Essays on the reorganization of firms and on the interplay of hesitant agents

Inauguraldissertation zur Erlangung des Grades eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.) an der Fakultät für Wirtschaftswissenschaften der Universität Bielefeld

Dipl.-Vw., Dipl.-Kaufm. Martin Lunge

April 2019

Erster Gutachter: Prof. Dr. Herbert Dawid

Zweiter Gutachter: Prof. Dr. Antoine Mandel

Gedruckt auf alterungsbeständigem Papier nach DIN-ISO 9706.

Acknowledgment

Now, while looking back at the end of my doctoral time, I have to admit that, as often in life, things developed in various ways differently than once planned. On the one hand, some things turned out not to be as demanding and as much reason of concerns than believed in the beginning of the doctoral time. On the other hand I experienced frequently the situation that suddenly new challenges occurred or that certain aspects of the research work which were expected to run smoothly developed in their own ways and were often not easy to get back on track. Finishing my thesis successfully, despite all these unexpected turns and obstacles, is also due to the engagement of a lot of people that accompanied me during this endeavor.

First of all I want to thank my family and especially my parents. With their constant and unconditional support during the doctoral time as well as before, they did not only enable me to focus on my research but also to see and to learn so much about the world. Considering that I am a very curious person I cannot put in words how much that means to me and how grateful I am.

Next, I want to thank my supervisors Prof. Herbert Dawid and Prof. Antoine Mandel for leaving me the academic freedom to develop my own research ideas and for their patience when I, as often happened, chose an all but straightforward path in order to put them in motion.

In challenging times not less important than subject-specific guidance is a source for recreation and distraction and general advice. The following very good friends Marc-André Bürgel, Vahe Chinaryan, Robin Fischer, Tobias Krug, Friedrich Schmidt and Ronny Voigt were this source for me. I am deeply grateful that they were always there when I needed them, and for believing in me especially when I did not.

I also want to thank all members of the IMW (Center for Mathematical Economics) and particularly its director Prof. Frank Riedel for creating such a nice research environment from which I benefit a lot. I also owe special thanks to my friends from the academic community Elena Orlova, Oliver Claas, Erdenebulgan Damdinsuren, Jasper Hepp, Torben Koch, Vincenzo Platino and Patrick Schuhmann for various support and for teaching me many things about research work and methods and life.

In my first days as a doctoral student I met four other students Yuliia Stupnytska, Tobias Hellmann, Philipp Külpmann and Johannes Tiwisina who became very important to me. I appreciated a lot the warm social environment you created during our time together in Paris and Bielefeld. I consider myself as very lucky that I started my PhD together with you. Additionally I want to mention that there were only a few people who supported me as much as Johannes no matter whether there were things of academic or private nature to talk through, programming tasks to do or new and mostly vague ideas to argue about. He was always willing to listen, to help, to get involved. My deepest thanks and appreciation for accompanying me through all these years.

Another reason to consider myself lucky is that I met Prof. Volker Böhm who was always willing to discuss my research, advising and providing orientation when I was lacking it. Despite being an emeritus professor for quite some time, his unbroken passion for research, teaching and guiding is impressive and inspiring. I am very grateful for all his support and advice.

Moreover, I thank all those people who helped and supported me often without even knowing. Some of them I met in classes, some at the administrative departments of the university, others at the university gym or while being part of the student charity organization "sozial lokal".

Last but not least I thank the German Research Foundation (DFG) for the financial support during the first three years of my doctoral studies.

Contents

Chapter 1

Introduction

Economic agents find themself very often in a decision situation in which they have to balance strongly contrary effects in order to generate with their decision an outcome that is beneficial. Whereby the term economic agents is meant in a very broad sense. In principle all single persons or institutions are at some point in such a decision situation. For example a government has to decide whether it announces a law in the interest of one or in the interest of another group of voters, voters them self might have to decide whether to confirm a government or not with witch they agree on some point of its political agenda and disagree on some others, a firm has to decide which group of workers it promotes or to balance various cost effects that might come along with the reorganization of its operations. All those agents are in a dilemma situation, thus they need to handle a certain trade-off that might origin in a scarce resource that can be allocate just ones or in other interdependencies originating from preferences or physical constraints only to name a few.

There are as many different ways to model such decision situations as different situations themself. One can think of dynamic or static set-ups, of agents that base their decision on heuristics or on an optimization approach or of decision situations in which agents operate in an uncertain or a certain environment to name only some possible criteria. The contribution of this work is to shed some light on the way such decision situations could be modeled and on explicit underlying mechanisms that occur in some of those situations. Therefore, the work is divided into two parts. The first part has a broad view on such situations. The goal is to derive general characteristics that many of those decision situations have in common. Furthermore, I describe how these characteristics are connected and I make suggestions how to capture them in the scope of an agent-based model. This part of the work attributes therefore to the methodological structure of economics. In the second part the focus is on two specific decision situations. The economic agent is in both cases a firm that reorganizes because of technological progress. Thereby the firm has to balance contrary cost effects. The goal of this part of the work is to examine and illustrate the way firms deal with those cost trade-offs and which consequences this has for the worker employed by the firm. In the following I will give a brief overview about each single project.

In the second chapter I introduce four characteristics that a lot of decision situations have in common. First the decisions agents make, in other words the manifestations of their choice variables, will feed back to them over time, along certain steps. Where each step comprises a decision of another agent. This is, what I will call a feedback loop. Furthermore there is always at least one agent that is involved in two or more of such feedback loops.

The second characteristic is that agents decisions, thus the feedback loops are interdependent. This can be caused by different reasons for example a limited amount of money, time or space, because of complementarity of consumption goods or input factors or by technical constraints.

The third characteristic concerns coordination. The common thing in all examples in terms of coordination is that agents interact sequentially with each other. For example a government announces a budget plan or a policy mix or a firm sets wages and prices in one period their counterparts, e.g. voters, workers or costumers, however will process this in the next period and act accordingly, afterwards this causes a reaction of the agents next on the feedback loops. At some point the initial impulses gets back to the first agents who then may adjust again depending on the reaction of their social environment. The main consequence from this kind of interaction is that there is no direct coordination between the agents. From this follows that a mismatch of the interests of the interacting agents is rather the standard case than an exception.

This leads directly to the fourth characteristic, agents hesitant adjustment behavior. Because of the lack of information and no assurance of coordination an agent can not connect perfectly her behavior and that of the other agents. Thus an agent is uncertain how to respond to a change in her social environment. On the one side an agent shows at first strong hesitation regarding adjustments to behavioral changes of her social environment. Even if changes keep going, either she does only small adjustments of her behavior or no changes at all. On the other side, at a particular point once the imbalance between her environment and her own behavior becomes too large she overcomes the reasons causing her hesitation. The following adjustments are more intense than the recent environmental changes would suggest.

In the second part of the second chapter I develop a small-scale agent based model that incorporates those characteristics. Afterwards the effect of the hesitant adjustment behavior on the dynamic of the model is analyzed.

The starting point for the third chapter are two empirical phenomena. A topic to which for decades social science devoted frequently much attention is the development of labor incomes, more precisely of the inequality between the incomes of different groups of workers. Empirical results suggest that there are fundamental mechanisms underlying the economy that slowly but consistently cause incomes to become more unequal. This can be observed not only in the entire economy but also on the firm level. Among the various metrics to measure or to express income dispersion, the Gini coefficient will be important in this work.

Another slow but not less persistent phenomenon is the increase of the variety of professions and necessarily of educational opportunities. This trend towards a more heterogeneous labor force always accompanied economic progress, but in recent decades developments like computerization and digital transformation made this trend even more visible. Firms regularly reorganize their operations that is, they reallocate the tasks or activities, which together form those operations, between their workers. Usually, this leads to a higher degree of labor decision, thus the range of tasks resp. activities that each worker conducts becomes smaller or in other words workers become more specialized.

The contribution of this work is to show the connection between those two phenomena, the increase in labor division and its effect on the income distribution of the work force of a single firm. For this, I develop a model of a single firm that produces a final good and that takes the price of the good and the various wages of differently skilled workers as given. The firm forms working groups and allocates the tasks to them. Thus the firm has not only to decide which group conducts which set of tasks, as it is the case in the task-based models, but also which number of working groups is optimal, which is similar to Becker's approach. Both decisions imply the optimal organizational structure of the firm. We will see that the firm has to consider two contrary effects when increasing the number of working groups. On the one hand, this lowers the total wage costs, but on the other hand increases complexity, i.e. the costs of coordination on the other side. In the event of technical progress the firm needs to reorganize to balance both effects again. How that changes the optimal organizational structure and moreover how that again influences the income distribution depends on the production function of the firm and on the assumed outcome of the labor market.

It is well documented that in western countries the daily working time decreased substantially during the last century. In the last decades it seems, even so that the general trend is still in place, that the change of the daily working is unequally distributed. This makes the labor force more and more heterogeneous in terms of the time they spend at work. Nowadays there are workers who work only a couple of hours each day and often even have to do more than one job. At the same time unions, representing a huge part of the labor force, demand to reduce the weekly working time step by step. Also their is a small but increasing group of workers with very high workloads accumulating extra hours on a scale not know before. Additionally, workers who fall into the last category pursue professions that comprises various highly interdependent tasks. Such workers are for example managers, programmers, designers or consultants. While workers of the first category are for example employed in the low paid service sector conducting simple routinized tasks. Another phenomenon of the recent time is that because of new technological trends and innovations like digital transformation or cloud computing it becomes easier to coordinate workers even in large numbers. Which raises the issue of the economic impact of those developments on the labor force in terms of working time and income.

The contribution of the fourth chapter is to examine the connection of those phenomena, the heterogeneity in the daily working time of various professions, which is correlated with the structure of the work processes inherent to those professions, and the decreasing coordination efforts caused by technological progress. I illustrate and analyze the underlying mechanisms from the perspective of a single firm and the corresponding workforce. For this purpose I develop a model of a firm that is confronted with two decision problems. First, it has to allocate the daily working time between tasks that a single worker is supposed to conduct. And second, the firm has to determine its optimal combination of workers and working time.

Chapter 2

Interdependent feedbacks – An agent's dilemma

2.1 Introduction

In most real live situations agents, such as consumers, voters, governments or firms, neither have the luxury to observe the behavior of all agents of their social environment and perceive all properties of the social and physical world nor are they able to make their decisions one by one. Usually agents have to do plenty of decisions that depend on each other. At the same time based on some general knowledge and information they received through interactions with a certain group of agents during past periods. These interactions take place in a lot of different combinations. One agent might be influenced by one or a couple of the decisions that are made by a certain agent in her environment. Moreover she might directly respond with an action, what implies a preceding decision, that in turn affects the first agent or she interacts with a third one whose reaction then again is considered by the first agent. Just as one can think of a situation where both happens at the same time. It is easy to imagine that there are uncountable many possibilities of interaction schemes that might occur in the real world. In this context making a decision means that an agent responds resp. adjusts to changes in her environment caused by actions of some of the other agents in the past periods and in turn causing changes herself in the perceived environment of some other agents in the next period. A crucial point in this work is how agents conduct these adjustments. In the real world a whole range of adjustment behavior is observable from agents that immediately adjust to changes to situations in which agents delay their adjustments until a certain point is reached and finally adjust by a few big steps.Before going into more details, lets have a brief look at some examples.

A decision regularly made by a government is how to distribute a limited budget to possible uses. Lets say a government has to decide in which proportion to split a certain budget that might either be used for security purposes such as payments for the police force or it can be spent for education.[1] Obviously the goal of the ruling

[1]There are plenty of other examples where a government has to decide how to use a scarce resource to meet contrary demands. This dilemma does not necessarily have to arise from a shared budget. In the model of the second part the limiting factor is a strip of land that needs to be allocated between two modes of usage. Also time as in situations where during the day certain activities are or are not allowed e.g. operating a airport or a factory is a resource of such kind as is the pollution capacity of

parties is to split the budget in a way that satisfies the population, that is, the voters most. Even so that a government usually knows that there are voters who prefer that more money is spend for security and those who tend to higher education expanses it neither necessarily knows the proportion of these groups nor their exact preferences over different implementable states concerning the security force and the education system. Additionally, new circumstances might occur, e.g. new security threads or new technologies that demand changes in the education system, which very likely would make a redistribution of the budget necessary. Beside its general knowledge about the voter the government will base its decision to adjust the budgets for the next period on the signals received from the voters, their satisfaction levels, during the last periods. Lets assume a government thinks it is reasonable to shift some money from the education to the security budget to meet changes in the needs of the society. If the voters who benefit from such a policy show a positive signal of satisfaction while those who are worse of do not signal their dissatisfaction with the same intensity the government will very likely continue to change the budgets arguing that according to the received signals it will improve the overall situation of the population. This will last until the dissatisfaction of the second group reaches a point that makes them overcome the reason for delaying their responses to recent developments. What could have been the instance that they are uncertain about the sustainability of the budget change or that they are unsure about additional consequences that a reaction from their side my cause, an effect that is described in the last example below. Once those agents start to signal their actual satisfaction level they will send strong signals even so that there might have been only small changes over the last periods. Receiving strong negative signals from this group will cause the government to over think its policy. Depending on how secure the government feels about the recent trend of the signals and about the consequences a contrary change of the budget will cause it might react immediately, thus changing the budgets in favor of the second group, as it might conduct only small changes, that is, delaying adjustments until it feels more convinced of the recent trend change. Of course strategies in between these two opposite ends of a continuum in

an environmental entity. The differences and the shared characteristics of cases with such resources involved are briefly sketched below.

terms of the assumed adjustment aversion are thinkable as well. In case the reaction of the government is strong enough to trigger a change in the announced satisfaction levels, furthermore if both voter groups are similar in their way to respond to budget changes, that is, to positive ones within a narrower time frame than to negative ones, and if the government considers recent signals more than such from earlier periods than the proportion of the budgets will move in opposite direction as before. This will continue until a group of voters, what would be this time the first one, intervenes and the whole cycle starts again. Whether at some point in time a stable split of the overall budget will be reached or whether the proportion of the budgets continues to change or whether even a very different dynamic will occur depends on the interplay of the adjustment and learning behavior of the voters and those of the government.

Changing the system of legal rules, thus changing laws and regulations, is beside the budget another important tool a government may use to balance the interests of the citizens of its administrative area. An often announced political goal is the support and promotion of a certain social group or of a particular region. Usually this is only achievable by a well aligned interplay of various specific regulations and investments. Lets say the government want to improve the employment situation in an underdeveloped area by settling firms of an promising economic sector. Normally the authorities do not found and run such businesses by themselves but creating incentives for firms to settle by setting up a promising business environment. For example the government might lower local taxes and change the orientation of the education system and that of its research spendings towards professions and projects connected to the economic sector in mind. As in the first example it is unavoidable that such changes will worsen the situation of some people. In terms of tax reduction it will very likely effect the employment situation in other areas since firms settling in the latter have a competitive disadvantage or they move themselves. If the tax reduction applies also to already operating firms it will reduce the state revenues and consequently the expenditures for uses certain groups profited from. Another possible side effect is that some parts of the population consider a lower participation of firms in bearing the costs of a functioning state as unjustified. Considering the education and research system another orientation goes necessarily along with less fonds for other projects and studies, what most likely will not be appreciated by the affected people. In a nutshell there will be voters who are not approving such policy changes. On the other hand an increase of the employment level in the underdeveloped region will have a positive effect on the approval rate. Thus the goal of the government is to make sure that in the eye of the entire votership this effect overcompensates the negative ones. Whether this is the case depends on the ability of the government to harmonize both policies. Only lowering the local taxes may induce firms to settle in the respective area but the employment effects would probably be small if they can not employ enough suitable educated people or collaborate with state research institutions. Firms might as well settle only administrative units to benefit from the tax subsidies or they operate on a high level of automation while the mentioned negative effects occur to a large extent. On the other hand changing the education system but reducing the taxes only by a small amount would provide well qualified workers but it might be altogether still not profitable to operate in the particular area. In the end the government would lose approval from those voters who disagree with the new orientation in the education system without attracting new voters who were supposed to benefit from such policy mix. What dynamic will occur while trying to find a good set of policies will highly depend on the patience and the aversion to change of all involved groups. If the voters who are affected by the negative consequences of a policy change respond immediately there will be probably not much of a change since the positive effects will not have time to unfold. This is especially the case if firms are risk avers, hence it takes longer before they are convinced that a new and sustainable trend is established and hence before they are willing to invest. Unless the government itself is patient enough to continue its policy even so that there will be an increasing number of opposing voters for a while. In the case where voters might fear that their negative signals will cause unintended ripple effects, that is they do not signal their dissatisfaction and furthermore if with its changed policy mix the government is able to create positive effects for enough other voters who show their estimation, then a political agenda with a new focus will emerge until the disagreeing voters overcome their indecisiveness and demand corrections. How this continues will also depend on the ability of the government to improve their trial and error procedure to find a widely accepted policy mix.

Cyclical changes of key variables can be observed not only in many different ways in the political sphere of a society but also on the firm level. Even so that there are contracts between firms and workers not all terms of their working relationship can be clearly defined in advance or certain deviations are not verifiable. Imagine a firm consist of two different departments. Usually the contract between the firm and the workers of the departments lay down the working time, wages and several aspects of the work content, while to a certain degree the engagement of the workers or the fluctuation rate are not controllable. Furthermore the working environment also consist of other things, that affect the commitment of the workers e.g. bonuses, staff training or physical improvements of workplaces. To conduct the latter limited resources like money and time are needed, thus a firm has to decide how to use these resources in an optimal way therefore in a way that workers are most dedicated to their tasks. For example if the management decides to change their bonus policy because of some structural changes in the company or a changed profit situation most likely the workers of the department relatively being better off in comparison to the last bonus payments would send a positive signal while the others might wait before signaling their dissatisfaction.[2] Later we will see their might be different reasons why workers do not respond immediately to changes, so far lets assume again that this is because the second group is afraid what consequences a negative signal would have in this case for their future employment situation. Based on the perceived reactions of the workers the management would continue to shift bonus payments or continue to improve the working environment of the first group by other means still assuming to improve the overall efficiency of its workforce. But the growing dissatisfaction of the discriminated workers will lead at some point to a behavioral change. Workers will work less efficient, changing jobs or report sick, that is, sending a negative signal causing the management to reevaluate their policy. Whether in the following one would observe the opposite trend concerning changes of the working environment towards a stable allocation of the available resources balancing the interests of the workers, no changes at all or even

[2] The initial impulse might also be caused by the workers themself, that is, they signal an altered satisfaction level, for example because they do not perceive the current allocation to be fair anymore or they believe to be recently in a stronger position to put such claims forward.

fluctuations over a longer period depends on the interplay of the workers signaling behavior and that of the management which might show delayed reactions as well.[3]

There are plenty of other situations in which a firm interacts responsibly with other groups of agents and where it has to balance the interests of these groups to reach its own goals. Obviously one of the most prominent interconnected interaction of a firm are its simultaneous activities on several markets for example on a labor and on a product market which because of its importance I want to briefly introduce. In the real world such activities take place along a lot of different paths. Lets have a look on a particular manifestation where on the one side a firm has to renew a certain number of its labor contracts in each period because of retirements, contract duration or general fluctuations. To compensate for this and for changes in its overall labor demand the firm announces a wage offer each period. Subsequently the potential workers decide whether they are willing to work for this wage. On the other side the firm also announces a product price each period and again subsequently the other market side responds by expressing a certain demand. There are two features inherent to this kind of market interactions. First it is rather unlikely that demand and supply on one market match not to mention on both at the same time. And second agents neither do know exactly how other agents respond once they change their behavior nor do they know whether a behavioral change of their counterparts is only caused by a temporary effect or a longer lasting development. Additionally, consumers, workers just as firms do effect with their decisions different aspects of their "live" at the same time with often contradicting consequences in terms of their overall goals. In this example a firm might be confronted with an higher demand for its product than expected if the supply of labor is higher as well this could work out. Otherwise if it is lower or even lower than expected the firm experiences a lost of potential revenue and has to decide whether to respond by increasing the product price, the wage, a combination of it or doing nothing assuming that the deviation is caused by an indeterminable temporary effect. Once the firm changes its behavior it may happen again that costumers and

[3] As in all these examples the dynamic of the social situation depends also whether and in which frequency exogenous stimuli occur. In this example that could be the bonus payments of an exogenous peer group, demand shocks or changed legal regulations.

workers do not respond with the intended intensity or not even in the direction it thought of. Again either the firm concludes that the other agents are less sensitive concerning the change of the wage and the product price than expected and adjusts accordingly or it assumes that they are are as well indecisive about the sustainability of the recent changes, thus delaying their adjustments. Also the potential workers have to handle interdependent interests. They have to decide whether to work for a certain wage even if it is not as high as expected what means less income for the duration of the contract or to wait hoping the firm will do a better offer next period. Otherwise even in case of a lag of labor supply the firm may not increase the wage but reduces its labor demand. Furthermore, beside the fact that workers would lose one period of income there is no guarantee that they will be choose by the firm in the next period if many other workers decided to wait as well causing an excess supply of labor. On the other hand the costumers of a firm facing a quite similar problem. If prices are high shall they wait or buy the product considering that the firm might not adjust the prices but the supply or that in the next period an excess demand could occur in case of a price drop or that the firm holds the price thinking costumers only delaying their purchases. Considering that people usually assuming both social roles the interconnected interests uncover especially when durable goods and multi-period payments and labor contracts are in place. Thus a worker who may not be a costumer from her employee but most likely from another firm has to consider her consumption decisions while deciding whether to work for a certain wage. Each decision agents have to make separately takes place in an uncertain environment the linkage between them even amplifies the uncertainty agents experience. Agents might show different shema how to handle uncertainty, for example they may continuously adjust once they noticed changes of relevant variables like the wage or an excess demand for the product or they delay decisions until a certain trigger value is reached followed by periods of bigger adjustments. Anyway the interplay of the agent specific adjustment processes will determine the dynamics of the markets.

A lot of decisions that single individuals have to make fit as well the general picture of situations drawn above. For example an individual assuming the role of a voter who evaluates the political initiatives of the recent time. Lets consider a case that there are regular poles where people have to decide what level of satisfaction they announce sending a signal to the ruling government. Usually voters do not evaluate a single policy but a set of policies that do not even need to be related to the same issue. Therefore sending a positive signal having a recent development of a certain policy in mind means automatically evaluating the other policies as well even so that these might not have been pointing in a beneficial direction. One might also think of the opposite situation. Anyway, in both cases voters will find themselves in a dilemma situation while supporting an favorable policy change they must also fear to cause another development that in the end can make them to be worse off. This dilemma will probably cause some voters not to respond to changes of their environment immediately but to wait till a sustainable trend seems to be established. On the other side the government also experiences a challenging maybe an even more serious dilemma situation. By necessity it does not explicitly know to which policy change a signal refers. Additionally a government has to balance the interests of heterogeneous voters since it does not know how sensitive several voter groups react to changes. Consequently, it also does not know for sure how well the pole results reflect their actual opinions about the current political agenda. This dilemma of the government is illustrated in the first example. Furthermore the second example above shows that interdependencies between several policies will even intensify the dilemma situation of the government. The way voters and government deal with the uncertainty about the different consequences of their actions defines how they respond to changes in their environment. Thus, each agent shows an individual adjustment shema. Imagine a specific policy can be captured by a variable e.g. a tax rate or a budget size than its path over time depends on the interplay of the various shemata.

There are several aspects resp. mechanisms that occur in different manifestations in all those examples. At this point I will introduce four of them, since they are the conceptional cornerstones of the following considerations. First the decisions agents make, in other words the manifestations of their choice variables, will feed back to them over time along certain steps. Where each step comprises a decision of another agent. This is, what I will call a feedback loop. Furthermore there is at least one agent that is involved in two or more of such feedback loops. The second aspect is that agents decisions, thus the feedback loops are interdependent. This comes for different reasons. In the first example the state is confronted with the decision how to divide a budget into two sub-budgets to meet the demands of two voter groups. Since the overall budget is limited the budget decisions interdependent. A limited budget is also causing the interdependency in the firm-worker example. While in the last one circumstances compel a similar evaluation about two policies. Imagining this example as one with homogeneous voters shows that in such a case two feedback loops are already established since because of the two policies the interaction between voters and government is 2-dimensional. However, in the market example the feedback loops of the firm are interdependent because of a technical input-output relation. If labor supply and product demand are not harmonized one constrains the other with consequential effects on the firms objectives. The second example illustrates also well how structurally complicated a model may become while trying to grasp a multi-layered real interaction shema. For a start there are feedback loops consisting of two steps between several groups of voters and the government concerning the education and the tax policy. In addition there are two more feedback loops that also include firms hence their investment behavior what is the actual target of the governments policy mix. The interdependency between these two loops arises from the circumstance that the policies enfold a positive effect only in the right proportion. What is actually the same kind of interdependency as in the example where a firm tries to coordinate its activities on the input and the output market.

The third aspect concerns coordination. The common thing in all examples in terms of coordination is that agents interact sequentially with each other. For example a government announces a budget plan or a policy mix or a firm sets wages and prices in one period their counterparts, e.g. voters, workers or costumers, however will process this in the next period and act accordingly, afterwards this causes a reaction of the agents next on the feedback loops. At some point the initial impulses gets back to the first agents who then may adjust again depending on the reaction of their social environment. The main consequence from this kind of interaction is that there is no direct coordination between the agents. From this follows that a mismatch of the interests of the interacting agents is rather the standard case than an exception. For instance, between supply and demand on a market or in the case when the satisfaction levels expected by a government, while determining a budget plan, and the one actually announced by different groups of voters do not coincide. Furthermore, with the lack of direct coordination and the very likely mismatch of agents interests comes along a high degree of uncertainty since knowing to be coordinated would give them otherwise additional information that they so do not have.

This leads directly to the fourth common aspect, agents hesitant adjustment behavior. Because of the lack of information and no assurance of coordination an agent can not connect perfectly her behavior and that of the other agents. Thus an agent is uncertain how to respond to a change in her social environment. She does not exactly know whether a behavioral change of the other agents is temporary or persistent, to which extent this change was caused by her own behavior in past periods or to what feedback her present behavior will lead. As in the initial examples illustrated, agents seem to handle such decision situations in similar way. All their adjustments follow a typical pattern. Looking on a single agent, on the one side an agent shows at first strong hesitation regarding adjustments to behavioral changes of her social environment. Even if changes keep going, either she does only small adjustments of her behavior or no changes at all. On the other side, at a particular point once the imbalance between her environment and her own behavior becomes too large she overcomes the reasons causing her hesitation. The following adjustments are usual more intense than the recent environmental changes would suggest. Thus she seems to try to make up for the missed adjustments in previous periods. Eventually this will induce the agents with whom she interacts to readjust. Even when this happens she does not conduct immediately a corresponding readjustment as well, that means she shows a resistance to change in both directions. For example a voter responds barely to a changed policy mix, but at some point when a certain policy is changed too much in a non-beneficial direction she will signal her dissatisfaction strongly. Even if the government turns back recent policy changes, for the same reasons as before, the voter might not immediately signal that she agrees with this new development. Also a firm (e.g. prices, input factor demand) or the government (e.g. budgets) might show such erratic adjustment behavior. We will later see there are several economic as well as psychological reasons that might cause agents hesitation leading to such adjustment pattern.

This work has two goals. The first goal (section two) is to develop a deeper understanding of those four components and how they are connected. For this I do suggestions how to model them and illustrate those suggestions by referring to the described examples above. The second goal (section three) is to analyze how especially the hesitant adjustment behavior of the agents effect the dynamic of a model that is constructed in the way suggested in the first part. For this purpose I present a small scale agent-base model of a government and two types of voter that interact with each other concerning an environmental state. Furthermore, I analyze the interplay of different combinations regarding the reactions of the voters to a change of the environmental state and that of the government in terms of the feedback it receives from the voters. Section four concludes.

2.2 Essential components of a model of self-organizing, heterogeneous agents with hesitant adjustment behavior

2.2.1 Input variables, choice variables and feedback loops

Every agent in a society has to make plenty of decisions often various at the same time considering different aspects of their lives. Making a decision means to determine the value of a specific choice variable that is supposed to represent the agents decision about one specific aspect. Agents reassess their decisions frequently to responds to changes in their physical and social environment. Different frequencies are imaginable agents might set some choice variables once per period while they fix others irregularly for a couple of periods. Consequently there exists for every period and for every agent i ($i \in A = \{1, \cdots, a\}$ the set of agents of the modeled part of the world) a vector of choice variables $X_t^i = (x_{1,t}^i, ..., x_{N,t}^i)$ that the agent either determines in the actual period t or she has fixed in a past period $t-q$ for $q > 0$. As described in the introductory examples the state sets a tax rate or a specific budget while voters signal their satisfaction or firms announce a wage and offer a certain number of products for a certain price. Through her choice variables X_t^i agent i (un-)intentionally influences her environment starting from period $t+1$. Of course this works also the other way around, that is, every agent i has a specific vector of input variables $Y_t^i = (y_{1,t}^i, \cdots, y_{M,t}^i)$ in period t composed of choice variables of other agents whose values were determined in $t-1$ or even in a previous period. For example agent i interacts with two other agents j and h what means that agent i processes not necessarily all but some of their choice variables, lets say three of agent j's and two of agent h's, then the vector of input variables might look as follows $Y_t^i = (x_{3,t-1}^j, x_{5,t-1}^j, x_{6,t-1}^j, x_{2,t-1}^h, x_{3,t-1}^h)$. After receiving this vector at the end of period $t-1$ agent i conducts activities, which in the following I will call her internal processes, to be more precise the utilization of Y_t^i, the updating of her set of information I_t^i and based on this set of information the determination of her choice variables X_t^i. I will describe those processes in greater detail below. The important question at this point is, along which steps does an impulse induced by X_t^i influences Y_{t+p}^i for some $p > 1$. Where a step stands for a single or a group of agents. Agents form a group if they have the same vectors of input and choice variables in terms of the type of the variables and the vector size, e.g. workers with the same profession consuming the same product, firms using the same input factors to produce similar products or voters concerned about the same political topics. For the moment not the actual values of the future Y^i's is of interest but the specific sequence of steps that an impulse passes through on its way back to its origin. I call such a specific sequence a feedback loop. To structure the interaction of agents in a specific social situation with the help of feedback loops, I suggest to distinguish between a horizontal and a vertical dimension of a feedback loop system. Lets say one choice variable x_l^i of agent i is an input variable of a certain group of agents and being an agent of this group means to pass at least one choice variable to another group of agents where again every single agent of this group in turn determines at least one choice variable that is an element of Y^i the vector of input variables of the agent from whom the impulse originated. Additionally lets assume the same agents interact in the same order but at least at one step another choice variable so a variable of different type is transmitted. Moreover imagine now an interaction shema like the one before but where one agent is subsidized by an agent of a different group but who sets the same kind of choice variable for example a firm of a different type demanding the same kind of labor or a consumer who demands the same kind of product as the one before but also consumes other products the original one was not interested in. Furthermore let be there a fourth interaction which goes along the same steps as in the last case except that there is now an additional agent of a new group involved before the impulse feeds back to agent i. The described interactions form four vertical distinctive feedback loops. Technical speaking a feedback loop is a sequence of specific choice variables so that the last choice variable is an input variable of the agent who sets the first choice variable of the sequence, e.g. $(x_t^i, x_{t+p_1}^j, x_{t+p_2}^k, x_{t+p_3}^h)$, where $x_{t+p_3}^h$ is an element of $Y_{t+p_3+1}^i$ and i, j, k, h are distinct elements of A and $1 \leq p_1 < p_2 < p_3$.[4] Two sequences are in a vertical

[4]An interaction shema based on such a sequence of actions implies that agents do not receive immediately a feedback concerning their actions. This is a crucial aspect in this framework.

sense of different type, assuming that the first element of both sequences is set by the same agent resp. by agents of the same group, if for at least one element of a sequence the counterpart of the other sequence is of different type, is determined by an agent of a different group or does not exist meaning the sequences do not have the same number of elements. Obviously, since there are groups of agents involved and a single feedback loop refers to one agent of a group at every step there are plenty of possible combinations, that is, plenty of feedback loops with the same vertical structure but different elements resp. different manifestations of the same choice variable type. Such feedback loops form a family. To understand why and in which way the values of different feedback loops of the same family differ even so that they necessarily have the same vertical structure one has to take a look on the horizontal dimension of a family of feedback loops which refers to their inner-heterogeneity. Even if two agents belong to the same group they might process the same input variables in different ways due to different preferences e.g. in terms of consumption or political agendas, or diverging parameters of an otherwise similar decision method or they actually applying various decision rules while processing the same information or in case of firms they use the same inputs but processing them differently to produce the same goods.

At this point one has to admit that probably in real live do not exist two agents, not to mention a whole group of agents, with identical input and choice variable vectors. This becomes obvious once reminding that agents usually take a lot of different social roles. For example an agent who buys a final good might beside being a consumer also being a worker, a voter or a social volunteer. Very likely the decisions she does within her various social roles are interdependent. Same is true for a firm that typically interacts with a diverse group of stakeholders, e.g. workers, investors, policymakers and different interest groups. But even letting this aside taking only firms operations in a narrow sense into consideration the doubt, whether there are identical firms, still remains. Using the same inputs, what is highly questionable considering the complexity of production procedures, but processing them differently to produce the same goods means that at least in the past, and most likely will be in the future, other inputs were used to gain the knowledge and technologies particular for each single firm. In a nutshell, considering this variety of the real world, that is, the infinity many manifestations of agents characteristics and their way to interact developing a model along clearly defined feedback loops and distinguished agent groups is a method to reduce complexity. It is part of the modeling process to form groups of agents by concentrating on the crucial characteristics resp. variables and to select the feedback loops that are supposed to be the driver of the dynamic of the examined social interactions. To what extend one keeps certain degree of real live heterogeneity and interactions aside depends on the purpose of the model and the subjective decisions of the modeler.

Although the examples mentioned so far capture very different parts of a society all of them have in common that there is an agent, the state in the first example or a firm in the third one, who is involved in at least two of the explicitly modeled feedback loops connecting those. This is one of the main ingredient of the modeling approach in this work.

2.2.2 Interdependent feedback loops

The second aspect concerns the interdependency of the feedback loops. To see the importance of this point one needs to consider that the agents take interest first of all not in their choice variables (X^i) but in their input variables (Y^i) the ones that will be utilized. Assuming $\pi^i(Y^i_t)$ is the objective function of agent i then $\Pi^i_t = \pi^i(Y^i_t)$ is the realization of agent i's objective function in period t this might be for example profits or utility depending on the type of agent i.[5] In most real life situations agents can not simply determine and set their input variables on values that optimize their rationals resp. their objective function and utilizing them. The only possibility of agent i in period t to affect Y^i_{t+p} is through an impulse initialized by X^i_t passing through the

[5] Beside the case $\Pi^i_t = \pi^i(Y^i_t)$, there are more complex situations where Π^i_t is the result of an optimization. In such cases the input variables constitute constraints of the corresponding optimization problem. While chapter 2.4 considers the general case in more detail, it is at this point sufficient to focus on the more simple one, what is a special case of the latter. Nevertheless a firm synchronizing demand for their good and its input factor flows is an example for an utilization with an optimizing agent. Alternatively, the utilization process of a government receiving signals from voters evaluating its policies takes place without optimization, since the government has to process its input variables to the full extent. Obviously there is not the option to chose from a certain choice set the optimal value of voters signal.

feedback loops back to agent i in period $t+p$, where $p > 1$[6]. This leads to the question, whether agent i considers, while determining the value of a certain choice variable $x^i_{l,t}$, only the expected influence that this variable will have on a certain input variable $y^i_{m,t+p}$ along one specific feedback loop, or does agent i also has to consider effects that $x^i_{l,t}$ has on $y^i_{m,t+p}$ along other feedback loops, just as the influence that her other choice variables ($x^i_{n,t}$ for $n \neq l$) might have on this particular input variable? Additionally, does agents i has to consider a potential impact that any choice variable including $x^i_{l,t}$ of agent i might have on her other input variables ($y^i_{o,t+p}$ is an element of Y^i_{t+p} for $o \neq m$) and by that on the realization of the objective function in $t+p$. To put it briefly, answering those questions means to answer the question, are there between the feedback loops connected by agent i interdependencies that need to be considered while determining $x^i_{l,t}$?

In the examples presented so far – as probably in most real live decision situations – agents operate along interdependent feedback loops. Such interdependencies might exist for various reasons. One that is present in many decision situations is due to the fact that choice variables may be subject to constraints. For example, let $x^i_{l,t}$ and $x^i_{n,t}$ be two choice variables of agent i constrained by $\boldsymbol{\upsilon}(x^i_{l,t}, x^i_{n,t})\ \boldsymbol{\mathcal{R}}\ \boldsymbol{\theta}$ where $\boldsymbol{\upsilon} = (\upsilon_1(x^i_{l,t}, x^i_{n,t}), \ldots, \upsilon_W(x^i_{l,t}, x^i_{n,t}))$ is a set of W constraint functions, while $\boldsymbol{\mathcal{R}} = (\mathcal{R}_1, \ldots, \mathcal{R}_W)$ symbols a vector of certain relations usually (in-)equality relations and $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_W) \in \mathbb{R}^W$ is a given parameter vector, then feedback loops with either x^i_l or x^i_n as an element are interdependent. General speaking, such a set of constraints could exist for any subset of choice variables. One of the most prominent examples is the budget constraint that occurs in various forms affecting the possibilities of a lot of different agents. In the first introductory example the state has to decide how to divide a limiting amount of money between two competing purposes the security force and the education system. While the firm in example three finds itself in the dilemma in which proportion to split a fixed budget for extra payment, work place improvements or working education between various groups of workers. A further example is the

[6]Because of the sequential characteristic of feedback loops even in case of the smallest possible interaction shema, one with two agents, it takes at least two periods before agents receive a feedback on their actions.

well-known consumer budget constraint. Moreover, in case a budget is interpreted as a term that refers to a general, limited resources as e.g. time, space or certain capacities, that at least in the short run often prove to be scarce, then there are plenty of other situations with interdependencies of the kind caused by monetary budgets.

Beside limiting budgets, constraints may also arise from technical causalities. A firm that operates in a mode of co-production, what is the case for most firms of the industrial sector, considering also all kind of pollutions, needs to pay attention to the overall consequences of its various outputs while determining its production plan.

Another potential source for interdependencies is the objective function itself. It is usually the case that certain combinations of input variables are more beneficial than others. This trivial statement becomes more delicate once one remembers the often present inability of agents to chose directly one of those combinations meaning to optimize. But rather an agent tries to make the consecutive agents along the feedback loops, to act, that is, to set their choice variables, in a way that she finally receives the favored input variables. Since an agent can only presume how other agents will respond there is the risk that not the intended combination of input variables is received with corresponding consequences for the realization of the objective function. In other words agents act in an uncertain environment.[7]. The next three chapters will shed some more light on these issues.

Apparently, a firm has to synchronize many feedback loops in order to be profitable, for example its flow of input factors and the demand for its product. Otherwise it either experiences lost potential sales and revenues or bears the costs but does not generate the revenues to make its endeavor a successful business. Thus a firm is confronted with at least two feedback loops whose interdependency is caused by its profit function.

These are not the only reasons for interdependencies, they are simply some that agents have typically to deal with. At each step of agents internal processes, thus while utilizing the input variables Y_t^i, processing the thereby received information and

[7]Consequently the challenge that arises from the vague influence an agent has on its input variables is how to consider a mismatch between the intended and the realized input variables in past periods while determining present choice variables. We will see agents find themselves in a permanent adjustment process based on simple optimization procedures and heuristic rules

finally while evaluating the effects that X_t^i will have on future input variables (Y_{t+p}^i), hence determining the choice variables, might occur plenty of other mechanisms causing interdependent feedback loops. Moreover agents have to take interdependencies into account that are originated in the processes of other agents on the feedback loops. This might be the case when at least two agents interact along the same two or more feedback loops. If one agents experiences interdependent feedback loops because of its internal processes the loops are interdependent for the other agent as well. The second example in the introduction describes such situation. Apparently a government conducting an economic policy for a certain region needs to coordinate several policies to create an promising investment climate for firms to act accordingly. Therefore on the one hand the interdependencies are caused by the way a firm processes its input variables. On the other hand those variables are set by the government.[8]

Evidently, since the introduced mechanisms can be inherent to any single connecting agent of a feedback loops system, the interdependency of such loops is rather the standard case then a rare exception. What kind of interdependencies occur depends on the way the internal processes of the agents and the feedback loop structure itself are modeled, that is, on the assumptions made about agents behavior and about the social environment in which they operate. The important point concerning the kind of social situations focused in this work is that in all of them at least one connecting agent has to balance two interdependent feedback loops at the minimum.

2.2.3 Sequential interaction and coordination

Usually when we talk about coordination or coordinated agents, we talk about situations in which the individual plans of the agents fit to each other in a way that first it is possible to conduct those plans and second that agents do not have an incentive to deviate from their individual plan of action. In a nutshell, the part of the social world (e.g. markets, different interacting groups inside a firm or the interplay between citizens and government) that is modeled is in an equilibria state. This can be achieved

[8]This example also shows that in case of two feedback loops with at least one common element the loops are necessarily interdependent, since the other connecting agents can not distinguish how this common signal of one agent is influenced by the distinct signals they send along the feedback loops.

either by a central coordination device as the Waldensian auctioneer constitutes one or as in strategic decision situations by the agents themselves. Especially models that belong to the first group assume that agents do not act before a plan of action assuring equilibria is elaborated. Since there is no trading, no production or, in general terms, no activities at all out-of-equilibria one might say that coordination happens in zero-time. Once such a plan is established agents conduct their part of it without further consultations. In modeling concepts that are based on such kind of coordination time implicitly loses its meaning.

As one can see such coordination process does not take place in the social situations described in the introduction. That is why in the modeling approach at hand agents are supposed to act sequentially since they base their decisions in the actual period t on the information received in past periods, thus there is no further interaction between agents in the current period influencing agents decision in this particular period. Therefore agent i's only possibility to influence her social environment, thus to make other agents to behave in a certain beneficial way, is to set her choice variables (X_t^i), to wait until they transmit along the feedback loops, to evaluate the feedback (Y_{t+p}^i) and most likely to adjust her choice variables (X_{t+p}^i) to generate a more beneficial feedback in future periods. For example a government announces a state budget plan, citizens signal their satisfaction through polls, followed by the first rethinking the budgets depending on the reactions of the second. Or a firm distributing a limited amount of bonus payments, waiting and observing how this effects motivation and work results of the single groups of workers and eventually adjusting the payments. Up to now in all mentioned examples, even so that agents act every period, exists, considering a certain impulse caused by a certain realization of a choice variable, a repeating sequence of acting, waiting, evaluating feedback and acting again initializing a new impulse. For those sequential interactions the term coordination has to be interpreted in a broader sense. It implies that agents respond to their environment trying to fit, to adjust and even trying to influence others to induce from their perspective a better fit. But in this broader sense coordination does not necessarily imply that interacting agents reach a state where individual plans resp. their activities match to each other perfectly as stated in the beginning. Hence coordination is first of all understood as a process of mutual influencing than as a state of balance, hence a state where agents have no incentives to change their choice variables. If such state would occur between a group of interacting agents and particular if it does on the global level (the entire feedback loop system) it would be rather a random side effect than an intended outcome.

This does not mean that common agreements leading future activities of the involved agents can not be achieved. It can and of course it happens but not in a way where all activities stop until such an agreement is achieved. The rest of the 'world', more precisely the interactions along the not involved feedback loops will continue to take place. Hence agents may bargain an agreement for some of their choice variables while continuing to set sequentially others not included ones. As a consequence not only choosing the optimal set of agents and choice variables but also determining the duration of the agreement as well as the time agents are willing to spend for bargaining are decisions that need to be made while negotiating an agreement. In such case coordination is understood in the sense of a balanced state. To compare interactions in cases of common agreements with those taking place in social situations where agents interact sequentially one may account for the directness of their interactions. Directness can be understood as a two dimensional criteria. On the one hand directness refers to the transmission of a choice variable between two agents of a feedback loop system. Agents who send and receive from each other a variable, thus constituting the smallest possible feedback loop, are most directly interacting. In case of a feedback loop consisting of three agents, all of them are still interacting with each other along this particular feedback loop but their interactions are one-sided. On a feedback loop with even more steps there are for each agent other participating agents with whom they interact only through other agents. The least directly interacting agents are those that are not on a common feedback loop. This is in principle the same for both kind of interactions the previously arranged agreements and the sequential ones. But on the other hand, since they harmonize their choice variables, the first kind shows more directness in a contentual way what can be interpreted as the second dimension of directness.[9]

[9]In case of interactions based on agreements it is useful to account for the expanse of it. Locally, between two agents this refers to the proportion of choice variables covered by an agreement, while

Needless to say that the extreme case where it is optimal for all agents not to interact until a complete agreement is reached is certainly not going to take place in the real world. Rather, both kind of coordination occur simultaneously along different feedback loops. Both sets of feedback loops the ones coordinated sequentially and the ones coordinated through an agreement might evolve over time in terms of their expanse with corresponding consequences for the other. In this work the focus lies on the part of the world in which agents interact sequentially, hence contentual indirectly. Thus coordination in the narrow sense is not an aim of those agents. Actually we will see that it depends on the way the internal processes of agents are formulated whether agents reach a state of unintended equilibria, again a state where agents have no incentives to change their choice variables. Speaking of sequential interaction and internal processes, what distinctive features needs to be beared in mind while modeling agents internal processes in comparison to approaches that are either based on agreements or on equilibrium concepts?

Two such features are crucial in this work. First the circumstance that without intended coordination the realization of the input variables in the current period (Y_t) are not necessarily the one that agents had in mind when determining their choice variables in past periods. What also means that the various input variables ($y_{1,t}, \ldots, y_{M,t}$) do not need to match in the most beneficial way to each other. Actually this is the normal case. Since agents choose to interact sequentially, they neither communicate in the current period trying to set up an agreement about future actions nor is their a mechanism making sure that their planed actions match. This implies that an agent does not exactly know how the other agents will respond in future periods to the decisions she makes in the actual period. That suggests to have a separated look on the utilization of the input variables (Y^i) and the determination of the choice variables (X^i) of an agent. In other words to model them as distinguished processes, thus as two decisions

globally to the size of the subset of agents who are involved in this particular agreement. In case all agents and all their choice variables are part of a single (complete) agreement, it would occur a situation that might be close, depending on the mechanism leading the negotiation of such agreement, to an economy that is implicitly assumed in models based on equilibrium concepts. But, as mentioned above, whether an agent is participating in such an agreement is not systemic as in equilibrium models but up to the decision of each single agent whether it is beneficial to join or not.

that have to be made. Of course this does not mean that there are no conceptual interdependencies between both, of course there are. Below we will see that on the one side the utilization of Y_t^i might have, under certain conditions, an influence on the determination of X_t^i and on that of choice variables of future periods. On the other side, since in cases of sequential interactions agent i can neither determine her future input variables (Y_{t+q}^i, for $q > 2$) on her own nor can she anticipate the exact response of the other agents her influence in period t on Y_{t+q}^i is therefore indirect and imprecise what is necessarily also true for their utilization. Of course agent i has in mind this influence while determining X_t^i, actually to apply that influence is the only purpose of determining the choice variables, and most certainly she will succeed up to a certain level to affect her future input variables in a beneficial way but the final decision how to utilize Y_{t+q}^i has to be made in period $t + q$ the point in time when the actual value of the input variables is known to agent i.[10]

The second crucial feature concerns uncertainty. What influence has the assumption of sequential interactions, that is, the lag of coordination in the narrow sense on the level of uncertainty an agent has to cope with? Considering that a crucial source for uncertainty are agents limited information about her social environment coordination based on sequential interactions leads inevitably to high uncertainty. For one thing, the only information about other agents that agent i frequently updates are related to her neighboring agents on the feedback loops, meaning agent i's choice and input variables. Furthermore, even so that she might have some general knowledge about the network of feedback loops usually she does not oversee the network as a whole. When it comes to complex transmission channels agent i might not even be aware of all feedback loops she is part of. Therefore she also does not know the entire vector of input and choice variables of the other agents. The same is true for the information about the

[10] If agents intendedly coordinate their actions for a number of periods they necessarily incorporate the potential responses of their interaction counterparts (e.g. agents i and j). Since agent i exactly knows what consequences ($\hat{Y}_{t+q}^i = \hat{X}_{t+q-1}^j$) will be caused if she sets X_t^i at a certain value $\hat{X}_t^i$ she can optimize her utility in period $t + q$ by setting X_t^i accordingly. Thus a mismatch between the intended and the realized feedback is conceptually ruled out. Consequently, the determination of X_t^i and the utilization of Y_{t+q} would become a single decision. Again, this is only possible to conduct by means of an agreement or through a coordination device as the Walrasian auctioneer constitutes one.

internal processes of those agents. While agent i might have some knowledge about the way other agents next to her on the feedback loops process their input variables it seems reasonable to assume that this knowledge gets less reliable the further two agents are apart along a feedback loop. What is even more decent to assume for agents not interacting along a common feedback loop. For another thing, the only way for agents to develop some further understanding about the mechanisms driving other agents behavior is to form expectations based on the time series of their choice and input variables and on their general knowledge. The fact that agents have to form expectations itself is not extraordinary what makes it crucial for this kind of models is the circumstance that it goes along with a lack of direct coordination. This becomes apparent considering the role of expectations within the other two mentioned approaches of modeling coordination. In models where agents coordinate by means of an agreement expectations concerning the other agents variables do not play a role. While in models based on an external coordination mechanism they do, but such mechanism will also assure considering agents expectations that their individual plans match. An approach where agents know that their actions are perfectly harmonized implicitly assumes that agents have additional information. Thus agents have to cope with less uncertainty about future outcomes while forming expectations then they would need to in settings with sequential interactions. In a nutshell agents decide and act based on very limited information about the feedback loop structure as well as about other agents variables and internal processes, hence in a highly uncertain environment without direct coordination. An approach incorporating such features should take the consequences for agents decision making in terms of uncertainty explicitly into account.[11]

[11] Of course, there are other not considered features inherent to this approach that also origin from the possibility of not matching individual plans, e.g. which influence does a mismatch of expected and realized input variables of past periods have on the determination of the choice variables in the current period?

2.2.4 Agents internal processes

Lets keep these two crucial aspects in mind, the potential mismatch of the intended and the realized input variables and agents highly uncertain environment, while having a closer look on the internal processes of the agents. As already mentioned the activities of an agent are divided into three groups that in a condensed form are understood as abstract internal processes namely utilization, information updating and determination of choice variables. Though this division follows a chronological and contentual logic it is, of course, to a certain extent arbitrary.

2.2.4.1 First process: Utilization of the input variables

Concerning the utilization process agent i literally has to live in period t with the input variables ($Y_t^i = (y_{1,t}^i, \ldots, y_{M,t}^i)$) that she received in period $t-1$ and the her state variables ($S_{t-1}^i = (s_{1,t-1}^i, \ldots, s_{\hat{M},t-1}^i)$) accumulated in the past. There is no possibility to change Y_t^i even if it is not the intended feedback she had in mind while determining her choice variables in past periods. In chapter 2.2 a special case of utilization was already introduced, one without actual choosing hence optimizing. The general form of the utilization process of agent i in period t is to be understood as a constraint optimization of her objective function π^i over $\tilde{Y}_t^i = (\tilde{y}_{1,t}^i, \ldots, \tilde{y}_{M,t}^i)$ and $\tilde{S}_t^i = (\tilde{s}_{1,t}^i, \ldots, \tilde{s}_{\hat{M},t}^i)$ under consideration of a set of grouped constraints (2.2), (2.3), (2.4), (2.5) and (2.6).[12]

$$\Pi_t^i = \max_{\tilde{Y}_t^i,\, \tilde{S}_t^i} \pi^i(\tilde{Y}_t^i, \tilde{S}_t^i) \tag{2.1}$$

$$\text{s.t. } \check{\boldsymbol{\upsilon}}(\tilde{Y}_t^i)\, \check{\boldsymbol{\mathcal{R}}}\, \check{\boldsymbol{\theta}}, \tag{2.2}$$

$$\tilde{y}_{m,t}^i\ \check{\mathcal{Q}}_m\ y_{m,t}^i \ \text{ for } m \in \{1, ..., M\} \tag{2.3}$$

$$\text{and s.t. } \hat{\boldsymbol{\upsilon}}(\tilde{S}_t^i)\, \hat{\boldsymbol{\mathcal{R}}}\, \hat{\boldsymbol{\theta}}, \tag{2.4}$$

$$\tilde{s}_{\hat{m},t}^i\ \hat{\mathcal{Q}}_{\hat{m}}\ s_{\hat{m},t-1}^i \ \text{ for } \hat{m} \in \{1, ..., \hat{M}\} \tag{2.5}$$

$$\text{and s.t. } \bar{\boldsymbol{\upsilon}}(\tilde{Y}_t^i, \tilde{S}_t^i)\, \bar{\boldsymbol{\mathcal{R}}}\, \bar{\boldsymbol{\theta}}. \tag{2.6}$$

The first group consists of constraints that are assumed to be exogenously given for the part of the world that is explicitly modeled, for example working time limits, quotas

[12] The constraints of the group (2.2), (2.4) and (2.6) are of the same structure as the ones constraining the determination of the choice variables in chapter 2.2.

to employ people with certain characteristics or consumption restrictions. While (2.3) can be contentually interpreted as a potential, conditioned by agent i's input variable $y^i_{m,t}$ and the corresponding relation $\check{\mathcal{Q}}_m$, that agent i might use. For example a firm has in period t a certain number of workers $y_{m,t}$ on hand. This is the maximum number of workers it is able to deploy. But, in case of a low expected demand ($y^i_{o,t+2}$) in the next period (thus less sold products) it might be more profitable, depending on the production process and the storability of the produced good, to deploy less workers despite the wage costs. Or the opposite is true, the demand is evaluated to be higher and the firm would like to deploy even more workers, but obviously this is not feasible, since $\check{\mathcal{Q}}_m$ represents an equal-or-less relation. In contrast to this $\check{\mathcal{Q}}_m$ becomes an equal relation if agent i has no choice but to process the complete amount of its input variable ($\tilde{y}^i_{m,t} = y^i_{m,t}$). As it is in case of a government being evaluated by its voters or for a firm and the demand for its product in the last period $t-1$ ($y^i_{o,t}$). There is no possibility to choose, thus to optimize.[13]

Also the second vector of arguments $\tilde{S}^i_t$ of π^i needs to be understood as the amounts of potentials that are actually used by agent i. Those potentials are put at disposal by agent i's state variables S^i_{t-1} e.g. physical capital, stocks and inventory, knowledge, pollution rights or past-depending safety requirements hence accumulating variables.[14] The constraining nature of the state variables depends on the specific form of (2.5). In case where $s^i_{\hat{m}}$ represents capital $\hat{\mathcal{Q}}_{\hat{m}}$ becomes most likely an equal-or-less relation. On the other hand knowledge is probably best modeled with $\hat{\mathcal{Q}}_{\hat{m}}$ being an equality relation and in case of safety requirements it might be reasonable that $\hat{\mathcal{Q}}_{\hat{m}}$ represents a greater-or-equal relation. Additionally, there may be as well constraints (2.4) that

[13] This special case of the utilization process was introduced in section 2.2.

[14] In models without equilibrium concepts the role of state variables becomes from a methodological point of view more crucial. It is most likely that the input variables do not take the most beneficial values in each period. If, in addition to that, it is technically possible to store the corresponding object in some way the input variable is associated with a complementary state variable. One can argue that this is the case for most input variables. The following example illustrates the difference between immediate utilization and a specific kind of storing. Imagine a firm i has a certain labor force at hand $y^i_{m,t}$ but it utilizes only $\tilde{y}^i_{m,t} < y^i_{m,t}$ in period t because of a lack of expected demand in $t+1$. Either there is only immediate utilization, thus a part of $y^i_{m,t}$ stays unused or in case working-time accounts exist $y^i_{m,t} - \tilde{y}^i_{m,t}$ can be stored in a corresponding state variable $s^i_{\hat{m},t}$.

are exogenously given, for example capacities that have to be accessible at any time like in case of the power grid, or inventories of gas, oil and oil products in case of refineries. There might be also constraints (2.6) which bring input and state variables into relation, for example safety requirements in terms of operating machines that need proper supervision or the opposite constellation that features of the physical capital limiting the number of workers that can operate at the same time.

It remains the question, how S_t^i and $\tilde{S}_t^i$ are determined? Depending on the context answering this question may get easily very complicated. Because of limited space and considering the general nature of this first part, only some basic conceptual aspects are sketched below. There are two crucial features to bear in mind while distinguishing between different implementations of state variables. First whether it is technically possible to chose the value of $\tilde{s}_{\hat{m},t}^i$ within certain boundaries or does it equal $s_{\hat{m},t-1}^i$, in other words does $\hat{\mathcal{Q}}_{\hat{m}}$ represents an equality relation or does it not. Furthermore changes of s_m^i from period $t-1$ to t might be caused among others by the extent it is used in the utilization process in period t and by the transformation of a specific y_m^i in period t. Therefore the second feature refers to the issue whether there is a trade-off between utilization of $y_{m,t}^i$ or using it to shift $s_{\hat{m}}^i$ to a certain value ($s_{\hat{m},t}^i$) at the end of period t. Obviously the most simplest situation appears if there is no trade-off and $s_{\hat{m}}^i$ is completely used in the utilization process in each period, that is $y_{m,t-1}^i = s_{\hat{m},t-1}^i = \tilde{s}_{\hat{m},t}^i$. If this is not the case (2.1) would become part of an intertemporal optimization problem. Since agent i can now either increase Π_t^i by utilizing more of $y_{m,t}^i$ in period t ($\tilde{y}_{m,t}^i$) or using it to shift $s_{\hat{m}}^i$ to a certain value $s_{\hat{m},t}^i$ and eventually utilizing it in form of $\tilde{s}_{\hat{m},t+1}^i$, hence realizing a higher level for Π_{t+1}^i. Another kind of intertemporal problem exists if $\tilde{s}_{\hat{m},t}^i \neq s_{\hat{m},t-1}^i$, that is, agent i can choose to which extent she uses the potential emerging from this specific state variable. In such a case it may happen that, depending on the utilization process, a trade off between $\tilde{s}_{\hat{m},t}^i$ and $\tilde{s}_{\hat{m},t+1}^i$ arises. Of course both dilemmas can occur at the same time.

Modeling a situation that requires such kind of intertemporal reasoning makes it necessary during the utilization process to take the determination of X_t^i into account. Since, at least partly, setting X_t^i is the only way to influence future values of Y^i. Below we will see that the determination process of the choice variables without such intertemporal reasoning is already a huge challenge for a single agent with limited cognitive abilities operating in a decentralized organized and uncertain environment. This suggests to use heuristic rules in order to reduce complexity. For example allocating $y^i_{m,t}$ according to a fixed proportion between its utilization $\tilde{y}^i_{m,t}$ and its transformation into additional $s^i_{\hat{m},t}$ in each period. A simple way for a firm to handle the second dilemma, could be to implement a rule so that does not fall $s^i_{\hat{m},t}$ below a certain threshold while choosing $\tilde{s}^i_{\hat{m},t}$ under consideration of $y^i_{m,t}$.[15]

2.2.4.2 Second process: Updating of the information set

In general agent i's information set I^i_t in period t has the following structure

$$I^i_t = \{(X^i_{t-1}, \ldots, X^i_{t-T}), (Y^i_{t-1}, \ldots, Y_{t-T}), (S^i_t, \ldots, S^i_{t-T}),$$
$$(\tilde{Y}^i_t, \ldots, \tilde{Y}^i_{t-T}), (\tilde{S}^i_t, \ldots, \tilde{S}^i_{t-T}), \mathcal{T}^i, (\dot{A}^i, \boldsymbol{\omega}^i), \Theta^i\}.$$

Of course agents choice, input and state variable vectors are elements of this set as well as the vectors of the input and state variables utilized in each period.[16] It is assumed that agent i cares about resp. memorizes the past T periods. Furthermore agent i might have some additional general knowledge about her social environment which is part of her information set. For one thing agent i may presume to have knowledge about the internal processes of some of the other agents $\ddot{A}^i = \{1, \ldots, \ddot{a}^i\} \subset A$ represented by $\mathcal{T}^i = (\boldsymbol{\tau}^i_1(\cdot), \ldots, \boldsymbol{\tau}^i_{a^i}(\cdot))$, where $\boldsymbol{\tau}^i_j(\cdot) = (\tau^i_{j,1}(\cdot), \ldots, \tau^i_{j,J_j}(\cdot))$ for $j \in \ddot{A}^i$ and $J_j \in \mathbb{N}$. Those J_j functions could represent a presumed profit or utility function as well as a constraint or a heuristic rule to determine choice variables that agent i believes agent j is applying. Real agents usually have incomplete information about other agents preferences, the way those agents reason or the set of information they use, to name only a few. A possibility to take this into account while modeling agent i's general knowledge

[15] In the event that modeling a social situation demands to consider intertemporal dependencies and no heuristic rules are employed the suggested simplifying chronological order of the internal processes needs to be suspended.

[16] Depending on the kind of agent state variables could be involved in the determination process of agents choice variables. Consequently their values alter in the same period again resp. are at all updated at the end of the period. This will be the case in the introductory model illustrated in chapter 3.

is to assume that agent i knows the general form of these functions but is uncertain about the parameters. Consequently they need to be updated based on the overall current information set. But of course there are social situations where it is reasonable to assume, while modeling agents knowledge, that they know the right parameters as well as situations where agents presume a wrong general form. For another thing agents are considered to have limited knowledge about the overall feedback loop system. At least for a certain subset of agents $\dot{A}^i = \{1, \ldots, \dot{a}^i\} \subset A$ agent i presumes to know how these agents are linked. This is expressed for agent i by a graph $(\dot{A}^i, \boldsymbol{\omega}^i)$, that consist of the set of nodes $\dot{A}^i$ and a $\dot{a}^i \times \dot{a}^i$ matrix $\boldsymbol{\omega}^i$, where each matrix element $\omega^i_{j,h}$ (for $j, h \in \dot{A}^i$) is either 0 in case there is no assumed connection or a vector of choice variables that is transmitted from agent j to agent h. Additionally each agent knows about a certain set of parameters Θ^i, e.g. the parameter vector $\hat{\boldsymbol{\theta}}^i$ that is part of the constraint (2.4) or the parameters specifying $\mathcal{T}^i$.

2.2.4.3 Third process: Determination of the choice variables

Based on the information set agent i conducts its third internal process the determination of its choice variables. In its most general form the determination is a mapping from the space of the information set to the space of the choice variables. Depending on the assumption about the cognitive abilities of agent i concerning the way she processes information and copes with uncertainty, many different ways are imaginable to fill this general expression with life. To illustrate the range of possibilities in the following two very different approaches are briefly sketched.

a: choice variables based on an optimization

The first requires agents to have fairly powerful cognitive abilities. Initially, based on its information set agent i derives expectations about how other agents may respond to her potentially possible actions. In other words, what kind of feedback in future periods would different values of her choice variables induce? Technical speaking agent i assumes in period t that the input variable $\bar{y}^i_{m,t+p}$, thus the feedback in period $t+p$ (for $p > 2$), follows a specific distribution conditional on her information set (I^i_t) and her choice variables (X^i_t), which she is supposed to set in the current period t. There are two assumptions to make that specify the way agent i processes information. First, the general form of the distribution that agent i applies and second the rules $\boldsymbol{\psi}^i_{1,m}$ that she uses to determine the moments of this distribution.[17] For example, let agent i assume that $\bar{y}^i_{m,t+p} \sim \mathcal{N}(\mu_{m,t+p}, \sigma^2_{m,t+p})$ and that $(\mu_{m,t+p}, \sigma^2_{m,t+p}) = \boldsymbol{\psi}^i_{1,m}(I^i_t, X^i_t)$ for $m \in \{1, \dots, M\}$. This leads to the probability density function $\rho^i_{\bar{Y}^i_{t+p}|I^i_t,X^i_t}(Y^i_{t+p})$ for $Y^i_{t+p} = (y^i_{1,t+p}, \cdots, y^i_{M,t+p})$, where $X^i_t \in C^i_t$ and $C^i_t = \{X_t \in \mathbb{R}^n : \boldsymbol{\upsilon}(X_t)\, \boldsymbol{\mathcal{R}}\, \boldsymbol{\vartheta}(I^i_t)\}$[18] is agent i's choice set in period t. That is,

$$\begin{gathered}\rho^i_{\bar{Y}^i_{t+p}|I^i_t,X^i_t} : \mathbb{R}^M \to \mathbb{R}_+, \ \ (y^i_{1,t+p}, \dots, y^i_{M,t+p}) \mapsto \rho^i_{\bar{Y}^i_{t+p}|I^i_t,X^i_t}(y^i_{1,t+p}, \dots, y^i_{M,t+p}) \\ \text{s. t.} \\ 1 = \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \rho^i_{\bar{Y}^i_{t+p}|I^i_t,X^i_t}(y^i_{1,t+p}, \dots, y^i_{M,t+p}) dy^i_{1,t+p} \dots dy^i_{M,t+p} \ \forall X^i_t \in C^i_t.\end{gathered} \tag{2.7}$$

The mapping (2.7) can be interpreted as a condensed form of agents i understanding about the way the considered part of the world works. In a second step agent i uses her subjective imagination of the world to determine the optimal vector of her choice variables, hence the vector ($\overset{\star}{X}{}^i_t \in C^i_t$) that optimizes the expected value of her objective function

$$\begin{gathered}\overset{\star}{X}{}^i_t = \psi^i_2(I^i_t) = \underset{X^i_t \in C^i_t}{\arg\max}\, \mathbb{E}[\pi^i(\bar{Y}^i_{t+p})|I^i_t, X^i_t] = \\ \underset{X^i_t \in C^i_t}{\arg\max} \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \pi^i(\bar{Y}^i_{t+p})\, \rho^i_{\bar{Y}^i_{t+p}|I^i_t,X^i_t}(y^i_{1,t+p}, \dots, y^i_{M,t+p}) dy^i_{1,t+p} \dots dy^i_{M,t+p}.\end{gathered} \tag{2.8}$$

The advantage of this approach is that the decision of agent i is based on an optimization procedure. Therefore she shows at least in the second step a certain rational behavior. On the other hand it is somehow arbitrary which distribution agent i assumes and what rule she uses to update the moments of this distribution. Hence it is not simple to show that agent i acts also in the first step according to a certain rationality concept, which in itself is already difficult to establish.[19] As already mentioned this approach has a big disadvantage the high cognitive demands agents need to meet while compiling (2.7) and computing (2.8). This becomes even more demanding once agent i assumes that, contrary to the illustration above, her choice variables set in a certain

[17]The kind of distribution and the form of the functions $\boldsymbol{\psi}^i_{1,m}$ reflects how agent i is assummed to scope with uncertainty.

[18]Those constraints are of the same structure as the one introduced in chapter 2.2.

[19]See Gilboa (2010).

period influence their input variables not in one but in a few consecutive future periods. Furthermore agent i has to take diverse intertemporal dependencies into account which will do their part in making the computation of the optimal value $\overset{\star}{X}{}_t^i$ a real challenge. That is because agent i determines her choice variables in every period, thus a decision today might influence the decision situations and in a consequence the optimal choices in future periods. One example for such interdependencies are the described effects that come along as soon as state variables are involved. ***b: choice variables based on heuristic rules***

The second approach takes up those critics. Agents modeled in this sense are confronted with less complexity and need far smaller cognitive abilities. Again it is a two step procedure. At first agent i determines for each choice variable $x_{l,t}^i$ with $X_t^i = (x_{1,t}^i, \ldots, x_{N,t}^i)$ a reference value $\hat{x}_{l,t}^i$ with $\hat{X}_t^i = (\hat{x}_{1,t}^i, \ldots, \hat{x}_{N,t}^i)$. Aware of their cognitive limitations agent i reduces complexity by using only a subset of the available information $\tilde{I}_t^i \subset I_t^i$. The vector of functions $\boldsymbol{\psi}_1^i = (\psi_{1,1}^i, \ldots, \psi_{1,N}^i)$ expresses how agent i process those information, so $\hat{X}_t = \boldsymbol{\psi}_1^i(\tilde{I}_t^i)$. Where $\boldsymbol{\psi}_1^i$ might represent agent specific heuristic rules or simple optimization procedures. As previously outlined it is inherent that in a world with decentral coordination agents overview only a very small part of this world. Aware of this limitation they are uncertain about the consequences of their decision to change their choice variables. Since this is a very crucial aspect in this work an additional structure is needed to capture this explicitly. That is why agent i uses in a second step the heuristic rules $\boldsymbol{\psi}_2^i = (\psi_{2,1}^i, \ldots, \psi_{2,N}^i)$ to determine the actual values of her choice variables $X_t^i = \boldsymbol{\psi}_2^i(\hat{X}_t^i, X_{t-1}^i; \Phi^i)$, based on the choice variables of the last period and the reference points.[20] Furthermore there is a set of parameter vectors $\Phi^i = \{\boldsymbol{\phi}_1^i, \ldots, \boldsymbol{\phi}_N^i\}$ that defines the specific form of $\boldsymbol{\psi}_2^i$. What goes along with the assumptions about agent i's preferences and among other things about how she copes with uncertainty or adjustment costs. The next chapter looks at this in more detail. Additionally, one might also ask how the decision behavior changes along with the experiences an agent makes over time, so whether the parameters change depending on the subset of information, $\Phi_t^i = \boldsymbol{\varphi}^i(\tilde{I}_t^i)$ where $\boldsymbol{\varphi}^i = (\varphi_1^i, \ldots, \varphi_N^i)$. Each φ_l^i

[20]Since an agent i is usually constrained in her choices one has to make sure that $\boldsymbol{\psi}_2^i$ maps into the choice set of X^i, meaning if $\hat{X}_t^i \in C_{\hat{X}_t^i} \subset \mathbb{R}^N$ then $\boldsymbol{\psi}_2^i : C_{\hat{X}_t^i} \times C_{X_{t-1}^i} \to C_{X_t^i}$.

represents a function determining the parameters ($\boldsymbol{\phi}^i_{l,t}$) of a particular choice variable x^i_l (for $l \in \{1, \ldots, N\}$). Unfortunately this work can not further discuss that important aspect of heuristic-based decision making.

Nevertheless their is another important point to note. While specifying the decision rule $\boldsymbol{\psi}_1(\tilde{I}_t)$, thus defining the way agents reason, one has to decide to which extent agents are aware of the causality between their choice and input variables and how actively they make use of this knowledge. If agents are assumed to have sufficient knowledge about the feedback loop system ($\dot{A}$, $\boldsymbol{\omega}$) and the other agents internal processes ($\mathcal{T}$) agent i's decision rule $\boldsymbol{\psi}_1$ should connect the feedback in t (Y_t) explicitly with her choice variables set in past periods. Thus it would imply that agent i aims in t for a specific future feedback. Lets look at a single loop consisting of two agents. Because of the sequential structure the decision rule $\boldsymbol{\psi}^i_1$ of agent i connects, the input variable $y^i_{m,t}$ with a choice variable $x^i_{l,t-q}$ (for $q > 2$) determined at least two periods ago, meaning it computes a causal relationship between the both. Making it more realistically, since agent j processes, alone because of her hesitant adjustment behavior, a input variable not only in one but in a couple of consecutive periods, $\boldsymbol{\psi}^i_1$ should consider that $y^i_{m,t}$ is accordingly influenced by more than the choice variable of one past period or the other way around that her choice variables do not influence the value of her input variables in only one period. Since agent i updates in every period her information set she also updates the assumed causal relation between her choice and input variables. Based on this agent i computes ($\boldsymbol{\psi}^i_1$) her reference value $\hat{x}^i_{l,t}$. Where $\boldsymbol{\psi}^i_1$ would have to take into account that the values of x^i_l set in the last periods might have been computed based on another understanding of this causality and whose influence on agent j's decisions (x^j_n) has not completely transmitted back to i. Those effects can be either compensated or strengthened by $\hat{x}^i_{l,t}$. If agent i regularly updates her adjustment parameters (Φ^i) equivalent considerations are necessary while determining the associated rule $\boldsymbol{\varphi}^i$. Generally speaking, the more detailed such causalities are considered the more complex $\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$ and $\boldsymbol{\varphi}^i$ become, hence the stronger the implied cognitive abilities of the agents have to be and the more the interplay of $\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$ and $\boldsymbol{\varphi}^i$ become conceptually close to the first approach, contradicting the reasons why to use simplifying decision rules in the first place.

A more simple alternative, whose general idea the model of part three will follow, is to implement a decision rule ($\boldsymbol{\psi}_1$) that assumes that agents do not have the cognitive abilities or the information to compute the causal relationship needed to determine the values of the choice variables that are supposed to trigger a specific future feedback. Agents following such a rule might know about the general structure of the feedback loops and the reasoning of the other agents, but loosely, so the only intention while setting their choice variables is to push the input variables in the favored direction. For example an agent i might determine $\hat{x}^i_{l,t}$ based only on her input variables Y^i of the last q periods. She is not processing the past choice variables or trying to connect their choice and input variables, thus she acts in a more or less reactive way. In case agent i represents a voter she might evaluate (signaling her satisfaction level) the policy of a government (agent j) only based on the outcome of its actions of the last period ($y^i_{m,t}$). Since, if agent i is a firm, their is not a given optimal value resp. a scale to compare $y^i_{m,t}$ with, as it is the case for voters satisfaction level, i might use the information carried by $y^i_{m,t}$ to decide whether to decrease or increase the reference value of the product price ($\hat{x}^i_{l,t}$) by a fixed value. In this more simple approach, $\phi^i_{l,t}$ would be either fixed or the outcome of a heuristic rule φ^i_l that is also based on the recent values of y^i_m. The circumstances that the causal relationships between X^i and Y^i are hardly and impulses that are already set in motion are not at all considered by agent i while determining X^i_t may have stabilizing or destabilizing effects on the system, thus on the dynamics of her choice variables (X^i). Either way, another important aspect, that also has ambiguous effects, is whether or not $\boldsymbol{\psi}_1$ accumulates environmental changes, hence considers in t also the difference between the reference point ($\hat{x}^i_{l,t-1}$) and the actual choice variable ($x^i_{l,t-1}$) of the last period. On the one hand accumulation smoothens the effects of single environmental changes on $\hat{x}^i_l$ but in combination with a decision rule $\boldsymbol{\psi}_2$, that delays adjustments, it might also amplify the causes that lead to periods with unproportionally strong changes of x^i_l in relation to the recent environmental changes. But of course the dynamics of the system depends also highly on the interplay with agent j's decision rules and on that with the agents along the other feedback loops.

The advantages and disadvantages of the second approach are basically opposite to those of the first one. On the one hand, because of the number of heuristic rules applied, the decision process is far more arbitrary. But on the other hand agents are assumed to need less cognitive efforts to determine the values of their choice variables. What makes this approach more suitable for the kind of model set-up introduced here.[21]

2.2.5 Hesitant adjustment behavior

On the one side, as in the initial examples introduced, agents show strong hesitation regarding adjustments to changes of their environment. Looking on a single agent i, even if her input variables continue to change, she does either only small adjustments of her choice variables or no changes at all. On the other side, at a particular point when the imbalance between the changed input variables and, considering the new circumstances, the not suitable choice variables, so her actions, becomes too large agent i overcomes the reasons causing the hesitation. The following adjustments of the choice variables from X_{t-1}^i to X_t^i are usually more intense than the changes of Y^i in recent periods would suggest. Thus agent i seems to try to make up for the missed adjustments in previous periods. Eventually this will induce the agents with whom agent i interacts to readjust their choice variables, hence agent i's input variables. Even when this happens and Y^i moves towards its initial values agent i does not conduct immediately a corresponding readjustment as well, that means she shows a resistance to change in both directions. Beside the phases of inertia and the larger adjustments following at some point, the fact that this might occur both ways is the third empirical aspect describing the qualitative properties of agents general adjustment behavior in social situation focused in this work.

There are different explanations in the literature that try to figure out why agents to a certain extent resist to adjust immediately to changes in their environment. Some are based on economic reasoning some are founded psychologically. The latter are usually connected with the framing of decision situations meaning preference irrelevant aspects influence agents decision. A kind of framing (status quo framing) that might explain the stickiness of decisions states that, in comparison to the canonical model of decision

[21] The two criteria cognitive demand and traceability constitute a two-dimensional continuum. This can be used to classify and compare the two described, but of course also other procedures agents are assumed to apply to determine their choice variables.

making, one out of agent i's set of alternatives is labeled as status quo alternative. Thus sticking to the previous decision is in most cases an option. In a lot of various set ups such labeling leads agents to choose this alternative with a higher likelihood than the canonical decision model would predict, suggesting that it exists a so called status quo bias. Divers reasons that may cause such decision behavior are quoted e.g. habits, customs, innate conservatism, policies, convenience or fear.[22] In all introduced examples agents always have the opportunity to maintain their previous choice, hence they might experience a status quo bias. Conservatism or habits may influence voting behavior, market interactions may be driven to some extent by customs and firms internal allocation decisions by established policies. Furthermore, agents do not find themselves in a decision situation where they are certain about the outcome of their alternatives. Indeed they interact with each other in a way where an agent i tries to induce resp. to influence with her decisions in each period (X^i) the future feedback (Y^i) she will obtain, meaning the result of the choose alternative, but agent i neither knows the complete network of feedback loops but only a part of it (the graph ($\dot{A}^i$, $\boldsymbol{\omega}^i$)) nor does she have all information about the internal processes of the other agents but only about those ($\mathcal{T}^i$) of a subset of agents ($\ddot{A}^i$). That is why she can not compute the exact causal relationship between X^i and Y^i making the outcome of her actions uncertain. This may also cause or intensify a status quo bias.[23] Depending on the specific set up the status quo bias might prevent positive as well as negative adjustments of agents choice variables, as it is in the examples presented in the introduction. Whether at some point agents overcome the psychological reasons causing the status quo bias and, if so, what does define this point are questions that are not finally answered.

Another group of models, the so called (S,s)-models, explaining why agents show periods of strong inertia in their responses to changes of their environment followed by a relatively large change considers the interplay of different costs that come along with the decision how to respond to such environmental changes. All these models focus decision situations that have three crucial features in common. First and second, there are two

[22]See Samuelson and Zeckhauser (1988) and Kahneman et al. (1991).

[23]Fernandez and Rodrik (1991) show this regarding a government uncertain about voters response to a policy change.

factors (or variables) influencing the objectives of an agent where one is absent control (environmental changes), agents are even uncertain about its future development, while the other can be set by the agent. Third, controlling for the latter raises fixed costs. Thus agents have to handle a trade off between responding to changes of the first factor continuously with adequate changes of the second hence staying well adjusted but bearing high adjustment costs or adjusting less often therefore internalizing losses of non-adjustment but lower adjustment costs. If adjustment costs are high and the losses of nonadjustment relatively low than most likely agents optimal adjustment strategy is characterized by periods of inertia.[24] Such reasoning can be find in a broad variety of decision situations. Initially, questions of optimal inventory holding inspired the development of the (S,s)-models.[25] Later it was conceptually extended to applications in monetary policy[26], to recruitment behavior of firms[27] and to optimal pricing under consideration of fixed menu costs[28], to name a few.[29]

Labor demand and pricing strategies in the light of adjustment costs could be an explanation for the hesitation a firm shows while trying to match the feedbacks they obtain from the factor and product markets, as in the introduction illustrated. The difference between the (S,s)-models and this kind of social interaction, just as with all others described so far, is that in the case of the latter the factors (input variables) influencing agents objectives are not divided into those which can be set by the agent and those considered to be exogenously given. Agent i is able to control the values of Y^i_{t+p} (for $p > 1$) to a certain extent by setting X^i_t accordingly but, as stated above, there are also mechanisms in place she can not control for, often does not even know about, making the actual realization of Y^i_{t+p} uncertain. Since the changing or adjusting of each single choice variable could trigger adjustment costs agent i still needs to handle

[24]See Caplin and Leahy (2010).

[25]See Arrow et al. (1951) and Arrow et al. (1958).

[26]Caplin and Spulber (1987) and Caplin and Leahy (1991).

[27]Hamermesh (1989) and Caballero et al. (1997).

[28]See Barro (1972) and Sheshinski and Weiss (1977).

[29]Real-options is another cost related approach explaining the emergence of hysteresis in agents adjustments. In this case not adjustment costs but irreversible investments so potential sunk costs are causing agents to hesitate, therefore to wait until more information are available before acting (Dixit et al. (1994)).

a trade off between losses caused by mismatch, for instance in case the size of a firms workforce and the demand for their product does not match, and adjustment costs. What might give rise to an erratic adjustment behavior.

If an agents decision behavior is modeled in the sense of the first approach of chapter 2.4 it is conceptually most reasonable to consider eventually occurring causes for a status quo bias while determining $\rho^i_{Y^i_{t+p}|I^i_t,X^i_t}$. More specifically the rule $\boldsymbol{\psi}^i_{1,m}(I^i_t, X^i_t)$ should be of a form that allow to account for mechanisms leading to such bias. Since it is not even completely understood how those psychological mechanisms work it is undoubtedly very demanding to incorporate them in form of a mathematical structure in such a way that the stated characteristics of agents adjustment behavior can be reproduced. On the other hand adjustment costs would be taking account of while determine the values of the choice variables ($\overset{\star}{X}{}^i_t$) that maximize the expected value of the pay of function of agent i ($\mathbb{E}[\pi^i(\bar{Y}^i_{t+p})|I^i_t, X^i_t]$).

Nevertheless, because in the real world agents have to obey to the fact that they have limited cognitive abilities as well as limited information about their environment the second approach of chapter 2.4 will be in the following of greater importance for this work. After agent i determined the reference values ($\hat{X}^i_t$) of her choice variables (e.g. product prices, budget sizes, satisfaction levels, offered amount of labor or policy variables), it is assumed that she uses the heuristic rule $\boldsymbol{\psi}^i_2$ to incorporate the causes of inertia. Therefore, what general form of $\boldsymbol{\psi}^i_2$ would be able to capture the three crucial behavioral characteristics an agent shows, i.e. at first hesitation to adjust, followed by large or lumpy adjustments, just as the circumstance that this takes place for positive as well as for negative changes of agents choice variables. In the following, two general heuristics fulfilling those requirements, that are applied to one particular choice variable, will be discussed. The first (2.9) is, because of its discontinuity, suitable to capture inertia effects that mainly originate from adjustment costs considerations.

$$x^i_{l,t} = \psi^i_{2,l}(\hat{x}^i_{l,t}, x^i_{l,t-1}; \boldsymbol{\phi}^i_l) = x^i_{l,t-1} + \tilde{\psi}^i_{2,l}(\hat{x}^i_{l,t} - x^i_{l,t-1}; \boldsymbol{\phi}^i_l) \tag{2.9}$$

$$\tilde{\psi}^i_{2,l}(\hat{x}^i_{l,t} - x^i_{l,t-1}; \boldsymbol{\phi}^i_l) = \begin{cases} \hat{x}^i_{l,t} - x^i_{l,t-1} & \text{if } (\hat{x}^i_{l,t} - x^i_{l,t-1}) \geq \bar{\phi}^i_l \\ \hat{x}^i_{l,t} - x^i_{l,t-1} & \text{if } (\hat{x}^i_{l,t} - x^i_{l,t-1}) \leq \underline{\phi}^i_l \\ 0 & \text{else} \end{cases} \tag{2.10}$$

$$\text{where } \boldsymbol{\phi}^i_l = (\bar{\phi}^i_l, \underline{\phi}^i_l) \text{ and } \underline{\phi}^i_l \leq 0 \leq \bar{\phi}^i_l$$

According to this heuristic agent i adjusts her choice variable x^i_l only if the suggested change in t, this is, the difference between the reference value ($\hat{x}^i_{l,t}$) and the value of the particular choice variable in the last period ($x^i_{l,t-1}$) exceeds a certain threshold $\bar{\phi}^i_l$ ($underline\phi^i_l$) in case of an increase (decrease) of x^i_l respectively. Otherwise agent i shows the hesitant behavior ($x^i_{l,t} = x^i_{l,t-1}$) that is supposed to be characteristic for her adjustments. The larger the adjustment costs the more takes the heuristic implicitly account for this through larger thresholds. The symmetry of $\psi^i_{2,l}$ is another defining characteristic, hence whether $\bar{\phi}^i_l$ and $|\underline{\phi}^i_l|$ are equal or differ. The latter would be the case if the adjustment costs for positive or negative changes of x^i_l differ from each other. Beside the adjustment costs also agent i's risk attitude and the degree of uncertainty influence the size of the thresholds. This is briefly sketched below commonly for both heuristic rules.

Technically speaking, the second heuristic (2.12) can be understood as a generalization of the first or the first as a special case of the second. Again the value of a choice variable $x^i_{l,t}$ of agent i is determined based on its value of the previous period and its reference value ($\hat{x}^i_{l,t}$).[30] As illustrated below the heuristic is also shaped in a way that it captures agent i hesitation to adjust for small differences between $\hat{x}^i_{l,t}$ and $x^i_{l,t-1}$ while at some point with increasing $\hat{x}^i_{l,t}$ the adjustments would become more or less abruptly

[30]There are economic situations where it is reasonable for an agent to signal a change resp. an intended change of an underlying variable. For example it could be, as in the illustrative model of chapter 3, that signaling a change of a variable might enable an agent to pass on more resp. more precise information. In such case agent i's reference point $\hat{x}^i_l$ and her choice variable x^i_l represent not a certain value, as in (2.12), but a certain change of the underlying variable e.g. the change resp. the signaled change of agent i's satisfaction level. Therefore, agent i's adjustment behavior is entirely

larger in their amounts converging to their maximum the actual suggested change. As one can see the main difference between the two heuristics is the smooth form of the latter. This implies that agents, who are assumed to base decisions on this heuristic, conduct at least small adjustments even in times when they show hesitation towards adjustments. That is why (2.12) is very suitable to capture psychological effects that could cause a status quo bias.

$$x^i_{l,t} = \psi^i_{2,l}(\hat{x}^i_{l,t}, x^i_{l,t-1}; \boldsymbol{\phi}^i_l) = x^i_{l,t-1} + \tilde{\psi}^i_{2,l}(\hat{x}^i_{l,t} - x^i_{l,t-1}; \boldsymbol{\phi}^i_l) \tag{2.12}$$

$$\tilde{\psi}^i_{2,l}(\hat{x}^i_{l,t} - x^i_{l,t-1}; \boldsymbol{\phi}^i_l) =$$

$$\begin{cases} (\hat{x}^i_{l,t} - x^i_{l,t-1}) \cdot \left(1 - e^{-\left|(\bar{\phi}^{i,2}_l \cdot (\hat{x}^i_{l,t} - x^i_{l,t-1}))^{\bar{\phi}^{i,1}_l}\right|}\right) & \text{if } (\hat{x}^i_{l,t} - x^i_{l,t-1}) \geq 0 \\ (\hat{x}^i_{l,t} - x^i_{l,t-1}) \cdot \left(1 - e^{-\left|(\underline{\phi}^{i,2}_l \cdot (\hat{x}^i_{l,t} - x^i_{l,t-1}))^{\underline{\phi}^{i,1}_l}\right|}\right) & \text{if } (\hat{x}^i_{l,t} - x^i_{l,t-1}) < 0 \end{cases} \tag{2.13}$$

$$\text{where } \boldsymbol{\phi}^i_l = (\bar{\phi}^{i,1}_l, \bar{\phi}^{i,2}_l, \underline{\phi}^{i,1}_l, \underline{\phi}^{i,2}_l)$$

Agents individual adjustment behavior, following (2.12) can be understood as a combination of two general modes of reasoning, a conscious, deliberate and an unaware, intuitive one. The first refers to agents ability to reflect about the way they do decisions and about the consequences of those decisions. If agents to a certain extent use explicitly their knowledge about the feedback loop system and about the other agents they are able to develop an image of their environment and of the effects their actions will cause, that is, about the costs that would come along with their choices. Comparing the computed optimal action with the one they actually would like to choose might reveal a difference between the both. Such consciously reasoning enables agents

captured by the second part of (2.12), thus

$$x^i_{l,t} = \tilde{\psi}^i_{2,l}(\hat{x}^i_{l,t}; \boldsymbol{\phi}^i_l) = \begin{cases} \hat{x}^i_{l,t} \cdot \left(1 - e^{-\left|(\bar{\phi}^{i,2}_l \cdot \hat{x}^i_{l,t})^{\bar{\phi}^{i,1}_l}\right|}\right) & \text{if } \hat{x}^i_{l,t} \geq 0 \\ \hat{x}^i_{l,t} \cdot \left(1 - e^{-\left|(\underline{\phi}^{i,2}_l \cdot \hat{x}^i_{l,t})^{\underline{\phi}^{i,1}_l}\right|}\right) & \text{if } \hat{x}^i_{l,t} < 0 \end{cases} \tag{2.11}$$

$$\text{where } \boldsymbol{\phi}^i_l = (\bar{\phi}^{i,1}_l, \bar{\phi}^{i,2}_l, underline\phi^{i,1}_l, underline\phi^{i,2}_l).$$

This case is not to be confused with a situation where agents hesitation refers to the signaled change itself and not to the underlying variable as above. That is, an agent hesitates to adjust a variable that expresses a change of another variable. If this is the case then again (2.12) needs to be applied.

Of course, in an equivalent way it is also possible to apply the discrete adjustment heuristic.

not only to recognize and understand but also partly to overcome the psychological mechanisms causing a status quo bias. Therefore, this mode of reasoning leads for all values of $(\hat{x}^i_{l,t} - x^i_{l,t-1})$ to the same proportional adjustments of $x^i_{l,t}$. If an agents decision is entirely based on this mode or she does not experience those psychological mechanisms and her decision is not affected by uncertainty she would adjust her choice variable as it is indicated by $(\hat{x}^i_{l,t} - x^i_{l,t-1})$. According to the second mode of reasoning it is assumed that an agent does not adjust until $(\hat{x}^i_{l,t} - x^i_{l,t-1})$ reaches a certain threshold triggering complete adjustment. This implies that an agent notice the difference between the indicated change of x^i_l and her actual choice but, despite that, she is not able or willing to understand in a structured way how she does her decision. But of course she will process such information in a intuitive, not traceable way what will lead, at her specific trigger level of mismatch, to the awareness that it is in her interest to adjust. The actual shape of (2.12) depends on the proportion of both modes of reasoning an agent is assumed to apply.[31,32]

To characterize agents individual adjustment behavior, hence the shape of $\tilde{\psi}^i_{2,l}$ the part of the adjustment heuristic that controls for the actual change of x^i_l, it is useful to divide $\tilde{\psi}^i_{2,l}$ roughly into three parts for negative and positive changes respectively. There is a flat one in the beginning, followed by a steeper one while the third is flatter again. The change of the average slope between the single parts, the approximate size of $(\hat{x}^i_{l,t} - x^i_{l,t-1})$ at which the steep one starts and ends and the symmetry between positive and negative adjustments are a conclusive set of criteria to describe and compare

[31]Since agents i decision behavior is supposed to be captured by the interplay of $\boldsymbol{\psi}^i_1$ and $\boldsymbol{\psi}^i_2$ some combinations are more reasonable then others. For example if a decision rule $\boldsymbol{\psi}^i_1$ is based only on the input variables of the last periods meaning it does not determine a causal relation between X^i and Y^i, hence assuming low cognitive abilities, it is convenient to assume the same for $\boldsymbol{\psi}^i_2$. In this case the second mode would dominate agents adjustment decision.

[32]If the adjustment parameters (Φ^i) are constant agents decision making does not change over time. Therefore, agents do not exhibit any kind of learning. In every period agents act according to the same adjustment rule $\boldsymbol{\psi}^i_2$ no matter whether the past decisions proved to be adequate or not. In contrast to that, assuming agent i does learn from her past choices, than for example in case Y^i tend to be a satisfying realization agent i would increasingly trust in her ability to understand the way she does decisions reducing the status quo bias, as well as in her understanding of her environment decreasing uncertainty. What might coincide, depending on the decision situation, with a change of Φ^i.

qualitatively agents adjustments. The figures (2.1) and (2.2) show how the parameters $\boldsymbol{\phi}_l^i$ control for these properties. Referring to the first mode of adjustment, as above stated, it is assumed that the more the agents are able to reflect about their environment and their decision making (small $\bar{\phi}_l^{i,1}, underline\phi_l^{i,1}$) the less the slope changes. While, concerning the second mode, the more sensitive agents are (large $\bar{\phi}_l^{i,2}, underline\phi_l^{i,2}$) the smaller the values of $(\hat{x}_{l,t}^i - x_{l,t-1}^i)$ that trigger larger adjustments. If this works the same way for an increase or decrease of x_l^i the adjustment behavior is symmetric.

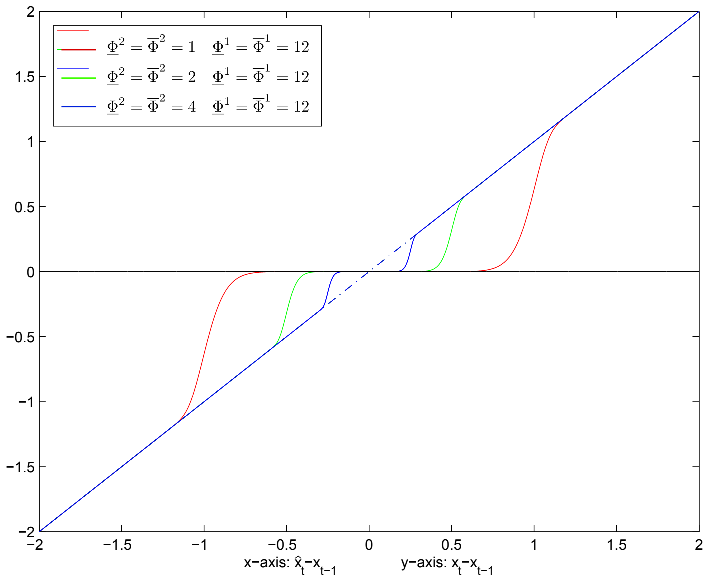

Figure 2.1: Adjustment function 1

Those effects might be intensified or compensated depending on the kind of expected costs and their proportion caused in case of a mismatch between the actual $x_{l,t}^i$ and the expected optimal value in past periods for period t, on agent i's risk attitude and on the level of uncertainty she has to cope with.

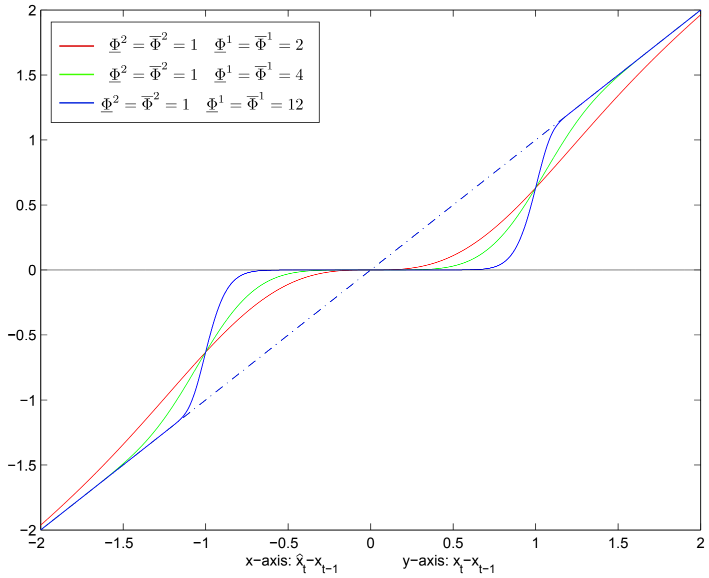

Figure 2.2: Adjustment function 2

2.3 A basic model

2.3.1 Introduction

The goal of this section is to present a first model whose key elements follow the approach outlined above to examine what influence their interplay has on the dynamic of the key variables of a particular social situation. To develop a first basic understanding of the involved mechanisms a very simple setting will be introduced.

As in the introduction already sketched a well known conflict in societies and a very suitable case for the kind of modeling approach at hand is the allocation of a limited resource between competing purposes. Presumably the most obvious is the state budget but also pollution capacities, time or as in the following case the use of a limited space might become such a resource. It is also well known that such policy variables may show very different dynamics over time. Sub-budgets, pollution thresholds and proportions of time and space usage might for example do not change at all or move gradually for a certain period and suddenly change rapidly in opposite direction and continue to fluctuate before settling around a particular value or develop again a smooth trend. There are also times when such variables seem to fluctuate constantly and slowly within a corridor. We will see how this will turn out in the following example.

To be more precise the model consist of two kind of agents a state resp. a government and a group of voters. The state has to decide how to use a certain strip of land while voters have different preferences concerning the usage of the land. Thus there will be two feedback loops that are interdependent because of the limited space. Agents decide how to act based on their information about other agents past behavior as about their own conducted actions. As a consequence the government as well as the voters act sequentially without direct coordination. This makes them uncertain about the reaction of the others to their own actions resulting in an erratic adjustment behavior. What will be the driver of the dynamic of the policy variable the proportion between the different land usages, that will be analyzed.[33]

2.3.2 The object of interest

To begin with, before explaining the way agents resp. voters and the government operate, it is necessary to introduce the object that is in the eyes of the agents of interest namely the limited strip of land and how it can be used and altered. There are only two possible modes of use, on the one side as a nature reserve for recreational purpose or on the other side as grassland for example to feed cattle, where $x^e_{l,t}$ for $l \in \{1, 2\}$ captures the size of land that is allocated to each mode in period t, respectively. Assuming that the size of the overall land strip is captured by $L > 0$, then $x^e_{l,t} \in (0, L)$ s.t. $x^e_{1,t} + x^e_{2,t} = L$. Consequently a change of one variable necessarily implies a change of the other, so the modes of use form a partition of the strip of land. This is actually equal to other scenarios e.g. time limits to operate an industrial plant or an airport or a rule that allows for driving with a certain speed also a change of pollution thresholds like such in inner city areas causes contrary effects on rival interests.[34]

[33] The lag of direct coordination has this type of models in common with most agent-based approaches. For a survey seeTesfatsion and Judd (2006).

[34] Even so that the examples made so far which consider sub-budgets for competing purposes seem of the same nature there is a decisive difference. In these cases the dependency emerges only from the limited size of the budget but not from some physical interdependencies between the objects of interest for which it is spend. If additionally a change of the objects can be caused externally without having a change of the sub-budgets a change in one object of interest does not necessarily go along with that of the other. For instance the safety situation in a certain administrative unit can change

Since the land strip is completely allocated between both modes, it is sufficient to model how one evolves over time to describe the overall state of the land strip in each period. Before modeling the way the reference mode of use x_1^e the nature reserve is altered by the state some assumptions about constraints imposed by nature are needed to be made. First of all the state decides in each period t how much effort it want to invest (x_t^s) to change x_1^e for example a monetary budget. Technically it can be used to adjust x_1^e in both directions, thus either for re-naturalization increasing the size of the nature reserve or for transforming it into grassland. In addition it is assumed that there is a particular size of the nature reserve $\bar{x}_1^e$ that defines a land allocation from which the state experiences a decreasing effect of its effort to change x_1^e in either direction. Thus every additional unit of effort used to change the nature reserve in the same direction will have a smaller effect on x_1^e than the previous one. Depending on the states decision whether to increase or decrease the size of the nature reserve, x_1^e would continuously converge either to L or to 0 with every additional unit of effort. Furthermore, the land allocation is also affected in each period t by an exogenous factor ($\epsilon_t \in (0,1)$) e.g. certain climate conditions, a natural growth rate of one of the biotopes or deterioration caused by the usage itself. The following type of a logistic function

because of external reasons, thus despite constant spendings for the security infrastructure, without effecting for example the quality of the education system or that of the public administration. This is also true the other way around, an education system confronted with new demands caused by societal or technological changes or a public administration that has to enforce new legal and more work intensive regulations, would not lead necessarily to a change of the safety situation again assuming constant spendings.

To have the same effect in the first group of examples one needs to include an additional dimension allowing for a change of benefits agents can gain from the usage of the scared resource without a change of the allocation. In case of the leading example that could be an increasing agricultural productivity what would subsequently induce a reallocation of the land or a change of voters preferences. Nevertheless, real and usually complex situations consist of both kind of interdependencies, using a budget to change a land strip means that the budget can not be spend for other purposes like the education system, or more police force would consequently lead to the effect that some s feel more safe while others feel at the same time more suppressed.

captures those effects.

$$x^e_{1,t+1} = \frac{L}{1 + e^{-\gamma(x^s_t - \hat{x}_{t+1})}} \text{ where} \tag{2.14}$$

$$\hat{x}_{t+1} = -\frac{ln\left(\frac{L}{x^e_{1,t} - (L - x^e_{1,t})\epsilon_{t+1}} - 1\right)}{\gamma} \tag{2.15}$$

A particular chronological order concerning the influence of the state and that of the exogenous factor on x^e_1 underlies (2.14) and (2.15). During each period t the state processes its information and finally decides at the end of t how much effort (x^s_t) it want to invest to change x^e_1. At the same time the nature reserve evolves reaching $x^e_{1,t}$ at the end of period t. Thus the state knows $x^e_{1,t-1}$ but not the current size of the nature reserve since those processes taking place simultaneously. In the next period ($t+1$) the state effort (x^s_t) unfolds its effect on x^e_1, as well as the exogenous factor (ϵ_{t+1}). To keep things simple it is assumed that this happens consecutively. At first ϵ_{t+1} affects x^e_1 starting from the value of the nature reserve at the end of period t hence $x^e_{1,t}$ and leading to x. As stated in (2.15) the absolute size of the exogenous factor depends on the effect itself and on the size of the modes of usages. In the present case ϵ_t can be interpreted as the natural growth rate in t of the grassland (x^e_2). It could be a random variable or constant over time. Afterwards x^s_t continues to change x^e_1 through to its final value $x^e_{1,t+1}$ at the end of period $t+1$. This is captured by (2.14) where $\gamma \in \mathbb{R}_+$ is a slope parameter and $\hat{x}_{t+1}$ is the hypothetical effort equivalent to achieve the value that x^s_1 reaches after the influence of ϵ_{t+1} but before the state effort (x^s_t) unfolds its effect.[35] The amount of land allocated in $t+1$ to each mode ($X^e_{t+1} = (x^e_{1,t+1}, x^e_{2,t+1})$) are the input variables that voters will process in period $t+2$.

2.3.3 The voters

As mentioned in the introduction there are rival interests among the voters concerning the usage of the limited land (L). More precisely, voters are divided into two groups

[35] Technical speaking, (2.14) is constructed so that $dx^e_{1,t+1}/dx^s_t$ is symmetric at $x^s_t = 0$ for $\hat{x}_{t+1} = 0$, from what follows that $\bar{x}^e_1 = L/2$. It also means that $\hat{x}_{t+1}$ shifts (2.14) in such a way that the value of x^e_1 after the influence of (ϵ_{t+1}) is at the origin referring to x^s_t. If x^s_t is positive the nature reserve expands while for a negative value it shrinks. The size of the grassland (x^e_2) changes respectively each period.

of different type ($i = \{1, 2\}$) with regard to their preferences. For simplification it is assumed that voters of type one only concern is about the nature reserve, where those of type two obtain benefits exclusively from the grassland. The first might be residents of cities looking for a place to relax and recover while the latter are most likely farmers or people working in the agricultural sector. Again for simplification it is further assumed that at this point voters of the same type are in all aspects identical and that both groups equal in terms of their size. Thus in the following each group is represented by a single voter. Since both voters are only interested in one of the modes of usage, their input vector is composed of a single element, $Y_t^i = (y_t^i) = (x_{l,t-1}^s)$ for $i = l$. Additionally both voters experience a positive but decreasing marginal return in their preferred usage of the land, respectively. It is also assumed that $\tilde{y}_t^i = y_t^i = x_{l,t-1}^s$, therefore an objective function (π^i) of the kind below follows for both type of voters.

$$\Pi_t^i = \max_{\tilde{y}_t^i} \pi^i(\tilde{y}_t^i) = \alpha^i \cdot (y_t^i)^{\beta^i} \tag{2.16}$$

$$\text{for } \alpha^i > 0 \text{ and } \beta^i \in (0, 1)$$

In case of the voters Π_t^i represents the utility that voter of type i has realized in period t. As one can see this utilization process follows the special case introduced in chapter 2.2 and 2.4, where the agent has no possibility to conduct an optimization. This is because here the allocation of the land is not constraining a choice set from which voters can choose the one that fits best. In fact the allocation is for technical reasons fixed in each period hence voters can not change the allocation in the actual period and have to utilize the current state of the land as a whole. Consequently there is also no ground for the intertemporal problems mentioned in chapter 2.4. Neither is their the choice between utilizing y_t^i in form of $\tilde{y}_t^i$ or storing it to some extend to utilized it in a future period nor is thus there a interdependency between the utilization of y_t^i and the utilization of y_{t+p}^i for $p > 0$.

The allocation of the land in period $t-1$ hence voters input variables and also the size of the actual utilized land in t which are in the example at hand the same are, beside others which will be introduced below, part of voter i's information set. Therefore the updated information set of voter i in period t is

$$I_t^i = \{(y_t^i, y_{t-1}^i), (\tilde{y}_t^i), (\Delta\mathcal{S}_{t-1}^i), (x_{t-1}^i)\}. \tag{2.17}$$

In the next step voters use their information sets to derive suitable actions, hence determining their choice variables, what means to send a signal to the government evaluating its work. Voters do not signal their utility realized each period to evaluate the work of the government but the signal is based on the change of their satisfaction level what is basically the change of their normalized utility. The reasoning behind this assumption is that voters belief the state would not remember the utility resp. the satisfaction level from the past periods of each single voter and they are also not aware how much the state knows about their preferences. Does the government know that voters are only interested in one of the two modes of usage? If the state does not it can neither conclude from a single satisfaction level hence not at all from an utility level to which type a specific voter belongs nor how sensitive this voter will respond to a change in the allocation of the land. Consequently sending a signal based on the change of their satisfaction level provides more information enabling the state to conclude on voters preferences.
To do so, first of all voters derive their actual satisfaction level. For this a reference point is needed, which is at $\tilde{y}^i = L$, since $d\pi^i(\tilde{y}^i_t)/d\tilde{y}^i_t > 0$. Therefore voter i's satisfaction level in period t is

$$\mathcal{S}^i_t = \frac{\Pi^i_t}{\pi^i(L)}. \tag{2.18}$$

This leads to

$$\lhd\mathcal{S}^i_t = (\mathcal{S}^i_t - \mathcal{S}^i_{t-1}) + \lambda^i(\lhd\mathcal{S}^i_{t-1} - x^i_{t-1}), \tag{2.19}$$

where $\lhd\mathcal{S}^i_t$ is voter i's unannounced change of satisfaction in period t after utilization but before determination of her choice variable (x^i_t).[36] The first term refers to the change of the actual satisfaction of voter i based on the change of the realized utility between period t and $t-1$. While the second captures the impact of the hesitation the voter showed in the last period. Where x^i_{t-1} is voter i actual announced change of satisfaction at the end of period $t-1$. Thus the difference is the amount of accumulated change of satisfaction that voter i was for various reasons not willing to communicate.

[36]In the spirit of the general notation outlined in chapter 2.4 the unannouced satisfaction can be also expressed as $\lhd\mathcal{S}^i_t = \psi^i_1(\tilde{I}^i_t)$, where in the present case $\tilde{I}^i_t = I^i_t$.

The parameter $\lambda^i \in [0,1]$ on the other hand varies with the weight a voter put on the past development of her satisfaction. The more λ^i tends to 0 the less voter i takes changes of satisfaction caused in the past into consideration. This is, because either she shows a certain degree of acceptance that her environment changes or she does not put less weight on the past not to risk to send a wrong signal. Even so that it is based mainly on earlier changes the state might connect the signal with recent changes of the land allocation hence drawing false conclusions about voters interests. As stated before in this heuristic-based approach voters incorporate uncertainty about the consequences of their actions by determine their actual choice variable applying a second rule ψ_2^i to their reference point ($\lhd\mathcal{S}_t^i$).

$$x_t^i = \psi_2^i(\lhd\mathcal{S}_t^i) \tag{2.20}$$

The actual form of ψ_2^i depends on the reason that is supposed to cause agents i hesitation to adjust. If agent i has to consider habits or conservatism inducing a status quo bias her voting behavior (x_t^i) follows the heuristic rule (2.11). While in case her aversion to change is mainly originating from adjustment costs e.g. time or efforts in general to get informed about the recent political decisions that presumably lead to y_t^i she adjusts according to the discontinuous version of (2.11). As described above those main drivers for agent i's adjustment patterns can be enhanced or weakened by agents risk attitude and the degree of uncertainty she is confronted with. The next chapter expresses how those effects are represented by different combinations of the parameters of the heuristic rules. Furthermore, it is illustrated how those combinations influence the dynamic of the model.

2.3.4 The state

Beside the voters and if you want the land as imaginary agent the state is the third type of agent completing this model. The state forms with each of the representative voters a separated feedback loop of the following kind $(x_t^s, x_{t+1}^e, x_{t+p}^i)$, for $i = 1, 2$ and $p > 1$. The states goal is a beneficial response from the voters. Since it is ruled out that agents negotiate an ideal land allocation the state can only decide based on the past feedback how much effort to invest (x^s_t) in period t to change the allocation of the land (x^e_{t+1}) in the next period what leads to voters satisfaction signal (x^i_{t+p}), thus the agents interact sequentially and coordinate indirectly. From the feedback loop structure follows for one thing that both feedback loops constitute a family since the variables are of the same type and for another thing that the state is the connecting agent. Additionally because of the scarcity of the land the feedback loops are interdependent.

The activities resp. the processes of the state more precisely that of a government will be introduced according to the same three step shemata as in case of the voters. It is likewise assumed that the state is endowed with very limited cognitive abilities what is why it processes, again as well as the voters, only a few basic information and bases its decision first of all on simple heuristics. First of all the government utilizes in each period t its input variables $Y^s_t = (y^s_{1,t}, y^s_{2,t}) = (x^1_{t-1}, x^2_{t-1})$, here voters change of satisfaction signals received in the previous period. Thus the government is more satisfied the more it can improve the well being of the voters, what also means that it can experience negative utilities.[37] Again as it is for the voters the input variables (Y^s) do not condition a potential so that it is up to the government to decide how to use it ($\tilde{Y}^s_t$), thus an admissible set of options from which it can choose the one that maximizes its objectives. For obvious reasons governments utilization reduces to processing voters signals as it received them $(x^1_{t-1}, x^2_{t-1}) = (y^s_{1,t}, y^s_{2,t}) = (\tilde{y}^s_{1,t}, \tilde{y}^s_{2,t})$, or one might for technical reasons to think of an maximization over a choice set consisting of a single element.[38] Furthermore it is assumed that the state has an additive utility function with positive but decreasing marginal returns in both arguments and it might have a higher interest in the improvement of one of the voter groups (depending on α). This leads to the following state objective function

$$\Pi^s_t = \max_{\tilde{y}^s_{1,t}, \tilde{y}^s_{2,t}} \pi^s(\tilde{y}^s_{1,t}, \tilde{y}^s_{2,t}) = \alpha \cdot (y^s_{1,t})^{\beta^s_1} + (1-\alpha) \cdot (y^s_{2,t})^{\beta^s_2} \tag{2.21}$$

$$\text{for } \alpha \in [0,1] \text{ and } \beta^s_1, \beta^s_2 \in (0,1).$$

[37]Below we will see that even if the government would receive the actual satisfaction level from the voters and utilize them a heuristic decision rule based on satisfaction levels may demand higher cognitive skills than one that is grounded on the change of voters satisfaction.

[38]Therefore the utilization of the state is another example of the special case introduced in chapter 2.2. Also there are no further constraints based on state variables that the government has to consider.

Because of the additivity there is no interdependency between the feedback loops originating from the utility function of the state. Also the state is not confronted with any intertemporal optimization problem, that is, how to allocate its input variables to different kinds of usage e.g. immediate utilization or storage hence future utilization or transformation to some other type of variable and subsequent storage.

Considering the input variables received in the last period, the utilization process as well as the model structure introduced so far and assuming that the state memorizes the last $T+1$ values of each type of variable its information set exhibits the following structure

$$I_t^s = \{(Y_t^s, \ldots, Y_{t-T}^s), (\tilde{Y}_t^s, \ldots, \tilde{Y}_{t-1-T}^s), (X_{t-1}^s, \ldots, X_{t-1-T}^s), (X_{t-1}^e, \ldots, X_{t-1-T}^e),$$
$$(S_{t-1}^s, \ldots, S_{t-1-T}^s), (\Pi_t^s, \ldots, \Pi_{t-T}^s), \mathcal{T}^s, (\dot{A}^s, \boldsymbol{\omega}^s), \Theta^s\}. \quad (2.22)$$

Beside the input variables and the actual utilized input variables, the size of the budget the government invested to change the land allocation and the allocation itself which the state can observe is a element of the set.[39] Furthermore its utility (Π^s) is as well an information the government obviously has as is S^s. The later is the approved but so far not used budget or rather budget line to change the land allocation. As the index indicates it is a variable that will be updated after the determination of the states choice variable X^s at the end of each period. In accordance with the general assumption about agents information the government knows very little about the internal processes of the voters $\mathcal{T}^s$. It is only aware of the fact that voter of type one (two) is always interested in a bigger nature reserve (grassland) respectively, thus it knows that $d\pi^i(\tilde{y}_t^i)/d\tilde{y}_t^i = d\pi^i(x_{i,t}^e)/dx_{i,t}^e > 0$ (for $i = 1, 2$). In terms of the feedback loop the state knows the complete system hence $\dot{A}^s = A$ and $\boldsymbol{\omega}^s$ is a matrix with an entry at any coordinate with the state involved. The size of the land (L) and the set

[39] Being more precise, if one understands the government and the environmental state as one agent whose actions need two periods to be executed X^s is to be seen, depending on the determining rule, as an internal choice variable set by the state. While the one that is observable by the other agents but also not entirely controlled by the state is X^e. Alternatively and probably more consistent one could interpreted the environmental state as an separated agent. In this case X^e would be also an input variable of the state and an additional feedback loop is formed.

of agents (A) are the only parameters known by the state (Θ^s).

Since in this simple set-up are no interdependencies between the utilization process and the determination of the choice variables the state conducts the latter chronologically as the last process in each period based on its information set. More precisely because of the states low cognitive abilities on the following subset of information

$$\tilde{I}_t^s = \{Y_t^s, S_{t-1}^s, \mathcal{T}^s, (\dot{A}^s, \boldsymbol{\omega}^s), \Theta^s\}. \tag{2.23}$$

In accordance to that it is assumed that the state is not able to optimize over a computed distribution of its future input variables conditional on the information subset and its potential choice variable of the actual period but follows a two step heuristic as outlined in chapter 2.4. Furthermore it is assumed that the state is not even able to derive heuristically a causality between its choice and input variables based on their past values that it might used for an optimization. Thus first of all the government uses a much more simple heuristic rule (2.24) to derive the reference value of its choice variable ($\tilde{x}_t^s$), which captures the budget line the government is authorized to use to enlarge the nature reserve in period t.

$$\tilde{x}_t^s = \lambda^s \cdot S_{t-1}^s + \bar{s} \quad \text{for } \bar{s} = \begin{cases} s & \text{if } |y_{1,t}^s| > |y_{2,t}^s| \\ -s & \text{if } |y_{1,t}^s| < |y_{2,t}^s| \\ 0 & \text{else} \end{cases} \tag{2.24}$$

It is composed of the unused budget line from the previous period (S_{t-1}^s) and of a fixed amount $\bar{s}$ by which it might be adjusted each period. While again the parameter λ^s controls to which extent the state takes the past budget decision into account. Depending on the internal budgeting process it can be the case that unused budget lines of one period can not or just partly be transfered to the next period. Also the government might presume that the voters show a certain level of acceptance in terms of a changing land allocation towards one direction. Another effect suggesting to reduce the weight on the past is that in each period only the stronger signal is considered no matter whether the other voter satisfaction changed almost in the same way or not at all. This can lead in both cases over time to the same accumulated budget line even so that the overall signaled satisfaction of the population is very different. The second component ($\bar{s}$) in (2.24) is to be understood as a rule of thumb in terms of how to adjust to recent changes in the states environment, that is, voters reactions. If the signal of the voter of type one is stronger than that of the other voter the state adjust the budget line in favor of this first type of voter while in the opposite case it reduces it.[40] Hence the approved budget line can become negative what is equal to a positive budget line committed to the extension of the grassland. On the one hand the rule incorporates that the state has presumed information about the feedback loop structure and knows the general form of voters utility function. On the other hand the rule refers only to the last value of the states input variables implying that there was a change of the land allocation in the recent past that triggered a proportional stronger signal of one voter and that it is best, based on the processed information ($\tilde{I}^s_t$) to assume that this will continue for further similar changes. Consequently, because of its simplicity, the rule does not consider in period t the changes of the land allocation initialized by states recent actions which are not yet embodied in voters feedback (Y^s_t). Furthermore one might ask why the states decision rule does not refer directly to the size of the nature reserve ($x^e_{1,t}$). For one thing, if the implementation of a decision to adjust the environmental state takes time and further if several external factors are in place influencing the effectiveness of those efforts it could be reasonable to focus in t on a variable that is entirely in the governments range of influence and that correlates with the actual target variable. For another thing, this is even more the case considering that usually a government has to allocate a limited budget to several competing purposes concentrating on budget variables makes it easier to compare between those usages

[40] As mentioned above one might assume that voters signal not the change but the actual levels of their satisfaction. Depending on the decision rule this would lead in a lot of cases to higher demands concerning the employed information set ($\tilde{I}^s$) or the cognitive abilities of the state. For example, to apply the introduced heuristic rules the government would have to memorize the last two values of voters satisfaction signals. A decision rule based on the actual levels of voters satisfaction would be only reasonable if the state has more precise information about the form of voters utility function or is able to generate this knowledge. For example the state might compute based on the signaled satisfaction levels and past land allocations the parameter of voters utility function and determine the updated presumed optimal land allocation in each period.

and to justify a certain budget plan. But of course this implies that a certain causality between budgets and the corresponding purposes are kept in mind while distributing the available financial resources of a period.[41]

Once the state has computed the reference value, the second step follows determining the actual choice variable (x_t^s) hence the size of the budget that is finally used in the next period to alter the land allocation.

$$x_t^s = \psi_t^s(x_{t-1}^s, \tilde{x}_t^s) \tag{2.25}$$

The functional form of (2.25) depends on the mechanisms that the government is presumed to be subject to, preventing it from adjusting to changes immediately. In case the government would have to take fixed costs into account as soon as it attempts to alter the land allocation ψ_t^s would represent a heuristic rule characterized by (2.9). While in case the decision behavior of the government is driven by conservatism, habits or other psychological factors causing a status quo bias ψ_t^s will take the form of (2.12). Nevertheless for both main driver behind governments decision making is true that the higher respectively the stronger they are the bigger the difference ($\tilde{x}_t^s - x_{t-1}^s$) between the amount of the budget spend in the last period for alterations and the reference value of the recent period has to be before the current budget (x_t^s) changes or in the second case becomes substantially adjusted in comparison to the previous one. Moreover, as described above, there are other factors that might reinforce or mitigate agents hesitation. On the one side there is governments level of uncertainty about the future responses of the voters to changes of the land allocation and on the other side, and strongly connected, the risk attitude of the government. Since a change of one mode of usage leads consequently to a change of the other one governments single decision heuristic has to combine uncertainty and risk attitude effects from both feedback loops.[42] For example assuming a risk averse government a stronger hesitation regarding an increase of x^s could be caused either by a higher uncertainty in terms of

[41] Given different circumstances it might be also more realistic to model the government in a way that it considers the size of the nature reserve as its decision variable. In such a case the budget line constitutes in each period the choice set of the state (C_t^s).

[42] So far, the states risk attitude and the degree of uncertainty it experiences are only implicitly considered. There is no structure that connects for example the shape of the utility function (β_1^s, β_2^s) as a measure for its risk attitude with the shape parameter of its adjustment heuristic. This is as

the feedback of the type one voter or by a lower one related to the response of type two. The next chapter will shed more light on the influence various constellations of agents parameter, defining the shape of their adjustment heuristics, have on the dynamic of their interactions and on the reasons that might lead to those different parameter values.

Nevertheless, the part of the available budget line that the government decided in t to apply (x_t^s) will alter along with the exogenous effect the land allocation in $t+1$. This decision also leads to

$$S_t^s = \tilde{x}_t^s - x_t^s,$$

the size of the unused budget line at the end of each period.

2.3.5 Simulation

Even so that the actual model captures already only some set-ups out of a range of variations introduced in the general part further restrictions are necessary to be made to keep the analysis focused on the most important mechanisms and parameters. More precisely on the way the intensity of each single agents hesitant adjustment behavior influences their choices once they interact. Therefore the dynamic that the choice variables of the state and of the voters follow is of interest in particular that of the environmental state. Therefore, the emphasis of the simulations lies on the parameters characterizing the adjustment heuristics and whether particular combinations come along with certain regularities in the dynamic of agents choice variables. In contrast, it will be only briefly dealt with the implications other parameters have for the dynamics of the choice variables. Furthermore, the simulations are based on the version of the model where agents decision are characterized by a status quo bias caused by certain psychological mechanisms, hence agents decision rules (ψ_2^i, ψ_2^s) follow the smooth variant (2.9). In fact this specification is justifiable because the model is supposed to serve as an illustrative example for other social interactions that have the same inherent structural elements concerning the interdependent feedback loops. Since adjustment

well the case for the degree of uncertainty about voters behavior which is obviously connected to the amount and quality of information ($\tilde{I}^s$) the government makes use of.

costs do not play a role in all such situations status quo bias based on psychological effects, that could always occur when doing nothing is an option as it usually is the case in decision situations that regularly take place, is the more comprehensive explanation for agents hesitation. Additionally, giving more structure to the parameter analysis, a distinction is made between two basic scenarios one where voters care about their past satisfaction levels ($\lambda^i \neq 0$) and another one, that is of primary concern in the analysis, with not past-oriented voters ($\lambda^i = 0$). Regarding the latter scenario there are three interesting cases that are schematically presented below. But before some additional constraining assumptions are needed. At first voters vary only in $\bar{\phi}^{i,2}$ and $\underline{\phi}^{i,2}$ while $\bar{\phi}^{i,1}$ and $\underline{\phi}^{i,1}$ are kept equal and fixed. That means a constant and equal proportion between the two modes of reasoning is assumed comparing positive and negative adjustments conducted by a single voter as well as comparing those of voters of different types. Therefore voter might only differ in the sensitivity in terms of the size of potential mis-adjustments. The less sensitive voters are the more the psychological mechanisms causing hesitation unfold their effect consequently leading to a status quo bias. At the moment the state as well does not consider the past so voters former signals ($\lambda^s = 0$) in its decision in the current period and shows a symmetric adjustment behavior, meaning the status quo bias is of the same extent for intended increasing or deceasing changes of the environmental state. With regard to the other parameters, it is assumed that the size of the land strip is normalized to one, that voters have the same preferences ($\beta^1 = \beta^2$) concerning the preferred mode of usage respectively and that the exogenous factor (ϵ_t) is constant and small but sufficiently big to stimulate the model as is ($\bar{s}$). Now lets take a look on the three potential cases which are illustrated in the following figures.

The magnitude of the signal of each voter depends on the recent changes of their satisfaction levels and the shape of their adjustment heuristic (ψ_2^i). As the figures show a shift of x^e changes voters satisfaction in opposite directions, furthermore the changes themselves alter contrarily. Thus an increasing nature reserve leads in absolute terms to a declining satisfaction change in case of voter one and a growing one in case of voter two. If x^e shrinks the effects are reverse. That is why there are two values of the nature reserve $\bar{x}^e$ and $\underline{x}^e$ at which the positive signal of one voter equals the negative one of the

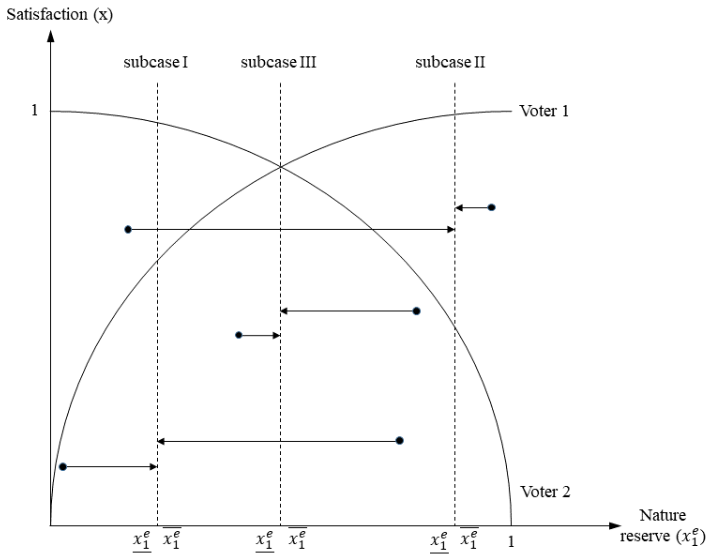

Figure 2.3: Case 1

other type making them threshold values of the system. At $\bar{x}^e$ it is the positive signal of voter one and the negative one of voter two that equal in case of a small increase of x^e, hence x^s was positive two periods earlier. So far, if $x^e > \bar{x}^e$ voter one sends the stronger signal consequently the nature reserve expands till it equals $\bar{x}^e$. While $\underline{x}^e$ determines the value of x^e where its reduction causes in absolute terms signals of the same magnitude a negative one from voter of type one and positive one from type two. In the range between $\underline{x}^e$ and 1 the signal of the second voter dominates inducing x^e to converge to $\bar{x}^e$. The actual value of both thresholds depends on the respective, involved adjustment parameters, while their constellation predefines the dynamic of x^e, and accordingly the case that is present.

The decisive feature of the first case (2.3) is the identity of $\bar{x}^e$ and $\underline{x}^e$. One can differentiate between three sub-cases. In the first (I) both types of voters show the same adjustment behavior for opposite changes of their satisfaction levels ($\bar{\phi}^{1,2}=\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}=\bar{\phi}^{2,2}$). While the second sub-case (II) is characterized by a bigger status quo bias that voter of type two show in terms of signaling positive as well as negative changes of their satisfaction levels ($\bar{\phi}^{1,2}>\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}>\bar{\phi}^{2,2}$). The third sub-case (III) is the reverse version of the second. It is true for both parameter sets, $\bar{\phi}^{1,2}$ and $\underline{\phi}^{2,2}$ for an increasing x^e and $\underline{\phi}^{1,2}$,$\bar{\phi}^{2,2}$ for a decreasing one, that not all parameter combinations fulfilling those

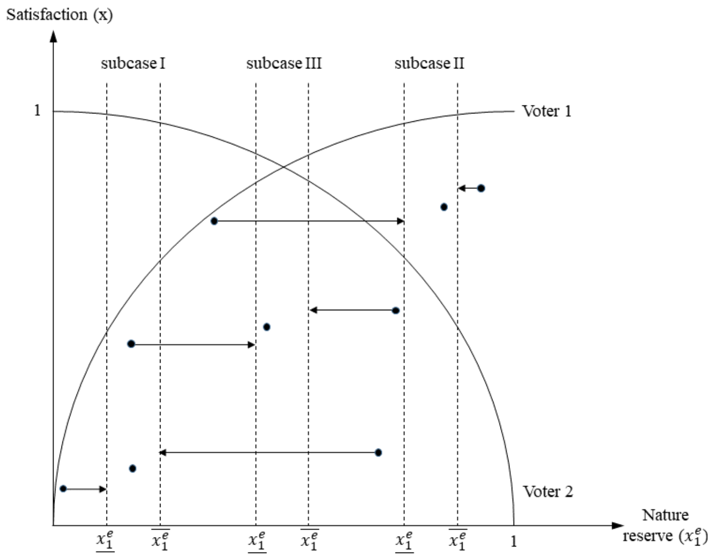

Figure 2.4: Case 2

necessary but not sufficient conditions producing, if at all, satisfaction signals of the same size for an increase and decrease of x^e at a certain value of x^e. It is also true that any common value of the thresholds could be the outcome of infinitely many, adequate parameter combinations. Therefore, the question poses what those possible parameter combinations say about the presumed decision making of both type of voters. In order to do so one more assumption is made to separate the different effects behind voters individual adjustment behavior. Namely, that the main drivers behind the status quo bias, that voters of both type show in their adjustment behavior, are the above mentioned psychological mechanisms like habits or conservatism. What implies that those mechanisms might affect a single voter to different extents in case of increasing or decreasing adjustments of their choice variables, that is, of their signaled change of satisfaction. This assumption is, of course, questionable and raises the demand for further empirical confirmation.

In the first sub-case a voter of type one exhibits the same sensitivity in terms of signaling a positive (negative) change of satisfaction as type two does for negative (positive) satisfaction changes respectively. But a single voter might be unequally sensitive when it comes to respond to contrary changes of x^e, thus $\bar{\phi}^{i,2}$ could be equal, smaller or bigger than $\underline{\phi}^{i,2}$) depending on the presumed intensity of the psychological mechanisms. However, one qualification that ought to be made, is that voters of different type need

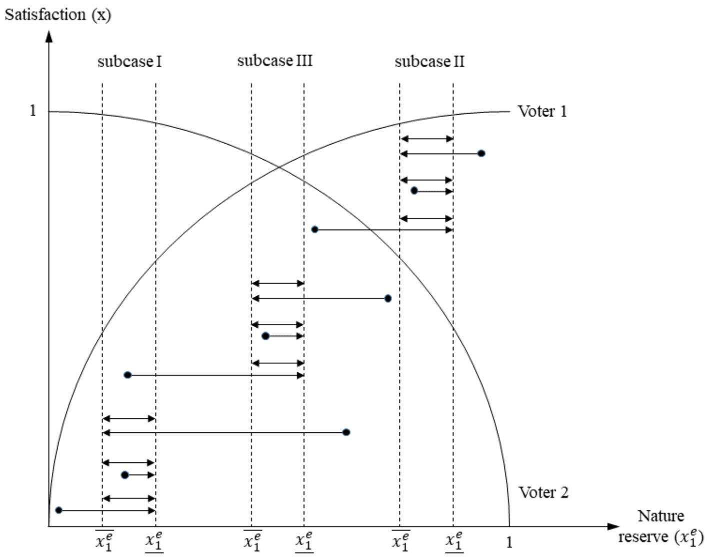

Figure 2.5: Case 3

to have a reverse disparity in terms of their sensitivities while signaling an increase or decrease of their satisfaction change to make sure that the above identity conditions of voters adjustment parameter sets hold. Now lets also consider voters risk attitude. Starting from a certain risk attitude being more (less) risk averse resp. less (more) risk loving leads to a bigger (smaller) $\bar{\phi}^{i,2}$ and a smaller (bigger) $\underline{\phi}^{i,2}$ respectively. If there are no additional constrains concerning voters sensitivities then for all types of voters risk attitude and intensities one can think of a combination of sensitivities so that the adjustment parameters hence voters status quo biases resulting from both effects are equal ($\bar{\phi}^{1,2}$=$\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}$=$\bar{\phi}^{2,2}$). But if additional constraints on voters sensitivities in terms of signaling changes exist this would limit the set of possible combinations of both effects drastically. For example, it seems reasonable to assume that a single voter shows the same sensitivity while responding to increases or decreases of x^e. This might also be true comparing the respective reactions of voters of both types to positive or negative changes of their satisfaction levels. If the first situation occurs for both types there is a small range of sensitivity where if type one is risk avers and type two is risk loving as well as a second range for the opposite case so that the overall effect fulfills the conditions of this sub case. In addition to those cases there are further sensitivity combinations such that the second situation occurs. Nevertheless, in all of them one voter has to be risk avers and the other risk loving.

In general, the difference between the first and the second sub-case is that the latter assumes voter of type one to respond more sensitive to changes of x^e regardless whether the nature reserve shrinks or expands, that is, $\bar{\phi}^{1,2}<\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}<\bar{\phi}^{2,2}$. Therefore, to make sure that both types send the same signal the satisfaction change of type one has to be bigger than that of the voters of type two. Consequently, the value of x^e where the threshold values $\bar{x}^e$ and $\underline{x}^e$ equal has to be bigger than the one of the first sub-caste. As before looking at a single voter the sensitivity that comes along with positive signals and thus the corresponding status quo bias might be bigger, equal or smaller compared with that for negative signals. Adding voters risk attitude as a potential explanation of the status quo bias and looking at the same two situations regarding the symmetry of voters sensitivities as before shows that in the situation where single voters are assumed to have symmetric sensitivities suitable parameter constellations imply that either voter one is risk avers and two has a risk loving attitude or that it has to be the other way around. In both incidents the sensitivity of type two voters is bigger than that of type one. In case of symmetric sensitivities concerning both type of voters responses when it comes either to increases or to decreases of x^e the same general pattern in the possible parameter combinations, that is, of voters risk attitudes occurs.
The third sub-case is basically the reverse version of the second one. Now voters of type one respond less sensitive to changes of x^e, that is, $\bar{\phi}^{1,2}<\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}<\bar{\phi}^{2,2}$ and the threshold values $\bar{x}^e$ and $\underline{x}^e$ lie to the left of the ones of the first subcase. The dynamic of the first case is in all three sub cases qualitatively the same. Regardless of the initial value the system converges always straight to the value of x^e where the threshold values coincide and stays there resp. fluctuates in a small neighborhood around it.

The second case, illustrated in (2.4), pools those parameter set-ups, that is, certain combinations of voters decision making behavior, which have in common that they imply $\bar{x}^e$ to be bigger than $\underline{x}^e$. Again there are three interesting sub-cases worthy to be mentioned. The first (I) captures situations where voters of type one exhibit a smaller status quo bias concerning their signaled satisfaction change than those of type two whether or not x^e increases or decreases, in this way $\bar{\phi}^{1,2}>\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}>\bar{\phi}^{2,2}$. The

reverse parameter constellation is typical for the second sub-case (II). While in case of the third (III) type one voters show a stronger status quo bias in the event of an increasing x^e and type two voter do when x^e decreases, thus $\bar{\phi}^{1,2}<\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}>\bar{\phi}^{2,2}$. As before each pair of $\bar{x}^e$ and $\underline{x}^e$, that is, the corresponding satisfaction changes might come along with several parameter constellations that bring about not the same signals but signals, comparing voters of both types, that are respectively equal. Under consideration of the general restrictions of the second case concerning potential parameter combinations one can derive the following scenarios in terms of the adjustment behavior of the single voters. At first, lets suppose that the resulting status quo biases of each voter and their proportion arise from voters sensitivity to signal satisfaction changes as the consequence of the assumed psychological mechanisms.
Regarding the first sub-case, there are possible situations with both types of voter exhibiting a stronger sensitivity in case of an increasing than in the event of a decreasing x^e. Reducing proportionally those sensitivities in favor of an increasing x^e would lead to a scenario where type one responds less and type two more sensitive in the event of an increase of the nature reserve than in case of a decrease. Continuing to alter the sensitivities would imply that also voters of type two respond less sensitive to satisfaction changes caused by an expanding x^e comparing to a shrinking one. Raising now the two symmetry constraints regarding the sensitivities of each single voter one has to include voters risk attitude to be able to explain the before supposed status quo bias constellations. Since also the status quo difference between voters of different type is bigger in case of a decrease of x^e and, furthermore, looking at symmetric individual sensitivities there are three general scenarios, one with type one being risk averse and two risk loving, the reverse one and such one where both are risk averse. Thereby, in terms of the proportion of each voters resulting status quo biases the first comes a long with $\bar{\phi}^{1,2}<\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$, in the second $\bar{\phi}^{1,2}>\underline{\phi}^{1,2}$ while $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$ and finally in the third scenario $\bar{\phi}^{1,2}<\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$. Voters of type one always respond less sensitive to changes than type two. Assuming that voters show symmetric sensitivities for arbitrary changes of x^e type one voter would have to be risk averse and type two risk loving. Actually each of the feasible status quo bias constellations of this sub-case could be explained by this kind of general combination of sensitivities and risk atti-

tudes. Also here voters of type one are always the less sensitive one.

The second and the first sub-case have in common that there are potential scenarios with both types exhibiting a stronger or a smaller status quo bias at the same time in case of an increase and consequently for a decrease of x^e as well while signaling their respective satisfaction change. Additionally, there are also scenarios where type one shows proportionally a smaller and type two a bigger status quo bias when x^e increases than when it decreases. Both sub-cases are also contrary in the sense that the absolute difference between the status quo biases of a single voter is regarding a type one voter smaller in the first and bigger in the second sub-case comparing with those of a type two voter. In turn they have in common that those status quo bias constellations could be explained by purely considering sensitivities or by a combination of sensitivities and voters risk attitudes. In terms of the first symmetry constraint that refers to single voters the possible scenarios concerning voters risk attitude and thus the resulting status quo bias constellations are qualitatively the same as in the previous sub-case. But in case of the second symmetry constraint now voter one would have to be risk loving and two risk averse. What may coincide with all general status quo bias constellations of the second sub-case. Moreover, voter one is in all scenarios less sensitive than voter two.

Looking at the difference between the status quo biases of a single voter and comparing those between voters of different type shows that there are parameter combinations for three qualitatively different cases that fulfill the requirements of the third sub-case. Namely, one can think of scenarios where both types exhibit a smaller, an equal or such where a type one voter shows a bigger and type two a smaller status quo bias, in terms of signaling satisfaction changes, when it comes to an expanding nature reserve than in case of a shrinking one. As before all these scenarios meaning the respective parameter constellations are entirely explainable by corresponding assumptions about voters sensitivities to signal changes of their satisfaction or by considering voters risk attitude as well. Again, by limiting the set of sensitivity combinations through raising the two symmetry constraints, one can distinguish for both between three general scenarios, one with type one being risk averse and two risk loving, the reverse one and such one where both are risk averse. Whereby, in case of the symmetry constraint that refers to a single voter the proportion of voters resulting status quo biases are as following, in the first scenario $\bar{\phi}^{1,2}<\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}>\underline{\phi}^{2,2}$, in the second $\bar{\phi}^{1,2}>\underline{\phi}^{1,2}$ while $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$ and finally in the third scenario $\bar{\phi}^{1,2}<\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$. Fulfilling the second symmetry constraint does not limit the possible parameter combinations in the sense that all above mentioned general status quo bias constellations could occur.

Qualitatively, one can draw a distinction between three patterns that can occur in the dynamics of each single sub-case. Depending on the initial value (x_0^e) the system converges either to $\bar{x}^e$ if $x_0^e < \bar{x}^e$ or to $\underline{x}^e$ if it starts from a value bigger than $\underline{x}^e$. In both cases the size of the nature reserve stays, as in the first case, in a close neighborhood around the threshold values respectively. A third pattern emerges if x_0^e lies between both thresholds. In this instance the land allocation does not change at ll over time.

The third case, shown schematically in (2.5), covers those parameter combinations, which imply $\bar{x}^e$ to be smaller than $\underline{x}^e$. There are also three interesting general sub-cases that are to a certain extent the counterparts to those of the second case. The first (I) captures situations where voters of type one exhibit a smaller status quo bias concerning their signaled satisfaction change than those of type two whether or not x^e increases or decreases, in this way $\bar{\phi}^{1,2}>\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}>\bar{\phi}^{2,2}$. The reverse general parameter constellation is typical for the second sub-case (II). While in case of the third (III) type one voters show a smaller status quo bias in the event of an increasing x^e and type two when x^e decreases, thus $\bar{\phi}^{1,2}>\underline{\phi}^{2,2}$ and $\underline{\phi}^{1,2}<\bar{\phi}^{2,2}$. As before for each combination of $\bar{x}^e$ and $\underline{x}^e$, that is, for the corresponding satisfaction changes there are several parameter constellations causing not the same signals but signals, comparing voters of both types, that are respectively equal. Under consideration of the specific restrictions of this third general case concerning suitable parameter combinations one can derive for each sub-case several scenarios in terms of voters adjustment behavior that underlays the corresponding status quo biases.

Regarding the first sub-case, there are potential scenarios with both types of voters exhibiting a stronger or a smaller, as well as such with type one showing a smaller and type two a stronger status quo bias in case of an increasing x^e than in the event of a decreasing x^e. Assuming voters adjustment behavior is determined by their risk attitude and certain psychological mechanisms causing an individual degree of sensitivity to signal changes of their satisfaction and raising the two symmetry constraints regarding those sensitivities continues to reduce the set of potential scenarios. With symmetric individual sensitivities in place there are three general scenarios possible, one with type one being risk averse and two risk loving, the reverse one and such where both are risk loving. Thereby, in terms of the proportion of each voters resulting status quo biases the first comes a long with $\bar{\phi}^{1,2}<\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}>\underline{\phi}^{2,2}$, in the second $\bar{\phi}^{1,2}>\underline{\phi}^{1,2}$ while $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$ and finally in the third scenario $\bar{\phi}^{1,2}>\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}>\underline{\phi}^{2,2}$. Voters of type one always respond more sensitive to changes than those of type two. Now assuming that voters show symmetric sensitivities for arbitrary changes of x^e either type one voters would have to be risk averse and those of type two risk loving or it has to be the other way around. Actually each of the feasible status quo bias constellations of this sub-case could be explained by this kind of combination of sensitivities and risk attitudes. Also here voters of type one are always the more sensitive one.

The same general scenarios in terms of the proportion of voters status quo biases that are possible in the first sub-case can also occur within the limits of the second sub-case. Also in the second sub-case those status quo bias constellations could be explained by simply considering sensitivities or by a combination of sensitivities to changes and voters risk attitudes. Furthermore, in terms of the first symmetry constraint that refers to single voters the possible scenarios concerning voters risk attitude and thus the resulting individual status quo bias constellations are qualitatively the same as in the previous sub-case. This is also true in case of the second symmetry constraint, what again may coincide with all feasible status quo bias constellations of the second sub-case. In contrast, voter one is now in all scenarios less sensitive than voter two.

The third sub-case (III) comes also along with three qualitatively different parameter combinations comparing voters of different types in terms of the difference between their individual status quo biases. That is, one can think of scenarios where both types exhibit a smaller, a bigger or such where type one voter show a smaller and type two a bigger status quo bias, in terms of signaling satisfaction changes, when it comes to an increase of x^e than in case of a decrease of x^e. As before all these scenarios meaning the respective parameter constellations are entirely explainable by corresponding assumptions about voters sensitivities to signal changes of their satisfaction or by considering also voters risk attitude. Again limiting the set of sensitivity combinations by raising the two symmetry constraints, one can distinguish in both instances between three general scenarios, one with type one being risk averse and two risk loving, the reverse one and such one where both are risk loving. Whereby in case of the symmetry constraint referring to a single voter the proportion of voters resulting status quo biases are as following in the first scenario $\bar{\phi}^{1,2}<\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}>\underline{\phi}^{2,2}$, in the second $\bar{\phi}^{1,2}>\underline{\phi}^{1,2}$ while $\bar{\phi}^{2,2}<\underline{\phi}^{2,2}$ and finally in the third scenario $\bar{\phi}^{1,2}>\underline{\phi}^{1,2}$ and $\bar{\phi}^{2,2}>\underline{\phi}^{2,2}$. While fulfilling the second symmetry constraint does not limit the possible parameter combinations in the sense that all above mentioned general status quo bias constellations could occur. Once again, also in the third case one can draw qualitatively a distinction between three patterns that can occur in the dynamics of each single sub-case. Depending on the initial value (x^e_0) the system converges either to $\bar{x}^e$ if $x^e_0<\underline{x}^e$ or to $\underline{x}^e$ if it starts from a value bigger than $\bar{x}^e$. Once the threshold $\bar{x}^e$ ($\underline{x}^e$) is reached the signal of the voter of type two (one) becomes bigger hence x^e starts to decrease (increase). In the further course the type two voter continues to send the bigger signal therefore the system moves to $\underline{x}^e$ ($\bar{x}^e$) where the reverse signal switch happens thus again the nature reserve expands (shrinks) back to $\bar{x}^e$ ($\underline{x}^e$). In the following x^e keeps oscillating between both thresholds. The third pattern occurs if the system starts from a value between the thresholds, thus $\bar{x}^e>x^e_0>\underline{x}^e$. The only difference to the other two patterns is that now x^e oscillates between both thresholds from the start. Whether x^e shrinks or expands at first depends on the initial impulse caused by ϵ_t.

One has to admit that not all of the described parameter combinations are in the same way plausible. Combinations with both type of voters being risk avers and combinations with turning points that are close to each other seem to describe the behavior of real agents best. Furthermore, numerical simulations also show that in case of relaxing the restrictions on the other parameter the described regularities fast vanish. In most cases the adjustment behavior of the agents compensate each other forcing the environmental state to stay at a stable path.

2.4 Conclusion

In the first part of this work I described various examples for social situations that all have certain characteristics in common, namely the existence of feedback loops, the interdependency of those loops, a lack of direct coordination and the circumstance that the interacting agents show a hesitant adjustment behavior. Afterwards suggestions were made how to capture those characteristics in an adequate modeling approach. Based on this, in the second part, a small scale agent-based model was presented. With this model first insights could be drawn how the adjustment behavior of the interacting agents influence each other and thus the dynamic of the model. From all different combinations of the adjustment parameter, representing different intensities of hesitation, only a few were economically plausible and lead to a dynamic of the model with stable regularities.

Bibliography

Kenneth J Arrow, Theodore Harris, and Jacob Marschak. Optimal inventory policy. *Econometrica: Journal of the Econometric Society*, pages 250–272, 1951.

Kenneth Joseph Arrow, Samuel Karlin, Herbert E Scarf, et al. *Studies in the mathematical theory of inventory and production*. Stanford University Press, 1958.

Robert J Barro. A theory of monopolistic price adjustment. *The Review of Economic Studies*, 39(1):17–26, 1972.

Ricardo J Caballero, Eduardo M.R.A. Engel, and John Haltiwanger. Aggregate employment dynamics: Building from microeconomic evidence. *The American Economic Review*, 87(1):115–137, 1997.

Andrew Caplin and John Leahy. State-dependent pricing and the dynamics of money and output. *The Quarterly Journal of Economics*, 106(3):683–708, 1991.

Andrew Caplin and John Leahy. Economic theory and the world of practice: A celebration of the (s, s) model. *Journal of Economic perspectives*, 24(1):183–202, 2010.

Andrew S Caplin and Daniel F Spulber. Menu costs and the neutrality of money. *The Quarterly Journal of Economics*, 102(4):703–725, 1987.

Avinash K Dixit, Robert K Dixit, Robert S Pindyck, and Robert Pindyck. *Investment under uncertainty*. Princeton university press, 1994.

Raquel Fernandez and Dani Rodrik. Resistance to reform: Status quo bias in the presence of individual-specific uncertainty. *The American economic review*, pages 1146–1155, 1991.

Itzhak Gilboa. *Rational choice*. MIT press, 2010.

Daniel S Hamermesh. Labor demand and the structure of adjustment costs. *The American Economic Review*, pages 674–689, 1989.

Daniel Kahneman, Jack L Knetsch, and Richard H Thaler. Anomalies: The endowment effect, loss aversion, and status quo bias. *Journal of Economic perspectives*, 5(1): 193–206, 1991.

William Samuelson and Richard Zeckhauser. Status quo bias in decision making. *Journal of risk and uncertainty*, 1(1):7–59, 1988.

Eytan Sheshinski and Yoram Weiss. Inflation and costs of price adjustment. *The Review of Economic Studies*, 44(2):287–303, 1977.

Leigh Tesfatsion and Kenneth L Judd. *Handbook of computational economics: agent-based computational economics*, volume 2. Elsevier, 2006.

Chapter 3

The reorganization of firms and income inequality

3.1 Introduction

A topic to which for decades social science devoted frequently much attention is the development of labor incomes, more precisely of the inequality between the incomes of different groups of workers. Recently, besides others, it was especially the work of Thomas Piketty that brought the issue income inequality back into the spotlight of the academic and the public discourse.[1] The new empirical results presented by Piketty suggest that there are fundamental mechanisms underlying the economy that slowly but consistently cause incomes to become more unequal. This can be observed not only in the entire economy but also on the firm level. Among the various metrics to measure or to express income dispersion, the comparison of quantiles and the Gini coefficient are the two most popular ones,[2] where the latter will be important in this work.

Social science looked at this empirical phenomenon, increasing income inequality, from various angels. One of the most spread set of explanations focuses on the production side of the firms, more specifically on how technological change causes firms to alter the way they produce goods. The first models in this area of research, which are often subsumed under "skill biased technological change models", claim that new technologies first of all favor high-skilled workers, increase their relative productivity and incomes,[3] hence worsening the situation of low-skilled workers accordingly.[4] Following empirical and theoretical work showed the limitation of this approach,[5] what lead to a new set of production-oriented models to explain the demand for different types of labor and the rise of income inequality. Those models assume production, as the model below does, to be a combination of tasks that are conducted by various skilled workers. Furthermore, they also assume that tasks are differently affected by new technologies in terms of automation and that all workers are able to perform those

[1]Besides the academic work e.g. Piketty and Saez (2003) and Atkinson et al. (2011), his book "Capital in the Twenty-First Century" (Piketty (2014)) made the insights of this research accessible to a broad non-academic audience.

[2]The comparison of quantiles is extensively used in the work of Piketty. For the Gini coefficient see (Atkinson et al. 2017).

[3]See e.g. Krusell et al. (2000) and Acemoglu (2002)

[4]See e.g. Katz and Murphy (1992), Murphy and Welch (1992) and Berman et al. (1998).

[5]See Card and DiNardo (2002) and Autor et al. (2008).

tasks but that their productivity depends on their skill level. Therefore, the possibility of substitution between workers and capital and between differently skilled workers is inherent to those models.[6]

Another slow but not less persistent phenomenon is the increase of the variety of professions and necessarily of educational opportunities. This trend towards a more heterogeneous labor force always accompanied economic progress, but in recent decades developments like computerization and digital transformation made this trend even more visible. On the one side this is simply caused by a broader set of products people consume, and of course that have to be manufactured. In the event of technological progress the productivity of input factors increases. Therefore, one can either produce more of the same products with the same resources or additional, new goods. In the latter case new skill combinations, i.e. professions, are needed. On the other side firms regularly reorganize their operations that is, they reallocate the tasks or activities, which together form those operations, between their workers. Usually, this leads to a higher degree of labor decision, thus the range of tasks resp. activities that each worker conducts becomes smaller or in other words workers become more specialized.[7]

The contribution of this work is to show the connection between those two phenomena, the increase in labor division and its effect on the income distribution of the work force of a single firm. The following two research questions will serve as a guideline for the paper. First, what influence does technological change have on the way a firm organizes its operations and, second, how does such change affect the incomes of the work force, more precisely does it make the firm-specific income distribution more or less equal. To answer those questions I develop a model of a single firm that produces a final good and that takes the price of the good and the various wages of differently skilled workers as given. The firm forms working groups and allocates the tasks to them. Thus the firm has not only to decide which group conducts which set of tasks, as it is the case in the task-based models, but also which number of working groups is optimal, which is similar to Becker's approach. Both decisions imply the optimal

[6] One of the first and very influential task-based models was Autor et al. (2003). A further refined and very well presented task-based model is Acemoglu and Autor (2011). See also Autor et al. (2006) and Autor and Handel (2013).

[7] A very influential paper regarding labor division is Becker and Murphy (1992).

organizational structure of the firm. We will see that the firm has to consider two contrary effects when increasing the number of working groups. On the one hand, this lowers the total wage costs, but on the other hand increases complexity, i.e. the costs of coordination on the other side. In the event of technical progress the firm needs to reorganize to balance both effects again. How that changes the optimal organizational structure and moreover how that again influences the income distribution depends on the production function of the firm and on the assumed outcome of the labor market.

The paper is organized in the following way. In Section 3.2 the focus is on the properties of the labor force, especially on the skills inherent to each worker. In contrast, the concern of Section 3.3 is the firm. At first an activity-based production function is introduced. Afterwards the components of the firm's organizational structure are explained in detail. The section closes with an explicit illustration of the cost trade-off the firm is facing. Section 3.4 finally describes the optimization problem the firm has to solve to determine its optimal organizational structure. First some general results are derived before through an illustrative example it is explicitly analyzed how the functional form of the production function and the skill referring wage function as well as technological progress influence the optimal number of working groups and their degree of specialization. Before Section 3.6 concludes, Section 3.5 is dedicated to discuss how a change of the optimal organizational structure of a firm influences the income equality of the work force.

3.2 The labor force

It is obvious that humans and therefore workers differ widely regarding their cognitive, social and physical abilities that are either innate to each single person or that is actively obtained for example through learning on the job or through schooling, that is, abilities in the sense of an investment good. This very general classification can be refined further. Nevertheless, the important point of such classifications is that different combinations of those abilities might lead to the same outcome. This indeed depends on the task a worker is supposed to conduct. To be more precise combinations refer to the actual abilities and the specific levels of these abilities that a worker possesses.

To simplify the comparison of workers regarding their abilities, a couple of assumptions is essential. First of all, workers cannot compensate their lack of abilities needed to accomplish a certain task by simply spending more time on it. Thus there are minimum requirements for each ability concerning a specific task. Additionally, it is assumed that workers, even so that they might use different combinations of abilities resp. of ability levels, do not perform tasks with different speed. That is, if two workers are able to conduct a task, both need the same amount of time to do so. Furthermore, in real life it is quite likely that confronting the same workers with different tasks will lead to a situation where one is able to perform successfully a certain task and the other is not while for another task the opposite happens. In this work such situations are excluded.

Based on the assumptions made so far, one can think of an abstract one-dimensional measure ranking workers according to their individual combination of abilities. Thus, to each worker a value $\phi \in [0, 1]$ is assigned to what in the following is referred to as the skill level that defines a workers type. The higher the skill level, the more demanding tasks she is able to conduct. This implies, as we will see below, that each task that is part of a firm's operations, is ranked as well according to the minimum skill level that it requires. Arranging all workers along the skill interval gives an economy-specific skill level distribution $\mathcal{L}(\phi)$. So far it is only assumed that there are workers of every skill type, hence $\mathcal{L}(\phi) : [0, 1] \to \mathbb{R}_{++}$.

Each worker supplies inelastically one unit of labor in each period. The following set L^s_ϕ captures all potential combinations of skill levels that a worker of skill type ϕ can offer

$$L^s_\phi = \left\{ l^s(\tilde{\phi}) : [0, \phi] \to \mathbb{R}_+ : \int_0^\phi l^s(\tilde{\phi}) d\tilde{\phi} = 1 \right\}. \tag{3.1}$$

As one can see there are, beside a workers specific skill level, no further constraints concerning the skill content of her labor supply. Neither, for example, is she obliged to cover all possible skill levels nor can she only offer a convex sub-interval of the skill range she is able to cover.

3.3 The firm

3.3.1 The firm's operations as a task-based concept

As often mentioned in the economic literature, to produce goods many distinguishable tasks need to be combined. A task is usually understood in the sense of a process, hence as a set of various activities that exhibit strong content- and time-related dependencies. An activity is assumed to be the smallest technically possible or reasonable work content unit. Considering the same skill measure that was introduced to evaluate the abilities of the labor force, the activities of a task might differ highly in the minimum skill requirements workers need to meet to be able to conduct them. As will be shown later, it is to a certain extent beneficial for the firm to divide tasks into subtasks. That is, into sets of activities, in a way that the range of the minimum skill levels demanded by the activities of the single subtasks becomes narrower than the one of the initial task. Continuing to split subtasks would finally lead to sets that consist of activities with the same minimum skill requirement or even only of a single activity.

Let us assume that arranging the activities that a firm has to combine to produce a good y, in terms of their skill requirements form a subinterval $[0, \bar{\phi}]$ (for some $0 < \bar{\phi} < 1$) of the total skill range. Furthermore, single tasks are assumed to be complementary. One can think of cases where certain tasks can be substitutes. For example putting more effort into an operative production planning task could improve the work flow hence increase the output of a certain group of workers. But on the one side such substitution effects are usually very limited and on the other side for most task combinations do not exist at all. Therefore, "combining activities" takes place according to the Leontief production function

$$y = \max\left\{\min_{\phi\in[0,\bar{\phi}]} \tilde{y}(\phi) - \theta, 0\right\} = \max\left\{\min_{\phi\in[0,\bar{\phi}]} \{(g(\phi)\cdot \tilde{f}(\phi))^{h(\phi)}\} - \theta, 0\right\}, \text{ for } \theta \geq 1. \tag{3.2}$$

The potential output level $\tilde{y}(\phi)$ yielded by the activities of type ϕ is determined by the effort $\tilde{f}(\phi)$ allocated to those activities and $g(\phi)$ and $h(\phi)$ characterizing, besides the complementarity assumption, the technology applied by the firm. The skill level with the lowest potential output level defines the actual realized output level $y \in \mathbb{R}_+$.

Since workers of various types do not differ in their productivity to perform certain activities, as long as they have a sufficient skill level, and because their labor supply is inelastic, the effort $\tilde{f}(\phi)$ can synonymously be understood as working time or number of workers allocated to the activities of type ϕ, therefore $\tilde{f} : [0, \bar{\phi}] \to \mathbb{R}_{++}$.

The function $g : [0, \bar{\phi}] \to \mathbb{R}_{++}$ assigns to every skill level a skill-specific technology parameter which expresses how effective a unit of working time at a certain skill level is in providing a potential output level. The higher $g(\phi)$, the more productive are workers at skill level ϕ regarding $\tilde{y}(\phi)$. Additionally, it is assumed that the higher the skill level, the higher the technology parameter, thus $g_\phi > 0$. Whether this is because at higher skill levels, there are less activities to be executed or because workers are able to conduct them faster than those requiring lower skill levels, depends highly on the way the various tasks at the beginning were splitted into the respective activities. Since there is no equal natural unit to measure various activities capturing them in units of skill-specific input factors, either working time or the number of workers necessary to conduct them makes them, as an aggregate at each skill level, comparable. This becomes clearer when looking at the inverse of the production function, what is done hereafter. Therefore, the only thing that is relevant at the moment is that deploying the same amount of working time or the same number of workers at a higher skill level provides a higher potential output level than at a lower skill level.

A further important feature that characterizes a production function is the way the marginal products of its input factors vary along with the deployed amount of those factors. In the case of the Leontief production function (3.2), this is captured by $h : [0, \bar{\phi}] \to \mathbb{R}_{++}$, which determines the marginal product of the effective working time $g(\phi) \cdot \tilde{f}(\phi)$ at each skill level ϕ. Besides the fact that the marginal products are positive, there are two further important assumptions made concerning h. First, assuming the same amount of effective working time, the marginal products at higher skill levels are also higher than those at lower skill levels, thus $h_\phi > 0$. Second, in the case of an increase of the amount of effective working time, the marginal products at lower skill levels are diminishing while at higher ones they are rising, therefore $h(0) < 1$ and $h(\bar{\phi}) > 1$. Thereby, since $g(\phi)$ is a given parameter, an increase of the effective working time at skill level ϕ implies an increase of the working time $f(\phi)$ the firm allocates to the respective activities at ϕ. Looking at the diversity of the firm sector of an economy, it becomes obvious that, as in the case of g, there are plenty of mechanisms, firm-specific as well as industry-specific ones, that would demand for various forms of h. The two most important ones featured in the present activity-based model are the scalability of the outcome of the single activities and the extent of the coordination efforts at different skill levels. The first refers to the assumption that the higher the skill level the more activities are conducted which outcome is being used independently of the actual realized output level. For example, a top manager's the strategic decisions or the product improvements done by the engineers of the development department are necessary to make and of the same use at any level of y. Therefore, they are highly scaleable in terms of the output level. While in the case of production planners, a significant amount of the activities they perform are directly connected with the level of y, other planning activities again might be more general, hence scaleable. On the lower end of the skill interval, these activities are located whose extent depends directly on y, like those producing the actual final good. Their scaleability is consequently close to zero. Concerning the second mechanism, coordinating the activities, it is assumed that to a certain extent each worker has to contribute to the overall coordination efforts of a firm. That is, all workers have to perform activities that coordinate other activities, for example communicating the results of activities they have carried out or harmonizing beforehand the details of the work content of their activities with the ones executed by other workers of comparable skill level. Furthermore, it is also assumed that in the case of low skill activities such coordination efforts increase overproportionally with the number of workers. Considering the potential output level the coordination efforts have a contrary effect than the scalability of the activities. At the lower end of the skill range, the coordination effect dominates while at higher skill levels the latter does. The described form of h captures both effects commonly expressing the differences between the skill levels in terms of the marginal returns resp. the change of the marginal returns of the skill-specific effective working time.

The last assumption regarding the firm's production function is that a certain amount of working time of each skill level is needed to conduct the activities necessary to keep the firm running. This is captured by the number θ. The higher θ the more workers are involved in maintaining operations. The value of θ can be interpreted either as the equivalent quantity of the final good that could be produced with those working time units otherwise or as actual output that is used internally. In the latter case y would not denote the absolute output level of the firm, but the share that is meant for sale.[8]

For obvious reasons it is important for a firm to know how much of the good y can be produced with various working time distributions over the skill range. But it is the reverse relation that serves as a basis for the firm's effort to work out its optimal organizational structure, which will be discussed in more detail below. Thus we are looking for a function $f : [0, \bar{\phi}] \times \mathbb{R}_+ \to \mathbb{R}_+$ such that for any given $y \in \mathbb{R}_+$ and for all $\phi \in [0, \bar{\phi}]$, $y = (g(\phi) \cdot f(\phi, y))^{h(\phi)} - \theta$ holds. This condition ensures that the firm allocates the right amount of working time to each single activity type to produce y without wasting resources. Solving for $f(\phi, y)$ yields

$$f(\phi, y) = \frac{1}{g(\phi)} \cdot (y + \theta)^{\frac{1}{h(\phi)}}. \tag{3.3}$$

The function f assigns to any skill level ϕ and any intended output level y the necessary amount of working time or number of workers. From the assumptions made so far concerning the production technology, one can derive that at any skill level, the amount of working time is strictly increasing in y and at the same time that it is strictly decreasing in ϕ for all output levels, hence $f_\phi < 0$ and $f_y > 0$.[9] Furthermore, the monotonicity properties also imply that the proportion of the total amount of working time below any arbitraryly chosen level of ϕ and that above is increasing in y. In other words the more goods the firm produces, the more proportionally low skill activities are performed by the workers.

Later we will see that to specify the optimal organizational structure it is essential for the firm to determine the number of workers it has to employ to conduct all activities up to a skill level ϕ for some given output level y. The following expression captures

[8]For technical reasons θ is assumed to be bigger than one.

[9]For the derivatives, see the Appendix Section 3.7.

this relation

$$F(\phi, y) := \int_0^{\phi} f(\tilde{\phi}, y) d\tilde{\phi} = \int_0^{\phi} \frac{1}{g(\tilde{\phi})} \cdot (y+\theta)^{\frac{1}{h(\tilde{\phi})}} d\tilde{\phi}. \tag{3.4}$$

3.3.2 The firm's organizational structure

Knowing the right number of workers or the amount of working time to perform the activities of each type necessary to produce y, thus knowing $f(\phi, y)$, is the first step for the firm to find its optimal organizational structure. The second step is to assign workers to those skill-specific activities behind the working time amounts. In principle the firm can choose from an infinite set of combinations of workers and activities regarding type and numbers, but considering that these come along with different coordination and wage costs, some are more beneficial than others. The assumptions and mechanisms behind these cost considerations will be discussed in the next section, while in the following, the resulting general characteristics of the firm's optimal structure are introduced. The first characteristic is that it is optimal for a firm to form groups of workers with each group conducting all activities of a certain range of skill levels. Second, these skill levels are adjacent. Technically speaking, the firm partitions the skill interval $[0, \bar{\phi}]$, hence the corresponding activities, into a set of n subintervals $\{\Phi_i\}_{i=1}^n$ that are indexed in increasing order in terms of their skill levels. Thus, for any $\phi' \in \Phi_j$, $\phi'' \in \Phi_k$ with $j > k$ and $\Phi_j, \Phi_k \in \{\Phi_i\}_{i=1}^n$ it holds that $\phi' > \phi''$. The subintervals are mutually exclusive and exhaustive, that is, every skill level $\phi \in [0, \bar{\phi}]$ belongs to one and only one subinterval. Therefore, except for the first subinterval, which is compact, subintervals are assumed to be open to the left side and closed to the right side. Thus, there is a skill level vector $\widehat{\boldsymbol{\phi}} = (\widehat{\phi}_1, \ldots, \widehat{\phi}_n)$ with $\widehat{\phi}_n = \bar{\phi}$, where $\widehat{\phi}_i$, $i \in \{1, \ldots, n-1\}$, separates subinterval i from subinterval $i+1$. Therefore, $\phi' \leq \widehat{\phi}_j$ holds for every $\phi' \in \Phi_j$ if $\widehat{\phi}_j \in \Phi_j$. These skill levels are called the cut-off skill levels of the corresponding subintervals. Economically this is the minimum skill level a worker needs to exhibit to be part of the working group that is supposed to perform the activities belonging to this subinterval.

Consequently, this means that the organizational structure of a firm is embraced by the number of cut-offs n, that is, the number of working groups or subintervals and by the cut-off skill level vector $\widehat{\boldsymbol{\phi}}$ itself. But this does not say much about the internal structure of the working groups. In fact, beside the minimum skill levels, there are no further constraints, thus a firm could combine workers of several skill types to form a working group. Actually, from a technical point of view, given a certain n, $\widehat{\boldsymbol{\phi}}$ and an amount of goods y, there are infinitely many combinations of workers in terms of number and skill types for each working group, with corresponding aggregated demands l^d over all skill types, that would enable a firm to perform the respective activities. The set of functions L^d represents those aggregated demands[10]

$$L^d_{\widehat{\boldsymbol{\phi}}} = \left\{ l^d : [\widehat{\phi}_1, 1] \mapsto \mathbb{R}_+ : \int_{\widehat{\phi}_i}^{1} l^d(\phi)d(\phi) \geq \int_{\widehat{\phi}_{i-1}}^{\widehat{\phi}_n} f(\phi, y)d(\phi),\ \forall i \in \{1, \ldots, n\} \right\}. \quad (3.5)$$

Nevertheless, as we will see later, from the same assumptions concerning the labor supply that among others lead to the two above mentioned characteristics of a firm's structure, it also follows that there is only one optimal aggregated demand level for any given organizational structure.

3.3.3 The firm's coordination and wage costs

To produce a final good a firm has not only to perform activities that vary considerably in terms of work content, skill level and in number, but also that needs to happen not independently but in a harmonized way. Every activity produces information which is relevant for other activities or The outcome of every activity is, beside in some cases a kind of material output, a huge variety of information which is important to be known by other workers conducting similar activities so that they are matched. This usually happens in both directions or in plenty of information circles that connect all activities with each other resp. the performing workers. In a nutshell, activities need to be coordinated. Before describing how coordination takes place, the concept of a working group has to be clarified. A group of workers to whom a certain set of activities is assigned form a working group. Regarding their skill levels, all workers are supposed to be able to execute all those activities and are familiar with the corresponding work content. They might for example rotate through the single activities, perform the same kind of activities or conducting activities which are, because of work content-related

[10] For the derivation, see the Appendix Section 3.7.

dependencies, highly connected. It is assumed that coordination is a two-dimensional process on the one side coordination between the working groups and on the other side within the working group. Considering the similarity and the linkage between the work content of the activities performed by the workers of the same working group, their coordination is supposed to be managed entirely through the coordinating activities at each skill level. Thus there are no additional efforts in terms of working time or costs necessary.

This is not the case for the coordination between the working groups. The members of one working group are not familiar with the activities of another. They even might not know necessarily what the work content of those activities comprises. Therefore, a firm has to handle the information flow at the interfaces between the working groups in a standardized way. This means information, that come along with the various activities, needs to be gathered, processed, transformed and saved, that is, it needs to be made accessible and processible for the workers of the other working groups. Therefore, a firm requires a suitable IT infrastructure. The resulting investments are referred to as coordination costs. Since such technical systems provide a certain capacity range, it is an acceptable simplification at this point to assume that the coordination costs C^c depend only on the number of working groups n and on the used technology a, thus

$$C^c = c^c(n, \delta),\ c^c : \mathbb{N} \times \mathbb{R}_+ \to \mathbb{R}_+ \text{ with } c^c_n > 0,\ c^c_{nn} > 0 \text{ and } c^c_\delta < 0. \tag{3.6}$$

The number of interfaces between working groups grows exponentially with their number. This explains why the coordination costs are strictly increasing in n. A positive technological change on the other side is expressed by an increasing δ. In practice, this could be caused by a lot of different developments. For example, besides improvements of the physical devices, one may think of better or new possibilities to measure processes hence to collect data or enhanced algorithms processing those data or cloud computing, that is, all such technologies that are typically part of recent themes like digital transformation, industry 4.0 and artificial intelligence.

In this work the firm is assumed to take the technology level as exogenously given. A positive technological change means either that with the same investment the firm can afford an IT infrastructure that is able to support a more sophisticated firm structure thus coordinating more working groups or that the coordination of the same number of working groups comes now along with less investments thus lower coordination costs. Either way, technological progress causes the firm to reevaluate its organizational structure.

Lets have a closer look on the second component of the firm's overall cost considerations, the wage costs. Due to the labor market being competitive, the firm acts as a price taker, thus it takes the wages as given and there is no constraint from the supply side regarding the number of workers of any skill type a firm can employ. Furthermore, it is assumed that the labor market is highly differentiated in terms of skills thus there is a continuum of wages corresponding to the workers' skill levels. This is captured by

$$W = w(\phi),\ w : [0, 1] \rightarrow \mathbb{R}_+ \text{ with } w_\phi > 0,\ w_{\phi\phi} \geq 0. \tag{3.7}$$

The assumed downwards substitutability regarding the minimum skills prevents wages form falling along the skill range, since otherwise less skilled workers would be substituted by workers with higher skills but lower wages. Furthermore, the circumstance that in the past decades the demand for higher skilled workers increased more than their supply[11] and the assumption that workers demand higher wages the more skilled they are underlies the suggested increasingly rise of the wages with the skill level.

It becomes apparent that solving the trade-off between coordination costs and wage costs is the challenge a firm has to meet to find its optimal organizational structure, hence its optimal number of working groups and cut-off values $(\widehat{\boldsymbol{\phi}}^{\star}, n^{\star})$. On the one side a firm can reduce the overall wage costs by allocating the activities necessary to produce a certain output to a larger number of working groups exploiting the wage difference between differently skilled workers. That is, it allocates less demanding activities to lower skilled workers with lower wage demands and thus making higher skilled and better paid workers to concentrate on more the challenging activities in terms of required minimum skills. But on the other side increasing the number of working groups will also increase the coordination costs a firm has to bear, up to a point, where the wage cost reduction of an additional working group is smaller than the rise of the coordination costs.

[11]This is a well established fact for a lot of industrialized countries.

How the firm solves the trade-off problem to find the optimal way to organize its operations and how the organizational structure changes in the case of a technological progress is the main concern of the next section. There are three general characteristics, two were already mentioned above, that all such solutions have in common. First, from the labor market structure it follows that a firm forms a working group only with workers that are exactly of the type of the corresponding cut-off skill levels. Thus, in terms of skills, working groups are homogeneous. Therefore, for a given vector $\widehat{\boldsymbol{\phi}}$ only the aggregated labor demand function out of L^d, where the entire demand for each working group is concentrated at the cut-off skill levels, is of potential interest for the firm. Consequently, finding firms optimal aggregated labor demand is equivalent with finding the optimal cut-off skill levels. Second and third, that a working group performs all activities belonging to a convex subset of the skill range, can be explained in one. Suppose the skill levels of the activities conducted by a working group form a convex subset, but not all activities within this subset are conducted by this group. Then there must be another working group performing those activities. In case such a group has a lower skill type, it would reduce the overall wage costs if those workers execute all activities up to their skill level. The opposite effect takes place if the skill level of such a group is higher than that of the first group. Now, to reduce the wage costs, the first group should perform all activities up to their skill type which were performed before by the group with the higher skills. While in the case there is a second group of workers of the same type, the coordination costs can be reduced if the second group performs all activities of the first group as well. This is because the coordination costs depend on the number of working groups n, but not on the amount of work or activities each group performs. Finally, it might be that the activities assigned to the first group form two disjunct subsets regarding the skill level. But if a group of lower skilled workers performs those activities that demand a skill level between the two subsets, this working group should also perform the activities belonging to the subset with lower skill levels reducing the wage costs. Otherwise, if those activities are executed by a working group of a higher skill type the first group should also perform those activities, forming a big convex skill set, in order to decrease the wage costs. From those characteristics concludes the following general form of the wage costs

$$C^w = c^w(\widehat{\boldsymbol{\phi}}, n, y) = \sum_{i=1}^{n} (F(\widehat{\phi}_i, y) - F(\widehat{\phi}_{i-1}, y)) \cdot w(\widehat{\phi}_i) \tag{3.8}$$
$$\text{for } 0 = \widehat{\phi}_0 < \widehat{\phi}_1 < \cdots < \widehat{\phi}_{n-1} < \widehat{\phi}_n = \bar{\phi}.$$

3.4 Finding a firm's optimal organizational structure

3.4.1 The general case

Usually firms try to achieve with their operations a certain objective what is for this particular firm, as in most cases, to generate high profits $\Pi^\star$. As well as on the labor market the firm is assumed to act on the good market as a price taker. Therefore, to maximize profits the firm chooses its optimal output level $y^\star$. In order to produce this output level, the firm has to deploy the input factors optimally, thus forming working groups that differ in number and skills of the respective workers and assigning those groups to the task-forming activities in the best possible way considering the total costs. In other words producing the profit maximizing output level $y^\star$ means to implement the corresponding optimal organizational structure $(\widehat{\boldsymbol{\phi}}^\star, n^\star)$. This implies that the firm is confronted with the following optimization problem

$$\Pi^\star = \max_{\widehat{\boldsymbol{\phi}}, n, y} \pi(\widehat{\boldsymbol{\phi}}, n, y; p, \delta) = \max_{\widehat{\boldsymbol{\phi}}, n, y} \Big(p \cdot y - c^c(n; \delta) - c^w(\widehat{\boldsymbol{\phi}}, n, y) \Big) \tag{3.9}$$
$$\text{for } 0 = \widehat{\phi}_0 < \widehat{\phi}_1 < \cdots < \widehat{\phi}_{n-1} < \widehat{\phi}_n = \bar{\phi}.$$

On the one side, the revenue of the firm increases in y for a given price p which is normalized to one. On the other side, we have the coordination costs and the wage costs, where the latter also increases in y. One can also see the second conflict of interest caused by the contrary effects of n on the costs, hence on the profits, that a firm has to consider while solving (3.9). Additionally, since $\widehat{\boldsymbol{\phi}}$, $f(\phi, y)$ and $w(\phi)$ effect only the wage costs C^w and δ only the coordination costs C^c it is beneficial to rewrite (3.9) as a three layer optimization problem

$$\Pi^\star = \max_{\widehat{\boldsymbol{\phi}}, n, y} \pi(\widehat{\boldsymbol{\phi}}, n, y; \delta) = \max_{y} \left(y - \min_{n} \left(c^c(n; \delta) + \min_{\widehat{\boldsymbol{\phi}}} c^w(\widehat{\boldsymbol{\phi}}; n, y) \right) \right) \tag{3.10}$$
$$\text{for } 0 = \widehat{\phi}_0 < \widehat{\phi}_1 < \cdots < \widehat{\phi}_{n-1} < \widehat{\phi}_n = \bar{\phi}.$$

This makes it easier to examine which general properties potential solutions of (3.9) resp. (3.10) have and how they are affected by a change of the functional forms of f and w as well as by a change of the technology level δ. This will be done in the next section for an illustrative example. But first of all, since the most basic elements of the firm's structure are the cut-offs, some general properties inherent to an optimal cut-off vector and its use to guide the organizational efforts of the firm are highlighted. For this, let us have a look at the first layer of the optimization problem

$$C^{w\star} = \min_{\widehat{\boldsymbol{\phi}}} c^w(\widehat{\boldsymbol{\phi}}; n, y) = \min_{\widehat{\boldsymbol{\phi}}} \sum_{i=1}^{n} (F(\widehat{\phi}_i, y) - F(\widehat{\phi}_{i-1}, y)) \cdot w(\widehat{\phi}_i) \tag{3.11}$$
$$\text{for } 0 = \widehat{\phi}_0 < \widehat{\phi}_1 < \cdots < \widehat{\phi}_{n-1} < \widehat{\phi}_n = \bar{\phi}.$$

This expression captures the firm's attempt to minimize the wage costs for a given output level and a fixed number of working groups. The following lemma gives some first insights about potential optimal cut-off vectors.

Lemma 1. *For any given output level y and number of working groups n, there exists a cut-off vector $\widehat{\boldsymbol{\phi}}^\star$ that minimizes the wage costs c^w. The components $\widehat{\phi}_i^\star$ (for all $i = 1, \ldots, n-1$) fulfill the following condition*

$$(F(\widehat{\phi}_i^\star, y) - F(\widehat{\phi}_{i-1}^\star, y)) \cdot w_{\phi_i}(\widehat{\phi}_i^\star) = (w(\widehat{\phi}_{i+1}^\star) - w(\widehat{\phi}_i^\star)) \cdot f(\widehat{\phi}_i^\star, y). \tag{3.12}$$

Furthermore, a solution $\widehat{\boldsymbol{\phi}}^\star$ has the following order: $\widehat{\phi}_{i-1}^\star < \widehat{\phi}_i^\star$ for all $i \in \{1 \ldots n\}$.

Proof. See the Appendix Section 3.7. □

Based on the optimality condition (3.12) the firm can draw two other conclusions that are useful in case circumstances change demanding a reorganization of its operations. The first refers to a situation where a firm decides to reorganize the operations or the activities of a single working group while the second is of use if the firm reevaluates its entire organizational structure.

Lemma 2. *Assume a given wage function w, a function f expressing the demand for skill-specific working time and a set of skill intervals $\{\Phi_i\}_{i=1}^n$ with a corresponding optimal vector of cut-off values $\widehat{\boldsymbol{\phi}}$, all defined as stated above. For any given output* *level* y *and any* $\Phi_j \in \{\Phi_i\}_{i=1}^{n}$ *a newly introduced cut-off* $\widehat{\phi}' \in (\widehat{\phi}_{j-1}, \widehat{\phi}_j)$ *reduces the total wage costs. Additionally, there exists only one cut-off* $\widehat{\phi}'^{\star}$ *that minimizes the wage costs for this sub-problem.*

Proof. See the Appendix Section 3.7. □

Imagine the firm can, for some exogenous reason, reorganize only one working group Φ_j. Reorganizing means the activities that were performed by a single group are now allocated to two groups. While the number and the work content of the activities, hence the working time or number of workers needed, does not change only the assigned work force in terms of skills changes. There is now a group of lower skilled workers performing the less demanding activities and one composed of workers who conduct the more demanding ones exhibiting the same skill level as the workers that belonged to the former single working group. To decide how to allocate the activities or in other words at which skill level to split the skill interval Φ_j, the firm has to consider two contrary effects on the wage costs. Increasing the skill level $\widehat{\phi}'$ of the splitting point will on the one side allocate more activities to the lower paid working group reducing the wage costs attributed to some activities, but on the other side lift the wage, hence the costs, for each worker of this group. The first effect is positive while the latter is negative if the firm raises $\widehat{\phi}'$. The optimality condition (3.12) states that the firm should implement the skill level where both effects caused by a marginal change of the skill level balance each other. The right-hand side of (3.12) captures the wage cost reduction when a marginal small amount of activities $f(\widehat{\phi}'^{\star}, y)$ is additionally allocated to the low-skilled working group. While the left-hand side expresses the increase of the total wage costs for the worker, of the low skilled group $F(\widehat{\phi}'^{\star}, y) - F(\widehat{\phi}'^{\star}_{j-1}, y)$ caused by an marginal raise of the wage $w(\widehat{\phi}'^{\star})$ which they receive. The following lemma presents a much more broadly applicable guideline for the firm.

Lemma 3. *Assume again a wage function* w *and a demand function* f *for skill-specific working time with the stated properties. For any skill interval* $\Phi = (\widehat{\phi}_1, \widehat{\phi}_2]$*, with* $\widehat{\phi}_1 \geq 0$ *and* $\widehat{\phi}_2 < \bar{\phi}$*, there is at most one* $\widehat{\phi}_3 \in (\widehat{\phi}_2, \bar{\phi}]$ *such that* $\widehat{\phi}_2$ *fulfills the optimality condition* (3.12)*, i.e. such that* $\widehat{\phi}_2$ *is optimal on the interval* $(\widehat{\phi}_1, \widehat{\phi}_3]$*. The same holds for the lower interval: there is at most one* $\widehat{\phi}_3 \in [0, \widehat{\phi}_1)$ *such that* $\widehat{\phi}_1$ *fulfills* (3.12)*.*

Proof. See the Appendix Section 3.7. □

In case the firm wants to reorganize or reevaluates its entire structure, it could use the following scheme, based on Lemma 3, to compare different set-ups in terms of structure and business plan. The later refers to the intended output level. This is done extensively in the illustrative example below. Before the general procedure is sketched. Therefore, for a given y and n the firm first, sets the first cut-off preferably at a skill level close to 0. Afterwards, second, it sets stepwise all other cut-offs. Since for each next cut-off the defining interval is already determined the skill level for this particular cut-off, according to Lemma 3, is as well. Thus setting the first cut-off implicitly determines the whole cut-off vector. Assuming that the first cut-off was set such that $\widehat{\phi}_n < \bar{\phi}$, as a third step the firm has to increase the skill level of the first cut-off until $\widehat{\phi}_n = \bar{\phi}$. Thereby it is not clear that an increase of $\widehat{\phi}_1$ always causes an increase of the other cut-offs, except for the second, but at some point they have to rise since otherwise they would eventually coincide with their predecessor contradicting the optimality condition (3.12). The vector of cut-offs generated in this way minimizes, according to Lemma 1, the total wage costs. Now let us focus on the illustrative example to gain further insights regarding the optimal organizational structure of a firm.

3.4.2 An illustrative example

The production function of the explicitly modeled firm exhibits, of course, the properties stated above. More precisely the firms technology parameter at each skill level is determined by $g(\phi) = \frac{1}{(1-\phi^{\alpha})}$ for $\alpha > 0$. The higher α the more effective are working units at each skill level.

The marginal returns along the skill range of the effective working time are captured by $h(\phi) = \frac{1}{(1-\phi)\beta}$ with $\beta > 1$. The parameter β controls for the size of the marginal returns, for example an increase of β lowers the marginal returns at each skill level. Consequently, it also raises the skill level above which increasing ($h(\phi) > 1$) and below diminishing ($h(\phi) < 1$) marginal returns appear. Furthermore, the output equivalent that is needed to ensure the firm's operations θ is set to one. This leads to the following inverse production function

$$f(\phi, y) = (1 - \phi^{\alpha}) \cdot (y + 1)^{(1-\phi)\beta}. \tag{3.13}$$

It denotes the amount of skill-specific working time or workers the firm requires to produce y. For simplification it is assumed that $\bar{\phi}$ is by an infinitesimal amount smaller than 1, hence $f : [0, 1) \times \mathbb{R}_+ \to \mathbb{R}_+$. Since $g_\phi(\phi) = \frac{1}{(1-\phi^\alpha)^2} \cdot \alpha\phi^{\alpha-1} > 0$ and $h_\phi(\phi) = \frac{1}{\beta\cdot(\beta-\beta\phi)^2} > 0$, the required working time becomes smaller the higher the skill level is and it increases with the output level. This is illustrated in Figure 3.1 for one manifestation of (3.13).

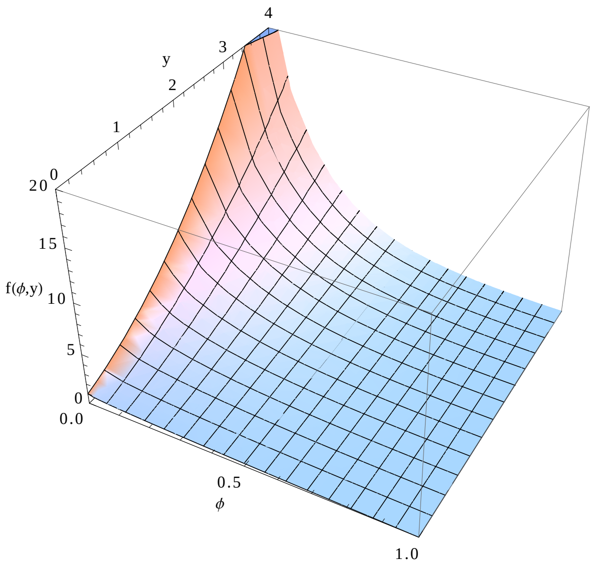

Figure 3.1: The inverse production function f ($\alpha = 0.5$ and $\beta = 2$)

The costs of operation that the firm has to consider, while deciding how much to produce (y) and how to set up its organizational structure ($\widehat{\boldsymbol{\phi}}, n$), depend on the wages at the single skill levels and on the firm-specific coordination costs. The exogenously given wages are denoted by $w(\phi; \gamma) = \phi^\gamma$ (for $\gamma \geq 1$) where γ determines the relation between wages of different skill levels. The higher γ, the stronger the wages increase along the skill interval, thus the more expansive are higher skilled workers proportionally to those with lower skills.[12] The firm's coordination costs are assumed to be $c^c(n; \delta) = \frac{1}{\delta} \cdot (n-1)^2$. The quadratic form incorporates that the number of intersec-

[12] For all $\phi, \tilde{\phi} \in [0, \bar{\phi})$ with $\phi < \tilde{\phi}$ and for all $\gamma, \gamma' \geq 1$ with $\gamma < \gamma'$ we have $\frac{\tilde{\phi}^\gamma}{\phi^\gamma} = \left(\frac{\tilde{\phi}}{\phi}\right)^\gamma < \left(\frac{\tilde{\phi}}{\phi}\right)^{\tilde{\gamma}} = \frac{\tilde{\phi}^{\tilde{\gamma}}}{\phi^{\tilde{\gamma}}}$.

tions usually, depending on the firm-specific tasks, increases overproportionally with the number of interacting working groups (n).

The following simulation illustrates the way the firm adjusts its optimal output level $y^{\star}$ and its optimal organizational structure $(\widehat{\boldsymbol{\phi}}^{\star}, n^{\star})$ in case the conditions under which it operates change. Those changes are captured by variations of the above introduced parameters α, β, γ and δ. The previously made suggestion to examine the firms decision problem stepwise will also guide this simulation. At first the focus lies on the optimal cut-offs, in step two and three the influence of the parameters on $n^{\star}$ and $y^{\star}$ are analyzed. On the one hand this enables to develop a better understanding of the direct effects as well as for the indirect cross-effects a parameter change causes. On the other hand it shows how a firm decides best in case additional constraints might prevent the adjustment of the number of working groups or of the output level.

Step 1: The optimal cut-off vector $\widehat{\boldsymbol{\phi}}^{\star}$

In the first step, one might think of a situation where the firm has a fixed plan of sales, for example due to contract manufacturing, and there is no possibility to invest in the IT infrastructure. In such cases a firm takes n and y for a certain time period as given. Therefore, the aim of the firm is to deploy the "scarce resource", number of working groups, best such that the wage costs are minimal. This depends on the operating conditions, that is, on α, β, γ and n, y. In each of the following figures one of those parameters is varied (red) in comparison to the baseline case (blue) with $\alpha = 0.5$, $\beta = 2$, $\gamma = 1$, $n = 7$ and $y = 1$.

At first a variation of α is examined. This can be interpreted as a comparison between different firms applying different, skill-specific technologies. Thus a distinct value of α captures another composition of required working time along the skill range. Figure 3.2 shows immediately that in both cases the distance between the cut-offs is increasing. This is plausible recalling the two effects balanced by the optimality condition (3.12). Since more workers are located in the lower part of the skill range, splitting the total skill interval there yields a higher reduction of wage costs by a change of the respective wage, it is better to differentiate the total skill interval there more than in the upper part. In case of the latter larger subintervals means higher wages for a

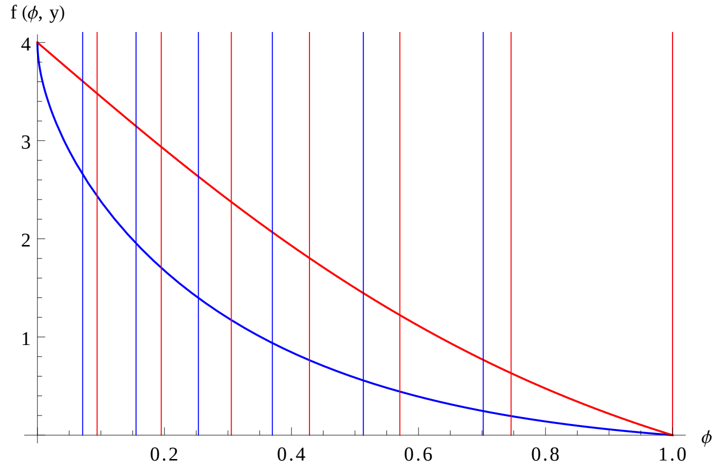

Figure 3.2: The inverse production function f for different levels of α (blue $\alpha = 0.5$, red $\alpha = 2$)

bigger group of workers but since this group is still proportionally small differentiating more would mean wasting the "scarce resource" cut-offs. The more low-skilled working time is proportionally needed, the stronger is the effect. Comparing the normalized antiderivatives[13] of both cases shows that for $\alpha = 0.5$ proportionally more working time needs to be allocated to the lower half of the skill range and less to the higher one than in the case of $\alpha = 2$. Consequently, in case of the baseline scenario a firm differentiates its operations stronger in the lower part of the demanded skill range and less in the upper part than the firm in the second scenario does. Therefore, on the one hand the values of the cut-offs of the latter are bigger than that in the baseline case. On the other hand the skill intervals of the second scenario, constituted by two adjacent cut-offs, increase at first and shrink again at higher skill levels relatively to those of the baseline scenario. Economically speaking, in the baseline case working groups are more heterogeneous in terms of their specialization, thus regarding the skill demands of the activities that they perform.

Figure 3.3 illustrates the influence of β on the skill-specific working time. Different values of β can be, as in the case of β, interpreted as a comparison between firms using different technologies. The firm with the higher value of α exhibits smaller scale effects at all skill levels, hence operations become more labor intensive. Such a

[13]For an illustration of the normalized antiderivatives, see the Appendix Section 3.7.

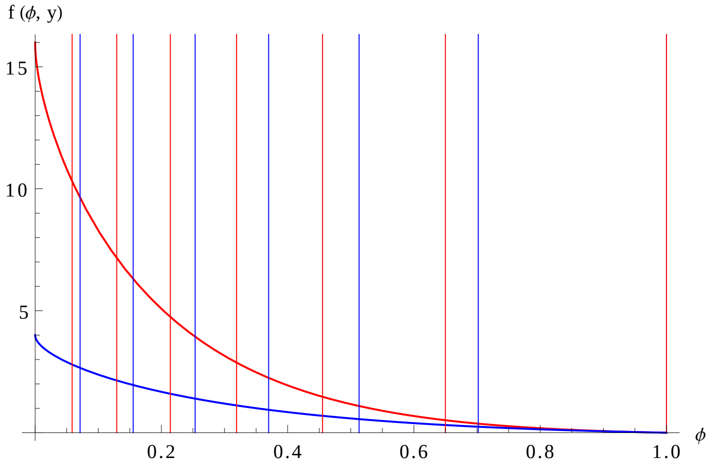

Figure 3.3: The inverse production function f for different levels of β (blue $\beta = 2$, red $\beta = 4$)

change of β alters not only the marginal returns of a unit of working time at each skill level it also alters the proportion between the working time at different skill levels necessary to produce y. As the normalized antiderivatives[14] indicate, an increase of β has qualitatively the same effect on the working time proportion as a decrease of α, thus the lower the skill level, the higher the proportional increase of the corresponding working time. Consequently, one can observe also a similar effect on the relative size of the skill intervals. In the case of $\beta = 4$ a firm differentiates the lower skill range more than in the baseline case. Thus the values of the optimal cut-off vector for $\beta = 4$ are smaller and the skill intervals shrink at first and increase at higher skill levels relatively to those of the baseline scenario. Therefore, in case of a high β working groups cover a wider range of activities regarding the respective skill levels in the upper part of the skill range than in case of low β. At the lower part of the skill range it is the other way around. Thus the higher β the more specialized lower skill and the less higher skill professions become.

After investigating several technologies applied by firms, the simulation results in Figure 3.4 visualize the influence that different constellations of the labor market have on the distribution of the cut-offs. In the case of the baseline scenario, the wage changes constantly with the skill level, thus looking at the optimality condition (3.12)

[14]See the Appendix Section 3.7.

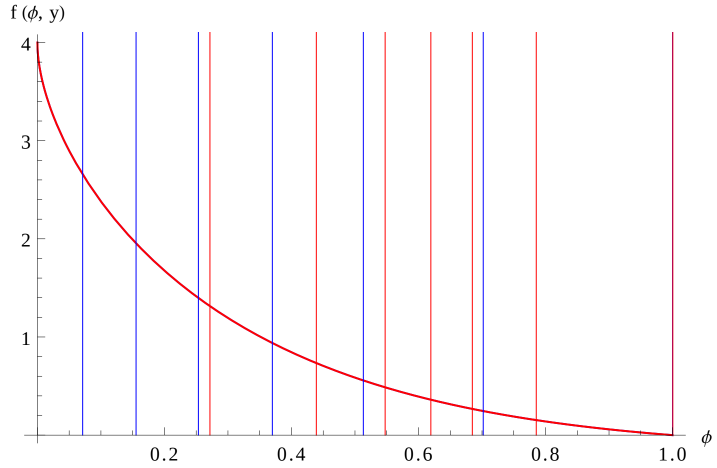

Figure 3.4: The inverse production function f for different levels of γ (blue $\gamma = 1$, red $\gamma = 4$)

makes clear that the relative size of two adjacent skill intervals and the variation of this proportion along the skill range is driven by the form of f. As long as f is strictly decreasing, the same change of the wage affects more workers the lower their skill levels are. Therefore, firms differentiate stronger at lower skill levels, hence subintervals with larger skill levels become larger. Once the wages are not increasing constantly, as when $\gamma > 1$ e.g. caused by an economy wide demand shift for higher skilled workers, the firm has still to consider two effects, but now with contrary trend. Consider a subinterval of fixed size. Shifting it, the skill range upwards reduces the number of workers belonging to the hypothetical working group, but it also increases the wage gap between the two cut-offs constituting the subinterval. Thus although at lower skill levels more workers are affected, the firm does not differentiate this part of the skill range intensively because the increases of the wages are proportionally to the total wage range still small. The same is true for the highest skill levels where the interplay occurs the other way around. There the wages increase strongly with only a few workers being affected by these wages. In the middle part, where wages already increase strongly, a substantial yet declining amount of workers is to consider. Thus to realize low total wage costs the firm uses the limited number of cut-offs best by differentiating the middle part stronger and the upper and lower part of the skill range increasingly less with the respective consequences for the degree of specialization of the working groups.

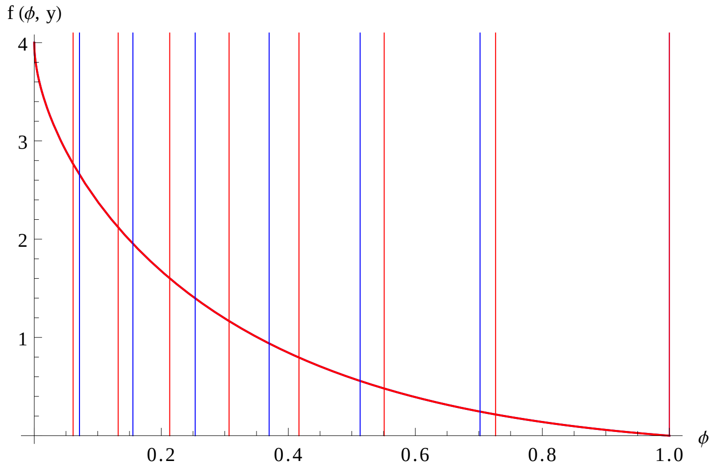

Figure 3.5: The inverse production function f for different levels of n (blue $n = 7$, red $n = 8$)

As one can see in Figure 3.5 simulations indicate that a firm differentiates lower skill levels more than higher ones independently of the number of cut-offs or working groups. Since one more cut-off means the firm has can in general differentiate the skill range more, it is plausible that with the additional interval all the other intervals (for $n = 8$ the first seven) shrink. Therefore the skill range is also as a whole more differentiated not only in certain parts, that is, all workers are more specialized. Consequently, the same is true for the wage structure of the entire work force of the firm. Due to one additional wage level, the differences between the levels are becoming smaller with the number of working groups.

The last parameter variation that is analyzed concerns the fixed output level. Figure 3.6 shows the optimal arrangements of cut-offs for two different values of y. The higher the output level, the more the differences in the marginal returns along the skill range enfold their effect on the proportions of the firm's skill-specific working time demands. Qualitatively, an increase of y shifts the cut-offs in the same way as an increase of β, thus both parameter changes have the same effect on the size of the cut-off values and on the relative size of the skill intervals, i.e. on the degree of specialization of the working groups.

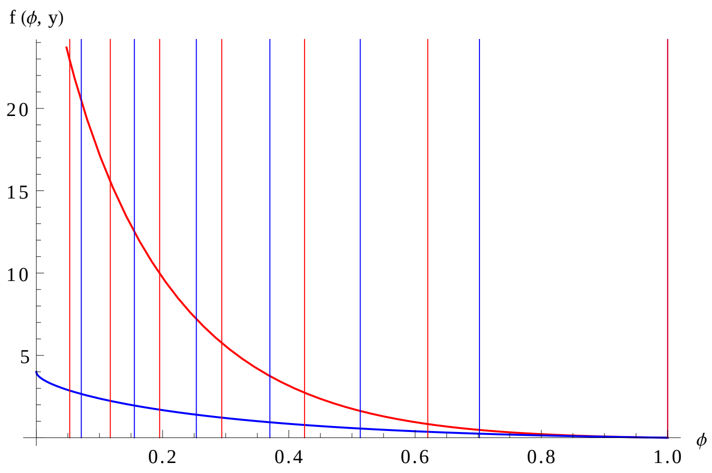

Figure 3.6: The inverse production function f for different levels of y (blue $y = 1$, red $y = 5$)

Step 2: The optimal number of cut-offs $n^{\star}$

Now the second element, besides the cut-offs, of the organizational structure of a firm, namely the number of cut-offs or working groups n is endogenized as well, while the output level is still kept constant. Thus, this part of the simulation addresses the second component that is needed to answer the first research question which was raised in the beginning, namely how does a cost of coordination reducing technological change affect the optimal organizational structure of a firm?

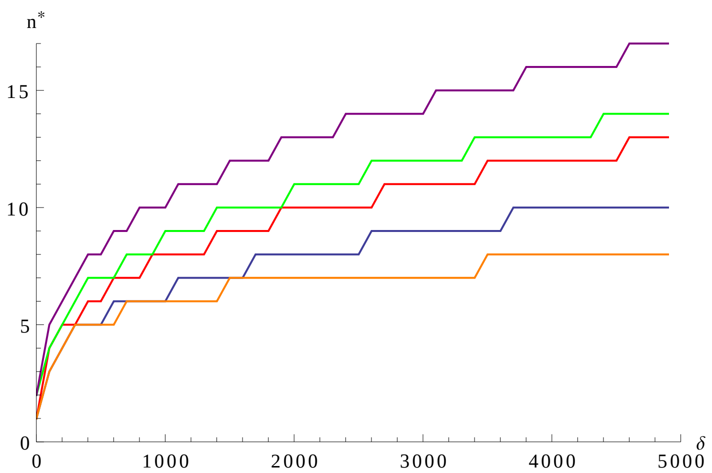

Figure 3.7: The Optimal number of working groups (red $\alpha = 2$, green $\beta = 4$, orange $\gamma = 4$, purple $y = 5$)

Figure 3.7 shows the relation between the technology parameter δ and the optimal number of cut-offs n for the same economic set-ups as in step one concerning effective working time (α), marginal returns (β), the wage structure (γ), the coordination technology (δ) and the output level (y). All scenarios have in common that the number of cut-offs increases if δ rises. Considering the discussed effects that a change of n causes, a rise of δ also implies a growing specialization of the working groups. As stated above, the cut-offs of an optimal arrangement are distinct and each additional cut-off reduces the total wage costs. Thus with a rising δ the marginal costs of coordination decline to the point where they are smaller than the marginal reduction of the total wage costs making it beneficial to introduce a new cut-off. The only difference between the scenarios is how strongly a change of δ affects the optimal number of cut-offs. Since an increase of α, β and y increases the number of required working hours at each skill level compared to those of the baseline case, the wage reduction of each additional cut-off refers to a bigger group of workers, thus it is optimal to differentiate the skill range more for the same value of δ. In contrast, in the case of an increase of γ, the firm increases the number of working groups slower than in the baseline scenario. Although the number of required working hours are the same, the firm can reduce the total wage costs less with each additional cut-off since now the wages are lower at each skill level. Therefore, the coordination costs have to decline more, that is, δ has to rise to a higher level before increasing the number of working groups becomes beneficial.

With n being endogenous it is also insightful to briefly look at scenarios where δ is fixed and one of the other parameters continuously changes to understand the way they, beside δ, influence the firms optimal organizational structure $(\widehat{\boldsymbol{\phi}}^{\star}, n^{\star})$. Regarding the number of working groups, as described, an increase of α, β and y reduces their optimal number while in the case of an increase of γ, it shrinks. Thus the main difference to a change of δ concerns $\widehat{\boldsymbol{\phi}}^{\star}$. In the case of an alteration of δ the values of the optimal cut-offs only shift when the number of working groups changes. In contrast in the instance of a change of the other parameters they, as illustrated in step one, continuously alter also in between the discrete jumps that are caused by a change of n.

Step 3: The optimal output level $y^{\star}$

Despite that the model analyzes a single firm and assumes fixed wages and a constant price, the results of the simulations reveal causal relations that would also affect the behavior of a firm that is dealing with flexible wages and a varying price for the final good. This gives at least some qualitative insights how such a firm makes decisions. For all three scenarios in Figure 3.8 exists at each level of δ an optimal output level. Recalling the general structure of the production function, there is always a skill level above which increasing ($h(\phi) > 1$) and below diminishing ($h(\phi) < 1$) marginal returns occur. In the case of an increase of y, the first will reduce and the latter increase the marginal costs of production. Since with higher levels of y the effect of the part of the skill range with diminishing marginal returns increasingly dominates, the marginal costs are also increasing. Thus at some level of y they equal the fixed price.

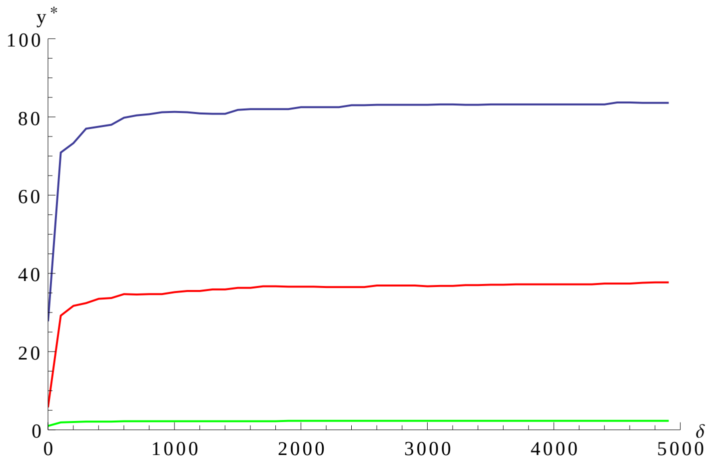

Figure 3.8: The optimal output level (red $\alpha = 2$, green $\beta = 4$)

The different scenarios also have in common that already after small changes of δ the firm's output reaches an almost constant level, implying that the number of working groups continuous to rise. This is pictured in Figure 3.9. This is because with an additional cut-off the firm can reduce the wage costs and, accordingly, the marginal costs, which makes it profitable to increase the output level. This again means the wage reduction effect of an additional cut-off affects more workers while the additional coordination costs do not depend on y. Thus n and y mutually reinforce but while for a low number of working groups each additional cut-off reduces the wages for some workers drastically once the skill range is more differentiated the effects of an additional cut-off become marginal as does the reinforcement. Therefore, also the continuing decrease of the coordination costs and the resulting increase of n have almost no effect on the optimal output level.

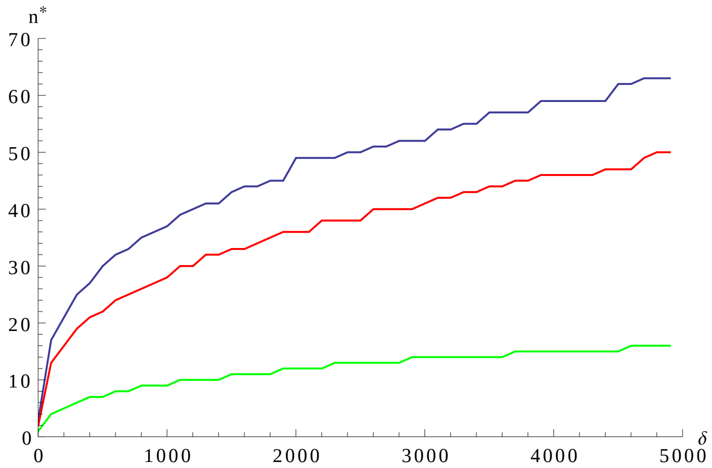

Figure 3.9: red $\alpha = 2$, green $\beta = 4$

A third and distinctive feature is that in all scenarios the output converges to different values. Since in the cases of $\alpha = 2$ and $\beta = 4$, production is more labor-intensive, thus more costly, the output level of the baseline scenario is higher. Considering $\beta = 4$, the increase of β causes besides a rise of the labor intensity additional lower scale effects at all skill levels. Therefore, β extents the part of the skill range that exhibits diminishing marginal returns and shifts the marginal costs upwards which explains why the output level is particularly low.

3.5 Technological change and income inequality

Finally, this section is dedicated to the second research question, how does a variation of the parameters, i.e. of the functional form of f, w and the output level y in connection with technological change, affects the incomes of the firm's workforce. Since the change of the whole income distribution is of interest, a measure of statistical dispersion is needed. The Gini coefficient[15] was developed for this purpose and is also the most

[15] See Gini (1912).

applied one. The following equation determines the Gini coefficient of the firm for a continuous change of δ (technological change)

$$\begin{aligned} Gini &= \frac{0.5 - B}{0.5} \text{ with} \\ B &= \sum_{i=1}^{n} \frac{\frac{1}{2}\sum_{j=1}^{i-1}[F(\phi_j) - F(\phi_{j-1})]w(\phi_i) + \frac{1}{2}\sum_{j=1}^{i}[F(\phi_j) - F(\phi_{j-1})]w(\phi_i)}{\sum_{j=1}^{n}[F(\phi_i) - F(\phi_{i-1})]w(\phi_i)} \\ &\quad \cdot \frac{F(\phi_i) - F(\phi_{i-1})}{F(1)} \end{aligned} \tag{3.14}$$

In Figure 3.10 the Gini coefficients of the above scenarios are illustrated.[16] All of them have two features in common. First, the Gini coefficients decreases rapidly, that is, the income inequality increases. Secondly they converge to scenario-specific values. To understand those phenomena one has to take a look at Figures 3.5 and (3.7). On the one hand the number of cut-offs is fast increasing in δ and on the other hand a cut-off vector with more cut-offs exhibits qualitatively the same arrangement as one with less cut-offs. Therefore, the more cut-offs are optimal the more the firm differentiates all parts of the skill range, hence the more specialized all workers become. Moreover, since with each additional cut-off the total wage costs decrease, the higher n and the lower the skill level of an activity the bigger the part of the wage reduction that the worker, who conducts that activity, has to burden. Thus the higher the skill level of a worker the bigger the share of the total wage bill that this worker gets. This explains the general worsening of the Gini coefficient. Starting from $n = 1$, the case of equality (Gini = 0), the first additional cut-offs lead to big wage changes effecting big working groups, what causes the rapid increase of the Gini coefficient for small δ's. With increasing n the change of the wages and the working groups become smaller so does the change of the Gini coefficient. In the limit workers perform only activities of their individual skill level and gain a corresponding wage, thus the unequally distributed wage reduction reaches the maximum and the Gini coefficient the convergence value.

The differences between those values are caused by a change of the proportion of the relative share of the total workforce and the relative share of the total wage bill for the workers at each skill level. In case of an increase of β and y workers of all skill

[16] The Gini coefficients are computed for a given output level (y) but for the optimal number of cut-offs ($n^\star$) and the optimal respective values of the cut-offs ($\widehat{\boldsymbol{\phi}}^\star$).

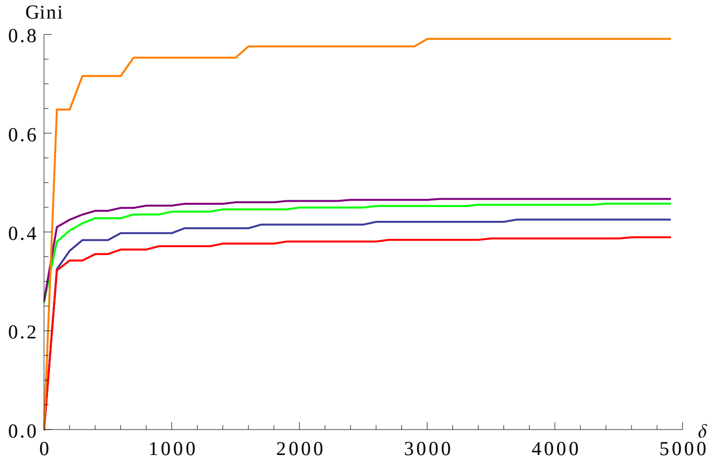

Figure 3.10: The Gini coefficient (red $\alpha = 2$, green $\beta = 4$, orange $\gamma = 4$, purple $y = 5$)

levels are more demanded. Despite, as the normalized antiderivatives show, that the relative share of the total workforce of workers with lower skill levels increases, because of the increasing wage function, their relative share of the total wage bill is decreasing. An increase of α causes the opposite effect. In case of γ the change of the convergence value is entirely driven by the changes of the relative shares of the wage bill of the workers at each skill level. An increase of γ decreases the wages for all workers but proportionally especially for those with lower skill level. Thus their relative share of the total wage bill decreases.

Thus one can conclude that depending on the functional form of the production function and the wage function a firm has a specific inherent maximum level of income inequality. The difference between this value and the actual level of income inequality constitutes a potential of further deterioration. Technological progress in turn only activates, through the change of the organizational structure, this potential step by step. Therefore, technological change that affects the coordination costs causes only indirectly an increase of the income inequality concerning a firm's workforce. In contrast technological progress that would change the productivity of the workers, thus causing a change of the form of $g(\phi)$ and $h(\phi)$, would directly alter the firm's maximum level of income inequality. Such kind of technological change is usually assumed in the task-based models.

3.6 Conclusion

In the beginning I raised the following questions. First, what influence does technical change have on the way a firm organizes its operations and, second, how does such change affect the incomes of the work force, more precisely does it make the firm-specific income distribution more or less equal. Regarding the first question, the model showed that in the event of technological progress that reduces the coordination costs, a firm reorganizes its operations. That is, it increases the number of working groups and reallocates the activities among them. Whereby the firm differentiates not all parts of the skill range equally but in a way that with increasing number of working groups all workers become more specialized. Which parts of the skill range are more and which less differentiated, depends on the functional form of the production function and on the assumed labor market outcome.

In a nutshell, concerning the second question, technological change increases the income inequality within a firm. With every additional working group the income distribution becomes more unequal. At first this happens rapidly while with increasing degree of differentiation the Gini coefficient changes in smaller steps. In the limit it converges to a value that is determined by the functional form of the production and the wage function.

3.7 Appendix

Monotonicity properties of f:

Since $g(\phi) > 0$, $h(\phi) > 0$ for $\phi \in [0, \bar{\phi}]$ and $y \in \mathbb{R}_+, \theta > 1$

- $f_\phi(\phi, y) = -\frac{(y+\theta)^{\frac{1}{h(\phi)}}\left(h(\phi)^2 g_\phi(\phi) + g(\phi)\log(y+\theta)h_\phi(\phi)\right)}{g(\phi)^2 h(\phi)^2} < 0$
- $f_y(\phi, y) = \frac{1}{g(\phi)} \cdot \frac{1}{h(\phi)} \cdot (y+\theta)^{\frac{1}{h(\phi)}-1} > 0.$

Derivation of L^d:

The condition for the functions l^d follows step by step from the following considerations: First, all tasks which need a skill level between $\widehat{\phi}_{n-1}$ and $\widehat{\phi}_n$ can be done by workers with a skill level between $\widehat{\phi}_n$ and 1. Therefore, the aggregated demand for workers with skill level in $(\widehat{\phi}_n, 1]$ must be at least as high as the amount of working time units belonging to skill levels in $(\widehat{\phi}_{n-1}, \widehat{\phi}_n]$. Hence, we have that

$$\int_{\widehat{\phi}_n}^{1} l^d(\phi) d(\phi) \geq \int_{\widehat{\phi}_{n-1}}^{\widehat{\phi}_n} f(\phi, y) d(\phi).$$

In a second step, we that the firm needs enough workers of skill level $(\widehat{\phi}_{n-1}, 1]$ to compensate the workers they need with skill level $(\widehat{\phi}_{n-2}, \widehat{\phi}_{n-1}]$. Hence, the demand has to fulfill

$$\int_{\widehat{\phi}_{n-2}}^{\widehat{\phi}_{n-1}} f(\phi, y) d(\phi) \leq \int_{\widehat{\phi}_n}^{1} l^d(\phi) d(\phi) - \int_{\widehat{\phi}_{n-1}}^{\widehat{\phi}_n} f(\phi, y) d(\phi) + \int_{\widehat{\phi}_{n-1}}^{\widehat{\phi}_n} l^d(\phi) d(\phi)$$

where the first difference is the number of workers of the first step, which are not used for activities with skill level in $(\widehat{\phi}_{n-1}, \widehat{\phi}_n]$.

Rearranging terms leads to

$$\int_{\widehat{\phi}_{n-1}}^{1} l^d(\phi) d(\phi) \geq \int_{\widehat{\phi}_{n-2}}^{\widehat{\phi}_n} f(\phi, y) d(\phi).$$

Following this procedure, we obtain that the demand-function has to fulfill

$$\int_{\widehat{\phi}_i}^{1} l^d(\phi) d(\phi) \geq \int_{\widehat{\phi}_{i-1}}^{\widehat{\phi}_n} f(\phi, y) d(\phi)$$

for all $i \in \{1, \ldots, n\}$.

Proof of Lemma 1:

Proof. We begin to show the existence of a minimizing $\widehat{\boldsymbol{\phi}}^\star$. Assume y and n to be fixed. The optimal cutoff $\widehat{\boldsymbol{\phi}}^\star$ is an element in $[0, \bar{\phi}]^{n-1} \times \{\bar{\phi}\}$. Since this set is compact, we can apply Weierstrass' Theorem and obtain that the element exists.
Furthermore, we need to show that $\widehat{\phi}^\star_{i-1} < \widehat{\phi}^\star_i$ for all $i \in \{1 \ldots n\}$. For simplicity, we look at the case of $n = 3$. Assume that we have an optimal vector $\widehat{\phi} = (0, \widehat{\phi}_1, \widehat{\phi}_2, \bar{\phi})$ such that $\widehat{\phi}_1 > \widehat{\phi}_2$ and $\bar{\phi}$ resp. 0 are the given boundaries. Let further be $\tilde{\phi} := (0, \tilde{\phi}_1, \tilde{\phi}_2, \bar{\phi}) = (0, \widehat{\phi}_2, \widehat{\phi}_1, \bar{\phi})$ a second optimal vector with reversed order. We compare the corresponding costs, for a fixed y:

$$\begin{aligned}
c^w(\widehat{\phi}; 3, y) &= (F(\widehat{\phi}_1, y) - F(0, y)) \cdot w(\widehat{\phi}_1) + (F(\widehat{\phi}_2, y) - F(\widehat{\phi}_1, y)) \cdot w(\widehat{\phi}_2) \\
&\quad + (F(\bar{\phi}, y) - F(\widehat{\phi}_2, y)) \cdot w(\bar{\phi}) \\
&= (F(\widehat{\phi}_1, y) - F(\widehat{\phi}_2, y)) \cdot w(\widehat{\phi}_1) + (F(\widehat{\phi}_2, y) - F(0, y)) \cdot w(\widehat{\phi}_1) \\
&\quad + (F(\widehat{\phi}_2, y) - F(\widehat{\phi}_1, y)) \cdot w(\widehat{\phi}_2) + (F(\bar{\phi}, y) - F(\widehat{\phi}_1, y)) \cdot w(\bar{\phi}) \\
&\quad + (F(\widehat{\phi}_1, y) - F(\widehat{\phi}_2, y)) \cdot w(\bar{\phi}) \\
&= (F(\widehat{\phi}_2, y) - F(0, y)) \cdot w(\widehat{\phi}_1) + (F(\widehat{\phi}_1, y) - F(\widehat{\phi}_2, y)) \cdot w(\bar{\phi}) \\
&\quad + (F(\bar{\phi}, y) - F(\widehat{\phi}_1, y)) \cdot w(\bar{\phi}) + (F(\widehat{\phi}_1, y) - F(\widehat{\phi}_2, y)) \cdot (w(\widehat{\phi}_1) - w(\widehat{\phi}_2)) \\
&\geq (F(\widehat{\phi}_2, y) - F(0, y)) \cdot w(\widehat{\phi}_2) + (F(\widehat{\phi}_1, y) - F(\widehat{\phi}_2, y)) \cdot w(\widehat{\phi}_1) \\
&\quad + (F(\bar{\phi}, y) - F(\widehat{\phi}_1, y)) \cdot w(\bar{\phi}) + 0 \\
&= c^w(\tilde{\phi}; 3, y)
\end{aligned}$$

where we use that the wage function w is strictly increasing. Additionally, we observe that $\phi^\star_i \neq \phi^\star_j$ for all $i \neq j$ with $i, j \in \{1 \ldots n-1\}$ holds. This is clear since for $\phi^\star_i = \phi^\star_{i+1}$ an increase of $\phi^\star_{i+1}$ would decrease costs $-$ a contradiction to the optimality condition. The same idea can be used to generalize for the case $n > 3$. Therefore, we know that there exists a minimizing $\widehat{\phi}^\star$ such that $\widehat{\phi}^\star_{i-1} < \widehat{\phi}^\star_i$.
Equation (3.12) follows now immediately by taking the derivative and setting it equal to zero. Here we use that $\widehat{\phi}^\star_i \in (0, \bar{\phi})$, $i = 1, \ldots, n-1$ and therefore boundary solutions

are excluded. □

Proof of Lemma 2:

Proof. Assume we introduce a new cut-off $\widehat{\phi}' \in (\widehat{\phi}_{j-1}, \widehat{\phi}_j)$. We want to show that it reduces wage costs. The wage costs for this skill interval without the new cut-off are:

$$(F(\widehat{\phi}_j, y) - F(\widehat{\phi}_{j-1}, y)) \cdot w(\widehat{\phi}_j)$$

Introducing the cut-off leads to:

$$(F(\widehat{\phi}', y) - F(\widehat{\phi}_{j-1}, y)) \cdot w(\widehat{\phi}') + (F(\widehat{\phi}_j, y) - F(\widehat{\phi}', y)) \cdot w(\widehat{\phi}_j)$$

Taking the difference between the two costs simplifies to:

$$(F(\widehat{\phi}', y) - F(\widehat{\phi}_{j-1}, y)) \cdot (w(\widehat{\phi}_j) - w(\widehat{\phi}'))$$

This term is strictly positive since F and w are strictly increasing functions. We see that introducing the new cut-off reduces the costs. For its uniqueness we look at the optimality condition:

$$(F(\widehat{\phi}', y) - F(\widehat{\phi}_{j-1}, y)) \cdot w_{\phi'}(\widehat{\phi}') = (w(\widehat{\phi}_j) - w(\widehat{\phi}')) \cdot f(\widehat{\phi}', y)$$

Since the right-hand side is strictly increasing in $\widehat{\phi}'$ and the left-hand side strictly decreasing, there exists exactly one solution. □

Proof of Lemma 3:

Proof. To prove the existence of $\widehat{\phi}_3$ we need a solution for the following equation, given $\widehat{\phi}_1, \widehat{\phi}_2$:

$$(F(\widehat{\phi}_2, y) - F(\widehat{\phi}_1, y)) \cdot w(\widehat{\phi}_2) = (w(\widehat{\phi}_3) - w(\widehat{\phi}_2)) \cdot f(\widehat{\phi}_2, y).$$

Solving for $\widehat{\phi}_3$, this simplifies to:

$$w(\widehat{\phi}_3) = [(F(\widehat{\phi}_2, y) - F(\widehat{\phi}_1, y)) \cdot w(\widehat{\phi}_2) + w(\widehat{\phi}_2) \cdot f(\widehat{\phi}_2, y)] : f(\widehat{\phi}_2, y)$$

Since w is a strictly increasing function and the right-hand side is a constant real number, this equations has one and only one solution. However, we see that this solution does not necessarily lie in the interval $(\widehat{\phi}_2, \bar{\phi}]$, and thus we state that on this interval there exists at most one optimal cut-off. The same arguments hold for the symmetric case with $\widehat{\phi}_3 \in [0, \widehat{\phi}_1)$. □

Normalized antiderivative of f for different values of α:

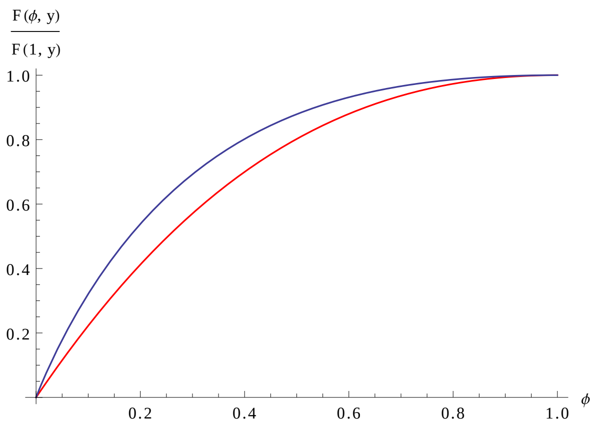

Figure 3.11: blue $\alpha = 0.5$, red $\alpha = 2$

Normalized antiderivative of f for different values of β:

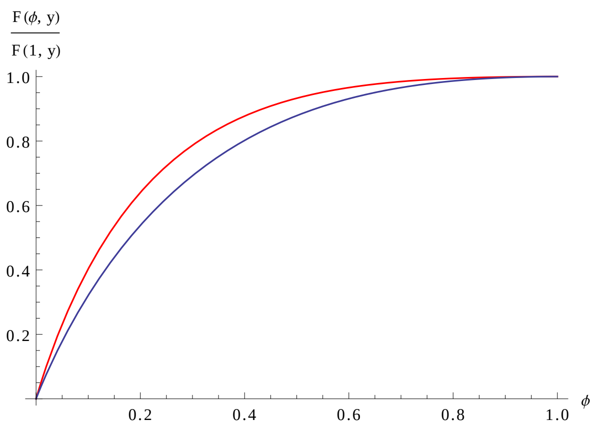

Figure 3.12: blue $\beta = 2$, red $\beta = 4$

Normalized antiderivative of f for different values of y:

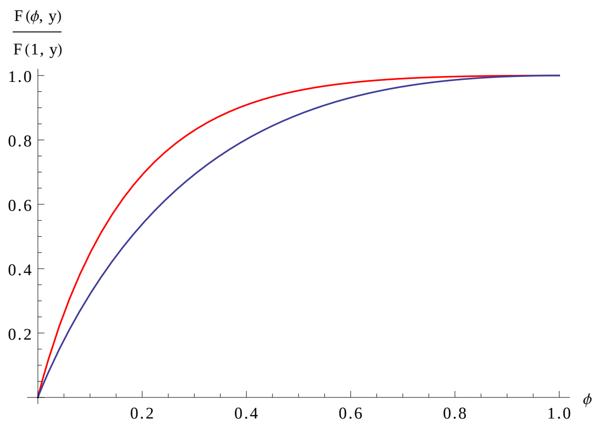

Figure 3.13: blue $y = 1$, red $y = 5$

Description of the simulation algorithm:

For the first part the algorithm coincides with the procedure based on Lemma 3 that the firm is assumed to apply to reorganize. Thus, for each run n and y are fixed. The first cut-off $\widehat{\phi}_1$ (for $\widehat{\boldsymbol{\phi}} = (\widehat{\phi}_1, \ldots, \widehat{\phi}_n)$ and $n > 1$) is set, depending on the step size, at a value marginal bigger than zero. The other cut-offs are set according to the first order conditions that are solved for the respective cut-off value. Then the first cut-off is constantly raised until $\widehat{\phi}_n$ is in an ϵ-interval around $\bar{\phi}$. Despite the computed approximation of an optimal cut-off distribution, the algorithm continues until $\widehat{\phi}_1 = \widehat{\phi}_1'^{\star}$ (for $\widehat{\boldsymbol{\phi}}'^{\star} = (\widehat{\phi}_1'^{\star}, \widehat{\phi}_2'^{\star})$) to test whether further cut-off vectors that fulfill the optimality conditions exist. The algorithm stops at $\widehat{\phi}_1'^{\star}$ because from Lemmas 1 to 3, it follows that $\widehat{\boldsymbol{\phi}}'^{\star}$ exists, that it is unique and that not all elements of $\widehat{\boldsymbol{\phi}}$ (for $n > 2$) can be bigger than $\widehat{\phi}_1'^{\star}$ and equal or smaller than $\bar{\phi}$. The simulations show that for all $\widehat{\phi}_n$ which are in an ϵ-interval around $\bar{\phi}$, the corresponding $\widehat{\phi}_1$ form a convex subset $\Phi_\epsilon \in (0, \bar{\phi})$. The results of all simulated combinations of n and y also show that if $\epsilon' < \epsilon$, then $\Phi_{\epsilon'} \subset \Phi_\epsilon$. All this together suggests that in the introduced general set-up exists only one optimal cut-off vector for each parameter combination.

Bibliography

Daron Acemoglu. Technical change, inequality, and the labor market. *Journal of Economic Literature*, 40(1):7–72, 2002.

Daron Acemoglu and David Autor. Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of Labor Economics*, volume 4, pages 1043–1171. Elsevier, 2011.

Anthony B Atkinson, Thomas Piketty, and Emmanuel Saez. Top incomes in the long run of history. *Journal of Economic Literature*, 49(1):3–71, 2011.

Tony Atkinson, Joe Hasell, Salvatore Morelli, and Roser Max. *The Chartbook of Economic Inequality.* Institute for New Economic Thinking at the Oxford Martin School, 2017.

David H Autor and Michael J Handel. Putting tasks to the test: Human capital, job tasks, and wages. *Journal of Labor Economics*, 31(S1):S59–S96, 2013.

David H Autor, Frank Levy, and Richard J Murnane. The skill content of recent technological change: An empirical exploration. *The Quarterly Journal of Economics*, 118(4):1279–1333, 2003.

David H Autor, Lawrence F Katz, and Melissa S Kearney. The polarization of the US labor market. *American Economic Review*, 96(2):189–194, 2006.

David H Autor, Lawrence F Katz, and Melissa S Kearney. Trends in US wage inequality: Revising the revisionists. *The Review of Economics and Statistics*, 90(2): 300–323, 2008.

Gary S Becker and Kevin M Murphy. The division of labor, coordination costs, and knowledge. *The Quarterly Journal of Economics*, 107(4):1137–1160, 1992.

Eli Berman, John Bound, and Stephen Machin. Implications of skill-biased technological change: international evidence. *The Quarterly Journal of Economics*, 113(4): 1245–1279, 1998.

David Card and John E DiNardo. Skill-biased technological change and rising wage inequality: Some problems and puzzles. *Journal of Labor Economics*, 20(4):733–783, 2002.

Corrado Gini. Variabilità e mutabilità (variability and mutability). *Reprinted in Memorie di metodologica statistica (Ed. Pizetti E, Salvemini, T). Rome: Libreria Eredi Virgilio Veschi (1955) ed. Bologna*, 1912.

Lawrence F Katz and Kevin M Murphy. Changes in relative wages, 1963–1987: supply and demand factors. *The Quarterly Journal of Economics*, 107(1):35–78, 1992.

Per Krusell, Lee E Ohanian, José-Víctor Ríos-Rull, and Giovanni L Violante. Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5):1029–1053, 2000.

Kevin M Murphy and Finis Welch. The structure of wages. *The Quarterly Journal of Economics*, 107(1):285–326, 1992.

Thomas Piketty. *Capital in the Twenty-First Century*. The Belknap Press of Harvard University Press, 2014.

Thomas Piketty and Emmanuel Saez. Income inequality in the United States, 1913–1998. *The Quarterly Journal of Economics*, 118(1):1–41, 2003.

Chapter 4

Working time allocation and the allocation of work

4.1 Introduction

It is well documented that in western countries the daily working time decreased substantially during the last century.[1] In the last decades it seems, even so that the general trend is still in place, that the change of the daily working is unequally distributed. This makes the labor force more and more heterogeneous in terms of the time they spend at work. Nowadays there are workers who work only a couple of hours each day and often even have to do more than one job. At the same time unions, representing a huge part of the labor force, demand to reduce the weekly working time step by step. Also there is a small but increasing group of workers with very high workload accumulating extra hours on a scale not know before. Additionally, workers who fall into the last category pursue professions that comprise various highly interdependent tasks. Such workers are for example managers, programmers, designers or consultants. While workers of the first category are for example employed in the low paid service sector conducting simple routinized tasks. Another phenomenon of the recent time is that because of new technological trends and innovations like digital transformation or cloud computing it becomes easier to coordinate workers even in large numbers.[2] Which raises the issue of the economic impact of those developments on the labor force in terms of working time and income.

The contribution of this work is to examine the connection of those phenomena, the heterogeneity in the daily working time of various professions, which is correlated with the structure of the work processes inherent to those professions, and the decreasing coordination efforts caused by technological progress. I illustrate and analyze the underlying mechanisms from the perspective of a single firm and the corresponding workforce. For this purpose I develop a model of a firm that is confronted with two decision problems. First, it has to allocate the daily working time between tasks that a single

[1] See OECD (1991) and (OECD 1999).

[2] There is a large literature examining the influence of information technology on organizational coordination costs and firm productivity. See, among others, Clemons et al. (1993), Cordella and Simon (1997), Cordella (2009), Chen et al. (2016).

worker is supposed to conduct.[3] And second, the firm has to determine its optimal combination of workers and working time.[4] Throughout the model the following three research questions serve as a guideline. First, what characterizes the production function of the firm and the generating two subtasks. Second, which optimal combination of number of workers and working time result from different subtasks characteristics. And third, which effect does coordination effort reducing technological progress have on firms production and thereby on the income situation of the workers.

The paper is organized in the following way. In section two the production function is determined and characterized. Section three introduces at first the specific form of the coordination efforts and afterwards the firms optimization problem, how to find the optimal combination of number of workers and working time. In section four the optimization problem is in the scope of an parametrized example numerically solved. Therefore, section four is dedicated to the second research question, while section five is to the third. The paper is concluded by section six.

4.2 Production as a combination of two tasks

In this work, as usual in the task-based literature, the operations of a firm are understood as a set of tasks which have to be performed,[5] in order to produce a certain good. Each of those tasks demands a specific set of skills that a worker needs to poses that is expected to perform the task. Furthermore, these sets are supposed to be exclusive in the sense that there are not two sets with one being a subset of the other. This makes sense considering that a skill might be beside basic physical and cognitive abilities also a task specific method, a work procedure or factual knowledge. Therefore each of those tasks, or the corresponding skill sets, defines a certain occupation. Additionally it is assumed that there are homogeneous groups of workers whose skills fit perfectly to one of the occupations. This at first restrictive appearing assumption is reasonable once

[3]Empirical research examples include Midttun (2007) that analyzes time-allocation for medical specialists between patient assignments, administrative and educational tasks, or Stoikov (1964) that searches for optimal research/teaching allocation of scientific effort.

[4]This trade-off usually occurs because of fix cost considerations, see (Cahuc et al. 2014).

[5]See e.g. Autor et al. (2003), Acemoglu and Autor (2011) and Becker and Murphy (1992).

taking into account that step by step most people get prepared, based on their innate abilities, for a certain kind of occupation while passing through the education system. Each of the basic tasks itself is a set of several activities that a worker has to conduct. To keep things simple those activities are grouped into two subtasks, a preparation task and a core task. The first embraces all activities that enable a worker to conduct the core task. Performing the latter leads to the final output of the task.
Consequently, a worker or rather the employing firm has to decide, first, what is the optimal daily working time T of a worker, and second how to allocate the daily working time between the preparation task and the core task best in order to produce an optimal output level of the overall task. While the first decision problem is the concern of a later chapter, the latter is discussed in the following one. Finding the optimal allocation of T coincides with the determination of the optimal production function $y(T)$ of a specific task or of a single worker conducting this task. Before discussing this in more detail let us have a look on the general form of the production function (4.1) for an arbitrary allocation of a given T.

$$\tilde{y}(t_1, t_2) = \alpha \cdot f(t_1) \cdot g(t_2) \;\text{ s.t. }\; t_1 + t_2 = T \;\text{ for }\; \alpha, T > 0 \tag{4.1}$$

The function $f : \mathbb{R}_+ \rightarrow [0, 1)$ corresponds to the preparation task. It assigns to every amount of working time t_1 a value $f(t_1)$ that states the activated support potential of the preparation task. As the examples above illustrate one has to think of a broad notion of preparation procedures with very different measure units. Assuming that there is for each of these procedures a maximum support level that a worker is able to activate, one can normalize its range to the interval $[0, 1]$. The actual impact level of the preparation task on the core task is captured by the technology parameter α. Thus a change of a feature that enables a worker to prepare better e.g. rearranged workplaces at an assembly line or additional information like reports or statistical data in case of an manager is expressed by an increase of α. While if workers are able to reach the same support potential in less time this would be captured by a change of f. This leads to the functional form of f. To begin with, the function f is twice differentiable in t_1 and satisfies

$$f'(t_1) \equiv \frac{\partial f(t_1)}{\partial t_1} > 0, \quad f''(t_1) \equiv \frac{\partial^2 f(t_1)}{\partial {t_1}^2} < 0 \text{ and} \tag{4.2}$$

$$f(0) = 0, \quad \lim_{t_1 \to \infty} f(t_1) = 1 \text{ and} \tag{4.3}$$

$$f'(0) = a \in \mathbb{R}_{++}. \tag{4.4}$$

Condition (4.2) assures that the marginal product of t_1 is positive and diminishing. In case of standard production functions (e.g. the Cobb-Douglas production function) it is argued that diminishing marginal products concerning input factors as capital and labor occur because an increase of a single factor, while keeping the others constant, implies that the factors are not deployed in the optimal proportion anymore. Or if there is only one factor a higher input of it increases also complexity of production overproportionally causing for example higher coordination efforts, lower utilization rates or longer downtimes. Such effects can not give rise to a diminishing marginal product in case of f since its only argument is working time of a single worker. But there are other reasons why condition (4.2) is an appropriate approximation of the real processes taking place. On the one side, concerning the actual activities conducted, for technical reasons the longer a worker prepares her work place or analyses certain data the less improvements or the less useful information she can gain from it. On the other side, taking physiological constraints into account, the productivity of workers decreases during the day because they simply become tired. Both effects depend strongly on the specific task a worker performs. Moreover, from the first part of (4.3) follows that there is no given support potential without investing effort in form of working time. This makes the preparation task essential. The second part assures that a worker is able to almost activate the entire support potential as long as she spend enough time on the preparation task. Assumption (4.4) is needed for technical reasons.

The following standard assumptions are imposed on the core task denoted by $g : \mathbb{R}_+ \to \mathbb{R}_+$ which is twice differentiable and satisfies

$$g'(t_2) \equiv \frac{\partial g(t_2)}{\partial t_2} > 0, \quad g''(t_2) \equiv \frac{\partial^2 g(t_2)}{\partial {t_2}^2} < 0 \text{ and} \tag{4.5}$$

$$\lim_{t_2 \to \infty} g(t_2) = \infty, \quad \lim_{t_2 \to \infty} g'(t_2) = 0 \text{ and} \tag{4.6}$$

$$g(0) = 0. \tag{4.7}$$

Assumption (4.5) implies positive and diminishing marginal returns. Obviously, what is true for the preparation task is also true for the core task, the longer a worker performs the core task the less productive she becomes. Beside this physiological effect there are other mechanisms that influence the productivity change in the course of the working time spend on the core task. For example learning effects might, especially in the case of a cognitive demanding core task, affect the productivity positively. But it is assumed that the fatigue effect is increasingly dominating. Condition (4.7) demands that at least a small amount of T has to be spend on the core task in order to have an output bigger than zero. While assumption (4.6) is again owed to technical reasons.

After specifying the subtasks we can come back to the actual decision a firm is confronted with, how to allocate a given amount of time T optimally between the preparation and the core task in terms of a single worker. Thus the firm has to solve the following optimization problem

$$\max_{t_1} \tilde{y}(t_1, T - t_1) = \max_{t_1} \alpha \cdot f(t_1) \cdot g(T - t_1). \tag{4.8}$$

Differentiating (4.8) with respect to t_1 leads to

$$\frac{\partial \tilde{y}(t_1, T - t_1)}{\partial t_1} = \alpha \cdot f'(t_1) \cdot g(T - t_1) + \alpha \cdot f(t_1) \cdot g'(T - t_1)(-1). \tag{4.9}$$

Equating (4.9) to zero and solving for t_1 gives the optimal working time ($t_1^\star$) for the preparation task.

Lemma 1: For every $T > 0$ there is a unique $t_1^\star, t_2^\star > 0$ maximizing (4.1) *and $t_1^\star$ as well as $t_2^\star$ are unbounded and strictly increasing in T.*
Proof: See appendix.

Thus the optimal working time $t_1^\star$ is a function over T. Considering that T is the actual variable of interest for a firm to set up its production by plugging $t_1^\star(T)$ into (4.1) one expresses the optimal production of a single worker also as a function depending on T.

$$\tilde{y}(t_1^\star, T - t_1^\star) = y(T) = \alpha \cdot f(t_1^\star(T)) \cdot g(T - t_1^\star(T)) \tag{4.10}$$

To keep things simple, throughout the following chapters, I assume that a firm performs only one main task, hence it employs a homogeneous workforce. Furthermore assuming that the firm exhibits constant return to scale in the number of workers N firms overall production function is expressed by

$$Y(T, N) = y(T) \cdot N, \text{ for } N \geq 0. \tag{4.11}$$

Looking at (4.11) brings two questions to mind. First, what is the optimal $N - T$ combination, thus how many worker should a firm hire and how long should they be supposed to work each day? Or in other words should a firm employ less workers who worker longer each day or is the other way around the better alternative. And second, which general characteristics does Y have and, based on that, which qualitative conclusions can be drawn concerning the first question?

Lemma 2: Let ϵ_f and ϵ_g be the elasticities of the task based production functions f and g. If $\epsilon_g + \epsilon_g > 1$, then $y(T)$ is s-shaped in an environment of the origin.
Proof: See appendix.

Lemma 2 gives already a hint which $N - T$ combination are reasonable to consider as being potentially optimal. If the additional conjecture is made that Y exhibits only one inflection point, then f is s-shaped and thus it exists a unique $\bar{T} > 0$, for all N, for which a line through the origin exists that has a tangent point with Y at $\bar{T}$. Without anticipating results, if the overall costs are linear in T, what is the case as we will see in the next chapter, then $\bar{T}$ constitutes a lower boundary for the set of possible solutions concerning the firms second optimization problem, the $N - T$ trade-off. Therefore, it is crucial to understand how the s-shape of Y, the special feature of this kind of production function, changes when varying the functional properties of the constituting functions f and g. In chapter four as part of the comparative analysis this will be further discussed, but before it is essential to take a look on the cost structure of a firm.

4.3 Coordination effort and firms optimal production

Costs of production:

There are two kinds of costs that the firm has to consider. First of all, the wages of the employees which are expressed by $C^w = NTw_1$, with $w_1 \geq 0$. The wage costs depend on the number of employed workers (N), the given wage (w_1) and on the number of daily working hours (T).

The second source of costs are the efforts of a firm to coordinate the workers. For the coordination of its workforce the firm uses an IT infrastructure that for example consist of a data warehouse, data collecting devices, data processing software and algorithms or an external cloud system only to name a few. The corresponding costs are denoted by

$$\begin{gathered} C^c = E(N, \gamma) \cdot T \cdot w_2, \text{ with } w_2 \geq 0 \text{ and} \\ E_N(N, \gamma) > 0, \ E_{NN}(N; \gamma) \geq 0 \text{ and} \\ E_\gamma(N, \gamma) > 0, \ E_{\gamma\gamma}(N; \gamma) \geq 0. \end{gathered} \tag{4.12}$$

Lets assume a measure capable to capture the capacity of the total IT system or vice versa to express the capacity that is needed. This capacity demands are determined by $E(N; \gamma) \cdot T$, with γ being a technology parameter and $E : \mathbb{R}_+ \times (1, \infty) \to \mathbb{R}_+$. Those capacity demands concerning the IT infrastructure are measured in effective units. Since E is increasing in γ technological progress, captured by a decrease of γ, does not reduce the amount of coordination processes but the number of effective IT units that are needed. Furthermore, it is assumed that the coordination effort is increasingly rising with the number of workers, hence E is convex in N. The workload of a single worker, represented by the working time, effects the amount of coordination effort linear. Moreover, it is assumed that the specific characteristic of the coordination processes in a firm depends on the interplay of the workers and the coordination technology. The specific functional form of E takes that into account. This will be further examined below. The last component is the cost parameter w_2, which captures the given price for an unit of IT infrastructure. The cost parameter can vary between firms with different operations or tasks that are consequently conducted by different kind of workers. This implies that different tasks demand not only differently skilled workers but different coordination technologies, that is, IT infrastructure as well. Thus the price also represents the general technical quality that an IT system has to exhibit regarding a certain firm. Therefore, the capacity demands could be the same comparing two firms but the quality demands, hence the price, might differ.

Optimal production:
Considering the costs and assuming that the price for the final good is given, the firm has to solve the following maximization problem

$$\max_{N,T} \Pi(N,T) = \max_{N,T} p \cdot Y(T) - N \cdot w_1 \cdot T - E(N;\gamma) \cdot T \cdot w_2 \tag{4.13}$$

Solving, implies to find the optimal $N-T$ combination, that is, balancing the trade-off between working time and number of workers. First of all, dividing by p leads to

$$\max_{N,T} \tilde{\Pi}(N,T) = \frac{\Pi(N,T)}{p} = y(T) \cdot N - N \cdot T \cdot \tilde{w}_1 - E(N;\gamma) \cdot T \cdot \tilde{w}_2 \tag{4.14}$$

with $\tilde{w}_2$ ($\tilde{w}_2$) being the proportion of the wages (IT cost parameter) and the price of the final good respectively. Differentiating (4.14) with respect to N and T and afterwards equating to zero yields the following first order conditions

$$0 = \frac{y(T)}{T} - \tilde{w}_1 - E_N(N;\gamma) \cdot \tilde{w}_2 \tag{4.15}$$

and

$$0 = y'(T) - \tilde{w}_1 - \frac{E(N;\gamma)}{N} \cdot \tilde{w}_2. \tag{4.16}$$

Dividing (4.15) by (4.16) yields

$$\frac{N E_N(N;\gamma)}{E(N;\gamma)} = \epsilon_E = \frac{\frac{y(T)}{T} - \tilde{w}_1}{y'(T) - \tilde{w}_1}. \tag{4.17}$$

This means that if the coordination cost function has a constant elasticity with respect to its argument N, then the optimal T will not depend on N or γ at all. To be precise, for every γ there will be an N solving the optimisation problem, while the optimal T will be constant. There exists an optimal T because since the coordination costs are convex in N the corresponding elasticity ϵ_E will be greater than 1. Because for the

optimal T values we have $\frac{y(T)}{T} > y'(T)$, which means that the term on the right hand side of (4.17) will continuously range from 1 to ∞.
Based on the assumptions made so far, for the general case no further conclusions can be drawn, neither about the interplay of N and T nor about the affect that technological progress, thus an increase of γ, has on the optimal $N-T$ combinations.

4.4 An illustrative example

So far, remembering the questions raised in the introduction, the specific form of the production function is characterized. The second and third question concerning the influence of the generating subtasks and that of technological progress on the specific $N-T$ combination will be illustrated in the scope of the following parameterized model.

Firms production function:
Lets assume that a firms preparation task and the core task are expressed by

$$f(t_1) = \frac{t_1}{t_1 + \beta_1} \text{ and } g(t_2) = t_2^{\beta_2} \text{ for } \beta_1 > 0,\ \beta_2 \in (0,1). \tag{4.18}$$

Both f and g fulfilling the conditions stated above respectively. They are monotonously increasing, strictly concave and $f(0), g(0) = 0$. Furthermore, f is bounded at 1. Since g is assumed to be a power function its elasticity (ϵ_g) is constant and equals β_2 while in case of f the elasticity ($\epsilon_f = \frac{\beta_1}{\beta_1 + t_1}$) is 1 at $t_1 = 0$ and strictly decreasing in t_1.
Plugging the parameterized variants of f and g into (4.1) gives the corresponding output function of a single worker.

$$\tilde{y}(t_1, t_2) = \alpha \cdot \frac{t_1}{t_1 + \beta_1} \cdot t_2^{\beta_2} \text{ s.t. } T = t_1 + t_2 \tag{4.19}$$

Substituting t_2 with $T - t_1$, differentiating with respect to t_1, equating the resulting first order condition to 0 and solving for t_1 yields the optimal working time for the preparation task ($t_1^\star(T)$) depending on T. Furthermore, by plugging $t_1^\star(T)$ into (4.19) the production function of a single worker can be expressed as

$$y(T) = \alpha \cdot \frac{t_1^\star(T)}{t_1^\star(T) + \beta_1} \cdot (T - t_1^\star(T))^{\beta_2}, \text{ where} \tag{4.20}$$

$$t_1^\star(T) = -\frac{\beta_1 + \beta_1 \cdot \beta_2}{2\beta_2} + \sqrt{\left(\frac{\beta_1 + \beta_1 \cdot \beta_2}{2\beta_2}\right)^2 + \frac{\beta_1}{\beta_2} \cdot T}. \tag{4.21}$$

Looking at this particular realization of the general model one can show that the statements about the optimal working time for the single sub-tasks and the overall production function made so far hold, beyond that additional conclusions can be drawn. From (4.21) follows that $t_1^\star(T)$ is unbounded and strictly increasing in T, thus it is always optimal if workers spend a certain amount of working time additionally for the preparation task improving the efficiency of the core task, if the total working time is extended. What is true as well for $t_2^\star(T)$. But, differentiating (4.21) with respect to T,

$$\frac{\partial t_1^\star(T)}{\partial T} = \frac{\beta_1 \cdot \beta_2^{-1}}{2\sqrt{\left(\frac{\beta_1+\beta_1\cdot\beta_2}{2\beta_2}\right)^2 + \frac{\beta_1}{\beta_2}\cdot T}}, \tag{4.22}$$

shows that the slope of $t_1^\star$ converges to 0 with increasing T, thus proportionally a worker spends less working time on the preparation and more on the core task. What is not surprising since once $t_1^\star(T)$ gets close to its boundary any additional working time leads only to diminishing changes of the support effect of the preparation task for the core task. This is the case even so that the latter also exhibits diminishing marginal returns, since it is not bounded. Hence the effect of increasing the productivity of the core task through a higher t_1 is increasingly compensated by the output effect that an increase of t_2, that is, of the core task itself has.

The already made conjecture that the production function of a single worker, hence that of the total firm as well, is s-shaped can at least for the specific example at hand be shown. In fact, differentiating (4.20) twice with respect to T, equating to 0 and solving for T yields the inflection point of the production function,

$$\hat{T}_0 = \frac{2\cdot\beta_1\cdot\beta_2 - \beta_1\cdot\beta_2^3}{4(\beta_2-1)^2}. \tag{4.23}$$

Thus it exists for any combination of the parameter β_1 and β_2 only one $T > 0$ that is the root of $y''(T)$. Since lemma 2 applies to the production function, it follows that $y(T)$ is s-shaped.

Coordination-effort functions:

To express the efforts to coordinate the firms workers three different effort functions are considered.

$$(a) \quad E_N(N;\gamma) = N^{\Phi_1} \cdot \gamma^{\Phi_2} \quad with \quad \Phi_1, \Phi_2 \geq 1 \tag{4.24}$$

$$(b) \quad E_N(N;\gamma) = (N+1)^{\gamma} - 1 \tag{4.25}$$

$$(c) \quad E_N(N;\gamma) = (N+1)^{\gamma} + e^{-N\gamma} - 2 \tag{4.26}$$

The first function is strictly convex in its arguments N and γ. It exhibits a constant elasticity with respect to N. Function (b) is also strictly convex in both N and γ if $\gamma > 1$ and $N > 0$. Function (c) is very similar to function (b). However, (c) has the property $\frac{\partial E_N}{\partial N}\big|_{N=0} = 0$ while for (b) we have $\frac{\partial E_N}{\partial N}\big|_{N=0} = \gamma$.

Firms optimal N-T combination:

Firms optimize

$$\tilde{\Pi}(N,T) = \alpha \cdot \frac{t_1^{\star}(T)}{t_1^{\star}(T) + \beta_1} \cdot (T - t_1^{\star}(T))^{\beta_2} N - \tilde{w}_1 T N - \tilde{w}_2 T E_N(N;\gamma). \tag{4.27}$$

In this profit function we plug in the three different functions for the coordination effort described above. In Figure 4.1 three different graphs using the above cost functions are shown, where N is plotted over T while β_1 is varied. All plots exhibit qualitatively the same pattern when β_1 and thus the elasticity of $f(t_1)$ is increased. A higher β_1, corresponding to a higher amount of time needed for the preparation task before working gets efficient, leads the optimal time per worker to go up and the optimal number of workers to go down. This makes sense because as workers have to invest more time to prepare, they become more expensive wich makes a larger workforce less attractive.

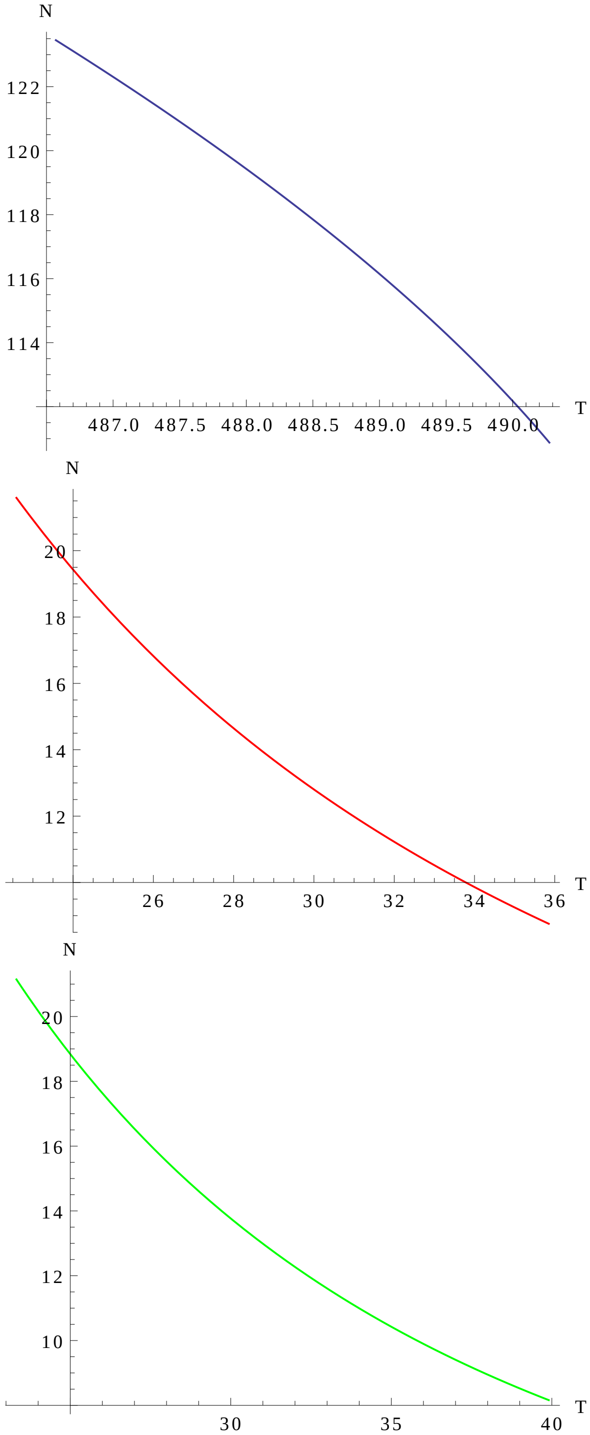

Figure 4.1: Optimal N-T combinations using the effort functions (a), (b) and (c) [with $\Phi_1 = \Phi_2 = 2$ and parameters $\tilde{w}_1 = \tilde{w}_2 = 0.01$, $\beta_2 = 0.4$ and a range of β_1 between 5 and 10

4.5 How does technological change affects the firm and the work force?

In this section the effect of technological change on the size of the workforce and its optimal working time are discussed. In the considered model technological change is primarily described by the parameter γ which controls the effort needed to coordinate a workforce of a specific size. Figures 4.2 to 4.6 discuss the effect of a γ variation in the N-T space, always combining two different curves in order to gain an understanding of the influence of the other model parameters.

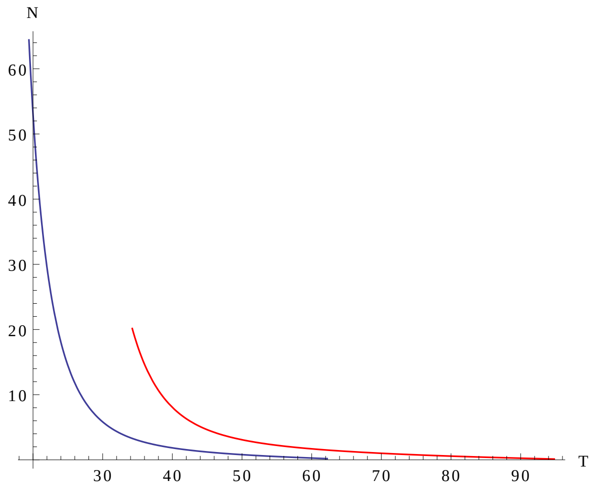

Figure 4.2: Optimal N-T combinations using effort function (c) and parameters $\tilde{w}_1 = \tilde{w}_2 = 0.01$, $\beta_1 = 5$ (blue), $\beta_1 = 10$ (red), $\beta_2 = 0.4$ and a range of γ between 1.4 and 4

The most important thing about Figures 4.2 to 4.5 is that as γ is increased, the firms optimal N decreases and the optimal T increases. The reason for this effect is that a higher effort needed to coordinate workers forces the firm to reduce the size of its workforce. However, since the firm still wants to produce (using its production function which itself did not change) it tends to decide to let the individual worker work for longer times. One can simply say that a higher γ makes a smaller workforce with higher working times more attractive.

In Figure 4.2 one can also observe what happens when the time needed for the preparation task is increased by doubling the value for β_1. As intuition suggests and in accordance with the results of the preceding section a higher β_1 favours higher working times and a smaller workforce, because it makes a worker preparing the same time less efficient. Hereby the curve is vertically shrinked and shifted to the right.

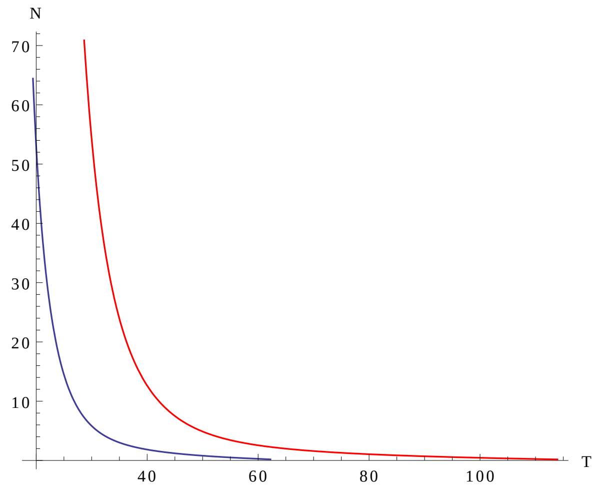

Figure 4.3: Optimal N-T combinations using effort function (c) and parameters $\tilde{w}_1 = \tilde{w}_2 = 0.01$, $\beta_1 = 5$, $\beta_2 = 0.4$ (blue), $\beta_2 = 0.45$ (red) and a range of γ between 1.4 and 4

In Figure 4.3 the parameter β_2 is shifted. A higher value for this parameter decreases the effect of diminishing efficiency on the personal level. This means that a worker becomes more effective. In the graph one can observe that this leads to a larger optimal T and also a slightly larger N as curve is shifted to the top right.

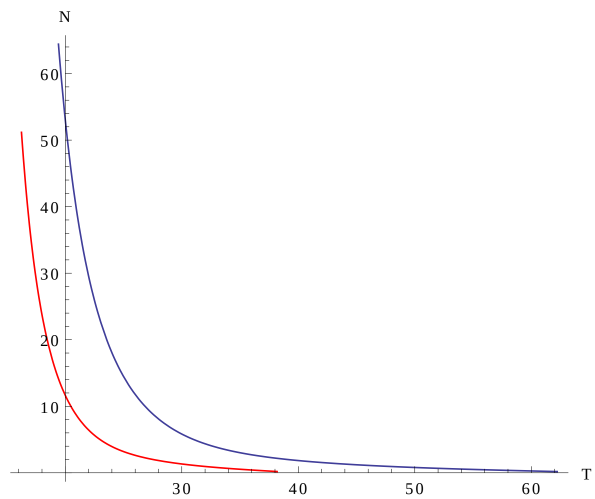

Figure 4.4: Optimal N-T combinations using effort function (c) and parameters $w_1 = 0.01$ (blue), $\tilde{w}_1 = 0.02$ (red), $\tilde{w}_2 = 0.01$, $\beta_1 = 5$, $\beta_2 = 0.4$ and a range of γ between 1.4 and 4

Next, Figure 4.4 increases the wage w_1 that is payed to the workers for their time. This change leads the firm to decrease its optimal T. Interestingly this is especially true for large values of γ. Also a slight decrease in the size of the workforce can be observed.

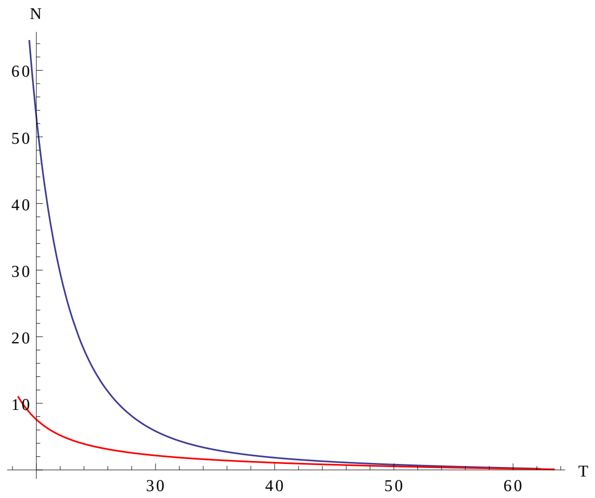

Figure 4.5: Optimal N-T combinations using effort function (c) and parameters $\tilde{w}_1 = 0.01$, $\tilde{w}_2 = 0.01$ (blue), $\tilde{w}_2 = 0.02$ (red), $\beta_1 = 5$, $\beta_2 = 0.4$ and a range of γ between 1.4 and 4

The last parameter that is investigated is the coordination cost parameter $\tilde{w}_2$. Figure 4.5 shows that a higher premium that is payed for coordinating its workers leads the firm to massively decrease the size of the workforce. The optimal T values are not even slightly influenced by this effect resulting in a curve that is only clinched vertically.

In Figure 4.6 we depart from the use of cost function (c) in order to illustrate the qualitatively different behaviour when using a cost function of the form (b). As can be seen, an increasing γ makes the curve in the N-T diagram look similar to the ones from above for low γ values at first, but after a critical value of γ the curve turns horizontally in the other direction suggesting lower T values for higher coordination efforts opposing what was observed above. The reason for this qualitative difference when using the seemingly very similar cost functions (b) and (c) is suspected to be its derivative with respect to N in the origin. Using Function (c), coordination efforts rise only very moderately for small N while using function (b) they rise with slope γ. So

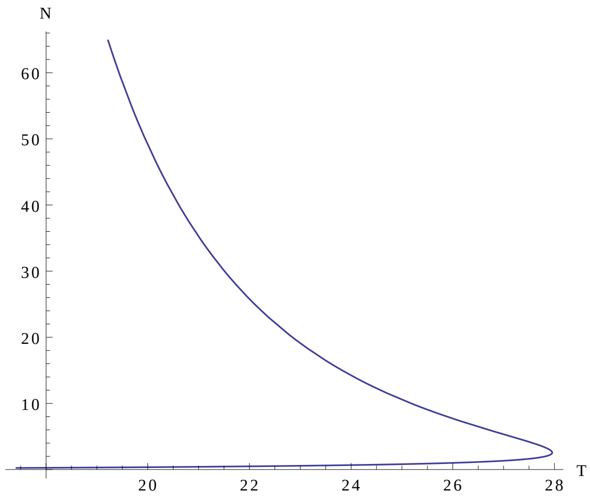

Figure 4.6: Optimal N-T combinations using effort function (b) and parameters $\tilde{w}_1 = 0.01$, $\tilde{w}_2 = 0.01$, $\beta_1 = 5$, $\beta_2 = 0.4$ and a range of γ between 1.4 and 4

if γ becomes large, this effect comes into play more strongly.

When using cost function (a) the curve takes the form of a vertical line as was discussed further above.

4.6 Conclusion

I raised three research questions. First, what characterizes the production function of the firm and the generating two subtasks. It was shown that with specific assumptions regarding the concavity of the functions which represent the generating subtasks the production function exhibits a s-shape. This implies that the production function has a minimum value for the working time. Second, which optimal combination of number of workers and working time result from different subtasks characteristics. It was also shown for various effort functions that the lower the change of the productivity along the daily working time, thus the higher the elasticity, of the subtasks the higher the optimal daily working time and the less worker are employed. And third, which effect does coordination effort reducing technological progress have on firms production and thereby on the income situation of the workers. The effect of technological change on the optimal N-T combination highly depends the functional form of the effort functions. On the one side γ has always a negative effect on the number of workers but on the other side it might have a neutral, a negative or a positive effect on the optimal working time. So far the results concerning the effects of γ on workers income are inconclusive. To draw final conclusions on this issue, especially while distinguishing between different groups of workers, not only the effects of the different effort functions need further considerations but also the interplay of different combinations of the final good price, the wage, the technology parameter and the effort cost parameter.

4.7 Appendix

Proof lemma 1:

Proof. Equating (4.9) to zero and rearanging terms gives $\frac{f'(t_1)}{f(t_1)} = \frac{g'(T-t_1)}{g(T-t_1)}$. Since $t_1 \in [0, T]$, $T > 0$ and because of $f'(0) = a \in \mathbb{R}_{++}$ and $f(0) = 0$ we have $\lim_{t_1 \to 0} \frac{f'(t_1)}{f(t_1)} = \infty$. What is larger then $\lim_{t_1 \to 0} \frac{g'(T-t_1)}{g(T-t_1)} = b \in \mathbb{R}_{++}$, since $g'(T-t_1) = c \in \mathbb{R}_{++}$ and $g(T-t_1) = d \in \mathbb{R}_{++}$. Also, from the concavity of g and $g(0) = 0$ follows, that $\lim_{t_1 \to T} \frac{g'(T-t_1)}{g(T-t_1)} = \infty$. Furthermore, since f and g are strictly concave $\frac{f'(t_1)}{f(t_1)}$ is strictly decreasig and $\frac{g'(T-t_1)}{g(T-t_1)}$ is strictly increasing in t_1. Therefore, for each T exists only one $t_1^\star \in [0, T]$ that fullfills the optimality condition $f(t_1)' \cdot g(T-t_1) - f(t_1) \cdot g(T-t_1)' = 0$. Thus, since $T = t_1 + t_2$ and $t_2 \geq 0$ there is also a unique $t_2^\star$ for each $T > 0$.

Next we show that $t_1^\star$ and $t_2^\star$ are strictly increasing in T. Think of any $T \in \mathbb{R}_{++}$ and the corresponding $t_1^\star$, for any $T' > T$ applies that $\frac{f'(t_1^\star)}{f(t_1^\star)} = \frac{g'(T-t_1^\star)}{g(T-t_1^\star)} > \frac{g'(T'-t_1^\star)}{g(T'-t_1^\star)}$. Because of the uniqueness of $t_1^{\star\prime}$ and since $\frac{f'(t_1)}{f(t_1)}$ is strictly decreasing and $\frac{g'(T'-t_1)}{g(T'-t_1)}$ is strictly increasing in t_1 it follows that $t_1^\star < t_1^{\star\prime}$. Expressing firms first optimization problem in terms of t_2 leads to the corresponding optimality condition $f(T-t_2) \cdot g'(t_2) - f'(T-t_2) \cdot g(t_2) = 0$ or $\frac{f'(T-t_2)}{f(T-t_2)} = \frac{g'(t_2)}{g(t_2)}$. Following the same steps as above will show that $t_2^\star < t_2^{\star\prime}$ for any $T < T'$.

The last property that is left to be shown is the unboundedness of $t_1^\star$ and $t_2^\star$ in T. From above follows that it exists a function $\tilde{t}_1^\star$ so that $t_1^\star = \tilde{t}_1^\star(T)$. Lets assume there is a $\hat{t}_1 = \sup \tilde{t}_1^\star$, thus $\tilde{t}_1^\star$ is bounded. Since $\tilde{t}_1^\star$ is strictly increasing in T, $\lim_{T \to \infty} \tilde{t}_1^\star(T) = \hat{t}_1$. Furthermore, since $f(t_1)$ is strictly increasing and concave $\lim_{T \to \infty} \frac{f'(t_1^\star)}{f(t_1^\star)} = \frac{f'(\hat{t}_1)}{f(\hat{t}_1)} = m \in \mathbb{R}_{++}$. Because of the optimality condition this would imply $\lim_{T \to \infty} \frac{g'(T-t_1^\star)}{g(T-t_1^\star)} = \frac{g'(T-\hat{t}_1)}{g(T-\hat{t}_1)} = m$. From the boundedness of $t_1^\star$ also follows that $T - t_1^\star$ is strictly increasing in T and not bounded, since $T = t_1^\star + t_2^\star$. Consequently, because g is strictly increasing and concave, it follows that $\lim_{T \to \infty} \frac{g'(T-t_1^\star)}{g(T-t_1^\star)} = 0$. Thus assuming that $t_1^\star$ is bounded leads to a contradiction. The same logic applies to the assumption that $t_2^\star$ is bounded. While assuming that both $t_1^\star$ and $t_2^\star$ are bounded already contradicts $T = t_1^\star + t_2^\star$. □

Proof lemma 2:

Proof. We have $\tilde{y} = f(t_1)g(T-t_1)$. Optimizing with respect to the share spent on the

preparation activity yields
$\frac{\partial \tilde{y}}{\partial t} = f'(t_1)g(T-t_1) - f(t_1)g'(T-t_1) \stackrel{!}{=} 0$

$$\Rightarrow f'(t_1)g(T-t_1) = f(t_1)g'(T-t_1) \tag{4.28}$$

The elasticity of the production function is defined as

$$\epsilon_y = T \cdot \frac{\frac{\partial t}{\partial T} f'(t_1)g(T-t_1) + f(t_1)g'(T-t_1)(1-\frac{\partial t}{\partial T})}{f(t_1)g(T-t_1)}$$

using the optimality condition, we get

$$\epsilon_y = T \cdot \frac{g'(T-t_1)}{g(T-t_1)} = (T-t_1) \cdot \frac{g'(T-t_1)}{g(T-t_1)} + t \cdot \frac{f'(t_1)}{f(t_1)} = \epsilon_{g(T-t_1)} + \epsilon_{f(t_1)}$$

This means that if $\lim_{t\to 0} \epsilon_{g(t_1)} + \epsilon_{f(t_1)} > 1 \Rightarrow \lim_{T\to 0} \epsilon_y > 1$ which implies that y is convex in the origin.

The next step will be to show that y is concave for large values of T. Consider

$$\frac{\partial y}{\partial T} = \frac{\partial t}{\partial T} f'(t_1)g(T-t_1) + f(t_1)g'(T-t_1)(1-\frac{\partial t}{\partial T}) = f(t_1)g'(T-t_1)$$

$$\frac{\partial^2 y}{\partial T^2} = f'(t_1)g'(T-t_1)\frac{\partial t}{\partial T} + f(t_1)g''(T-t_1)(1-\frac{\partial t}{\partial T})$$

Since f has an upper bound and by Lemma 1 t_1 does not, $f'(t_1)$ will approach 0 for large values of T. Because $f'(t_1)$ is finite and decreasing and $\frac{\partial t}{\partial T} \in (0,1)$, the first part of the sum will approach 0 for large T. Also, one can easily see that the second part of the sum will always be negative since g is concave. Hence we have $\lim_{t_1\to\infty} \frac{\partial^2 y}{\partial T^2} < 0$ implying that y is concave for large values of T and hence is locally s-shaped. □

Bibliography

Daron Acemoglu and David Autor. Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, volume 4, pages 1043–1171. Elsevier, 2011.

David H Autor, Frank Levy, and Richard J Murnane. The skill content of recent technological change: An empirical exploration. *The Quarterly journal of economics*, 118(4):1279–1333, 2003.

Gary S Becker and Kevin M Murphy. The division of labor, coordination costs, and knowledge. *The Quarterly Journal of Economics*, 107(4):1137–1160, 1992.

Pierre Cahuc, Stéphane Carcillo, and André Zylberberg. *Labor economics*. MIT press, 2014.

Jengchung V. Chen, Bo-chiuan Su, and Timothy M. Hiele. The impact of IT-coordination costs on firm size and productivity: Transaction cost perspective. *International Journal of Electronic Commerce*, 21:99–127, 2016.

Eric K. Clemons, Sashidhar P. Reddi, and Michael C. Row. The impact of information technology on the organization of economic activity: The "move to the middle" hypothesis. *Journal of Management Information Systems*, 10:9–35, 1993.

Antonio Cordella. Transaction costs and information systems: does IT add up? In C.Avgerou et al., editor, *Bricolage, Care and Information*, pages 386–400. Palgrave Macmillan UK, 2009.

Antonio Cordella and Kai A. Simon. The impact of information technology on transaction and coordination cost. *Conference on Information Systems Research in Scandinavia*, pages 1–13, 1997.

Linda Midttun. Medical specialists' allocation of working time. *Health Policy*, 83: 114–127, 2007.

OECD. *OECD employment outlook*. Paris: OECD Publishing, 1991.

OECD. *OECD employment outlook*. Paris: OECD Publishing, 1999.

Vladimir Stoikov. The allocation of scientific effort: Some important aspects. *The Quarterly Journal of Economics*, 78:307–323, 1964.

Short Curriculum Vitae of Martin Lunge

Academic Education

since 10/2011	PhD student at the *Bielefeld Graduate School of Economics and Management* (BiGSEM) Member of the *Center for Mathematical Economics* (IMW) at Bielefeld University
02/2013–04/2014	Research stay at Université Paris 1 Panthéon-Sorbonne
04/2011–09/2011	Teaching and research assistant at University of Hohenheim
04/2003-05/2010	Diplom (master equivalent) in Economics at University of Bayreuth
10/2001–10/2009	Diplom (master equivalent) in Business Administration at University of Bayreuth
11/2007–08/2008	Study abroad at Irkutsk State Technical University