INTERNATIONAL ECONOMIC REVIEW
Vol. 61, No. 4, November 2020 DOI: 10.1111/iere.12477

FORMAL SEARCH AND REFERRALS FROM A FIRM'S PERSPECTIVE*

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We explore the relationship between firms' characteristics and recruitment strategies. We present a theoretical search model with two search channels: a costly formal channel and a costless informal channel (referrals). We empirically test the model predictions and find that: larger firms search more formally; firms search more formally for high-skilled workers; the positive relationship between firm's size and formal search intensity also holds for firms not using referrals; firms using referrals invest less into formal search compared to firms not using referrals; larger firms hire less often by referrals than smaller firms; and larger search effort leads to more applicants.

1. INTRODUCTION

There is a wide discussion in labor economics on worker and firm search with a particular focus on comparing referral hiring and formal search. Typically, the formal search channel includes several search strategies such as job advertisements in the media or the involvement of employment agencies and the informal channel includes recruitment by incumbent employees' referrals. The empirical literature suggests that both channels are equally important (see Topa, 2011). Since the seminal work by Granovetter (1995) workers' choices of search strategies including social networks were frequently analyzed. But there is only scarce literature that addresses firms' choices of search strategies. Our article fills this gap and explores the relationship between firms' characteristics and their recruitment strategies. We develop a model of firm search with two search channels—a formal channel that is costly for firms and a costless informal channel (network referrals). We use this model to guide empirical research and investigate the link between the frequency of referral hiring, formal search effort of firms, and their size.

The scarce availability of data is the reason that only a few studies shed light on the motivation, reasons, or outcomes of referrals relative to other search channels from the firms' perspective. Holzer (1987) published one of the first studies on firms' choices of recruitment channels and their economic outcomes, based on a firms survey. Studies using personnel data include, for example, Fernandez and Weinberg (1997), Fernandez et al. (2000), Castilla (2005),

*Manuscript received August 2017; revised December 2019.

[Correction added on September 28, 2020 after first online publication: DEAL funder statement added]

[1] We are thankful for useful comments and advice to Ronald Bachmann, Herbert Dawid, Roberto Fernandez, Alexander Kubis, Christian Merkl, Michael Neugart, and conference participants at the annual meetings of the American Economic Association in Philadelphia, the European Association of Labour Economists in St. Gallen, the European Economic Association in Cologne, the German Economic Association in Vienna, the annual Search and Matching conference in Barcelona, the IAB conference on "Vacancies, Hiring and Matching" as well as seminar participants at the University of Bielefeld, the Center for Interdisciplinary Research (ZIF) in Bielefeld, Technical University of Darmstadt, the Institute for Employment Research (IAB), the Friedrich-Alexander-University Erlangen-Nuremberg, and the RWI - Leibniz Institute for Economic Research. Please address correspondence to: Dr. Michael Stops, Institute for Employment Research (IAB), Regensburger Str. 100, 90479 Nuremberg, Germany. E-mail: *stops.michael@t-online.de*.

Fernandez and Sosa (2005), and Brown et al. (2016). Even though these studies answer a broad range of questions regarding referrals, their analysis is either restricted to only one firm (Castilla, 2005; Fernandez and Sosa, 2005), a selection of firms from certain regions (DeVaro, 2008), or branches (Fuller-Love, 2009; Berardi, 2013).

Due to the argument that formal search strategies are more costly, it is plausible to think that more productive firms have larger resources that can be spend on formal search and stronger incentives to use these resources to attract job applicants. Thus, firm characteristics such as firm productivity or firm size are important for understanding firms' choices of recruitment channels, which is only possible by using a representative data set on employers' recruitment strategies and processes. To the best of our knowledge, only Kramarz and Skans (2014) and Dustmann et al. (2016) used data with firm characteristics to analyze referral hiring; however, none of the two studies reported results for the firm size and search effort.

In our model, workers are ex ante homogeneous, whereas firms pay an entry cost and wait for a random productivity realization. If this productivity realization is sufficiently high, firms enter the market and postvacancies. There are two channels of job search: a formal and an informal channel. Firms can decide if they want to use the formal channel or not and how much search effort they want to exert through this channel. Their search effort is costly, for example, there are the costs of posting a job advertisement in the newspaper, paying the registration fee on the Internet job platform or paying for recruiting services of (private) employment agencies. Endogenous formal search effort is introduced as in Pissarides (2000). Informal job search is based on employee referrals, here we use the framework of Cahuc and Fontaine (2009). The probability of an employee referral depends on the size of the social network and the equilibrium fraction of unemployed contacts in the networks of incumbent employees; however, it cannot be influenced by firms.

In this framework, we show that there are two general equilibria. In the first equilibrium, less productive firms do not advertise their vacancies formally and rely exclusively on the costless informal channel. Due to the low job-filling rate, these firms are less successful in hiring than their competitors and remain small in the long run. In contrast, more productive firms use both search channels simultaneously, recruit more intensively, and become large in the long run. Considering this latter group of firms, we can prove that more productive firms face larger foregone profits and invest more effort into formal search activities. This means that formal matching is balanced, that is, workers are more likely to sample job advertisements from more productive firms. As a result, more productive firms achieve a higher job-filling rate, which leads to a positive correlation between employment, productivity, wages, and the fraction of workers hired via the formal channel. Therefore, larger firms do not only pay higher wages, which is a common "firm size effect" (see, for example, Burdett and Mortensen, 1998) but also have a smaller fraction of applicants hired by referrals. The second equilibrium is a special case of the first one where all firms use both search channels.

We test several theoretical predictions of the model by using the Job Vacancy Survey of the Institute for Employment Research (IAB JVS). The IAB JVS is a representative survey among human resource managers or managing directors in German firms reporting information about their search strategies with detailed answers about the most recent successful recruitment case and the most recent unsuccessful recruitment case. Since most firms are observed only once, our sample is a repeated cross section of firms and includes more than 39,500 observations over the period 2012–2016.

The advantage of our empirical data is that we have detailed information about search activities of firms prior to hiring. Therefore, we do not only estimate an empirical link between the firm size and referral hiring but additionally control for the ex ante search activities of firms, which is often unobserved in other data sets (e.g., labor force surveys) and considered as an omitted variable. We summarize this ex ante information in an intensity index that is our empirical proxy for the formal search effort of firms. This search index is an ordinal variable equal to the number of formal search activities used by the firm. The list of such activities includes: placed ads in newspapers or magazines, posted vacancy in the Internet, posted vacancy

on the firm's own Web site, and contacted the federal employment agency. Because of the ordinal nature of this index variable, we use an ordered logistic regression with time fixed effects to predict which firms are using these formal search strategies more intensively.

We find that large firms are more likely to use formal search compared to small firm. Next, there is a positive link between the firm size and the formal search index. This relationship is observed regardless of the fact whether the firm is using referrals or not, but the formal search index is lower on average if employee referrals are used, indicating that there is substitution between the two channels. Further, we show that more intensive formal search by larger firms is one of the reasons for lower frequencies of referral recruiting. These empirical findings are inline with our theoretical predictions. Even though we consider a submarket with homogeneous workers in our model, we can perform comparative statics with respect to the workers' skill level. This analysis reveals that firms exert more formal search effort in submarkets with higher worker skills. This prediction is also supported by our empirical analysis. We find that the formal search index is higher for positions requiring vocational training and even higher for positions requiring a university degree.

A broad set of firm- and vacancy-specific control variables are considered and a number of robustness checks corroborate our results. One of them is based on using monetary expenses of firms for recruitment processes that can be seen as another proxy for the search effort of firms. In addition, we provide new empirical evidence that successful job candidates coming from unemployment are less likely to enter jobs by referrals compared to previously employed job candidates making a job-to-job transition.

Our article is related to a number of theoretical and empirical studies on referrals as search strategy. The seminal contribution in this field was by Montgomery (1991). He shows that employee referrals may serve as a useful screening device if worker's ability is not observed by the potential employer. Hensvik and Skans (2016) confirm this result empirically by using Swedish data.

Purely theoretical studies on referrals can be generally divided into two groups, those using network theory (Calvo-Armengol and Jackson, 2004, 2007; Ioannides and Soetevent, 2006; Horvath, 2014) and those using a search and matching framework (Fontaine, 2008; Cahuc and Fontaine, 2009; Zaharieva, 2013, 2015; Stupnytska and Zaharieva, 2015). While the first group of studies shows that exchange of job information within the network of friends leads to better employment chances, the second group of studies has a stronger focus on worker and firm heterogeneity.

For example, Fontaine (2008) shows that heterogeneity in the size of social networks leads to different bargaining positions of workers and generates wage dispersion. Similar to our study, Zaharieva (2015) considers a model with heterogeneous firms and identical workers. In this model, workers continue searching on-the-job and accept job offers from more productive firms, whereas forwarding offers from less productive firms to their social contacts. Thus, referral job offers are associated with a wage penalty compared with the formal channel. Another study focusing on referral hiring and on-the-job search is by Arbex et al. (2019). These authors provide model-consistent evidence that workers switching jobs are more likely to use network search compared to unemployed applicants. Our empirical findings confirm this view.

In the group of theoretical studies, Galenianos (2013) is the most closely related work. This study uses a search and matching framework with two search channels, formal and informal, and two firm types, where workers direct their job search to only one type of the firm. This approach becomes restrictive when a continuum of heterogeneous firms should be considered. Hence, in this article, we extend and modify the theoretical approach of Galenianos (2013) to the more realistic case where workers send simultaneous applications to different firms and we empirically test our theoretical predictions.

Empirical studies using personnel data (Simon and Warner, 1992; Castilla, 2005; Fernandez and Sosa, 2005; Brown et al., 2016) typically report that workers who apply due to a referral are more likely to be invited for the interview than workers who apply via the formal channel. These studies also find that workers hired due to referrals receive a slight

wage premium over workers who were hired via the formal channel, even though this premium disappears over time. On the other hand, panel data studies based on worker surveys report wage penalties associated with referrals (Pistaferri, 1999; Addison and Portugal, 2002). Pellizzari (2010) concludes that in the European Union, premiums and penalties to finding jobs through personal contacts are equally frequent and are of about the same size.

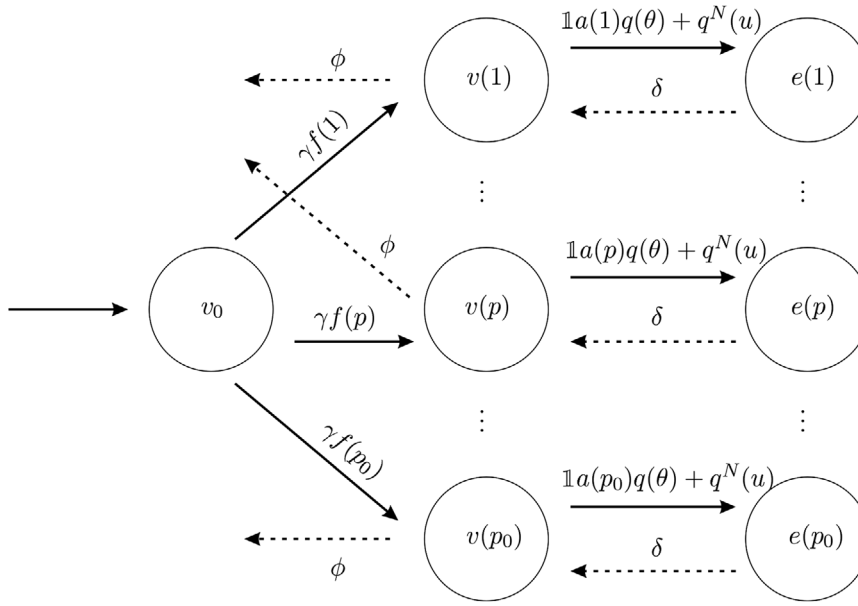
Other studies contain both empirical and theoretical analyses of referral hiring. Similar to our study, Holzer (1987) considers the problem from the perspective of firms and finds that those hired through referrals from current employees are more productive and stay longer than those hired through other search ways. Kugler (2003) finds evidence that referred workers are more productive, because of the peer pressure and internal monitoring by the incumbent employee who provided recommendations. This mechanism leads to lower monitoring costs for firms that rely on referral hiring and stimulates firms to pay efficiency wages. Galeotti and Merlino (2003) consider workers' search via networks over the business cycle and show that the search intensity is characterized by the inverse U-shape, that is, workers initially increase their search effort in the recession but get discouraged later on if the recession gets more severe. Bentolila et al. (2010) develop a model that can explain wage penalties associated with referrals by linking referral hiring and occupational mismatch. Specifically, in their model, unemployed workers may follow the recommendations by friends and relatives and accept mismatch jobs in alternative occupations.

Our study is also close to the early search literature on wages, productivities and firm sizes with the seminal contribution by Burdett and Mortensen (1998) who show that firms offering higher wages are able to attract and keep more workers, resulting in a positive link between wages and firm sizes. Bontemps et al. (2000) complement this model with heterogeneous firms and show a positive correlation between productivities, wages, and firm sizes. Both studies are based on the assumption of random matching, which means that all firms have an equal probability of being sampled by workers independent of their size. Though the finding of a positive correlation between productivity, wage level, and firm size is similar, in our model, larger firms invest more search effort and are more likely to be matched with workers compared to smaller firms. In this respect, our study continues the balanced matching approach of Burdett and Vishwanath (1988) with the difference that positive association between the matching rate and firm size is endogenous in our model, whereas it is assumed to be exogenous in Burdett and Vishwanath (1988).

Even more closely related is the study by Mortensen and Vishwanath (1994), who introduce two sampling distributions. Specifically, in their model, workers are randomly matched to firms via the formal search channel but can also be matched with a help of incumbent employees, which yields a superior distribution of wage offers. Therefore, referral matching is balanced and workers are more likely to be matched to jobs offering higher wages than jobs with lower wages. Despite this similarity, our model is different from Mortensen and Vishwanath (1994) because in our model, firms have different productivities and there is an explicit choice of search intensity by firms in our setting.

Finally, our study establishes new sources of heterogeneities in the matching efficiency; see for this discussion Davis et al. (2013) and, more recent, Gavazza et al. (2018). In particular, we show how firm properties or job requirements can lead to varied decisions on search strategies. The aforementioned literature emphasizes that different search strategies and intensities can result in different search outcomes in terms of wages or matching efficiency. So, our results shed light on the sorting of job applicants across heterogeneous firms, and therefore, can enable deeper analysis of the underlying reasons for wage inequality in future research.

The plan of the article is as follows: Section 2 presents our theoretical model with a number of testable predictions. Section 3 describes the data sample. Section 4 contains our empirical results. Section 5 presents additional robustness checks and Section 6 concludes.



Notation: v_0—pool of homogeneous vacancies waiting for the productivity draw, γ—Poisson arrival rate of productivity types, $F(p)$—cdf of the productivity distribution, ϕ—vacancy abolishment rate, $v(p)$—number of vacancies in firm p, $e(p)$—number of employees in firm p, $\mathbb{1}a_j q(\theta) + q^N(u)$—total job-filling rate of the firm, where $\mathbb{1}$ is an indicator variable for using the formal channel of search, a_j—individual search intensity of firm j, $q(\theta)$—market component of the job-filling rate, $q^N(u)$—network job-filling rate, δ—job destruction rate, p_0—the least productive firm type in the market.

FIGURE 1

LABOR MARKET DYNAMICS

2. THEORETICAL FRAMEWORK

2.1. *Labor Market Framework.* The labor market is characterized by the following properties. There is a continuum of infinitely lived risk neutral workers and firms discounting future at a common discount rate r. When opening a new vacancy every firm has to pay an entry cost K. Vacancies are ex ante homogeneous at the entry stage and form a pool of vacancies v_0. At the Poisson arrival rate, γ vacancies are assigned a productivity level p that is a random draw from an exogenous productivity distribution $F(p)$, with the corresponding density $f(p)$. To keep the model tractable, we assume that the productivity distribution is uniform, so that $F(p) = p$, $p \in [0..1]$. This mechanism is illustrated in Figure 1 and generates ex post heterogeneity of vacancies with different productivity levels.

Every vacancy with an assigned productivity level can be either filled with a worker or abandoned without hiring. The latter happens at rate ϕ and the abandoned vacancies exit the market. Let $v(p)$ denote the number of vacancies with productivity p and $e(p)$ denote the number of workers employed in jobs with productivity p. All vacancies $v(p)$ and filled jobs $e(p)$ form a firm of type p; thus, $e(p)$ can be interpreted as a firm size in terms of the total number of employees working in firm p. Each filled job can be destroyed at an exogenous rate δ. After the job destruction shock, the worker becomes unemployed and the filled job becomes an open vacancy with the same productivity level as before. Note that some productivity draws can be low and not acceptable to firms. Let p_0 denote an endogenous reservation productivity cutoff corresponding to the least productive firm in the market. Thus, vacancies with productivity realizations below p_0 remain in the pool of unassigned vacancies v_0 and wait for a new productivity realization in the future. In contrast, vacancies with productivities above p_0 belong to one of the firms in the market and can be filled with workers.

Even though our model builds on the seminal contribution by Mortensen and Pissarides (1994), heterogeneous productivities are generated in a different way. In Mortensen and Pissarides (1994), vacancies are identical before matching and productivity draws are realized after the worker and the firm are matched. Thus, heterogeneous productivities are an outcome of different match qualities and it is not specified whether it is because the worker is more productive or the firm. This is different in our setting, where vacancies are already heterogeneous before matching, thus higher productivity is attributable to the firm. This property is important for our empirical analysis where we can fully exploit rich firm characteristics contained in our data.

Workers are ex ante homogeneous and contribute part b to the total output. This means that any worker matched with a firm of type p produces output bp, where b is the contribution of the worker and p is the contribution of the firm. Even though all workers are identical in the considered labor market, changing b in the form of comparative statics will allow us to analyze the implications of the model for labor markets with more educated workers (higher b) or workers with lower qualification (lower b). Furthermore, more educated workers are more productive. Employed workers receive the wage $w(p)$, which is obtained by Nash bargaining between the worker and the firm. Worker's bargaining power is denoted by β and an exogenous reservation wage is $\bar{w} < b$.

Firms can hire workers via the formal search channel or via referrals from incumbent employees. We model referral search like in Cahuc and Fontaine (2009). Let every worker have l social contacts. The firm sends its vacancy to one randomly chosen person in the population. This person is unemployed with probability u and employed with probability $1 - u$. Unemployed workers always accept the job offer (formal proof is provided later in the article). Since there is no on-the-job search, employed workers cannot make use of the open vacancy and consider their social network. All their social contacts are already employed with probability $(1 - u)^l$, but the employee has at least one unemployed friend in the social network with probability $1 - (1 - u)^l$; the offer is then forwarded to this friend. Thus, the network job-filling rate q^N is given by:

$$q^N(u) = s(u + (1 - u)(1 - (1 - u)^l)) = s(1 - (1 - u)^{l+1}).$$

The variable s denotes an indicator taking value 1 in a setting with referral hiring and 0 otherwise. This variable is helpful in distinguishing between the two settings with and without referrals. Even though in some cases, s can be an outcome of the decision process, we assume that s is exogenous in our model. The reason is that some industries such as public administration have limitations and are not allowed to use referral hiring. Thus, the choice of s is not necessarily a free choice by all firms but can also be driven by institutional and legislative restrictions.

The above equation shows that $q^N(u)$ is an increasing function of u. Intuitively, this means that higher unemployment raises the probability that incumbent employees recommend one of their friends for the job and the position is filled by referral. At the same time, $q^N(u)$ is increasing in the network size l, so that larger social networks lead to more frequent job recommendations. Further, we assume that referral hiring is free of cost for firms. This means that all firms use referral hiring in our model if $s = 1$. Note that we do not limit referral hiring only to the employees of a given firm. This approach is a simplification that was already used in the previous literature (Cahuc and Fontaine, 2009; Stupnytska and Zaharieva, 2017). One justification for this assumption is the fact that also in our empirical data, referrals include recommendations from current employees of the firm and other social contacts who could be employees from affiliated companies, friends, and acquaintances of external consultants and long-term customers.

In addition to the costless referral hiring, firms may decide to advertise their vacancies in a formal way. The decision of firms to use or not to use this formal search channel is

endogeneous. Let $a(p)$ denote a formal search intensity or effort of firm p per vacancy conditional on advertising. Then $a(p)v(p)$ denotes efficiency units of vacancies $v(p)$ (see Pissarides, 2000). We assume that the formal search effort is costly for firms with a corresponding cost function $C(a) = c + a^2/(2c_0)$. This is a flow cost of advertising a vacancy per unit time that has a fixed and a proportional component. Thus, formal advertising is an endogenous decision of firms that depends on the size of the fixed-cost component c. According to our model, this results in two different equilibrium types:

- General equilibrium I. More productive firms with productivities in the range $[p_c..1]$ use formal advertising in addition to referral hiring, whereas less productive firms in the range $[p_0^I..p_c]$ find that formal advertising is too expensive for them and use referral hiring as the only search channel. Here, p_0^I is the least productive firm type in the equilibrium. The total number of vacancies in efficiency units in this equilibrium is given by:

$$\tilde{v} = \int_{p_c}^{1} a(p)v(p)dp.$$

- General equilibrium II. All firms in the market use formal advertising in addition to referral hiring. We denote the least productive firm type in this equilibrium by p_0^{II}. The total number of vacancies in efficiency units in this equilibrium is given by:

$$\tilde{v} = \int_{p_0^{II}}^{1} a(p)v(p)dp.$$

Matching via the formal channel is achieved by means of a standard matching function $m(u, \tilde{v})$, which is increasing in both arguments and homogeneous of degree 1. Thus, $m(u, \tilde{v})$ is the total number of matches between searching unemployed workers and open vacancies created via the formal channel per period of time. Let $\theta = \tilde{v}/u$ denote the market tightness. The number of formal matches between unemployed workers and advertising type p vacancies is given by:

$$m(u, \tilde{v})\frac{a(p)v(p)}{\tilde{v}}.$$

Consider some firm j of type p with the search intensity a_j, so the probability of being matched with the worker via the formal channel for this firm is given by:

$$m(u, \tilde{v})\frac{a(p)v(p)}{\tilde{v}}\frac{a_j}{a(p)v(p)} = a_j\frac{m(u, \tilde{v})}{\tilde{v}} = a_j m\left(\frac{1}{\theta}, 1\right).$$

Let $q(\theta) \equiv m(\frac{1}{\theta}, 1)$, so the flow probability of being matched for firm j is equal to $a_j q(\theta)$. This means that higher search effort a_j raises the probability of hiring a worker. At the same time, a larger θ reduces this probability so that $q'(\theta) < 0$ and $\lim_{\theta\to 0} q(\theta) = \infty$. This can be either because the number of unemployed workers is smaller or because all other vacancies are searching more intensively. Note that this functional specification rules out general equilibria without formal advertising. For example, if all firms in the market rely exclusively on referral hiring and $a(p) = 0\ \forall p$, then $\theta = 0$ and the job-filling rate $q(\theta)$ becomes infinitely large, which creates infinite incentives for firms to invest into formal advertising. Since potentially every firm can use both search channels simultaneously, the total job-filling rate for firm j is given by: $\mathbb{1}a_j q(\theta) + q^N(u)$, see Figure 1, where the indicator variable $\mathbb{1}$ is equal to 1 if the firm is advertising its vacancies in a formal way and 0 otherwise.

2.2. *Choice of the Formal Search Intensity.* Let $V(p)$ be the present value of an open vacancy and $J(p)$ be the present value of a filled job in a firm of type p. We follow a standard approach in the search and matching literature and analyze per vacancy decisions of firms.[2] Consequently, consider some vacancy j of firm p with the corresponding present value $V_j(p)$ that is given by:

$$(1) \qquad rV_j(p) = \max_{a_j \geq 0} a_j q(\theta)(J(p) - V_j(p)) - C(a_j) + q^N(u)(J(p) - V_j(p)) - \phi V_j(p),$$

where $a_j q(\theta)$ is the job-filling rate through the formal search channel and $q^N(u)$ is the job-filling rate through the network of social contacts. Vacancies are abandoned without hiring at rate ϕ. The firm chooses its formal search intensity a_j to maximize the present value of an open vacancy $V_j(p)$. Thus, the firm will advertise its vacancy only if the expected gain from advertising $a_j q(\theta)(J(p) - V_j(p))$ is larger than the cost $C(a_j)$:

$$(2) \qquad a_j > 0 \Leftrightarrow a_j q(\theta)(J(p) - V_j(p)) > c + a_j^2/(2c_0),$$

otherwise the firm will not use formal advertising and $a_j = 0$. In the former case $a_j > 0$, the optimal advertising effort can be found from the first-order condition $C'(a_j) = q(\theta)(J(p) - V_j(p))$, which means that the marginal cost $C'(a_j)$ should be equal to the marginal gain $q(\theta)(J(p) - V_j(p))$. With a quadratic cost function $C(a_j) = c + a_j^2/(2c_0)$, the optimal search intensity of firm j is given by:

$$a_j(p) = c_0 q(\theta)(J(p) - V_j(p)).$$

This implies that the optimal search effort of firm j is increasing in the expected net gain from filling the job with a worker $(J(p) - V_j(p))$. Since all vacancies are identical, the firm will choose the same search effort for all vacancies, so that $a_j(p) = a(p)\ \forall j$. Insert the optimal $a(p)$ into Equation (1) to get:

$$(3) \quad (r + \phi)V(p) = -c + 0.5c_0 q^2(\theta)(J(p) - V(p))^2 + q^N(u)(J(p) - V(p)) \quad \text{if } a(p) > 0$$

$$(4) \qquad (r + \phi)V^0(p) = q^N(u)(J(p) - V^0(p)) \quad \text{if } a(p) = 0.$$

Here, we use an upper index "0" if the firm refrains from formal advertising. Next, consider the present value of a filled job $J(p)$, which can be written as:

$$rJ(p) = bp - w(p) - \delta(J(p) - V(p)) \quad \Rightarrow \quad J(p) - V(p) = \frac{bp - rV(p) - w(p)}{r + \delta}.$$

The variable $w(p)$ is the wage and $bp - w(p)$ is the firm's flow profit. At rate δ, the job is destroyed and the firm begins to search for a new worker, which yields the present value $V(p)$. Further, recall that $\bar{w}$ denotes the reservation wage of the worker and β is the bargaining power. Nash bargaining over the wage between the firm and the worker can be written as:

$$\max_{w(p)} (w(p) - \bar{w})^{\beta} (bp - rV(p) - w(p))^{1-\beta} \quad \Rightarrow \quad w(p) = \beta(bp - rV(p)) + (1 - \beta)\bar{w}.$$

[2] The underlying assumption here is that the total output of the firm is linear in the number of employees and the total hiring cost is linear in the number of vacancies, see Pissarides (2000) for a more detailed treatment of this issue.

Later, we will show that the term $bp - rV(p)$ is increasing in the productivity p. Therefore, our wage equation indicates that more productive firms pay higher wages. Insert this wage equation into the firm rent $J(p) - V(p)$ to get:

$$J(p) - V(p) = (1-\beta)\frac{bp - rV(p) - \bar{w}}{r+\delta}.$$

To simplify the notation, let $y \equiv bp - \bar{w}$ denote the net flow output. Note that the job surplus $J(p) - V(p)$ can be expressed as a function of y. Thus, in the following, we will use this linear transformation for the ease of notation and suppress the dependence of all firm-specific variables on p. Next, consider firms with a positive advertising intensity and insert the job surplus into Equation (3) to obtain:

$$(r+\phi)V(y) = -c + 0.5c_0 q^2(\theta)(1-\beta)^2\frac{(y - rV(y))^2}{(r+\delta)^2} + q^N(u)(1-\beta)\frac{(y - rV(y))}{(r+\delta)}. \tag{5}$$

This is a quadratic equation in $V(y)$ for advertising firms that can be rewritten as:

$$A(\theta)r^2V^2(y) - V(y)[2rA(\theta)y + rB(u) + r + \phi] + A(\theta)y^2 + B(u)y - c = 0, \tag{6}$$

(6)

$$\text{where} \qquad A(\theta) \equiv \frac{c_0 q^2(\theta)(1-\beta)^2}{2(r+\delta)^2} \qquad B(u) \equiv \frac{q^N(u)(1-\beta)}{(r+\delta)}. \tag{6}$$

Here, we use auxiliary variables $A(\theta)$ and $B(u)$ to simplify the notation. Note that $A'(\theta) < 0$, whereas $B'(u) > 0$. We find the following solution to this quadratic equation:

$$rV(y) = y - \frac{1}{2A(\theta)r}\left(\sqrt{(rB(u) + r + \phi)^2 + 4rA(\theta)[y(r+\phi) + rc]} - (rB(u) + r + \phi)\right). \tag{7}$$

PROPOSITION 1. *The present value of an open vacancy $V(y)$ is increasing in the net output y but decreasing in the fixed cost c. $V(y)$ is increasing in the auxiliary variables $A(\theta)$ and $B(u)$, which means that $V(y)$ is increasing in $q(\theta)$ and $q^N(u)$. Moreover,* $\lim_{\theta\to 0} rV(y) = y$ *and:*

$$\frac{\partial V(y)}{\partial y} > \frac{B(u)}{rB(u) + r + \phi}.$$

PROOF. Appendix A.1. □

This proposition and the above equation for $rV(y)$ imply that more productive firms expect to get a higher present value of profits in the future. At the same time, this present value is increasing in the job-filling rates $q(\theta)$ and $q^N(u)$. The easier it is to hire workers, the higher is the expected present value of profits. At the same time, note that $V(p)$ is increasing in u but decreasing in θ because $q'(\theta) < 0$. On the one hand, if unemployment u is higher, firms can easier hire workers by referrals. On the other hand, a higher market tightness θ implies more competition between firms in the search for applicants, which reduces the hiring chances of every firm. Note also that $y - rV(y)$ is increasing in y, which proves that more productive firms pay higher wages.

Another result in Proposition 1 concerns the case when θ is converging to 0. This is the limiting case when search frictions become negligible because $q(\theta) \to \infty$. In this situation, firms can hire workers immediately, so $V(y) = y/r$, which means that workers receive their reservation wage $\bar{w}$ and do not have any effective bargaining power.

Our findings regarding the optimal search intensity $a(y)$ are summarized in Proposition 2:

PROPOSITION 2. *The optimal search intensity of advertising firms $a(y)$ is given by:*

$$a(y) = \frac{r+\delta}{rq(\theta)(1-\beta)}\left(\sqrt{(rB(u)+r+\phi)^2 + 4rA(\theta)[y(r+\phi)+rc]} - (rB(u)+r+\phi)\right),$$

$a(y)$ is increasing in y and c. It is also increasing in $q(\theta)$ but decreasing in $q^N(u)$.

PROOF. Appendix A.2. □

Proposition 2 shows one of our main results. Considering firms that use formal advertising ($a(y) > 0$), we can see that more productive firms exert more formal search effort. This is because more productive firms face higher foregone profit if the position is not filled, so their incentives to search formally are stronger. Also, we can see that firms advertise more if the fixed cost component c is higher. At the same time, our model predicts that $a(y)$ is decreasing in $q^N(u)$. This shows that there is substitution between the two channels. Firms reduce their search effort if hiring by referrals becomes easier. On the other hand, $a(y)$ is increasing in $q(\theta)$. If the formal channel is more efficient, it becomes more attractive to firms and leads to more search effort $a(y)$.

So far, we could show that the firm will advertise its vacancy only if the expected profit gain is larger than the cost of advertising (see condition 2). From Proposition 1, we already know that the expected profit of a filled job is increasing in the output of this job, that is, $(1-\beta)(y - rV(y))/(r+\delta)$ is increasing in y. This means that the gain from advertising a vacancy is increasing in y; therefore, there exists an output cutoff y_c such that the firm is indifferent between advertising or not. This cutoff output is given by the following indifference condition:

$$\frac{0.5c_0q^2(\theta)(1-\beta)^2(y_c - rV(y_c))^2}{(r+\delta)^2} = c \quad \Rightarrow \quad y_c = \frac{\sqrt{c}(rB(u)+r+\phi)}{(r+\phi)\sqrt{A(\theta)}}, \qquad \frac{\partial y_c}{\partial u} > 0 \quad \frac{\partial y_c}{\partial \theta} > 0.$$

We obtained this condition after inserting the present value of an open vacancy $V(y)$ and solving for y_c (note that our linear transformation implies that $y_c = bp_c + \bar{w}$). This equation shows that firms with output y larger than y_c find it optimal to advertise their vacancies in a formal way in addition to referral hiring. Recall that $y_0 = bp_0 - \bar{w}$ is output of the least productive firm in the market. So, if the cutoff output y_c is larger than this output y_0, then firms with output values in the range $y_0 < y < y_c$ are present in the market but they will not formally advertise their position; thus, they rely exclusively on referral hiring, which gives rise to equilibrium I. In contrast, if the cutoff output y_c is smaller than the output of the least productive firm y_0, then all firms advertise their vacancies, which gives rise to equilibrium II. In the next two sections, we characterize each of these equilibria separately. Note that the two reservation outputs are different in the two equilibria, so we use notation y_0^I for the first equilibrium and y_0^{II} for the second equilibrium.

2.3. *Equilibrium I.* We already know that in equilibrium I, low productivity firms do not advertise their positions. This holds for firms in the output range $y_0^I < y < y_c$. So, the necessary condition for this equilibrium to arise is $y_0^I < y_c$. To characterize this condition, we need to derive a formal expression for y_0^I. Let V denote the discounted present value of profits for vacancies expecting their type p to be assigned (present value of waiting). Productivity draws are uniformly distributed in [0..1] with a cdf function $F(p) = p$. This also implies that the net output per vacancy y is uniformly distributed in the range $[-\bar{w}..b - \bar{w}]$ with a cdf function y/b.

Based on this linear transformation, the present value of an open vacancy V becomes:

$$(8)\quad rV = \gamma \int_{-\bar{w}}^{b-\bar{w}} \max(V(y) - V, 0)\frac{1}{b}dy = \frac{\gamma}{b}\int_{y_0^I}^{y_c}(V^0(y) - V)dy + \frac{\gamma}{b}\int_{y_c}^{1}(V(y) - V)dy.$$

This equation takes into account that more productive firms use both search channels, so their present values of open vacancies are given by $V(y)$ described in Proposition 1. In contrast, less productive firms are not advertising their positions, their present values of open vacancies are given by $V^0(y)$ (see Equation (4)). Rewriting this equation by inserting the job surplus and solving for $V^0(y)$ yields:

$$rV^0(y) = \frac{rB(u)y}{rB(u) + r + \phi} \quad \Rightarrow \quad \frac{\partial V^0(y)}{\partial y} = \frac{B(u)y}{rB(u) + r + \phi} < \frac{\partial V(y)}{\partial y}.$$

This shows that the curve $V^0(y)$ is linearly increasing in y. From an economic perspective, this means that more productive firms expect higher profits irrespective of whether they are advertising their positions or not. At the same time, the curve $V^0(y)$ is flatter than the curve $V(y)$ for all positive y. Given that $V^0(y)$ is increasing in y, there exists a cutoff value y_0^I such that $V^0(y_0^I) = V$, which describes another indifference condition for firms. Intuitively, it means that low-output realizations below y_0^I are insufficient to cover the entry cost of a vacancy. Therefore, in the equilibrium, no firm exists with such low-output values. Free entry implies that vacancies enter the market as long as they are profitable for firms, that is, $V \geq K$. Thus, in the equilibrium, we should have $V = K$, which also means $V^0(y_0^I) = K$. This allows us to find the reservation output cutoff y_0^I:

$$y_0^I = \frac{K(rB(u) + r + \phi)}{B(u)} > rK \quad \Rightarrow \quad \frac{\partial y_0^I}{\partial u} < 0.$$

We summarize our findings in the following proposition:

PROPOSITION 3. *Consider a given tuple $\{\theta, u\}$. Then there exists a partial equilibrium (PEI) of the model such that low-productivity firms with $y_0^I < y < y_c$ rely exclusively on referral hiring, whereas more productive firms with $y > y_c$ use both search channels simultaneously. The necessary conditions for this partial equilibrium are denoted by (R), ($\tilde{R}$):*

$$(R)\qquad \frac{\sqrt{c}}{\sqrt{A(\theta)(r+\phi)}} > \frac{K}{B(u)} \qquad (\tilde{R})\qquad \frac{\sqrt{c}(rB(u) + r + \phi)}{(r+\phi)\sqrt{A(\theta)}} < b - \bar{w}.$$

The first condition (R) requires that $y_0^I < y_c$, whereas the second condition $(\tilde{R})$ implies $y_c < b - \bar{w}$. Note that the highest productivity realization in our setting is $p = 1$, so the highest net output of the most productive firm is $b - \bar{w}$. For given values of the unemployment rate u and the market tightness θ, it is possible to find parameters such that $y_c > b - \bar{w}$. This would lead to a third partial equilibrium where no firm is advertising its vacancies ($PE0$). However, further below, we find that this situation cannot happen in the general equilibrium when u and θ are endogenous. Intuitively, if less and less firms use formal advertising, then open vacancies created via the formal channel will sharply decrease ($\tilde{v} \to 0$), which implies that the market tightness will decrease; thus, $\theta \to 0$, and therefore, $A(\theta) \to \infty$. Note that $A(\theta)$ is in the denominator of y_c, which implies that the second condition is always satisfied. Therefore, in the following, we focus on the first condition.

Figure 2 illustrates equilibrium I in terms of the net output y that is measured on the horizontal axis. Both present value curves $V^0(y)$ and $V(y)$ are increasing in y; however, the curve

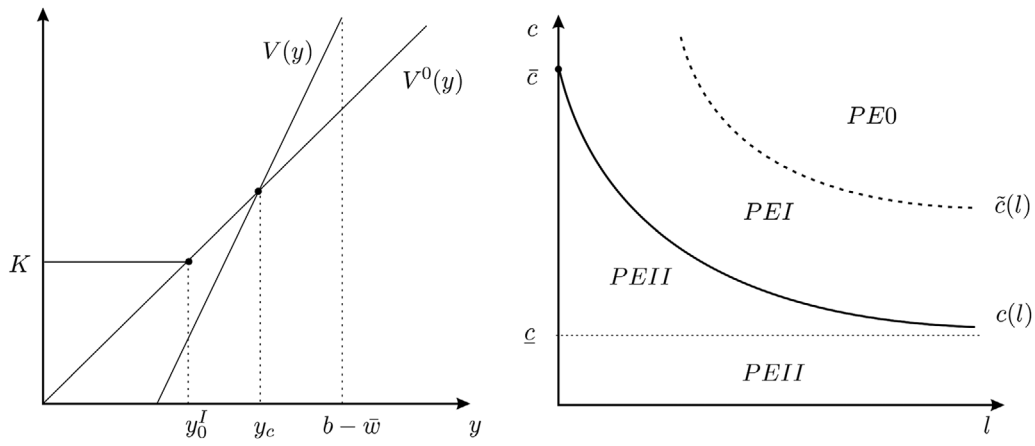


FIGURE 2

LEFT PANEL: PARTIAL EQUILIBRIUM I (*PEI*). RIGHT PANEL: PARTIAL EQUILIBRIA *PE*0, *PEI*, AND *PEII* DEPENDING ON THE FIXED COST COMPONENT c AND THE SIZE OF SOCIAL NETWORKS l.

$V(y)$ is steeper and cuts the horizontal axis at some positive value because $V(0) < 0$. In contrast, we know that $V^0(0) = 0$, so the curve $V^0(y)$ starts at the origin. The reservation output cutoff y_0^I is obtained at the point $V^0(y_0^I) = K$ and indicates that lower productivities do not cover the entry cost K. For firms in the range $y_0^I < y < y_c$, we can see that $V^0(y) > V(y)$. Thus, the costs of advertising are too high and these firms are better off without advertising. The firm with $y = y_c$ is exactly indifferent between advertising or not since $V(y_c) = V^0(y_c)$. For more productive firms with $y > y_c$, it holds that $V(y) > V^0(y)$. The cost of advertising is relatively low for these firms since their output is sufficiently high and these firms are using both search channels then.

Next, consider a special case $y_c = y_0^I$. This equality can be rewritten and expressed as:

$$c(l) = \frac{0.5c_0 q^2(\theta) K^2 (r+\phi)^2}{s^2 (1-(1-u)^{l+1})^2}. \tag{9}$$

Therefore, the lowest cost c, necessary for equilibrium I to prevail, depends on the network size l; thus, $y_c > y_0^I$ if $c > c(l)$. The boundary $c(l)$ decreases with the size of the network in a given market situation $\{\theta, u\}$. If social networks are small (low l), then referral hiring has low efficiency, that is, $q^N(u)$ is low, then firms have strong incentives to advertise their vacancies. Then equilibrium I will only prevail if the fixed cost of advertising c is relatively high, which means that the costs are too high for less productive firms. In contrast, if social networks are large (high l) and referral hiring generates many job applicants ($q^N(u)$ is high), then firms have low incentives to use formal advertising and even a low cost c may generate equilibrium I, where some firms rely exclusively on referral hiring. This explains the downward-sloping curve $c(l)$, which is illustrated on the right panel of Figure 2. Note that Equation (9) has a lower and an upper limit:

$$\underline{c} = \lim_{l\to\infty} \frac{0.5c_0 q^2(\theta) K^2 (r+\phi)^2}{(su)^2} \qquad \bar{c} = \lim_{l\to 0} \frac{0.5c_0 q^2(\theta) K^2 (r+\phi)^2}{s^2}.$$

This means that equilibrium I does not exist for very low costs $c < \underline{c}$ irrespective of the network size l. This is plausible, because if costs are so low, then all firms find it optimal to advertise their positions, which leads to partial equilibrium II (*PEII*). We describe this equilibrium

in the next section. For higher values of c such that $\underline{c} < c$, the type of equilibrium depends directly on the network size l. Only if the network size is sufficiently large and referral hiring is efficient, some firms will avoid the cost of advertising giving rise to equilibrium I. Note that the necessary condition $(\tilde{R})$ leads to another boundary $\tilde{c}(l)$ such that:

$$\tilde{c}(l) = \frac{(b-\bar{w})(r+\phi)\sqrt{A(\theta)}}{rB(u)+r+\phi} > c(l) \quad \text{for} \quad K < \frac{(b-\bar{w})B(u)}{rB(u)+r+\phi}.$$

This implies that, for a given market situation $\{\theta, u\}$, a third partial equilibrium ($PE0$ on the figure) arises in which no firm is using formal advertising. This equilibrium would appear for $c > \tilde{c}(l)$. However, in the next section, we will show that this situation is not compatible with a general equilibrium since the boundary $\tilde{c}(l)$ becomes infinite once the market tightness θ becomes endogenous. Overall, Figure 2 shows that equilibrium I is more likely to appear if parameters c and l are relatively large; that is, the fixed cost component is too high for some firms and referral hiring alone generates sufficiently many applicants.

Next we analyze the size of firms with different productivities. Recall that v_0 is a number of vacancies waiting for their type to be assigned, so that $\gamma v_0 f(p)$ is a number of vacancies receiving type p per unit time (see Figure 1). With a uniform distribution of productivities, we have $f(p) = 1$ for $0 < p < 1$. From above, we already know that high productivity vacancies with $y > y_c$ are matched with workers at rate $a(y)q(\theta) + q^N(u)$. At the same time, all vacancies can be abolished without hiring and exit the market at rate ϕ. This gives rise to the following system of dynamic equations for $v(y)$ and $e(y)$ (here again, we suppress the dependence of y on p for the ease of notation):

$$\dot{v}(y) = \gamma v_0 + \delta e(y) - (a(y)q(\theta) + q^N(u))v(y) - \phi v(y) \quad \text{if} \quad y > y_c,$$
$$\dot{e}(y) = (a(y)q(\theta) + q^N(u))v(y) - \delta e(y) \quad \text{if} \quad y > y_c.$$

The term $\gamma v_0 + \delta e(y)$ is an inflow of vacancies to firm y at every point in time. γv_0 are completely new vacancies available for the first time and $\delta e(y)$ are vacancies that became open after a worker displacement. Further, the term $(a(y)q(\theta) + q^N(u))v(y) + \phi v(y)$ is the outflow of vacancies from firm y. It consists of new hires $(a(y)q(\theta) + q^N(u))v(y)$ and abandoned vacancies $\phi v(y)$. The second equation shows changes in the firm size $e(y)$, where the inflow of workers consists of newly hired employees $(a(y)q(\theta) + q^N(u))v(y)$ and the outflow consists of displaced workers $\delta e(y)$. In the steady state, it should be the case that $\dot{v}(y) = 0$ and $\dot{e}(y) = 0$ $\forall y > y_c$, which yields $v(y) = \gamma v_0/\phi$. The size of more productive firms is then given by:

$$e(y) = (a(y)q(\theta) + q^N(u))\frac{v(y)}{\delta} = (a(y)q(\theta) + q^N(u))\frac{\gamma v_0}{\phi\delta} \quad \text{for} \quad y > y_c. \tag{10}$$

This equation shows that more productive firms invest more resources into formal recruitment, which makes them more successful in hiring. In the long run, these firms grow faster and become larger in the steady state, so that $\partial e(y)/\partial y > 0$ for $y > y_c$. Thus, our model predicts a positive relationship between productivities, wages, formal advertising, and the firm size.

Next consider low productivity firms. Their vacancies are filled exclusively by referral hiring and are matched with workers at rate $q^N(u)$, so the dynamic equations for $v(y)$ and $e(y)$ are given by:

$$\dot{v}(y) = \gamma v_0 + \delta e(y) - q^N(u)v(y) - \phi v(y) \quad \text{if} \quad y_0^I < y < y_c,$$
$$\dot{e}(y) = q^N(u)v(y) - \delta e(y) \quad \text{if} \quad y_0^I < y < y_c.$$

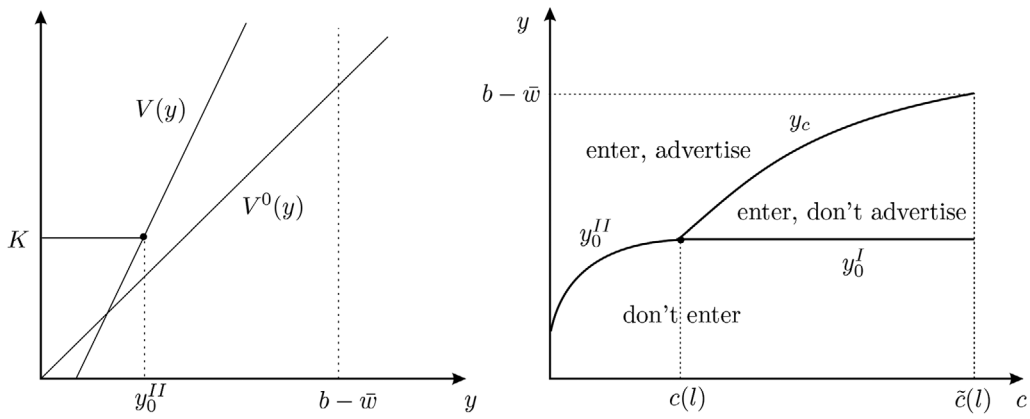


FIGURE 3

LEFT PANEL: PARTIAL EQUILIBRIUM II (*PEII*).RIGHT PANEL: ENDOGENOUS BOUNDARIES y_0^I, y_c, AND y_0^{II} DEPENDING ON THE FIXED COST COMPONENT c.

In the steady state, it should again be the case that $\dot{v}(y) = 0$ and $\dot{e}(y) = 0$, which yields $v(y) = \gamma v_0/\phi$. The size of less productive firms is then given by:

$$e(y) = q^N(u)\frac{v(y)}{\delta} = q^N(u)\frac{\gamma v_0}{\phi\delta} \quad \text{for} \quad y_0^I < y < y_c.$$

We can see that all these firms have the same employment size that is smaller than the size of advertising firms. Hence, our model predicts that nonadvertising firms grow slowly and remain small in the steady state. Moreover, small firms use referral hiring as the only search channel, whereas larger firms use both search channels simultaneously.

2.4. *Equilibrium II.* Equilibrium II arises when condition (R) is not satisfied, in this case, the fixed cost of advertising a vacancy is sufficiently small $(c < c(l))$, so that all firms find it optimal to advertise their vacancies to some extent. This equilibrium is illustrated on the left panel of Figure 3. We can see that the intersection between the two present value curves $V^0(y)$ and $V(y)$, giving rise to the output cutoff y_c, is substantially below the reservation output y_0^{II}, so it is not relevant in equilibrium II. Moreover, we can see that $V(y) > V^0(y)$ in the whole range $[y_0^{II}..b - \bar{w}]$. Thus, all firms are better off by advertising their vacancies. The least productive firm in the market can then be found from the following condition $V(y_0^{II}) = K$, which means that this firm is indifferent between paying the entry cost K combined with obtaining an expected present value of profits $V(y_0^{II})$ and not entering the market at all. In addition, the present value of open vacancies before their productivity type is assigned is given by:

$$rV = \gamma \int_{-\bar{w}}^{b-\bar{w}} \max[(V(y) - V), 0]\frac{1}{b}dy = \frac{\gamma}{b}\int_{y_0^{II}}^{b-\bar{w}} (V(y) - V)dy.$$

To obtain this condition, we rely on our result from proposition 1 that the present value of an open vacancy $V(y)$ is increasing in y, which generates a unique reservation productivity y_0^{II} based on the boundary condition $V(y_0^{II}) = K$. Given Equation (7), this boundary condition leads to the following quadratic equation:

$$A(\theta)y_0^2 - y_0(2KrA(\theta) - B(u)) - K(rB(u) + r + \phi) + A(\theta)r^2K^2 - c = 0,$$

Solving this equation for y_0, we can find the reservation productivity in the second equilibrium y_0^{II}. Our results are summarized in the following proposition:

PROPOSITION 4. *Consider a given tuple $\{\theta, u\}$. Then there exists a partial equilibrium (PEII) of the model such that all firms with output in the range $[y_0^{II}..b - \bar{w}]$ open new vacancies and advertise them formally in addition to referral hiring. This partial equilibrium arises if condition (R) is violated. The reservation output of firms y_0^{II} is given by:*

$$(11)\quad y_0^{II} = rK + \frac{(r+\delta)}{c_0 q^2(\theta)(1-\beta)}\left(\sqrt{(q^N(u))^2 + 2c_0 q^2(\theta)[K(r+\phi)+c]} - q^N(u)\right) > rK.$$

This function is strictly decreasing in $q(\theta)$ and $B(u)$, which means that y_0^{II} is increasing in θ but decreasing in u. Moreover, $\lim_{\theta\to 0} y_0^{II} = rK$.

PROOF. Appendix A.3. □

Proposition 4 shows that the reservation cutoff y_0^{II} is increasing in the cost c, so that a higher advertising cost reduces the number of active firms in the market. This is different from equilibrium I where the reservation cutoff y_0^I does not directly depend on the cost c. The reason is that in the first equilibrium, low-productivity firms do not advertise their vacancies, and therefore, their entry decision does not depend on the cost c. This is illustrated on the right panel of Figure 3.

Figure 3 illustrates and compares the three threshold variables y_0^I, y_c, and y_0^{II} for different values of the advertising cost c. For low values of the cost parameter such that $c < c(l)$, low-productivity firms with $y < y_0^{II}$ do not enter the market, whereas all firms with output values above y_0^{II} enter and use both search channels simultaneously. Moreover, the reservation output y_0^{II} is positive even when $c = 0$ and it is increasing in c meaning that in equilibrium II, less and less firms enter the market when the advertising cost is higher. For the special case $c = c(l)$, both partial equilibria coexist and several conditions are satisfied at the same time. In the first equilibrium, we have the reservation output condition $V^0(y_0^I) = K$ and the advertising indifference condition $V^0(y_0^I) = V(y_c)$. In addition, we know that $y_0^I = y_c$ for $c = c(l)$. In the second equilibrium, we have the reservation output condition $V(y_0^{II}) = K$. So, we get:

$$\text{if}\quad c = c(l)\quad \text{then}\quad V(y_0^{II}) = K = V^0(y_0^I) = V(y_c)\quad \Rightarrow\quad y_0^{II} = y_c.$$

Hence, all three cutoff values coincide, that is, $y_0^{II} = y_0^I = y_c$. This means that in this special case, firms with output $y = y_0^{II} = y_0^I = y_c$ are indifferent between entering or not and, given the market entry, between advertising or not. For larger values of the cost $c > c(l)$, the reservation output y_0^I remains constant since it does not depend on c but the threshold y_c is increasing in c (y_c is proportional to the square root of c). This means that the number of entering firms remains constant in equilibrium I, but the number of advertising firms is decreasing with the higher advertising cost c.

Note that $y_0^{II} > rK$ implies that all workers optimally accept all jobs in this economy. To see this, note that $y_0^{II} > rK$ is equivalent to:

$$y_0^{II} = bp_0^{II} - \bar{w} > rV(bp_0^{II} - \bar{w}) = rK \Leftrightarrow bp_0^{II} - rV(bp_0 - \bar{w}) > \bar{w} \Leftrightarrow w(p_0^{II}) > \bar{w}.$$

This equation implies that even the least productive firm in the market is able to pay a wage that is higher than the reservation wage of workers. Thus, the optimum for workers is to accept all jobs. Intuitively, this is because firms pay an entry cost K in this market, which makes them relatively selective, so the threshold productivity p_0^{II} is sufficiently high. In other words, unproductive firms that cannot even pay the reservation wage to workers do not enter the

market. The same holds in equilibrium I where we could show that $y_0^I > rK$ meaning that $w(p_0^I) > \bar{w}$.

Finally, we consider the firm size. Since all firms are advertising their vacancies in equilibrium II, the steady-state firm size is given by:

$$e(y) = (a(y)q(\theta) + q^N(u))\frac{v(y)}{\delta} = (a(y)q(\theta) + q^N(u))\frac{\gamma v_0}{\phi\delta} \quad \text{for} \quad y > y_0^{II}.$$

Here, again, we can see that more productive firms spend more resources on formal recruitment, grow faster, and become larger in the steady state. This implies a positive relationship between the firm size and the intensity of formal advertising.

2.5. *The Market Tightness.* In this section, we analyze how the market tightness is determined in each of the equilibria. Note that all endogenous variables in the previous sections depend on the tuple $\{\theta, u\}$, even though we suppressed this dependence so far for the ease of notation. In this section, we use extended notation for the present values of open vacancies $V(y, \theta, u)$ and $V^0(y, u)$. We already know that both present values are increasing with a higher unemployment rate u because a larger pool of unemployed individuals makes network hiring faster and more efficient. $V^0(y, u)$ does not depend on the market tightness θ since it is a present value of firms that do not advertise their vacancies. In contrast, $V(y, \theta, u)$ is decreasing in θ because the probability of formal hiring is lower when the market is tighter and more competitive from the perspective of firms.

Note that the present value of waiting $V(\theta, u)$ also depends on the tuple $\{\theta, u\}$. So, the equilibrium market tightness θ can be found from the free-entry condition $V(\theta, u) = K$, which says that firms open vacancies up to the point where the expected gain of opening a new vacancy $V(\theta, u)$ is equal to the entry cost K. This allows us to rewrite Equation (8) in equilibrium I in the following way:

$$rV(\theta, u) = \frac{\gamma}{b}\int_{y_0^I(u)}^{y_c(\theta,u)} (V^0(y, u) - K)dy + \frac{\gamma}{b}\int_{y_c(\theta,u)}^{b-\bar{w}} (V(y, \theta, u) - K)dy, \tag{12}$$

Note that the cutoff values $y_c(\theta, u)$ and $y_0^I(u)$ also depend on the market situation. In particular, $y_c(\theta, u)$ is increasing in the unemployment rate u, whereas $y_0^I(u)$ is decreasing in u, which means that in the times of high unemployment, fewer firms advertise their positions in a formal way and more firms rely on referral hiring as the only search channel. The reason is that network hiring is relatively efficient when the unemployment rate u is high. Proposition 5 below shows that the present value of waiting $V(\theta, u)$ is decreasing in θ, so if there exists a solution to equation $K = V(\theta, u)$, then it is unique.

Next, consider the free-entry condition $V(\theta, u) = K$ in equilibrium II:

$$rV(\theta, u) = \frac{\gamma}{b}\int_{y_0^{II}(\theta,u)}^{b-\bar{w}} (V(y, \theta, u) - K)dy. \tag{13}$$

Note here that the lower output boundary $y_0^{II}(\theta, u)$ is increasing in the market tightness θ, which means that stronger competition in formal advertising between firms reduces the number of firms or vacancies in the market. Again, Proposition 5 proves that the present value of waiting $V(\theta, u)$ in equilibrium II is decreasing in θ, so if there exists a solution to Equation (13), then this solution is unique.

Figure 4 is illustrating our results. Consider a very high value of θ such that $y_c(\theta, u) > y_0^I(u)$, which implies that the labor market is in equilibrium I. If we now reduce the market tightness, the cutoff $y_c(\theta, u)$ decreases. This means that more and more firms use formal advertising in addition to referral hiring. For some θ, we will reach a situation when $y_c(\theta, u) = y_0^I(u)$ which

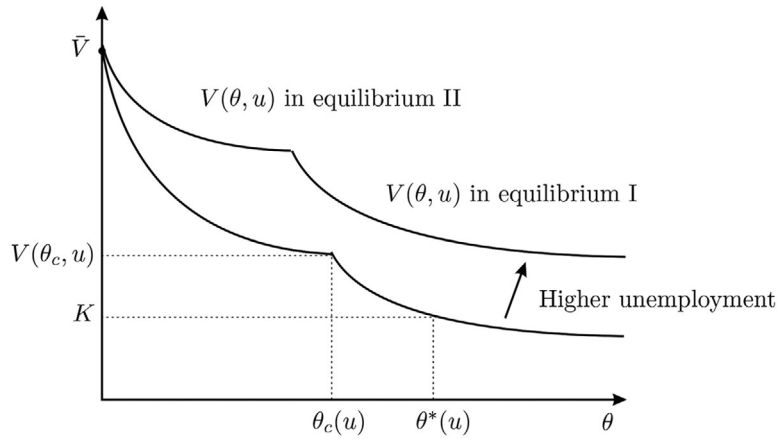


FIGURE 4

SOLUTION TO THE FREE-ENTRY CONDITION $K = V(\theta, u)$ FOR LOW AND HIGH UNEMPLOYMENT

is the equilibrium switching point. We denote the corresponding market tightness by θ_c, then we get:

$$y_c(\theta_c, u) = y_0^I(u) = y_0^{II}(\theta_c, u) \quad \Rightarrow \quad q(\theta_c) = \frac{q^N(u)\sqrt{c}}{\sqrt{0.5c_0}K(r+\phi)} \quad \Rightarrow \quad \theta_c(u).$$

At this point, the market is moving from equilibrium I to equilibrium II and the present value of open vacancies in both equilibria is given by $V(\theta_c, u)$. Next, we analyze what happens if the market tightness converges to 0 in the limit in equilibrium II. From Proposition 1, we know that $\lim_{\theta\to 0} rV(y, \theta, u) = y$, and from Proposition 4, we know that $\lim_{\theta\to 0} y_0^{II} = rK$. This leads to the following expression for $\lim_{\theta\to 0} V(\theta, u)$ based on Equation (13):

$$\lim_{\theta\to 0} rV(\theta, u) = \lim_{\theta\to 0} \frac{\gamma}{b} \int_{rK}^{b-\bar{w}} \left(\frac{y}{r} - K\right) dy = \frac{\gamma}{b}\frac{0.5}{r}(b - \bar{w} - rK)^2 \equiv r\bar{V}.$$

Intuitively, this is the highest expected present value of profits from opening a vacancy in the market, let it be denoted by $r\bar{V}$. This directly implies that there is no equilibrium in our model if the entry cost K is larger than the maximum expected profit $\bar{V}$ since in this case, there is no solution to the free-entry condition $K = V(\theta, u)$. We summarize these results in Proposition 5:

PROPOSITION 5. *For $K < \bar{V}$, the market tightness $\theta^*(u)$ is uniquely determined by the free-entry condition $K = V(\theta, u)$. For $K < V(\theta_c, u)$, equilibrium I prevails and the present value $V(\theta, u)$ is given by Equation (12); for $V(\theta_c, u) < K < \bar{V}$, equilibrium II prevails and the present value $V(\theta, u)$ is given by Equation (13). In both cases, $\partial\theta^*(u)/\partial u > 0$. For $K > \bar{V}$, there is no equilibrium.*

PROOF. Appendix A.4. □

Given the results in Proposition 5, we can determine the equilibrium unemployment and analyze the impact of unemployment on the endogenous variables in the model. However, the focus of our article is on the cross-sectional heterogeneity of firms, so we continue by summarizing the cross-sectional predictions of our model in the next section. At the same time, the business cycle properties of the model could be interesting and particularly unemployment is

one of the most frequently used indicators of the business cycle. That is why we decided to present these properties in Appendix A.5.

2.6. *Summary of Model Predictions.* In this section, we summarize a number of predictions from our model. As we know from above, equilibrium I captures stronger heterogeneity of firms, so we use the results obtained from this equilibrium to summarize our predictions. As we show later in the empirical part of the study, this equilibrium selection is supported by the data since there is a substantial number of firms that do not use the formal search channel. First, our model indicates that small firms do not use formal advertising and rely on referral hiring for filling their positions. The reason is that small firms are less productive and the fixed cost of advertising c is too high for them. In contrast, larger firms find it optimal to advertise their vacancies. These firms are more productive and can afford to pay a higher cost; moreover, the foregone profit from open vacancies is higher for more productive firms, so they have stronger incentives to make their vacancies visible to workers. We formulate these results as hypothesis H0:

> **Hypothesis H0**: *Smaller firms with output in the range $y_0^I < y < y_c$ and total employment equal to $e(y_c)$ do not use formal advertising and rely exclusively on referral hiring. Larger firms with employment $e(y) > e(y_c)$ use both search channels—formal advertising and referral hiring.*

Next, we consider large firms with output values $y > y_c$ and employment $e(y) > e(y_c)$. All these firms are advertising their vacancies; furthermore, from Proposition 2, we know that their search effort a is increasing in y, that is, $da/dy > 0$. Given that in our empirical data, we observe wages but not outputs, we reformulate this result in terms of wages for the purpose of testing. Recall that the wage equation is given by $w(y) = \beta(y - rV(y)) + \bar{w}$. From Proposition 1, we know that the term $y - rV(y)$ is increasing in y, which means that more productive firms pay higher wages: $dw/dy > 0$. Next, we want to understand the relationship between the firm size and wages predicted by our model, so we differentiate the firm size e with respect to w for the group of advertising firms based on Equation (10), we get:

$$\frac{de}{dw} = \frac{\partial e}{\partial a} \cdot \frac{\partial a}{\partial w} > 0, \quad \text{because} \quad \frac{\partial e}{\partial a} = \frac{\gamma v_0 q(\theta)}{\phi \delta} > 0 \quad \text{and} \quad \frac{\partial a}{\partial w} = \frac{da/dy}{dw/dy} > 0.$$

This equation shows that more productive firms pay higher wages and invest more resources into formal recruitment. This makes more productive firms more successful in hiring workers than less productive firms. So, these firms grow faster and become larger in the equilibrium. This means the model describes a positive relationship between the firm size e and advertising intensity a combined with a positive relationship between the firms size e and wages w. We summarize the former part of this finding in hypothesis H1, whereas all the findings concerning wages are summarized and discussed in the end of this section.

> **Hypothesis H1**: *Considering the group of firms with $a > 0$, larger firms invest more effort into formal search activities than smaller ones, that is, $\partial a/\partial e > 0$.*

Next notice that firm's output $y = bp - \bar{w}$ not only depends on the firm's type p but it also depends on the type of the worker b. That is, firms face even higher foregone profits in labor markets for more educated workers, so they invest more into formal job search in these markets. We summarize this finding in hypothesis H2.

> **Hypothesis H2**: *Considering the group of firms with $a > 0$, firms invest more formal search effort in labor markets for more educated workers, that is, $\partial a/\partial b = p(da/dy) > 0$.*

Suppose now that some firms do not use referrals as a search channel, that is, $s = 0$. This could be an explicit choice of the firm or an institutional restriction. Their search intensity via

the formal search channel then becomes:

$$a = \frac{r+\delta}{rq(\theta)(1-\beta)}\left(\sqrt{(r+\phi)^2 + 4rA(\theta)[y(r+\phi)+rc]} - (r+\phi)\right) \quad \Rightarrow \quad \frac{\partial a}{\partial w} = \frac{da/dy}{dw/dy} > 0,$$

which means that even if referrals are not used by the firm, there is still a positive relationship between output, wages, and formal search intensity. Moreover, the employment size of these firms is given by:

$$e = aq(\theta)\frac{\gamma v_0}{\phi\delta} \quad \Rightarrow \quad \frac{\partial e}{\partial a} = \frac{\gamma v_0 q(\theta)}{\phi\delta} > 0,$$

so again, there is a positive relationship between the advertising intensity a and the firm size e. We formulate this prediction in hypothesis H3:

> **Hypothesis H3**: *The positive relationship between the firm size and formal search intensity should be observed even for firms that do not use referrals, that is*, $\partial a/\partial e = \phi\delta/(\gamma v_0 q(\theta)) > 0$ *even if* $s = 0$.

Next, Proposition 2 shows that $a(y)$ is decreasing in $q^N(u)$. This means that firms that use referrals as a search channel invest less effort into formal search activities compared to firms that do not use referrals. To put it in other words, there is substitution between these two channels. We summarize this prediction in hypothesis H4.

> **Hypothesis H4**: *Firms that use referrals as a search channel invest less into formal search compared to firms that do not use referrals, that is, the formal search intensity a is lower when* $s = 1$ *compared to firms with* $s = 0$.

So far, we have focused on the ex ante characteristics of hiring. Next, we consider firms that use both search channels and calculate a fraction of workers hired by referrals in these firms, denoted by F. Thus, this fraction refers to an ex post perspective on hiring; F is given by:

$$F = \frac{q^N(u)}{aq(\theta) + q^N(u)} \quad \Rightarrow \quad \frac{\partial F}{\partial a} < 0 \quad \Rightarrow \quad \frac{dF}{de} = \frac{\partial F/\partial a}{\partial e/\partial a} < 0.$$

Since larger firms invest more into the formal channel (i.e., a is increasing in e), the fraction of workers hired by referrals should be decreasing with the firm size. This result is our next hypothesis.

> **Hypothesis H5**: *Considering the group of firms with* $a > 0$, *larger firms are less likely to hire an applicant by referral than smaller firms, that is*, $dF/de < 0$.

Even though our model differs considerably from the model by Galenianos (2013), both models conclude that low-productivity firms exhibit greater prevalence of referrals than high productivity firms (Proposition 3.4). In particular, the model in Galenianos (2013) is based on the perfect substitution mechanism assuming that more productive firms (i.e., larger firms) invest more effort into the formal search channel but at the same time reduce their search intensity through the referral channel. Therefore, the decision of firms to invest more effort into the formal channel does not affect the total number of matches or applicants, respectively. In contrast, our model implies that larger firms invest more formal search effort into the hiring process and, on average, get a larger number of applications per vacancy per unit time than smaller firms. Indeed, in our model, firms also increase the proportion of applications from the formal channel by increasing their search effort a. But the difference to Galenianos (2013) is that the absolute number of applicants is allowed to change, because firms do not necessarily reduce their search effort associated with the referral channel. Recall that $aq(\theta) + q^N(u)$ is

the Poisson arrival rate of applicants, so this term is also the average number of applications per vacancy over a given unit of time. We summarize this result in hypothesis H6:

> **Hypothesis H6**: *Larger firms invest more effort into formal search activities* $\partial e/\partial a > 0$ *and receive a larger number of applications* $aq(\theta) + q^N(u)$ *per vacancy per unit time than smaller firms.*

Note that the total number of applications per vacancy mentioned in hypothesis 6 is a hypothetical construct since in the model, firms always accept the first candidate applying for the job. However, if firms decide to wait and accumulate applications over a prespecified unit of time, then larger firms would accumulate more applications than smaller firms. Formally, the reason is that job applicants arrive to vacancies according to the Poisson process and the intensity of this process $aq(\theta) + q^N(u)$ is equal to the average number of successful arrivals over a given unit of time. We formulate this hypothesis in terms of the number of applications to bring it closer to the empirical data.

Finally, our model has several predictions concerning wages. First, wages are increasing with the skill level of the worker, and second, wages are increasing with the firm size, that is, $\partial e/\partial w > 0$. This holds for all firms irrespective of their recruitment strategies. Even though there is some support for this "firm size effect" for the United States (Oi and Idson, 1999; Fox, 2009), it is not obvious whether this link also holds for Germany. We tested this relationship by using the Establishment History Panel (BHP) and find that it is strongly supported by the German empirical data. Also, the data reveal that wages are increasing with the skill requirements of the position. However, the focus of our study is on recruitment strategies instead of wages, so our wage regressions are delegated to Appendix A.6. Thus, in the following, we focus on hypothesis H0–H6 containing our findings with respect to recruitment strategies and search channels of firms and turn to the description of the main empirical data in the next section.

3. DATA

Our empirical analysis is based on data from the Job Vacancy Survey of the Institute for Employment Research (IAB JVS) for the years 2012–2016. The IAB JVS is a representative survey among human resource managers or managing directors in German firms from all sectors of economic activities and from all company size classes. One of the main objectives of the survey is to explore the recruiting processes. In each wave of the survey, a representative sample of companies and administrations with at least one employee subject to social security contributions is taken from the employment statistics of the Federal Employment Agency. The survey is stratified disproportionately, structured into 28 economic activities and seven company size classes, and is drawn separately for Eastern and Western Germany. The reasons for the disproportional stratification are twofold. First, there are still considerable differences between the labor markets in East and West Germany, for example, in the section structure or the labor supply structure. Second, around 95% of German firms are small or very small. Accordingly, there is a need to oversample large firms to come to a valid data base. Additionally, to ensure that the frequency in each cell of this sample matrix is sufficiently large, the drawing generally considers the response rates, the permissible sample error, and the maximum error tolerance for sampling rates to be expected (Brenzel et al., 2016).

The gross sample comprises between 75,000 and 85,000 addresses and the response rate is about 20% (for more information, see Kettner et al., 2011; Brenzel et al., 2016). To ensure the representativity of the survey data, the distribution of the share of establishments and employees subject to social insurance contributions in each cell of the stratified sample is steadily monitored. Additionally, a nonresponse analyses on each wave of the survey returned only marginal corrections (see Brenzel et al., 2016). Generally, the sample for the different waves is newly drawn each year. This implies that most firms are observed only once during the

observation period and only a small part of firms appears more than once. Since these firms cannot be explicitly identified, our data have the characteristics of a cross-sectional sample, observed in different years.

To get information about recruiting processes, a part of the questionnaire focusses on the last case of hiring in the previous 12 months prior to the survey. The firms are asked for detailed information about their search strategies for one successful and one unsuccessful recruitment case. In the case of successful recruitment, there is also information asked for the one strategy that finally turned out to be successful and yielded the applicant. To ensure a random sample of successful recruitment cases, firms are requested to identify the last person that was hired on a position that is subject to social insurance. If more than one person was hired, firms should fill in information on the person whose name comes first in the alphabet. In principle, the procedure is the same for the selection of an unsuccessful recruitment process. Since in this case no person was hired, there are no further sampling instructions.[3]

To bring the data and the theoretical model close to each other, we merged several formal search strategies used by firms into one formal search channel, and the alternative is the informal search channel consisting of employee's referrals. The formal channel includes (i) placing ads in newspapers or magazines, (ii) posting the vacancy in the Internet, (iii) posting a vacancy on the firm's own Web site, and (iv) contacting the federal employment agency.[4] One variable that is crucial for our theoretical analysis is search effort of firms attributed to the formal channel. Even though search effort is generally unobservable, we constructed a proxy variable for the formal search intensity of firms a_i. This variable takes five discrete values in the range [0..4], where $a_i = j > 0$ implies that firm i used j formal strategies from the above list to fill the position. In the following, we investigate which factors are driving the formal search intensity of firms and use it as an explanatory variable determining which channel generated a new match.

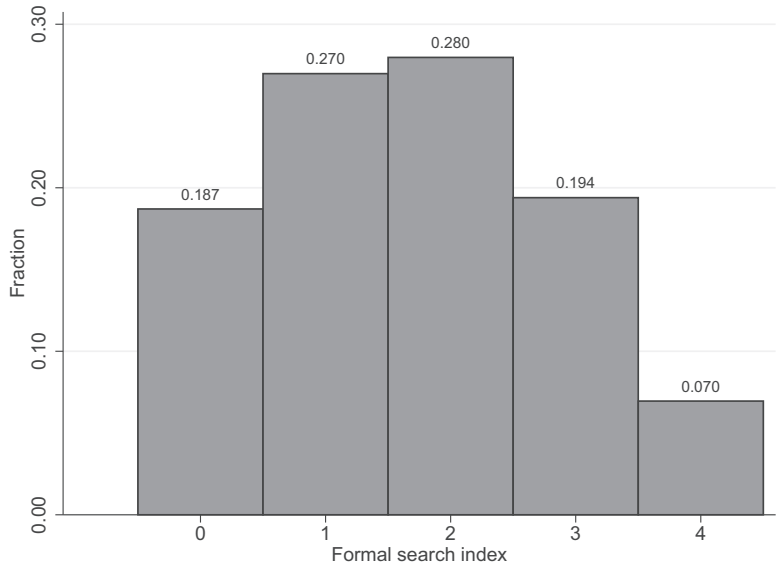
The idea to use the formal search effort as an explanatory variable received some attention in the literature but was not implemented so far. For example, Mouw (2003) refers to the works of Montgomery (1992) and discusses the problem of "multiple methods of job search" when considering the process of search from the perspective of workers. In particular, he points out that comparing ex post frequencies of finding a job formally or by referral is "a misleading way to determine the effectiveness of job search methods if workers use multiple methods of job search" (Mouw, 2003, p. 870), because workers' ex ante search effort attached to these channels can be different and may lead to biased results. The same problem may arise in the context of firms' search, and therefore, we follow the proposal of Krug and Rebien (2012) and use formal search effort of firms as an explanatory variable instead of the search channel that was successful to find a new employee.

Figure 5 shows the histogram of the formal search intensity a_i. We can see that most of the firms use one, two, or three search strategies, followed by firms that do not use the formal search channel at all ($a_i = 0$). Note that $a_i = 0$ implies that the firm was exclusively relying on referrals in the search process. The histogram reveals that very few firms use four search strategies (approximately 7%).

If the firm used at least one of the formal strategies to find job candidates, we set $y_i^F = 1$; otherwise, $y_i^F = 0$. This is a binary indicator variable for the formal job search. We use a similar indicator $s_i = 1$ if the firm was using referrals to fill the position; otherwise, $s_i = 0$.

[3] One potential drawback of the data is that it is not possible to derive information on the probability of how successful firms and their recruitment strategies are, that is, there is no information on the total number of firm's vacancies and how many of them were successfully filled over a given period. However, this is not of importance for our article.

[4] We assigned the most frequent formal strategies to the formal channel. However, the survey distinguishes a few further search strategies, for example, contacting a private employment agency, considering the pool of unsolicited applications, internal job postings, selection of apprentices, leased workers, or interns. We excluded firms using these search strategies from our analysis because these strategies are considered by the survey only for certain years and because these strategies cannot be clearly assigned to the formal or the informal channel.



NOTE: $N = 39{,}735$.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

FIGURE 5

DISTRIBUTION OF THE NUMBER OF CHOSEN FORMAL SEARCH CHANNELS BY FIRMS (FRACTIONS)

Note that $a_i = 0$ automatically implies that $s_i = 1$ but not vice versa. Finally, y_i^S equals 1 if the worker was recruited by referral and y_i^S equals 0 if it was the formal channel. This ex post information is only available for successful recruitment cases.[5]

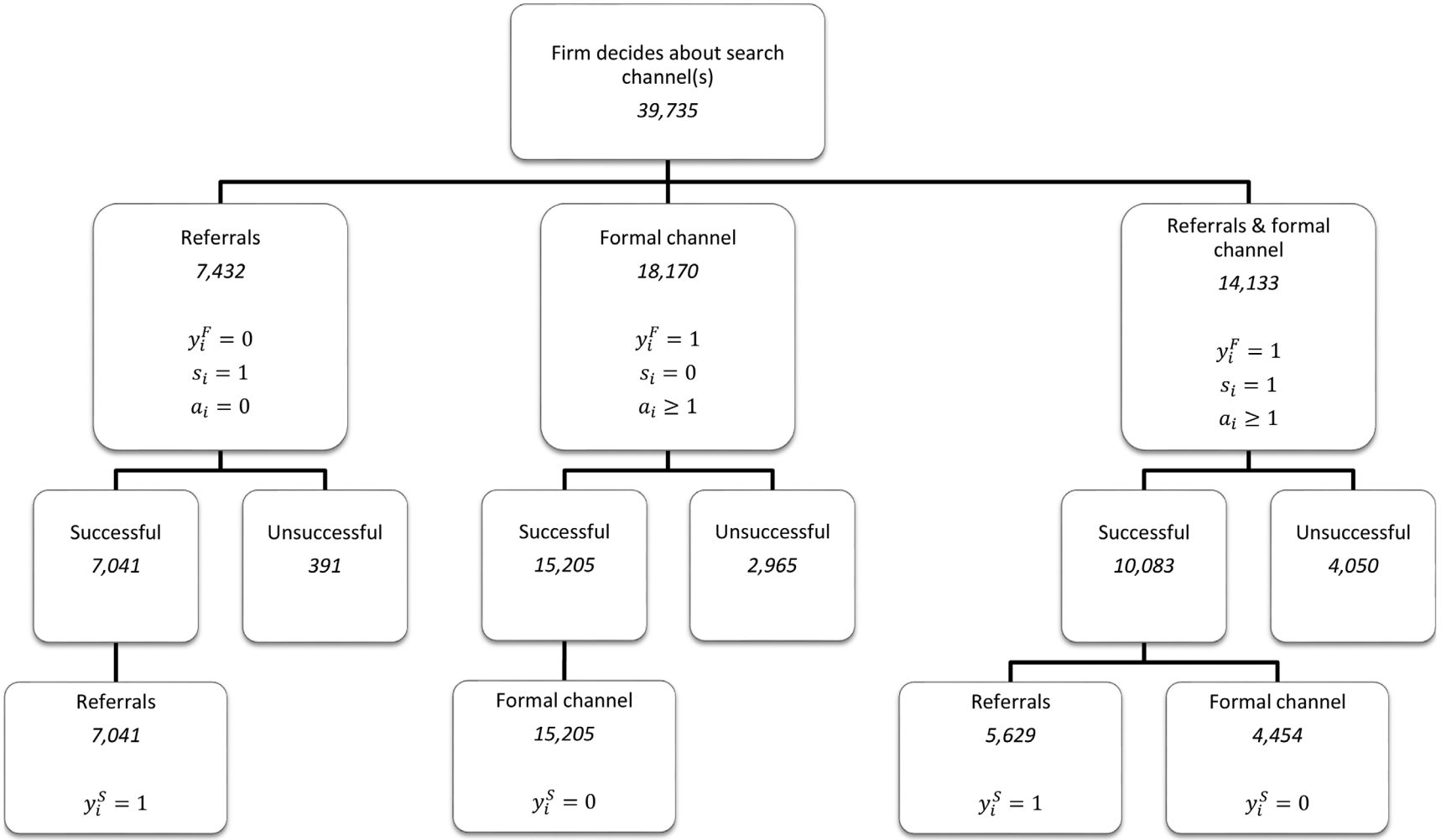
The data for successful recruitment give information on the characteristics of the vacancy to be filled, such as whether the position requires additional skills, for example, leadership abilities or knowledge of foreign languages; additionally, information about the previous employment status of hired workers is available.[6] All in all, about 9,000 companies answer this part of the questionnaire in each year, from which around 1,000 report unsuccessful recruiting attempts. For our main analysis, we use data from the years 2012–2016, because information about abortions of recruitment processes is only available for this observation period. Although due to the extensive use of control variables we lose a number of observations, our restricted sample remains representative. We checked this by comparing the frequency statistics of the stratification variables for each constructed data set.

Figure 6 illustrates the number of observed recruitment cases depending on the search channel and separating between successful and unsuccessful recruiting. This also illustrates the logic of our analysis; at first, firms decide to use referrals, to search via the formal channel or to use both channels simultaneously. Then the following recruiting process is either unsuccessful, thus the firm has to abort the recruitment activity without hiring, or the recruiting process is successful. In the latter case, we can observe by which of the two channels the worker was hired.

Figure 6 shows that there is a substantial number of firms that do not use the formal search channel, indicating that our equilibrium selection in the theoretical part of the study was correct. Indirectly, this evidence suggests that the fixed costs of using the formal channel are pro-

[5] Note that we can also observe two further formal strategies for the sample of firms that report successful recruitment cases. These two strategies are (v) contacting a private employment agency and (vi) considering the pool of unsolicited applications. Based on this sample and where possible, we conducted analyses with the six formal strategies in addition to our main analysis that we present in Section 4; the results with seven discrete values of the formal search index [0..6] are qualitatively the same and can be obtained from the authors upon request.

[6] The data are not available for unsuccessful recruitment cases.



NOTES: Number of observation is not equal to number of firms since some firms report one successful and one unsuccessful case of recruitment. Number of firms in the sample is 34,519.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

FIGURE 6

NUMBER OF CASES OF SUCCESSFUL AND UNSUCCESSFUL RECRUITMENT REPORTED BY FIRMS

hibitively high for some firms in the market and/or social networks are sufficiently large and yield suitable job candidates. Table 1 shows descriptive statistics of survey variables that are most important for testing hypotheses H0–H6 from the theoretical part. The table consists of seven panels; each of these panels refers to one of the hypotheses (see the headlines).

Hypothesis H0 postulates that there are some firms in the market that rely exclusively on referral hiring and do not use the formal channel; moreover, these firms are smaller than those using the formal channel. First panel of Table 1 presents relative frequencies of firms that search formally (y_i^F) depending on the firm size ('1–9 employees," "10–49 employees," "50–249 employees," and "250 and more employees"). The third column contains absolute numbers of observed firms. We can see that even among the smallest firms with less than 10 employees, 67% use formal search, at the same time, this fraction is increasing with the firm size. This shows that hypothesis H0 is supported in the broader sense that larger firms are more likely to use the formal search channel than smaller firms but it is not supported in the strict sense that none of the smaller firms is using the formal channel.

Next, we analyze descriptive evidence related to hypothesis H1 saying that larger firms invest more effort into formal advertising of their positions. The second panel of Table 1 shows that the average formal search index a_i is increasing from 1.87 in the class of firms with less than 10 employees to 2.49 among firms with more than 250 employees. This finding is in line with hypothesis H1. Note here that we restricted the sample to firms with $a_i > 0$ to make it comparable with the theoretical model.

The third panel refers to hypothesis H2 and shows the average formal search index a_i depending on the required skill level for the position. Skill requirements include the following three categories: "no qualification necessary," "vocational training," and "university degree." The average formal search index increases with higher qualification requirement. This is in line with hypothesis H2.

Further, hypothesis H3 implies that the positive relationship between the firm size and formal search intensity should be observed even for firms that do not use referrals. The fourth panel of Table 1 presents averages of the formal search index depending on the firm size. The difference to the second panel is that the sample is restricted to firms that do not use referrals; thus, all firms in the sample search exclusively through the formal channel, that is, $s_i = 0$ and $a_i > 0$. We find again that this search index increases on average with an increasing firm size. This finding corroborates hypothesis H3.

In the fifth panel, related to hypothesis H4, we compare the average formal search index of firms that either use referrals $s_i = 1$ or do not use referrals $s_i = 0$ for the four firm size classes. We find that the average intensities of firms that use referrals are smaller compared to firms that do not use referrals; this is in line with hypothesis H4.

The sixth panel refers to Hypotheses H5 and presents relative frequencies of referral ($y_i^S = 1$) and formal placements ($y_i^S = 0$) for the four firm size classes. We see that these frequencies become smaller with larger firm size. Here, the sample is restricted to firms that used both search channels following the theoretical model. The descriptive evidence is in line with hypothesis H5.

The seventh panel provides descriptive statistics referring to hypothesis H6. In our data, we have information about the number of applications received by firms. However, a direct comparison of the number of applications across firms can be misleading because firms may choose different time spells between the moment when the position is posted and the application deadline. Thus, we control for these differences by dividing the number of applications by the search duration. This is to guarantee that we compare the number of applications over the same time period for all firms. The search duration is measured in days and it is defined as the period from the start of the firm's search until cancelation of the firm's search or a firm's positive recruitment decision; in our sample, the average search duration is 83 days (with a standard deviation of 107). Therefore, the last panel reports average numbers of applications per vacancy and search duration for the four firm size groups. We can see that the average

TABLE 1
DESCRIPTIVES OF THE MOST IMPORTANT VARIABLES FOR HYPOTHESES H0–H6

Hypothesis H0: Formal Search and Firm Size

	Use of Formal Channels y_i^F Rel. Freq.	N
Number of employees (four firm size classes)		
$1 \leq ee < 10$	0.67	5313
$10 \leq ee < 50$	0.79	20272
$50 \leq ee < 250$	0.88	9945
$250 \leq ee$	0.95	4205
Total	0.81	39735

Hypothesis H1: Formal Search and Firm Size, Restricted to a Sample of Firms that Used Both Referral and Formal Search

	Formal Search Index a_i Mean	N
Number of employees (four firm size classes)		
$1 <= ee < 10$	1.87	1759
$10 <= ee < 50$	2.02	7377
$50 <= ee < 250$	2.26	3634
$250 <= ee$	2.49	1363
Total	2.11	14133

Hypothesis H2: Formal Search and Skill Requirements, Restricted to a Sample of Firms that Used Both Referral and Formal Search Strategies

	Formal Search Index a_i Mean	N
Skill requirements		
w/o qualification	1.98	1901
Vocational training	2.09	9863
University degree	2.29	2369
Total	2.11	14133

Hypothesis H3: Formal Search and Firm Size, Restricted to Firms that Use Only the Formal Channel and Don't Use Referrals

	Formal Search Index $a_i > 0$ Mean	N
Number of employees (four firm size classes)		
$1 \leq ee < 10$	1.77	1818
$10 \leq ee < 50$	1.92	8572
$50 \leq ee < 250$	2.20	5161
$250 \leq ee$	2.42	2619
Total	2.06	18170

Hypothesis H4: Formal Search Intensity by Firms that Either Use Referrals Or Not

	Use of Referrals s_i			
	No	Yes	All	
	Mean of Formal Search Index a_i			N (for All)
Number of employees (four firm size classes)				
$1 \leq ee < 10$	1.77	0.94	1.22	5313
$10 \leq ee < 50$	1.92	1.27	1.55	20272
$50 \leq ee < 250$	2.20	1.71	1.97	9945
$250 \leq ee$	2.42	2.14	2.31	4205
Total	2.06	1.38	1.69	39735

Continued

TABLE 1
CONTINUED

Hypothesis H5: Placement with Referral by Firm Size, Restricted for Successful Recruitment Cases and Firms that Used Both Referral and Formal Search Strategies	Placement by Referral ($y_i^S = 1$) Rel. Freq.	N
Number of employees (4 firm size classes)		
$1 \leq ee < 10$	0.63	1051
$10 \leq ee < 50$	0.60	5294
$50 \leq ee < 250$	0.50	2710
$250 \leq ee$	0.41	1028
Total	0.56	10083

Hypothesis H6: Numbers of Applicants Per Vacancy and Search Duration by Firm Size	Average Number of Applicants Per Vacancy and Search Duration	N
Number of employees (four firm size classes)		
$1 \leq ee < 10$	0.26	3144
$10 \leq ee < 50$	0.35	13428
$50 \leq ee < 250$	0.52	7150
$250 \leq ee$	0.71	3224
Total	0.43	26946

SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

number of applications per day increases with the size of the firm. This finding is in line with hypothesis H6.

We continue the empirical analysis by evaluating different regression models. The list of control variables allow us to consider temporal shocks and cross-sectional heterogeneity of firms. For the analysis based on all recruitment cases (with and without successful recruitment), these controls consist of the share of employees without any qualification, the share of employees with vocational training and the share of employees with a university degree. In the survey, the data are available for 32,360 of 39,735 observations. We imputed missing values for the remaining 7,375 observations with sector and firm size averages of the employment shares.[7] The upper part of Table 2 presents means and standard deviations of these variables. With an average of 72%, the share of employees with vocational training in a firm is much larger than the share of employees without qualification or employees with a university degree in the same firm. According to the standard deviations, there is huge variation of these shares in the sample.

For the analysis based on successful recruitment processes, the set of control variables is larger and includes additional information on further skill requirements such as leadership ability, adequate experience, skills from a training program beyond the standard vocational training, social skills, and foreign languages. We compare relative frequencies and standard deviations of these variables in the middle part of Table 2. Here, we see that social skills (35% of all successful recruitment cases) or experience (34% of all successful recruitment cases) are rather often asked for, whereas leadership abilities and foreign languages are rarely required

[7] In doing so, we computed the total numbers of employees belonging to a certain skill level (i.e., without any qualification, vocational training, or university degree) within each of 23 economic sectors and one of seven firm size classes ("1≤ ee 10," "10≤ ee < 20," "20≤ ee < 50," "50≤ ee < 200," "200≤ ee < 500," "500≤ ee < 1,000," "1,000≤ ee "). Then we divided each of these numbers by the total of all employees within the same economic sector and the same firm size class. Therefore, we get an average share of employees with a certain skill level by economic sectors and firm size classes that we use as a proxy for the employee share in a firm without information about the employment share and that belongs to the adequate economic sector and size class.

TABLE 2
DESCRIPTIVE STATISTICS OF FURTHER CONTROL VARIABLES

	Mean	Stand. Dev.
Employee shares by skill level		
Share of unskilled workers	0.10	0.18
Share of skilled workers with vocational training	0.72	0.25
Share of skilled workers with university degree	0.17	0.22
N	39735	
Only for successful recruitment cases	Rel. Frequency	Stand. Dev.
Additional skill requirements (multiple answers allowed)		
Leadership ability	0.08	0.29
Experience	0.34	0.47
Skills acquired from a training program outside the standard vocational training	0.20	0.40
Foreign languages	0.08	0.28
Social skills	0.35	0.48
Former employment status of recruited workers		
Employed	0.66	0.47
Unemployed	0.26	0.44
Out of labor force	0.08	0.27
N	32329	

SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

(both at 8% of all successful recruitment cases). Again, the standard deviations are relatively large and indicate large variation of the further skill requirements in the sample of successful recruitment cases.

For successful placements, we also have information on the former employment status of recruited workers; this status is either "employed," "unemployed." or "out of the labor force." The lower part of Table 2 reports relative frequencies and standard deviations of these dummy variables. We can see that most of the recruited workers were "employed" before, followed by workers that were "unemployed," and those "out of the labor force." Given that in our theoretical model, there is no on-the-job search, we pay special attention to workers hired from unemployment and report our empirical results separately for the full sample and for the restricted sample of previously unemployed applicants.

Finally, all our specifications contain dummy variables for the German federal states, for the 23 economic sectors, for the observation years, and for the distinction of firms that report either a successful or an unsuccessful case of recruitment versus firms that report both.

4. EMPIRICAL STRATEGY AND RESULTS

For the exploration of hypotheses H0, H1, and H2, we use a simple logistic model:

$$\text{Prob}(y_i^F = 1|\mathbf{x_i}) = \frac{\exp(b_0 + \mathbf{b_1 x_i})}{1 + \exp(b_0 + \mathbf{b_1 x_i})} = \frac{1}{1 + \exp[-(b_0 + \mathbf{b_1 x_i})]}. \tag{14}$$

The term $\text{Prob}(y_i^F = 1|\mathbf{x_i})$ denotes firm's i probability of using the formal search channel (y_i^F equals 1 in case of using the formal search channel and 0 otherwise), conditional on the column vector $\mathbf{x_i}$ containing explanatory variables. At this stage, firm's i size and the required skill level are the explanatory variables we are interested in.

Regarding the firm size, we defined four groups (less than 10 employees, 10 to less than 50 employees, 50 to less than 250 employees, and 250 and more employees) and assigned every firm to one of these groups. This corresponds to three size dummy variables (the group of firms with less than 10 employees is the reference group, and therefore, excluded).

Regarding the skill level, we can distinguish the jobs that do not require any formal qualification from jobs that require vocational training and jobs that require a university degree. This gives us two skill level dummy variables with "no formal qualification requirement" as reference category.

All these dummy variables are stacked with a number of further covariates in the column vector $\mathbf{x_i} = (x_{i1}x_{i2}\ldots x_{ik})'$. The corresponding coefficients are stacked in row vector $\mathbf{b_1} = (\beta_{11}\beta_{12}\ldots\beta_{1k})$. In all our estimations, we include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment. We also add stepwise firm, vacancy, and worker-specific characteristics; further details about the covariates follow along the description of the result tables. We report cluster robust standard errors by federal states and we generally report odds ratios[8].

The main analysis is based on a sample that includes both successful and unsuccessful recruitment processes. This gives us the opportunity to analyze the probability of formal search, regardless whether the recruitment process ended up with a new hire or not.

Table 3 presents the estimates. Results in column (1) stem from a specification where the right-hand side consists only of three firm size class dummies (with the smallest firm size class between one and nine employees) and the mentioned controls. Column (2) presents results of a specification that contains information about the skill-level requirement of the job; that is, there is a dummy in the case that the job requires vocational training and a dummy in the case that the job requires a university degree. The reference category is equivalent to cases the job requires no qualification. Results in column (3) are based on a specification with both firm size class dummies and skill requirement dummies. Column (4) presents the results of the regression estimates based on the specification before and complemented by information about the employment structure in terms of the share of employed who completed vocational training and the share of employed university graduates (reference category is the share of employees without qualification; this share would sum up with the other shares to 1 and must be excluded from the regression equation).

In columns (5) and (6), we restrict the sample and exclude cases of unsuccessful search. The specification in column (5) is the same as in column (4) and the specification in column (6) considers further control variables that refer to additional skill requirements, more precisely leadership ability, experience, skills from training beyond formal qualification, foreign languages, and social skills.

According to the results reported in Table 3, the probability of using the formal channel by firms is higher in larger firms since the coefficients are increasing with the firm size. These results are robust throughout all presented specifications.[9] This finding supports our theoretical conclusion summarized in hypothesis H0 that larger firms are more likely to use formal advertising; however, this hypothesis is not supported in the strict sense that small firms rely exclusively on referral hiring.

Hypotheses H1 and H2 refer to the formal search intensity of firms instead of the probability of using the formal channel. Though we are not able to construct the ideal continuous measure, our data allow us to compute an ordinal indicator that is equal to the sum of the search strategies that firms decided to use and that are assigned to the formal channel. This indicator variable was defined above and is denoted by a_i. In our sample firms used between 1 and

[8] Given that $P_i \equiv \text{Prob}(y_i = 1)$, the odds are $\frac{P_i}{(1-P_i)} = \exp(b_0)\exp(\beta_{11}x_{i1})\cdot\cdots\cdot\exp(\beta_{1k}x_{ik})$. Thus, for a certain covariate (x_{ij} with $1 \le j \le k$) that is increased by 1, the change in the log-odds ratio ($\ln \frac{P_i}{(1-P_i)}$) is equal to (β_{1j}). In all result tables, we report $\exp(\beta_{1j})$; values below 1 (thus, $\beta_{1j} < 0$) indicate a negative effect on the odds and values larger than 1 indicate a positive effect (thus, $\beta_{1j} > 0$).

[9] Table A3 in Appendix A.7 contains the coefficients on further control variables. For example, we can see that firms are less likely to use the formal channel when they are looking for workers with leadership abilities and experience.

TABLE 3
EMPIRICAL RESULTS: PROBABILITY OF USING THE FORMAL CHANNEL IN RECRUITING PROCESSES AS A DEPENDENT VARIABLE, PROB($y_i^F = 1$)

	All Observations				Only Successful Recruitment Cases	
	(1) logit	(2) logit	(3) logit	(4) logit	(5) logit	(6) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)						
10 ≤ ee < 50	1.773***		1.724***	1.731***	2.015***	2.022***
	(0.061)		(0.060)	(0.060)	(0.078)	(0.076)
50 ≤ ee < 250	3.629***		3.454***	3.482***	4.163***	4.167***
	(0.252)		(0.234)	(0.241)	(0.288)	(0.288)
250 ≤ ee	7.886***		7.041***	7.151***	8.668***	8.568***
	(0.520)		(0.408)	(0.416)	(0.563)	(0.549)
Skill requirements, reference: w/o qualification						
Vocational training		1.901***	1.832***	1.775***	1.716***	1.667***
		(0.115)	(0.110)	(0.103)	(0.098)	(0.096)
University degree		2.995***	2.295***	2.077***	1.948***	1.947***
		(0.207)	(0.131)	(0.120)	(0.112)	(0.113)
Further control variables						
Employee shares by skill level				x	x	x
Additional skill requirements						x
Observations	40944	39736	39736	39735	32329	32329
Pseudo-R^2	0.105	0.0785	0.112	0.112	0.101	0.103

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where a zero effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used exclusively referrals, exclusively formal ways, or both referrals and formal ways as search channels ($s_i \geq 0, 0 \leq a_i \leq 4$). Employee shares by skill level comprise three groups: unskilled workers, skilled workers with vocational training, and skilled workers with university degree. Additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in Appendix A.7, Table A3.
SOURCE: IAB Vacancy Survey 2012–2016.

4 of these strategies,[10] with a median of two chosen search strategies. So we computed an ordinal indicator for the formal search intensity, based on the assumption that the formal search intensity is higher, the more formal strategies are used. In what follows, we analyze the influence of firm size and skill level requirements on the search intensity indicator with an ordered logit model:

$$\text{Prob}(a_i = j) = \text{Prob}(\kappa_{j-1} < b_0 + \mathbf{b_1 x_i} + u_i \leq \kappa_j). \tag{15}$$

The probability $\text{Prob}(a_i = j)$ of the outcome $j = 1, \ldots, 4$ corresponds to the probability that the estimated linear function, plus the error term,[11] u_i, is within the range of the cutpoints $\kappa_0, \kappa_1, \kappa_2$. These are simultaneously estimated with the coefficients $b_0, \beta_{11}, \beta_{12}, \ldots \beta_{1k}$. In our analysis, κ_{-1} goes to $-\infty$ and κ_3 to ∞. We report our estimation results as odds ratios: Note that the ordered logit model estimates a single equation (regression coefficients) over (dis-

[10] The maximum must be 4, because we assign four strategies (placing ads in newspapers or magazines, posting the vacancy in the Internet, posting a vacancy on the firm's own Web site, and contacting the federal employment agency) to the formal channel in the survey throughout the observation period. The arithmetic mean for the sample of firms that use both referrals and formal search channels is 2.08 with a standard deviation of 0.95. This shows that the index values vary sufficiently across the firms.

[11] The error term is assumed to be logistically distributed.

TABLE 4
EMPIRICAL RESULTS: SEARCH INTENSITY THROUGH THE FORMAL CHANNEL, a_i, AS A DEPENDENT VARIABLE

	All Observations				Only Successful Recruitment Cases	
	(1) ord. logit	(2) ord. logit	(3) ord. logit	(4) ord. logit	(5) ord. logit	(6) ord. logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)						
10 ≤ ee < 50	1.271***		1.254***	1.249***	1.263***	1.274***
	(0.056)		(0.048)	(0.049)	(0.058)	(0.059)
50 ≤ ee < 250	1.940***		1.901***	1.890***	1.884***	1.899***
	(0.095)		(0.079)	(0.076)	(0.079)	(0.080)
250 ≤ ee	3.017***		2.768***	2.739***	2.738***	2.744***
	(0.192)		(0.168)	(0.174)	(0.190)	(0.199)
Skill requirements, reference: w/o qualification						
Vocational training		1.435***	1.432***	1.451***	1.332***	1.288***
		(0.102)	(0.106)	(0.101)	(0.104)	(0.107)
University degree		2.235***	1.922***	2.027***	1.794***	1.703***
		(0.174)	(0.150)	(0.135)	(0.129)	(0.145)
Further control variables						
Employee shares by skill level				x	x	x
Additional skill requirements						x
Observations	14497	14133	14133	14133	10083	10083
Pseudo-R^2	0.0400	0.0333	0.0425	0.0426	0.0327	0.0339

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, ** $p<0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$). Employee shares by skill level comprise three groups: unskilled workers, skilled workers with vocational training, and skilled workers with university degree. Additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in the Appendix A.7, Table A4.
Estimated cutpoints for specification in column (4): $\kappa_0 = 0.8114301$, $\kappa_1 = 2.297124$, $\kappa_2 = 3.898334$.
SOURCE: IAB Vacancy Survey 2012–2016.

tinct) levels of the dependent variable. The change in levels is interpreted in a cumulative sense. Taking the coefficients in odds ratios implies a comparison of the observation units that are in groups greater than a specific κ versus those that are in groups less than or equal to κ. A one unit change in the predictor variable induces that the odds for cases in a group that is greater than κ versus less than or equal to κ are the proportional odds times larger.

Table 4 reports the estimates, based on the sample of firms that use both referrals and formal strategies as search channel ($s_i = 1, 1 \leq a_i \leq 4$). This is close to hypothesis H1 in the theoretical model that is formulated for firms that use both channels. The different model specifications in columns (1)–(6) are analogous to the previous Table 3.

We can see that in all specifications, the formal search indicator increases with a larger firm size and with a higher qualification requirement of the position. According to our results, the search intensity is low for job positions without a qualification requirement, it is somewhat higher with a requirement of vocational training, and still higher for jobs that require a university degree. Thus, hypotheses H1 and H2 cannot be rejected.

However, the survey also includes the firms that reported using only the formal channel for recruitment, that is $s_i = 0$. According to hypothesis H3 derived from our theoretical model, the positive relationship between the firm size and formal search intensity should be preserved also for this subsample of firms. To validate this, we reestimated the last specifications of the ordered logistic regression based on the sample of firms that searched exclusively via the formal channel. Table 5 contains the results. Again, the

TABLE 5
EMPIRICAL RESULTS: SEARCH INTENSITY THROUGH THE FORMAL CHANNEL AS A DEPENDENT VARIABLE, a_i, RESTRICTED SAMPLE

	All Observations				Only Successful Recruitment Cases	
	(1) ord. logit	(2) ord. logit	(3) ord. logit	(4) ord. logit	(5) ord. logit	(6) ord. logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)						
10 ≤ ee < 50	1.351***		1.308***	1.307***	1.377***	1.385***
	(0.047)		(0.046)	(0.046)	(0.056)	(0.056)
50 ≤ ee < 250	2.369***		2.224***	2.226***	2.359***	2.352***
	(0.121)		(0.119)	(0.124)	(0.134)	(0.133)
250 ≤ ee	3.515***		3.056***	3.057***	3.143***	3.112***
	(0.305)		(0.264)	(0.274)	(0.273)	(0.270)
Skill requirements, reference: w/o qualification						
Vocational training		1.942***	1.907***	1.898***	1.877***	1.717***
		(0.149)	(0.134)	(0.124)	(0.128)	(0.116)
University degree		3.117***	2.555***	2.562***	2.486***	2.162***
		(0.238)	(0.148)	(0.173)	(0.168)	(0.155)
Further control variables						
Employee shares by skill level				x	x	x
Additional skill requirements						x
Observations	18698	18171	18171	18170	15205	15205
Pseudo-R^2	0.0357	0.0283	0.0418	0.0418	0.0397	0.0421

Federal states cluster robust standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used exclusively formal ways as search channels ($s_i = 0, 1 \leq a_i \leq 4$). Employee shares by skill level comprise three groups: unskilled workers, skilled workers with vocational training, and skilled workers with university degree. Additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in the Appendix A.7, Table A5. Estimated cutpoints for specification in column (4): $\kappa_1 = 0.9824285$, $\kappa_2 = 2.600384$, $\kappa_3 = 4.447658$.
SOURCE: IAB Vacancy Survey 2012–2016.

formal search intensity, a_i, increases with the firm size. Thus, hypothesis H3 cannot be rejected.

Another prediction generated by our theoretical model is that search by referrals and formal search are regarded by firms as substitutes. To test if this is the case, we used the same ordered logit specification as before but added the referral dummy, s_i, as an additional explanatory variable. Our findings are presented in Table 6. The first column presents the results of a specification that includes only the dummy variable for referrals. Afterward we gradually add the set of further control variables. The results in columns (2)–(7) are based on specifications that are similar to columns (1)–(6) in the previous result tables.

We can see from this table that using referrals is associated with lower formal search intensity. This corroborates the prediction that formal search and referrals are regarded as substitutes, even though using referrals is a costless search channel. However, based on this analysis, we cannot rule out that the measured effect is pure mechanical in the sense that a more intensive usage of any of the search channels results in a less intensive usage of all other search channels, independent of the fact whether the selected channel is formal or informal. We explore this in detail in Section 5.1 and find a substitution effect only for the referral channel against the formal search channel. Thus, hypothesis H4 cannot be rejected.

We now test hypothesis H5, containing the key prediction of our theoretical model, and estimate the probability of referral hiring with a logit model. Our results refer to the restricted sample of firms that decided to search by using both the formal channel and referrals

TABLE 6
SEARCH INTENSITY IN FORMAL WAYS, a_i, AND THE USAGE OF REFERRALS

	All Observations					Only Successful Recruitment Cases	
	(1) ord. logit	(2) ord. logit	(3) ord. logit	(4) ord. logit	(5) ord. logit	(6) ord. logit	(7) ord. logit
Usage of referrals	0.323***	0.347***	0.337***	0.359***	0.359***	0.272***	0.271***
	(0.013)	(0.014)	(0.015)	(0.016)	(0.016)	(0.012)	(0.012)
Further control variables							
Number of employees (four firm size classes)		x		x	x	x	x
Skill requirements			x	x	x	x	x
Employee shares by skill level					x	x	x
Additional skill requirements							x
Observations	40944	40944	39736	39736	39735	32329	32329
Pseudo-R^2	0.0631	0.0801	0.0691	0.0835	0.0835	0.0922	0.0938

Federal states cluster robust standard errors in parentheses.
***p<0.01, **p<0.05, *p<0.1.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both or either exclusively formal ways or referrals as search channels ($s_i \geq 0, 0 \leq a_i \leq 4$). The number of employees comprises four firm size classes: '1–9 employees" (reference group), "10–49 employees," "50–249 employees," and "250 and more employees." The employee shares by skill level comprise three groups: unskilled workers, skilled workers with vocational training, and skilled workers with university degree. The additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in the Appendix A.7, Table A6. Estimated cutpoints for specification in column (5): $\kappa_1 = 0.0253469, \kappa_2 = 1.597462, \kappa_3 = 2.999108, \kappa_4 = 4.698802$.
SOURCE: IAB Vacancy Survey 2012–2016.

simultaneously. This restriction is necessary because for firms that use exclusively only one of these two channels, the successful channel would be predetermined. Note that in these regression specifications, we can additionally control for the former employment status of the worker (employed or unemployed, reference is the group of workers who come from out of the labor force). Table 7 presents our results.

We clearly see that larger firms are less likely to hire a worker by referral (column (1)). The question now is whether this effect is explained by higher formal search intensity a_i (as predicted by our theoretical model) or not. From column (2) of this table, we can see that the formal search intensity a_i has a significant negative effect on the probability of hiring a worker by referral. Moreover, the coefficients on the three firm size variables are higher in column (3) and closer to 1. Given that we report the model in terms of odds ratios, this means that the negative effect of the firm size on the probability of referral hiring is largely mitigated and is closer to 0 once we control for the higher formal search intensity of larger firms. Therefore, the mechanism described in our model is supported by empirical data even though the firm size coefficients remain significant.[12]

Table 7 also shows that the coefficient on the dummy that denotes unemployment as former employment status is smaller than 1. Therefore, referral hiring is more widespread among previously employed applicants instead of unemployed. There can be several explanations for this effect. One explanation can be found in Arbex et al. (2019) who consider referral hiring combined with on-the-job search. In their model, employees have higher reservation wages than unemployed workers and network job offers are superior to direct offers from firms.

[12] However, this also implies that there can be additional reasons leading to the lower probability of referral hiring in larger firms that are beyond our model. Furthermore, larger firms may search harder in the ways that are not captured by our formal search index; for example, spending more money for each formal strategy.

TABLE 7
EMPIRICAL RESULTS: PROBABILITY TO RECRUIT WITH REFERRALS AS A DEPENDENT VARIABLE (y_i^S)

	(1) logit	(2) logit	(3) logit	(4) logit	(5) logit	(6) logit	(7) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)							
10 ≤ ee < 50	0.839***		0.875**	0.880*	0.868**	0.871**	0.870*
	(0.056)		(0.058)	(0.064)	(0.058)	(0.060)	(0.064)
50 ≤ ee < 250	0.553***		0.617***	0.624***	0.615***	0.618***	0.620***
	(0.049)		(0.057)	(0.059)	(0.058)	(0.057)	(0.058)
250 ≤ ee	0.370***		0.445***	0.462***	0.447***	0.452***	0.458***
	(0.046)		(0.058)	(0.063)	(0.058)	(0.057)	(0.063)
Formal search index		0.648***	0.671***	0.674***	0.671***	0.672***	0.673***
		(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
Former employment status, reference: out of labor force							
Unemployed	0.578***	0.586***	0.578***	0.562***	0.572***	0.574***	0.558***
	(0.052)	(0.054)	(0.052)	(0.053)	(0.051)	(0.051)	(0.052)
Employed	0.936	0.938	0.974	0.990	0.977	0.976	0.987
	(0.065)	(0.065)	(0.065)	(0.073)	(0.066)	(0.065)	(0.073)
Further control variables							
Skill requirements				x			x
Employee shares by skill level					x		x
Additional skill requirements						x	x
Observations	10348	10348	10348	10083	10348	10348	10083
Pseudo-R^2	0.0310	0.0448	0.0540	0.0554	0.0552	0.0563	0.0578

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$). The skill requirements comprise three levels: w/o qualification (reference group), vocational training, or university degree. The employee shares by skill level comprise three groups: unskilled workers (reference group), skilled workers with vocational training, and skilled workers with university degree. The additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in the Appendix A.7, Table A7.
SOURCE: IAB Vacancy Survey 2012–2016.

In this situation, unemployed workers accept almost all job offers, but employed workers switching jobs accept only high wage offers, so job-to-job transitions occur more frequently via the network channel. Another explanation consistent with our empirical evidence is that employees value their reputation in the company and are reluctant to recommend formerly unemployed job candidates since unemployment can be perceived as a signal of low ability and/or motivation.

In the next step, we restrict the sample to previously unemployed successful candidates to bring closer our theoretical model and empirical data. Our results based on this restricted sample are presented in Table 8. In column (2), we can see that the coefficient on the formal search index remains almost the same and it is significant, indicating that more formal search activities reduce the probability of referral hiring. The coefficient of the second smallest firm size class is insignificant. The coefficient of the next larger firm size class variable becomes insignificant at 5% significance level once we control for the search intensity. This implies that higher frequency of referral hiring in smaller firms is fully explained by the lower formal search intensity of these firms. This is in line with hypothesis H5. Only for very large firms with more than 250 employees, the firm size coefficient remains significant at 5% significance level implying other factors at play that are beyond our model.

Our results so far indicate that larger firms exert more effort trying to fill their vacancies in a formal way and this explains a part of the lower probability of referral hiring in larger firms. Even though the coefficient of the formal search index is statistically significant one remaining question is whether it is also meaningful from an economic perspective. Considering again columns (1) and (2) of Table 7, we can see that the pseudo-R^2 coefficient is only 3.1% when we control for the firm size but not for the higher search intensity, whereas it rises to 4.48% when we control for the formal search intensity but not for the firm size. This is the first indication that the search intensity index is a driving force for the lower probability of referral hiring and not the firm size per se.

To get a better notion about the relative impact of the firm size classes and the search intensity, we apply a Shorrocks–Shapley decomposition (Shorrocks, 1982) of the pseudo-R^2. According to this methodology, we find for the full specification in column (7) of Table 7 that the firm size accounts for about 18% and the search intensity accounts for about 46% of the pseudo-R^2. We find similar results for the full specification in column (7) of Table 8. Here, the firm size accounts for about 14% and the search intensity accounts for about 46% of the pseudo-R^2. We conclude that the negative relationship between referral hiring and the firm's size is to a very large extent due to the higher search effort of larger firms, which is in line with the prediction of our model related to hypothesis H5.

Finally, we test hypothesis H6, and thus, how firm size and the number of applicants are related. The number of applicants is a count variable, so to find an adequate model, we have to consider the distribution of this variable. Recall that in the theoretical model, workers' applications arrive to a firm with an open vacancy according to a Poisson process and positions are filled as soon as the applicant is matched to the job. This is a simplifying assumption underlying many search and matching models. To compare the numbers of applications across firms in the data, we normalize them by the corresponding vacancy duration in days. The histogram of this normalized variable is presented in Figure 7.

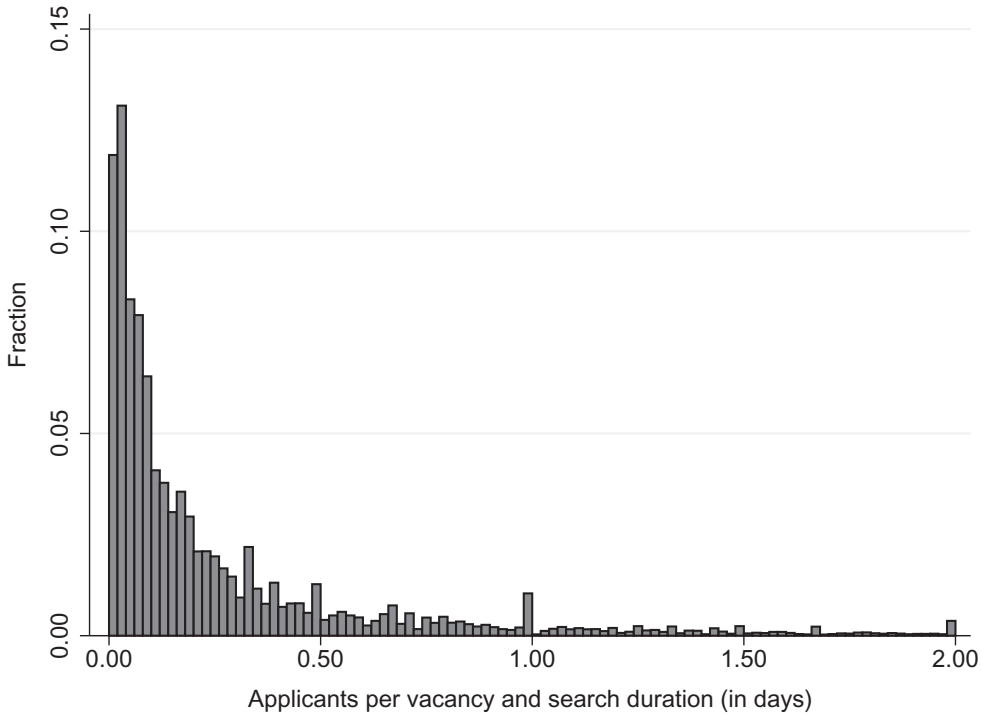
Recall that the Poisson process requires equality of mean and variance for the number of applicants, but the distribution here clearly reveals overdispersion with a mean of 0.43 applicants per vacancy and day with a 10 times larger variance of 3.87. Thus, we continue by using a negative binomial distribution that is well suited for count data with large variances. The empirical model we use is based on the negative Bbinomial II distribution with the following first two moments: the expected value and the variance (Cameron and Trivedi, 2005, p. 676):

$$E[y|\mu,\alpha] = \mu \qquad \mathrm{Var}[y|\mu,\alpha] = \mu(1+\alpha\mu). \tag{16}$$

TABLE 8
EMPIRICAL RESULTS: PROBABILITY TO RECRUIT UNEMPLOYED WORKERS WITH REFERRALS AS A DEPENDENT VARIABLE (y_i^S), RESTRICTED SAMPLE

	(1) logit	(2) logit	(3) logit	(4) logit	(5) logit	(6) logit	(7) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)							
10 ≤ ee < 50	0.833		0.866	0.861	0.867	0.860	0.852
	(0.126)		(0.137)	(0.143)	(0.136)	(0.136)	(0.140)
50 ≤ ee < 250	0.652***		0.745*	0.749*	0.744*	0.738*	0.741*
	(0.108)		(0.127)	(0.130)	(0.127)	(0.123)	(0.126)
250 ≤ ee	0.350***		0.439***	0.425***	0.439***	0.439***	0.422***
	(0.070)		(0.091)	(0.092)	(0.092)	(0.091)	(0.092)
Formal search index		0.655***	0.674***	0.671***	0.674***	0.688***	0.681***
		(0.032)	(0.033)	(0.034)	(0.033)	(0.031)	(0.034)
Further control variables							
Skill requirements				x			x
Employee shares by skill level					x		x
Additional skill requirements						x	x
Observations	2618	2618	2618	2551	2618	2618	2551
Pseudo-R^2	0.0265	0.0437	0.0486	0.0537	0.0489	0.0536	0.0572

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$). Sample is further restricted to cases with recruitment of former unemployed. The skill requirements comprise three levels: w/o qualification (reference group), vocational training, or university degree. The employee shares by skill level comprise three groups: unskilled workers (reference group), skilled workers with vocational training, and skilled workers with university degree. The additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in the Appendix A.7, Table A8.
SOURCE: IAB Vacancy Survey 2012–2016.



NOTES: Left censored at more than 0 applicants and right censored at 2 applicants, $N = 24{,}575$
SOURCE: IAB Vacancy Survey 2012–2016. Own computations.

FIGURE 7

DISTRIBUTION OF THE NUMBER OF APPLICANTS PER VACANCY PER DAY (FRACTIONS)

The variance, Var[.], exceeds the mean in case $\alpha > 0$ and $\mu > 0$. To estimate the model, we define μ as:

$$\mu_i \equiv y_i = \exp(b_0 + \mathbf{b_1 x_i}). \tag{17}$$

Due to the nonlinearity, we use maximum likelihood to estimate parameters b_0, $\mathbf{b_1}$, and α:

$$\ell_i(b_0, \mathbf{b_1}, \alpha) = \alpha^{-1} \log\left[\frac{\alpha^{-1}}{\alpha^{-1} + \exp(b_0 + \mathbf{b_1 x_i})}\right] + y_i \log\left[\frac{\exp(b_0 + \mathbf{b_1 x_i})}{\alpha^{-1} + \exp(b_0 + \mathbf{b_1 x_i})}\right] \tag{18}$$

$$+ \log\left[\frac{\Gamma(y_i + \alpha^{-1})}{\Gamma(\alpha^{-1})\Gamma(y_i + 1)}\right]. \tag{18}$$

$\Gamma(.)$ denotes the gamma function $\Gamma(r) = \int_0^\infty z^{r-1} \exp(-z) dz$, $r > 0$.

In this model, the dependent variable, y_i, denotes the number of applicants per vacancy and search duration. All explanatory variables are the same as before and we additionally estimate parameter α by the maximum-likelihood estimation. The estimates of the coefficients in β have to be interpreted as semielasticities: for example, a one-unit change in x_i changes the conditional mean, μ_i, by the multiplier $1 + \beta_i$. Table 9 presents the main results.

The results in columns (1)–(6) are based on the unrestricted sample of firms. The results in columns (7) and (8) are based on a restricted sample of successful recruitment cases and firms

TABLE 9
EMPIRICAL RESULTS: NUMBER OF APPLICANTS PER VACANCY AND SEARCH DURATION AS A DEPENDENT VARIABLE

	(1) Neg. Bin. II	(2) Neg. Bin. II	(3) Neg. Bin. II	(4) Neg. Bin. II	(5) Neg. Bin. II	(6) Neg. Bin. II	(7) Neg. Bin. II	(8) Neg. Bin. II
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)								
10 ≤ ee < 50	0.325***		0.276***	0.276***	0.291***	0.297***	0.324***	0.303***
	(0.086)		(0.087)	(0.088)	(0.081)	(0.083)	(0.091)	(0.087)
50 ≤ ee < 250	0.669***		0.558***	0.560***	0.550***	0.570***	0.718***	0.688***
	(0.104)		(0.104)	(0.103)	(0.098)	(0.098)	(0.139)	(0.133)
250 ≤ ee	0.951***		0.807***	0.832***	0.783***	0.845***	1.165***	1.152***
	(0.109)		(0.110)	(0.114)	(0.109)	(0.108)	(0.166)	(0.161)
Formal search index		0.216***	0.164***	0.177***	0.163***	0.177***	0.038	0.047
		(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.041)	(0.038)
Further control variables								
Skill requirements				x		x	x	x
Employee shares by skill level					x	x	x	x
Additional skill requirements								x
α	0.914***	0.923***	0.895***	0.895***	0.886***	0.886***	1.038***	1.022***
	(0.087)	(0.092)	(0.085)	(0.085)	(0.085)	(0.086)	(0.135)	(0.133)
Number of observations	27394	27394	27394	26947	27392	26946	7335	7335
Pseudo-R^2	0.0613	0.0581	0.0661	0.0683	0.0687	0.0711	0.0973	0.102
Log likelihood	−22345	−22424	−22233	−21887	−22170	−21820	−5933	−5904

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as semielasticities, a one-unit change in x_i changes the conditional mean, μ_i, by the multiplier $1 + \beta_i$. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. The skill requirements comprise three levels: w/o qualification (reference group), vocational training, or university degree. The employee shares by skill level comprise three groups: unskilled workers (reference group), skilled workers with vocational training, and skilled workers with university degree. The additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Results in columns (7) and (8) are from a specification that are based on a restricted sample of firms that report successful recruitment and used both formal channels and referrals ($s_i = 1, 1 \leq a_i \leq 4$). Detailed results on the further control variables are presented in the Appendix A.7, Table A9.
SOURCE: IAB Vacancy Survey 2012–2016.

that used both search channels for recruitment. Table 9 also shows that the estimates of parameter α are significantly positive. This finding confirms the overdispersion.[13]

From the estimates in columns (1)–(6), we can conclude that higher search effort of firms leads to a higher number of applications. This is in line with hypothesis H6.[14] For the restricted sample of successful recruitment cases and firms that used both referrals and formal search (columns (7) and (8)), the coefficient on the formal search index becomes insignificant, although it remains positive. Note, however, that the number of observations is much smaller in the restricted sample, which can lead to the insignificance.[15]

Even though our model is not linear, the first three columns of Table 9 reveal an omitted variable bias. Column (1) shows that larger firms receive more applications on average, whereas column (2) shows that firms exerting more search effort also receive more applications. Column (3) with the results of a specification with both variables shows that the magnitude of the effects is lower, and thenthis suggests that the estimates of the separate effects in column (1) and column (2) are upward biased. This means that higher search effort explains a part of the positive correlation between the size of the firm and the number of applications, which confirms hypothesis H6. But the firm size coefficients remain significant in columns (1)–(6), suggesting that other factors beyond search intensity may also be responsible for the positive correlation between the size and the number of applications. For example, the literature on directed search, for example, Moen (1997), shows that workers anticipate higher wages in larger establishments and are more likely to send their applications to larger firms.

5. FURTHER ROBUSTNESS CHECKS

5.1. *Are Referrals and Formal Strategies Substitutes?* Hypothesis H4 of our theoretical model suggests that the informal channel (referral hiring) and the formal channel are substitutes. We could show already that in the data, there is a negative relationship between the probability of using referrals and formal search intensity of firms (see Table 6 in the main article). However, the question remains, whether this substitution effect is purely mechanical and would be observed between any two search channels of firms or there is a deeper economic reason underlying this negative relationship.[16]

If substitution is a mechanical outcome, then it should also be observed if we select any other formal search strategy as a separate search channel and pool referrals and the remaining formal strategies into a "placebo search intensity index." We conducted these placebo tests and sequentially selected each of the four formal search strategies (ads in newspapers and journals, own Webpage, Internet platforms and databases, public employment agency) as a separate channel. With this data at hand, we reestimated ordered logit models as they are

[13] The test statistic of α results from computing twice the difference in log-likelihoods between the negative binomial model and the Poisson model. The statistic is assumed to be χ^2-distributed with one degree of freedom. The usual asymptotic would not apply, because the null hypothesis ($\alpha = 0$) lies on the lower bound of the parameter space ($\alpha >= 0$). However, Gutierrez et al. (2001) argue that the statistic should be treated as "50 : 50" mixture of 0 and a χ^2-distribution with 1 degree of freedom. This test is feasible for a Poisson and negative binomial model with conventionally computed standard errors. We computed the test based on the specification in column (6) of Table 9. According to the test statistic, we have clearly to reject the null hypothesis of $\alpha = 0$ ($\bar{\chi}_1^2 = 7,916.28$, Prob $>= \bar{\chi}_1^2 = 0.000$).

[14] Our results are robust to an alternative of the presented model that is based on the negative Bbinomial II distribution. This alternative is based on the so-called negative binomial I distribution with the expectation and variance given by Cameron and Trivedi (2005, p. 676): $E[y|\mu, \delta] = \mu$, $\text{Var}[y|\mu, \delta] = (1 + \delta)\mu$. The variance, Var[.], exceeds the mean in case of $\delta > 0$ and $\mu > 0$. Details and results are presented in Appendix A.8.

[15] Given that our theoretical model does not allow for on-the-job search, we performed further estimations based on a restricted sample of successful recruitment cases with unemployed job applicants. The results in Tables 3–6 and 9 remain qualitatively the same. One exception is column (7) of Table 9, where the coefficient on the formal search index becomes negative but remains insignificant. Overall, we prefer the estimations based on the sample containing both types of hires from employment and unemployment because in reality when firms choose their search channels and search effort, they anticipate both types of applicants. Thus, restricting the sample based on the ex post type of the successfully hired applicant can be misleading, if we want to understand the ex ante decisions of firms.

[16] We thank two anonymous referees for raising the question and attracting our attention to this point.

TABLE 10
PLACEBO TEST SUMMARY: THE USAGE OF EACH OF THE FIVE SEARCH STRATEGIES AND THE "PLACEBO SEARCH INTENSITY INDICATOR" OF THE REMAINING FOUR SEARCH WAYS

	Dependent Variable: "Placebo Search Intensity Indicator" of the Remaining Four Search Ways				
Search Strategy, Selected as Explanatory Variable	(1) Ord. logit	(2) Ord. logit	(3) Ord. logit	(4) Ord. logit	(5) Ord. logit
Reference: Usage of referrals	0.359*** (0.016)				
Search with advertisements in newspapers and journals		1.209*** (0.059)			
Firm Web site			2.464*** (0.071)		
Search with Internet job data bases				2.124*** (0.082)	
Search with employment agency					1.152*** (0.063)
Further control variables					
Number of employees (four firm size classes)	x	x	x	x	x
Skill requirements	x	x	x	x	x
Employee shares by skill level	x	x	x	x	x
Observations	39735	39735	39735	39735	39735
Pseudo-R^2	0.0835	0.0561	0.0532	0.0538	0.0526

Federal states cluster robust standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include dummy variables for four firm size classes, skill requirements, employee shares by skill level, year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used at least one of five search channels. The number of employees comprises four firm size classes: "1–9 employees" (reference group), "10–49 employees," "50–249 employees," and "250 and more employees." The employee shares by skill level comprise three groups: unskilled workers, skilled workers with vocational training, and skilled workers with university degree. Detailed results on the further control variables are available from the authors upon request.
SOURCE: IAB Vacancy Survey 2012–2016.

presented in Table 6. Table 10 summarizes the results of specifications based on the data sample that includes all recruitment cases, successful and unsuccessful. Each of the columns refers to a specification with a selected search strategy as an explanatory variable and the "placebo search intensity index" of the other four search strategies as a dependent variable. For example, column (2) reports the results of the specification with "search with advertisements in newspapers and journals" as an explanatory variable and the "placebo search intensity index" constructed from the remaining four search strategies (referrals, firm's web page, search with Internet job databases, or search with employment agency).

Column (1) repeats the results from the specification in column (5) of Table 6. We see that the coefficient (measured as odds ratio) is smaller than 1, which implies a negative relationship between the probability of using referrals and the formal search index. Each of columns (2)–(5) show a coefficient larger than 1 revealing a positive relationship between the probability of using the selected search strategy and the compound index of the remaining strategies. Therefore, this placebo test shows that only referrals are negatively correlated with the pooled index of the formal strategies; moreover, this substitution is not mechanical, rather it is specific to referral hiring and is not observed for any other formal search strategy of firms. Thus, we cannot reject the finding that referrals and formal strategies are substitutes.

5.2. *An Alternative Measure for the Formal Search Indicator.* Our data offer an alternative measure for the intensity of search via the formal channel; this measure is based on the additional recruitment costs (in EUR) reported by firms for successful recruitment cases. This in-

formation is only available for two consecutive years 2014 and 2015, so the number of observations for this variable is much smaller than our main sample. Yet, it is sufficient, so we take these costs as another proxy for the intensity of formal search and use it for the robustness check under the assumption that these monetary costs are fully attributed to formal search strategies instead of referrals. In the sample of successful recruitment cases, 11,207 firms reported these additional costs and the other relevant information for our analysis.[17]

Table 11 shows average additional recruiting costs by the intensity of formal search, the (successful) recruitment channel, firm size classes, and skill requirements. We see that the recruitment costs increase with the formal search intensity. Particularly, firms reported very low recruitment costs (48.53 Euros) in the case when they used only referrals.[18] This shows that additional recruitment costs are positively associated with the formal search intensity and supports the idea of using it as another proxy for the formal search effort. We can also see that additional recruitment costs increase with the firm size and with the skill requirement of the position. These observations are in line with our theoretical expectations.

Table 12 presents our results for alternative specifications with additional recruitment costs as a dependent variable and Table 13 presents the results for alternative specifications with additional recruitment costs as an explanatory variable.

Column (1) of Table 12 shows that the results are qualitatively similar to the results in column (6) of Table 4: recruitment costs increase with the firm size and with higher skill requirements of the position. The same is true for column (2) of Table 12 that refers to the specification in column (6) of Table 5. Thus, these results support our findings regarding hypotheses H1, H2, and H3 from the previous section. Column (3) of Table 12 confirms the results of Table 6 (see column (7)) that the use of referrals is negatively correlated with recruitment costs. Thus, this result supports our findings regarding hypothesis H4 from the previous section.

Columns (1) and (2) of Table 13 show that additional recruitment costs are negatively correlated with the probability of referral hiring—this confirms the qualitative result of the negative correlation between the formal search effort and the probability of referral placement as it was shown in Table 7. In columns (3) and (4), where the specifications are based on a sample that is restricted to recruitments of former unemployed, like in Table 8, the same coefficient is lower but the standard errors are much larger. The magnitude of the firm size coefficients is similar to the equivalent specifications in Tables 7 and 8. The firm size coefficients are also only partly significant and a comparison of the standard errors suggests that the lower number of observations leads to the loss of precision. But all in all, the results rather support our findings regarding hypothesis H5 from the previous section.

Columns (5)–(8) of Table 13 show that the coefficients on additional recruitment costs are insignificant at the 5% level; even though the point estimates suggest a positive correlation with the number of applicants per vacancy and search duration. The specifications in columns (2), (4), (6), and (8) of Table 13 additionally contain the formal search index. We included this variable to see how the coefficient estimates change in the case when both proxies of search effort are used as explanatory variables. We see a weaker impact of additional recruitment costs after including the formal search index, whereas the influence of the formal search index remains robust and positive.

All in all, using additional recruitment costs as an alternative proxy for the formal search effort does not contradict our results in the main analysis. However, our regression models based on the additional recruitment costs suffer from a smaller number of observations, and

[17] In detail, the survey asked for additional recruitment costs in 2014 and 2015. In 2015, 5,647 firms reported additional costs. To make these values comparable with the costs reported by 5,560 firms in 2014, we deflated the values in 2015 considering a change of consumer prices of +0.3% from 2014 to 2015, compare with the press release 018 by the Federal Statistical Office from 19th January 2016 (https://www.destatis.de/EN/PressServices/Press/pr/2016/01/PE16_018_611.html).

[18] Note that the structure of our data implies that firms exclusively use referrals in the case when they do not use formal ways for the search of workers. This means that in this case, the additional recruitment costs must be linked to referrals.

TABLE 11
DESCRIPTIVE STATISTICS FOR ADDITIONAL RECRUITMENT COSTS

Additional Recruitment Costs and Formal Search Intensity

	Additional Recruitment Costs (in EUR) Mean	*N*
Formal search intensity a_i		
0	48.53	2490
1	427.56	3046
2	839.92	3000
3	1457.22	2004
4	2304.04	667
Total	749.53	11207

Additional Recruitment Costs and Successful Recruitment Channel

	Additional Recruitment Costs (in EUR) Mean	*N*
Recruitment way		
Formal way	1081.09	6727
Referrals	251.69	4480
Total	749.53	11207

Additional Recruitment Costs and Firm Size

	Additional Recruitment Costs (in EUR) Mean	*N*
Number of employees(four firm size classes)		
$1 \leq ee < 10$	258.84	1369
$10 \leq ee < 50$	518.12	5797
$50 \leq ee < 250$	948.95	2853
$250 \leq ee$	1965.28	1188
Total	749.53	11207

Additional Recruitment Costs and Skill Requirements

	Additional Recruitment Costs (in EUR) Mean	*N*
Skill requirements		
w/o qualification	176.30	1610
Vocational training	521.98	7559
University degree	2046.40	2038
Total	749.53	11207

SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

therefore, a loss of precision. Another implication of the results above is that the formal effort of human resource managers and their actions is relatively more important in generating new applicants than additional monetary expenses. Finally, we want to note that monetary expenses for recruitment depend on prices that could be different across regions, which renders this variable less suitable for comparing heterogeneous firms. On the contrary, the formal search index is constructed based on the actions of human resource managers and is better comparable across firms. However, the identification of further components that fully describe workers' and firms' search effort goes widely beyond this article and we have to leave these tasks for future research.

TABLE 12
ROBUSTNESS CHECKS: OLS REGRESSION WITH ADDITIONAL RECRUITMENT COSTS AS A PROXY FOR THE INTENSITY OF FORMAL SEARCH

	(1)	(2)	(3)
	Based on a Sample of Firms that Report Only Successful Recruitment Cases and that Used		
	Both Referrals and Formal Strategies	Only Formal Strategies	Either Referrals or Formal Strategies or both
	$(s_i = 1, 1 \leq a_i \leq 4)$	$(s_i = 0, 1 \leq a_i \leq 4)$	$(s_i \geq 0, 0 \leq a_i \leq 4)$
	OLS	OLS	OLS
Usage of referrals			−437.281***
			(64.636)
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)			
10 ≤ ee < 50	85.539	280.372***	171.133***
	(89.886)	(91.287)	(43.631)
50 ≤ ee < 250	352.419***	601.989***	448.039***
	(107.081)	(83.685)	(62.188)
250 ≤ ee	868.658***	1169.397***	1053.971***
	(144.390)	(189.562)	(169.134)
Skill requirements, reference: w/o qualification			
Vocational training	20.871	91.032	37.242
	(74.616)	(70.799)	(33.126)
University degree	1237.275***	1523.811***	1199.763***
	(286.989)	(201.437)	(170.540)
Further control variables			
Employee shares by skill level	x	x	x
Additional skill requirements	x	x	x
Observations	3589	5128	11207
R^2	0.099	0.135	0.111

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: The specification and the sample restriction for column (1) refer to the specification and the sample restriction in column (6) of Table 4. The specification and the sample restriction for column (2) refer to the specification and the sample restriction in column (6) of Table 5. The specification and the sample restriction for column (3) refer to the specification and the sample restriction in column (7) of table of Table 6. All specifications include a constant, year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Employee shares by skill level comprise three groups: unskilled workers, skilled workers with vocational training, and skilled workers with university degree. Additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Detailed results on the further control variables are presented in the Appendix A.7, Table A10.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

6. CONCLUSIONS

This study explores the relationship between firms' characteristics and their recruitment strategies. These strategies include a formal search channel that consists of advertisements in newspapers and magazines, Internet postings as well as employment agencies, and an informal search channel, that is, referrals by incumbent employees. The formal search channel and the referral search induce different costs in terms of money and time; generally, the formal channel is more expensive than the informal channel. More productive firms have stronger incentives to fill their positions and they also have larger human, financial, and material resources, relative to less productive firms. Therefore, more productive firms invest more effort and can spend more resources on formal search to attract job applicants. This implies that firm characteristics are important for understanding firms' choices of recruiting channels.

We propose a model based on a search and matching framework with two search channels: the formal and the informal channel. In contrast to previous studies, there is a continuum of

TABLE 13
ROBUSTNESS CHECKS: SPECIFICATIONS WITH ADDITIONAL RECRUITMENT COSTS IN 1,000 EUROS AS A PROXY FOR THE INTENSITY OF FORMAL SEARCH AS AN EXPLANATORY VARIABLE

Dep. Variable	Probability to Recruit with Referrals				Applicants Per Vacancy and Search Duration			
Model	(1) logit	(2) logit	(3) logit	(4) logit	(5) Neg. Bin. II	(6) Neg. Bin. II	(7) Neg. Bin. II	(8) Neg. Bin. II
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)								
10 ≤ ee < 50	0.837	0.872	0.696	0.740	0.393***	0.307**	1.445***	1.382**
	(0.102)	(0.0982)	(0.187)	(0.197)	(0.122)	(0.121)	(0.187)	(0.188)
50 ≤ ee < 250	0.618***	0.681**	0.757	0.861	0.661***	0.502***	2.153***	2.023***
	(0.100)	(0.107)	(0.206)	(0.240)	(0.131)	(0.135)	(0.343)	(0.345)
250 ≤ ee	0.366***	0.413***	0.358**	0.404*	1.151***	0.934***	4.308***	3.902***
	(0.0965)	(0.108)	(0.166)	(0.188)	(0.104)	(0.124)	(0.871)	(0.791)
Recruitment effort	0.909***	0.947**	0.864	0.903	0.0271*	0.0120	1.022	1.014
(Additional costs in 1000 Euros)	(0.0286)	(0.0207)	(0.119)	(0.0874)	(0.0142)	(0.0140)	(0.0170)	(0.0172)
Formal search index		0.674***		0.779***		0.254***		1.178***
		(0.0242)		(0.0592)		(0.0347)		(0.0403)
Further control variables	x	x	x	x	x	x	x	x
Observations	3589	3589	916	916	7761	7761	2619	2619
Pseudo-R^2	0.0449	0.0657	0.0676	0.0749	0.0618	0.0727	0.112	0.115

Federal states cluster robust standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
NOTES: The samples for the specifications in columns (1)–(8) are generally restricted to successful recruitment cases because the additional recruitment cost is only reported for successful recruitment cases. The specifications and further sample restrictions for columns (1) and (2) refer to the specification and the sample restriction in column (7) of Table 7. Columns (2) and (4) additionally contain the formal search index as an explanatory variable. The specifications and further sample restrictions for columns (3) and (4) refer to the specification and the sample restriction in column (7) of Table 8. Coefficients in columns (1)–(4) are reported as odds ratios, where the null effect is 1. The specifications and further sample restrictions for columns (5)–(8) refer to the specifications and the sample restrictions in columns (6) and (8) of Table 9. Columns (6) and (8) additionally contain the formal search index as an explanatory variable. Coefficients in columns (5)–(8) are reported as semielasticities, a one-unit change in x_i changes the conditional mean, μ_i, by the multiplier $1 + \beta_i$. More detailed results on the further control variables are presented in the Appendix A.7, for columns (1)–(4) in Table A11 and for columns (5)–(8) in Table A12.
SOURCE: IAB vacancy survey 2014 and 2015.

heterogeneous vacancies in our model where every firm with an open vacancy chooses optimal search effort to attract job candidates. Workers send simultaneous applications to open positions. Similar to the previous studies, we prove that firm size and productivity are positively correlated. This finding is important for our empirical assessment because there is a lack of (representative) data with valid information on individual firm productivity and recruitment strategies.

We derive seven hypotheses based on the model: (0) Larger firms use both search channels, whereas small firms rely exclusively on referral hiring; (1) larger firms invest more effort into formal search activities; (2) firms invest more formal search effort in labor markets for more educated workers; (3) the positive relationship between the firm size and formal search intensity should be observed even for firms that do not use referrals; (4) firms that use referrals as a search channel invest less into formal search compared to firms that do not use referrals (substitution); (5) larger firms are less likely to hire an applicant by referral than smaller firms; and (6) more intensive search effort leads to a larger number of applications.

We assess these hypotheses by using the Job Vacancy Survey of the Institute for Employment Research (IAB JVS). The IAB JVS is a representative survey among human resource managers and managing directors in German firms reporting information about their search strategies with detailed answers about the most recent successful and the most recent unsuccessful recruitment case. Since most firms are observed only once, our sample is a repeated cross section of firms and includes more than 39,500 observations over the period 2012–2016.

To obtain an empirical proxy for the intensity of formal job search, we define an ordinal variable equal to the number of formal search activities used by the firm. The list of such activities includes: placed ads in newspapers or magazines, posted vacancy in the Internet, posted vacancy on the firm's own Web site, and contacted the federal employment agency.

Our empirical results confirm hypotheses (1)–(6) derived from our model. More generally, we show that firm characteristics and requirements of posted vacancies, like skill requirements, are central to understanding firms' choice of search channels. For example, we find that firms are less likely to use formal search if they need an applicant with leadership abilities or large experience. In addition, our analysis reveals that unemployed workers are more likely to enter employment through the formal channel, whereas employed workers changing jobs are more likely to be hired by referrals. Regarding hypothesis H0, we do find that larger firms are more likely to use the formal channel in addition to referral hiring but it is not the case that small firms do not use the formal channel at all. To account for this data property, the theoretical model can be adjusted with an idiosyncratic cost component, such that even small firms sometimes take formal actions to fill their positions if their idiosyncratic cost realization is sufficiently small. We leave this extension for future research.

There are two further issues that we consider important for future research. First, our theoretical model does not consider the intensity of referral search. Second, as we can see from the data, there is more than one formal activity that firms undertake to find new workers. In our model, we merge these activities into one formal channel; measuring the efficiency of each of these single activities remains also a task for further research.

APPENDIX

A.1 *Proof of Proposition 1.* Opening the square bracket in Equation (5) yields:

$$\frac{0.5c_0q^2(\theta)(1-\beta)^2}{(r+\delta)^2}r^2V^2(y) - V(y)\left[\frac{0.5c_0q^2(\theta)(1-\beta)^2}{(r+\delta)^2}2ry + \frac{q^N(u)(1-\beta)}{r+\delta}r + r + \phi\right]$$
$$+ \frac{0.5c_0q^2(\theta)(1-\beta)^2}{(r+\delta)^2}y^2 + \frac{q^N(u)(1-\beta)}{r+\delta}y - c = 0.$$

Using the definition of variables $A(\theta)$ and $B(u)$, this equation can be rewritten as:

$$A(\theta)r^2V^2(y) - V(y)[A(\theta)2ry + B(u)r + r + \phi] + A(\theta)y^2 + B(u)y - c = 0.$$

The discriminant of this quadratic equation is given by:

$$\begin{aligned} D &= [A(\theta)2ry + B(u)r + r + \phi]^2 - 4A(\theta)r^2[A(\theta)y^2 + B(u)y - c] \\ &= 4A^2(\theta)r^2y^2 + 4A(\theta)ry(B(u)r + r + \phi) + (B(u)r + r + \phi)^2 + 4A(\theta)r^2c \\ &- 4A^2(\theta)r^2y^2 - 4A(\theta)r^2B(u)y = (B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]. \end{aligned}$$

This equation has two roots; however, the upper root exceeds y/r and cannot be the equilibrium solution. So, we get:

$$V(y) = \frac{A(\theta)2ry + B(u)r + r + \phi - \sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]}}{2A(\theta)r^2}$$

so that $rV(y)$ is given by:

$$rV(y) = y - \frac{1}{2A(\theta)r}\left[\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]} - (B(u)r + r + \phi)\right] < y.$$

Differentiating $rV(p)$ with respect to $y(p)$, we obtain:

$$\begin{aligned} \frac{\partial rV(y)}{\partial y} &= 1 - \frac{4rA(\theta)(r + \phi)}{4rA(\theta)\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(p)(r + \phi) + rc]}} \\ &> 1 - \frac{(r + \phi)}{\sqrt{(B(u)r + r + \phi)^2}} = \frac{rB(u)}{r + \phi + rB(u)} > 0. \end{aligned}$$

Differentiating $rV(y)$ with respect to $A(\theta)$, we obtain:

$$\begin{aligned} \frac{\partial rV(y)}{\partial A(\theta)} &= \frac{1}{2rA^2(\theta)}\left[\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]} - (B(u)r + r + \phi)\right] \\ &- \frac{4r[y(r + \phi) + rc]}{4rA(\theta)\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]}} \\ &= \frac{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc] - 2rA(\theta)[y(r + \phi) + rc]}{2rA^2(\theta)\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]}} \\ &- \frac{(B(u)r + r + \phi)\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]}}{2rA^2(\theta)\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]}}. \end{aligned}$$

The denominator of this expression is positive, so we focus on the analysis of the numerator, which is given by:

$$\begin{aligned} &- (B(u)r + r + \phi)\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r + \phi) + rc]} + (B(u)r + r + \phi)^2 \\ &+ 2A(\theta)r[y(r + \phi) + rc] \\ &= -\sqrt{(B(u)r + r + \phi)^4 + 4A(\theta)r[y(r + \phi) + rc](B(u)r + r + \phi)^2 \pm [2rA(\theta)[y(r + \phi) + rc]]^2} \end{aligned}$$

$$+ (B(u)r + r + \phi)^2 + 2A(\theta)r[y(r+\phi) + rc]$$
$$= -\sqrt{[(B(u)r + r + \phi)^2 + 2rA(\theta)[y(r+\phi) + rc]]^2 - [2rA(\theta)[y(r+\phi) + rc]]^2}$$
$$+ (B(u)r + r + \phi)^2 + 2A(\theta)r[y(r+\phi) + rc] > 0,$$

which means that $\partial rV(y)/\partial A(\theta) > 0$. Next, we consider the derivative of $rV(y)$ with respect to $B(u)$:

$$\frac{\partial rV(y)}{\partial B(u)} = \frac{1}{2rA(\theta)}\left(r - \frac{2r(rB(u) + r + \phi)}{2\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r+\phi) + rc]}}\right)$$
$$= \frac{1}{2A(\theta)} \frac{\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r+\phi) + rc]} - (B(u)r + r + \phi)}{\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r+\phi) + rc]}} > 0.$$

Finally, we consider the limiting case when $\theta \to 0$, which means that $q(\theta) \to \infty$ and $A(\theta) \to \infty$. We get:

$$\lim_{A(\theta)\to\infty} rV(y) = y - \lim_{A(\theta)\to\infty} \frac{1}{2r}\left[\sqrt{\frac{(B(u)r + r + \phi)^2}{A^2(\theta)} + \frac{4r[y(r+\phi) + rc]}{A(\theta)}} - \frac{(B(u)r + r + \phi)}{A(\theta)}\right] = y,$$

which completes the proof of Proposition 1.

A.2 *Proof of Proposition 2.* Insert $V(p)$ into the equation for $a(p)$ to get:

$$a(y) = c_0 q(\theta)(J(y) - V(y)) = \frac{c_0 q(\theta)(1-\beta)}{r+\delta}(y - rV(y)) =$$
$$= \frac{c_0 q(\theta)(1-\beta)}{(r+\delta)2rA(\theta)}\left[\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r+\phi) + rc]} - (B(u)r + r + \phi)\right]$$
$$= \frac{(r+\delta)}{rq(\theta)(1-\beta)}\left[\sqrt{(B(u)r + r + \phi)^2 + 4A(\theta)r[y(r+\phi) + rc]} - (B(u)r + r + \phi)\right]$$
$$= \frac{(r+\delta)}{r(1-\beta)}\left[\sqrt{\frac{(B(u)r + r + \phi)^2}{q^2(\theta)} + \frac{2r[y(r+\phi) + rc]c_0(1-\beta)^2}{(r+\delta)^2}} - \frac{(B(u)r + r + \phi)}{q(\theta)}\right].$$

Let $X = (B(u)r + r + \phi)/q(\theta)$; obviously, X is increasing in $B(u)$ but decreasing in $q(\theta)$. Search intensity $a(y)$ can then be rewritten as:

$$a(y) = \frac{(r+\delta)}{r(1-\beta)}\left[\sqrt{X^2 + \frac{2r[y(r+\phi) + rc]c_0(1-\beta)^2}{(r+\delta)^2}} - X\right].$$

Differentiating $a(y)$ with respect to X, we get:

$$\frac{\partial a(y)}{\partial X} = \frac{(r+\delta)}{r(1-\beta)}\left(\frac{2X}{2\sqrt{X^2 + \frac{2r[y(r+\phi)+rc]c_0(1-\beta)^2}{(r+\delta)^2}}} - 1\right)$$

$$= \frac{(r+\delta)}{r(1-\beta)} \left(\frac{X - \sqrt{X^2 + \frac{2r[y(r+\phi)+rc]c_0(1-\beta)^2}{(r+\delta)^2}}}{\sqrt{X^2 + \frac{2r[y(r+\phi)+rc]c_0(1-\beta)^2}{(r+\delta)^2}}} \right) < 0.$$

This means that $\partial a(y)/\partial B(u) < 0$ and $\partial a(y)/\partial q(\theta) > 0$.

A.3 *Proof of Proposition 3.* Rewrite equation for y_0 as follows:

$$A(\theta)y_0^2 - y_0(2rKA(\theta) - B(u)) + A(\theta)r^2K^2 - K(rB + r + \phi) - c = 0.$$

The discriminant of this quadratic equation is:

$$\begin{aligned} D &= (2rKA(\theta) - B(u))^2 - 4A(\theta)(A(\theta)r^2K^2 - K(rB(u) + r + \phi) - c) \\ &= 4r^2K^2A^2(\theta) - 4rKA(\theta)B(u) + B^2(u) - 4A^2(\theta)r^2K^2 + 4A(\theta)KrB(u) \\ &+ 4A(\theta)K(r+\phi) + 4A(\theta)c = B^2(u) + 4A(\theta)K(r+\phi) + 4A(\theta)c. \end{aligned}$$

This equation has two roots; however, the lower root is less than rK that cannot be an equilibrium outcome. This yields:

$$\begin{aligned} y_0^{II} &= \frac{2rKA(\theta) - B(u) + \sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]}}{2A(\theta)} \\ &= rK + \frac{1}{2A(\theta)} \left(\sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]} - B(u) \right) \\ &= rK + \frac{(r+\delta)^2}{c_0q^2(\theta)(1-\beta)^2} \left(\sqrt{\frac{(q^N(u))^2(1-\beta)^2}{(r+\delta)^2} + 4A(\theta)[K(r+\phi)+c]} - \frac{q^N(u)(1-\beta)}{r+\delta} \right) \\ &= rK + \frac{r+\delta}{c_0q^2(\theta)(1-\beta)} \left(\sqrt{q^N(u)^2 + 2c_0q^2(\theta)[K(r+\phi)+c]} - q^N(u) \right). \end{aligned}$$

Differentiating y_0^{II} with respect to $A(\theta)$, we get:

$$\begin{aligned} \frac{\partial y_0^{II}}{\partial A(\theta)} &= -\frac{\sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]} - B(u)}{2A^2(\theta)} + \frac{1}{2A(\theta)} \frac{4[K(r+\phi)+c]}{2\sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]}} \\ &= \frac{-(B^2(u) + 4A(\theta)[K(r+\phi)+c]) + B(u)\sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]} + 2A(\theta)[K(r+\phi)+c]}{2A(\theta^2)\sqrt{B^2(u) + 4A(\theta)]K(r+\phi)+c]}}. \end{aligned}$$

Since the denominator is positive, we continue analyzing the sign of the numerator:

$$\begin{aligned} &\sqrt{B^4(u) + 4A(\theta)[K(r+\phi)+c]B^2(u)} - (B^2(u) + 2A(\theta)[K(r+\phi)+c]) \\ &= \sqrt{B^4(u) + 4A(\theta)[K(r+\phi)+c]B^2(u) \pm (2A(\theta)[K(r+\phi)+c])^2} - (B^2(u) + 2A(\theta)[K(r+\phi)+c]) \\ &= \sqrt{(B^2(u) + 2A(\theta)[K(r+\phi)+c])^2 - (2A(\theta)[K(r+\phi)+c])^2} - (B^2(u) + 2A(\theta)[K(r+\phi)+c]) < 0. \end{aligned}$$

This means that $\partial y_0^{II}/\partial A(\theta) < 0$. Next, we differentiate y_0^{II} with respect to $B(u)$:

$$\frac{\partial y_0^{II}}{\partial B(u)} = \frac{1}{2A(\theta)} \left(\frac{2B(u)}{2\sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]}} - 1 \right)$$

$$= \frac{1}{2A(\theta)}\left(\frac{B(u) - \sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]}}{\sqrt{B^2(u) + 4A(\theta)[K(r+\phi)+c]}}\right) < 0.$$

And finally, we consider the limiting case when $\theta \to 0$:

$$\lim_{\theta\to 0} y_0^{II} = \lim_{A(\theta)\to\infty} y_0^{II} = rK + \frac{1}{2}\left(\sqrt{\left(\frac{B(u)}{A(\theta)}\right)^2 + \frac{4[K(r+\phi)+c]}{A(\theta)}} - \frac{B(u)}{A(\theta)}\right) = rK.$$

A.4 *Proof of Proposition 5.* Consider, at first, first equilibrium I and differentiate $V(\theta, u)$ with respect to θ:

$$\begin{aligned}\frac{rb}{\gamma}\frac{\partial V(\theta,u)}{\partial\theta} &= \frac{\partial y_c(\theta,u)}{\partial\theta}(V^0(y_c(\theta,u),u) - K)\\ &\quad - \frac{\partial y_c(\theta,u)}{\partial\theta}(V(y_c(\theta,u),\theta,u) - K) + \int_{y_c(\theta,u)}^{b-\bar{w}} \frac{\partial V(y,\theta,u)}{\partial\theta}dy < 0,\end{aligned}$$

because $V^0(y_c) = V(y_c)$ and $\partial V(y,\theta,u)/\partial\theta < 0$. This proves that $V(\theta,u)$ is a decreasing function of θ in equilibrium I. Next, we differentiate $V(\theta,u)$ with respect to u:

$$\begin{aligned}\frac{rb}{\gamma}\frac{\partial V(\theta,u)}{\partial u} &= -\frac{\partial y_0^I(u)}{\partial u}(V^0(y_0^I(u),u) - K) + \frac{\partial y_c(\theta,u)}{\partial u}(V^0(y_c(\theta,u),u) - K)\\ &\quad - \frac{\partial y_c(\theta,u)}{\partial u}(V(y_c(\theta,u),\theta,u) - K) + \int_{y_0^I(u)}^{y_c(\theta,u)} \frac{\partial V^0(y,u)}{\partial u}\\ &\quad + \int_{y_c(\theta,u)}^{b-\bar{w}} \frac{\partial V(y,\theta,u)}{\partial u}dy > 0,\end{aligned}$$

because $V^0(y_0^I) = K$, $V^0(y_c) = V(y_c)$, $\partial V^0(y,u)/\partial u > 0$, and $\partial V(y,\theta,u)/\partial u > 0$. This proves that equation $K = V(\theta,u)$ implies $\partial\theta/\partial u > 0$ in equilibrium I. Next consider equilibrium II and differentiate $V(\theta,u)$ with respect to θ:

$$\frac{rb}{\gamma}\frac{\partial V(\theta,u)}{\partial\theta} = -\frac{\partial y_0^{II}(\theta,u)}{\partial\theta}(V(y_0^{II}(\theta,u),\theta,u) - K) + \int_{y_0^{II}(\theta,u)}^{b-\bar{w}} \frac{\partial V(y,\theta,u)}{\partial\theta}dy < 0,$$

because $V(y_0^{II}) = K$ and $\partial V(y,\theta,u)/\partial\theta < 0$. This proves that $V(\theta,u)$ is a decreasing function of θ in equilibrium II. Next, we differentiate $V(\theta,u)$ with respect to u:

$$\frac{rb}{\gamma}\frac{\partial V(\theta,u)}{\partial u} = -\frac{\partial y_0^{II}(\theta,u)}{\partial u}(V(y_0^{II}(\theta,u),\theta,u) - K) + \int_{y_0^{II}(\theta,u)}^{b-\bar{w}} \frac{\partial V(y,\theta,u)}{\partial u}dy > 0,$$

because $V(y_0^{II}) = K$ and $\partial V(y,\theta,u)/\partial u > 0$. This proves that equation $K = V(\theta,u)$ implies $\partial\theta/\partial u > 0$ in equilibrium II.

A.5 *Equilibrium Unemployment and Business Cycle Properties.* Proposition 5 in Section 2.5 indicates that the threshold output values can be expressed as functions of unemployment, for example, $y_c^*(u) = y_c(u,\theta^*(u))$. This shows that y_c is increasing stronger with unemployment when the market tightness is endogenously derived from the free-entry condition rather than the market tightness is kept exogenous. On the one hand, higher unemployment makes referral hiring more attractive, so more firms decide to use referral hiring

as the only search channel. This is the direct effect of unemployment. In addition, if referral hiring becomes more attractive, more firms enter the market and open vacancies. Therefore, the number of vacancies increases, which makes competition in formal advertising tougher. This indirect effect via the market tightness implies that even less firms decide to advertise. What about the intensity of advertising? Here, we get $a^*(y,u) = a(y,\theta^*(u),u)$. We can see that both effects—direct and indirect via the market tightness—reduce the amount of time and resources that firms invest into formal advertising when the unemployment rate increases. Formally, we can summarize these results in the following way:

$$\frac{dy_c^*(u)}{du} = \frac{\partial y_c(u,\theta^*(u))}{\partial u} + \frac{\partial y_c(u,\theta^*(u))}{\partial \theta}\frac{\partial \theta^*(u)}{\partial u} > 0,$$
$$\frac{da^*(y,u)}{du} = \frac{\partial a(y,\theta^*(u),u)}{\partial u} + \frac{\partial a(y,\theta^*(u),u)}{\partial \theta}\frac{\partial \theta^*(u)}{\partial u} < 0.$$

In addition, note that the fraction of nonadvertising firms in equilibrium I is given by:

$$N(u) = 1 - \frac{b-\bar{w}-y_c^*(u)}{b-\bar{w}-y_0^I(u)} \quad \Rightarrow \quad \frac{\partial N(u)}{\partial u} > 0.$$

We can conclude for recessions with high unemployment rates: more firms rely on referral hiring as the only search channel ($\partial N(u)/\partial u > 0$), the number of advertising firms $1 - N(u)$ is lower and those firms that advertise their vacancies invest less effort ($\partial a^*(y,u)/\partial u < 0$). This also means that there are more applicants hired via referrals and the average number of search channels used by firms is lower in recessions. Testing business cycle properties of the model is beyond the scope of this article, but we can still compare these theoretical findings with the empirical results found in other studies. For example, Granovetter (1995) finds that "in recessions, word-of-mouth recruitment increases" (p. 159). This result based on the U.S. data is confirmed by Behtoui (2008) who reports a very high negative value of the Pearson correlation coefficient (−0.933) between the employment ratio and the use of informal search methods in the Swedish labor market. These results are in line with our theoretical conclusions. In addition, Shimer (2004) finds that in recessions when finding jobs is difficult for workers, they search more intensively and use a larger number of search channels to find jobs. We do not have workers' search in our model but one can expect the opposite result for firms, because finding applicants in recessions is an easy task for firms. So, firms should reduce their search intensity and use a lower number of search channels. Our results so far are consistent with this view even though they should be taken with care because unemployment itself is endogenous and there are several other indicators for the business cycle used in the literature such as technological progress, matching efficiency, and job destruction. For example, Galeotti and Merlino (2003) find a nonlinear relationship between the job-destruction rate and the fraction of referral hires in the British labor market. At this point, we leave a deeper investigation of the issue for future research since a proper model of search intensity and referral hiring over the business cycle should include endogenous choices of workers which we do not have in our model.

In equilibrium II, the situation is ambiguous. With endogenous market tightness, the reservation cutoff value is given by $y_0^*(u) = y_0^{II}(\theta^*(u),u)$. This expression shows that the direct and the indirect effects tend to neutralize each other. On the one hand, higher unemployment makes referral hiring more attractive, so more firms want to enter the market, but on the other hand, stronger competition in formal advertising makes new entry less attractive.

Finally, we determine the equilibrium values of unemployment and vacancies. Considering networks, every open position is filled at rate $q^N(u) = s(1-(1-u)^{l+1})$, which implies that the total number of matches via the network is given by $sv(1-(1-u)^{l+1})$ in every period of time. The total number of matches via the formal channel is equal to $m(u,\tilde{v}) = um(1,\theta)$.

So, the total number of individuals finding jobs in a given period of time is a sum: $um(1,\theta) + sv(1-(1-u)^{l+1})$. At the same time, jobs can be destroyed at rate δ, so that $\delta(1-u)$ is an average number of workers losing jobs. In the steady state, it should be that the inflow of workers into unemployment is equal to the outflow ($\dot{u}=0$):

$$um(1,\theta) + sv(1-(1-u)^{l+1}) = \delta(1-u).$$

Inserting $\theta^*(u)$ from above, we obtain a modified version of the Beveridge curve that describes a negative relationship between unemployment u and vacancies v:

$$v = \frac{\delta(1-u) - um(1,\theta^*(u))}{s(1-(1-u)^{l+1})} \quad \Rightarrow \quad \frac{\partial v}{\partial u} < 0. \tag{A.1}$$

Next recall that $v(y) = \gamma v_0/\phi$, so another way of counting total vacancies v that should also hold in the equilibrium is given by:

$$v = \int_{y_0(u)}^{b-\bar{w}} v(y)dy = \frac{\gamma v_0}{\phi}(b-\bar{w}-y_0(u)) \quad \Rightarrow \frac{v(y)}{v} = \frac{1}{(b-\bar{w}-y_0(u))},$$

where $y_0(u)$ is given by $y_0^I(u)$ in equilibrium I and by $y_0^*(u)$ in equilibrium II. In the first case, the definition of the market tightness θ implies:

$$\theta = \frac{\tilde{v}}{u} = \frac{1}{u}\int_{y_c^*(u)}^{b-\bar{w}} a^*(y,u)v(y)dy = \frac{v}{u}\int_{y_c^*(u)}^{b-\bar{w}} \frac{a^*(y,u)}{b-\bar{w}-y_0^I(u)}dy = \frac{v}{u}\bar{a}(u). \tag{A.2}$$

Here, $\bar{a}(u)$ is the average formal search intensity in the market, it can be expressed as a fraction of firms using formal advertising $(b-\bar{w}-y_c^*(u))/(b-\bar{w}-y_0^I(u))$ multiplied by the average search intensity of advertising firms plus the fraction of nonadvertising firms multiplied with 0:

$$\bar{a}(u) = \int_{y_c^*(u)}^{b-\bar{w}} \frac{a^*(y,u)}{b-\bar{w}-y_0^I(u)}dy = \frac{b-\bar{w}-y_c^*(u)}{b-\bar{w}-y_0^I(u)} \cdot \int_{y_c^*(u)}^{b-\bar{w}} \frac{a^*(y,u)}{b-\bar{w}-y_c^*(u)}dy \quad \Rightarrow \quad \frac{\partial \bar{a}(u)}{\partial u} < 0.$$

We already know from above that in equilibrium I, the fraction of advertising firms $(b-\bar{w}-y_c^*(u))/(b-\bar{w}-y_0^I(u))$ is decreasing with higher unemployment and those firms that continue advertising invest less resources. All of these effects imply that the average search intensity in the market $\bar{a}(u)$ is a decreasing function of u, which means that equation $v = \theta^*(u)u/\bar{a}(u)$ is strictly increasing in u. In the second equilibrium, we know that all participating firms are advertising their positions, so the average search intensity in the market is given by:

$$\bar{a}(u) = \int_{y_0^*(u)}^{b-\bar{w}} \frac{a^*(y,u)}{b-\bar{w}-y_0^*(u)}dy.$$

Here, the effect of u on the average search intensity is ambiguous because we do not know the exact change in the reservation productivity $y_0^*(u)$. Nevertheless, from Proposition 5, we know that variable $y_0^*(u)$ is likely to be not very sensitive to the unemployment rate u. So, if changes in $y_0^*(u)$ are not very pronounced and all participating firms reduce their search intensity $a^*(y,u)$, we would again expect that the average search intensity $\bar{a}(u)$ is decreasing in u. Therefore, if $\gamma > 2r^2bK/(b-\bar{w}-rK)^2$, the equilibrium unemployment rate is uniquely determined at the intersection between the decreasing Beveridge curve and the increasing curve

$v = u\theta^*(u)/\bar{a}(u)$ (Equations (A.1) and (A.2)).

$$\frac{\delta(1-u) - um(1,\theta(u))}{s(1-(1-u)^{l+1})} = \frac{\theta^*(u)u}{\bar{a}(u)}.$$

Here, $\gamma > 2r^2bK/(b-\bar{w}-rK)^2$ is a necessary condition for the equilibrium existence identified in Proposition 5 and equivalent to $K < \bar{V}$. It means that productivity draws should arrive sufficiently often to firms, so that firms can recover their ex ante entry cost K.

A.6 *Firm Size and Wages.* Referring to our theoretical findings about the relationship between firm size and wages (see end of Section 2.6), we use information from the BHP to explore this relationship for Germany. The BHP is a cross-sectional data set collected since 1975 for West Germany and since 1992 for East Germany. Every wave contains all the establishments in Germany, which are covered by the IAB Employment History (BeH) on June 30th. These are all establishments with at least one employee liable to social security on the reference date. Establishments with no employee subject to social security but with at least one marginal part-time employee are included since 1999. Annual waves can be combined to form a panel. The BHP contains information about the branch of industry and the location of an establishment. Furthermore, the BHP reports median and average daily wages, the numbers of employees liable to social security, and marginal part-time employees, both in total and broken down by gender, age, occupational status, qualification, and nationality. Additional data providing information about foundations and closures of establishments are also available (for details, see also https://fdz.iab.de/en/FDZ_Establishment_Data/Establishment_History_Panel.aspx, Hethey and Schmieder, 2010; Schmucker et al., 2018).

In a first step, we conducted an ordinary least square (OLS) regression to test the relationship between median daily wages for full-time employees and firm size for all firms in Germany that are listed in BHP between 1993 and 2016; we restricted the analysis to this observation period because valid data for East Germany are not available for 1993. We also generally restricted the analysis to full-time employees because we do not have exact information about part time hours, and therefore, cannot adequately control for working time in our sample.

Columns (1)–(4) of Table A1 report the results of specifications with various groups of explanatory variables to explain the median daily wages for full-time employees in real terms[19]. The specification in column (5) uses the median daily wages in Euros for full-time employees in nominal terms as variable to be explained as it is reported in the BHP. All specifications control for year fixed effects and federal states. We calculate Huber/White robust standard errors.

Column (1) refers to the specification that contains firm size classes with the smallest size class as reference. We see that, as expected and according to the theoretical model, the daily wage increases with the firm size class. Column (2) refers to a specification that, beside the firm size classes, additionally includes the employees' skill level, the share of female full-time employees, the average age, and the squared average age of all full-time employees. The coefficients of the firm size class are getting smaller but the positive pattern remains. The daily wages also increase with the skill level and decrease with the share of female employees. Furthermore, the daily wages increase with the average age and (slightly) decrease with the squared average age implying a concave functional relationship between average age and daily wage. Column (3) includes firm size classes, the firm age, and the squared firm age in month. Again, the daily wage increases with the firm size and, again, with even smaller coefficients than in columns (1) and (2), implying that further controls have empirical relevance.

[19] We deflated the reported nominal daily wages in Euros by consumer price indices provided by the Statistisches Bundesamt, see https://www.destatis.de/DE/Themen/Wirtschaft/Preise/Verbraucherpreisindex/_inhalt.html#sprg238918.

TABLE A1
FIRM SIZE AND MEDIAN DAILY WAGES

Dep. Var.	(1) Real Wage	(2) Real Wage	(3) Real Wage	(4) Real Wage	(5) Nominal Wage
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)					
10 ≤ ee < 50	11.181***	8.242***	8.927***	6.451***	7.327***
	(0.011)	(0.010)	(0.014)	(0.013)	(0.014)
50 ≤ ee < 250	19.880***	15.019***	14.891***	10.909***	12.067***
	(0.021)	(0.018)	(0.035)	(0.030)	(0.032)
250 ≤ ee	32.136***	25.193***	26.210***	20.457***	22.311***
	(0.055)	(0.046)	(0.131)	(0.108)	(0.114)
Employee shares by skill level, reference: share of unskilled workers					
Share of full-time ee with vocational training		18.344***		16.953***	19.247***
		(0.017)		(0.019)	(0.021)
Share of full-time ee with university degree		59.285***		56.382***	62.563***
		(0.038)		(0.043)	(0.047)
Share of female full-time ee		-15.724***		-15.552***	-17.742***
		(0.010)		(0.012)	(0.014)
Average age of full-time ee		1.859***		1.685***	1.902***
		(0.004)		(0.004)	(0.005)
Average age of full-time ee, squared		−0.020***		−0.019***	−0.022***
		(0.000)		(0.000)	(0.000)
Firm age, months			0.017***	0.022***	0.026***
			(0.000)	(0.000)	(0.000)
Firm age, months, squared			−0.000***	−0.000***	−0.000***
			(0.000)	(0.000)	(0.000)
Constant	27.070***	−25.803***	23.946***	−22.522***	-16.599***
	(0.023)	(0.075)	(0.037)	(0.087)	(0.096)
Observations	41795066	41360276	28838061	28444147	28444147
R^2	0.216	0.368	0.184	0.337	0.274

Robust standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
SOURCES: Establishment History Panel (BHP) 1993–2016, IAB Job Vacancy Survey 2012–2016. Own computations.

The firm age has a positive significant impact and the squared firm age has a very small but significant negative impact. Column (4) includes the results of a specification with all explanatory variables. Generally, the coefficients change only slightly compared with the previous specifications and the main findings are not altered. Column (5) includes the results of a specification with median daily wages for full-time employees in nominal terms (Euros). The magnitude of the coefficients slightly increases and the results do not alter our main findings.

Further, we restrict the full sample to firms observed in the IAB Job Vacancy Survey and reduce the observation period to 2012–2016 to match the data we used for the empirical analysis. First, we restrict the BHP sample to firms with their full observed employment histories and that are observed in at least one year in the observation period between 2012 and 2016. Columns (1)–(5) from Table A2 report results from specifications analogous to the previous table. We generally find that the magnitude of the coefficients and the size of the standard errors only slightly change;[20] thus, our main findings remain robust.

Second, we take a sample from the BHP with only those firms that were included in the IAB-JVS from 2012 to 2016, and, again, we observe their full reported employment histories. Columns (6)–(10) report the estimates that are based on this sample. The number of observations is much smaller now, resulting in higher standard errors compared with the previous results. The coefficients on the firm size classes "$10 \leq$ ee < 50" and "$50 \leq$ ee < 250" are also smaller; but the qualitative result that the wage increases with the firm size class is not altered. Vice versa the magnitudes of the coefficients on the skill level shares ("Share of full-time ee with vocational training," "Share of full-time ee with university degree") and the average age of the employees are larger but the qualitative findings that the wage increases with the skill level and the age are corroborated. The coefficient on firm age is smaller compared with previous results, but it remains significantly positive.

Third, we reconducted our analysis based on a sample with BHP information but only for those years when the observed firms were included in the IAB-JVS, thus in one of the years from 2012 to 2016. Note that, systematically, most of the firms are now observed for only one year (compare with the data section in the main article); therefore, all specifications based on this sample do not include dummy variables for the observation periods. Results are reported in columns (11)–(15) and, regarding the magnitudes of coefficients, do reveal only small differences to the results in columns (5) to (10). As a consequence of the further reduced number of observation, the estimates are less precise. This is reflected in larger standard errors. This is also the reason why the coefficient on firm age in the full specification reported in column (9) is not significant, though its magnitude and sign are the same as in column (14).

We conclude that there is a positive relationship between wages, firm size, and firm age in Germany as it is predicted by our theoretical model. Our results are robust to alternative sample restrictions and the choice of the dependent variable in terms of the daily wage for full-time employees either in real or nominal terms.

A.7 *Tables with Detailed Results.*

A.8 *Alternative Specification for the Model Based on the Negative Binomial II Distribution That Explains the Number of Applicants Per Vacancy and Search Duration (see Section 4).* Another negative binomial distribution is the so-called negative binomial I with the expectation and variance given by (Cameron and Trivedi, 2005, p. 676):

$$E[y|\mu, \delta] = \mu \qquad \text{Var}[y|\mu, \delta] = (1 + \delta)\mu \tag{A.3}$$

[20] Regarding the ladder, this is the case although the number of observations decreased, see at the bottom of Tables A1 and A2.

TABLE A2
FIRM SIZE AND MEDIAN DAILY WAGES

Dep. Var.	(1) Real Wage	(2) Real Wage	(3) Real Wage	(4) Real Wage	(5) Nominal Wage	(6) Real Wage	(7) Real Wage	(8) Real Wage	(9) Real Wage	(10) Nominal Wage	(11) Real Wage	(12) Real Wage	(13) Real Wage	(14) Real Wage	(15) Nominal Wage
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)															
10 ≤ ee < 50	10.967***	8.007***	8.824***	6.326***	7.071***	10.078***	5.746***	7.952***	4.045***	4.088***	10.010***	6.176***	7.748***	4.452***	4.463***
	(0.013)	(0.012)	(0.017)	(0.015)	(0.016)	(0.275)	(0.263)	(0.324)	(0.300)	(0.302)	(0.568)	(0.538)	(0.679)	(0.624)	(0.632)
50 ≤ ee < 250	19.864***	14.815***	14.773***	10.658***	11.643***	18.736***	11.977***	11.482***	6.383***	6.450***	18.489***	12.300***	11.356***	6.920***	6.944***
	(0.024)	(0.021)	(0.039)	(0.033)	(0.035)	(0.298)	(0.284)	(0.378)	(0.339)	(0.340)	(0.617)	(0.583)	(0.796)	(0.701)	(0.709)
250 ≤ ee	32.334***	25.021***	26.239***	20.307***	21.984***	36.896***	27.421***	28.865***	22.148***	22.355***	36.210***	27.494***	27.640***	21.836***	22.100***
	(0.059)	(0.050)	(0.141)	(0.116)	(0.122)	(0.385)	(0.351)	(0.771)	(0.627)	(0.630)	(0.819)	(0.737)	(1.672)	(1.359)	(1.373)
Employee shares by skill level, reference: share of unskilled workers															
Share of full-time ee		19.782***		18.333***	20.052***		43.968***		33.894***	34.359***		44.399***		33.967***	34.501***
with vocational training		(0.023)		(0.025)	(0.027)		(0.695)		(0.774)	(0.778)		(1.537)		(1.725)	(1.740)
Share of full-time ee		62.939***		59.676***	64.217***		111.563***		100.433***	101.426***		111.143***		99.429***	100.802***
with university degree		(0.047)		(0.053)	(0.056)		(0.783)		(0.915)	(0.920)		(1.736)		(2.077)	(2.100)
Share of female full-time ee		−16.522***		−16.500***	−18.255***		−15.631***		−17.428***	−17.594***		−16.251***		−17.892***	−17.972***
		(0.013)		(0.016)	(0.017)		(0.234)		(0.305)	(0.306)		(0.505)		(0.665)	(0.672)
Average age of		1.926***		1.772***	1.941***		2.875***		2.784***	2.787***		2.707***		2.535***	2.569***
full-time ee		(0.005)		(0.006)	(0.006)		(0.119)		(0.144)	(0.145)		(0.248)		(0.303)	(0.306)
Average age of		−0.020***		−0.020***	−0.022***		−0.026***		−0.029***	−0.029***		−0.025***		−0.027***	−0.027***
full-time ee, squared		(0.000)		(0.000)	(0.000)		(0.001)		(0.002)	(0.002)		(0.003)		(0.004)	(0.004)
Firm age,			0.012***	0.019***	0.017***			0.042***	0.011***	0.013***			0.037***	0.011	0.013*
months			(0.000)	(0.000)	(0.000)			(0.004)	(0.003)	(0.003)			(0.009)	(0.007)	(0.007)
Firm age,			0.000***	−0.000***	−0.000			−0.000***	0.000	0.000			−0.000**	0.000	0.000
months, squared			(0.000)	(0.000)	(0.000)			(0.000)	(0.000)	(0.000)			(0.000)	(0.000)	(0.000)
Constant	27.019***	−28.923***	22.606***	−27.285***	−18.535***	60.702***	−59.314***	54.164***	−41.766***	−41.760***	59.345***	−56.550***	53.521***	−37.215***	−38.017***
	(0.032)	(0.098)	(0.064)	(0.117)	(0.125)	(0.318)	(2.461)	(0.623)	(2.913)	(2.929)	(0.667)	(5.107)	(1.283)	(6.084)	(6.144)
Observations	29655317	29359320	20416035	20143338	20143338	168713	168487	82119	81912	81912	34594	34546	17012	16971	16971
R^2	0.196	0.354	0.163	0.322	0.268	0.189	0.461	0.133	0.437	0.439	0.190	0.463	0.132	0.432	0.433

Robust standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

NOTES: Columns (1)–(5) are based on a restricted sample from the BHP only to firms observed in the IAB Job Vacancy Survey. Columns (6)–(10) are based on a restricted sample with only those firms that were included in the IAB Job Vacancy Survey from 2012 to 2016. Columns (11)–(15) are based on a restricted sample with BHP information but only for those years when the observed firms were included in the IAB-JVS, thus in one of the years from 2012 to 2016.

SOURCES: Establishment History Panel (BHP) 1993–2016, IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A3
DETAILED EMPIRICAL RESULTS OF TABLE 3: PROBABILITY OF USING THE FORMAL CHANNEL IN RECRUITING PROCESSES AS A DEPENDENT VARIABLE, PROB($y_i^F = 1$)

	All Observations				Only Successful Recruitment Cases	
	(1) logit	(2) logit	(3) logit	(4) logit	(5) logit	(6) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)						
10 ≤ ee < 50	1.773***		1.724***	1.731***	2.015***	2.022***
	(0.061)		(0.060)	(0.060)	(0.078)	(0.076)
50 ≤ ee < 250	3.629***		3.454***	3.482***	4.163***	4.167***
	(0.252)		(0.234)	(0.241)	(0.288)	(0.288)
250 ≤ ee	7.886***		7.041***	7.151***	8.668***	8.568***
	(0.520)		(0.408)	(0.416)	(0.563)	(0.549)
Skill requirements, reference: w/o qualification						
Vocational training		1.901***	1.832***	1.775***	1.716***	1.667***
		(0.115)	(0.110)	(0.103)	(0.098)	(0.096)
University degree		2.995***	2.295***	2.077***	1.948***	1.947***
		(0.207)	(0.131)	(0.120)	(0.112)	(0.113)
Employee shares by skill level, reference: share of unskilled workers						
Share of skilled workers with vocational training				1.195	1.188	1.183
				(0.137)	(0.144)	(0.143)
Share of skilled workers with university degree				1.579***	1.667***	1.595***
				(0.224)	(0.229)	(0.224)
Additional skill requirements						
Leadership ability						0.712***
						(0.048)
Experience						0.996
						(0.033)
Further skills from training outside formal qualification						1.171***
						(0.042)
Foreign lang.						1.042
						(0.049)
Social skills						1.183***
						(0.032)
Observations	40944	39736	39736	39735	32329	32329
Pseudo-R^2	0.105	0.0785	0.112	0.112	0.101	0.103

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where a zero effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used exclusively referrals, exclusively formal ways, or both referrals and formal ways as search channels ($s_i \geq 0$, $0 \leq a_i \leq 4$).
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A4
DETAILED EMPIRICAL RESULTS OF TABLE 4: SEARCH INTENSITY IN FORMAL WAYS, a_i, AS A DEPENDENT VARIABLE

	All Observations				Only Successful Recruitment Cases	
	(1) ord. logit	(2) ord. logit	(3) ord. logit	(4) ord. logit	(5) ord. logit	(6) ord. logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)						
10 ≤ ee < 50	1.271***		1.254***	1.249***	1.263***	1.274***
	(0.056)		(0.048)	(0.049)	(0.058)	(0.059)
50 ≤ ee < 250	1.940***		1.901***	1.890***	1.884***	1.899***
	(0.095)		(0.079)	(0.076)	(0.079)	(0.080)
250 ≤ ee	3.017***		2.768***	2.739***	2.738***	2.744***
	(0.192)		(0.168)	(0.174)	(0.190)	(0.199)
Skill requirements, reference: w/o qualification						
Vocational training		1.435***	1.432***	1.451***	1.332***	1.288***
		(0.102)	(0.106)	(0.101)	(0.104)	(0.107)
University degree		2.235***	1.922***	2.027***	1.794***	1.703***
		(0.174)	(0.150)	(0.135)	(0.129)	(0.145)
Employee shares by skill level, reference: share of unskilled workers						
Share of skilled workers with vocational training				0.934	0.889	0.877
				(0.098)	(0.099)	(0.098)
Share of skilled workers with university degree				0.796*	0.792	0.796
				(0.109)	(0.114)	(0.119)
Additional skill requirements						
Leadership ability						1.092
						(0.089)
Experience						0.940
						(0.039)
Further skills from training outside formal qualification						1.261***
						(0.065)
Foreign lang.						0.977
						(0.079)
Social skills						1.042
						(0.060)
Observations	14497	14133	14133	14133	10083	10083
Pseudo-R^2	0.0400	0.0333	0.0425	0.0426	0.0327	0.0339

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$).
Estimated cutpoints for specification in column (4): $\kappa_0 = 0.8114301, \kappa_1 = 2.297124, \kappa_2 = 3.898334$.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A5
DETAILED EMPIRICAL RESULTS OF TABLE 5: SEARCH INTENSITY IN FORMAL WAYS AS A DEPENDENT VARIABLE, a_i, RESTRICTED SAMPLE

	All Observations				Only Successful Recruitment Cases	
	(1) ord. logit	(2) ord. logit	(3) ord. logit	(4) ord. logit	(5) ord. logit	(6) ord. logit
Number of employees (four firm size classes, reference: 1≤ ee < 10)						
10 ≤ ee < 50	1.351***		1.308***	1.307***	1.377***	1.385***
	(0.047)		(0.046)	(0.046)	(0.056)	(0.056)
50 ≤ ee < 250	2.369***		2.224***	2.226***	2.359***	2.352***
	(0.121)		(0.119)	(0.124)	(0.134)	(0.133)
250 ≤ ee	3.515***		3.056***	3.057***	3.143***	3.112***
	(0.305)		(0.264)	(0.274)	(0.273)	(0.270)
Skill requirements, reference: w/o qualification						
Vocational training		1.942***	1.907***	1.898***	1.877***	1.717***
		(0.149)	(0.134)	(0.124)	(0.128)	(0.116)
University degree		3.117***	2.555***	2.562***	2.486***	2.162***
		(0.238)	(0.148)	(0.173)	(0.168)	(0.155)
Employee shares by skill level, reference: share of unskilled workers						
Share of skilled workers with vocational training				1.046	1.007	1.014
				(0.119)	(0.125)	(0.127)
Share of skilled workers with university degree				1.001	1.007	1.022
				(0.152)	(0.161)	(0.166)
Additional skill requirements						
Leadership ability						1.081
						(0.068)
Experience						1.140***
						(0.052)
Further skills from training outside formal qualification						1.203***
						(0.048)
Foreign lang.						0.989
						(0.082)
Social skills						1.125***
						(0.043)
Observations	18698	18171	18171	18170	15205	15205
Pseudo-R^2	0.0357	0.0283	0.0418	0.0418	0.0397	0.0421

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used exclusively formal ways as search channels ($s_i = 0, 1 \leq a_i \leq 4$).
Estimated cutpoints for specification in column (4): $\kappa_1 = 0.9824285$, $\kappa_2 = 2.600384$, $\kappa_3 = 4.447658$.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A6
DETAILED EMPIRICAL RESULTS OF TABLE 6: SEARCH INTENSITY IN FORMAL WAYS, a_i, AND THE USAGE OF REFERRALS

	All Observations					Only Successful Recruitment Cases	
	(1) ord. logit	(2) ord. logit	(3) ord. logit	(4) ord. logit	(5) ord. logit	(6) ord. logit	(7) ord. logit
Usage of referrals	0.323***	0.347***	0.337***	0.359***	0.359***	0.272***	0.271***
	(0.013)	(0.014)	(0.015)	(0.016)	(0.016)	(0.012)	(0.012)
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)							
10 ≤ ee < 50		1.588***		1.544***	1.545***	1.749***	1.758***
		(0.036)		(0.034)	(0.034)	(0.038)	(0.037)
50 ≤ ee < 250		2.857***		2.720***	2.723***	3.131***	3.129***
		(0.124)		(0.114)	(0.116)	(0.138)	(0.133)
250 ≤ ee		4.395***		3.904***	3.909***	4.475***	4.430***
		(0.267)		(0.218)	(0.230)	(0.293)	(0.288)
Skill requirements, reference: w/o qualification							
Vocational training			1.789***	1.751***	1.747***	1.625***	1.524***
			(0.097)	(0.095)	(0.091)	(0.084)	(0.079)
University degree			2.796***	2.259***	2.227***	1.993***	1.825***
			(0.142)	(0.095)	(0.104)	(0.091)	(0.093)
Employee shares by skill level, reference: share of unskilled workers							
Share of skilled workers with vocational training					1.006	1.011	1.007
					(0.093)	(0.081)	(0.082)
Share of skilled workers with university degree					1.064	1.143	1.123
					(0.121)	(0.121)	(0.121)
Additional skill requirements							
Leadership ability							0.921**
							(0.030)
Experience							1.032
							(0.027)
Further skills from training outside formal qualification							1.245***
							(0.039)
Foreign lang.							1.002
							(0.059)
Social skills							1.172***
							(0.042)
Observations	40944	40944	39736	39736	39735	32329	32329
Pseud-R^2	0.0631	0.0801	0.0691	0.0835	0.0835	0.0922	0.0938

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both or either exclusively formal ways or referrals as search channels ($s_i \geq 0$, $0 \leq a_i \leq 4$). Estimated cutpoints for specification in column (5): $\kappa_1 = 0.0253469$, $\kappa_2 = 1.597462$, $\kappa_3 = 2.999108$, $\kappa_4 = 4.698802$.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A7
DETAILED EMPIRICAL RESULTS OF TABLE 7: PROBABILITY TO RECRUIT WITH REFERRALS AS A DEPENDENT VARIABLE (y_i^S)

	(1) logit	(2) logit	(3) logit	(4) logit	(5) logit	(6) logit	(7) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)							
10 ≤ ee < 50	0.839***		0.875**	0.880*	0.868**	0.871**	0.870*
	(0.056)		(0.058)	(0.064)	(0.058)	(0.060)	(0.064)
50 ≤ ee < 250	0.553***		0.617***	0.624***	0.615***	0.618***	0.620***
	(0.049)		(0.057)	(0.059)	(0.058)	(0.057)	(0.058)
250 ≤ ee	0.370***		0.445***	0.462***	0.447***	0.452***	0.458***
	(0.046)		(0.058)	(0.063)	(0.058)	(0.057)	(0.063)
Formal search index		0.648***	0.671***	0.674***	0.671***	0.672***	0.673***
		(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
Skill requirements, reference: w/o qualification							
Vocational training				0.886**			0.891*
				(0.054)			(0.061)
University degree				0.807***			0.876*
				(0.064)			(0.069)
Employee shares by skill level, reference: share of unskilled workers							
Share of skilled workers with vocational training					0.950		1.039
					(0.121)		(0.140)
Share of skilled workers with university degree					0.617***		0.758*
					(0.090)		(0.113)
Additional skill requirements							
Leadership ability						1.186***	1.230***
						(0.068)	(0.076)
Experience						1.049	1.059
						(0.059)	(0.064)
Further skills from training outside formal qualification						1.022	1.021
						(0.031)	(0.026)
Foreign lang.						0.820**	0.859**
						(0.071)	(0.067)
Social skills						0.806***	0.822***
						(0.036)	(0.036)
Former employment status, reference: out of labor force							
unemployed	0.578***	0.586***	0.578***	0.562***	0.572***	0.574***	0.558***
	(0.052)	(0.054)	(0.052)	(0.053)	(0.051)	(0.051)	(0.052)
employed	0.936	0.938	0.974	0.990	0.977	0.976	0.987
	(0.065)	(0.065)	(0.065)	(0.073)	(0.066)	(0.065)	(0.073)
Observations	10348	10348	10348	10083	10348	10348	10083
Pseudo-R^2	0.0310	0.0448	0.0540	0.0554	0.0552	0.0563	0.0578

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$).
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A8
DETAILED EMPIRICAL RESULTS OF TABLE 8: PROBABILITY TO RECRUIT UNEMPLOYED WORKERS WITH REFERRALS AS A DEPENDENT VARIABLE (y_i^S), RESTRICTED SAMPLE

	(1) logit	(2) logit	(3) logit	(4) logit	(5) logit	(6) logit	(7) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)							
10 ≤ ee < 50	0.833		0.866	0.861	0.867	0.860	0.852
	(0.126)		(0.137)	(0.143)	(0.136)	(0.136)	(0.140)
50 ≤ ee < 250	0.652***		0.745*	0.749*	0.744*	0.738*	0.741*
	(0.108)		(0.127)	(0.130)	(0.127)	(0.123)	(0.126)
250 ≤ ee	0.350***		0.439***	0.425***	0.439***	0.439***	0.422***
	(0.070)		(0.091)	(0.092)	(0.092)	(0.091)	(0.092)
Formal search index		0.655***	0.674***	0.671***	0.674***	0.688***	0.681***
		(0.032)	(0.033)	(0.034)	(0.033)	(0.031)	(0.034)
Skill requirements, reference: w/o qualification							
Vocational training				0.699***			0.742**
				(0.081)			(0.105)
University degree				0.649**			0.767
				(0.112)			(0.152)
Employee shares by skill level, reference: share of unskilled workers							
Share of skilled workers with vocational training					0.876		1.043
					(0.179)		(0.238)
Share of skilled workers with university degree					0.731		0.999
					(0.252)		(0.374)
Additional skill requirements							
Leadership ability						0.973	1.029
						(0.191)	(0.191)
Experience						0.952	0.988
						(0.077)	(0.083)
Further skills from training outside formal qualification						0.732***	0.751***
						(0.078)	(0.082)
Foreign lang.						0.778	0.794
						(0.179)	(0.168)
Social skills						0.896	0.907
						(0.095)	(0.102)
Observations	2618	2618	2618	2551	2618	2618	2551
Pseudo-R^2	0.0265	0.0437	0.0486	0.0537	0.0489	0.0536	0.0572

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$). Sample is further restricted to cases with recruitment of former unemployed.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A9
DETAILED EMPIRICAL RESULTS OF TABLE 9: NUMBER OF APPLICANTS PER VACANCY AND SEARCH DURATION AS A DEPENDENT VARIABLE

	(1) Neg. Bin. II	(2) Neg. Bin. II	(3) Neg. Bin. II	(4) Neg. Bin. II	(5) Neg. Bin. II	(6) Neg. Bin. II	(7) Neg. Bin. II	(8) Neg. Bin. II
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)								
10 ≤ ee < 50	0.325***		0.276***	0.276***	0.291***	0.297***	0.324***	0.303***
	(0.086)		(0.087)	(0.088)	(0.081)	(0.083)	(0.091)	(0.087)
50 ≤ ee < 250	0.669***		0.558***	0.560***	0.550***	0.570***	0.718***	0.688***
	(0.104)		(0.104)	(0.103)	(0.098)	(0.098)	(0.139)	(0.133)
250 ≤ ee	0.951***		0.807***	0.832***	0.783***	0.845***	1.165***	1.152***
	(0.109)		(0.110)	(0.114)	(0.109)	(0.108)	(0.166)	(0.161)
Formal search index		0.216***	0.164***	0.177***	0.163***	0.177***	0.038	0.047
		(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.041)	(0.038)
Skill requirements, reference: w/o qualification								
Vocational training				−0.352***		−0.326***	−0.478***	−0.448***
				(0.070)		(0.070)	(0.107)	(0.117)
University degree				−0.385***		−0.493***	−0.705***	−0.667***
				(0.076)		(0.088)	(0.136)	(0.160)
Employee shares by skill level, reference: share of unskilled workers								
Share of skilled workers with vocational training					−0.534***	−0.344***	−0.262*	−0.245*
					(0.137)	(0.126)	(0.134)	(0.135)
Share of skilled workers with university degree					0.039	0.381***	0.632***	0.581***
					(0.098)	(0.123)	(0.182)	(0.184)
Additional skill requirements								
Leadership ability								−0.367***
								(0.097)
Experience								0.021
								(0.103)
Further skills from training outside formal qualification								−0.376***
								(0.079)
Foreign lang.								0.200**
								(0.096)
Social skills								0.122*
								(0.073)
α	0.914***	0.923***	0.895***	0.895***	0.886***	0.886***	1.038***	1.022***
	(0.087)	(0.092)	(0.085)	(0.085)	(0.085)	(0.086)	(0.135)	(0.133)
Number of observations	27394	27394	27394	26947	27392	26946	7335	7335
Pseudo-R^2	0.0613	0.0581	0.0661	0.0683	0.0687	0.0711	0.0973	0.102
Log likelihood	−22345	−22424	−22233	−21887	−22170	−21820	−5933	−5904

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

NOTES: Coefficients are reported as semielasticities, a one-unit change in x_i changes the conditional mean, μ_i, by the multiplier $1 + \beta_i$. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Results in columns (7) and (8) are from specifications that are based on a restricted sample of firms that report successful recruitment and used both formal channels and referrals ($s_i = 1, 1 \leq a_i \leq 4$).

SOURCE: IAB Job Vacancy Survey 2012−2016. Own computations.

TABLE A10
DETAILED EMPIRICAL RESULTS OF TABLE 12: ORDINAL LEAST SQUARE REGRESSION FOR THE ADDITIONAL RECRUITING COSTS IN EURO

	(1)	(2)	(3)
	Based on a Sample of Firms that Report Only Successful Recruitment Cases and that Used		
	Both Referrals and Formal Strategies ($s_i = 1, 1 \leq a_i \leq 4$) OLS	Only Formal Strategies ($s_i = 0, 1 \leq a_i \leq 4$) OLS	Either Referrals or Formal Strategies or Both ($s_i \geq 0, 0 \leq a_i \leq 4$) OLS
Usage of referrals			−494.924***
			(81.853)
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)			
10 ≤ ee < 50	85.539	280.372***	171.133***
	(89.886)	(91.287)	(43.631)
50 ≤ ee < 250	352.419***	601.989***	448.039***
	(107.081)	(83.685)	(62.188)
250 ≤ ee	868.658***	1169.397***	1053.971***
	(144.390)	(189.562)	(169.134)
Skill requirements, reference: w/o qualification			
Vocational training	20.871	91.032	37.242
	(74.616)	(70.799)	(33.126)
University degree	1237.275***	1523.811***	1199.763***
	(286.989)	(201.437)	(170.540)
Employee shares by skill level, reference: share of unskilled workers			
Share of skilled workers with	637.811***	459.491**	321.158***
vocational training	(164.272)	(182.368)	(107.287)
Share of skilled workers with	−156.560	207.415	−111.970
University degree	(344.931)	(383.145)	(196.771)
Additional skill requirements			
Leadership ability	1033.690**	1923.081***	1171.295***
	(362.938)	(352.614)	(216.820)
Experience	187.129*	448.958***	268.595***
	(104.511)	(90.799)	(55.551)
Further skills from training outside formal qualification	115.333	28.718	60.977
	(97.040)	(168.263)	(87.970)
Foreign lang.	202.423	663.778**	401.102***
	(190.584)	(277.699)	(110.845)

(*Continued*)

TABLE A10
CONTINUED

	(1)	(2)	(3)
	Based on a Sample of Firms that Report Only Successful Recruitment Cases and that Used		
	Both Referrals and Formal Strategies ($s_i = 1, 1 \leq a_i \leq 4$) OLS	Only Formal Strategies ($s_i = 0, 1 \leq a_i \leq 4$) OLS	Either Referrals or Formal Strategies or Both ($s_i \geq 0, 0 \leq a_i \leq 4$) OLS
Social skills	9.795	−25.395	40.455
	(125.404)	(137.529)	(92.492)
Constant	−583.001***	−745.738***	−254.770**
	(175.297)	(232.064)	(103.543)
Observations	3589	5128	11207
R^2	0.099	0.135	0.111

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: The specification and the sample restriction for column (1) refer to the specification and the sample restriction in column (6) of Table 4. The specification and the sample restriction for column (2) refer to the specification and the sample restriction in column (6) of Table 5. The specification and the sample restriction for column (3) refer to the specification and the sample restriction in column (7) of table of Table 6. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

TABLE A11
DETAILED EMPIRICAL RESULTS OF COLUMNS (1)–(4) OF TABLE 13: PROBABILITY TO RECRUIT WITH REFERRALS AS A DEPENDENT VARIABLE (y_i^S)

	(1) logit	(2) logit	(3) logit	(4) logit
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)				
10 ≤ ee < 50	0.837	0.872	0.696	0.740
	(0.102)	(0.0982)	(0.187)	(0.197)
50 ≤ ee < 250	0.618***	0.681**	0.757	0.861
	(0.100)	(0.107)	(0.206)	(0.240)
250 ≤ ee	0.366***	0.413***	0.358**	0.404*
	(0.0965)	(0.108)	(0.166)	(0.188)
Recruitment effort (Additional costs in 1000 Euros)	0.909***	0.947**	0.864	0.903
	(0.0286)	(0.0207)	(0.119)	(0.0874)
Formal search index		0.674***		0.779***
		(0.0242)		(0.0592)
Skill requirements, reference: w/o qualification				
Vocational training	0.937	0.999	0.884	0.888
	(0.114)	(0.131)	(0.188)	(0.196)
University degree	0.855	0.914	0.654	0.661
	(0.140)	(0.158)	(0.297)	(0.313)
Employee shares by skill level, reference: share of unskilled workers				
Share of skilled workers with vocational training	1.338*	1.222	1.682	1.615
	(0.230)	(0.253)	(0.587)	(0.549)
Share of skilled workers with university degree	1.246	1.119	3.688**	3.571**
	(0.276)	(0.292)	(2.303)	(2.215)
Additional skill requirements				
Leadership ability	1.300**	1.329***	0.465*	0.485*
	(0.145)	(0.145)	(0.185)	(0.192)
Experience	1.090	1.085	1.451	1.442
	(0.0942)	(0.0863)	(0.339)	(0.329)
Further skills from training outside formal qualification	1.132	1.192*	0.585***	0.625**
	(0.103)	(0.115)	(0.115)	(0.117)
Foreign lang.	0.806*	0.780**	0.401**	0.406**
	(0.0972)	(0.0977)	(0.150)	(0.164)
Social skills	0.714***	0.721***	1.040	1.039
	(0.0701)	(0.0761)	(0.217)	(0.235)
Former employment status, reference: out of labor force				
unemployed	0.572***	0.564***		
	(0.0597)	(0.0694)		
employed	0.965	0.982		
	(0.0608)	(0.0726)		
Constant	2.679***	4.937***	1.600	2.395*
	(0.884)	(1.952)	(0.787)	(1.160)
Observations	3589	3589	916	916
Pseudo-R^2	0.0449	0.0657	0.0676	0.0749

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as odds ratios, where the null effect is 1. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Firms size class based on all employees, regardless whether employees are subject to social security contributions or not. Firms are considered that used both referrals and formal ways as search channels ($s_i = 1, 1 \leq a_i \leq 4$). Sample is restricted to successful recruitment cases. Estimation results of the specification in columns (3) and (4) are based on a sample that is further restricted to cases with recruitment of former unemployed.
SOURCE: IAB vacancy survey 2014 and 2015.

TABLE A12
DETAILED EMPIRICAL RESULTS OF COLUMNS (5)–(8) OF TABLE 13: NUMBER OF APPLICANTS PER VACANCY AND SEARCH DURATION AS A DEPENDENT VARIABLE, NB II MODEL

	(5) Neg. Bin. II	(6) Neg. Bin. II	(7) Neg. Bin. II	(8) Neg. Bin. II
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)				
10 ≤ ee < 50	1.482***	1.359**	0.368***	0.324**
	(0.180)	(0.164)	(0.129)	(0.136)
50 ≤ ee < 250	1.937***	1.651***	0.767***	0.704***
	(0.253)	(0.222)	(0.159)	(0.171)
250 ≤ ee	3.162***	2.545***	1.460***	1.362***
	(0.330)	(0.316)	(0.202)	(0.203)
Recruitment effort (Additional costs in 1000 Euros)	1.027*	1.012	0.022	0.014
	(0.015)	(0.014)	(0.017)	(0.017)
Formal search index		1.290***		0.163***
		(0.045)		(0.034)
Skill requirements, reference: w/o qualification				
Vocational training	0.788**	0.715***	−0.575***	−0.636***
	(0.077)	(0.071)	(0.148)	(0.151)
University degree	0.623***	0.573***	−1.046***	−1.093***
	(0.062)	(0.056)	(0.241)	(0.234)
Employee shares by skill level, reference: share of unskilled workers				
Share of skilled workers with vocational training	0.742	0.736	−0.591**	−0.567*
	(0.175)	(0.165)	(0.290)	(0.290)
Share of skilled workers with university degree	1.159	1.122	0.144	0.144
	(0.290)	(0.266)	(0.256)	(0.239)
Additional skill requirements				
Leadership ability			−0.032	−0.024
			(0.150)	(0.162)
Experience			−0.054	−0.046
			(0.113)	(0.112)
Further skills from training outside formal qualification			−0.361***	−0.370***
			(0.065)	(0.067)
Foreign lang.			0.276*	0.272*
			(0.162)	(0.160)
Social skills			0.087	0.074
			(0.100)	(0.093)
α	1.019***	0.976***	1.166***	1.150***
	(0.111)	(0.116)	(0.132)	(0.134)
Number of observations	7761	7761	2619	2619
Pseudo-R^2	0.0618	0.0727	0.112	0.115
Log likelihood	−7242	−7158	−2287	−2280

Federal states cluster robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as semielasticities, a one-unit change in x_i changes the conditional mean, μ_i, by the multiplier $1 + \beta_i$. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. Sample is restricted to successful recruitment cases.
SOURCE: IAB vacancy survey 2014 and 2015.

The variance, $Var[.]$, exceeds the mean in case of $\delta > 0$ and $\mu > 0$. We use this alternative specification for a further robustness check; again we define:

$$\mu_i \equiv y_i = \exp(b_0 + \mathbf{b_1 x_i}). \tag{A.4}$$

The maximum likelihood to estimate parameters $b_0, \mathbf{b_1}$, and the additional parameter δ is then:

$$\ell_i(b_0, \mathbf{b_1}, \delta) = \delta^{-1} \exp(b_0 + \mathbf{b_1 x_i}) \log\left[\frac{\delta^{-1}}{\delta^{-1}+1}\right] + y_i \log\left[\frac{1}{\delta^{-1}+1}\right] \tag{A.5}$$

TABLE A13
ROBUSTNESS CHECKS: NUMBER OF APPLICANTS PER VACANCY AND SEARCH DURATION AS A DEPENDENT VARIABLE, NB I MODEL

	(1) Neg. Bin. I	(2) Neg. Bin. I	(3) Neg. Bin. I	(4) Neg. Bin. I	(5) Neg. Bin. I	(6) Neg. Bin. I	(7) Neg. Bin. I	(8) Neg. Bin. I
Number of employees (four firm size classes, reference: 1 ≤ ee < 10)								
10 ≤ ee < 50	0.223***		0.191***	0.193***	0.199***	0.206***	0.129***	0.124***
	(0.029)		(0.029)	(0.027)	(0.029)	(0.026)	(0.032)	(0.033)
50 ≤ ee < 250	0.463***		0.387***	0.396***	0.385***	0.405***	0.345***	0.338***
	(0.026)		(0.029)	(0.030)	(0.029)	(0.030)	(0.035)	(0.037)
250 ≤ ee	0.699***		0.588***	0.614***	0.575***	0.626***	0.664***	0.652***
	(0.035)		(0.038)	(0.037)	(0.039)	(0.036)	(0.046)	(0.050)
Formal search index		0.161***	0.128***	0.135***	0.127***	0.136***	0.065***	0.070***
		(0.009)	(0.008)	(0.008)	(0.008)	(0.008)	(0.021)	(0.021)
Skill requirements, reference: w/o qualification								
Vocational training				−0.166***		−0.170***	−0.226***	−0.222***
				(0.036)		(0.037)	(0.043)	(0.050)
University degree				−0.220***		−0.323***	−0.357***	−0.360***
				(0.045)		(0.050)	(0.075)	(0.088)
Employee shares by skill level, reference: share of unskilled workers								
Share of skilled workers with					−0.188***	−0.101*	−0.204***	−0.201***
vocational training					(0.059)	(0.054)	(0.066)	(0.065)
Share of skilled workers with					0.191***	0.410***	0.358***	0.306***
university degree					(0.073)	(0.080)	(0.116)	(0.117)
Additional skill requirements								
Leadership ability								−0.189***
								(0.047)
Experience								0.002
								(0.038)
Further skills from training outside formal qualification								−0.161***
								(0.033)
Foreign lang.								0.137***
								(0.049)

(*Continued*)

TABLE A13
CONTINUED

	(1) Neg. Bin. I	(2) Neg. Bin. I	(3) Neg. Bin. I	(4) Neg. Bin. I	(5) Neg. Bin. I	(6) Neg. Bin. I	(7) Neg. Bin. I	(8) Neg. Bin. I
Social skills								0.134***
								(0.029)
δ	0.422***	0.424***	0.415***	0.419***	0.412***	0.415***	0.579***	0.574***
	(0.0753)	(0.0763)	(0.0747)	(0.0759)	(0.0748)	(0.0760)	(0.161)	(0.160)
Number of observations	27394	27394	27394	26947	27392	26946	7335	7335
Pseudo-R^2	0.0317	0.0302	0.0357	0.0365	0.0368	0.0382	0.0300	0.0320
Log likelihood	−23051	−23086	−22956	−22634	−22927	−22594	−6375	−6362

Robust standard errors in parentheses.
***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
NOTES: Coefficients are reported as semielasticities, a one-unit change in x_i changes the conditional mean, μ_i, by the multiplier $1 + \beta_i$. All specifications include year dummies, dummies for the federal states where the firms are located, industrial sectors, and a dummy that indicates firms reporting both the last case of a successful and an unsuccessful recruitment case. The skill requirements comprise three levels: w/o qualification (reference group), vocational training, or university degree. The employee shares by skill level comprise three groups: unskilled workers (reference group), skilled workers with vocational training, and skilled workers with university degree. The additional skill requirements are a set of dummy variables that denote whether the vacancy requires leadership ability, experience, further skills from training outside formal qualification, foreign languages, and/or social skills. Results in columns (7) and (8) are from a specification that are based on a restricted sample of firms that report successful recruitment and used both formal channels and referrals.
SOURCE: IAB Job Vacancy Survey 2012–2016. Own computations.

$$+ \log\left\{\frac{\Gamma[y_i + \delta^{-1}\exp(b_0 + \mathbf{b_1}\mathbf{x_i})]}{\Gamma[\delta^{-1}\exp(b_0 + \mathbf{b_1}\mathbf{x_i})]\Gamma(y_i + 1)}\right\}. \tag{A.5}$$

Table A13 presents the results. Parameter δ is significantly positive as we expected, indicating overdispersion. The point estimates of the coefficients for the firm size classes are slightly smaller than the point estimates based on the negative binomial II model, but the differences are not significant (see Table 9). Thus, these results corroborate our findings regarding hypothesis H6 in Section 4.

ACKNOWLEDGMENTS

Open access funding enabled and organized by Projekt DEAL.

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