Periodic points in random substitution subshifts

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Abstract

We study various aspects of periodic points for random substitution subshifts. In order to do so, we introduce a new property for random substitutions called the disjoint images condition. We provide a procedure for determining the property for compatible random substitutions—random substitutions for which a well-defined abelianisation exists. We find some simple necessary criteria for primitive, compatible random substitutions to admit periodic points in their subshifts. In the case that the random substitution further has disjoint images and is of constant length, we provide a stronger criterion. A method is outlined for enumerating periodic points of any specified length in a random substitution subshift.

Random substitutions are a generalisation of the classical notion of a substitution on a finite alphabet. In the classical setting, letters are mapped to words over the same alphabet, and then this map is iterated. Dynamical systems associated with these classical substitutions are well studied and there is a large community devoted to solving some of the few remaining big problems in this area [1,5,8]. In the setting of random substitutions, letters have a *set of possible words* (often with an accompanying probability distribution) to which they may be independently mapped. There, one must contend with all possible outcomes of iteration, where each letter of a word is mapped independently of all others. This leads to an exponential growth in the number

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