theta\right)$$

By maximizing this kernel with respect to $\theta$, we obtain the mode of the posterior distribution. Unfortunately, it is difficult to obtain the closed-form solution for the posterior distribution (3.B.2). Therefore, the distribution is approximated numerically, using the Metropolis-Hastings (MH) sampling method (a heuristic rejection sampling algorithm). The MH is a MCMC method, which generates draws from a distribution that is unknown at the outset, eventually exploring the target posterior distribution (black lines in Figures 3.C.1–3.C.3). The posterior mode, obtained through the maximum likelihood method, is used as a starting value to generate the draws. In the following, the implementation of the MCMC-HC procedure is briefly described. An alternative, detailed description of the solution strategy is also reported by An and Schorfheide (2007), Griffoli (2007), Adjemian et al. (2011), and Marto (2014).

Step 1: Starting from an arbitrary point (this is usually the posterior mode), steps 2–4 are run in a loop large enough to build a histogram of retained draws.

Step 2: Draw a proposal from a jumping distribution

$$J\left(\theta^{*}|\theta_{t-1}\right)=\mathcal{N}\left(\theta_{t-1},c\sum\nolimits_{m}\right),$$

where $c$ is a scale factor and $\sum_{m} = H\left(\theta_{m}|Y_{T}\right)^{-1}$ is the inverse of the Hessian computed at the posterior mode $\theta_m$.

Step 3: Compute the acceptance ratio

$$\Omega=\frac{p\left(\theta^{*}|Y_{T}\right)}{p\left(\theta^{t-1}|Y_{T}\right)}=\frac{\mathcal{K}\left(\theta^{*}|Y_{T}\right)}{\mathcal{K}\left(\theta^{t-1}|Y_{T}\right)}$$

Step 4: Dependent on $\Omega$, accept or discard $\theta^*$ according to:

$$\theta^{t}=\begin{cases}\theta^{*} & \text{with probability}\min(\Omega,1)\\ \theta_{t-1} & \text{otherwise.}\end{cases}$$

Step 5: After the loop, compute the mean of the histogram of retained draws, reflecting the posterior distribution of $\theta$.

We implement the procedure on four chains with 100,000 iterations each (removing the first 50,000 observations from each chain to avoid any dependency from the initial conditions). The scale factor $c$ has been chosen in such a way that the acceptance rate $\Omega$ of each chain is around 25%, which comes close to the desired acceptance rate of 23% (Roberts et al., 1997).

## 3.C Priors and Posteriors Distributions

Figure 3.C.1: Priors and posteriors 1.

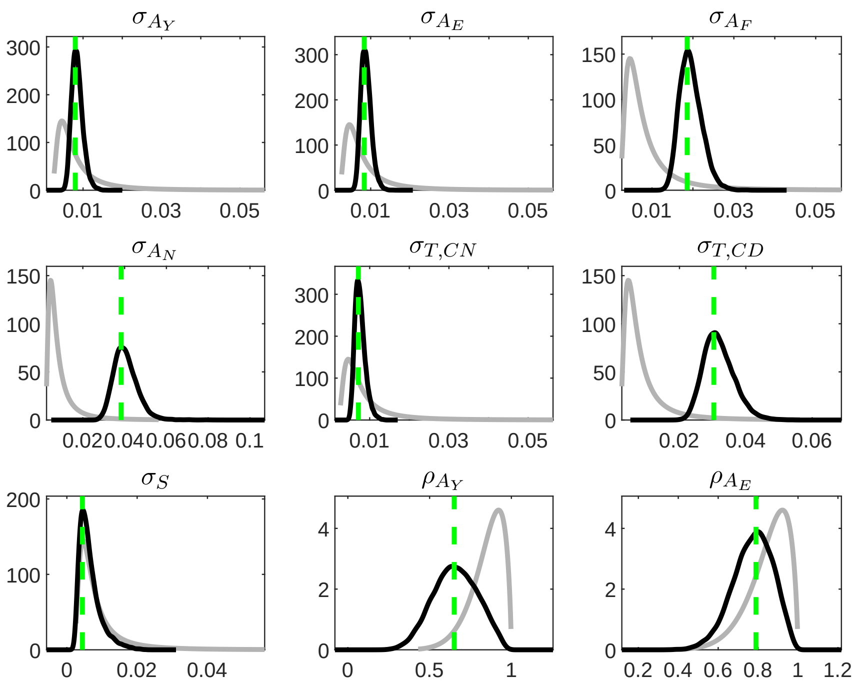



Figure 3.C.2: Priors and posteriors 2.

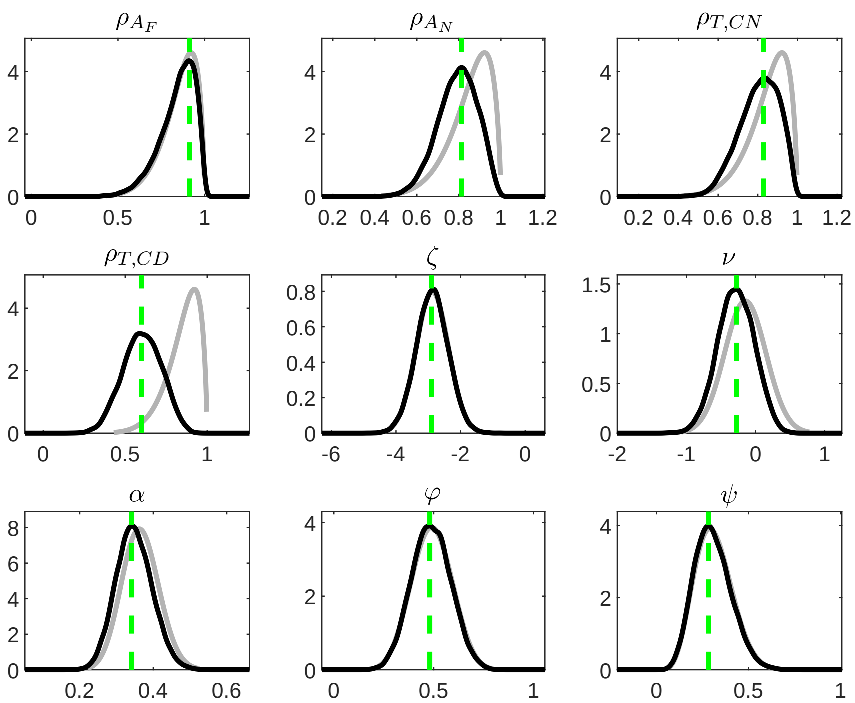
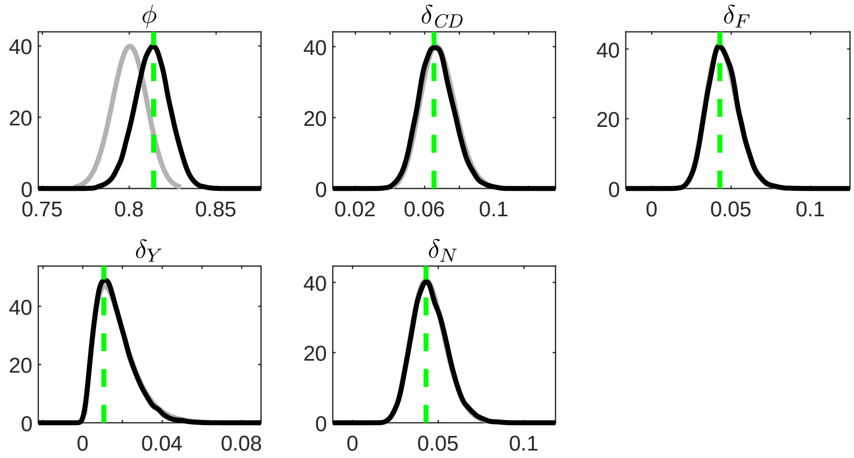
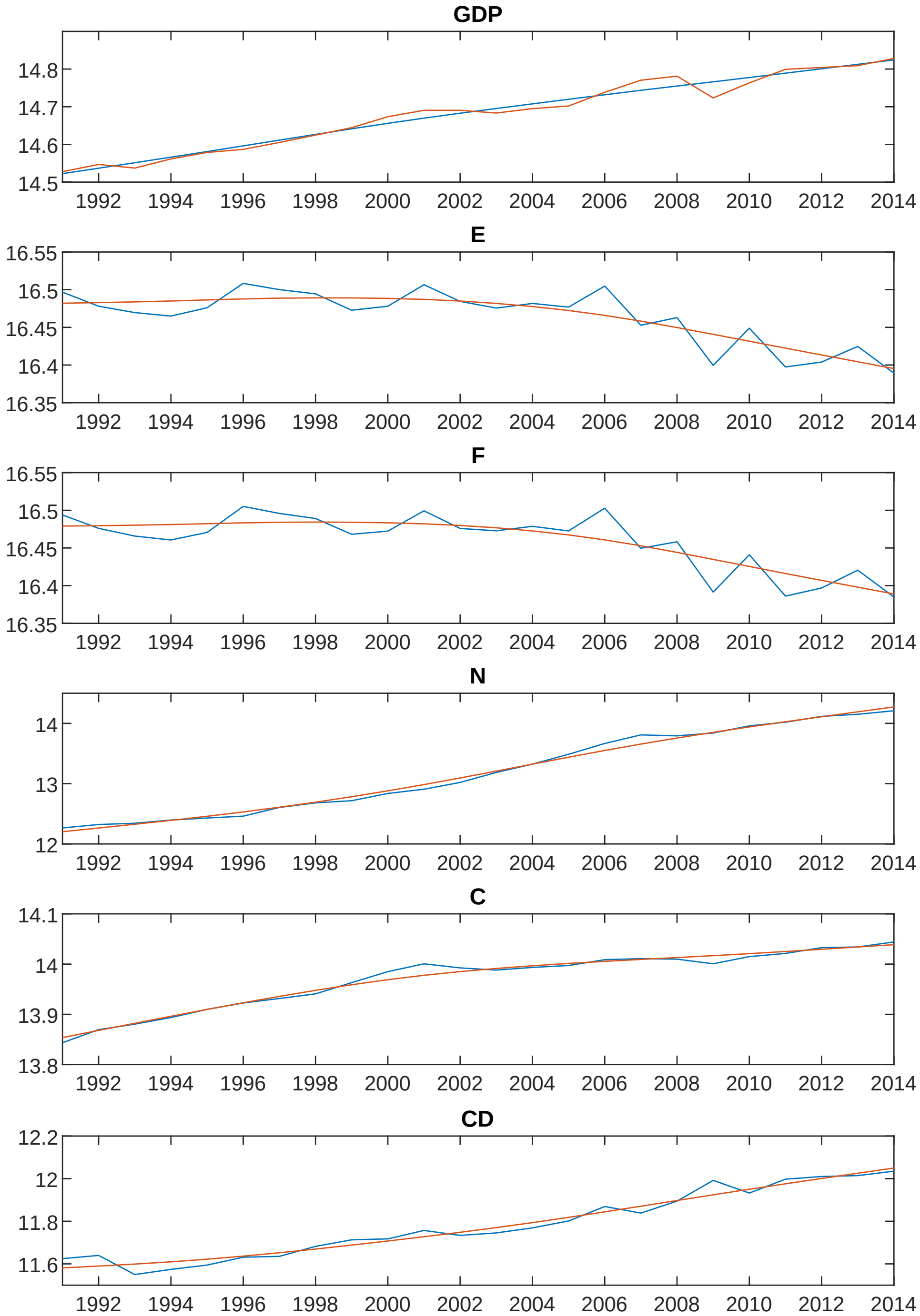


Figure 3.C.3: Priors and posteriors 3.



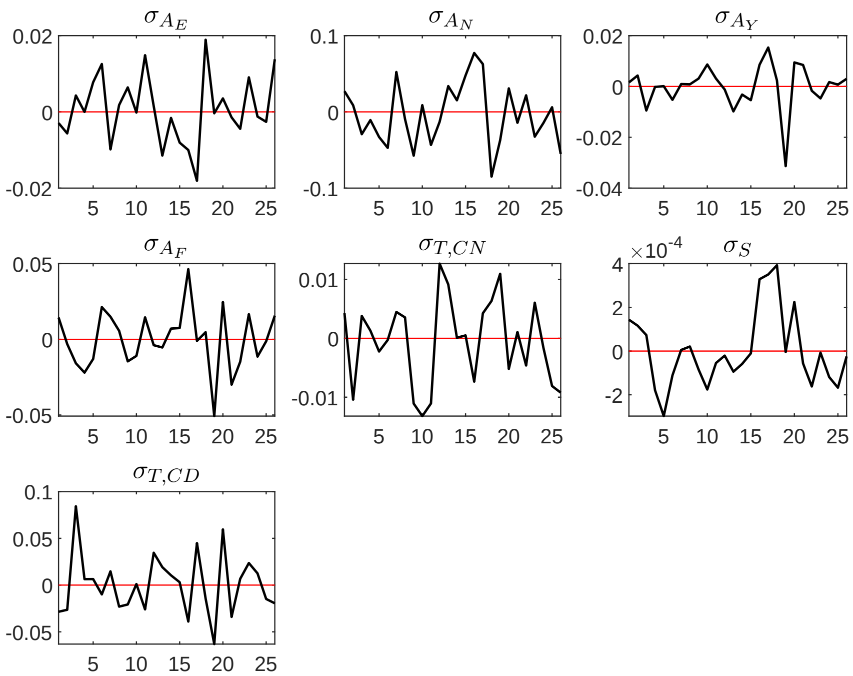

The gray line shows the prior density defined in section 3.5.4 while the black line shows the density of the posterior distribution. The dashed green line marks the posterior mode.

## 3.D Derivation of Business Cycles




Business cycles are derived by using an HP-filter with $\lambda = 100$ applied to the annual data of GDP, total energy (E), fossil energy (F), renewable energy (N), non-durable consumption (C), and durable consumption (CD).

Figure 3.D.2: Smoothed shocks.




The smoothed shocks plots show the best guess for the structural shocks given the observed data which is derived from the Kalman smoother at the posterior mean.

## 3.E Estimation Results of Model with Replenishment

Table 3.E.1: Results from Metropolis-Hastings (parameters).

| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $\rho_{A_{\mathrm{Y}}}$ | beta | 0.850 | 0.1000 | 0.645 | 0.1348 | 0.4387 | 0.8825 |
| $\rho_{A_{\mathrm{E}}}$ | beta | 0.850 | 0.1000 | 0.774 | 0.1009 | 0.6148 | 0.9414 |
| $\rho_{A_{\mathrm{F}}}$ | beta | 0.850 | 0.1000 | 0.841 | 0.1041 | 0.6908 | 0.9936 |
| $\rho_{A_{\mathrm{N}}}$ | beta | 0.850 | 0.1000 | 0.795 | 0.0931 | 0.6509 | 0.9492 |
| $\rho_{\mathrm{T,CN}}$ | beta | 0.850 | 0.1000 | 0.798 | 0.1016 | 0.6388 | 0.9599 |
| $\rho_{\mathrm{T,CD}}$ | beta | 0.850 | 0.1000 | 0.603 | 0.1196 | 0.4049 | 0.7979 |
| $\zeta$ | norm | -2.875 | 0.5000 | -2.886 | 0.5012 | -3.6811 | -2.0324 |
| $\nu$ | norm | -0.150 | 0.3000 | -0.274 | 0.2671 | -0.7133 | 0.1638 |
| $\alpha$ | beta | 0.365 | 0.0500 | 0.346 | 0.0484 | 0.2703 | 0.4282 |
| $\varphi$ | beta | 0.490 | 0.1000 | 0.475 | 0.0980 | 0.3149 | 0.6362 |
| $\psi$ | beta | 0.310 | 0.1000 | 0.309 | 0.1013 | 0.1422 | 0.4704 |
| $\phi$ | beta | 0.800 | 0.0100 | 0.813 | 0.0098 | 0.7976 | 0.8296 |
| $\upsilon$ | invg | 2.000 | 0.2000 | 2.005 | 0.1980 | 1.6909 | 2.3309 |
| $\delta^{CD}$ | beta | 0.068 | 0.0100 | 0.067 | 0.0097 | 0.0507 | 0.0824 |
| $\delta^{F}$ | beta | 0.045 | 0.0100 | 0.045 | 0.0098 | 0.0286 | 0.0602 |
| $\delta^{Y}$ | beta | 0.017 | 0.0100 | 0.016 | 0.0089 | 0.0025 | 0.0289 |
| $\delta^{N}$ | beta | 0.045 | 0.0100 | 0.045 | 0.0100 | 0.0286 | 0.0610 |

HPD inf (HPD sup) correspond to the lowest (highest) points of the highest posterior density with a 95% confidence interval.

Table 3.E.2: Results from Metropolis-Hastings (standard deviation of structural shocks).

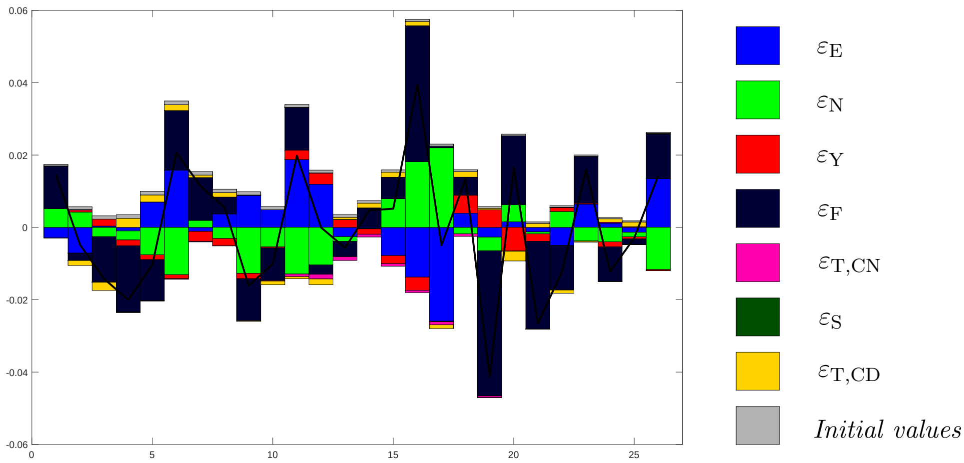
| | Prior | | | Posterior | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | Mean | Stdev. | Mean | Stdev. | HPD inf | HPD sup |
| $\sigma_{A_\mathrm{Y}}$ | invg | 0.010 | Inf | 0.008 | 0.0014 | 0.0062 | 0.0107 |
| $\sigma_{A_\mathrm{E}}$ | invg | 0.010 | Inf | 0.009 | 0.0013 | 0.0067 | 0.0108 |
| $\sigma_{A_\mathrm{F}}$ | invg | 0.010 | Inf | 0.019 | 0.0027 | 0.0151 | 0.0236 |
| $\sigma_{A_\mathrm{N}}$ | invg | 0.010 | Inf | 0.040 | 0.0057 | 0.0314 | 0.0490 |
| $\sigma_{\mathrm{T,CN}}$ | invg | 0.010 | Inf | 0.008 | 0.0013 | 0.0055 | 0.0096 |
| $\sigma_{\mathrm{T,CD}}$ | invg | 0.010 | Inf | 0.032 | 0.0046 | 0.0245 | 0.0393 |
| $\sigma_{\mathrm{S}}$ | invg | 0.010 | Inf | 0.009 | 0.0075 | 0.0023 | 0.0186 |
| $\sigma_{\mathrm{V}}$ | invg | 0.010 | Inf | 0.008 | 0.0048 | 0.0024 | 0.0141 |

HPD inf (HPD sup) correspond to the lowest (highest) points of the highest posterior density with a 95% confidence interval.

# 3.F Variance Decomposition

## 3.F.1 Baseline model

Figure 3.F.1: Historical shock decomposition.




(a) Historical shock decomposition: $E$.

The black line depicts the business cycle of the corresponding variables (Figure 3.D.1), given the specified parameter set. The colored bars correspond to the contribution of the respective smoothed shocks. Grey colored initial values are part of the business cycles which are not explained by the smooth shocks, but rather by unknown initial values of the state variables.

### 3.F.2 Model with replenishment

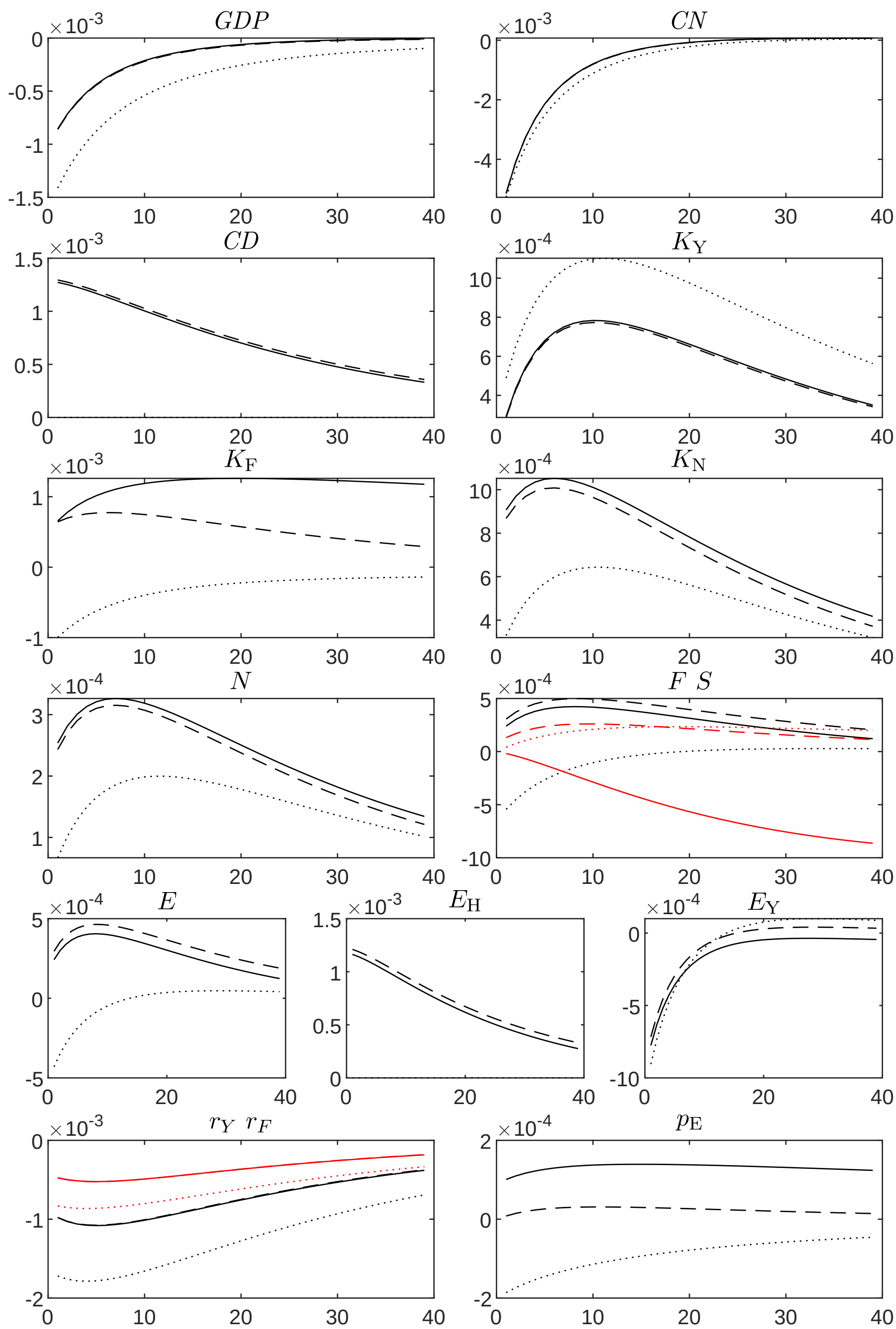
Table 3.F.1: Conditional variance decomposition: model with replenishment.

| | $\varepsilon_{\mathrm{E}}$ | $\varepsilon_{\mathrm{N}}$ | $\varepsilon_{\mathrm{Y}}$ | $\varepsilon_{\mathrm{F}}$ | $\varepsilon_{\mathrm{T,CN}}$ | $\varepsilon_{\mathrm{S}}$ | $\varepsilon_{\mathrm{V}}$ | $\varepsilon_{\mathrm{T,CD}}$ |
|---|---|---|---|---|---|---|---|---|
| | | | | short-term horizon | | | | |
| GDP | 0.26 | 0.2 | 96.72 | 0.13 | 1.06 | 0 | 0 | 1.64 |
| Non-durable goods | 0.26 | 0.19 | 60.8 | 0.14 | 28.27 | 0 | 0 | 10.34 |
| Durable goods | 20.07 | 16.37 | 21.81 | 0.36 | 25.28 | 0 | 0 | 16.11 |
| Total energy | 25.02 | 19.18 | 5.5 | 49.34 | 0.09 | 0 | 0 | 0.87 |
| Finite resources | 3.18 | 2.57 | 9.13 | 83.81 | 0.11 | 0 | 0 | 1.19 |
| Renewable resources | 0.06 | 99.88 | 0.01 | 0 | 0.01 | 0 | 0 | 0.04 |
| | | | | mid-term horizon | | | | |
| GDP | 0.3 | 0.23 | 95.91 | 0.12 | 1.71 | 0 | 0 | 1.72 |
| Non-durable goods | 0.43 | 0.33 | 66.42 | 0.12 | 25.04 | 0 | 0 | 7.67 |
| Durable goods | 10.85 | 8.83 | 47.64 | 0.23 | 22.61 | 0 | 0 | 9.84 |
| Total energy | 25.87 | 20.22 | 9.74 | 42.69 | 0.41 | 0 | 0 | 1.08 |
| Finite resources | 4.16 | 3.44 | 15.09 | 75.25 | 0.54 | 0 | 0 | 1.51 |
| Renewable resources | 0.07 | 99.7 | 0.15 | 0 | 0.03 | 0 | 0 | 0.06 |
| | | | | long-term horizon | | | | |
| GDP | 0.2 | 0.21 | 97.15 | 0.09 | 0.91 | 0 | 0 | 1.45 |
| Non-durable goods | 0.55 | 0.57 | 76.2 | 0.11 | 16.36 | 0 | 0.01 | 6.2 |
| Durable goods | 5.81 | 6.2 | 65.24 | 0.2 | 15.02 | 0.03 | 0.11 | 7.39 |
| Total energy | 20.1 | 20.96 | 14.21 | 42.73 | 0.66 | 0.04 | 0.14 | 1.16 |
| Finite resources | 2.97 | 3.31 | 19.79 | 71.28 | 0.83 | 0.07 | 0.24 | 1.5 |
| Renewable resources | 0.05 | 99.45 | 0.4 | 0 | 0.04 | 0 | 0 | 0.06 |

The short-term horizon is defined by the decomposition after four periods, the mid-term horizon after twelve periods. The long-term horizon is computed by an unconditional variance decomposition.

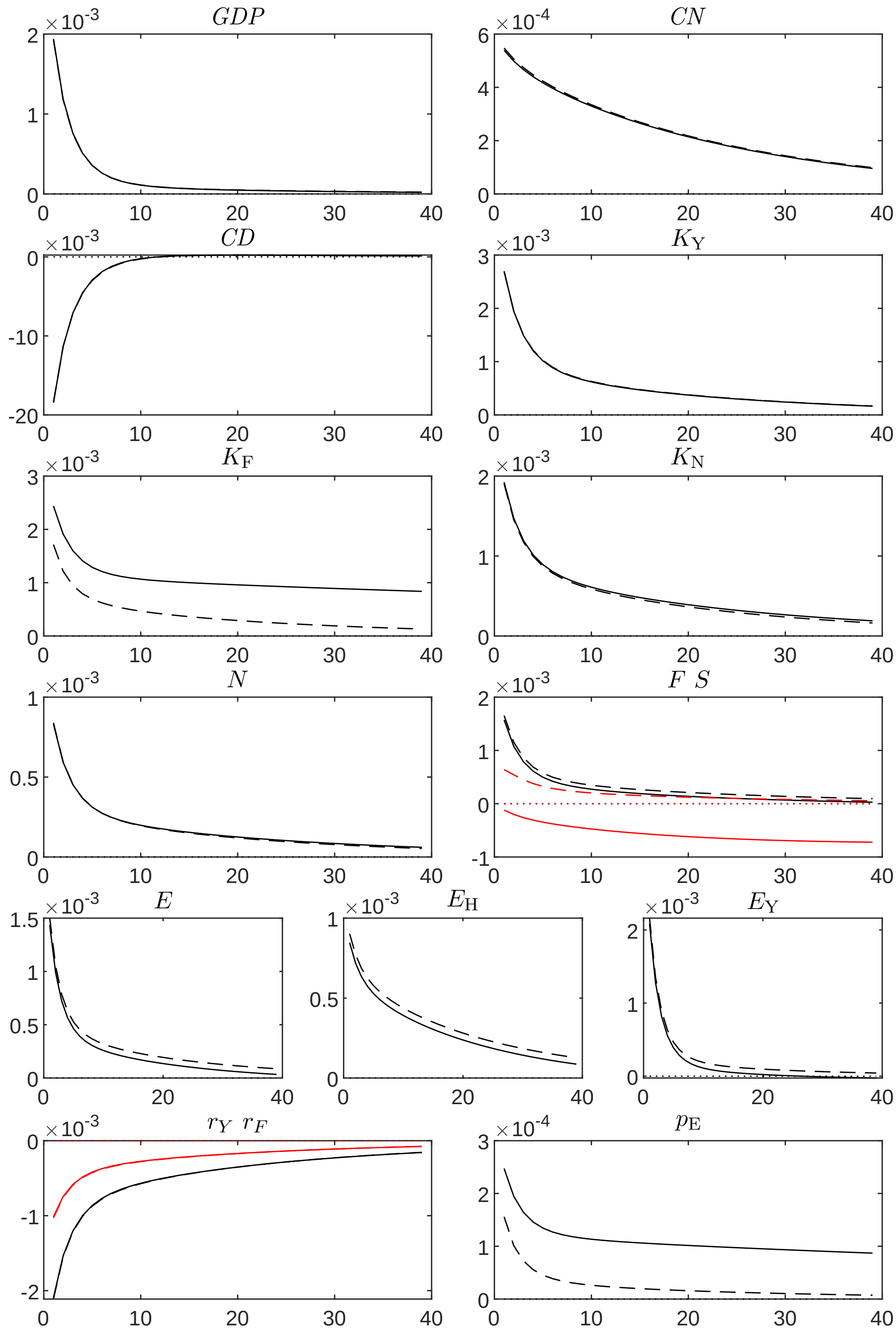
## 3.G Additional IRFs of Shocks in Consumer Taste

Figure 3.G.1: Bayesian IRF: orthogonalized shock to $\varepsilon_{T,CN}$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

Figure 3.G.2: Bayesian IRF: orthogonalized shock to $\varepsilon_{T,CD}$.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the baseline model, the dashed lines display the model with replenishment of the reserve stock while the dotted lines display the model without durable goods and energy consumption by the final good sector only. In figure (F S), the black lines displays the response of variable F, the red lines display the response of variable S. In figure ($r_Y$ $r_F$), the black lines displays the response of variable $r_Y$, the red lines display the response of variable $r_F$.

## 3.H Mathematical Appendix

### 3.H.1 Overall concavity

**Definition.** *Overall concavity*
*A twice continuously differentiable function of several variables is concave on a concave set iff its Hessian matrix of second partial derivatives is negative (semi)definite on the interior of the concave set. According to the Sylvester's criterion, a Hermitian matrix M is negative-(semi)definite if the leading principle minors of the LxL matrix are of an alternating sign starting with a minus sign, hence,*

$$\text{for strict concavity if} \quad (-1)^r \; {}_rH_r(x) > 0 \qquad \text{with } r = 1, ..., L$$
$$(\text{for concavity if} \quad (-1)^r \; {}_rH_r(x) \leq 0 \qquad \text{with } r = 1, ..., L)$$

*where ${}_rH_r(x)$ is the leading principle minor of order r.*

#### 3.H.1.1 Proof of Lemma 1

*Proof.* For the utility function (3.1) and the corresponding Hessian $H(x)$ (a 4x4 matrix), there are four leading principle minors given by the determinant:

$${}_1H_1 = \begin{bmatrix} \frac{-1+\vartheta}{(-1+L)^2} \end{bmatrix}$$

$$2H_2 = \begin{bmatrix} \frac{-1+\vartheta}{(-1+L)^2} & 0 \\ 0 & -\frac{\vartheta\gamma}{CN^2} \end{bmatrix}$$

$${}_3H_3 = \begin{bmatrix} \frac{-1+\vartheta}{(-1+L)^2} & 0 & 0 \\ 0 & -\frac{\vartheta\gamma}{CN^2} & 0 \\ 0 & 0 & \frac{\vartheta(\gamma-1)\theta CD^{\zeta}(\theta CD^{\zeta}+(1-\theta)(E_{\mathrm{H}})^{\zeta}-\zeta(1-\theta)E^{\zeta})}{CD^2(\theta CD^{\zeta}+(1-\theta)(E_{\mathrm{H}})^{\zeta})} \\ 0 & 0 & -\frac{\vartheta(\gamma-1)\theta CD^{\zeta}(\theta-1)\zeta(E_{\mathrm{H}})^{\zeta}}{CD(\theta CD\zeta+(1-\theta)(E_{\mathrm{H}})^{\zeta})^2 E_{\mathrm{H}}} \end{bmatrix}$$

$${}_4H_4 = \begin{bmatrix} \frac{-1+\vartheta}{(-1+L)^2} & 0 & 0 & 0 \\ 0 & -\frac{\vartheta\gamma}{CN^2} & 0 & 0 \\ 0 & 0 & \frac{\vartheta(\gamma-1)\theta CD^{\zeta}(\theta CD^{\zeta}+(1-\theta)(E_{\mathrm{H}})^{\zeta}-\zeta(1-\theta)E^{\zeta})}{CD^2(\theta CD^{\zeta}+(1-\theta)(E_{\mathrm{H}})^{\zeta})} & -\frac{\vartheta(\gamma-1)\theta CD^{\zeta}(\theta-1)\zeta(E_{\mathrm{H}})^{\zeta}}{CD(\theta CD\zeta+(1-\theta)(E_{\mathrm{H}})^{\zeta})^2 E_{\mathrm{H}}} \\ 0 & 0 & -\frac{\vartheta(\gamma-1)\theta CD^{\zeta}(\theta-1)\zeta(E_{\mathrm{H}})^{\zeta}}{CD(\theta CD\zeta+(1-\theta)(E_{\mathrm{H}})^{\zeta})^2 E_{\mathrm{H}}} & \frac{\vartheta(\gamma-1)(\theta-1)(E_{\mathrm{H}})^{\zeta}(\theta(\zeta-1)CD^{\zeta}+(\theta-1)(E_{\mathrm{H}})^{\zeta})}{-(E_{\mathrm{H}})^2(\theta CD^{\zeta}+(1-\zeta)(E_{\mathrm{H}})^{\zeta})^2} \end{bmatrix}$$

Overall strict concavity is satisfied when

$$\begin{aligned} &{}_1H_1 < 0 \\ &{}_2H_2 > 0 \qquad \text{if } \vartheta < 1, \quad \gamma > 0 \\ &{}_3H_3 < 0 \qquad \text{if } \vartheta > 0, \quad \theta, \gamma < 1, \quad \zeta < 1 \\ &{}_4H_4 > 0 \qquad \text{if } \gamma > 0, \quad \text{either } \theta, \zeta < 1. \end{aligned}$$

□

#### 3.H.1.2 Proof of Lemma 2

*Proof.* For the final good production function (3.4) and the corresponding Hessian $H(x)$ (a 3x3 matrix), there are three leading principle minors given by the determinant:

$$
{}_1H_1 = \begin{bmatrix} a_{K,K} \end{bmatrix}
$$
$$
{}_2H_2 = \begin{bmatrix} a_{K,K} & a_{K,E} \\ a_{E,K} & a_{E,E} \end{bmatrix}
$$
$$
{}_3H_3 = \begin{bmatrix} a_{K,K} & a_{K,E} & a_{K,L} \\ a_{E,K} & a_{E,E} & a_{E,L} \\ a_{L,K} & a_{L,E} & a_{L,L} \end{bmatrix}
$$

where

$$
a_{L,L} = \frac{L^{1-\alpha}(\alpha-1)\alpha(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^{\frac{\alpha}{\nu}}}{L^2}
$$
$$
a_{L,K} = a_{K,L} = -\frac{L^{1-\alpha}(\alpha-1)\alpha\eta(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^{\frac{\alpha}{\nu}}(K_{\mathrm{Y}})^{\nu}}{LK_{\mathrm{Y}}(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})}
$$
$$
a_{L,E} = a_{E,L} = -\frac{L^{1-\alpha}(\alpha-1)\alpha(\eta-1)(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^{\frac{\alpha}{\nu}}(E_{\mathrm{Y}})^{\nu}}{LE_{\mathrm{Y}}(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})}
$$
$$
a_{K,K} = \frac{L^{1-\alpha}(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^{\frac{\alpha}{\nu}}\alpha\eta(K_{\mathrm{Y}})^{\nu}(\eta(\alpha-1)(K_{\mathrm{Y}})^{\nu} - \eta\nu(E_{\mathrm{Y}})^{\nu} + \eta(E_{\mathrm{Y}})^{\nu} + (\nu-1)(E_{\mathrm{Y}})^{\nu})}{(K_{\mathrm{Y}})^2(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^2}
$$
$$
a_{K,E} = a_{E,K} = -\frac{L^{1-\alpha}(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^{\frac{\alpha}{\nu}}\alpha(\eta-1)(E_{\mathrm{Y}})^{\nu}\eta(K_{\mathrm{Y}})^{\nu}(\alpha-\nu)}{E_{\mathrm{Y}}((\eta-1)(E_{\mathrm{Y}})^{\nu} - \eta(K_{\mathrm{Y}})^{\nu})^2 K_{\mathrm{Y}}}
$$
$$
a_{E,E} = \frac{L^{1-\alpha}(\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)(E_{\mathrm{Y}})^{\nu})^{\frac{\alpha}{\nu}}\alpha(\eta-1)(E_{\mathrm{Y}})^{\nu}(\alpha\eta(E_{\mathrm{Y}})^{\nu} - \eta(K_{\mathrm{Y}})^{\nu}\nu - \alpha(E_{\mathrm{Y}})^{\nu} - \eta(E_{\mathrm{Y}})^{\nu} + \eta(K_{\mathrm{Y}})^{\nu} + (E_{\mathrm{Y}})^{\nu})}{(E_{\mathrm{Y}})^2(-\eta(K_{\mathrm{Y}})^{\nu} - (1-\eta)(E_{\mathrm{Y}})^{\nu})^2}
$$

Overall concavity is satisfied when

$$
\begin{aligned}
{}_1H_1 &< 0 && \text{if } \alpha < 1 \\
{}_2H_2 &> 0 && \text{if } \{\nu \leq 1, \eta \leq 1\}, \{\nu > 1, \eta \geq 1\}, \{\nu = 1, \eta > 1\} \\
{}_3H_3 &= 0 && \text{for all } \nu, \eta.
\end{aligned}
$$

□

### 3.H.2 Optimization

Under the assumption that prices for final energy are equal for households and final goods-producing firms $p_{\mathrm{E}} = p_{\mathrm{H}} = p_{\mathrm{Y}}$, the household's problem, the decision making of the remaining sectors, and the corresponding first order conditions with respect to the decision variables are:

**Household sector**

$$\begin{aligned}\mathcal{L}_t^H &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \vartheta \ln \Big[ CN_t^{\gamma} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}} \Big] + (1-\vartheta) \ln [1 - L_t] \\
&\quad + \lambda_t^H \, [w_t L_t + (r_{\mathrm{Y}})_t (K_{\mathrm{Y}})_{t-1} + (r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1} + (r_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1} + X_t + (\pi_{\mathrm{Y}})_t + (\pi_{\mathrm{N}})_t \\
&\quad + (\pi_{\mathrm{F}})_t - CN_t - (p_{\mathrm{E}})_t (E_{\mathrm{H}})_t - CD_t + (1-\delta^{CD}) CD_{t-1} - (K_{\mathrm{Y}})_t + (1-\delta^{Y})(K_{\mathrm{Y}})_{t-1} \\
&\quad - (K_{\mathrm{F}})_t + (1-\delta^{F})(K_{\mathrm{F}})_{t-1} - (K_{\mathrm{N}})_t + (1-\delta^{N})(K_{\mathrm{N}})_{t-1} ] \Big\}\end{aligned} \tag{3.H.1}$$

• Non-durable goods:

$$\begin{aligned}\frac{\partial \mathcal{L}_t^H}{\partial CN_t} &= \beta^t \vartheta \gamma \frac{CN_t^{\gamma-1} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}}}{CN_t^{\gamma} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}}} - \beta^t \vartheta \gamma \frac{1}{CN_t} - \beta^t \lambda_t^H \\
&= \beta^t \vartheta \gamma \frac{1}{CN_t} - \beta^t \lambda_t^H \overset{!}{=} 0 \\
\Leftrightarrow \quad & \vartheta \gamma \frac{1}{CN_t} - \lambda_t^H \overset{!}{=} 0 \qquad \Leftrightarrow \qquad \lambda_t^H = \vartheta \gamma \frac{1}{CN_t}\end{aligned} \tag{3.H.2}$$

• Durable goods:

$$\begin{aligned}\frac{\partial \mathcal{L}_t^H}{\partial CD_t} &= \beta^{t+1} \frac{\vartheta (1-\gamma) \zeta \theta}{\zeta} \mathbb{E} \left\{ \frac{CN_{t+1}^{\gamma} \left( \theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta} \right)^{\frac{1-\gamma}{\zeta}} CD_t^{\zeta-1}}{CN_{t+1}^{\gamma} \left( \theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta} \right)^{\frac{1-\gamma}{\zeta}}} \right\} \\
&\qquad - \beta^t \lambda_t^H + \beta^{t+1} \mathbb{E} \left\{ \lambda_{t+1}^H \left(1 - \delta^{\mathrm{CD}}\right) \right\} \\
&= \beta^{t+1} \vartheta (1-\gamma) \theta \mathbb{E} \left\{ \frac{CD_t^{\zeta-1}}{\theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta}} \right\} \\
&\qquad + \beta^{t+1} \mathbb{E} \left\{ \lambda_{t+1}^H \left(1 - \delta^{CD}\right) \right\} - \beta^t \lambda_t^H \overset{!}{=} 0 \\
\Leftrightarrow \quad \lambda_t^H &= \vartheta (1-\gamma) \theta \beta \mathbb{E} \left\{ \frac{CD_t^{\zeta-1}}{\theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta}} \right\} + \beta \mathbb{E} \left\{ \lambda_{t+1}^H \left(1 - \delta^{CD}\right) \right\}\end{aligned} \tag{3.H.3}$$

• Energy consumption of households:

$$\begin{aligned}\frac{\partial \mathcal{L}_t^H}{\partial (E_{\mathrm{H}})_t} &= \vartheta (1-\gamma) \frac{CN_t^{\gamma} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}}}{CN_t^{\gamma} \left( \theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta} \right)^{\frac{1-\gamma}{\zeta}}} (1-\theta) E_{\mathrm{H}}{}_{t-1}^{\zeta-1} - \lambda_t^H (p_{\mathrm{E}})_t \\
&= \vartheta (1-\gamma)(1-\theta) \frac{(1-\theta)(E_{\mathrm{H}})_t^{\zeta-1}}{\theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta}} - \lambda_t^H (p_{\mathrm{E}})_t \overset{!}{=} 0 \\
\Leftrightarrow \quad \lambda_t^H (p_{\mathrm{E}})_t &= \vartheta (1-\gamma)(1-\theta) \frac{(1-\theta)(E_{\mathrm{H}})_t^{\zeta-1}}{\theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta}}\end{aligned} \tag{3.H.4}$$

- Labor supply:

$$\frac{\partial \mathcal{L}_t^H}{\partial L_t} = \frac{1-\vartheta}{1-L_t} - \lambda_t^H w_t \quad \stackrel{!}{=} 0 \tag{3.H.5}$$

- Euler equations for capital stocks:

$$\frac{\partial \mathcal{L}_t^H}{\partial (K_\mathrm{Y})_t} = \beta^{t+1}\mathbb{E}\left\{\lambda_{t+1}^H \left(1+(r_\mathrm{Y})_{t+1}-\delta^Y\right)\right\} - \beta\lambda_t^H \quad \stackrel{!}{=} 0 \tag{3.H.6}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial (K_\mathrm{F})_t} = \beta^{t+1}\mathbb{E}\left\{\lambda_{t+1}^H \left(1+(r_\mathrm{F})_{t+1}-\delta^F\right)\right\} - \beta\lambda_t^H \quad \stackrel{!}{=} 0 \tag{3.H.7}$$

$$\frac{\partial \mathcal{L}_t^H}{\partial (K_\mathrm{N})_t} = \beta^{t+1}\mathbb{E}\left\{\lambda_{t+1}^H \left(1+(r_\mathrm{N})_{t+1}-\delta^N\right)\right\} - \beta\lambda_t^H \quad \stackrel{!}{=} 0 \tag{3.H.8}$$

**Final goods production sector**

$$\begin{aligned}(\pi_\mathrm{Y})_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ &(A_\mathrm{Y})_t \left[\eta (K_\mathrm{Y})_{t-1}^{\nu} + (1-\eta)(E_\mathrm{Y})_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{1-\alpha} \\ &-(r_\mathrm{Y})_t (K_\mathrm{Y})_{t-1} - w_t L_t - (p_\mathrm{E})_t (E_\mathrm{Y})_t \Big\}\end{aligned} \tag{3.H.9}$$

- Capital demand of final goods production:

$$\frac{\partial (\pi_\mathrm{Y})_0}{\partial (K_\mathrm{Y})_{t-1}} = (A_\mathrm{Y})_t \alpha\eta \left[\eta (K_\mathrm{Y})_{t-1}^{\nu} + (1-\eta)(E_\mathrm{Y})_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L^{1-\alpha} (K_\mathrm{Y})_{t-1}^{\nu-1} - (r_\mathrm{Y})_t \quad \stackrel{!}{=} 0 \tag{3.H.10}$$

- Energy consumption of final goods production:

$$\frac{\partial (\pi_\mathrm{Y})_0}{\partial (E_\mathrm{Y})_t} = (A_\mathrm{Y})_t \alpha\eta \left[\eta (K_\mathrm{Y})_{t-1}^{\nu} + (1-\eta)(E_\mathrm{Y})_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L^{1-\alpha} (E_\mathrm{Y})_t^{\nu-1} - (p_\mathrm{E})_t \quad \stackrel{!}{=} 0 \tag{3.H.11}$$

- Labor demand of final goods production:

$$\frac{\partial (\pi_\mathrm{Y})_0}{\partial L_t} = (A_\mathrm{Y})_t (1-\alpha) \left[\eta (K_\mathrm{Y})_{t-1}^{\nu} + (1-\eta)(E_\mathrm{Y})_t^{\nu}\right]^{\frac{\alpha}{\nu}} L^{-\alpha} - w_t \quad \stackrel{!}{=} 0 \tag{3.H.12}$$

**Final energy sector**

$$(\pi_\mathrm{E})_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ (p_\mathrm{E})_t (A_\mathrm{E})_t F_t^{\phi} N_t^{1-\phi} - (p_\mathrm{F})_t F_t - (p_\mathrm{N})_t N_t \right\} \tag{3.H.13}$$

- Demand for finite intermediate energy:

$$\frac{\partial(\pi_{\mathrm{E}})_0}{\partial F_t} = \phi(p_{\mathrm{E}})_t(A_{\mathrm{E}})_t F_t^{\phi-1}N^{1-\phi} - (p_{\mathrm{F}})_t \quad \stackrel{!}{=} 0 \tag{3.H.14}$$

- Demand for renewable intermediate energy:

$$\frac{\partial(\pi_{\mathrm{E}})_0}{\partial N_t} = (1-\phi)\,(p_{\mathrm{E}})_t(A_{\mathrm{E}})_t F_t^{\phi}N_t^{-\phi} - (p_{\mathrm{R}})_t \quad \stackrel{!}{=} 0 \tag{3.H.15}$$

**Finite intermediate energy sector**

$$\begin{aligned}\mathcal{L}^F =&\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{(p_{\mathrm{F}})_t(A_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1}^{\varphi}S_{t-1}^{1-\varphi} - (r_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1} - \left(\frac{D_t}{V_t}\right)^{\upsilon}\\ &+\lambda_t^S\left[S_{t-1} - \underbrace{(A_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1}^{\varphi}S_{t-1}^{1-\varphi}}_{F_t} + \omega D_t - S_t\right]\\ &+\lambda_t^V\left[V_t - D_t - V_{t+1}\right]\Bigg\}\end{aligned} \tag{3.H.16}$$

- Capital demand of finite intermediate energy generation:

$$\begin{aligned}&\frac{\partial\mathcal{L}^F}{\partial(K_{\mathrm{F}})_{t-1}} = \varphi(p_{\mathrm{F}})_t(A_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1}^{\varphi-1}S_{t-1}^{1-\varphi} - (r_{\mathrm{F}})_t - \lambda_t^S\varphi(A_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1}^{\varphi-1}S_{t-1}^{1-\varphi} \quad \stackrel{!}{=} 0\\ \Leftrightarrow\quad &(r_{\mathrm{F}})_t = \varphi\frac{F_t}{(K_{\mathrm{F}})_{t-1}}\left((p_{\mathrm{F}})_t - \lambda_t^S\right) \quad\Leftrightarrow\quad \lambda_t^S = (p_{\mathrm{F}})_t - \frac{(r_{\mathrm{F}})_t(K_{\mathrm{F}})_{t-1}}{\varphi F_t}\end{aligned} \tag{3.H.17}$$

- Optimal reserve stock:

$$\begin{aligned}&\frac{\partial\mathcal{L}^F}{\partial S_t} = \beta^{t+1}\mathbb{E}\left\{\left((p_{\mathrm{F}})_{t+1} - \lambda_{t+1}^S\right)\left[(1-\varphi)\,(A_{\mathrm{F}})_{t+1}(K_{\mathrm{F}})_t^{\varphi}S_t^{-\varphi}\right] + \lambda_{t+1}^S\right\} - \beta^t\lambda_t^S \stackrel{!}{=} 0\\ \Leftrightarrow\quad &\lambda_t^S = \beta\mathbb{E}\left\{\left((p_{\mathrm{F}})_{t+1} - \lambda_{t+1}^S\right)\left[(1-\varphi)\,(A_{\mathrm{F}})_{t+1}(K_{\mathrm{F}})_t^{\varphi}S_t^{-\varphi}\right] + \lambda_{t+1}^S\right\}\end{aligned} \tag{3.H.18}$$

- Exploration rate (only in model allowing for exploration):

$$\begin{aligned}&\frac{\partial\mathcal{L}^F}{\partial D_t} = -\upsilon D_t^{\upsilon-1}V_t^{-\upsilon} + \lambda_t^S - \lambda_t^V \quad \stackrel{!}{=} 0\\ \Leftrightarrow\quad &\lambda_t^V = \lambda_t^S - \upsilon D_t^{\upsilon-1}V_t^{-\upsilon}\end{aligned} \tag{3.H.19}$$

- Optimal resource stock (only in model allowing for exploration):

$$\begin{aligned}&\frac{\partial\mathcal{L}^F}{\partial V_t} = \upsilon D_t^{\upsilon}V_t^{-\upsilon-1} + \beta^t\lambda_t^V - \lambda_t^V \quad \stackrel{!}{=} 0\\ \Leftrightarrow\quad &\lambda_t^V = \upsilon D_t^{\upsilon}V_t^{-\upsilon-1} + \beta^t\lambda_t^V\end{aligned} \tag{3.H.20}$$

**Renewable intermediate energy sector**

$$(\pi_{\mathrm{N}})_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ (p_{\mathrm{N}})_t (A_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1}^{\psi} - (r_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1} \right\} \tag{3.H.21}$$

- Capital demand of renewable intermediate energy generation:

$$\frac{\partial(\pi_{\mathrm{N}})_0}{\partial(K_{\mathrm{N}})_t} = \psi (p_{\mathrm{N}})_t (A_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1}^{\psi-1} - (r_{\mathrm{N}})_t \quad \stackrel{!}{=} 0 \tag{3.H.22}$$

By rearranging the conditions above, the optimized decisions as well as the market clearing equations are calculated which define the complete model (34 equations).

**Household sector**

- Durable Euler equation: combining (3.H.2) and (3.H.3)

$$\begin{aligned} \vartheta\gamma \frac{1}{CN_t} &= \beta\vartheta(1-\gamma)\theta\mathbb{E}\left\{ \frac{CD_t^{\zeta-1}}{\theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta}} \right\} + \beta\mathbb{E}\left\{ \vartheta\gamma\frac{1}{CN_{t+1}}\left(1-\delta^{CD}\right) \right\} \\ \Leftrightarrow \qquad 1 &= \beta\frac{(1-\gamma)}{\gamma}\theta\mathbb{E}\left\{ \frac{CD_t^{\zeta-1}CN_t}{\theta CD_t^{\zeta} + (1-\theta)(E_{\mathrm{H}})_{t+1}^{\zeta}} \right\} + \beta\mathbb{E}\left\{ \frac{CN_t}{CN_{t+1}}\left(1-\delta^{CD}\right) \right\} \end{aligned} \tag{3.H.23}$$

- Non-durables vs. energy: combining (3.H.3) and (3.H.4)

$$\begin{aligned} \vartheta\gamma\frac{(p_{\mathrm{E}})_t}{CN_t} &= \vartheta(1-\gamma)(1-\theta)\frac{(E_{\mathrm{H}})_t^{\zeta-1}}{\theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta}} \\ \Leftrightarrow \qquad (p_{\mathrm{E}})_t &= \frac{(1-\gamma)(1-\theta)}{\gamma}\frac{CN_t (E_{\mathrm{H}})_t^{\zeta-1}}{\left(\theta CD_{t-1}^{\zeta} + (1-\theta)(E_{\mathrm{H}})_t^{\zeta}\right)} \end{aligned} \tag{3.H.24}$$

- Labor supply: combining (3.H.2) and (3.H.5)

$$w_t = \frac{CN_t}{1-L_t}\frac{1-\gamma}{\gamma\vartheta} \tag{3.H.25}$$

- Euler equation for capital of final goods production: combining (3.H.2) and (3.H.6)

$$\begin{aligned} \vartheta\gamma\frac{1}{CN_t} &= \beta\mathbb{E}\left\{ \vartheta\gamma\frac{1}{CN_{t+1}}\left(1+(r_{\mathrm{Y}})_{t+1}-\delta^{Y}\right) \right\} \\ \Leftrightarrow \qquad 1 &= \beta\mathbb{E}\left\{ \frac{CN_t}{CN_{t+1}}\left(1+(r_{\mathrm{Y}})_{t+1}-\delta^{Y}\right) \right\} \end{aligned} \tag{3.H.26}$$

• Euler equation for capital of finite intermediate energy generation: combining (3.H.2) and (3.H.7)

$$\begin{aligned} \vartheta\gamma\frac{1}{CN_t} &= \beta\mathbb{E}\left\{\vartheta\gamma\frac{1}{CN_{t+1}}\left(1+(r_{\mathrm{F}})_{t+1}-\delta^F\right)\right\} \\ \Leftrightarrow \qquad 1 &= \beta\mathbb{E}\left\{\frac{CN_t}{CN_{t+1}}\left(1+(r_{\mathrm{F}})_{t+1}-\delta^F\right)\right\} \end{aligned} \tag{3.H.27}$$

• Euler equation for capital of renewable intermediate energy generation: combining (3.H.2) and (3.H.8)

$$\begin{aligned} \vartheta\gamma\frac{1}{CN_t} &= \beta\mathbb{E}\left\{\vartheta\gamma\frac{1}{CN_{t+1}}\left(1+(r_{\mathrm{N}})_{t+1}-\delta^N\right)\right\} \\ \Leftrightarrow \qquad 1 &= \beta\mathbb{E}\left\{\frac{CN_t}{CN_{t+1}}\left(1+(r_{\mathrm{N}})_{t+1}-\delta^N\right)\right\} \end{aligned} \tag{3.H.28}$$

• Investments in durable goods:

$$(I_{CD})_t = CD_t - \left(1-\delta^{CD}\right)CD_{t-1} \tag{3.H.29}$$

• Investments in capital of final production:

$$(I_{\mathrm{Y}})_t = (K_{\mathrm{Y}})_t - \left(1-\delta^{Y}\right)(K_{\mathrm{Y}})_{t-1} \tag{3.H.30}$$

• Investments in capital of finite intermediate energy generation:

$$(I_{\mathrm{F}})_t = (K_{\mathrm{F}})_t - \left(1-\delta^{F}\right)(K_{\mathrm{F}})_{t-1} \tag{3.H.31}$$

• Investments in capital of renewable intermediate energy generation:

$$(I_{\mathrm{N}})_t = (K_{\mathrm{N}})_t - \left(1-\delta^{N}\right)(K_{\mathrm{N}})_{t-1} \tag{3.H.32}$$

**Final goods production sector**

• Final goods production output (= non-durable goods)

$$Y_t = (A_{\mathrm{Y}})_t\left[\eta(K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{1-\alpha} \tag{3.H.33}$$

• Capital demand of final goods production: rearranging (3.H.10)

$$(r_{\mathrm{Y}})_t = (A_{\mathrm{Y}})_t\alpha\eta\left[\eta(K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha}(K_{\mathrm{Y}})_{t-1}^{\nu-1} \tag{3.H.34}$$

• Energy demand of final goods production: rearranging (3.H.11)

$$(p_{\mathrm{E}})_t = (A_{\mathrm{Y}})_t\alpha\eta\left[\eta(K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha}(E_{\mathrm{Y}})_t^{\nu-1} \tag{3.H.35}$$

- Labor demand of final goods production: rearranging (3.H.12)

$$w_t = (A_{\mathrm{Y}})_t (1-\alpha) \left[\eta (K_{\mathrm{Y}})_{t-1}^{\nu} + (1-\eta) (E_{\mathrm{Y}})_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{-\alpha} \tag{3.H.36}$$

**Final energy sector**

- Amount of final energy generation:

$$E_t = (A_{\mathrm{E}})_t F_t^{\phi} N_t^{1-\phi} \tag{3.H.37}$$

- Price for finite intermediate energy: rearranging (3.H.14)

$$(p_{\mathrm{F}})_t = \phi (p_{\mathrm{E}})_t (A_{\mathrm{E}})_t F_t^{\phi-1} N_t^{1-\phi} \tag{3.H.38}$$

- Price for renewable intermediate energy: rearranging (3.H.15)

$$(p_{\mathrm{R}})_t = (1-\phi) (p_{\mathrm{E}})_t (A_{\mathrm{E}})_t F_t^{\phi} N_t^{-\phi} \tag{3.H.39}$$

**Finite intermediate energy sector**

- Amount of finite intermediate energy:

$$F_t = (A_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1}^{\varphi} S_{t-1}^{1-\varphi} \tag{3.H.40}$$

- Finite reserve constraint:

$$S_t = S_{t-1} - F_t + \omega D_t - e^{\varepsilon_{\mathrm{S},t}} \tag{3.H.41}$$

- Cost-function of exploration:

$$C(D_t, V_t) = \left(\frac{D_t}{V_t}\right)^{\upsilon} = X_t \tag{3.H.42}$$

- Finite resource constraint:

$$V_t = V_{t-1} - D_t - e^{\varepsilon_{\mathrm{V},t}} \tag{3.H.43}$$

- Capital demand of finite intermediate energy generation: combining equations (3.H.17) and (3.H.18)

$$(p_{\mathrm{F}})_t = \frac{(r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1}}{\varphi F_t} + \beta \mathbb{E}\left\{(p_{\mathrm{F}})_{t+1}\right\} + \beta \mathbb{E}\left\{\frac{(r_{\mathrm{F}})_{t+1} (K_{\mathrm{F}})_t}{\varphi F_{t+1}} \left[(1-\varphi) \frac{F_{t+1}}{S_t} - 1\right]\right\} \tag{3.H.44}$$

• Amount of exploration in finite intermediate energy generation: combining equations (3.H.19),(3.H.20), and (3.H.18)

$$\begin{aligned}(p_{\mathrm{F}})_t =& \frac{(r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1}}{\varphi F_t} + \upsilon \frac{D_t^{\upsilon}}{D_t V_t^{\upsilon}} - \beta \upsilon \mathbb{E}\left\{\frac{D_{t+1}^{\upsilon}}{D_{t+1} V_{t+1}^{\upsilon}}\right\} \\ &+ \beta \mathbb{E}\left\{(p_{\mathrm{F}})_{t+1}\right\} - \beta \mathbb{E}\left\{\frac{(r_{\mathrm{F}})_{t+1}(K_{\mathrm{F}})_t}{\varphi F_{t+1}}\right\} + \upsilon \frac{D_t^{\upsilon}}{V_t^{(1+\upsilon)}}\end{aligned} \tag{3.H.45}$$

**Renewable intermediate energy sector**

• Amount of renewable intermediate energy:

$$N_t = (A_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1}^{\psi} \tag{3.H.46}$$

• Capital demand of renewable intermediate energy generation: rearranging (3.H.22)

$$(r_{\mathrm{N}})_t = \psi (p_{\mathrm{N}})_t (A_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1}^{\psi-1} \tag{3.H.47}$$

**Market Clearing**

• Aggregate market constraint:

$$\begin{aligned} GDP_t &= Y_t - (p_{\mathrm{E}})_t (E_{\mathrm{Y}})_t + (r_{\mathrm{N}})_t (K_{\mathrm{N}})_{t-1} + (r_{\mathrm{F}})_t (K_{\mathrm{F}})_{t-1} + X_t \\ &= CN_t + (p_{\mathrm{H}})_t (E_{\mathrm{H}})_t + (I_{CD})_t + (I_{\mathrm{Y}})_t + (I_{\mathrm{F}})_t + (I_{\mathrm{N}})_t \end{aligned} \tag{3.H.48}$$

• Energy market constraint:

$$E_t = (E_{\mathrm{H}})_t + (E_{\mathrm{Y}})_t \tag{3.H.49}$$

**Formation of shocks**

• Productivity shock in final goods production:

$$\ln(A_{\mathrm{Y}})_t = \rho_{\mathrm{Y}} \ln(A_{\mathrm{Y}})_{t-1} + \varepsilon_{\mathrm{Y},t} \tag{3.H.50}$$

• Productivity shock in final energy generation:

$$\ln(A_{\mathrm{E}})_t = \rho_{\mathrm{E}} \ln(A_{\mathrm{E}})_{t-1} + \varepsilon_{\mathrm{E},t} \tag{3.H.51}$$

• Productivity shock in finite intermediate energy generation:

$$\ln(A_{\mathrm{F}})_t = \rho_{\mathrm{F}} \ln(A_{\mathrm{F}})_{t-1} + \varepsilon_{\mathrm{F},t} \tag{3.H.52}$$

- Productivity shock in renewable intermediate energy generation:

$$\ln(A_{\mathrm{N}})_t = \rho_{\mathrm{N}} \ln(A_{\mathrm{N}})_{t-1} + \varepsilon_{\mathrm{N},t} \tag{3.H.53}$$

- Consumer taste shock in non-durable goods:

$$\ln(T_{\mathrm{CN}})_t = \rho_{\mathrm{T,CN}} \ln(T_{\mathrm{CN}})_{t-1} + \varepsilon_{\mathrm{T,CN},t} \tag{3.H.54}$$

- Consumer taste shock in durable goods:

$$\ln(T_{\mathrm{CD}})_t = \rho_{\mathrm{T,CD}} \ln(T_{\mathrm{CD}})_{t-1} + \varepsilon_{\mathrm{T,CD},t} \tag{3.H.55}$$

### 3.H.3 Steady states

Table 3.H.1: Parameter values and targeted moments

| Parameter | Value | Source | Parameter | Value | Source |
|---|---|---|---|---|---|
| $\alpha$ | 0.3650 | see Section 3.5.2 | $\frac{I_F}{GDP}$ | 0.0006 | OECD (2012) |
| $\beta$ | 0.9900 | see Section 3.5.2 | $\frac{I_N}{GDP}$ | 0.0030 | UNEP (2017) |
| $\delta^{CD}$ | 0.0682 | see Section 3.5.2 | $\frac{I_{CD}}{GDP}$ | 0.1071 | OECD (2012) |
| $\nu$ | -0.1500 | see Section 3.5.2 | $L_{\mathrm{L}}$ | 0.3 | see Section 3.5.2 |
| $\zeta$ | -2.8748 | see Section 3.5.2 | $GDP$ | 3043.65 | OECD (2012) |
| $\phi$ | 0.8000 | see Section 3.5.2 | $F$ | 10593 | IEA (2012) |
| $\varphi$ | 0.4900 | see Section 3.5.2 | $N$ | 1679 | IEA (2012) |
| $\psi$ | 0.3100 | see Section 3.5.2 | $\frac{F}{S}$ | 0.076238 | see Section 3.H.6 |
| $\frac{CD}{GDP}$ | 1.5853 | OECD (2012) | $\frac{D}{V}$ | 0.012594 | see Section 3.H.6 |
| $\frac{E_{\mathrm{H}}}{GDP}$ | 0.0449 | IEA (2012) | $\frac{D}{S}$ | 0.034757 | see Section 3.H.6 |
| $\frac{E_{\mathrm{Y}}}{GDP}$ | 0.0406 | Schmidt and Zimmermann (2005) | $\frac{D}{F}$ | 0.455904 | see Section 3.H.6 |
| $\frac{K_{\mathrm{F}}}{GDP}$ | 0.0133 | OECD (2012) | | | |

In the following, we construct the steady state conditions of the model in combination with the calibrated parameters in Section 3.H.5 and the targeted moments from Germany in Table 3.H.1.

**Household sector**

- Durable Euler equation (3.H.23)

$$1 = \beta\theta \frac{(1-\gamma)}{\gamma} \frac{CN\, CD^{\zeta-1}}{\theta CD^{\zeta} + (1-\theta)(E_{\mathrm{H}})^{\zeta}} + \beta\left(1 - \delta^{CD}\right) \tag{3.H.23.SS}$$

- Non-durables vs. energy (3.H.24)

$$p_{\mathrm{E}} = \frac{(1-\gamma)(1-\theta)}{\gamma} \frac{CN (E_{\mathrm{H}})^{\zeta-1}}{\left(\theta CD^{\zeta} + (1-\theta)(E_{\mathrm{H}})^{\zeta}\right)} \tag{3.H.24.SS}$$

- Labor supply (3.H.25)

$$w = \frac{CN}{1-L} \frac{1-\gamma}{\gamma\vartheta} \tag{3.H.25.SS}$$

• Euler equation for capital of final good production (3.H.26)

$$1 = \beta \left(1 + (r_{\rm Y})_{t+1} - \delta^{Y}\right)$$

consequently

$$r_{\rm Y} = \frac{1}{\beta} + \delta^{Y} - 1 \tag{3.H.26.SS}$$

• Euler equation for capital of finite intermediate energy generation (3.H.27)

$$r_{\rm F} = \frac{1}{\beta} + \delta^{F} - 1 \tag{3.H.27.SS}$$

• Euler equation for capital of renewable intermediate energy generation (3.H.28)

$$r_{\rm N} = \frac{1}{\beta} + \delta^{N} - 1 \tag{3.H.28.SS}$$

• Investments in durable goods (3.H.29)

$$I_{CD} = \delta^{CD} CD \tag{3.H.29.SS}$$

• Investments in capital of final goods production (3.H.30)

$$I_{\rm Y} = \delta^{Y} K_{\rm Y} \tag{3.H.30.SS}$$

• Investments in capital of finite intermediate energy generation (3.H.31)

$$I_{\rm F} = \delta^{F} K_{\rm F} \tag{3.H.31.SS}$$

• Investments in capital of renewable intermediate energy generation (3.H.32)

$$I_{\rm N} = \delta^{N} K_{\rm N} \tag{3.H.32.SS}$$

**Final goods production sector**

• Final goods production output (= non-durable goods) (3.H.33)

$$Y = (A_{\rm Y}) \left[\eta (K_{\rm Y})^{\nu} + (1-\eta) (E_{\rm Y})^{\nu}\right]^{\frac{\alpha}{\nu}} L^{1-\alpha} \tag{3.H.33.SS}$$

• Capital demand of final goods production (3.H.34)

$$r_{\rm Y} = (A_{\rm Y}) \alpha \eta \left[\eta (K_{\rm Y})^{\nu} + (1-\eta) (E_{\rm Y})^{\nu}\right]^{\frac{\alpha}{\nu}-1} L^{1-\alpha} (K_{\rm Y})^{\nu-1} \tag{3.H.34.SS}$$

- Energy demand of final goods production (3.H.35)

$$p_{\mathrm{E}} = (A_{\mathrm{Y}})\alpha\eta\left[\eta(K_{\mathrm{Y}})^{\nu} + (1-\eta)\,(E_{\mathrm{Y}})^{\nu}\right]^{\frac{\alpha}{\nu}-1} L^{1-\alpha}(E_{\mathrm{Y}})^{\nu-1} \tag{3.H.35.SS}$$

- Labor demand of final goods production (3.H.36)

$$w = (1-\alpha)\,\frac{Y}{L} \tag{3.H.36.SS}$$

**Final energy sector**

- Amount of final energy generation (3.H.37)

$$E = (A_{\mathrm{E}})F^{\phi}N^{1-\phi} \tag{3.H.37.SS}$$

- Price for finite intermediate energy (3.H.38)

$$p_{\mathrm{F}} = \phi(p_{\mathrm{E}})(A_{\mathrm{E}})F^{\phi-1}N^{1-\phi} \tag{3.H.38.SS}$$

- Price for renewable intermediate energy (3.H.39)

$$p_{\mathrm{R}} = (1-\phi)\,(p_{\mathrm{E}})(A_{\mathrm{E}})F_t^{\phi}N^{-\phi} \tag{3.H.39.SS}$$

**Finite intermediate energy sector**

- Amount of finite intermediate energy (3.H.40)

$$F = (A_{\mathrm{F}})(K_{\mathrm{F}})^{\varphi}S^{1-\varphi} \tag{3.H.40.SS}$$

- Finite reserve constraint (3.H.41)

$$F_t = \omega D_t \tag{3.H.41.SS}$$

- Cost-function of exploration (3.H.42)

$$C\,(D,V) = \left(\frac{D}{V}\right)^{\upsilon} = X \tag{3.H.42.SS}$$

- Capital demand of finite intermediate energy generation (3.H.44)

$$(1-\beta)\,p_{\mathrm{F}} = \frac{r_{\mathrm{F}}K_{\mathrm{F}}}{\varphi F} + \beta\frac{r_{\mathrm{F}}K_{\mathrm{F}}}{\varphi F}\left[(1-\varphi)\,\frac{F}{S} - 1\right] \tag{3.H.44.SS}$$

- Amount of exploration in finite intermediate energy generation (3.H.45)

$$p_{\mathrm{F}} = \left(\frac{r_{\mathrm{F}}K_{\mathrm{F}}}{\varphi F} + \upsilon\frac{D^{\upsilon}}{DV^{\upsilon}}\right) + \frac{1}{(1-\beta)}\upsilon\frac{D^{\upsilon}}{V^{(1+\upsilon)}} \tag{3.H.45.SS}$$

**Renewable intermediate energy sector**

- Amount of renewable intermediate energy (3.H.46)

$$N = (A_{\mathrm{N}})(K_{\mathrm{N}})^{\psi} \tag{3.H.46.SS}$$

- Capital demand of renewable intermediate energy generation (3.H.47)

$$r_{\mathrm{N}} = \psi(p_{\mathrm{N}})(A_{\mathrm{N}})(K_{\mathrm{N}})^{\psi-1} \tag{3.H.47.SS}$$

**Market Clearing & additional equations**

- Aggregate market constraint (3.H.48)

$$\begin{aligned} GDP &= Y - p_{\mathrm{E}}E_{\mathrm{Y}} + r_{\mathrm{N}}K_{\mathrm{N}} + r_{\mathrm{F}}K_{\mathrm{F}} + X \\ &= CN + p_{\mathrm{E}}E_{\mathrm{H}} + I_{CD} + I_{\mathrm{Y}} + I_{\mathrm{F}} + I_{\mathrm{N}} \end{aligned} \tag{3.H.48.SS}$$

- Energy market constraint (3.H.49)

$$E = E_{\mathrm{H}} + E_{\mathrm{Y}} \tag{3.H.49.SS}$$

- Steady state price for final energy

$$p_{\mathrm{E}} = \frac{1 - \beta + \beta\delta^{CD}}{\beta\theta\,(1-\theta)\left(\frac{E_{\mathrm{H}}}{CD}\right)^{\zeta-1}} \tag{3.H.56}$$

- Steady state price for finite intermediate energy

$$p_{\mathrm{F}} = \frac{p_{\mathrm{E}}\left(\frac{E_{\mathrm{H}}}{GDP} + \frac{E_{\mathrm{Y}}}{GDP}\right) - p_{\mathrm{N}}\frac{N}{GDP}}{\frac{F}{GDP}} \tag{3.H.57}$$

- Steady state price for renewable intermediate energy: rearranging (3.H.21)

$$p_{\mathrm{N}} = r_{\mathrm{N}}\frac{K_{\mathrm{N}}}{N} \tag{3.H.21.SS}$$

### 3.H.4 Log-linearized equations

We have log-linearized the necessary equations characterizing the equilibrium of the system as derived before. In doing so, we have used the first order Taylor approximation around the steady state. For more details about the procedure, we refer to Uhlig (1995). Steady state values are marked with an upper bar. Log-linearized values are marked with a hat.

**Household sector**

$$\text{with: } CD^{LL} = \left(1 + \frac{(1-\theta)}{\theta}\left(\frac{\overline{E_{\mathrm{H}}}}{\overline{CD}}\right)^{\zeta}\right)^{-1} \quad \text{and} \quad (E_{\mathrm{H}})^{LL} = \left(1 + \frac{\theta}{(1-\theta)}\left(\frac{\overline{CD}}{\overline{E_{\mathrm{H}}}}\right)^{\zeta}\right)^{-1}$$

- Durable Euler equation (3.H.23)

$$\left(1-\delta^{CD}\right)\left(\widehat{CN_{t+1}} - \widehat{CN}_t\right) = \frac{1-\gamma}{\gamma}\frac{\overline{CN}}{\overline{CD}} CD^{LL}\left[\widehat{CN}_t - \left(CD^{LL}\zeta\widehat{CD}_t + (E_{\mathrm{H}})^{LL}\zeta(\widehat{E_{\mathrm{H}}})_{t+1}\right) + (\zeta-1)\,\widehat{CD}_t\right] \tag{3.H.23.LL}$$

- Non-durables vs. energy (3.H.24)

$$(\widehat{p_{\mathrm{E}}})_t = \widehat{CN}_t + (\zeta-1)\,(\widehat{E_{\mathrm{H}}})_t - CD^{LL}\zeta\widehat{CD}_t - (E_{\mathrm{H}})^{LL}\zeta(\widehat{E_{\mathrm{H}}})_t \tag{3.H.24.LL}$$

- Labor supply (3.H.25)

$$\widehat{CN}_t = \widehat{w_t} - \left(\frac{\bar{L}}{1-\bar{L}}\right)\widehat{L_t} \tag{3.H.25.LL}$$

- Euler equation for capital of final production (3.H.26)

$$\widehat{CN_{t+1}} = \widehat{CN}_t + (\beta\overline{r_{\mathrm{Y}}})\,(\widehat{r_{\mathrm{Y}}})_{t+1} \tag{3.H.26.LL}$$

- Euler equation for capital of finite intermediate energy generation (3.H.27)

$$\widehat{CN_{t+1}} = \widehat{CN}_t + (\beta\overline{r_{\mathrm{F}}})\,(\widehat{r_{\mathrm{F}}})_{t+1} \tag{3.H.27.LL}$$

- Euler equation for capital of renewable intermediate energy generation (3.H.28)

$$\widehat{CN_{t+1}} = \widehat{CN}_t + (\beta\overline{r_{\mathrm{N}}})\,(\widehat{r_{\mathrm{N}}})_{t+1} \tag{3.H.28.LL}$$

- Investments in durable goods (3.H.29)

$$(\widehat{I_{CD}})_t = \frac{\overline{CD}}{\overline{I_{CD}}}\widehat{CD}_t - \left(1-\delta^{CD}\right)\frac{\overline{CD}}{\overline{I_{CD}}}\widehat{CD}_{t-1} \tag{3.H.29.LL}$$

- Investments in capital of final production (3.H.30)

$$(\widehat{I_{\mathrm{Y}}})_t = \frac{\overline{K_{\mathrm{Y}}}}{\overline{I_{\mathrm{Y}}}}(\widehat{K_{\mathrm{Y}}})_t - \left(1-\delta^{Y}\right)\frac{\overline{K_{\mathrm{Y}}}}{\overline{I_{\mathrm{Y}}}}(\widehat{K_{\mathrm{Y}}})_{t-1} \tag{3.H.30.LL}$$

- Investments in capital of finite intermediate energy generation (3.H.31)

$$(\widehat{I_{\mathrm{F}}})_t = \frac{\overline{K_{\mathrm{F}}}}{\overline{I_{\mathrm{F}}}}(\widehat{K_{\mathrm{F}}})_t - \left(1-\delta^{F}\right)\frac{\overline{K_{\mathrm{F}}}}{\overline{I_{\mathrm{F}}}}(\widehat{K_{\mathrm{F}}})_{t-1} \tag{3.H.31.LL}$$

- Investments in capital of renewable intermediate energy generation (3.H.32)

$$(\widehat{I_{\mathrm{N}}})_t = \frac{\overline{K_{\mathrm{N}}}}{\overline{I_{\mathrm{N}}}}(\widehat{K_{\mathrm{N}}})_t - \left(1-\delta^{N}\right)\frac{\overline{K_{\mathrm{N}}}}{\overline{I_{\mathrm{N}}}}(\widehat{K_{\mathrm{N}}})_{t-1} \tag{3.H.32.LL}$$

**Final goods production sector**

with: $(K_{\mathrm{Y}})^{LL} = \left(1+\frac{(1-\eta)}{\eta}\left(\frac{\overline{E_{\mathrm{Y}}}}{\overline{K_{\mathrm{Y}}}}\right)^{\nu}\right)^{-1}$ and $(E_{\mathrm{Y}})^{LL} = \left(1+\frac{\eta}{(1-\eta)}\left(\frac{\overline{K_{\mathrm{Y}}}}{\overline{E_{\mathrm{Y}}}}\right)^{\nu}\right)^{-1}$

- Final goods production output (= non-durable goods) (3.H.33)

$$\widehat{Y_t} = (\widehat{A_{\mathrm{Y}}})_t + \alpha\left[(K_{\mathrm{Y}})^{LL}(\widehat{K_{\mathrm{Y}}})_{t-1} + (E_{\mathrm{Y}})^{LL}(\widehat{E_{\mathrm{Y}}})_t\right] + (1-\alpha)\,\widehat{L_t} \tag{3.H.33.LL}$$

- Capital demand of final goods production (3.H.34)

$$(\widehat{r_{\mathrm{Y}}})_t = \widehat{Y_t} - \left[(K_{\mathrm{Y}})^{LL}\nu(\widehat{K_{\mathrm{Y}}})_{t-1} + (E_{\mathrm{Y}})^{LL}\nu(\widehat{E_{\mathrm{Y}}})_t\right] + (\nu-1)\,(\widehat{K_{\mathrm{Y}}})_{t-1} \tag{3.H.34.LL}$$

- Energy demand of final goods production (3.H.35)

$$(\widehat{p_{\mathrm{E}}})_t = \widehat{Y_t} - \left[(K_{\mathrm{Y}})^{LL}\nu(\widehat{K_{\mathrm{Y}}})_{t-1} + (E_{\mathrm{Y}})^{LL}\nu(\widehat{E_{\mathrm{Y}}})_t\right] + (\nu-1)\,(\widehat{E_{\mathrm{Y}}})_t \tag{3.H.35.LL}$$

- Labor demand of final goods production (3.H.36)

$$\widehat{w_t} = \widehat{Y_t} - \widehat{L_t} \tag{3.H.36.LL}$$

**Final energy sector**

- Amount of final energy generation (3.H.37)

$$\widehat{E_t} = (\widehat{A_{\mathrm{E}}})_t + \phi\widehat{F_t} + (1-\phi)\,\widehat{N_t} \tag{3.H.37.LL}$$

- Price for finite intermediate energy (3.H.38)

$$(\widehat{p_{\mathrm{F}}})_t = (\widehat{p_{\mathrm{E}}})_t + \widehat{E_t} - \widehat{F_t} \tag{3.H.38.LL}$$

- Price for renewable intermediate energy (3.H.39)

$$(\widehat{p_{\mathrm{N}}})_t = (\widehat{p_{\mathrm{E}}})_t + \widehat{E_t} - \widehat{N_t} \tag{3.H.39.LL}$$

**Finite intermediate energy sector**

- Amount of finite intermediate energy (3.H.40)

$$\widehat{F}_t = (\widehat{A_{\mathrm{F}}})_t + \varphi(\widehat{K_{\mathrm{F}}})_{t-1} + (1-\varphi)\,\widehat{S}_t \tag{3.H.40.LL}$$

- Finite reserve constraint (3.H.41)

$$\widehat{S}_t = \widehat{S_{t-1}} - \frac{\overline{F}}{\overline{S}}\widehat{F}_t + \frac{\overline{D}}{\overline{S}}\widehat{D_t} - \varepsilon_{\mathrm{S},t} \tag{3.H.41.LL}$$

- Cost-function of exploration (3.H.42)

$$\widehat{CO_t} = \upsilon\left(\widehat{D_t} - \widehat{V}_t\right) \tag{3.H.42.LL}$$

- Finite resource constraint (3.H.43)

$$\widehat{V}_t = \widehat{V_{t-1}} - \frac{\overline{D}}{\overline{V}}\widehat{D_t} - \varepsilon_{\mathrm{V},t} \tag{3.H.43.LL}$$

- Capital demand of finite intermediate energy generation (3.H.44)

$$\begin{aligned}(\widehat{p_{\mathrm{F}}})_t = {} & \frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{F}}\frac{1}{\varphi\overline{p_{\mathrm{F}}}}\left((\widehat{r_{\mathrm{F}}})_t + (\widehat{K_{\mathrm{F}}})_{t-1} - \widehat{F}_t\right) + (1-\varphi)\,\frac{\overline{F}}{\overline{S}}\left((\widehat{p_{\mathrm{F}}})_t + \widehat{F}_t - \widehat{S}_t\right) + \beta(\widehat{p_{\mathrm{F}}})_{t+1} \\ & -\beta\frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{F}}\frac{1}{\varphi\overline{p_{\mathrm{F}}}}\left((\widehat{r_{\mathrm{F}}})_{t+1} + (\widehat{K_{\mathrm{F}}})_t - \widehat{F}_{t+1}\right) - \beta\,(1-\varphi)\,\frac{\overline{F}}{\overline{S}}\left((\widehat{p_{\mathrm{F}}})_{t+1} + \widehat{F}_t - \widehat{S}_t\right) \\ & +\beta\frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{F}}\frac{1}{\varphi\overline{p_{\mathrm{F}}}}\frac{\overline{F}}{\overline{S}}\,(1-\varphi)\left((\widehat{r_{\mathrm{F}}})_{t+1} + (\widehat{K_{\mathrm{F}}})_t - \widehat{F}_{t+1} + \widehat{F}_t - \widehat{S}_t\right)\end{aligned} \tag{3.H.44.LL}$$

- Amount of exploration in finite intermediate energy generation (3.H.45)

$$\begin{aligned}(\widehat{p_{\mathrm{F}}})_t = {} & \frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{F}}\frac{1}{\varphi\overline{p_{\mathrm{F}}}}\left((\widehat{r_{\mathrm{F}}})_t + (\widehat{K_{\mathrm{F}}})_{t-1} - \widehat{F}_t\right) + \frac{\upsilon\overline{D^{\upsilon}}}{\overline{DV^{\upsilon}}}\frac{1}{\overline{p_{\mathrm{F}}}}\left((\upsilon-1)\widehat{D}_t - \upsilon\widehat{V}_t\right) \\ & -\beta\frac{\upsilon\overline{D^{\upsilon}}}{\overline{DV^{\upsilon}}}\frac{1}{\overline{p_{\mathrm{F}}}}\left((\upsilon-1)\widehat{D}_{t+1} - \upsilon\widehat{V}_{t+1}\right) + \beta(\widehat{p_{\mathrm{F}}})_{t+1} + \frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{F}}\frac{1}{\varphi\overline{p_{\mathrm{F}}}}\left((\widehat{r_{\mathrm{F}}})_{t+1} + (\widehat{K_{\mathrm{F}}})_t - \widehat{F}_{t+1}\right) \\ & + \frac{\upsilon\overline{D^{\upsilon}}}{\overline{V^{1+\upsilon}}}\frac{1}{\overline{p_{\mathrm{F}}}}\left((\upsilon)\widehat{D}_t - (1+\upsilon)\widehat{V}_t\right)\end{aligned} \tag{3.H.45.LL}$$

**Renewable intermediate energy sector**

- Amount of renewable intermediate energy (3.H.46)

$$\widehat{N_t} = (\widehat{A_{\mathrm{N}}})_t + \psi(\widehat{K_{\mathrm{N}}})_{t-1} \tag{3.H.46.LL}$$

- Capital demand of renewable intermediate energy generation (3.H.47)

$$(\widehat{r_{\mathrm{N}}})_t = (\widehat{p_{\mathrm{N}}})_t + \widehat{N_t} - (\widehat{K_{\mathrm{N}}})_{t-1} \tag{3.H.47.LL}$$

**Market Clearing**

- Aggregate market constraint (3.H.48)

$$\widehat{GDP} = \left(1 - \frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{GDP}} - \frac{\overline{r_{\mathrm{N}}K_{\mathrm{N}}}}{\overline{GDP}} + \frac{\overline{p_{\mathrm{E}}E_{\mathrm{Y}}}}{\overline{GDP}} - \frac{\overline{C}}{\overline{GDP}}\right)\widehat{Y_t} - \frac{\overline{p_{\mathrm{E}}E_{\mathrm{Y}}}}{\overline{GDP}}\Big((\widehat{p_{\mathrm{E}}})_t + (\widehat{E_{\mathrm{Y}}})_t\Big) + \frac{\overline{r_{\mathrm{F}}K_{\mathrm{F}}}}{\overline{GDP}}\Big((\widehat{r_{\mathrm{F}}})_t + (\widehat{K_{\mathrm{F}}})_{t-1}\Big) + \frac{\overline{r_{\mathrm{N}}K_{\mathrm{N}}}}{\overline{GDP}}\Big((\widehat{r_{\mathrm{N}}})_t + (\widehat{K_{\mathrm{N}}})_{t-1}\Big) + \frac{\overline{C}}{\overline{Y}}\widehat{C_t} \tag{3.H.48a.LL}$$

$$\widehat{GDP} = \left(1 - \frac{\overline{I_{CD}}}{\overline{GDP}} - \frac{\overline{I_{\mathrm{Y}}}}{\overline{GDP}} - \frac{\overline{I_{\mathrm{F}}}}{\overline{GDP}} - \frac{\overline{I_{\mathrm{N}}}}{\overline{GDP}} - \frac{\overline{p_{\mathrm{E}}E_{\mathrm{H}}}}{\overline{GDP}}\right)\widehat{CN}_t + \frac{\overline{p_{\mathrm{E}}E_{\mathrm{H}}}}{\overline{GDP}}\Big((\widehat{p_{\mathrm{E}}})_t(\widehat{E_{\mathrm{H}}})_t\Big) + \frac{\overline{I_{CD}}}{\overline{GDP}}(\widehat{I_{CD}})_t + \frac{\overline{I_{\mathrm{Y}}}}{\overline{GDP}}(\widehat{I_{\mathrm{Y}}})_t + \frac{\overline{I_{\mathrm{F}}}}{\overline{GDP}}(\widehat{I_{\mathrm{F}}})_t + \frac{\overline{I_{\mathrm{N}}}}{\overline{GDP}}(\widehat{I_{\mathrm{N}}})_t \tag{3.H.48b.LL}$$

- Energy market constraint (3.H.49)

$$\widehat{E_t} = \frac{\overline{E_{\mathrm{H}}}}{\overline{E}}(\widehat{E_{\mathrm{H}}})_t + \frac{\overline{E_{\mathrm{Y}}}}{\overline{E}}(\widehat{E_{\mathrm{Y}}})_t \tag{3.H.49.LL}$$

**Formation of shocks**

- Productivity shock in final goods production (3.H.50)

$$(\widehat{A_{\mathrm{Y}}})_t = \rho_{\mathrm{Y}}(\widehat{A_{\mathrm{Y}}})_{t-1} + \varepsilon_{\mathrm{Y}} \tag{3.H.50.LL}$$

- Productivity shock in final energy generation (3.H.51)

$$(\widehat{A_{\mathrm{E}}})_t = \rho_{\mathrm{E}}(\widehat{A_{\mathrm{E}}})_{t-1} + \varepsilon_{\mathrm{E}} \tag{3.H.51.LL}$$

- Productivity shock in finite intermediate energy generation (3.H.52)

$$(\widehat{A_{\mathrm{F}}})_t = \rho_{\mathrm{F}}(\widehat{A_{\mathrm{F}}})_{t-1} + \varepsilon_{\mathrm{F}} \tag{3.H.52.LL}$$

- Productivity shock in renewable intermediate energy generation (3.H.53)

$$(\widehat{A_{\mathrm{N}}})_t = \rho_{\mathrm{N}}(\widehat{A_{\mathrm{N}}})_{t-1} + \varepsilon_{\mathrm{N}} \tag{3.H.53.LL}$$

- Consumer taste shock in non-durable goods (3.H.54)

$$(\widehat{T_{\mathrm{CN}}})_t = \rho_{\mathrm{T,CN}}(\widehat{T_{\mathrm{CN}}})_{t-1} + \varepsilon_{\mathrm{T,CN}} \tag{3.H.54.LL}$$

- Consumer taste shock in durable goods (3.H.55)

$$(\widehat{T_{\mathrm{CD}}})_t = \rho_{\mathrm{T,CD}}(\widehat{T_{\mathrm{CD}}})_{t-1} + \varepsilon_{\mathrm{T,CD}} \tag{3.H.55.LL}$$

### 3.H.5 Calibration

Some parameters can be directly derived from their target moments. The remaining parameters are calculated from the steady state conditions of the model. All endogenous variables correspond to their steady-state values.

Rearranging (3.H.29.SS), we get:

$$\delta^{CD} = \frac{I_{CD}/GDP}{CD/GDP} \tag{3.H.58}$$

Similarly, rearranging (3.H.31.SS), we get:

$$\delta^{F} = \delta^{N} = \frac{I_{F}/GDP}{K_{\mathrm{F}}/GDP} \tag{3.H.59}$$

Plugging (3.H.24.SS) into (3.H.23.SS):

$$\begin{aligned}
\left(\theta CD^{\zeta} + (1-\theta)\,(E_{\mathrm{H}})^{\zeta}\right)^{-1} &= p_{\mathrm{E}}\frac{\gamma}{(1-\gamma)\,(1-\theta)}\frac{(E_{\mathrm{H}})^{1-\zeta}}{CN} \\
1 = \beta\theta\frac{(1-\gamma)}{\gamma}\frac{(E_{\mathrm{H}})^{1-\zeta}}{CN}CN\,CD^{\zeta-1}p_{\mathrm{E}}&\frac{\gamma}{(1-\gamma)\,(1-\theta)} + \beta\left(1-\delta^{CD}\right) \\
= \beta\frac{\theta}{(1-\theta)}p_{\mathrm{E}}\left(\frac{E_{\mathrm{H}}}{CD}\right)^{1-\zeta} &+ \beta\left(1-\delta^{CD}\right)
\end{aligned}$$

solving for $\theta$

$$\theta = \frac{1-\beta\left(1-\delta^{CD}\right)}{1-\beta\left(1-\delta^{CD}\right)+\beta\frac{CD}{E_{\mathrm{H}}}^{\zeta-1}} \tag{3.H.60}$$

Rearranging (3.H.32.SS), we get:

$$\frac{K_{\mathrm{N}}}{GDP} = \frac{I_{N}}{GDP}\frac{1}{\delta^{N}} \tag{3.H.61}$$

Similar to Dhawan and Jeske (2008), we target the aggregate capital stock relative to GDP to 12:

$$\frac{K_{\mathrm{Y}}}{GDP} = 12 - \frac{K_{\mathrm{F}}}{GDP} - \frac{K_{\mathrm{N}}}{GDP} \tag{3.H.62}$$

From (3.H.35.SS), we can solve for $\eta$:

$$\eta = \frac{\alpha\left(\frac{E_{\mathrm{Y}}}{GDP}\right)^{-1} - 1}{\alpha\left(\frac{E_{\mathrm{Y}}}{GDP}\right)^{-1} - 1 + \left(\frac{K_{\mathrm{Y}}}{E_{\mathrm{Y}}}\right)^{\nu}} \tag{3.H.63}$$

From (3.H.34.SS), we can derive the steady state value for $r_{\mathrm{Y}}$:

$$r_{\mathrm{Y}} = \frac{K_{\mathrm{Y}}}{GDP}^{-1}\left(\eta + (1-\eta)\frac{K_{\mathrm{Y}}}{E_{\mathrm{Y}}}^{-\nu}\right)^{-1}\alpha\eta \tag{3.H.64}$$

Rearranging (3.H.27.SS), we get:

$$r_{\mathrm{F}} = \delta^{F} + \frac{1}{\beta} - 1 \tag{3.H.65}$$

Similarly, rearranging (3.H.28.SS), we get:

$$r_{\mathrm{N}} = \delta^{N} + \frac{1}{\beta} - 1 \tag{3.H.66}$$

From (3.H.23.SS), we can solve for $\gamma$:

$$\gamma = \frac{1-\theta}{1-\theta + \frac{E_{\mathrm{H}}}{CN}\left(\theta\left(\frac{CD}{E_{\mathrm{H}}}\right)^{\zeta} + 1 - \theta\right)} \tag{3.H.67}$$

Rearranging (3.H.26.SS), we get:

$$\delta^{Y} = r_{\mathrm{Y}} - \frac{1}{\beta} + 1 \tag{3.H.68}$$

Rearranging (3.H.30.SS), we get:

$$\frac{I_{\mathrm{Y}}}{GDP} = \delta^{Y}\frac{K_{\mathrm{Y}}}{GDP} \tag{3.H.69}$$

Rearranging the expenditures approach of (3.H.48.SS), we get:

$$\frac{CN}{GDP} = 1 - \frac{E_{\mathrm{H}}}{GDP} - \frac{E_{\mathrm{Y}}}{GDP} - \frac{I_{\mathrm{Y}}}{GDP} - \frac{I_{\mathrm{F}}}{GDP} - \frac{I_{\mathrm{N}}}{GDP} - \frac{I_{CD}}{GDP} \tag{3.H.70}$$

Equalizing (3.H.25.SS) and (3.H.36.SS), we can solve for $\vartheta$:

$$\vartheta = \frac{1}{1 + \left(\frac{CN}{GDP}\right)^{-1}\gamma\,(1-\alpha)\,\frac{(1-L_{\mathrm{SS}})}{L_{\mathrm{SS}}}} \tag{3.H.71}$$

### 3.H.6 Calibration of steady states in primary energy mining

Steady state values concerning the mining of finite primary energy, their reserves and resources are based on panel data from BGR (2016). These observations are subject to considerable fluctuations, in particular the estimations of resources, mainly because of different estimation techniques but also economics reasons or political interventions to abandon reserves or resources. Hence, we calculated the steady states as averages of the whole observation time period comprising 1992–2014. We covered oil, gas, hard coal, and lignite coal as finite primary energy resources. Table 3.H.2 depicts the steady states for each resource and the weighted-average. Since primary energy resources differ by the amount of total energy they can release, we standardized the values using heating values (MJ/kg). This is why the share of oil and gas is considerable larger than expected because both energy substance have significantly larger heating values in comparison to coal.

Table 3.H.2: Steady states of finite primary energy

| | oil | gas | hard coal | lignite coal | standard weighted-average |
|---|---|---|---|---|---|
| F/S | 0.06655 | 0.07021 | 0.21544 | 0.00435 | 0.07624 |
| D/V | 0.07063 | 0.02971 | 0.00002 | 0.00382 | 0.01259 |
| D/F | 0.57679 | 0.32557 | -0.01191 | 0.77500 | 0.45590 |
| Heating values[a] | 42.8 | 46 | 32.7 | 8 | |
| Share[b] | 4.54% | 25.81% | 24.67% | 44.99% | ∼100% |

[a] Heating values are in MJ/kg.
[b] Shares are calculated based on mining in 2007 and adjusted according to the respective heating values.

# Chapter 4

# Heterogeneity in an RBC Model with Durable Goods and Energy

## 4.1 Introduction

This paper investigates the effects of total factor productivity and energy price shocks in a real business cycle (RBC) model with heterogeneous agents. It extends the model by Dhawan and Jeske (2008), including the distinction between durable goods and non-durable goods, by an incomplete market similar to Preston and Roca (2007). Furthermore, in our model a fixed proportion of agents has limited asset market participation as in Gali et al. (2003). As a result, this model can predict the evolution of inequality in income and wealth, unlike traditional homogeneous macroeconomic models with a representative agent.

Basic dynamic general equilibrium models with a single consumption good produced by a production sector predict a consumption volatility that is significantly lower than the one in observational data. Dhawan and Jeske (2008) have extended the RBC model by Kim and Loungani (1992) which includes energy, with the opportunity to gain utility from the consumption of accumulated durable goods. Although energy has smaller effects on output fluctuations compared to Kim and Loungani, enlarging the flexibility to re-balance an agent's portfolio improves the prediction of consumption volatility. By impacting consumption of durable goods and improving the prediction of consumption volatility significantly, the factor energy shows that it is not negligible in analyzing economic activities. Huynh (2016) goes beyond this by endogenizing the production process of durable goods and energy, bringing energy volatility closer to its empirical target values. Representative for other, but similar, homogeneous frameworks, both models ignore the existence of heterogeneity in human beings and their decision making. As a result, they are neither able to describe how inequality arises nor how it affects economic activity. However, such explanations become important, in particular when considering the role of government intervention to effectively correct market imperfections.

In this paper, we provide a theoretical framework to demonstrate the consequences of agents' heterogeneous labor supply and limited market participation. The framework explicitly models the consumption of durable and non-durable goods. Just as in Dhawan and Jeske (2008), we assume complementarity between energy and the usage of durable goods (in the utility function) and capital (in the goods production function). We use explicit aggregation as done by Den Haan and Rendahl (2010) in order to solve the cross-section capital distribution among Ricardian households and, consequently, the policy function for capital on the macro level of the model.

The aim of this work is to investigate the transmission mechanisms and characteristics of total-factor productivity (TFP) and energy price shocks, but also of external shocks of heterogeneity though labor supply. Moreover, we study to what degree the empirically observed inequality in income and wealth can be explained by the provided framework. Therefore, we calibrate the model to match the German economy. Furthermore, we consider not only how policy intervention through redistribution affects individuals' income and wealth, but also the inequality on the macro level of the economy.

We show that the distinction between non-durable and durable goods leads to a significant improvement in predicting most of the moments close to the one in observational data from Germany. Here, energy price shocks have a contractionary effect on economic activity, as they cause disruptions in particular in durable goods, as is similarly shown in Dhawan and Jeske (2008). Nevertheless, TFP is still the driving force of output volatility. The provided framework is able to match income inequality indices quite well, whereas inequality of wealth remains underestimated. This is justified given that we assume homogeneity in individuals' productivity as well as the exogenous process of labor supply. Furthermore, we find that energy price shocks lead to decreasing inequalities, with respect to both income and wealth. This happens due to the complementary relationship between durable goods and energy and sunk costs, which arise along with adjustments in the durable goods stock. We conclude that it is not the low-income agent who benefits from volatility in energy prices, but the high-income agent who looses in income and wealth due to higher absolute sunk costs.

Policy intervention in the form of redistribution of income decreases income inequality on the macro level, between both classes of agents, and within the class of rule-of-thumb agents, but leads to a slight increase among Ricardian agents. For wealth inequality, we notice a slight increase in overall inequality. This is due to decreasing saving rates, which widen the gap between savers. Accordingly, we conclude that policy instruments have to be evaluated carefully in order to successfully combat inequality.

The remainder of this paper is structured as follows: After a brief discussion of several sources of heterogeneity in Section 2, Section 3 describes the model economy.

Section 4 specifies the market equilibrium and examines theoretical literature in order to solve the aggregate capital stock. Section 5 presents the calibrated and estimated parameters. Section 6 presents the results of the model. In Sections 7, we conduct a sensitivity analysis. Section 8 discusses the policy implication of redistribution through income taxation. Section 9 concludes.

## 4.2 Theoretical Literature on Inequality

Many traditional neoclassical economic models often assume an economy populated by a representative consumer who operates in a perfectly competitive good, factor, and asset market. Aggregated shocks, e.g. in TFP, can cause uncertainty in the market, which affects the behavior of consumers in maximizing their utility. Even in models in which heterogeneous agents face idiosyncratic shocks, such as in labor supply or income, the assumption of a representative household can hold through aggregation of heterogeneous agents, when complete markets are present (e.g. in standard Arrow-Debreu economies). The reason is that idiosyncratic risk can fully be diversified away (e.g. by borrowing and saving) and hence become irrelevant for equilibrium outcomes. The market is self-regulating and volatility in aggregate economic activity in an efficient response to shocks (Christiano et al., 2018), while government interference is inefficient and worsen the state of the economy. However, when agents cannot fully insure against idiosyncratic risk, e.g. due to incomplete capital markets, inequality in the evolution of wealth occurs. Incomplete capital markets may for example exist when there is a borrowing constraint, preventing agents from holding debt, so that they cannot borrow against their future earnings. When markets fail, government intervention can be efficient for correcting market failures.

To understand how inequality occurs and evolves in macroeconomic models, we briefly focus on several sources of heterogeneity and how they are mapped in economic models. For an extended discussion of approaches on modeling heterogeneity, we refer to Heathcote et al. (2009) and Guvenen (2011). It is common to distinguish between fundamental inequality or inequality of opportunity and inequality of outcome, when analyzing inequality in general. The latter is usually the result of the former, economically often resulting in inequality in income, wealth, consumption, utility, or leisure time. While individual utility is the ultimate object of interest, this is difficult to measure and quantify. Therefore, attraction is mostly turned to the remaining variables that play an intermediate role, particularly income and wealth.

Fundamental inequality describes the heterogeneous nature of individuals such as health, education, social status, gender, preference or age. When considering these types of inequality from the economic modeling perspective, fundamental inequality influences the model selection, the formation of assumption about the model, and its ingredients such as variables or functions. The model just acts as an intermediate transmission system. Consequently, a model without fundamental inequality cannot explain inequality on the output side. There are several sources of heterogeneity that cause fundamental inequality within a society. In the following, we look at three of them: namely, capabilities, external shocks, and preferences.

The assumption of different capabilities among human beings has a significant impact on inequality. In traditional models including homogeneous agents, individuals do not differ in their decision making. However, in real life this homogeneity is not very likely, because human beings differ in their behavior or their individual skills and abilities. This has an influence on their levels of productivity, and hence also on earnings, which in turn impacts income and wealth. In models that include education, agents can increase their productivity by choosing different amounts of schooling. However, that decision depends on several factors, such as the ability to learn, the starting level of human capital, access to educational institutions, or simply the choice of studying or entering the labor market (e.g. Huggett et al., 2011).

A further variation in capability comes from the restricted access to further key institutions, such as financial markets. Under "limited asset market participation", there is a distinction between two classes of agents: Ricardian agents who have free access to the capital market, and rule-of-thumb agents who are excluded from this market. Economically, this means that the latter cannot insure against income disruption to smooth their consumption. Other models describe heterogeneity in capability by including choice of occupation, in which an individual decides about becoming an entrepreneur or a worker because of its individual risk aversion (e.g. Clemens, 2006; Kanbur, 1979; Lucas Jr, 1978).

Considering heterogeneity in external shocks has become popular with standard incomplete market models (SIM), also called Bewley models (Bewley, 1976). In the SIM approach, individuals are identical ex-ante but differ ex-post due to idiosyncratic shocks which are uninsurable. These shocks are unexpected or unpredictable events and follow a stochastic process. In contrast to aggregate shocks in traditional RBC models, which generally affect the entire economy including several sectors, these idiosyncratic shocks affect households individually. Initial models incorporate uninsurable idiosyncratic earning shocks (e.g. Aiyagari, 1994; Huggett, 1993; Imrohoroğlu, 1989) which translate into inequality of income and wealth through different saving decisions. Storesletten et al. (2004) and Shimer (2010) model heterogeneity in the labor market by including idiosyncratic shocks in labor supply to replicate job fluctuation. Next to a job loss, further idiosyncratic shocks can affect health and family (e.g. Greenwood and Guner, 2008; Hubbard et al., 1995). An individual's health status can become dramatically worse through a sudden disease. The family composition can change through marriage, divorce, the birth of a child, or death. Although these types of heterogeneity can involve some decision making and can be modeled endogenously, they also incorporate an exogenous component (risk).

Heterogeneity in preferences is closely connected to heterogeneity in capabilities, as various abilities, such as learning abilities and human capital, might subsequently influence individual behavior. Many models assume that once set, preferences remain constant throughout the entire lifetime. Alternatively, in models that feature discrete groups of agents, such as overlapping generation models, different preferences can be assigned to different stages of life (e.g. Benabou, 2000; Persson and Tabellini, 1994). Furthermore, preferences may vary at the individual level after introducing a statistical distribution over parameters such as time preference, risk aversion, or elasticity of substitution.

In this paper, we combine two sources of heterogeneity. We first introduce heterogeneity by using idiosyncratic shocks in labor supply that allow for income fluctuation among all agents and result in an incomplete market model. However, this type of model would not allow for dichotomy in the access to selected institutions. Consequently, households at the extreme ends of income and wealth distribution would not yet be different in any relevant way. Moreover, taxation policy would become complicated, as there would be no groups that can individually be taxed progressively, as is evident from real-life observations. Therefore, we also introduce limited asset market participation, by excluding a fixed proportion of agents from the capital market. We acknowledge that this may violate the permanent income hypothesis by Friedman (2018), stating that agents save in anticipation of possible future declines. However, the violation would only be partial, thanks to the presence of a further (durable) consumption goods that can be accumulated over time.

## 4.3 Model

The model consists of two sectors: a household sector and a sector with goods-producing firms. Moreover, the model is characterized by incomplete markets, aggregate uncertainty, as well as an infinite number of agents. The specific structure of heterogeneity comes from the household sector which faces a partly uninsurable idiosyncratic labor supply and hence labor shocks similar to Aiyagari (1994). Since employers cannot discriminate between agents by assumption, a shock on the demand side would not affect agents individually, but aggregately. As a result, the introduction of idiosyncratic shocks on the supply side allows introducing heterogeneity among agents.

In addition, we distinguish between two types of households which differ by their access to the capital market. Ricardian households can intertemporally allocate capital while rule-of-thumb households are excluded from this activity.[1] Besides that, both classes do not differ; hence, they face the same elasticities of substitution

[1] In fact, rule-of-thumb households can use durable goods to slightly intertemporally smooth consumption. However, using the durable goods stock is not as efficient as using the capital stock, due to additional adjustment costs, and hence it can be described as partly-illiquid wealth. Therefore, these household are considered as a light version of non-Ricardian households as we elaborate later.

in the utility function, the same time discount rate, and the same depreciation rate for durable goods.

As a result, we combine two approaches of inequality described before, namely 'incomplete market models' and 'limited asset market participation models'. By this, we create inequality within and between different groups which offers possibilities to model inequality more realistically. The infinity-lived households, indexed by $i$, are defined over an interval $i \in [0, 1]$ while Ricardian and rule-of-thumb households account for $[0, \lambda]$ and $(\lambda, 1]$ respectively. For a better differentiation, households are further indexed with their respective type, namely, Ricardian households $\{R\}$ and rule-of-thumb households $\{N\}$. Figure 4.A.1 in the Appendix depicts a graphical description of the model. Hereafter, the model is described in more detail.

### 4.3.1 Ricardian households

In the model, all households maximize their utility by choosing the optimal demand for consumption goods and energy given the budget constraints. Households can consume three type of goods: non-durable goods $CN$ which are provided by the goods production sector, durable goods $CD$ in which agents can invest and which is accumulated over time, and energy $E_{\mathrm{H}}$ which is provided exogenously.[2] The utility function is assumed to have constant elasticity of substitution (CES) between durable goods and energy which are nested within a Cobb-Douglas function with non-durable goods.[3] Furthermore, it includes a separate additive penalty function to fulfill the transversality condition which otherwise might be violated due to occasionally-binding inequality constraints. Ricardian households consider the following utility function:

$$U_{R,i,t} = \ln\left[CN^{\gamma}_{R,i,t}\left(\theta\, CD^{\zeta}_{R,i,t-1} + (1-\theta)\,(E_{\mathrm{H}})^{\zeta}_{R,i,t}\right)^{\frac{1-\gamma}{\zeta}}\right] - \phi P(S_{R,i,t}) \qquad (4.1)$$

$$\text{where} \quad P(S_{R,i,t}) = \frac{1}{(S_{R,i,t}+b)^2} \qquad \text{with } S_{R,i,t} \geq -b \qquad (4.2)$$

where $\theta \in (0, 1)$ determines the consumption share of the durable goods. Furthermore, to fulfill a complementary relationship between durable goods and energy, the inverse of the elasticity of substitution $\zeta < 0$ must hold while the substitutionary relationship between this consumption bundle and non-durable goods imply $\gamma \in (0, 1)$. $\phi > 0$ is a penalty parameter.

The asset market is incomplete because of having a heterogeneous agent model with idiosyncratic shocks. Hence, employment risks are only partially insurable and the budget constraint includes occasionally-binding inequality. By adding the penalty function (4.2) to agent's utility, this allows us to deal with the problem

[2] Hereafter, we omit the time index when describing variables.

[3] The elasticity of substitution between durable and non-durable goods is often set close to unity in empirical literature (e.g. Ogaki and Reinhart, 1998).

of non-negative constraints by formulating the optimization problem as an unconstrained one. Here, we take the penalty specification suggested by Preston and Roca (2007).[4] The idea is that any amount of consumption and asset holding is feasible but the objective function faces undesired outcome when the constraint is violated. When individual asset holding $S_{i,t}$ approaches the borrowing limit $b$, the penalty function approaches infinity. For small $\phi$, the borrowing constraint becomes similar to $S_{i,t} + b \geq 0$ as in Aiyagari (1994). $b \geq 0$ describes the natural borrowing limit which avoids Ponzi-schemes.

**Lemma 3.** *Strict concavity of utility*
*The partial derivatives for the utility function $U_R$ are:*

$$U'_{CN} > 0, U'_{CD} > 0, U'_{E_\mathrm{H}} > 0, U'_{S_R} > 0$$
$$U''_{CN\,CN} < 0, U''_{CD\,CD} < 0, U''_{E_\mathrm{H}E_\mathrm{H}} < 0, U''_{S_R,S_R} < 0$$
$$U''_{CN\,CD} = U''_{CD\,CN} > 0, U''_{CD\,E_\mathrm{H}} = U''_{E_\mathrm{H}\,CD} > 0, U''_{CN\,E_\mathrm{H}} = U''_{E_\mathrm{H}\,CN} > 0.$$

*Utility function $U_t$ is overall strictly concave in $CN, CD, E_\mathrm{H}$ iff all the following conditions hold:*

$$\phi > 0$$
$$0 < \gamma < 1$$
$$\zeta, \theta < 1.$$

*Proof: Analogously to H.1.1 in Bergmann (2018).*

According to Lemma 3, a rise in consumption of all three consumption goods increases utility but with a diminishing rate. The complementary relationship between durable goods and energy implies the expenses for a certain amount of energy which is require to consume the accumulated durable goods. Hence, energy can be considered to be consumed to enhance the consumption of durable goods in a non-perfect substitutable manner. Alternatively, the presence of energy is required to consume durable goods. Overall concavity of utility function $U$ is guaranteed if Proposition 3 holds.

The maximization problem of Ricardian households is restricted by the budget constraint below.

$$CN_{R,i,t} + (p_\mathrm{H})_t(E_\mathrm{H})_{R,i,t} + (I_{CD})_{R,i,t} + (I_\mathrm{Y})_{R,i,t} = w_t L_{R,i,t} + r_t S_{R,i,t-1} + \pi_t \quad (4.3)$$

According to that, Ricardian households gain wage income $w_t$ from the supply of labor and capital rents $r_t$ from their accumulated savings.[5] On the expenditure

[4] For further penalty approaches, see Den Haan and Ocaktan (2009).

[5] Under the assumption of perfect competition in the goods market, goods-producing firms gain zero profits, hence $\pi = 0$.

side are non-durable consumption goods and energy as well as investments in the capital stock and durable goods described by the following equations:

$$(I_{\mathrm{Y}})_{R,i,t} = S_{R,i,t} - \left(1 - \delta^{Y}\right) S_{R,i,t-1} \tag{4.4}$$

$$\begin{aligned}(I_{CD})_{R,i,t} &= CD_{R,i,t} - \left(1 - \delta^{CD}\right) CD_{R,i,t-1} + IC\left(CD_{R,i,t}, CD_{R,i,t-1}\right) \\ \text{where} \quad IC\left(CD_{R,i,t}, CD_{R,i,t-1}\right) &= \frac{\omega_{1cd}}{1+\omega_{2cd}} \left(\frac{CD_{R,i,t} - CD_{R,i,t-1}}{CD_{R,i,t-1}}\right)^{1+\omega_{2cd}}\end{aligned} \tag{4.5}$$

Both investments are each diminished by a fixed depreciation rate while durable goods investments also contain adjustment costs ($IC$).[6] These costs are assumed to be quadratic in nature, hence, investment in durable goods goes along with an increasing and convex cost of net investment. In other words, the costs of adjusting investments increase proportionally faster than the amount of durable goods which is adjusted. On the one side, adjustment costs help to lower the correlation between investments and economic activity (Hayashi, 1982). On the other side, it captures the fact that building up or changing durable goods is costly and takes time. So, it avoids excessive changes in investments in the short run. In the long run, households do not face much of adjustment cost when they keep investments infinity small. As a result, households will respond by adjusting their investment decision continuously and smoothly.

In this model, labor supply is determined exogenously by an idiosyncratic component following an autoregressive process proposed by Preston and Roca (2007). This is in contrast to Dhawan and Jeske (2008) and Bergmann (2018) who assume labor to be endogenously determined. Hence, individual agents cannot choose the amount of work they are likely to provide. Because the supply of labor is set exogenously, it does not depend on the wage rate. This might describe a situation where an employee is demanded to work short-time or over-time which is not compensated. The stochastic autoregressive term for individual $i$ follows:

$$L_{R,i,t} = (1 - \rho_L)\bar{L}_R + \rho_L L_{R,i,t-1} + \varepsilon_{L,R,i,t}, \tag{4.6}$$

comprising the steady state $\bar{L}$, adjustment coefficient $\rho_L$, labor opportunity of the previous period, and a normally distributed variable $\varepsilon_{L,R,i} \overset{\text{iid}}{\sim} N\left(0, \sigma^2_{L,R}\right)$ describing a bounded i.i.d. disturbance. This shock is not insurable and hence, it leads to a variation in the income of the individuals, which has an impact on consumption. However, by accumulating primary capital but also durable goods, the effect of

[6] By reason of the cross-sectional distribution of capital and the way to deal with it through approximate aggregation, we do not consider adjustment costs along with investments in the capital stock which holds the model simple.

disturbances can be mitigated. Under the assumption of $\rho_L < 1$, (4.6) describing labor supply is stationary distributed.

Opposite to traditional neoclassical models with homogeneous agents, the heterogeneous structure of the model allows us to examine the evolution of inequality of outcomes such as income or wealth. As explained before, Ricardian households can gain income from labor and capital supply. The (net-)income equation corresponds to the income approach of an agent's budget constraint (4.3) after depreciation and is described by:

$$INC_{R,i,t} = w_t L_{R,i,t} + r_t S_{R,i,t-1} + r_t S_{R,i,t-1} + \pi_t. \tag{4.7}$$

Agent's wealth consists of income in the current period, equal to equation 4.13 and the stock of accumulated assets. A further share of wealth is the stock of accumulated durable goods, in contrast to Gali et al. (2003), Kim et al. (2005), Preston and Roca (2007), and Den Haan and Ocaktan (2009). Both net-portfolios (after depreciation) increase wealth which is denoted by:

$$WLTH_{R,i,t} = w_t L_{R,i,t} + r_t S_{R,i,t-1} + \left(1 - \delta^{Y}\right) S_{R,i,t-1} + \left(1 - \delta^{CD}\right) CD_{R,i,t-1} + \pi_t. \tag{4.8}$$

### 4.3.2 Rule-of-thumb households

Rule-of-thumb households share the same utility function like Ricardian households by maximizing their consumption of non-durable goods, durable goods, and energy, denoted by the following equation:

$$U_{N,i,t} = \ln \left[ CN^{\gamma}_{N,i,t} \left( \theta CD^{\zeta}_{N,i,t-1} + (1-\theta) \left(E_{\mathrm{H}}\right)^{\zeta}_{N,i,t} \right)^{\frac{1-\gamma}{\zeta}} \right] \tag{4.9}$$

The definitions and properties of all parameters comply with those from (4.1). Consequently, overall strict concavity is satisfied as in Lemma 3. Theoretically, the utility function also includes the penalty function (4.2) as described before. However, because rule-of-thumb agents are excluded from the financial asset market, their asset holding is zero. Hence, they are not affected by it. The maximization problem is confronted with the budget constraint:

$$CN_{N,i,t} + (p_{\mathrm{H}})_t (E_{\mathrm{H}})_{N,i,t} + (I_{CD})_{R,i,t} = w_t L_{N,i,t}, \tag{4.10}$$

where rule-of-thumb households solely gain income from their labor supply. They are barred from any access to the capital market and hence the possibility of intertemporal substitution. However, in contrast to Krusell and Smith (1998), Gali et al. (2003), Den Haan and Ocaktan (2009), and Troch (2014), the possibility to invest in durable goods yields in an opportunity to smooth their consumption behavior according to

$$(I_{CD})_{N,i,t} = CD_{N,i,t} - \left(1 - \delta^{CD}\right) CD_{N,i,t-1} + IC\left(CD_{N,i,t}, CD_{N,i,t-1}\right)$$
$$\text{where} \quad IC\left(CD_{N,i,t}, CD_{N,i,t-1}\right) = \frac{\omega_{1cd}}{1+\omega_{2cd}} \left(\frac{CD_{N,i,t} - CD_{N,i,t-1}}{CD_{N,i,t-1}}\right)^{1+\omega_{2cd}} \tag{4.11}$$

Concerning rule-of-thumb agents, adjustment costs in durable investments also fulfill a further role. By construction, these households can use the durable goods stock to intertemporally smooth consumption despite the exclusion from the financial asset market. This is not only natural and legitimate but also reflects conditions from reality. However, in this theoretical framework, the lack of access to the capital market will cause excess volatility in durable goods investments. Hence, additional costs makes it less efficient to use this investment possibility.[7] In addition, because rule-of-thumb agents can accumulated durable goods over time, this also means that they are not equal to non-Ricardian agents which by definition consume their current disposable income and are not able to smooth consumption. As a result, rule-of-thumb agents are assumed to be a light version of non-Ricardian agents because their smoothing capability is clearly limited, having no access to the asset market and facing adjustment costs when using the durable goods stock.

There is a further difference between Ricardian and rule-of-thumb households in the determination of the idiosyncratic employment opportunity equation following Preston and Roca (2007) and Troch (2014). Unlike Ricardian households, the latter do not only responds to the employment opportunity from the previous period but also on variation in the productivity of the goods-producing sector.

$$L_{N,i,t} = (1-\rho_L)\bar{L}_N + \rho_L L_{N,i,t-1} + \rho_{L,A}\left(A_t - \bar{A}\right) + \varepsilon_{L,N,i,t} \tag{4.12}$$

Steady-state labor supply $\bar{L}_N$ is equal to its counterpart of Ricardian households, $\rho_L$ indicates the variation coefficients, $\bar{A}$ is steady state productivity and $\varepsilon_{L,N,i,t}$ a bounded i.i.d. disturbance with mean and variance $(0, \sigma^2_{L,N})$. From the specification of the idiosyncratic employment opportunity, it holds that $\text{Cov}(\varepsilon_{L,N,i,t}, A_t) > 0$ although the disturbances of this idiosyncratic shock and the productivity equation are uncorrelated, such that $\text{Cov}(\varepsilon_{L,N,i,t}, \varepsilon_{A,t}) = 0$. Hence, opposite to Preston and Roca (2007) and Troch (2014), rule-of-thumb households' income shocks are only partly uninsurable due to the existence of durable goods. But variation in their income stream are still more volatile than those of Ricardian households.

Rule-of-thumb households gain income from labor supply only according to:

$$INC_{N,i,t} = w_t L_{N,i,t}. \tag{4.13}$$

[7] Alternatively, durable goods can be described to be a less-liquid factor stock.

In contrast to pure non-Ricardian households, who consume all their current disposable income and do not hold any wealth, in this model, rule-of-thumb agents can hold wealth by accumulating a durable goods stock. However, adjustments of this stock go along with additional costs, which is why it can also be described as partly-illiquid. The wealth equation follows:

$$WLTH_{N,i,t} = \left(1 - \delta^{CD}\right) CD_{N,i,t-1}. \tag{4.14}$$

### 4.3.3 Production sector

The production sector produces goods that are consumed by all households as non-durable goods $CN$. Following Kim and Loungani (1992) and Dhawan and Jeske (2008), the production function in a perfect competitive market is given by:

$$Y_t = A_t \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{1-\alpha}, \tag{4.15}$$

where $A$ defines Hicks-neutral productivity, $\eta \in (0,1)$ measures the share of capital in terms of energy and $\nu$ the elasticity of the substitution between capital and energy. As $\nu < 0$, there is a complementary relationship between both input factors similar to (Dhawan and Jeske, 2008). Thus, the efficient use of capital $K$ to produce output requires some energy $E_{\mathrm{Y}}$. In addition, the company employs people supplied by households $L$. $\alpha \in (0,1)$ indicates the elasticity of substitution of the capital-energy bundle. As the elasticity of substitution between labor and the composition of physical capital and energy is one, non-durable goods are produced with constant returns to scale, characterizing a Cobb-Douglas production function.

**Lemma 4.** *Concavity of final production*
*The partial derivatives for the final production function are:*

$$\begin{gathered} Y'_K > 0, Y'_{E_{\mathrm{Y}}} > 0, Y'_L > 0, \\ Y''_{KK} < 0, Y''_{E_{\mathrm{Y}}E_{\mathrm{Y}}} < 0, Y''_{LL} < 0, \\ Y''_{KE_{\mathrm{Y}}} = Y''_{E_{\mathrm{Y}}K} > 0, Y''_{KL} = Y''_{LK} > 0, Y''_{E_{\mathrm{Y}}L} = Y''_{LE_{\mathrm{Y}}} > 0. \end{gathered}$$

*The production function $Y_t$ is overall concave in $A^Y, K, E_{\mathrm{Y}}, L > 0$ iff all the following conditions hold:*

$$\begin{aligned} &\nu, \eta \leq 1 \textit{ or } \nu > 1, \eta \geq 1 \textit{ or } \nu = 1, \eta > 1 \\ &\alpha < 1. \end{aligned}$$

*Proof: See H.1.1 in Bergmann (2018)*

According to Lemma 4, final output increases with installed physical capital, energy and labor but at a decreasing rate. Moreover, overall concavity of the production function is satisfied. Actual alteration of investments in real capital takes place with a one-period delay, which is analogous to fixed investment. However, capital is only supplied by Ricardian households.

Firms producing non-durable goods face the following profit function:

$$\pi_t = Y_t - r_t K_{t-1} - w_t L_t - (p_\text{Y})_t (E_\text{Y})_t. \tag{4.16}$$

The price of non-durable goods is normalized to one. Hence, revenues of firms are equal to $Y$. On the expenditure side, the input factors capital, labor, and energy are paid off with their respective marginal products $w$, $r_Y$, and $p_E$. As the production sector is modeled by consisting of infinitely small firms, the market participants act under perfect competition. Rents of the input factors labor and capital (wage and interest rate) are determined by the labor and capital market, while energy prices are determined by an exogenous process. Hence, the profit of the production sector is $\pi = 0$.

### 4.3.4 Market clearing

The model is in equilibrium when all markets clear. For the goods market, this means that production equals the aggregated demand of households for non-durable consumption, investment as well as exogenous expenditures that are made for energy consumption. Hence, the aggregate resource constraint follows:

$$Y_t - (p_\text{Y})_t (E_\text{Y})_t = CN_t + (p_\text{H})_t (E_\text{H})_t + (I_{CD})_t + (I_\text{Y})_t. \tag{4.17}$$

By assumption, energy prices, which are exogeneously determined, are the same for households and firms. Hence, $(p_\text{E})_t = (p_\text{H})_t = (p_\text{Y})_t$ holds. Furthermore, the simplification of the exogenous setting of energy prices is based on the assumption that Germany is a small country in terms of energy consumption.[8] Hence, it has little market power to affect the world price of energy.

Next to the goods market, all factor markets have to clear. Consequently, the labor market is in equilibrium when demand for labor by goods-producing firms equals the labor supplied by households at the market wage rate. In the presence of Ricardian and rule-of-thumb households, aggregate labor supplied is described by the weighted sum of labor supply of both types which is exogenously determined by idiosyncratic labor opportunity:

$$L_t = \int_0^\lambda L_{R,i,t} + \int_\lambda^1 L_{N,i,t}. \tag{4.18}$$

To ensure an equal wage rate for Ricardian and rule-of-thumb households and consequently the same labor productivity, labor market equilibrium is characterized by $L_t = L_{R,t} = L_{N,t}$. Coenen and Straub (2005) illustrate this as a consequence of unions which pool the wage income of both groups of households.

[8] According to BP (2017), Germany's share of total primary energy consumption is 2.4%.

Similarly, the aggregate demand for non-durable and durable goods is determined by their weighted sum of consumption:

$$CN_t = \int_0^{\lambda} CN_{R,i,t} + \int_{\lambda}^{1} CN_{N,i,t} \tag{4.19}$$

$$CD_t = \int_0^{\lambda} CD_{R,i,t} + \int_{\lambda}^{1} CD_{N,i,t}. \tag{4.20}$$

The market clearing condition for energy is satisfied when the sum of energy demand by the goods-producing sector and weighted sum of the household sector equal energy supply where the latter is determined by an exogenous price formation:

$$E_t = \int_0^{\lambda} (E_{\mathrm{H}})_{R,i,t} + \int_{\lambda}^{1} (E_{\mathrm{H}})_{N,i,t} + (E_{\mathrm{Y}})_t. \tag{4.21}$$

The physical capital market is in equilibrium when Ricardian households' supply of capital equals the demand of capital by goods-producing firms at the market rental rate:

$$K_t = \int_0^{\lambda} S_{R,i,t}. \tag{4.22}$$

Next to idiosyncratic labor supply shocks, there are two further shocks affecting aggregate TFP in a firm's production function and energy prices for all energy consuming entities. Both, Hicks-neutral TFP and the price of energy are assumed to be exogenous and follow stochastic AR(1) processes. The laws of motion are described by the following log-functions:

$$\ln A_t = \rho_{\mathrm{A}} \ln A_{t-1} + \varepsilon_{\mathrm{A},t} \tag{4.23}$$

$$\ln{(p_{\mathrm{E}})}_t = \rho_{\mathrm{P}} \ln{(p_{\mathrm{E}})}_{t-1} + \varepsilon_{\mathrm{P},t}, \tag{4.24}$$

where $\rho_A, \rho_P \in (0,1)$ measures the sensitivity coefficients of persistence and $\varepsilon_A, \varepsilon_P$ the disturbance which is independent and identically distributed with zero mean and variance $\sigma_i^2, i \in (A, P)$.

## 4.4 Competitive Equilibrium

### 4.4.1 Households

In the following, the dynamic optimization problem is solved by maximizing each actor's maximization problem. The equations are derived in detail in Appendix 4.C.1. All households decide about their consumption of non-durable goods, durable goods, and energy to optimize their expected lifetime utility. In contrast to Dhawan and Jeske (2008) and Bergmann (2018), households cannot choose their supply of labor in this model as it is fixed and only affected by an exogenously determined variation of labor opportunity. Furthermore, this economy contains a continuum of individuals who are ex-ante identical but ex-post different in their asset holding $S_{i,t}$ and employment opportunity $L_{R,i,t}$ and $L_{N,i,t}$. This leads to heterogeneity due to incomplete insurance markets.

From this, the Ricardian households face the following optimization problem:

$$\max_{\substack{CN_{R,i,t}, CD_{R,i,t},\\ (E_{\mathrm{H}})_{R,i,t}, S_{R,i,t}}} U_{R,i,0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[ CN_{R,i,t}^{\gamma} \left( \theta CD_{R,i,t-1}^{\zeta} + (1-\theta)\,(E_{\mathrm{H}})_{R,i,t}^{\zeta} \right)^{\frac{1-\gamma}{\zeta}} \right] - \phi \frac{1}{(S_{R,i,t}+b)^2} \right.$$
$$\left. + \lambda_t^H \{ CN_{R,i,t} + p_t^H (E_{\mathrm{H}})_{R,i,t} + I_{R,i,t}^{CD} + I_{R,i,t}^{Y} - w_t L_{R,i,t} - r_t S_{R,i,t-1} - \pi_t \} \right\}, \tag{4.1}$$

while the rule-of-thumb households' optimization problem is given by:

$$\max_{CN_{N,i,t}, CD_{N,i,t}, (E_{\mathrm{H}})_{N,i,t}} U_{N,i,0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[ CN_{N,i,t}^{\gamma} \left( \theta CD_{N,i,t-1}^{\zeta} + (1-\theta)\,(E_{\mathrm{H}})_{N,i,t}^{\zeta} \right)^{\frac{1-\gamma}{\zeta}} \right] \right.$$
$$\left. + \lambda_t^H \{ CN_{N,i,t} + p_t^H (E_{\mathrm{H}})_{N,i,t} + I_{N,i,t}^{CD} - w_t L_{N,i,t} \} \right\}. \tag{4.2}$$

According to this, the corresponding first order conditions are written as:

$$1 = \beta \frac{\theta\,(1-\gamma)}{\gamma} \mathbb{E} \left\{ \frac{CD_{R,i,t}^{\zeta-1} CN_{R,i,t}}{\theta CD_{R,i,t}^{\zeta} + (1-\theta)\,(E_{\mathrm{H}})_{R,i,t+1}^{\zeta}} \right\} + \beta \mathbb{E} \left\{ \frac{CN_{R,i,t}}{CN_{R,i,t+1}} \left(1 - \delta^{CD}\right) \right\} \quad \text{for } c \in (R,N) \tag{4.3}$$

$$(p_{\mathrm{H}})_t = \frac{(1-\gamma)\,(1-\theta)}{\gamma} \frac{CN_{R,i,t} (E_{\mathrm{H}})_{R,i,t}^{\zeta-1}}{\left( \theta CD_{R,i,t}^{\zeta} + (1-\theta)\,(E_{\mathrm{H}})_{R,i,t}^{\zeta} \right)} \quad \text{for } c \in (R,N) \tag{4.4}$$

$$1 = \beta \mathbb{E} \left\{ \frac{CN_{R,i,t}}{CN_{R,i,t+1}} \left( 1 + r_{R,i,t+1} - \delta^{Y} \right) \right\} \tag{4.5}$$

Equation (4.3) describes the intertemporal substitution of durable goods. Due to its complementary relationship, it depends positively on energy consumption while it is negatively affected by an increase in non-durable consumption. Equation (4.4) determines the demand for energy and (4.5) equals the Euler equation describing the intertemporal substitution of non-durable goods. The latter implies that current marginal utility of non-durable goods is equal to the discounted utility of future consumption. While (4.3) and (4.4) are the same for both types of households, rule-of-thumb households are excluded from the possibility to use the capital market for intertemporal substitution of non-durable goods. Hence, (4.5) is only valid for Ricardian households.

Aggregate supply of labor by the household sector can be derived with (4.18) in combination with equations (4.6) and (4.12). As a result, it is determined by

$$L_t = \bar{L} + \frac{(1-\lambda)\rho_{L,A}}{1-\rho_L}(A_t - \bar{A}). \tag{4.6}$$

Due to the properties of the variances $\sigma_{L,i}, i \in (R, N)$ of zero mean, idiosyncratic employment opportunity shocks are canceled out according to the law of large numbers.[9] This leaves aggregate labor supply to the steady state of labor supply plus the adjusted business cycle fluctuation of productivity. As a result, labor supply behaves pro-cyclically.

### 4.4.2 Production sector

Goods production is maximized by optimizing over the employment of input factors physical capital, labor, and energy whose prices equal their respective marginal productivities. As the price of the aggregated (non-durable) goods is normalized to one, all prices in the economy are real prices.

$$r_t = \alpha\eta A_t \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha} K_{t-1}^{\nu-1} \tag{4.7}$$

$$\left(p_{\mathrm{Y}}\right)_t = \alpha\eta A_t \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha} (E_{\mathrm{Y}})_t^{\nu-1} \tag{4.8}$$

$$w_t = (1-\alpha) A_t \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{-\alpha} \tag{4.9}$$

While the prices for capital and labor are regulated by the market to match demand and supply, energy prices are exogenously determined by (4.24). As commonly assumed, all factor rents are putting negative pressure on the respective demand.

To derive the equilibrium of the model, agents must forecast future prices of capital and labor to solve the optimization problem. Labor $L_t$, productivity $A_t$, and energy prices $P_t$, are exogenous stochastic processes, while demand for durable goods depends on the difference of households' incomes, consumption of non-durable goods, and energy. In contrast to that, the process that describes the evolution of capital still has to be determined. Additionally, due to heterogeneity among households, the stochastic properties of the stock of capital also depend on the distribution of

[9] Observing a large number of agents, the average of $\varepsilon$ obtained from a large number of trials should be close to the expected value, which is the mean of the variance.

capital wealth. As a result, the cross-sectional capital distribution becomes a state variable by its own which is described by:

$$\Gamma_{t+1} = H\left(\Gamma_t, CD_t, A_t, (p_\mathrm{E})_t\right) \tag{4.10}$$

where $H(\cdot)$ is the law of motion, including all state variables except labor. According to (4.6), the latter is excluded, as aggregate labor supply is only dependent on productivity fluctuations while all idiosyncratic labor shocks for households, as well as lagged labor supply, are canceled out. Opposite to Den Haan and Ocaktan (2009), in this model, capital distribution only concerns Ricardian households, as the remaining agents do not intertemporally transfer physical capital.

In equilibrium, the economy is determined by a set of allocation and price paths that satisfy the following conditions, where $i \in (R, N)$:

i) solving the households' problems $\{CN_i, CD_i, S_i, (E_\mathrm{H})_i\}$ given prices $\{r, w, (p_\mathrm{E})\}$.

ii) solving the firm's demand of $\{K, L, E_\mathrm{Y}\}$ maximizing the profit given the prices $\{r, w, (p_\mathrm{E})\}$.

iii) rents of input factors are equal to marginal productivity $\{r, w, (p_\mathrm{E})\}$ of each factor, determined by (4.7),(4.9),(4.8).

iv) all markets clear according to (4.17),(4.18),(4.19),(4.20),(4.21). This includes the aggregation of input factors for all agents j with $K = \int S_{R,j}dj$ and $L = \int L_j dj$.

v) the distribution of $(S_{t-1}, CD_t, A_t, (p_\mathrm{E})_t)$ and hence, the probability distribution function (4.10) as well as the aggregated and idiosyncratic shock processes (4.23), (4.24), (4.6), and (4.12) are stationary.

Next to uncertainty, this model includes non-linear and stochastic properties, which is why it is not possible to obtain analytical solutions. The equilibrium can alternatively be obtained using numerical methods. Unfortunately, the law of motion for the capital distribution (equation 4.10) is a high-dimensional object, and leads to a large state space.

### 4.4.3 Solution methods for capital stock

To deal with non-linear and stochastic properties of capital distribution, Krusell and Smith (1998) propose a simplification by relying on a finite and discrete set, as described in the following. Under the assumption of bounded rational agents, they show that the distribution can also be summarized by a few moments only. As described before, solving a macroeconomic model for an equilibrium is more difficult since heterogeneous agents have to be taken into account. In a simple framework, considering heterogeneity in the accumulation of capital, Krusell and Smith (1998) notice that *approximate aggregation* is a helpful tool to determine all aggregated variables, such as consumption and wealth. In their work, they notice that higher-order moments of wealth distribution do not affect the evaluation of total capital. The authors argue that the correlation between the marginal propensity to consume out of wealth and levels of households' wealth is close to zero. Only for very poor households does this not hold. But as the fraction of wealth stemming from very poor households is relatively small, a higher order of moments describing the wealth distribution does not significantly improve the determination of the accumulation of capital. In equilibrium, the agents' decisions of how to accumulate capital is almost independent of the distribution of aggregated wealth. With respect to the model at hand, future prices only depend on the moments of the physical capital stock but not on its distribution. As a result, it is sufficient to know the evolution of the total capital stock to forecast its price.

Accordingly, the corresponding general transition law of aggregate capital can also be described as:

$$K_t = \varrho_0(s) + \sum_{i=0}^{I} \varrho_i(s) M(i) + \varrho_2 A_{t-1} + \varrho_3 (p_{\mathrm{E}})_{t-1} + \varrho_4 CD_{R,t-1}, \tag{4.11}$$

where $M(i)$ describes the cross-sectional average of assets of individual i, while $s$ represents a vector of aggregate state variables. In this paper, there are two more state variables, in comparison to the general models by Den Haan and Rendahl (2010). These variables are durable goods and energy prices which enter the law of motion.

Given this extended and more precise law of motion, each individual household can compute its optimal choice of consumption. There are several approaches to solve for aggregate capital, of which we will present simplified summaries. For a detailed description, we refer to Algan et al. (2014). But besides these approaches, the remaining procedure is always similar and consists of the following five steps: (1) selecting the order of moments by determining the approximation methodology, (2) choosing the functional form of law of motion of aggregate capital, (3) calculating individual policy functions by solving the decision problem, (4) updating the law of motion of aggregate capital, (5) iterating steps (2)-(5) until convergence.

The order of moments is closely linked to the selection of algorithm to obtain the aggregate law of motion. Above, it was pointed out that a few moments are sufficient to numerically approximate the equilibrium of a macroeconomic model with heterogeneous agents. Even the first moment of the wealth distribution (mean), along with the aggregated productivity shock, can be sufficient to describe all aggregated variables very accurately, as first shown by Krusell and Smith (1998). According to them, it is disputable that the model's approximate equilibrium is significantly less accurate in comparison to the true theoretical equilibrium given agents' irrationality.

Alternatively, Preston and Roca (2007) investigate the approximation using the second order of moments. They confirm the accurate determination of endogenous variables by using first-order dynamics only. Furthermore, they show that aggregate variation is less affected by second order moments compared to first order moments because of the virtually linear saving decision of agents. Nevertheless, the second order moments can contribute to the determination of individual mean consumption and saving when considering non-linear properties in the solution. In Preston and Roca (2007)'s work, a comparison improves accuracy by 2%. On the downside, when using second order moments in combination with the perturbation method, we need auxiliary policy rules and new aggregate state variables, which complicates the model by increasing its dimensionality. Therefore, because in our model the propensity to save out of wealth is almost equal across all agents, first order moments will be used in the model at hand.

As the law of motion of aggregate capital is derived from the individual policy function, we will first focus on the determination of the latter. Numerically, there are two methods to solve the policy functions: using the projection technique or the perturbation technique. The first generally consists of three steps: defining a grid in the state variables, calculating the conditional expectation of the optimized decision equations such as the Euler equation by applying quadratic methods, followed by solving the equation to find the coefficients of the approximating function for which the errors on the grid are minimized. This procedure has a few advantages, especially with respect to heterogeneous agents, because it captures the distributional aspect. Furthermore, it can be applied to non-linear equations. However, the more state variables there are in the model, the more difficult it becomes to solve the policy function. The main difference between the projection and perturbation methods is that the projection method is designed to derive a global approximation, while perturbation techniques are designed to be a local method. Still the latter can also give very close global approximations.

Perturbation techniques approximate policy functions around their steady state values. Concurrent with that, there can be only one steady state, limiting the result to a local optimum. Furthermore, it can only be applied to sufficiently linear equations, otherwise the results may be less robust and explosive. Technically, perturbation methods use the Taylor expansion, whose order is also determined by the number of moments. In general, a higher-order approximation reduces the error of a Taylor series, bringing it closer to the analytic function. First order perturbation methods are widely used in economics as they are fast to compute and simple to apply. While the order is primarily a technical issue for calculating the Taylor approximation, the number of moments also describes the degree of rationality of an agent. In the present paper, the first order perturbation approach will be used, similar as in Troch (2014), and based on the findings of Krusell and Smith (1998). On the one side, there is only a low contribution by an extension to second order. On the other side, although using second moments increases the agents' degree of rationality, it also goes along with considering the evolution of cross-products, increasing the complexity of determining the aggregate capital stock. Hence, it is questionable whether agents consider such higher moments in their decision making about optimizing their asset accumulation.

Given the individual policy functions, we can derive the aggregate law of motion of capital (4.11). Next to limiting the set of moments, Krusell and Smith (1998) were also among the first to develop an approach to obtain the aggregate policy function from a simulation procedure. After each step they solve for the individual policy rule, they construct a time series of the cross-sectional moments. By applying least squares, new coefficients for the law of motion $\Xi$ can be estimated from it. This process is iterated until convergence. On the downside, this approach is very computational and introduces sampling noise due to the long-run simulations. Subsequent to the approaches described so far, further ones have been developed by Den Haan (1996), Den Haan (1997), and Algan et al. (2008, 2010), all building on Krusell and Smith (1998). The former Den Haan (1996) simulates the individual and aggregated policy function of capital, using parameterization of the conditional expectation to avoid the approximation of law of motion of the finite set of moments. The other two, instead of using simulations, use projection techniques. However, this requires knowledge of either the aggregate capital stock $K_{t+1}$ or the actual distribution, which again increases the number of state variables and the complexity.

In this paper, we apply the approach of *explicit aggregation* by Den Haan and Rendahl (2010). Compared to the simulation and projecting approaches, this approach is less computational. Moreover, with respect to the model at hand, it is much simpler, as we use first order moments only. In general, the idea is to derive the aggregated law of motion by integrating the individual policy functions. Along with that, further information on the cross-sectional distribution does not have to be considered.

The parameterized individual policy function of agent i is given by:

$$\begin{aligned} S_{R,i,t} = \varpi_0 + \varpi_1 S_{R,i,t-1} + \varpi_2 K_{t-1} + \varpi_3 A_{t-1} + \varpi_4 (p_{\mathrm{E}})_{t-1} \\ + \varpi_5 L_{R,i,t-1} + \varpi_6 CD_{R,i,t-1} + \varpi_7 \varepsilon_{P,t-1}. \end{aligned} \tag{4.12}$$

This function expresses the policy function of the individual capital stock (and hence, its physical capital wealth) at the end of period t, after any realization of shocks through labor participation, technological progress, and energy prices. Furthermore, the function holds for both types of households, whereby the state variable for all rule-of-thumb households is $S_t = 0$. With respect to the market clearing, we can transcribe the heterogeneous law of motion to the law of motion of aggregated capital by integrating (4.12):

$$\begin{aligned} \int S_{R,i,t} = \varpi_0 + \int \varpi_1 S_{R,i,t-1} + \varpi_2 K_{t-1} + \varpi_3 A_{t-1} + \varpi_4 (p_{\mathrm{E}})_{t-1} \\ + \int \varpi_5 L_{R,i,t-1} + \int \varpi_6 CD_{R,i,t-1} + \varpi_7 \varepsilon_{P,t-1}. \end{aligned} \tag{4.13}$$

Subsequently, the integrated terms can be substituted by the market clearing conditions (4.22), (4.20), and (4.6). Solving the equation for aggregated capital $K_t$ leads to:

$$K_t = \underbrace{\lambda\left(\varpi_0 + \varpi_5 \bar{L}_R\right)}_{\Xi_0} + \underbrace{(\varpi_1 + \lambda\varpi_2)}_{\Xi_1} K_{t-1} + \underbrace{\lambda\varpi_3}_{\Xi_2} A_{t-1} + \underbrace{\lambda\varpi_4}_{\Xi_3}(p_{\mathrm{E}})_{t-1} + \underbrace{\lambda\varpi_6}_{\Xi_4} CD_{R,i,t-1} + \underbrace{\lambda\varpi_7}_{\Xi_5} \varepsilon_{P,t-1}. \tag{4.14}$$

Since physical capital can only be accumulated by Ricardian households and considering the fact that idiosyncratic employment opportunity shocks are canceled out in equilibrium according to the law of large numbers, it holds that $\int L_{R,i,t-1} = \bar{L}$. By suppressing the constant coefficients, we derive the aggregated policy function for capital in a straightforward manner.

$$K_t = \Xi_0 + \Xi_1 K_{t-1} + \Xi_2 A_{t-1} + \Xi_3 (p_{\mathrm{E}})_{t-1} + \Xi_4 CD_{R,t-1} + \Xi_5 \varepsilon_{P,t-1} \tag{4.15}$$

Hereby, we have taken advantage of being faced with a linear policy function including first moments only. Considering higher order moments, further laws of motion, who determines those variables, needs to be added as pointed out by Den Haan and Rendahl (2010). Concurrent, this means that without any modification of the approximation process, an infinite set of moments is required to find a solution for those policy functions.

In accordance with the procedure to compute the optimal choice of consumers with respect to the correct aggregated in equilibrium, the previous steps are iterated until there is convergence within the coefficients of (4.15) (see Section 4.5.2).

## 4.5 Calibration & Determination of Law of Motion of Aggregated Capital

In the following, we determine the parameters for the model. To do so, we either calibrate the values by calculating the values from the model in steady state condition, by using empirical data to fit the model with plausible real data, or by obtaining the values from existing literature. A derivation from the steady states can be found in Appendix 4.C.1. The respective structural parameters which characterize the properties of the model are summarized in Table 4.1, while Table 4.3 summarizes the shock-related parameters.

The steady state condition of the model corresponds to the model with respect to its long run historical averages from data. Only for labor supply do we set its long-run steady state value to $\overline{L} = 0.3$ as it is also standard in the literature. This follows from the assumption that 30% of the available time of an agent is used for working. Although this goes along with Dhawan and Jeske (2008), it is also similar to the value assumed for Germany (see Hristov, 2016).

According to Dhawan and Jeske (2008), we set the time discount factor and the elasticity of substitution of the durable good/energy consumption bundle and non-durable goods in the utility function of households equal to $\beta = 0.99$ and $\zeta = -2.8748$. For the elasticity of substitution within the non-durable goods production function, we choose $\nu = -0.15$ as in Kemfert and Welsch (2000). While Dhawan and Jeske (2008) follow Kim and Loungani (1992) by choosing $\nu = -0.7$ and performing a sensitivity analysis for other values, Kemfert and Welsch (2000) estimate the elasticity of substitution specifically for Germany using alternative nesting structures. As $\zeta, \nu < 0$, this leads to a complementary relationship between these factors in the household and firm sectors. The capital income share in goods production is set to $\alpha = 0.36$, similar to Kydland and Prescott (1982), Hansen (1985), and Maußner (1994). Inversely, this corresponds to a labor income share of 64% and accounts for the average of the capital income parameter set by Marto (2014) and Flor (2014) for Germany. Compared to lower values in earlier literature, the reduction of labor income shares considers a more capital-intensive production which goes along with Schmalwasser and Schidlowski (2006) who argue that capital shock grows faster than production as labor is increasingly replaced with capital in recent time. The elasticity of substitution between the energy-durable bundle and non-durable goods is equal (unity), similar to Fernandez-Villaverde and Krueger (2011). According to the derivation of the model in steady state using targeted ratios from empirical data (see Appendix 4.C.1), $\gamma$ is set to 0.781 which puts a higher weight on non-durable goods. In comparison to Dhawan and Jeske (2008) who use target moments of the US for calibration, the value is slightly lower for Germany.

Due to an initial value problem, the parameters $\zeta$ and $\theta$ in the utility function and the production function cannot be simultaneously calibrated. Hence, either of those must be predetermined, in our case the elasticities in these functions. Subsequently, the particular share parameters are calculated to match empirical data. Furthermore, we take the same depreciation rate of durable goods from Dhawan and Jeske (2008) due to the assumption that the behavior of US households with respect to durable goods does not distinguish from German consumers significantly. Accordingly, $\delta^{CD}$ is set to 0.0683.

The cost function of accumulation of durable goods is assumed to be quadratic according to Bruno and Portier (1995) and Dhawan and Jeske (2008). As the proportional part of the adjustment costs $\omega_{2d}$ does not affect the steady state condition of the model, it is calibrated in order to closely match volatility of total fixed investment to that from data. The benchmark model does not involve any taxes, hence $\tau = 0$ holds.

Regarding the motion of the capital stock, used in the production sector, its depreciation rate is calculated from the time preference rate and the steady state interest rate while the latter is calculated from the long-run first order condition of the production function. The sensitivity parameter of the borrowing constraint is set to $\phi = 0.5$. According to Preston and Roca (2007), theory does not restrict the setting of this parameter with respect to its magnitude. Hence, it is chosen to ensure no essential violation of the borrowing constraint. By setting the natural borrowing limit to $b = 0$, a household's utility is negatively affected by any negative asset holding.

Considering the fraction of rule-of-thumb consumers, Mayer and Stähler (2013) assume its share to be 0.33 which satisfies a moderate crowding out of private consumption for Germany in 2011.[10] Stähler and Thomas (2012) surmise a higher share of 0.4 in Germany for the post-financial crisis period after 2008. For models covering the EU area as a whole, Coenen and Straub (2005) set the fraction of liquidity-constraint agents to 0.25 which is in line with Coenen et al. (2008) while others assume a significant higher fraction of 0.37 (Forni et al., 2009). In our model, the fraction of rule-of-thumb consumers is set to $1 - \lambda = 0.35$ which is between these two ranges. Such a sizable fraction of rule-of-thumb consumers helps to reconcile the model with empirical evidence, in particular with respect to inequality measurements. It is worth to mention that existing literature often assumes a balanced weight between Ricardian and rule-of-thumb households, frequently referred to the models by Gali et al. (2003) and Campbell and Mankiw (1989), while the latter mainly relate this share to the pre-1990 period in the USA. An equal weight distribution is usually taken as an initial value for further estimations. Hereby, the USA is often observed as the underlying economy which reasonably differs from the German economy in terms of capital and income distribution. For instance, Colciago (2011) reports a higher fraction of 0.5 which is also consistent with Mankiw (2000), Bilbiie and Straub (2013), Callegari (2007), Muscatelli et al. (2004), and Amato and Laubach (2003). Considering the estimates based on Markov-chain Monte Carlo methods by Di Bartolomeo et al. (2011), the fraction of rule-of-thumb consumers in the USA indeed are at the higher end of the G7 countries, only surpassed by France and the UK. Overall, as Gali et al. (2003) notify that the introduction of liquidity constraint consumers can alter the equilibrium dynamics of the model, we further do some sensitivity checks for several values of $\lambda$ in Section 4.7.

### 4.5.1 Calibration of shocks

The parameters for the shock process are summarized in Table 4.3. Technological progress follows an AR(1) process. It is a common practice to assume a persistent parameter of 0.95 as considered by Kydland and Prescott (1982) and Hansen (1985). With respect to that, Kydland and Prescott (1982) suggest a standard deviation of

[10] Di Bartolomeo et al. (2011) estimate a fraction of rule-of-thumb consumers which is at the lower end with 0.075. Finding similar results for Japan, they explain these findings with psychological and cultural factors of the countries as well as higher saving rates compared to other G7 countries. However, they also do not rule out measuring errors.

Table 4.1: Parameter values

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | 0.990 | discount factor |
| $\zeta$ | -2.875 | elas. of substitution between durable goods and energy in households |
| $\theta$ | 0.999 | share of durable goods in consumption good bundle |
| $\gamma$ | 0.781 | elas. of substitution of consumption |
| $\lambda$ | 0.650 | Ricardian household share |
| $\phi$ | 0.050 | sensitivity of penalty constraint |
| $\alpha$ | 0.365 | final output elas. of VA |
| $\eta$ | 0.949 | share of capital in capital-energy bundle |
| $\nu$ | -0.150 | elas. of substitution between capital and energy in production |
| $\delta_{CD}$ | 0.068 | depreciation rate of durable goods |
| $\delta_Y$ | 0.018 | depreciation rate of physical capital |
| $\omega_{1d}$ | 2.410 | parameter adjustment costs |
| $\omega_{2d}$ | 1.000 | parameter adjustment costs (proportional part) |
| $\tau$ | 0.000 | tax rate |
| $\Xi_0$ | -0.664 | coefficient of constant in aggregate capital accumulation |
| $\Xi_1$ | 0.963 | coefficient of $K_{t-1}$ in aggregate capital accumulation |
| $\Xi_2$ | 0.988 | coefficient of $A_{t-1}$ constant in aggregate capital accumulation |
| $\Xi_3$ | 0.007 | coefficient of $P_{t-1}$ constant in aggregate capital accumulation |
| $\Xi_4$ | 0.084 | coefficient of $CD_{R,t-1}$ constant in aggregate capital accumulation |
| $\Xi_5$ | 0.003 | coefficient of $\epsilon_{P,t-1}$ constant in aggregate capital accumulation |

0.007 for the growth rate of the Solow residual. But there is little formal analysis of that specific derivation. Gomme and Rupert (2007) re-estimate the Solow residual process using three different regressions with varying numbers of capital stocks, by also taking durable goods into account. The results are fairly similar, amongst others, to those of Kydland and Prescott (1982) or Hansen (1985) and confirm that a first order process provides a good rendering of the data. Furthermore, Gomme and Rupert (2007) argue that the results are not sensitive to the number of capital stocks.[11] Their autoregressive coefficient of 0.9641 and volatility of the shock of 0.0082 are somewhat larger than the usual values. For the present model, we choose a persistent coefficient of $\rho_A = 0.964$ for the technological process and a shock volatility of $\sigma_A = 0.0086$ which is in line with Flor (2014) for the German economy and only slightly differs from Gomme and Rupert (2007) who based their analysis on US data.

The structure of energy market and hence the price formation processes differ significantly among countries. Therefore, we cannot use the estimates from studies like Dhawan and Jeske (2008) who consider the US economy. Alternatively, we do a separate estimation to derive the energy price function based on German data. Since the mid-1990s, Germany has imported more than 60% of its energy use (The World Bank, 2019). To trace energy prices, we consider the evolution of monthly

[11] In fact, when calculating the Solow residual, Kydland and Prescott (1982) omit the capital stock completely. They justify that capital series has a smooth process and hence less effect on the Solow residuals.

import prices of energy between 2005 and 2018. Next to a conservative AR(1) process, we estimate an ARMA(1,1) process such as Dhawan and Jeske (2008) and Kim and Loungani (1992). The results are summarized in Table 4.2 together with the log-likelihood and Bayesian information criterion.

Table 4.2: Estimation of energy price function

| | AR(1) | ARMA(1,1) |
|---|---|---|
| constant | 110.300*** | 112.8*** |
| | (6.40) | (7.68) |
| AR(1) | 0.984*** | 0.974*** |
| | (73.09) | (54.52) |
| MA(1) | | 0.355*** |
| | | (5.00) |
| sigma | 5.340*** | 4.905*** |
| | (24.07) | (22.69) |
| LL[a] | -512.2475 | -498.3773 |
| BIC | 1039.813 | 1017.178 |

[a] Log-likelihood (LL) and Bayesian Information Criterion (BIC) are used as estimators of the relative quality of the statistical model-based.
level of significance: $*p < 0.1$, $**p < 0.05$, $***p < 0.01$.

According to the two-quality estimators, ARMA(1,1) is preferred over AR(1), although the dominance is only weak. Nevertheless, we continue with an ARMA(1,1) energy price function despite the increase of complexity, due to an additional state variable (in form of the moving average of the variance of energy prices). It is thoroughly reasonable to assume that agents consider the price fluctuation of energy as one of their essential products in their utility function to predict future prices and consequently make a decision about their saving behavior.

Similar to TFP, labor supply or more precisely the employment shock follows an AR(1) process. While Preston and Roca (2007) set a employment persistence of 0.93 and for thumb-to-rule agents a persistence rate of current aggregate market conditions of 0.7, most literature assume a higher persistence rate of individual employment between 0.95 and 0.97 (e.g. Den Haan and Ocaktan, 2009; Lee and Mukoyama, 2015; Lopez, 2010; Storesletten et al., 2004). In our model, we set $\rho_{\mathrm{L}} = 0.96$ for both groups of agents. For the variance, we follow general literature with a variance of $\sigma_{\mathrm{EMP}} = 0.05$ as in Den Haan and Ocaktan (2009) and Preston and Roca (2007). Lopez assumes a significantly higher variance of 0.12 in the base state and during recession an even greater variance to include a cyclical variation of the risk-premium. In our model, this fact is covered by the aggregate market condition term which has a persistence of $\rho_{\mathrm{L,A}} = 0.04$ for rule-of-thumb households.[12]

### 4.5.2 Law of motion of aggregated capital

The law of motion of aggregated capital is derived by the iteration algorithm described in Section 4.4.3. The initial function contains arbitrary values which respects

[12] It seems reasonable to assume a slightly higher persistence and lower cyclical behavior due to higher restrictions in the German labor market than in the USA.

Table 4.3: Parameter values of shocks

| Parameter | Value | Description |
|---|---|---|
| $\rho_{\mathrm{A}}$ | 0.964 | persistence technology shock of $A$ |
| $\rho_{\mathrm{P}}$ | 0.974 | persistence energy price shock of $P$ |
| $\rho_{\mathrm{P}_\sigma}$ | 0.355 | persistence energy price shock of $\sigma_P$ |
| $\rho_{\mathrm{L}}$ | 0.960 | persistence labor opportunity shock of $L$ |
| $\rho_{\mathrm{L,A}}$ | 0.040 | persistence technology shock in labor opportunity $L$ |
| $\sigma_{\mathrm{A}}$ | 0.0086 | volatility shock of technological progress |
| $\sigma_{\mathrm{P}}$ | 0.049 | volatility shock of energy prices |
| $\sigma_{\mathrm{L,R}}$ | 0.050 | volatility shock of labor (Ricardian agents) |
| $\sigma_{\mathrm{L,N}}$ | 0.050 | volatility shock of labor (rule-of-thumb agents) |

the steady state values of capital and ensures a stable condition of the model. In principle, the initial distribution should not influence the convergence of coefficients. This is because the stationary density of the probability distribution function (4.10) should be independent of the starting state variables as long as the steady state values are respected. After each optimization process, the law of motion and its coefficients are updated according to (4.14). We set the convergence speed of the updating process of 10% to avoid explosive structures and instability. This means that the former aggregated policy function of capital is updated by only 10% of the new estimated coefficients. The number of simulations should be sufficiently large to guarantee convergence to the stationary distribution. Altogether, the iteration process is run over 150 periods. Figure 4.1 depicts the convergence of each coefficient in the policy function of aggregate capital.

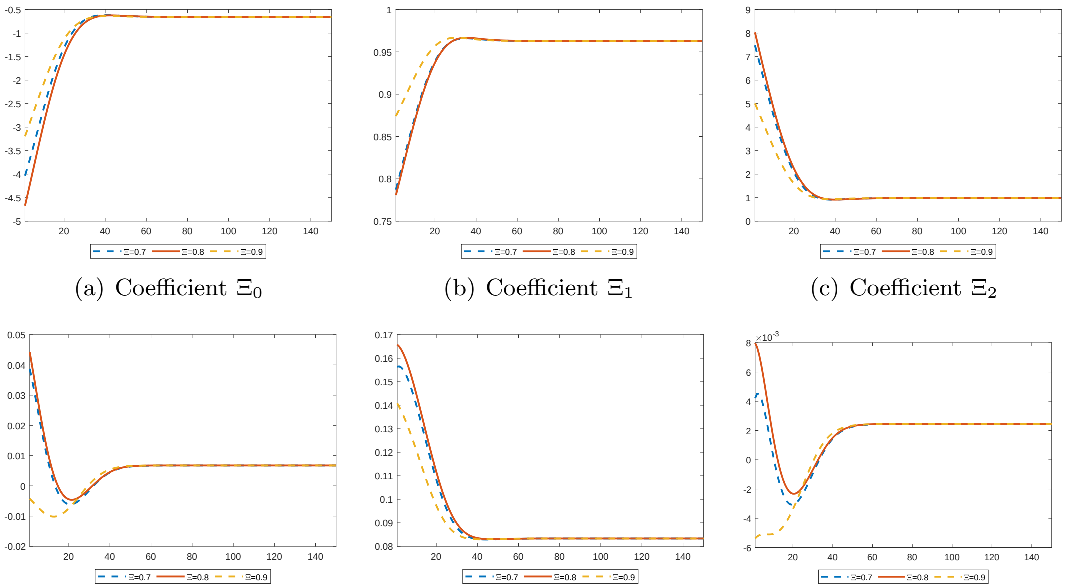
Clearly, convergence is reached after half of the iteration process. The same value is approached regardless of the selected initial starting points. As a result, the final law of motion of aggregate capital in consideration of the individual optimization behavior of households is given by:

$$K_t = -0.6549 + 0.9630K_{t-1} + 0.9782A_{t-1} + 0.0067(p_{\mathrm{E}})_{t-1} + 0.0833CD_{R,t-1} + 0.0024\epsilon_{P,t-1}. \tag{4.1}$$

## 4.6 Results

The analysis of results is separated into three parts. First, we look at the simulated moments of selected endogenous variables after running the model for several times. By comparing the results with observations from the German economy, we are able to validate and analyze the predictive power of the model. Because the relevant statistics for capturing business cycles is the standard deviation, we focus on $2^{\mathrm{nd}}$ moments to cover the volatility of variables. Second, we present some impulse response functions (IRFs) that show the expected propagated path of standard deviation of the endogenous variables, conditional on a one-time shock in the initial

Figure 4.1: Coefficients of law of motion of aggregated capital




(a) Coefficient $\Xi_0$ (b) Coefficient $\Xi_1$ (c) Coefficient $\Xi_2$

(d) Coefficient $\Xi_3$ (e) Coefficient $\Xi_4$ (f) Coefficient $\Xi_5$

The derivation of the coefficient of the final law of motion of aggregate capital is based on an iteration process about the decision making of agents. As long as the initial starting points respect the steady state value of capital, $\Xi_i$ converge to stable values.

period. We focus on both aggregate shocks, namely a temporary increase in TFP and energy prices, and the direction and shape of the response of selected model variables. Studying IRFs is a handy tool to evaluate the responses to aggregate exogenous shocks, which means that we can check the coherency with respect to economic theory. However, studying IRFs to evaluate consequences of idiosyncratic labor supply shocks does not work because of heterogeneous responses of agents. Hence, the third part concentrates on the analysis of the evolution of the income and wealth distributions, by inspecting several inequality metrics.

### 4.6.1 Simulated moments

Table 4.1 displays the simulated percent standard deviation of selected variables of various simulations of the model and the corresponding observations from Germany between 1991–2012 using an HP-filter to detrend the data. Next to the baseline model discussed so far, which includes all shocks (column 2), we show the moments of a simplified version of the model including Ricardian agents only (column 1) as well as the moments of the model without adjustment costs in durable goods investments (column 3). In addition, we look at the volatility or variables when the model is affected by each shock separately (column 5–7).

The model with one class of agents, namely Ricardian agents (column 1), corresponds to the baseline model with adjustment costs and durable goods by Dhawan and Jeske (2008), but with one exception. In contrast to Dhawan and Jeske (2008),

Table 4.1: Volatility of simulated variables (2nd moments)

| Variable | data | (1) simple | (2) baseline | (3) no IC | (4) diff wages | (5) only $\varepsilon_A$ | (6) only $\varepsilon_P$ | (7) only $\varepsilon_L$ |
|---|---|---|---|---|---|---|---|---|
| Output | 1.62 | 0.97 | 1.58 | 1.59 | 1.24 | 1.57 | 0.16 | 0.02 |
| Non-durable goods | 0.81 | 4.05 | 2.82 | 3.20 | 2.28 | 0.12 | 0.03 | 2.77 |
| $CN_R$ | | | 2.40 | 3.76 | 2.35 | 0.05 | 0.03 | 2.44 |
| $CN_N$ | | | 6.45 | 5.36 | 4.54 | 0.33 | 0.06 | 6.26 |
| Durable goods | 3.01 | 3.28 | 3.33 | 7.53 | 2.60 | 0.16 | 0.77 | 3.07 |
| $CD_R$ | | | 2.90 | 10.24 | 2.99 | 0.20 | 0.90 | 2.74 |
| $CD_N$ | | | 7.38 | 9.48 | 4.46 | 0.20 | 0.55 | 6.80 |
| Total energy | 1.76 | 3.00 | 2.95 | 2.97 | 2.60 | 0.74 | 3.08 | 0.09 |
| $E_y$ | | 2.92 | 2.86 | 2.86 | 2.52 | 0.74 | 2.98 | 0.01 |
| $E_{h,R}$ | | 0.14 | 0.13 | 0.28 | 0.14 | 0.00 | 0.10 | 0.08 |
| $E_{h,N}$ | | | 0.23 | 0.26 | 0.15 | 0.01 | 0.08 | 0.20 |
| Labor supply | 1.31 | 4.52 | 3.46 | 3.46 | 3.46 | 0.02 | 0.00 | 3.48 |
| $L_R$ | | | 4.61 | 4.61 | 4.61 | 0.00 | 0.00 | 4.70 |
| $L_N$ | | | 4.44 | 4.44 | 4.44 | 0.06 | 0.00 | 4.33 |
| Labor demand | 1.31 | 4.52 | 0.28 | 0.28 | 0.19 | 0.28 | 0.00 | 0.00 |
| Wage | 1.07 | 2.06 | 1.15 | 1.28 | 1.32 | 1.13 | 0.33 | 0.04 |
| Fix capital formation | 4.01 | 4.68 | 4.54 | 5.04 | 4.59 | 0.83 | 0.15 | 4.53 |
| Energy price | 5.28 | 5.42 | 5.30 | 5.30 | 5.30 | 0.00 | 5.56 | 0.00 |

Data is based on observations from Germany between 1991-2012 using an HP-filter to detrend.
All simulation results denote the percentage standard deviation (2nd moment) over 1000 periods using an HP-filter.
(1) includes Ricardian agents only (all agents have access to the asset market)
(2) shows results of baseline model
(3) shows results of model without adjustment costs in durable goods investments
(4) shows results under the assumption of different productivities for both classes of agents
(5-7) shows results of the baseline model with one specific shock only

labor supply is based on an idiosyncratic process and is determined exogenously in this model. While output volatility explains 60% of its empirical target, agents fail to smooth consumption because volatility of durable above all non-durable goods are higher than the data values. This is due to the high volatility in the supply of labor, which leads to fluctuation in income. All households have to choose between consumption and asset allocation in order to smooth consumption expenditures which are otherwise volatile due to the inconsistency of income because of the presence of shocks in TFP and energy prices. However, agents cannot fully insure against employment shocks, due to incomplete capital markets. This leads to income variations and affects those expenditures from which they can directly get utility, namely non-durable consumption goods.

Investigating the baseline model (column 2), the volatility of output is in line with historical values from Germany. The same applies to the standard deviation of aggregated durable goods which is only slightly higher. Here, fluctuations are significantly reduced due to the presence of adjustment costs. This is apparent in comparison with the model without those costs on doing investments in durable goods (column 3) in which volatility is more than twice as high. In contrast to that, volatility of total non-durable goods is far above their empirical targets which is mainly driven by the consumption fluctuation of rule-to-thumb agents. Similarly, volatility of durable goods of rule-of-thumb households is clearly above those of Ricardian households. However, its effect on the volatility of total durable goods is relatively low.

The reason for the high volatility of total non-durable goods is the missing possibility of intertemporal asset allocation to smooth consumption because rule-of-thumb households are hindered to postpone income to later periods considering their budget constraint (equation 4.10). The alternative of intertemporal income allocation via durable goods is constrained by adjustment costs but has also a lower attractiveness as it is linked to higher energy expenditures due to its complementary relationship. In cases where adjustments in durable goods are not constrained, fluctuations are significantly higher for durable goods whose consumption is easier to be experienced by households as it is only constrained with additional energy expenditures. But opposite to non-durable consumption goods, there is no production process that might limit the supply of durable goods. This is in line with Dhawan and Jeske (2008) who also find excess volatility for durable goods without the presence of adjustment costs. However, this does not apply to non-durable goods, due to endogenous labor supply in their model. Although volatility in idiosyncratic and exogenous labor supply is significantly lower in the present baseline model in comparison to the model without rule-of-thumb agents, volatility is still above its empirical target. As a result, the higher fluctuation of labor income is transmitted to consumption expenditures and physical capital investments (Ricardian agents). In sum, matching the fluctuation of durable goods comes at the expense of overestimating the fluctuation of non-durable goods similar to Alvarez-Parra et al. (2012).

The standard deviation of total (fixed) investment is close to that of the data as we choose $\omega_{1cd}$ to target the moments of durable goods and physical capital investments. Similarly, the volatility of energy prices matches its empirical target as we have estimated the energy price function by an ARMA(1,1) process. Similar to Dhawan and Jeske (2008), labor demand volatility is well below the empirical value, accounting for only 22% of its fluctuation.

Although we do not distinguish between Ricardian and rule-of-thumb agents in terms of wages, it is visible that the volatility of consumption goods differs significantly due to the missing possibility to smooth expenditures by the latter group. Therefore, we also check how the moments are affected by changing the labor productivity of rule-of-thumb agents and hence the labor income relative to wages of Ricardian households. The standard deviation of lowering wages of rule-of-thumb households by 1/3 are displayed in column 4. Of particular interest are the values of those variables that directly affect the agent's utility. Volatility of both the total durable and non-durable consumption goods, are far below those of the baseline model. The available income of rule-of-thumb agents decreases and hence leads to lower fluctuation values to impact the volatility of consumption goods. Hence, it is apparent that a different productiveness of labor improves the performance of the model to match empirical targets.

Next, we look at the degree of influence of the variables due to different shocks (columns 5–7). Unexpected shocks in TFP account for the origin of output volatility as energy price shocks can explain only 10% of fluctuation in GDP (see column 5). This is in line with the literature claiming that TFP is the main driver for business cycle fluctuation despite the presence of energy price or more specifically oil price shocks (Dhawan and Jeske, 2008; Finn, 2000; Kim and Loungani, 1992; Rotemberg and Woodford, 1996). Opposed to that, consumption is hardly affected by TFP shocks. Here, volatility in the supply of labor is the main driving force for durable goods which can almost fully explain volatility in comparison to 24% by energy price shocks. It is worth mentioning that the influence of TFP on consumption goods is also held down due to an exogenous labor supply. According to Dhawan and Jeske (2008), productivity shocks can attribute almost half to the volatility of non-durable goods by fully endogenizing the labor stock.

The energy price shock (column 6) plays a prevailing indirect role in influencing the utility function as well as the production process by increasing their costs. Furthermore, the energy share in producing output and generating utility for households is relatively small compared to capital, labor or consumption goods. But unsurprisingly, energy has a larger effect on durable goods in the utility function in comparison to capital investment in the production function as energy consumption has a larger share in the former. This is in line with Dhawan and Jeske (2008) who find the same results in a fully homogeneous economy model. In total, our results confirm the limited direct role of energy price fluctuation to output volatility. TFP is still the main driver of business cycles. However, they are not negligible in particular by explaining volatility in the consumption behavior of durable goods. Hence, as proposed by Hamilton (2008), energy price shocks affect the economy through other transmission channels such as postponing the purchase of durable goods.

### 4.6.2 Impulse response functions

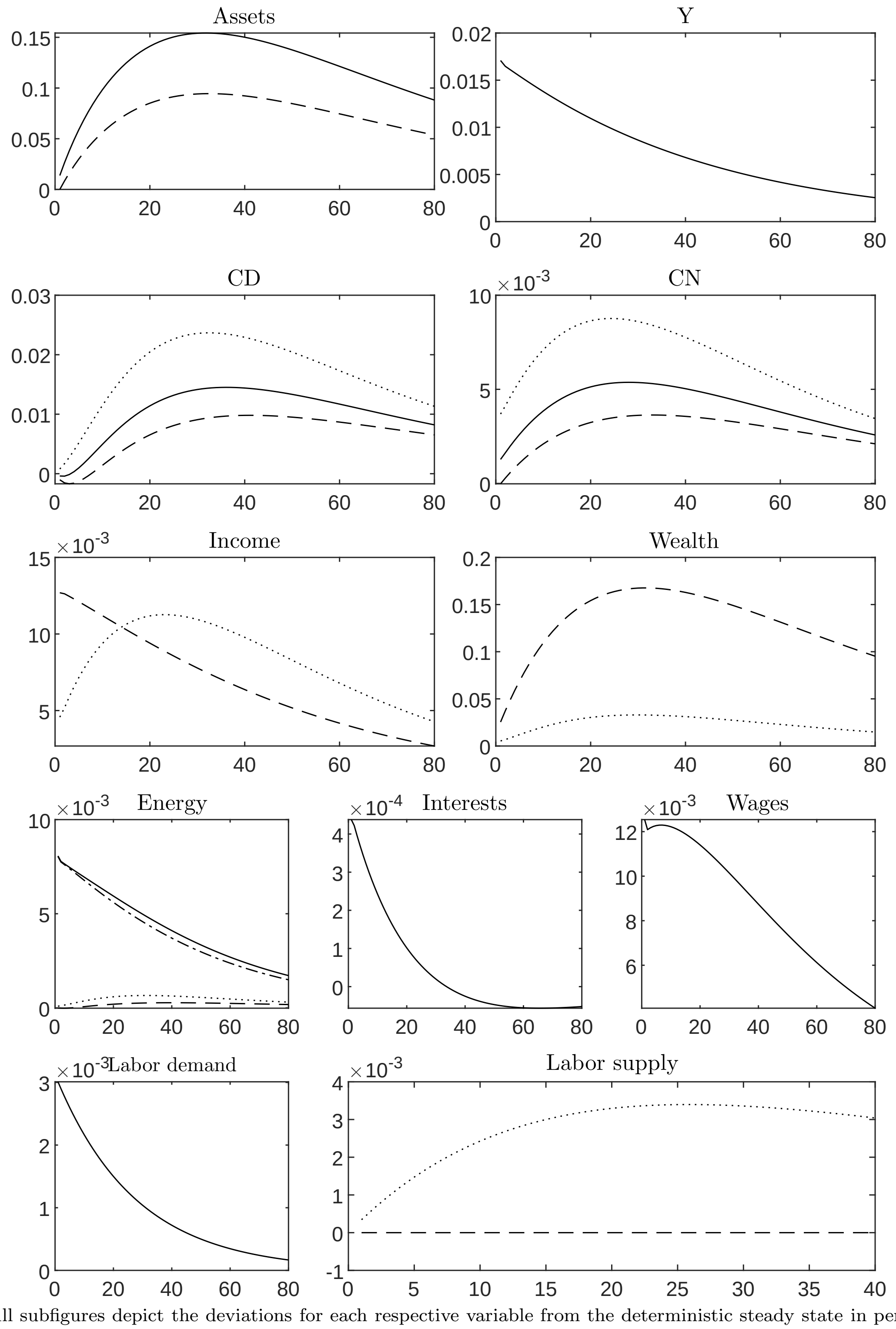
In the following, we analyze the impulse response function to changes in the productivity process $A$ and changes in the price of energy $P$. Responses are shown by their deviation from the balanced growth paths. As we use log-differences, fluctuations are mapped in percentages. The dynamic results are based on the calibrated values, hence the shocks are not normalized but correspond to the individual standard deviations of positive shocks $\sigma_A$ and $\sigma_P$ as described in Table 4.3. The graphs aim to explain two questions: Firstly, how do the endogenous variables respond to each shock. Secondly, to what extent do the responses differ with respect to different types of agents. For better comparability, we partly include the dynamics of both agents in the same graphs. Ricardian agents are marked by a dashed line, rule-of-thumb agents are marked by a dotted line, the production sector is marked by a dot-dashed line, while a solid line represents the overall dynamics of a respective variable.

#### 4.6.2.1 Shock to TFP in the (non-durable) goods production sector

In this section, we investigate a positive productivity shock to the (non-durable) goods production sector as depicted in Figure 4.1. Higher TFP lowers the marginal costs of goods producers and shifts up the supply of output. Along with a more productive production sector, marginal productivity of input factors increases as the same unit of all input factors becomes more productive, other things equal. For Ricardian agents, this has a positive income effect as wages increase in combination with a stable supply of labor. As the latter is exogenous, there is no substitution effect leading to a net increase in income. For rule-of-thumb agents, income also rises. However, labor supply positively responds to boom phases in business cycles, which additionally has a positive effect on their incomes. This stimulation is lagged progressively expanding, hence the persistent effects lead to an inverse u-shaped dynamic of labor supply. But opposite to Ricardian agents, they do not have access to the capital market. Hence, all income is split into expenditures in durable goods, in non-durable goods, and due to the complementary link in energy, which all share the same expanding dynamics as income.

Next to consumption expenditures, Ricardian households use the additional channel to enhance their asset investments because of the increase demand in capital due to lower marginal costs and an increase in the factor price. As an immediate response, a higher return on assets makes durable goods less attractive than capital. Consequently, Ricardian agents shift more of their investment portfolio towards capital investments, causing crowding out of durable goods investment in the short-term. Along with decaying productivity, the increase in capital stock eventually puts downwards pressure on the interest rate in the midterm, which diminishes the advantage of capital investments. Hence, durable purchases increase. After 30 periods, the capital market is saturated and households reduce their saving efforts. By reason of asset savings, the volatility of expenditures for consumption goods and energy for Ricardian households are relatively lower with respect to those of rule-to-thumb agents. Furthermore, as the former do not face increases in labor supply and positive income volatility declines until it returns to its balanced growth path.

As producers and households use more physical capital and durable goods, the overall demand for energy increases which is mainly traced back to the production sector due to their lower marginal costs. The price of energy which is equivalent to the world market price is inelastic and hence stays constant over the time. In total, we notice a significantly positive business cycle in the economy where all actors are positively affected. By reason of different earning channels, the income of Ricardian and rule-of-thumb agents evolves differently.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the respective aggregate variables, the dashed lines display the respective variables for Ricardian agents, the dotted lines display the respective variables for rule-of-thumb agents, and the dotted-dashed lines display the variables for firms (in the energy graph only).

#### 4.6.2.2 Shock to energy prices

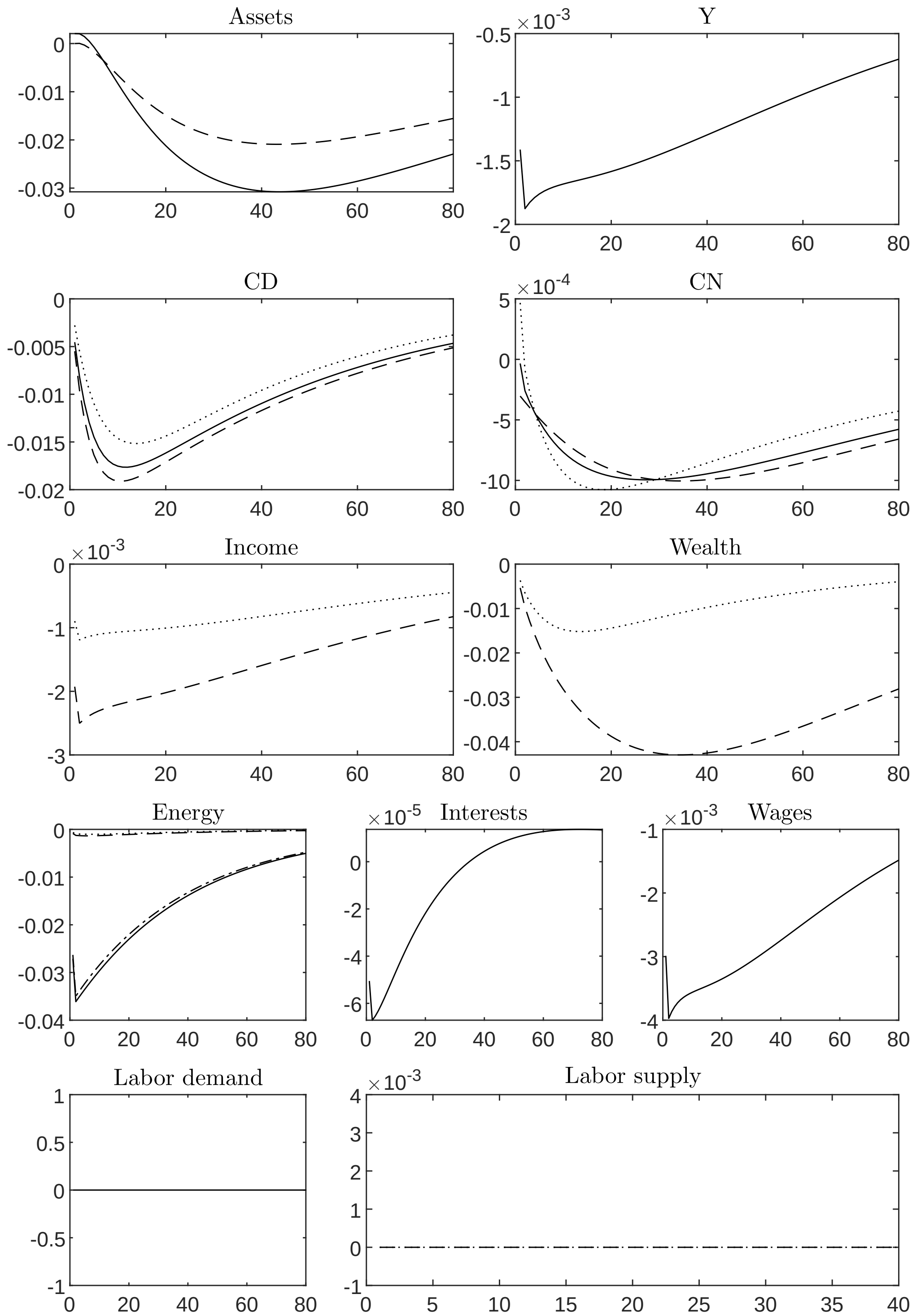
Energy prices react inelastically to a change in the demand of energy as we have seen in the previous scenario in Section 4.6.2.1. Hence, they behave like world market prices which do not vary significantly with changes in the demand of a small country. As a result, a positive energy price shock acts as an energy supply crunch in this framework. Similarly, this goes along with a decline in productivity in the energy generation process as in Bergmann (2018) or the traditional oil price setting by the OPEC.[13]

Figure 4.2 shows the responses to a temporary exogenous increase in the energy price. The instantaneous response of all economic entities leads to a reduction of the demand for energy, which leads to a drop in the quantity of energy used by households and producers. Note that according to the structure of the exogenous process of price determination, the same stochastic shock affects the energy price for two periods. Hence, in the second period, the quantity of energy continues to drop before converging back to its balanced growth path in the long term.

The producer faces higher marginal costs in energy leading to a reduction of output and consequently a fall in capital returns and wages. Due to the complementarity of capital and energy, there is less demand for assets. But opposite to Dhawan and Jeske (2008) and Huynh (2016), labor supply is exogenously determined (whose price formation process is not affected by energy price changes), whereas labor demand stays constant and wages clearly fall relative to interests to balance the market equilibrium. However, this also means that labor cannot be substituted for the loss in the capital-energy bundle which puts additional downward pressure on production.

For both groups of households, an increase in energy prices has impacts on both, the expense and the income sides of the budget constraint. On the one side, purchases of energy become more expensive which increase costs. On the other side, lower returns for production factors decrease the income. However, the negative income effect distinguishes between both types of agents. While rule-of-thumb households only suffer from lower wages, Ricardian households also face a decline in capital income, leading to income losses, which are about twice as large. As a consequence, consumption of durable and non-durable goods is cut by all agents which happens with a delay due to the lagged structure of durables within the utility function. Furthermore, the decline of the former is significantly higher given the high complementarity of energy and durables. Moreover, non-durable consumption goods can partly substitute the energy-consumption bundle which further increases the differences. Concerning Ricardian households, the combination of a lower interest rate and lower income budget also results in a reduction in its alternative investment, namely financial assets. This contraction is persistent which leads to a

[13]The Organization of the Petroleum Exporting Countries (OPEC) includes 15 countries accounting for 44% of global oil extraction and owning more than 80% of oil reserves.




All subfigures depict the deviations for each respective variable from the deterministic steady state in percentage. The solid lines display the respective aggregate variables, the dashed lines display the respective variables for Ricardian agents, the dotted lines display the respective variables for rule-of-thumb agents, and the dotted-dashed lines display the variables for firms (in the energy graph only).

decline in the negative impact of the interest rate. When the rental rate of capital exceeds its steady state values, which happens after 40 periods, the change in the reduction of physical capital investments turns around.

Figure 4.2 also depicts the various impacts on the demand of energy when comparing the household and the production sectors. Although both sectors use energy in their utility or production process and hence, are directly affected by price changes, the latter has the predominant share in the decline of total energy demand. This is essentially due to the weight of energy in the capital-energy bundle, which is clearly higher in comparison to the equivalent bundle in the utility function.

In sum, the response dynamics after a positive energy price shock confirm the results from Section 4.6.1 as output is significantly less influenced in comparison to its response following a change in productivity as in Section 4.6.2.1. However, both groups of agents clearly respond by dropping their investments in durable goods to optimize their utilities. Fluctuations are particularly faced by Ricardian households whose effective budget is stronger reduced due to the loss of capital returns.

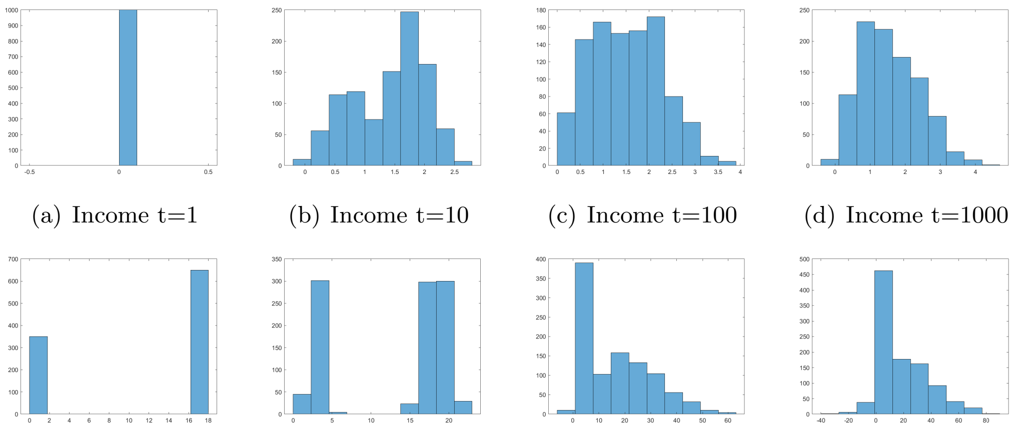
### 4.6.3 Inequality

In the previous section, we have investigated the dynamic responses of the economy, by looking at the impacts of each shock in isolation. Moreover, the exogenous stimuli have been temporary and occurred only once. Now, we simulate the economy by impacting the model with all shock simultaneously and continuously. We run the simulation for 1000 periods considering 1000 different agents which are split up into Ricardian and rule-of-thumb households and whose proportion is fix over the whole time sequence. Furthermore, we launch the simulation by endowing each agent with the same amount of labor, durable goods, and financial assets whereby the latter only holds for Ricardian agents. Next to the TFP and energy price shocks, each agent is additionally affected by an individual employment shock. The latter leads individuals' decisions to significantly differ from each other. The whole procedure gives insight about the development of the distribution of all endowment factors. Here, we are in particularly interested in the income and wealth distribution as they are the main factors from which utility is gained. Therefore, in the following, we focus the analysis by considering some inequality metrics which are often used in relevant literature but also in the public perception.

#### 4.6.3.1 Distribution

Figure 4.3 depicts the evolution of the income and wealth distribution after the $1^{\text{st}}$, $10^{\text{th}}$, $100^{\text{th}}$, and $1000^{\text{th}}$ period. By assuming that all shocks act on the model from the second period and capital returns are payed out with a lag of one period, all agents receive the same wage earnings which also account for total income in the initial period (see Figure 4.4(a)). Hence, the income is equally distributed. In

Figure 4.3: Distribution of income and wealth



(a) Income t=1 (b) Income t=10 (c) Income t=100 (d) Income t=1000

(e) Wealth t=1 (f) Wealth t=10 (g) Wealth t=100 (h) Wealth t=1000

Figures show distribution of income or wealth after t periods. Each bin-width is 10.

contrast, agents' wealth consists of labor income, the stock of durable goods, and possible assets. As only Ricardian agents are endowed with the latter, we can notice an imbalance in its distribution, but only between both groups. From the second period onwards, this does change as each agent is differently affected by employment stimuli. Furthermore, productivity shocks influence the response of both groups of households differently as rule-of-thumb employment is positively correlated with TFP. Even energy price shocks have different consequences for the decision making of agents as they affect the composition of utility by directly changing the price of one consumption good. The direct consequence of this is an alternation of the income distribution within both groups of agents, and thus also for wealth in subsequent periods as each agent is using the intertemporal smoothing channels differently. Particularly in the initial periods, income and wealth reflect the property of normal distribution from the idiosyncratic labor supply shocks within both groups. Households, which can invest in productive financial assets, can accumulated more capital in the long term due to positive capital returns. Hence, considering wealth, the distribution dispersion is larger among Ricardian households as we see clearly in Figure 4.4(f). In sum, we find convergence of the income and wealth distribution over the long run which is also reflected in the evolution of inequality indices as we will see later. Both, income and wealth distribution are right skewed which corresponds to reality whereby the size of skewness of the latter is explicitly larger in the long term.

Moreover, the wealth distribution of Figure 4.4(h) clearly shows the existence of agents who are indebted in the long term despite the borrowing constraint in the utility function, similar to Troch (2014). However, the share of those who have accumulated negative wealth is low with only 5% and relatively stable. Consequently, the economy does not collapse by a Ponzi scheme. Furthermore, we can identify agents who are indebted to be Ricardian households. Because individual households' policy functions are linear by construction, the same holds for the capital accumulation equation of households,[14] which is almost linear in their own holdings of assets. Consequently, the propensity to save out of wealth is the same for all Ricardian households as we discussed before in Section 4.4 and the saving behavior of agents does not differ at both ends of the wealth distribution (for very rich and very poor agents). However, this does hold for the poorest agents in real life. Nevertheless, capital aggregation still holds because the share of these agents and the fraction of wealth they hold are very small and have no significant implications on the qualitative outcome of the simulation.

#### 4.6.3.2 Inequality ratios

Inequality metrics are useful to determine the performance of the model to replicate the income and wealth distribution of Germany. Opposite to the whole distribution, ratios reflect the parts of the distribution with respect to each other and are a good measurement of between-class inequality. Hence, it is a relative measure and easier to interpret. Table 4.2 reports the 90/10, 90/50, and 50/10 percentile income ratios, comparing the income of the 90th, 50th, and 10th income group. According to the results, the model findings are only slightly above the income distribution of Germany but otherwise the model does a good job in predicting the income distribution. As the deviations are mostly visible in the P90/P50 and P50/P10 ratios, this means that the baseline model lightly overestimates the income inequality within the lower half. By looking at the income shares which show the income share of a sub-population relative to its size, we can see that for the bottom 20% and next 20% earners, the results are somewhat lower but still close the the empirical targets. Hence, the dispersion of income can attributed to the lower income groups or the low income tail. Nevertheless, the finding can be assessed as good, in particular with respect to the simplification of the model.

Furthermore, we consider the distribution of wealth by reporting the wealth shares of three sub-population groups (see Table 4.2). Here, we can see a significant deviation between the model predictions and the metrics reported by the OECD (2019b). While the results for the bottom 20% are similar to the data, the baseline model underestimates the actual inequality for the further sub-population groups. This become apparent by looking at the richest households. For instance, the top 5% of wealthiest agents own 46% of total wealth while the model predicts a share of only 18%. As a result, although the model's prediction of income distribution comes close to the empirical target and indicates a more unequal distribution for wealth, which goes along with properties of the German economy, the model fails to predict the targeted metrics for wealth inequality. In particular, the assumption of equal

[14] We use linear policy functions to numerically solve for the decision making of all agents on the basis of all state variables in the simulation.

Table 4.2: Inequality metrics

| | | Baseline | Data[a] | Source |
|---|---|---|---|---|
| Income ratios | P90/P10 | 5.93 | 3.58 | OECD (2019a) |
| | P90/P50 | 1.95 | 1.87 | OECD (2019a) |
| | P50/P10 | 3.04 | 1.93 | OECD (2019a) |
| Income shares | buttom 20% | 5.35 | 8.52 | The World Bank (2019) |
| | 2nd 20% | 10.67 | 13.2 | The World Bank (2019) |
| Wealth shares | buttom 20% | -0.2 | 0.0 | OECD (2019a) |
| | buttom 40% | 4.8 | 0.5 | OECD (2019a) |
| | top 10% | 31.1 | 59.8 | OECD (2019a) |
| | top 5% | 17.6 | 46.3 | OECD (2019a) |
| | top 1% | 4.5 | 23.7 | OECD (2019a) |
| Gini | Income | 0.327 | 0.330 | The World Bank (2019) |
| | Wealth | 0.553 | 0.667[b] | The World Bank (2019) |

[a] We use the average of the data from 1991 to 2013 to match the steady state moments of the calibrated model.
[b] This data refers to the value from 2000 which lies in the middle of the observed time period. More recent data indicate a significantly higher Gini index for wealth (77.5) in 2015.

productivity among agents but also the equal individual initial endowments among and within both groups of agents seems to be too simplistic.[15]

#### 4.6.3.3 Gini index

A further metrics to capture economy-wide dispersion of income and wealth is the Gini index. This synthetic measure is the most widely used single indicator of inequality because of its simplicity of calculation and interpretation (The World Bank, 2014). The index which is generally scaled between 0 and 1 where a value of zero expresses perfect equality and 1 shows maximal inequality.[16] It reflects half of the relative mean absolute difference or alternatively, the mean of the difference between every possible pair of agents, divided by the mean size, hence, the relative inequality within the economy:

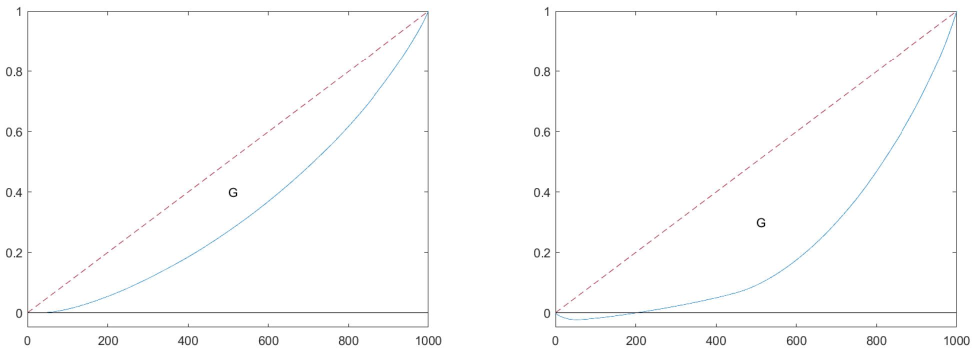
$$G = \frac{1}{2n^2\mu_y}\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j| \quad \text{with} \quad \mu_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

where $y_x$ denotes the income/wealth of agent $x$ and $n$ the total number of agents (in our model $n = 1000$). The Gini index and the Lorenz curve are closely related. The latter depicts the cumulative proportion of ordered individuals plotted onto the corresponding cumulative proportion of their size while the former describes the area between the Lorenz curve and its corresponding function with no inequality (see Figure 4.4). In case of no idiosyncratic shocks and a single class of households only, the Lorenz curve for income and wealth is described by a 45°-line (see red curve in Figures 4.5(a) and 4.5(b)). Consequently, there is no inequality. In the baseline

[15] By simulating the model with lower productivity for rule-of-thumb agents, wealth inequality indeed increases significantly.

[16] When allowing for negative values in the distribution (such as debt in case of wealth distribution), the Gini index can also theoretically become larger than 1.

Figure 4.4: Lorenz curves




(a) Income (b) Wealth

The dashed (dotted) curve denotes the 45°-line description line of equity while solid blue line depicts the Lorenz curve. The area between both curves amounts to the Gini coefficient G.

model, the income Gini index corresponds to the empirical target which confirms the good performance of the model in this regard. However, the wealth Gini index is significantly lower in comparison to the German data, which is in line with the finding from before.

#### 4.6.3.4 Theil index

The Gini index is handy to compare several distributions as it meets many axioms according to Cowell (1985). These include anonymity, scale of independence, population independence, and the transfer principle (Pigou-Dalton transfer principle). However, by not complying with the principle of decomposability, it can only describe the overall degree of inequality. Hence, it does provide any information about the distribution within the economy. This also means that it is possible to have two societies with the same Gini index but different distributions. Therefore, we also consider the income and wealth distribution of the baseline model with respect to the Theil index which allows to decompose inequality.

Applied to our model, the Theil coefficient describes inequality as the sum of inequality between groups (Ricardian vs. rule-of-thumb households) and inequality within these groups:

$$T_T = \sum_{i=1}^{n} \left[ \frac{y_i}{n\bar{y}} \ln \left( \frac{y_i}{\bar{y}} \right) \right] = \underbrace{S_R T_R}_{\substack{\text{within Ricardian} \\ \text{agents inequality}}} + \underbrace{S_N T_N}_{\substack{\text{within rule-of-thumb} \\ \text{agents inequality}}} + \underbrace{S_R \ln \frac{\bar{y}_R}{\mu} + S_N \ln \frac{\bar{y}_N}{\mu}}_{\substack{\text{between group} \\ \text{inequality}}}$$

$$\text{with} \quad S_m = \frac{n_m}{n} \frac{\bar{y}_m}{\mu}, \quad m \in (R, N) \qquad \text{and} \quad \mu_y = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{4.1}$$

where $T_R$ and $T_N$ are the decomposed Theil indices of both groups. This allows us to make up for the main factor contributing to overall inequality as we can associate the sources of inequality to the different parts of the decomposed Theil index. Table 4.3 includes the results for each component of the index. For the income distribution, within groups inequality mainly contributes to total inequality. They are at the same level as agents within both classes each share the same dispersion impacts through idiosyncratic shocks which significantly affect the individual income. These shocks have the same properties for both classes, so between groups inequality is hardly visible. On the contrary, within groups inequality significantly differs as Ricardian agents can accumulate assets by having access to the financial capital market. Income varies within this class whereby wealth accumulation evolves unequally. Rule-of-thumb households can only accumulate wealth through investments in durable goods. Hence, the possibility of different wealth evaluation is limited. By reason of the different market accesses, between groups inequality is naturally higher in comparison with its respective index of income inequality. The same hold for the total Theil index.

Table 4.3: Decomposition of Theil index

| | Theil index within | | between group | total |
|---|---|---|---|---|
| | Ricardian agents ($T_R$) | rule-of-thumb agents ($T_N$) | inequality ($T_B$) | Theil ($T_T$) |
| Income | 0.1486 | 0.1483 | 0.0300 | 0.1761 |
| Wealth[a] | 0.2686 | 0.0410 | 0.2023 | 0.4528 |
| | share of | | | total |
| | Ricardian agents ($S_R$) | rule-of-thumb agents ($S_N$) | | share |
| Income | 76.30% | 23.70% | - | 100% |
| Wealth | 92.04% | 7.96% | - | 100% |

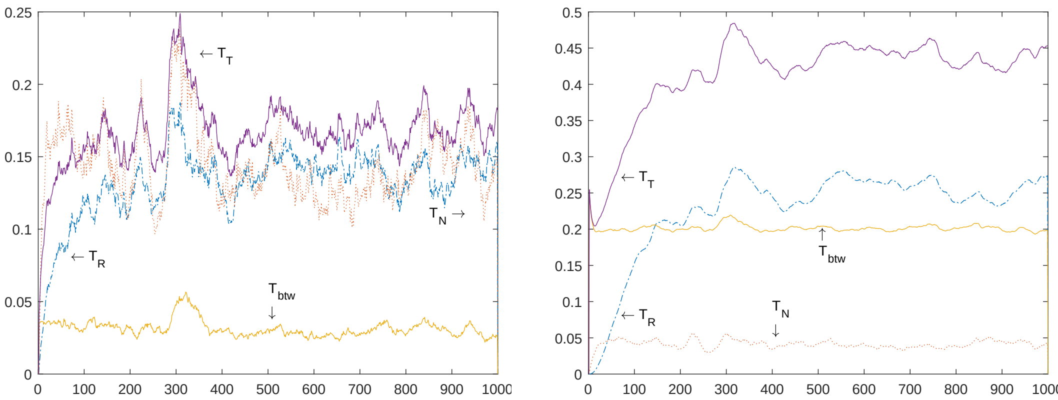
[a] As the Theil index cannot be calculated for data containing negative values, we have neutralized the debt (held by 5% of the population). Hence, the wealth Theil index of Ricardian agents and, consequently, the total Theil index, do not reflect the proper values which are slightly higher.

Furthermore, we can point out a significant difference in the shares of income and wealth held by both classes. Here, the latter is significantly shifted in favor for Ricardian households due to the exclusive access to the financial asset market. In the long term, the Theil indices even out at a constant level (see Figure 4.5). In sum, income and wealth inequality are closely related while the latter is usually at a higher level due to its higher persistency.

## 4.7 Sensitivity Analysis

In this study, we have set up a model that has been meant to present how inequality develops in an RBC framework by using two heterogeneous classes of agents. As such, we have particularly stylized the model i.e. by distinguishing between Ricardian and rule-of-thumb agents, while holding the share of the different classes of agents

Figure 4.5: Theil indices




(a) Income (b) Wealth

As the Theil index cannot be calculated with data containing negative values, we have neutralized the debt (hold by ~5% of population). Hence, the wealth Theil index of Ricardian agents and consequently, the total Theil index, do not reflect the proper values which are slightly higher.

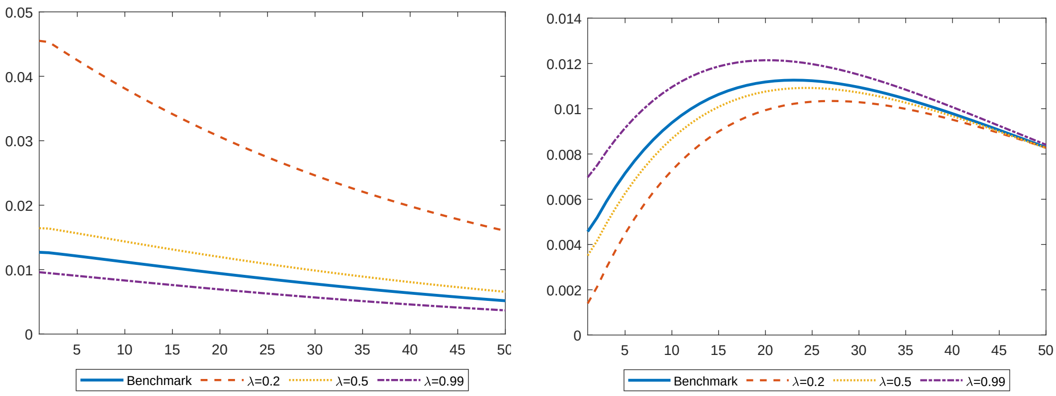
constant. This also means that the ability of intertemporal decision making of an agent cannot be altered, as switching to the other class is not possible.

Hence, we examine a sensitivity analysis to shed a further light on the general ability of this model to reproduce business cycles caused by exogenous stimuli. By using alternative specification of some parameters, we cannot only look at the dynamic responses but also on their effects on inequality measurements. Here, we focus on those parameters which are most likely to have an impact on an agent's decision making and which are not calibrated according to the equilibrium condition of the economy. These include the share of Ricardian households $\lambda$,[17] the presence of adjustment costs in making an investment in durable goods $\omega_{2cd}$, as well as the presence of each shock in isolation. The latter particularly allows us to find out the notable source of impact on inequality measurements originating from exogenous stimuli.

Because TFP is the main driver for business cycle fluctuation despite the presence of the energy price as we have previously confirmed in Sections 4.6.1 and 4.6.2.1, we consider the IRFs of a temporary exogenous increase in productivity to the production sector. Investigating the responses of income and wealth with various shares of Ricardian agents and rule-of-thumb agents after a positive impact through TFP, we can see that for the former, income is higher the less agents have access to the capital asset market (see Figure 4.1a). In particular, having a Ricardian household share of only 20% boosts earnings by 3.7 times larger in comparison to a household share of 99%, while the increase is disproportionate. On the opposite, the rule-of-thumb agents' income proportionally increases the higher the share of Ricardian

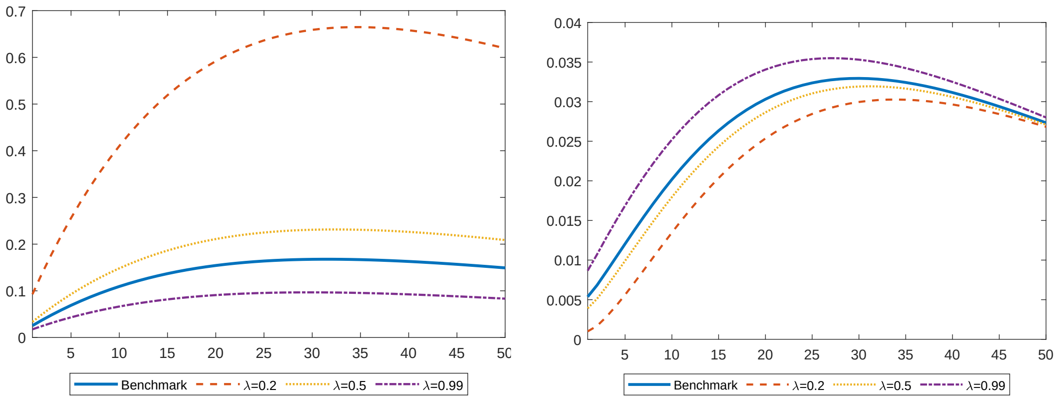
[17] Gali et al. (2003) point out that the distinction between Ricardian and rule-of-thumb households can alter the equilibrium dynamics of the model. In our model, we restrict the sensitivity analysis to $0.2 \leq \lambda < 1$ as we find indeterminacy when $\lambda < 0.2$ and $\lambda = 1$. Other models find different indeterminacy regions such as $\lambda < 0.36$ (Marto, 2014).

Figure 4.1: IRFs of income with various $\lambda$'s




(a) Ricardian agents (b) Rule-of-thumb agents

Figure 4.2: IRFs of wealth with various $\lambda$'s




(a) Ricardian agents (b) Rule-of-thumb agents

households. By reason of the correlation of labor employment and productivity, income diverges from its steady state value for several periods until it converges back. The dynamics can be explained as the more households can access the asset market, the more capital will be invested because it works as an additional channel of intertemporal consumption smoothing. Simultaneously, productivity of capital decreases and productivity of labor, hence wages, increases. The difference of both groups are intensified with respect to the development of wealth. At its peak, the dynamic of a Ricardian agent's wealth at $\lambda = 0.2$ is 5.9 times as large as at $\lambda = 0.99$ (see Figure 4.2).

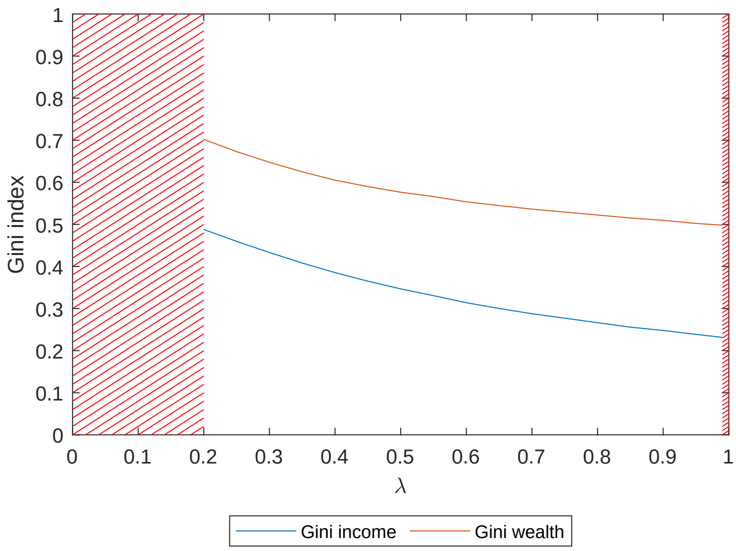
Table 4.1 depicts an overview of the results of the sensitivity analysis by showing the outcome for some inequality measurements. In particular, we look at the Gini coefficient of income and wealth, the income share of the bottom 10% of population, and two income ratios. By comparing the these results, we get an impression of what influences inequality and to what degree these changes appear. The results from the IRF analysis are confirmed by these inequality measurements. The lower the share of Ricardian households, the larger inequality in income according to the

Table 4.1: Sensitivity

| | Income | | | | Wealth |
|---|---|---|---|---|---|
| | Gini | share bottom 20% | P90/P10 | P90/P50 | Gini |
| baseline model | 0.3265 | 0.0535 | 5.9317 | 1.9499 | 0.5526 |
| $\lambda = 0.99$ | 0.3067 | 0.0526 | 5.6150 | 1.7491 | 0.4828 |
| $\lambda = 0.5$ | 0.3401 | 0.0538 | 6.2699 | 2.1069 | 0.5862 |
| $\lambda = 0.2$ | 0.4076 | 0.0500 | 8.5594 | 3.0293 | 0.6959 |
| $\omega = 1$ | 0.2692 | 0.0747 | 4.0821 | 1.7057 | 0.3912 |
| only $\sigma_A$ | 0.1246 | 0.1288 | 1.8538 | 1 | 0.2694 |
| only $\sigma_P$ | 0.1211 | 0.1308 | 1.8135 | 1 | 0.2693 |
| only $\sigma_L$ | 0.3371 | 0.0490 | 6.4927 | 1.9869 | 0.5671 |
| no energy price shocks | 0.3323 | 0.0510 | 6.2487 | 1.9697 | 0.5611 |

Values correspond to the respective equivalent from Table 4.2.

Figure 4.3: Gini coefficients with various $\lambda$'s




Shaded area mark indeterminacy when $\lambda < 0.2$ and $\lambda = 1$.

Gini index which is also reflected by the income share and income ratios (see also Figure 4.3). This is consistent for the value set $\lambda \in [0.2, 1)$. Furthermore, inequality in wealth rises even more strongly.

Moreover, inequality declines significantly when eliminating costs in doing investment adjustments in durable goods. This is mainly due to the fact that the omission of costs enables rule-of-thumb households to use this channel to smooth their consumption inter-temporally more effectively. In contrast, Ricardian households can also continuously use the financial asset market for intertemporal consumption smoothing.

By looking at the impact of each shock on the distribution of income and wealth, presented in the last three column of Table 4.1, we cannot recognize a significant difference between the results for a shock in TFP and energy prices. The Gini coefficients for income and wealth are 0.12 and 0.27 respectively and apparently deviate from the baseline model. These are the pure effects of distinguishing between Ricardian and rule-of-thumb households as individual employment variations are omitted. In contrast, by looking at the presence of a temporary exogenous increase in employment only, the inequality parameters are significantly higher and closer to the baseline model. Hence, we can identify these shocks as the main source of producing inequality in the distribution of income and wealth.

Since this model focuses on the presence of energy price shocks, we also look at the impact of volatility of energy prices on inequality metrics. The analysis of IRFs in Section 4.6.2.2 shows that the qualitative responses of both classes of agents are qualitatively similar, because in both cases there is an aggregated shock that does not differ between the entities. This particularly holds for responses to impacts from energy prices, because both classes are energy purchasers and both receive payments from the production sector (whose output is also affected by the quantity of energy and its prices). Consequently, it does not come as a surprise that differences in the income and wealth distribution in the scenario of no energy price fluctuations are relatively small in comparison to the baseline model with energy price shocks. The last row of Table 4.1 describes both distributions by their respective inequality indices. All measurements are slightly above those of the baseline model. This means that the presence of volatility in energy prices has a positive impact on income and wealth inequality, which means that both decrease marginally. The reason for these dynamics is the complementary relationship between durable goods and energy. The richer the agent, the higher the consumption of this consumption bundle. The presence of volatility in energy prices leads to adjustments in the durable goods stock, re-optimizing the agents' maximization problems (4.1) and (4.2). This goes along with additional expenditures due to the investment cost function (see equations (4.5) and (4.11)), which are sunk costs. As a result, the wealthier the agent and the larger the durable goods stock, the higher these sunk costs.

Based on this complementary relationship between durable goods and energy, and the consequences as described above, we conclude that it is not the low-income agent who benefits from volatility in energy prices, but instead it is the high-income agent who looses due to higher absolute sunk costs. However, this claim only holds when the proportionality coefficient of the complementary factors durable goods and energy increases when consumption of this bundle increases.[18]

## 4.8 Policy Implication Through Taxation

So far, existing RBC frameworks, which implement energy consumption through their main economic entities, are mainly based on the assumption of homogeneous agents. This model is predominantly designed to extend this field of literature by heterogeneity in the capability and endowments of agents. As we have seen in the

[18] The income effect is larger for durable goods than for energy. Hence, the ratio $CD/E_{\mathrm{H}}$ increases.

results of this model, different forms of heterogeneity can lead to inequality in income or wealth which each have implications on the current utility and intertemporal choice of agents. It is likely to avoid unequal distributions as they can lead to negative impacts for the society which can be transferred to many spheres in public life. For instance, inequality can enhance polarization, alienation and social friction but it can also encourage crime with negative consequences for the rest of society. As a result, all members of society would lose out. A free-market system does not necessarily respond to the wants and needs of individuals who are socially and economically worse off, especially not if they have insufficient economic votes to have any impact on market demand.

It should be the responsibility of policy makers of a country using its exceptional position by having organized control over a human community to meet these challenges. Hereby, various policy-making tools are available which have different effects but also a varying degree of support from miscellaneous community groups. However, we do not discuss the feasibility of a specific policy intervention with respect to its acceptance in society. There is a particular field in literature that is dealing not only with the electoral behavior of voters but also the decision making of policy makers with respect to upcoming elections (e.g. Nordhaus, 1975).

One of the government's most powerful tools to tackle inequality is fiscal policy. On the one hand, this directly affects households' decision making about consumption though taxes or transfers. On the other hand, this can not only indirectly influence agents by the provision of public goods and services but also promote the incentive to work. In particular, the role of tax policy plays an important role in times of increasing pre-tax inequality according to OECD (2019b). Therefore, we discuss the impacts of one possible instrument to induce a redistribution channel of income in a simplified form. By assuming that rule-of-thumb households belong to social group which is, by average, worse off in terms of income and wealth, we implement a one-way transmission channel of income from Ricardian households to rule-of-thumb households. This is done by taxing income of former agents whose revenues are directly re-distributed to the latter. We consider a non-progressive, constant tax rate which is put on overall income, hence capital returns plus wage income.

The budget constraints of both groups of agents (4.3) and (4.10) are altered accordingly:[19]

Ricardian households:

$$\begin{aligned} CN_{R,i,t} + (p_{\mathrm{H}})_t (E_{\mathrm{H}})_{R,i,t} + (I_{CD})_{R,i,t} + (I_{\mathrm{Y}})_{R,i,t} \\ = (1-\tau)\left(w_t L_{R,i,t} + r_t S_{R,i,t-1} + \pi_t\right) \end{aligned} \tag{5a}$$

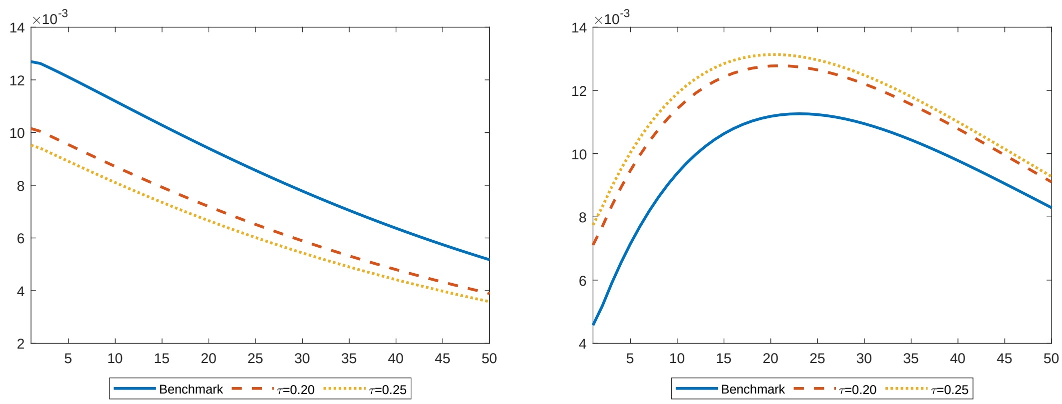
[19] Obviously, equations (4.7), (4.8), (4.13), and (4.14), describing the income and wealth of both classes of agents, alter respectively.

Rule-of-thumb households:

$$CN_{N,i,t} + (p_{\mathrm{H}})_t (E_{\mathrm{H}})_{N,i,t} + (I_{CD})_{N,i,t} \\ = w_t L_{N,i,t} + \tau \left( w_t L_{R,i,t} + r_t S_{R,i,t-1} + \pi_t \right). \tag{10a}$$

The direct responses of the income and wealth of both groups of agents are depicted in Figures 4.1 and 4.2. Again, we consider the IRFs after an exogenous impact on TFP. Unsurprisingly, taxation of income leads to a decline of Ricardian households' earnings in comparison to the baseline model. However, the positive impulse from the increase in the marginal productivity of capital and labor still results into a boost of income. A 0.86% increase in TFP leads to an immediate 1.02% increase in income at 20% income tax (0.95% when $\tau = 0.25$) while income increases by 1.27% with government intervention. The redistribution leads to an instantaneous increase of rule-of-thumb agents' budget by 0.71% ($\tau = 0.2$) and 0.77% ($\tau = 0.25$) while without taxes, income rises by only 0.46%. As clarified in Section 4.6.2.1, the rise of income is persistent for rule-of-thumb households due to the positive correlation of productivity and employment.

Figure 4.1: IRFs of income after taxation

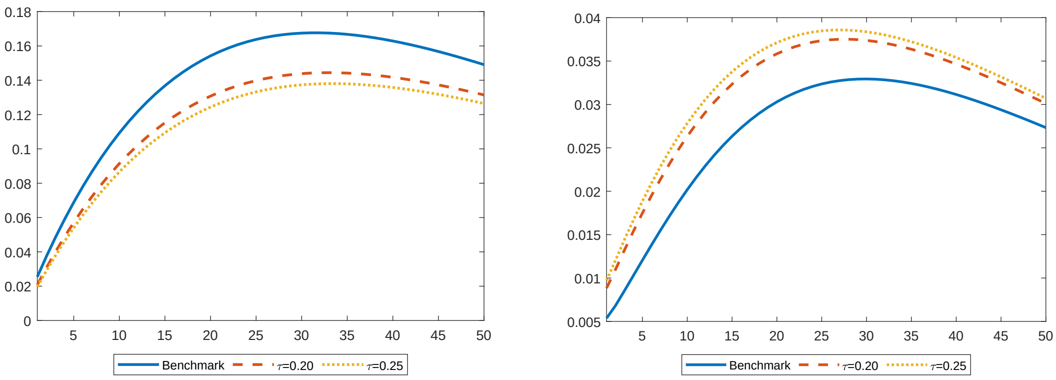



(a) Ricardian agents (b) Rule-of-thumb agents

Similarly, the wealth of Ricardian agents increases over a longer term. The difference between the scenarios with and without taxes are significantly visible in subsequent periods as the retraction of investments results in impacts of wealth only later. At its peak, wealth has reduced from +16.8% to +14.4% (20% income tax) and +13.8% (25% income tax) while the increase in the income of rule-of-thumb agents leads more investments in durable goods. As a consequence, wealth increases from +3.3% to +3.8% (20% tax) and +3.9% (25% tax).

This policy intervention led to redistribution between classes, and not within classes, and was thus aimed at reducing inequality between groups, not within groups. Nevertheless, within inequality is also affected because we consider a linear

Figure 4.2: IRFs of wealth after taxation




(a) Ricardian agents (b) Rule-of-thumb agents

Table 4.1: Taxation

| | Income | | | | Wealth | |
|---|---|---|---|---|---|---|
| | Gini | share bottom 20% | P90/P10 | P90/P50 | Gini | **Δ output** |
| baseline | 0.3265 | 0.0535 | 5.9298 | 0.5526 | 1.9496 | - |
| $\tau = 0.20$ | 0.2869 | 0.0636 | 4.5433 | 1.7169 | 0.5581 | -0.06% |
| $\tau = 0.25$ | 0.2811 | 0.0644 | 4.3995 | 1.6790 | 0.5620 | -0.08% |

Values correspond to the respective equivalent from Table 4.2.

tax rate. Consequently, each Ricardian agent has to pay the same taxes relative to its income while each rule-of-thumb agent receives the same amount of subsidy. This subsidy is relatively more valuable with respect to its total income for a poor recipient than for richer agent (see Table 4.A.1). Note that we look at average inequality measurements over a longer period. In Table 4.1, we present the results in accordance to the sensitivity analysis by looking at the same selected inequality parameters as well as at changes in economic performance. According to this, the Gini income index indicates a reduction by 3.96 points in case of a 20% income tax on Ricardian households (4.54 points for a tax rate of 25%).

The income share and both income ratios confirm the developments of lower income inequality. This is not surprising as the redistribution of income is permanent and hence rule-of-thumb households also profit by a positive investment portfolio in assets. As after-tax income decreases for Ricardian households, this also means that durable goods and asset investments declines and hence, personal savings diminish. Figure 4.A.2 depict the corresponding shifts of the Lorenz curve for both income and wealth.

According to Table 4.1, this goes along with an increase of the Gini wealth index by 0.55 points ($\tau = 0.2$) and 0.94 points ($\tau = 0.25$) stating a light rise in inequality of wealth. Previous literature confirms these findings (i.e. Berman et al., 2015; Cagetti and De Nardi, 2008). Doing a simulation, Berman et al. (2016) show that a positive average tax rate reduces net income and consequently savings. But simultaneously, the authors find that the saving rate for capital is essential to model wealth inequality. They detect that an imperfect correlation between wealth and income leads to the non-trivial effect of a reduction of the relative gap between deciles.

The following modified example by Berman et al. (2016) demonstrates these dynamics:
Assuming that Ricardian households' wealth is $W_{R,t+1} = W_{R,t} + sD_t + a_{R,t+1}W_{R,t}$ where $W_R$ is wealth (capital asset + durables), $sD$ is the share of income that is saved, and $a_R$ is the value change rate of wealth (i.e. gains from capital returns). Respectively, rule-of-thumb households' wealth is $W_{N,t} = \omega W_{R,t}$ with $\omega < 1$. Opposite to Berman et al. (2016), rule-of-thumb households have a different value change rate of wealth $a_N$ which is strictly smaller to the one of Ricardian households ($a_N < a_R$). This is because they are excluded from the asset market, so they cannot gain capital returns.[20]

As a result, the relative wealth is:

$$\frac{1}{\omega} = \frac{W_{R,t+1}}{W_{N,t+1}} = \frac{W_{R,t} + sD_t + a_{R,t+1}W_{R,t}}{\omega W_{R,t} + sD_t + \omega a_{N,t+1}W_{R,t}}$$

$$\Leftrightarrow \quad s = \frac{\omega W_{R,t}\left(a_{R,t+1} - a_{N,t+1}\right)}{\left(1-\omega\right)D_t} \qquad \text{with } s < 1.$$

In case of a decrease in the saving rate $s$, we can solve for the relative wealth that increases. Hence, the relative gap between the "rich" (or Ricardian) and the "poor" (or rule-of-thumb) agent becomes wider when personal savings from income are smaller with respect to the remaining terms. In case of an income tax, we can still have an increase in the ratio between individual wealth values despite a shift from the "poor" agents to the "rich" agents, dependent on the difference of $a_N$ and $a_R$. Of course, this example hold not only for the comparison between classes but also within a class, in particular the Ricardian agent class. However, the tax policy does not redistribute income among agents of the same group. Therefore, a decrease in $s$ always leads to an increase in the wealth ratio when $a_2 < a_1$.

We have seen that the implementation of a constant income tax rate to induce the redistribution of income from the high income group to the low income group indeed has impacts on economic inequality, but with different responses. Inequality in income can be diminished, while inequality in wealth expands, but to a lower extent. As a consequence of this trade-off, it should first be evaluated which of the two outcomes to aim at. In terms of output, the economy is only marginally harmed by 0.06% (20% tax) or 0.08% (25% tax). As reported by Piketty and Saez (2003), Neckerman and Torche (2007), and Biewen and Juhasz (2012), the minor change in wealth inequality is traced back to the fact that since the 1980s, disparate labor income has mainly driven inequality rather than capital gains. But in our model,

[20] Of course this assumption holds only if the capital returns for Ricardian households are strictly positive.

we assume an equal wage rate for both groups of agents. We leave this investigation for future research.

## 4.9 Conclusion

In this paper, we have constructed an RBC model with heterogeneous agents with idiosyncratic properties which belong to two sub-classes, namely either Ricardian households or rule-of-thumb households. Agents can consume durable goods, non-durable goods, and energy. Energy, provided endogenously and in unlimited quantity, is needed in order to either gain utility from durable goods, or to be able to produce goods through capital. To handle heterogeneity on the macro level of the model, explicit aggregation as developed by Den Haan and Rendahl (2010) is applied in order to solve the cross-sectional capital distribution, and hence the aggregate policy function for capital. The model has been calibrated on the basis of data from the German economy. Next to analyzing the model and its aggregated outputs, we have performed a separate simulation to investigate the evolution of individual heterogeneous outputs and their distributions.

We confirm findings from existing literature that TFP is the main driver for output fluctuation even though the presence of energy price shocks and resulting increases have a contractionary effect on output. Moreover, we show that the distinction between non-durable and durable goods leads to a significant improvement in matching most of the moments, with the exception of non-durable goods, because agents receive an additional channel of adjusting their investment decisions. It follows that energy mainly causes disruptions in durable goods investment. Thanks to heterogeneous characteristics of the model, we are able to make predictions of inequality in income that are close to the empirical target while inequality in wealth remains underestimated. This underestimation is mainly attributed to the absence of idiosyncratic productivity differences of agents, but also to the simplification that labor is supplied exogenously. With respect to energy price shocks, inequalities in income and wealth decrease, due to the complementary relationship between durable goods and energy, as well as sunk costs that arise along with adjustments in the durable goods stock. Hence, we conclude that it is not the low-income agent who benefits from volatility in energy prices. Instead, it is the high-income agent who looses in both income and wealth, due to higher absolute sunk costs.

In a brief policy analysis, we have taxed Ricardian agents' incomes with a constant tax rate, with revenues directly redistributed to rule-of-thumb agents, which is a simple possible policy instrument that induces a change in inequality. This has resulted in a reduction of income inequality, with the distribution of wealth becoming more unequal but to a lower extent. Furthermore, only between-inequality is affected because there has been no redistribution within agent classes. Aggregate economic performance in terms of output is only marginally harmed by this form of taxation. Nevertheless, we deduce that it should be evaluated carefully what kind of effect on inequality should be aimed for. Society can be quite sensible to the implementation of policy tools because agents are differently affected given their financial endowments, social standing, or education.

In future research, it would be worth analyzing the responses in the case that agents can optimize their labor supply, given the simplification of labor supply by agents in this model. In addition to uncertain idiosyncratic shocks, this might decrease the volatility of non-durable goods consumption in order to close the deviation from observational data. Furthermore, endogenizing energy generation such as in Bergmann (2018), while at the same time introducing a separate production of durable goods such as Baxter (1996) and Huynh (2016) can help to improve the description of energy within the economy. Both adaptations can have significant implications on the optimal allocation of labor and capital because agents would have further channels to shift mobile factors. With respect to modeling heterogeneity, robustness can further be verified by increasing the set of moments as done in Preston and Roca (2007). Additionally, a further subdivision of agents' classes in terms of their access to asset markets, but also a higher degree of differentiation between classes through heterogeneity in preferences, might help to improve the prediction of wealth inequality.

# Appendix

## 4.A Additional Figures and Tables

Figure 4.A.1: Model overview.

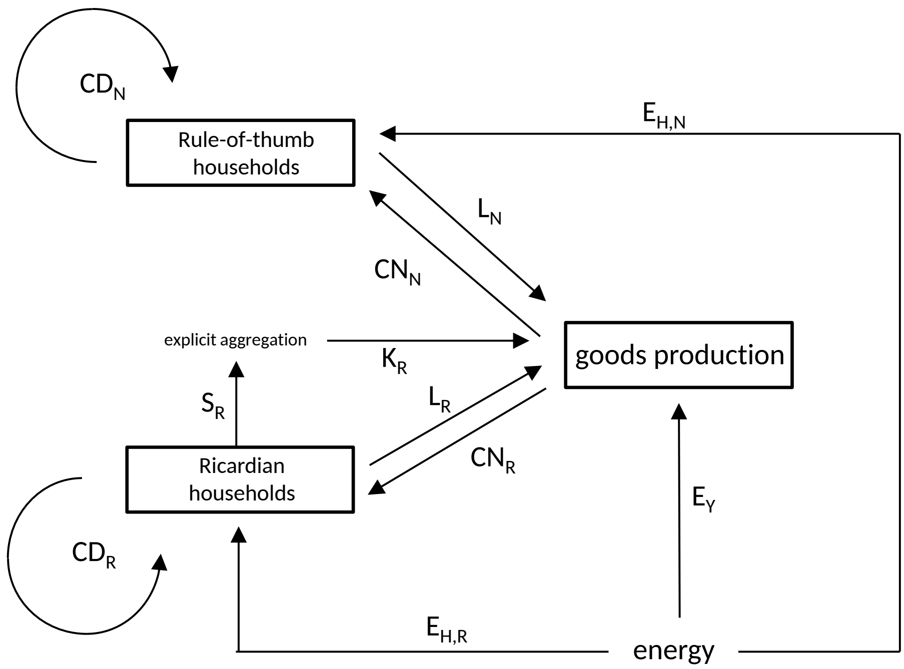
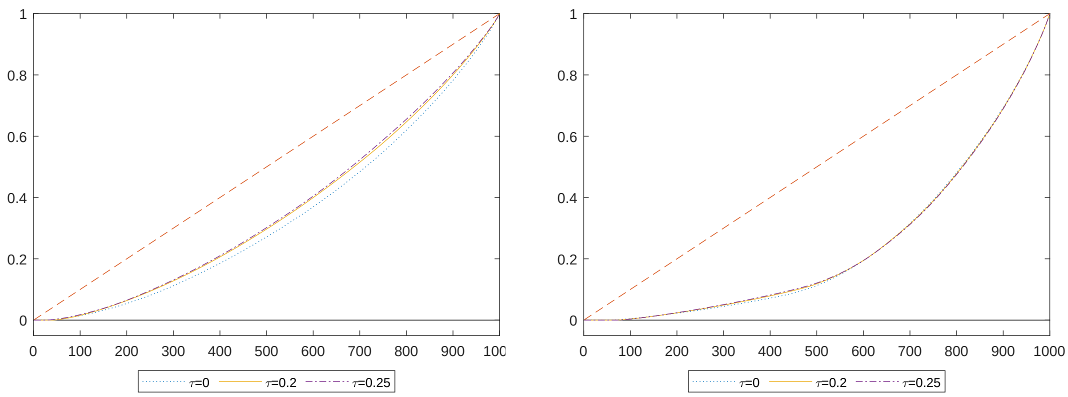



Figure 4.A.2: Comparison of Lorenz curves with taxation




(a) Income (b) Wealth

The dashed (dotted) curve denotes the 45°-line description line of equity while the remaining curves depict the Lorenz curves described by the legends.

Table 4.A.1: Decomposition of Theil index with taxation

| | | Theil index within | | between group | total |
|---|---|---|---|---|---|
| | | $T_R$[a] | $T_N$[b] | inequality ($T_B$) | Theil ($T_T$) |
| Income | $\tau = 0$ (baseline model) | 0.1486 | 0.1483 | 0.0300 | 0.1761 |
| | $\tau = 0.2$ | 0.1547 | 0.1003 | 0.0028 | 0.1403 |
| | $\tau = 0.25$ | 0.1566 | 0.0935 | 0.0004 | 0.1357 |
| | no energy price shocks | 0.1542 | 0.1450 | 0.0308 | 0.1827 |
| Wealth | baseline model | 0.2686 | 0.0410 | 0.2023 | 0.4528 |
| | no energy price shocks | 0.2776 | 0.0430 | 0.2034 | 0.4625 |
| | | share of | | | total |
| | | $S_R$[c] | $S_N$[d] | | share |
| Income | $\tau = 0$ | 76.30% | 23.70% | - | 100% |
| | $\tau = 0.2$ | 68.48% | 31.52% | - | 100% |
| | $\tau = 0.25$ | 66.22% | 33.78% | - | 100% |

Theil index is calculated according to (4.1).
[a] $T_R$ is Theil index of Ricardian agent class.
[b] $T_N$ is Theil index of rule-of-thumb agent class.
[c] $S_R$ is share of Ricardian agents.
[d] $S_N$ is share index of rule-of-thumb agents.

## 4.B Accuracy Checks

In general, it is difficult to find analytical solutions of stochastic general equilibrium models with rational expectations. Hence, simulations are helpful instruments to solve those models numerically. However, as a downside, simulations can also be inefficient numerical tools as they vanish sampling uncertainty as well as white noise which might arise along with approximation methods. Nevertheless, they are helpful to analyze complex models but need to be checked for accuracy. For instance, in our model, inaccuracies can appear in the process of explicit aggregation to determine the law of motion of aggregate capital as we look at first moments only. But also approximating the policy rules of each individual can contribute to inaccurate results.

A simple accuracy test proposed is the $R^2$ along with the standard error by Krusell and Smith (1998) who estimate the aggregate law of motion of capital with least-square regression. However, Den Haan (2010) has shown that this test is inadequate as it scales the errors which runs the risk of underestimating large errors. Therefore, we will perform two alternative accuracy tests to evaluate the accuracy of our solution method.[21] We concentrate on the $\chi^2$-test by Den Haan and Marcet (1994) and the Euler equation error test originally based on Judd (1992).[22]

[21] See Algan et al. (2014) for a discussion of the weakness of the $R^2$ accuracy test and an overview about several alternative accuracy tests.

[22] We refer to Fernández-Villaverde et al. (2016) for a detailed technical discussion of both accuracy tests.

Table 4.B.1: DHM-statistics with different simulation lengths

| | **t=200** | **t=500** | **t=1000** | **t=10000** |
|---|---|---|---|---|
| **DHM-statistics** | 0.85769 | 0.26129 | 0.13564 | 1.05466 |

Values present $\chi^2$-distribution with simulation length t.
Lower 5% critical value of $\chi^2$ is 0.0039, upper 5% critical values. of $\chi^2$ is 3.8415.

The DHM test considers the accumulated error residuals of the Euler equation along the simulated path. By theory, the residual that expresses the deviation of the Euler equation should be zero at *all* points in state space. Hence, it holds that:

$$f(\cdot) \equiv \mathbb{E}\left\{ CN_{R,i,t}^{-1} - \beta CN_{R,i,t+1}^{-1}\left(1 + r_{R,i,t+1} - \delta^{Y}\right) + \frac{2\phi}{\gamma} S_t^{-3} \right\} \tag{4.B.1}$$

$$\mathbb{E}\left\{ f(\cdot) h(x_t) \mid I_t \right\} = 0, \tag{4.B.2}$$

where $I_t$ is the information set of information available in the period $t$ and $h(\cdot)$ is an arbitrary function. We compute the residual of the model which has been simulated for $T$ periods to obtain an empirical distribution according to:

$$B_T = \frac{1}{T} \sum_{t=1}^{T} f(\cdot) \otimes h(x_t), \tag{4.B.3}$$

which converges to a $\chi^2$-distribution.[23] Subsequently, we check the closeness of this distribution to the $\chi^2$ distribution of the true policy function. However, according to Den Haan (2008), the DHM statistic is also limited in its accuracy power. On the one hand, the computational time for this test can be very high depending on the simulation length. On the other hand, this test is sensible to the simulation length as accurate solutions can be rejected more often than 5% for a high enough time range. Therefore, we compare the results of the DHM test with a different length of simulations. The results are summarized in Table 4.B.1. According to the outcomes, (4.B.2) is satisfied for all observed simulation lengths.

Due to the limitations of the DHM statistics, we do a second accuracy test by testing for one-period ahead forecast errors according to Den Haan (2010). It is similar to the definition of the normalized Euler equation error proposed by Judd (1992). To be more specific, we compute the Euler equation errors (EEE) by comparing the numerical approximation with the result of the optimal decision (equation 4.B.1) such as:

$$EEE(S_t) = 1 - \frac{u'_{CN,t}((\beta \mathbb{E}\left\{ u'_{CN,t+1}(1 + r_{t+1}) \right\} - \delta_Y) + 2\frac{\phi}{\gamma} S_t^{-3})}{\hat{c}_t}, \tag{4.B.4}$$

where $\hat{c}_t$ is the optimal decision under assumed calibration and $S_t$ denotes the set of states. In other words, we check the degree of irrationality of an agent to use the

[23] Note that if $T \to \infty$ than $B_T \to 0$.

approximation rule. To do so we compute the Euler equation error at *many* points in the state space defined by Gauss-Hermite nodes in the numerical integration. Den Haan and Ocaktan (2009) note that increasing the number of nodes has only negligible effects, hence we limit its amount to 10 for each state variable. The bounds of the grid defining the 8-D state space are obtained by the simulation to get reasonable values. The test statistics shows that agents make an average 1.233% error in their decision about consumption with a maximum of 3.263%. These values are higher than in standard homogeneous DSGE models. However, this is not surprising with respect to the higher amount of state variables as well as more shocks which increase the degree of uncertainty. In sum, the model seems to be an effective framework to consider incomplete markets, heterogeneous agents, and different consumption goods.

## 4.C Mathematical Appendix

### 4.C.1 Optimization

Under the assumption that prices for final energy are equal for households and final goods-producing firms $p_{\mathrm{E}} = p_{\mathrm{H}} = p_{\mathrm{Y}}$, the households' problems of Ricardian and rule-of-thumb agents, the decision making of firms, and the corresponding first order conditions with respect to the decision variables are:

**Household sector (Ricardian agents)**

$$\begin{aligned}\mathcal{L}^H_{R,i} =&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log\left[ CN^{\gamma}_{R,i,t} \left( \theta CD^{\zeta}_{R,i,t-1} + (1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}_{R,i,t} \right)^{\frac{1-\gamma}{\zeta}} \right] - \phi \frac{1}{(S_{R,i,t}+b)^2} \\ &+ \lambda^H_{R,i,t} \left[ w_t L_{R,i,t} + r_t S_{R,i,t-1} + \pi_t - CN_{R,i,t} - (p_{\mathrm{E}})_t (E_{\mathrm{H}})_{R,i,t} \right. \\ &-CD_{R,i,t} + \left(1-\delta^{CD}\right) CD_{R,i,t-1} - \frac{\omega_{1cd}}{1+\omega_{2cd}} \left( \frac{CD_{R,i,t} - CD_{R,i,t-1}}{CD_{R,i,t-1}} \right)^{1+\omega_{2cd}} \\ &\left. -S_{R,i,t} + \left(1-\delta^{Y}\right) S_{R,i,t-1} \right] \Bigg\}\end{aligned} \tag{4.C.1}$$

- Non-durable goods:

$$\begin{aligned}\frac{\partial \mathcal{L}^H_{R,i}}{\partial CN_{R,i,t}} &= \beta^t \vartheta \gamma \frac{CN^{\gamma-1}_{R,i,t} \left( \theta CD^{\zeta}_{R,i,t-1} + (1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}_{R,i,t} \right)^{\frac{1-\gamma}{\zeta}}}{CN^{\gamma}_{R,i,t} \left( \theta CD^{\zeta}_{R,i,t-1} + (1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}_{R,i,t} \right)^{\frac{1-\gamma}{\zeta}}} \\ &\qquad - \beta^t \vartheta \gamma \frac{1}{CN_{R,i,t}} - \beta^t \lambda^H_{R,i,t} \overset{!}{=} 0\end{aligned} \tag{4.C.2}$$

$$\Leftrightarrow \quad \vartheta \gamma \frac{1}{CN_{R,i,t}} - \lambda^H_{R,i,t} \overset{!}{=} 0 \quad \Leftrightarrow \quad \lambda^H_{R,i,t} = \vartheta \gamma \frac{1}{CN_{R,i,t}}$$

- Durable goods:

$$
\begin{aligned}
\frac{\partial \mathcal{L}^H_{R,i}}{\partial CD_{R,i,t}} &= \beta^{t+1}\mathbb{E}\left\{\frac{\vartheta(1-\gamma)\zeta\theta}{\zeta}\frac{CN^{\gamma}_{R,i,t+1}\left(\theta CD^{\zeta}_{R,i,t}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{t+1}\right)^{\frac{1-\gamma}{\zeta}}CD^{\zeta-1}_{R,i,t}}{CN^{\gamma}_{R,i,t+1}\left(\theta CD^{\zeta}_{R,i,t}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{t+1}\right)^{\frac{1-\gamma}{\zeta}}}\right\}\\
&\quad+\beta^{t+1}\mathbb{E}\left\{\lambda^H_{R,i,t+1}\left[1-\delta^{\mathrm{CD}}+\omega_{1cd}\frac{CD_{R,i,t+1}}{CD^2_{R,i,t}}\left(\frac{CD_{R,i,t+1}-CD_{R,i,t}}{CD_{R,i,t}}\right)^{\omega_{2cd}}\right]\right\}\\
&\quad-\beta^t\lambda^H_{R,i,t}\left[1+\frac{\omega_{1cd}}{CD_{R,i,t-1}}\left(\frac{CD_{R,i,t}-CD_{R,i,t-1}}{CD_{R,i,t-1}}\right)^{\omega_{2cd}}\right]\overset{!}{=}0\\
\Leftrightarrow\quad & \lambda^H_{R,i,t}\left[1+\frac{\omega_{1cd}}{CD_{R,i,t-1}}\left(\frac{CD_{R,i,t}-CD_{R,i,t-1}}{CD_{R,i,t-1}}\right)^{\omega_{2cd}}\right]\\
&=\beta\vartheta\theta(1-\gamma)\mathbb{E}\left\{\frac{CD^{\zeta-1}_{R,i,t}}{\theta CD^{\zeta}_{R,i,t}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{t+1}}\right\}\\
&\quad+\beta\mathbb{E}\left\{\lambda^H_{R,i,t+1}\left[1-\delta^{\mathrm{CD}}+\omega_{1cd}\frac{CD_{R,i,t+1}}{CD^2_{R,i,t}}\left(\frac{CD_{R,i,t+1}-CD_{R,i,t}}{CD_{R,i,t}}\right)^{\omega_{2cd}}\right]\right\}
\end{aligned}
\tag{4.C.3}
$$

- Energy consumption of households:

$$
\begin{aligned}
\frac{\partial \mathcal{L}^H_{R,i}}{\partial (E_{\mathrm{H}})_{R,i,t}} &= \beta^t\vartheta(1-\gamma)\frac{CN^{\gamma}_{R,i,t}\left(\theta CD^{\zeta}_{R,i,t-1}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{R,i,t}\right)^{\frac{1-\gamma}{\zeta}}}{CN^{\gamma}_{R,i,t}\left(\theta CD^{\zeta}_{R,i,t-1}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{R,i,t}\right)^{\frac{1-\gamma}{\zeta}}}(1-\theta)(E_{\mathrm{H}})^{\zeta-1}_{t-1}\\
&\qquad-\beta^t\lambda^H_{R,i,t}(p_{\mathrm{E}})_t\\
&=\beta^t\vartheta(1-\gamma)(1-\theta)\frac{(1-\theta)(E_{\mathrm{H}})^{\zeta-1}_{R,i,t}}{\theta CD^{\zeta}_{R,i,t-1}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{R,i,t}}-\beta^t\lambda^H_{R,i,t}(p_{\mathrm{E}})_t \quad\overset{!}{=}0\\
\Leftrightarrow\quad & \lambda^H_{R,i,t}(p_{\mathrm{E}})_t=\vartheta(1-\gamma)(1-\theta)\frac{(1-\theta)(E_{\mathrm{H}})^{\zeta-1}_{R,i,t}}{\theta CD^{\zeta}_{R,i,t-1}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{R,i,t}}
\end{aligned}
\tag{4.C.4}
$$

- Euler equations for asset stock:

$$
\frac{\partial \mathcal{L}^H_{R,i}}{\partial S_{R,i,t}}=\beta^{t+1}\mathbb{E}\left\{\lambda^H_{R,i,t+1}\left(1+r_{t+1}-\delta^Y\right)\right\}-\beta^t\lambda^H_{R,i,t}+\beta^t\frac{\phi}{\gamma}\frac{2}{(S_{R,i,t}+b)^3}\quad\overset{!}{=}0 \tag{4.C.5}
$$

**Household sector (Rule-of-thumb agents)**

$$
\begin{aligned}
\mathcal{L}^H_{N,i}=&\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\log\left[CN^{\gamma}_{N,i,t}\left(\theta CD^{\zeta}_{N,i,t-1}+(1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}\right)^{\frac{1-\gamma}{\zeta}}\right]\right.\\
&+\lambda^H_{N,i,t}\left[w_tL_{N,i,t}-CN_{N,i,t}-(p_{\mathrm{E}})_t(E_{\mathrm{H}})_{N,i,t}\right.\\
&\left.\left.-CD_{N,i,t}+\left(1-\delta^{CD}\right)CD_{N,i,t-1}-\frac{\omega_{1cd}}{1+\omega_{2cd}}\left(\frac{CD_{N,i,t}-CD_{N,i,t-1}}{CD_{N,i,t-1}}\right)^{1+\omega_{2cd}}\right]\right\}
\end{aligned}
\tag{4.C.6}
$$

- Non-durable goods:

$$\frac{\partial \mathcal{L}^H_{N,i}}{\partial CN_{N,i,t}} = \beta^t \vartheta\gamma \frac{CN^{\gamma-1}_{N,i,t}\left(\theta CD^{\zeta}_{N,i,t-1} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}\right)^{\frac{1-\gamma}{\zeta}}}{CN^{\gamma}_{N,i,t}\left(\theta CD^{\zeta}_{N,i,t-1} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}\right)^{\frac{1-\gamma}{\zeta}}} - \beta^t \vartheta\gamma \frac{1}{CN_{N,i,t}} - \beta^t \lambda^H_{N,i,t} \stackrel{!}{=} 0 \tag{4.C.7}$$

$$\Leftrightarrow \quad \vartheta\gamma\frac{1}{CN_{N,i,t}} - \lambda^H_{N,i,t} \stackrel{!}{=} 0 \qquad \Leftrightarrow \quad \lambda^H_{N,i,t} = \vartheta\gamma\frac{1}{CN_{N,i,t}}$$

- Durable goods:

$$\begin{aligned}
\frac{\partial \mathcal{L}^H_{N,i}}{\partial CD_{N,i,t}} &= \beta^{t+1}\mathbb{E}\left\{\frac{\vartheta(1-\gamma)\zeta\theta}{\zeta}\frac{CN^{\gamma}_{N,i,t+1}\left(\theta CD^{\zeta}_{N,i,t} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{t+1}\right)^{\frac{1-\gamma}{\zeta}} CD^{\zeta-1}_{N,i,t}}{CN^{\gamma}_{N,i,t+1}\left(\theta CD^{\zeta}_{N,i,t} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{t+1}\right)^{\frac{1-\gamma}{\zeta}}}\right\} \\
&+ \beta^{t+1}\mathbb{E}\left\{\lambda^H_{N,i,t+1}\left[1-\delta^{\mathrm{CD}} + \omega_{1cd}\frac{CD_{N,i,t+1}}{CD^2_{N,i,t}}\left(\frac{CD_{N,i,t+1}-CD_{N,i,t}}{CD_{N,i,t}}\right)^{\omega_{2cd}}\right]\right\} \\
&- \beta^t\lambda^H_{N,i,t}\left[1+\frac{\omega_{1cd}}{CD_{N,i,t-1}}\left(\frac{CD_{N,i,t}-CD_{N,i,t-1}}{CD_{N,i,t-1}}\right)^{\omega_{2cd}}\right] \stackrel{!}{=} 0 \\
\Leftrightarrow \quad & \lambda^H_{N,i,t}\left[1+\frac{\omega_{1cd}}{CD_{N,i,t-1}}\left(\frac{CD_{N,i,t}-CD_{N,i,t-1}}{CD_{N,i,t-1}}\right)^{\omega_{2cd}}\right] \\
&= \beta\vartheta\theta(1-\gamma)\mathbb{E}\left\{\frac{CD^{\zeta-1}_{N,i,t}}{\theta CD^{\zeta}_{N,i,t} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{t+1}}\right\} \\
&+ \beta\mathbb{E}\left\{\lambda^H_{N,i,t+1}\left[1-\delta^{\mathrm{CD}} + \omega_{1cd}\frac{CD_{N,i,t+1}}{CD^2_{N,i,t}}\left(\frac{CD_{N,i,t+1}-CD_{N,i,t}}{CD_{N,i,t}}\right)^{\omega_{2cd}}\right]\right\}
\end{aligned} \tag{4.C.8}$$

- Energy consumption of households:

$$\begin{aligned}
\frac{\partial \mathcal{L}^H_{N,i}}{\partial (E_{\mathrm{H}})_{N,i,t}} &= \beta^t\vartheta(1-\gamma)\frac{CN^{\gamma}_{N,i,t}\left(\theta CD^{\zeta}_{N,i,t-1} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}\right)^{\frac{1-\gamma}{\zeta}}}{CN^{\gamma}_{N,i,t}\left(\theta CD^{\zeta}_{N,i,t-1} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}\right)^{\frac{1-\gamma}{\zeta}}}(1-\theta)(E_{\mathrm{H}})^{\zeta-1}_{t-1} \\
&\quad - \beta^t\lambda^H_{N,i,t}(p_{\mathrm{E}})_t \\
&= \beta^t\vartheta(1-\gamma)(1-\theta)\frac{(1-\theta)(E_{\mathrm{H}})^{\zeta-1}_{N,i,t}}{\theta CD^{\zeta}_{N,i,t-1} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}} - \beta^t\lambda^H_{N,i,t}(p_{\mathrm{E}})_t \stackrel{!}{=} 0 \\
\Leftrightarrow \quad & \lambda^H_{N,i,t}(p_{\mathrm{E}})_t = \vartheta(1-\gamma)(1-\theta)\frac{(1-\theta)(E_{\mathrm{H}})^{\zeta-1}_{N,i,t}}{\theta CD^{\zeta}_{N,i,t-1} + (1-\theta)(E_{\mathrm{H}})^{\zeta}_{N,i,t}}
\end{aligned} \tag{4.C.9}$$

**Goods production sector**

$$\pi_0 = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{A_t\left[\eta K^{\nu}_{t-1} + (1-\eta)(E_{\mathrm{Y}})^{\nu}_t\right]^{\frac{\alpha}{\nu}} L^{1-\alpha}_t - r_tK_{t-1} - w_tL_t - (p_{\mathrm{E}})_t(E_{\mathrm{Y}})_t\right\} \tag{4.C.10}$$

- Capital demand:

$$\frac{\partial \pi_0}{\partial K_{t-1}} = A_t \alpha \eta \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha} K_{t-1}^{\nu-1} - r_t \quad \overset{!}{=} 0 \tag{4.C.11}$$

- Energy consumption of final goods production:

$$\frac{\partial \pi_0}{\partial (E_{\mathrm{Y}})_t} = A_t \alpha \eta \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}-1} L_t^{1-\alpha} (E_{\mathrm{Y}})_t^{\nu-1} - (p_{\mathrm{E}})_t \quad \overset{!}{=} 0 \tag{4.C.12}$$

- Labor demand:

$$\frac{\partial \pi_0}{\partial L_t} = A_t (1-\alpha) \left[\eta K_{t-1}^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)_t^{\nu}\right]^{\frac{\alpha}{\nu}} L_t^{-\alpha} - w_t \quad \overset{!}{=} 0 \tag{4.C.13}$$

By rearranging the conditions above, the optimized decisions as well as the market clearing equations are calculated which define the complete model (19 equations).

**Household sector (Ricardian agents)**

- Durable Euler equation: combining (4.C.2) and (4.C.3)

$$\begin{aligned} 1 + \frac{\omega_{1cd}}{CD_{R,i,t-1}} \left(\frac{CD_{R,i,t} - CD_{R,i,t-1}}{CD_{R,i,t-1}}\right)^{\omega_{2cd}} &= \beta \mathbb{E}\left\{\theta \frac{(1-\gamma)}{\gamma} \frac{CN_{R,i,t}\, CD_{R,i,t}^{\zeta-1}}{\theta CD_{R,i,t}^{\zeta} + (1-\theta)\left(E_{\mathrm{H}}\right)_{R,i,t+1}^{\zeta}}\right\} \\ + \beta \mathbb{E}\left\{\frac{CN_{R,i,t}}{CN_{R,i,t+1}} \left[1 - \delta^{\mathrm{CD}} + \omega_{1cd} \frac{CD_{R,i,t+1}}{CD_{R,i,t}^2} \left(\frac{CD_{R,i,t+1} - CD_{R,i,t}}{CD_{R,i,t}}\right)^{\omega_{2cd}}\right]\right\} \end{aligned} \tag{4.C.14}$$

- Non-durables vs. energy: combining (4.C.3) and (4.C.4)

$$\begin{aligned} \beta^t \vartheta \gamma \frac{(p_{\mathrm{E}})_t}{CN_{R,i,t}} &= \beta^t \vartheta (1-\gamma)(1-\theta) \frac{(E_{\mathrm{H}})_{R,i,t}^{\zeta-1}}{\theta CD_{R,i,t-1}^{\zeta} + (1-\theta)\left(E_{\mathrm{H}}\right)_{R,i,t}^{\zeta}} \\ \Leftrightarrow \qquad (p_{\mathrm{E}})_t &= \frac{(1-\gamma)(1-\theta)}{\gamma} \frac{CN_{R,i,t}(E_{\mathrm{H}})_{R,i,t}^{\zeta-1}}{\left(\theta CD_{R,i,t-1}^{\zeta} + (1-\theta)\left(E_{\mathrm{H}}\right)_{R,i,t}^{\zeta}\right)} \end{aligned} \tag{4.C.15}$$

- Labor supply: Ricardian agents

$$L_{R,i,t} = (1-\rho_L)\bar{L}_R + \rho_L L_{R,i,t-1} + \varepsilon_{L,R,i,t} \tag{4.C.16}$$

- Euler equation of capital in final production: combining (4.C.2) and (4.C.5)

$$\begin{aligned} \beta^t \vartheta \gamma \frac{1}{CN_{R,i,t}} &= \beta^{t+1} \mathbb{E}\left\{\vartheta \gamma \frac{1}{CN_{R,i,t+1}} \left(1 + r_{t+1} - \delta^Y\right)\right\} + \beta^t \frac{\phi}{\gamma} \frac{2}{\left(S_{R,i,t} + b\right)^3} \\ \Leftrightarrow \qquad \frac{1}{CN_{R,i,t}} &= \beta \mathbb{E}\left\{\frac{1}{CN_{R,i,t+1}} \left(1 + r_{t+1} - \delta^Y\right)\right\} + \frac{\phi}{\gamma} \frac{2}{\left(S_{R,i,t} + b\right)^3} \end{aligned} \tag{4.C.17}$$

**Household sector (Rule-of-thumb agents)**

- Durable Euler equation: combining (4.C.7) and (4.C.8)

$$1+\frac{\omega_{1cd}}{CD_{N,i,t-1}}\left(\frac{CD_{N,i,t}-CD_{N,i,t-1}}{CD_{N,i,t-1}}\right)^{\omega_{2cd}}=\beta\mathbb{E}\left\{\theta\frac{(1-\gamma)}{\gamma}\frac{CN_{N,i,t}CD^{\zeta-1}_{N,i,t}}{\theta CD^{\zeta}_{N,i,t}+(1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}_{N,i,t+1}}\right\}$$
$$+\beta\mathbb{E}\left\{\frac{CN_{N,i,t}}{CN_{N,i,t+1}}\left[1-\delta^{\mathrm{CD}}+\omega_{1cd}\frac{CD_{N,i,t+1}}{CD^{2}_{N,i,t}}\left(\frac{CD_{N,i,t+1}-CD_{N,i,t}}{CD_{N,i,t}}\right)^{\omega_{2cd}}\right]\right\} \tag{4.C.18}$$

- Non-durables vs. energy: combining (4.C.8) and (4.C.9)

$$\beta^{t}\vartheta\gamma\frac{(p_{\mathrm{E}})_t}{CN_{N,i,t}}=\beta^{t}\vartheta\left(1-\gamma\right)\left(1-\theta\right)\frac{(E_{\mathrm{H}})^{\zeta-1}_{N,i,t}}{\theta CD^{\zeta}_{N,i,t-1}+(1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}_{N,i,t}}$$
$$\Leftrightarrow\qquad (p_{\mathrm{E}})_t=\frac{(1-\gamma)\left(1-\theta\right)}{\gamma}\frac{CN_{N,i,t}(E_{\mathrm{H}})^{\zeta-1}_{N,i,t}}{\left(\theta CD^{\zeta}_{N,i,t-1}+(1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}_{N,i,t}\right)} \tag{4.C.19}$$

- Labor supply: rule-of-thumb agents

$$L_{N,i,t}=(1-\rho_L)\bar{L}_N+\rho_L L_{N,i,t-1}+\rho_{L,A}\left(A^{Y}_t-\bar{A}^{Y}\right)+\varepsilon_{L,N,i,t} \tag{4.C.20}$$

**Goods production sector**

- Final goods production output (= non-durable goods)

$$Y_t=A_t\left[\eta K^{\nu}_{t-1}+(1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}_t\right]^{\frac{\alpha}{\nu}}L^{1-\alpha}_t \tag{4.C.21}$$

- Capital demand of final goods production: rearranging (4.C.11)

$$r_t=A_t\alpha\eta\left[\eta K^{\nu}_{t-1}+(1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}_t\right]^{\frac{\alpha}{\nu}-1}L^{1-\alpha}_t K^{\nu-1}_{t-1} \tag{4.C.22}$$

- Energy demand of final goods production: rearranging (4.C.12)

$$(p_{\mathrm{E}})_t=A_t\alpha\eta\left[\eta K^{\nu}_{t-1}+(1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}_t\right]^{\frac{\alpha}{\nu}-1}L^{1-\alpha}_t(E_{\mathrm{Y}})^{\nu-1}_t \tag{4.C.23}$$

- Labor demand of final goods production: rearranging (4.C.13)

$$w_t=A_t\left(1-\alpha\right)\left[\eta K^{\nu}_{t-1}+(1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}_t\right]^{\frac{\alpha}{\nu}}L^{-\alpha}_t \tag{4.C.24}$$

**Market Clearing**

- Aggregate market constraint:

$$Y_t - (p_{\mathrm{E}})_t (E_{\mathrm{Y}})_t = CN_t + (p_{\mathrm{E}})_t (E_{\mathrm{H}})_t + CD_t - \left(1 - \delta^{CD}\right) CD_{t-1} + K_t - \left(1 - \delta^{Y}\right) K_{t-1} \tag{4.C.25}$$

- Aggregate non-durable goods:

$$CN_t = \int_0^{\lambda} CN_{R,i,t} + \int_{\lambda}^{1} CN_{N,i,t} \tag{4.C.26}$$

- Aggregate durable goods:

$$CD_t = \int_0^{\lambda} CD_{R,i,t} + \int_{\lambda}^{1} CD_{N,i,t}. \tag{4.C.27}$$

- Aggregate capital market:

$$K_t = \int_0^{\lambda} S_{R,i,t}. \tag{4.C.28}$$

- Aggregate labor market:

$$L_t = \int_0^{\lambda} L_{R,i,t} + \int_{\lambda}^{1} L_{N,i,t}. \tag{4.C.29}$$

with (4.C.16) and (4.C.20), the law of large numbers hold for $\forall t : \int_0^{\lambda} \varepsilon_{L,R,i,t} di \simeq \int_{\lambda}^{1} \varepsilon_{L,N,i,t} di \simeq 0$

$$\begin{aligned} L_t &= \int_0^{\lambda} \frac{(1-\rho_L)\bar{L}_N}{1-\rho_L} + \int_{\lambda}^{1} \left( \frac{(1-\rho_L)\bar{L}_N}{1-\rho_L} + \frac{\rho_{L,A}\left(A_t^Y - \bar{A}^Y\right)}{(1-\rho_L)} \right) \\ &= \lambda \bar{L} + (1-\lambda) \left( \bar{L} + \frac{\rho_{L,A}\left(A_t^Y - \bar{A}^Y\right)}{(1-\rho_L)} \right) \\ &= \bar{L} + \frac{(1-\lambda)}{(1-\rho_L)} \rho_{L,A} \left(A_t^Y - \bar{A}^Y\right) \end{aligned} \tag{4.C.30}$$

- Aggregate energy market:

$$E_t = \int_0^{\lambda} (E_{\mathrm{H}})_{R,i,t} + \int_{\lambda}^{1} (E_{\mathrm{H}})_{N,i,t} + (E_{\mathrm{Y}})_t \tag{4.C.31}$$

**Formation of shocks**

- Productivity shock in final goods production:

$$\ln A_t = \rho_{\mathrm{A}} \ln A_{t-1} + \varepsilon_{\mathrm{A},t} \tag{4.C.32}$$

- Productivity shock in final energy generation:

$$\ln\left(p_{\mathrm{E}}\right)_t = \rho_{\mathrm{P}} \ln\left(p_{\mathrm{E}}\right)_{t-1} + \varepsilon_{\mathrm{P},t} \tag{4.C.33}$$

### 4.C.2 Steady states

In the following, we can construct the steady state conditions from the model.

#### Household sector

- Durable Euler equation (4.C.14) and (4.C.18) (same for both types of agents):

$$1 = \beta\theta \frac{(1-\gamma)}{\gamma} \frac{CN\, CD^{\zeta-1}}{\theta CD^{\zeta} + (1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}} + \beta\left(1-\delta^{CD}\right) \qquad \text{(4.C.14.SS + 4.C.18.SS)}$$

- Non-durables vs. energy (4.C.15) and (4.C.19) (same for both types of agents):

$$p_{\mathrm{E}} = \frac{(1-\gamma)(1-\theta)}{\gamma} \frac{CN\left(E_{\mathrm{H}}\right)^{\zeta-1}}{\left(\theta CD^{\zeta} + (1-\theta)\left(E_{\mathrm{H}}\right)^{\zeta}\right)} \qquad \text{(4.C.15.SS + 4.C.19.SS)}$$

- Labor supply (4.C.30)

$$L_t = \bar{L} + \frac{(1-\lambda)}{(1-\rho_L)} \rho_{L,A}\left(A_t - \bar{A}\right) \tag{4.C.30.SS}$$

- Euler equation for capital of final production (4.C.17)

$$1 = \beta\left(1 + r - \delta^{Y}\right) + \frac{\phi}{\gamma} \frac{2}{(S+b)^3} CN$$

consequently

$$r = \frac{1}{\beta} - 1 + \delta^{Y} - \frac{\phi}{\beta\gamma} \frac{2}{(S+b)^3} CN \tag{4.C.17.SS}$$

#### Goods production sector

- Final goods production output (= non-durable goods) (4.C.21)

$$Y = A\left[\eta K^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}\right]^{\frac{\alpha}{\nu}} L^{1-\alpha} \tag{4.C.21.SS}$$

- Capital demand of final goods production (4.C.22)

$$r = A\alpha\eta\left[\eta K^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}\right]^{\frac{\alpha}{\nu}-1} L^{1-\alpha} K^{\nu-1} \tag{4.C.22.SS}$$

- Energy demand of final goods production (4.C.23)

$$p_{\mathrm{E}} = A\alpha\eta \left[\eta K^{\nu} + (1-\eta)\left(E_{\mathrm{Y}}\right)^{\nu}\right]^{\frac{\alpha}{\nu}-1} L^{1-\alpha}(E_{\mathrm{Y}})^{\nu-1} \tag{4.C.23.SS}$$

- Labor demand of final goods production (4.C.24)

$$w = (1-\alpha)\frac{Y}{L} \tag{4.C.24.SS}$$

**Market Clearing**

- Aggregate market constraint (4.C.25)

$$Y - p_{\mathrm{E}}E_{\mathrm{Y}} = CN + p_{\mathrm{E}}E_{\mathrm{H}} + \delta^{CD}CD + \delta^{Y}K \tag{4.C.25.SS}$$

- Aggregate non-durable goods market (4.C.26)

$$CN = \lambda CN_R + (1-\lambda)\, CN_N \tag{4.C.26.SS}$$

- Aggregate durable goods market (4.C.27)

$$CD = \lambda CD_R + (1-\lambda)\, CD_N \tag{4.C.27.SS}$$

- Aggregate asset market (4.C.28)

$$K = \lambda S_R \tag{4.C.28.SS}$$

- Aggregate energy market (4.C.31)

$$E = \lambda E_{\mathrm{H}} + (1-\lambda)\, E_{\mathrm{H}} + (E_{\mathrm{Y}}) \tag{4.C.31.SS}$$

### 4.C.3 Log-linearized equations

In contrast to Chapter 3, we feed Dynare with the numerical steady state values of all endogenous variables derived in Section 4.C.2, which makes the calculation of log-linearized equations obsolete.

### 4.C.4 Calibration

The calibration of parameters is carried out analogously to Bergmann (2018), except for the parameter $\delta_Y$.

Given $r$ and, we can rearrange (4.C.17.SS) to get:

$$\delta_Y = 1 + r - \frac{1}{\beta} + \frac{\phi}{\beta\gamma}\frac{2}{(S+b)^3}CN \qquad (4.C.34)$$

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