The impact of frequency distributions in a perceptual grouping oscillator network

Martin Meier, Robert Haschke and Helge J. Ritter

Neuroinformatics Group
Bielefeld University, 33501 Bielefeld, Germany
{mmeier,rhaschke,helge}@techfak.uni-bielefeld.de

The Kuramoto Model [2] is a recurrent network composed of limit cycle oscillators, whose dynamic is designed to facilitate phase synchronization among populations of oscillators. A single oscillator O_n in a population of N oscillators is described by it's phase θ_n and frequency ω_n. The recurrent update equation of the model is

$$\dot{\theta}_m = \omega_m + \frac{K}{N} \sum_{n=1}^{N} \sin(\theta_n - \theta_m). \tag{1}$$

Hence, the oscillators are globally coupled with a coupling strength K. The frequencies ω_m are constant and drawn from a random distribution. These frequencies introduce a separating force into the network model, which drives oscillators away from each other. The $K/N \sum \sin(\theta_n - \theta_m)$ term counteracts this separation, forcing the oscillators into a phase-synchronized state. This model can be used to describe different kinds of natural phenomena, for example synchronous flashing of fireflies or the synchronization of pacemaker cells in the heart. Please see [4] for a review of synchronization effects in Kuramoto models.

In [3], a network based on a hierarchical model of coupled Kuramoto oscillators is introduced, which is able to solve a broad spectrum of perceptual grouping tasks, for example texture based image segmentation and contour integration. The key principle of this network is to represent features from an input space in a one-to-one relation by Kuramoto oscillators. The coupling strength between the oscillators is chosen based on the similarity of the features according to some distance metric.

In contrast to the original Kuramoto model (1), the hierarchical model includes individual coupling strengths f_{mn} between oscillators O_m and O_n to facilitate feature-dependent synchronization of oscillators. It is described by the recurrent update equation

$$\dot{\theta}_m = \omega_m + \frac{K}{N} \sum_{n=1}^{N} f_{mn} \sin(\theta_n - \theta_m). \tag{2}$$

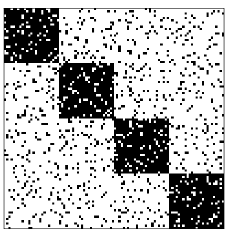
Using this equation, positively coupled oscillators, e.g. $f_{mn} = 1$, will attract each other and gather at a similar phase whilst negatively coupled oscillators ($f_{mn} = -1$) act repelling and spread in their phases.

In the original model, the frequencies ω_n are drawn from a random distribution and constant. We proposed to employ a set of discrete frequencies $\omega_0\alpha$,

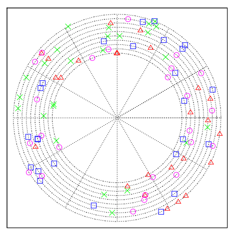
$\alpha = 1, \dots, L$ and update the frequency of each oscillator based on the support they gain from all oscillators. The support is calculated based on cosine similarity of oscillator phases, limited to the range $[0, 1]$ and weighted by their local coupling i.e. $S_m(\alpha) = \sum_{n \in \mathcal{N}(\alpha)} f_{mn} \cdot \frac{1}{2} \left(\cos(\theta_n - \theta_m) + 1\right)$. The new frequency w_m is then chosen to maximize the support, thus boosting phase synchronization:

$$\omega_m = \omega_0 \cdot \underset{\alpha}{\operatorname{argmax}} \Big(S_m(\alpha) \Big) \tag{3}$$

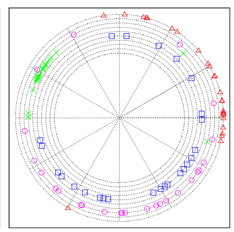
This adaptation allows an easy assignment of grouping results: Oscillators sharing a common frequency index α represent the same group. Additionally, adapting oscillators to the same frequency reduces the phase spread compared to randomly drawn, constant frequencies.



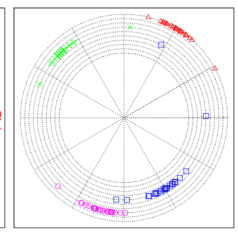
(a) f_{mn}



(b) Initialization.



(c) 10 updates.



(d) 20 updates.

Fig. 1: This figure displays an example of the dynamics for a network with 100 features, 10 discrete frequencies and 4 target groups. The leftmost panel shows the coupling matrix f_{mn} between features, where a black pixel indicates an attracting coupling while white represents a repelling coupling. Fig. 1b to 1d visualize the state of the oscillators in phase space (polar coordinates) at initialization, after 10 and 20 updates. The color and symbol of the oscillators represents the desired target state.

The behavior of the network in an artificial grouping task is shown in Fig. 1. The coupling matrix f_{mn} for 100 features, divided into four groups with 10% noise is shown in Fig. 1a, whilst the state of the oscillators in phase space is shown in Fig. 1b–1d. After 10 updates the network achieves a perfect grouping result in terms of frequency assignment. After 20 updates, oscillators representing the same target group have a high phase synchrony as well.

Although the recurrent update (2) appears intriguingly simple, the model and its' variations opened a wide spectrum of research, e.g. see [1] for an overview. By introducing a frequency adaption (3) in conjunction with discrete frequencies and recalling the original update equation (1), where the frequencies ω_n induce phase spread while the sin() term drives the oscillators towards a mean phase, the question arises whether the distribution of these frequencies has an impact on the network dynamics. To get a first insight into this question, we employed genetic algorithms (GA) to generate different sets of discrete frequencies and analyzed if the GA was able to improve the target state of the network by varying the discrete frequencies.

The initial grouping problem was similar to the settings in [3]. The compatibility f_{mn} was expressed as a matrix which encodes the couplings among 100 features, split into four target groups of 25 features. The matrices contained 25% noise. The oscillator networks contained 10 discrete frequencies. The crossover probability of the GA was set to 50% with a mutation probability of 3%. The population consisted of 100 chromosomes, where each chromosome encoded the discrete frequencies of an oscillator network. The fitness function was designed in terms of the grouping quality $q = [0, 1]$ and the oscillator order $r = [0, 1]$. A grouping quality q of one represents a perfect result compared to a given target labeling, whilst the order r represents the phase coherence of the oscillators. A value of one means that all oscillators share the same phase θ. For a more comprehensive description of both evaluation measures please refer to [3]. For the initial trials, two fitness functions are evaluated. On the one hand the product of quality and order $q * r$ and on the other hand the average of quality and order $\frac{q+r}{2}$. Additionally, two different ranges of frequencies are used, $[\frac{\pi}{2}, 2\pi]$ and $[\frac{\pi}{2}, 20\pi]$. The results are shown in table 1 for the different conditions. The last column shows the product of quality and order without a GA optimization, averaged over 1000 trials.

cond.	avg, 2π	avg, 20π	mul, 2π	mul, 20π	no GA
μ and σ	0.986 ± 0.001	0.989 ± 0.01	0.971 ± 0.001	0.979 ± 0.001	0.963 ± 0.03

Table 1: Mean and standard deviation of the fitness function over 1000 evolution steps for each condition. The leftmost column shows the result over 1000 trials for a non-optimized oscillator network.

These results suggest, that the frequency distribution does not have a significant impact on the grouping behavior of the oscillator network, at least for the considered artificial grouping problem. In contrast to the hierarchical model (2), the frequency adaption (3) reduces the phase spread induced by randomly drawn, fixed ω values in oscillator groups, which could counteract possible impacts of different ω distributions on the dynamics. Extending this investigation towards a mean phase analysis of oscillator groups will be of future interest.

References

1. Acebrón, J.A., Bonilla, L.L., Vicente, C.J.P., Ritort, F., Spigler, R.: The Kuramoto model: A simple paradigm for synchronization phenomena. Reviews of modern physics 77(1), 137 (2005)
2. Kuramoto, Y.: Chemical oscillations, waves, and turbulence. Courier Dover Publications (2003)
3. Meier, M., Haschke, R., Ritter, H.J.: Perceptual grouping through competition in coupled oscillator networks. Neurocomputing 141, 76–83 (2014)
4. Strogatz, S.H.: From Kuramoto to Crawford: Exploring the onset of synchronization in populations of coupled oscillators. Phys D 143(1), 1–20 (2000)