



New Examples on Lavrentiev Gap Using Fractals

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Received: 10 July 2019 / Accepted: 31 July 2020 / Published online: 24 September 2020
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Abstract

Zhikov showed 1986 with his famous checkerboard example that functionals with variable exponents can have a Lavrentiev gap. For this example it was crucial that the exponent had a saddle point whose value was exactly the dimension. In 1997 he extended this example to the setting of the double phase potential. Again it was important that the exponents crosses the dimensional threshold. Therefore, it was conjectured that the dimensional threshold plays an important role for the Lavrentiev gap. We show that this is not the case. Using fractals we present new examples for the Lavrentiev gap and non-density of smooth functions. We apply our method to the setting of variable exponents, the double phase potential and weighted p -energy.

Mathematics Subject Classification 35J60 · 46E35 · 35J20

1 Introduction

The Lavrentiev gap is a phenomenon that may occur in the study of variational problems. In particular, the minimum of the integral functional \mathcal{G} taken over smooth functions may differ from the one taken over the associated energy space.

Dedicated to Vasilii Vasil'evich Zhikov.

Communicated by T. Riviere.

Anna Kh.Balci and Lars Diening thank the German Research Foundation (DFG) for the support through the CRC 1283. The research of Mikhail Surnachev was supported by the Russian Science Foundation under grant 19-71-30004. Mikhail Surnachev acknowledges warm hospitality of Bielefeld University.

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