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The development of a domain map in probability for teacher education

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The importance of probability increased in German teacher education in the last decades. However, research about the professional knowledge of (prospective) teachers focused on other content such as number theory and functions rather than probability. The project "The development of professional knowledge of prospective teachers in probability" aims at developing a test in order to measure teacher knowledge in probability. In order to measure content knowledge and pedagogical knowledge in this domain, one has to identify categories, concepts and ideas and connections between. This paper shows the development of a domain map, which indicates categories, concepts and ideas. The theoretical framework, the procedure of developing and an outlook is given.

Keywords: Mathematics education, mathematics teachers, teacher competencies, teacher competency test, probability.

Introduction

In the state North Rhine-Westphalia in Germany, stochastics was not obligatory for the general qualification for university entrance until recently. Teachers could decide to exclude topics for the finals. Now stochastics is mandatory. The focus of studies about professional teacher knowledge was not on probability, which is part of stochastics. The main aim of this project is to develop a test to measure the development of content knowledge and pedagogical content knowledge of prospective teachers in probability. While developing items, one has to identify necessary categories in probability to develop items to measure content knowledge in that area. This paper identifies structures and categories by developing a domain map, which shows the content of probability. Those maps, which Hill and Bell developed for their study as well (2007), help distinguish categories in probability and draw important connections between categories in order to develop items. In order to develop this domain map, educational standards and curriculums were analyzed and an expert study was conducted.

The paper is organized as follows. In the first chapter, the theoretical frame will be presented. Afterwards, the two steps of the domain map development will be demonstrated. In the outlook, the domain map is integrated into the test design and the timetable of the test is presented.

Theoretical Framework

Models of Teacher Knowledge and Competencies

In this chapter, models of teacher knowledge and teacher competency tests are introduced. Shulman's categories of teacher knowledge, the studies COACTIV and TEDS-M are presented to determine the theoretical framework of this paper.

Initially, Lee S. Shulman (1986a, 1986b; 1987) developed categories of teacher knowledge: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational context and knowledge of educational ends. Several other researchers adopted Shulman's model and adapted it (e.g. Bromme, 1992, Hill et al., 2008 and Schumacher, 2017).

In the study COACTIV, conducted in 2003/2004 by Stefan Krauss et al. (2008), which is about professional competence of teachers, cognitively activating instruction and development of students' mathematical literacy, Shulman's category "content knowledge" was specified. Krauss et al. (2008, p. 876) declared content knowledge as "a teacher's understanding of structure" and pointed out possible notions of "content knowledge", while using (3):

1. The everyday mathematical knowledge that all adults should have.
2. The school-level mathematical knowledge that good students have.
3. Mathematical knowledge as a deep understanding of the contents of secondary school mathematics curriculum.
4. The university-level mathematical knowledge that does not overlap with the content of the school curriculum (e.g., Galois theory or functional analysis).

Another study is the first cross-national large-scale study, conducted by Sigrid Blömeke, Gabriele Kaise and Ralf Lehmann. The Teacher Education and Development Study in Mathematics (TEDS-M) had the main aim "to understand how national policies and institutional practices influence the outcomes of mathematics teacher education" (Döhrmann, Kaiser, & Blömeke, 2012, p. 325). The definition of teacher knowledge was also based on Shuman's definition. Pedagogical content knowledge was differentiated into (1) curricular knowledge and knowledge of planning for mathematics teaching and learning and (2) knowledge about enacting mathematics for teaching and learning. Content knowledge was differentiated into three cognitive elements, namely, knowing, applying and reasoning and was tested in four content domains, which were number theory, geometry, algebra and data (Döhrmann, Kaiser, & Blömeke, 2010).

Probability as a content domain was underrepresented in both COACTIV and TEDS-M, but the distinction of content knowledge of COACTIV and the definition of pedagogical content knowledge of TEDS-M is based on this project's definitions. However the definitions of content knowledge of COACTIV and pedagogical content knowledge of TEDS-M will be used as a working definition.

Domain map in probability for teacher education

While content domains like "number "was differentiated in eight categories at TEDS-M, data had only three categories, which included data organization and representation, data reading and interpretation and chance (Döhrmann et al., 2010). Only chance can be allocated to probability. In order to grasp possible developments in professional knowledge in this mathematical field, one has to distinguish it into more categories.

Methodology

The research question is which categories of (future) teacher knowledge the research area of probability can be distinguished. In order to answer this question a domain map was developed.

This distinction can be divided in two steps. First, one can raise the question what students in secondary level should learn, therefore what teachers also have to know. This can be assigned to the second notion of specification of content knowledge, which is the school-level mathematical knowledge that good students have. For carving out details, what students should learn, one will analyze educational standards and curriculums. For this paper, the German educational standards and the curriculum of the state NRW were being taken in to consideration. The statements being made to probability was first collected, analyzed and linked, so a first draft of a domain map can be presented.

The second step is to determine content knowledge in probability on the third notion, which is mathematical knowledge as a deep understanding of the contents of secondary school mathematics curriculum. On this notion, student knowledge is not adequate. In this step, content requirements for teacher education get augmented in the same way as in step 1.

After that, an expert study in a small frame was conducted. Mathematicians and Mathematics educators were being questioned about possible missing or redundant categories. In this study were three Mathematicians and ten Mathematics educators.

Step 1: Analysis of the German educational standards and curriculum of the state NRW in probability

The German educational standard in probability for students in secondary level was resolved by the Permanent Conference of the Ministers of Education and Cultural Affairs of the States in the Federal Republic of Germany. The key content "data and chance" states the following about probability (KMK, 2003, p. 12):

Students

- reflect and evaluate arguments, which are based on a data analysis
- describe appearance of randomness in everyday situations
- calculate probability at random experiments

While the first indent is still clearly located in statistics, it does have some relevance to probability. The second indent establishes the concept of randomness […] teachers should know about. In the third one, random experiments and probability are not further specified.

In the following, one curriculum, namely the one from North Rhine-Westphalia, will be analyzed to find out about notion 2 of probability. This analysis is one example for analyzing other curriculums to get a good idea of requirements for teachers in probability.

In the curriculum of the state NRW, the Ministry of Education and Training (2014, p. 16) stated more specified information about what students should know at the end of the secondary level:

- They [the students] calculate relative frequencies, mean values (arithmetic mean, median) and measures of variation (range, quartile) and interpret those.
- They [the students] calculate probabilities by using the Laplacian rules, tree diagrams and their rules, use frequencies to estimate probabilities and probabilities to predict frequencies.

Those two indents give a first idea of structuring the domain “probability” by differentiating it into the categories frequencies (relative and absolute), (Laplace) probabilities, graphic representation (tree diagram), as to be seen in Figure 1.

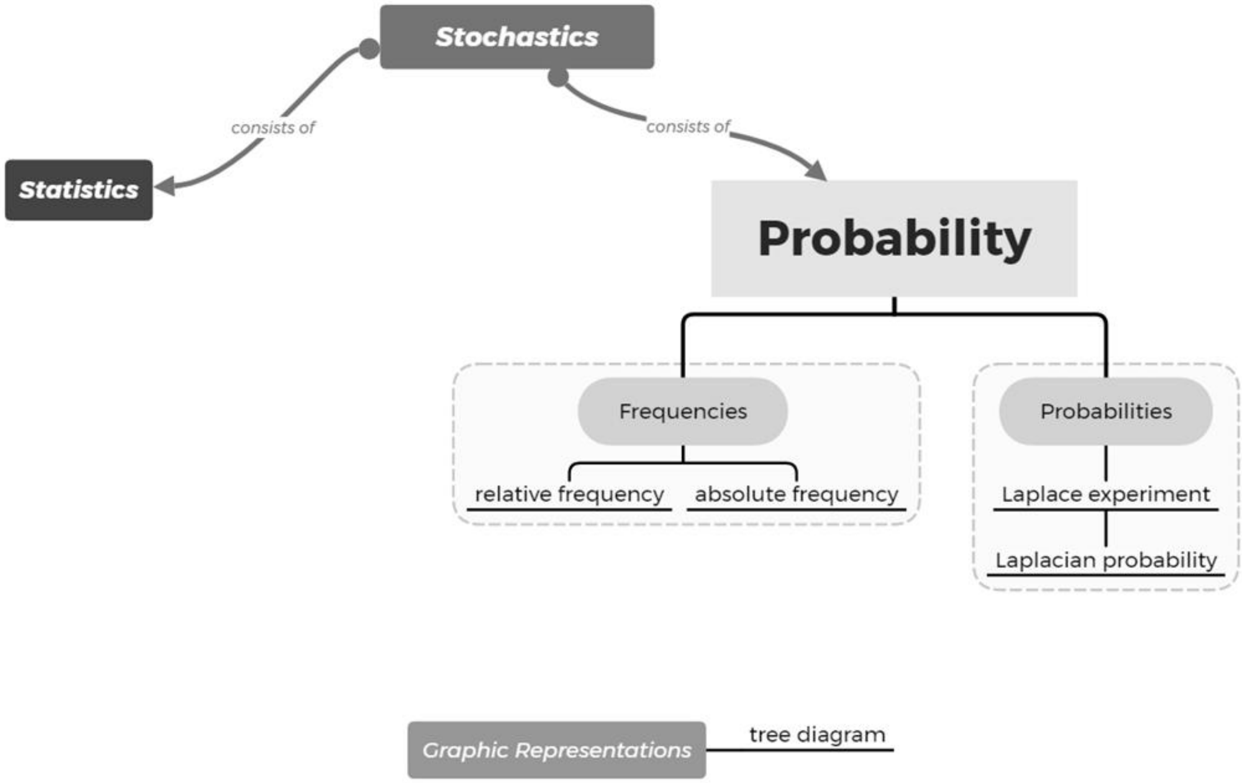

Figure 1: Domain map after step 1

Stochastics can be distinguished in to Statistics and Probability. Probability has the category “Frequencies”, which contains relative frequency and absolute frequency, the category “Probabilities”, which contains Laplace experiment and Laplacian probability and the category “Graphic Representations”, which includes tree diagrams.

After completing step 1, one can use Figure 1 and differentiate the categories further and adapt it to achieve a domain map for notion (3).

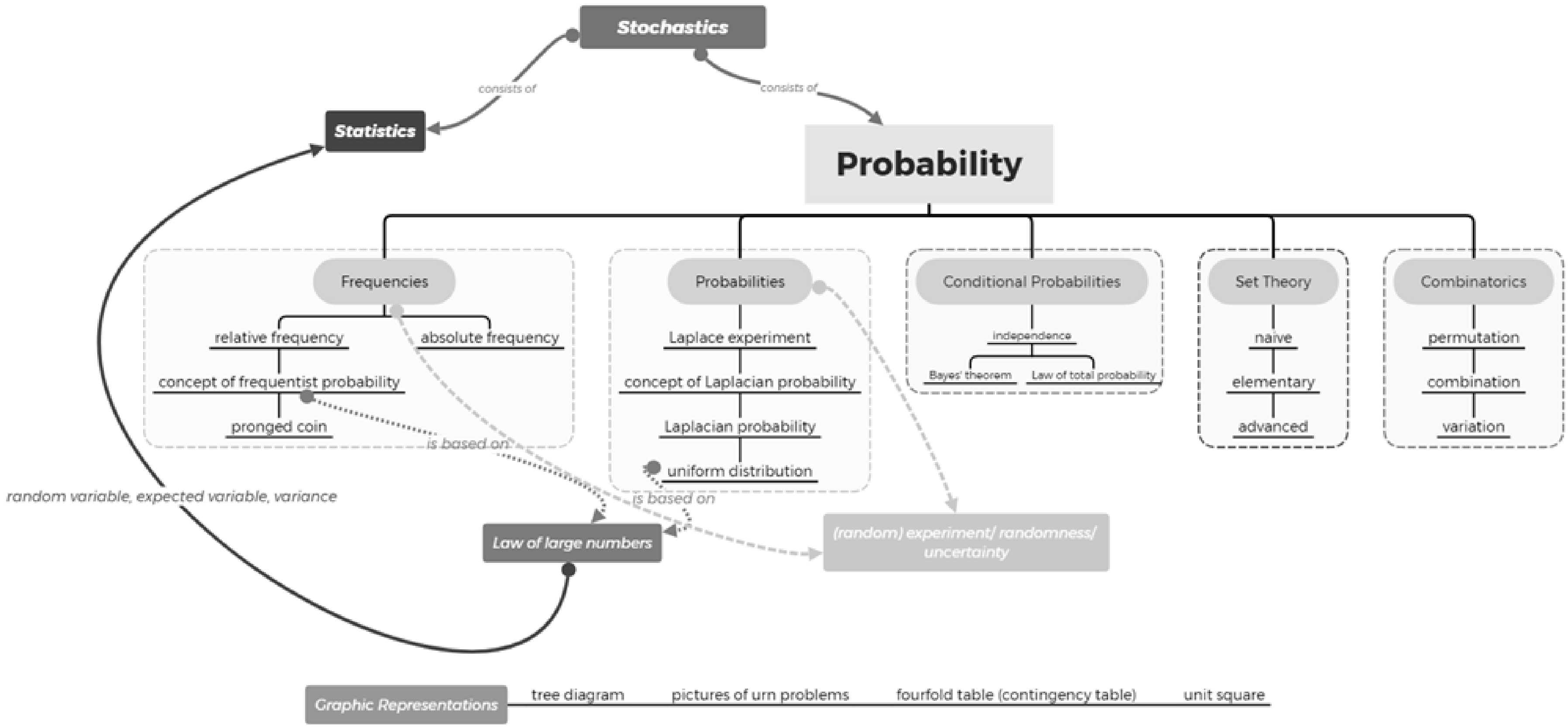

Figure 2: Domain map after step 2

Step 2: Obtaining a domain map for teacher knowledge as the deep understanding of the contents in probability

In order to obtain a domain map for a teacher knowledge as deep understanding of the contents in probability, one augments Figure 1 by analyzing the content requirements for teacher education (KMK, 2008). After that, Mathematicians and Mathematics Educators from the Mathematics department at Bielefeld University were invited to critically analyze the categories. Finally, literature such as Arbeitskreis Stochastik in der Gesellschaft für Didaktik der Mathematik e.V. (2018), Harten and Steinbring (1984), Krüger, Sill, and Sikora (2015), Kütting and Sauer (2011), Tietze, Klika, and Wolpers (2002) and Wolpers and Götz (2002) was taken into account. The results were the following (see Figure 2).

The category "Frequencies" was extended to the concept of frequentist probability and, as an example, a pronged coin, while "Probabilities" has the extension of the concept of Laplacian probability and uniform distribution. Both concepts are based on the law of large numbers, which is indicated by the dotted arrows. In order to obtain a deep understanding of both categories, one needs a profound concept of (random) experiments, randomness and uncertainty. Therefore, those are important connections between frequencies and probabilities.

The law of large number is linked to statistics through the concept of random variables, mathematical expectation and variance, because many concepts of statistics are modeled via random variables. This is indicated by a solid line in Figure 2.

The category "Graphic Representations" was extended by pictures of urn problems, fourfold table and unit squares to cover the main graphic representations used in probability. To achieve a deep understanding of tree diagrams, one should know about "conditional probabilities", because tree diagrams are based on the law of total probability and Bayes' theorem. The category "Graphic Representations" stands on its own, because as representations it isn't any traditional content of probability. However, graphic representations can help understand concepts of probability.

The category "Combinatorics", which is typically categorized as algebra, was added, because of the importance for random experiments. Teachers should know about permutations, combinations and variations.

The category "Set Theory" was added, because one needs naïve set theory to grasp the idea of probability and complementary probability. Set theory on an elementary level is in use for combinatorics. Advanced set theory is mandatory for conditional probabilities.

This domain map makes no claim to be complete. However, it is the foundation to work on developing items to test content knowledge regarding these topics.

Outlook

The study "The development of professional knowledge of prospective teachers in probability" is focusing on content knowledge and pedagogical content knowledge of prospective teachers. Prospective teachers for secondary school usually take their probability courses in one semester. They will participate in a pre- and post-test, so the development of knowledge in probability is visible. After a certain time, they will also participate at a follow-up-test to measure effectiveness of their probability education. They will also answer questions about emotions toward probability, demographics and self-efficacy.

The research questions of the study are the following:

- How is the development of professional knowledge of prospective teachers in probability?
- How do emotions influence the development?
- How is the self-efficacy changing during those classes?
- How effective is the university education in probability?

One important preparation for the pre- and post-test was the development of this domain map. Educational standards, curriculums and requirements for teacher education were analyzed and Mathematicians as well as Mathematics educators were being questioned in an expert study. One now has an overview of current research on probability knowledge of (prospective) teachers and is being able to develop items for the study mentioned above. One limitation is that the domain map was only analyzed by German standards. The expansion to an international level is planned. Another limitation is the aspect of the educational standards and teacher requirements. They already are developed and assessed by Mathematics Educators on basis of empirical results, but they are not empirical results themselves. However it is a good estimation for what prospective teachers should learn in probability.

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