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Human capital formation with heterogeneous agents, sustainable debt policies and growth: Who benefits from fiscal policy rules?

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Human capital formation with heterogeneous agents, sustainable debt policies and growth: Who benefits from fiscal policy rules?

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Abstract

With this paper our objective is to study the effects of different deficit policies in an endogenous growth model with publicly funded human capital accumulation and public debt, where we allow for heterogeneous households. Two types of households are considered. One household acquires human capital or skills through education while the other household remains low-skilled. Aggregate production is given by a function with physical capital and labor as input factors, where total labor input is modeled by a CES function with high-skilled and low-skilled labor as arguments. The government can run into debt, but, the primary surplus is a positive function of public debt which guarantees that public debt is sustainable. We study the characteristics and stability of the steady state and we investigate the effects of fiscal policy with regard to long-run growth and the distribution of welfare of the two households. Further, we analyze growth and welfare effects of switching from a balanced government budget to permanent public deficits taking into account transition dynamics.

Keywords: Human capital, heterogeneous agents, distribution, endogenous economic growth, fiscal policy, sustainable public debt

JEL Classification: E62; H52; H63; I28; J24

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1 Introduction

The financial crisis of 2007/08, the refugee crisis 2015 and the current Covid-19 crisis have been the main economic shocks, not only in Europe but even worldwide, during the last fourteen years. The question 'How to finance the losses of those crises?' has worried many governments and has often been answered by higher public deficits and debt, giving rise to public debt crises with in part drastically increasing national debt to GDP ratios in some countries. Public debt is particularly challenging when several economies cannot repay or refinance their government debt or do not stick to sustainable debt policy rules and need help from other countries. The frictions, in particular among the Member States of the EU, demonstrate not only the importance of fiscal policy but also the challenge of finding adequate solutions resulting from excessive debt policies in Europe. Despite clear guidelines, fixed in the Maastricht treaty stating that the public deficit and the public debt relative to GDP must not exceed 3% and 60%, respectively, for the countries of the European Economic and Monetary Union, some of the governments had to find rescue through the European Stability Mechanism trying to prevent bankruptcy (Greiner and Fincke, 2015).

Economists identify human capital to be an essential factor of economic growth for both developed and developing economies, in crisis as well. The perception that human capital is a leading force of sustained per capita growth in market economies began at the end of the last century with the contributions by Uzawa (1965) and by Lucas (1988). Besides those contributions, Romer (1986) has developed the endogenous economic growth theory, where fiscal policies do not only affect the level of variables in the long-run, but, the growth rate. In his paper Romer (1986) assumes that capital displays increasing returns to scale once new accumulation factors, such as knowledge embodied in human capital, are integrated.

Empirically, there is strong evidence that education is positively correlated with income growth. At the microeconomic level the positive correlation seems to be quite robust. On the macroeconomic level the findings are more fragile (Krueger and Lindahl, 2001) which, however, may be due to measurement errors. Krueger and Lindahl (2001) illustrate that cross-country regressions show that the change in education is positively correlated with economic growth if measurement errors are taken into account. Greiner et al. (2005) found in time series studies that the Uzawa-Lucas endogenous growth model with human capital can replicate the evolution of aggregate economic variables in the USA and in Germany once non-linearities in the generation of human capital is allowed for. Further, Levine and Renelt (1992) have shown that human capital, measured by the secondary enrollment rate, is a robust variable in growth regressions, so that building endogenous growth models with human capital as the engine of growth seems to be justified.

Basically, raising the level of human capital can impact productivity growth in the following ways: On the one hand, highly educated workers raise the stock of knowledge by developing new processes and new technologies and, on the other hand, education influences economic growth through the diffusion and transmission of knowledge, i.e. educated individuals exert positive externalities on their colleagues by social and professional interactions, making them more productive (European Commission Directorate-General for Economic and Financial Affairs, 2010).

Constructing human capital through various practices endures all lifetime. Schultz (1961) proposes two possibilities of human capital formation: the general formation through qualification by diploma without specialization and the specific formation with specialized experience that is often achieved during the working time. The professional experience is measured by age. In the educational system, a general educational background is achieved during the compulsory education path. After school time the more specific formations begin. An individual's input factors in the process of human capital formation are time, physical capital, such as equipment, human capital itself and, of course, funds to finance the construction of physical and human capital.

In the contributions by Uzawa (1965), Lucas (1988) and Rebelo (1991), public spending as an input factor in the process of generating human capital was not taken into account, whereas in Beauchemin (2001) and Blankenau and Simpson (2004) governments provide investments or public funds to build up human capital. Further, contributions considering that the public sector can stimulate the formation of human capital by devoting public resources to schooling, are for example Glomm and Ravikumar (1992), Ni and Wang (1994) and Greiner (2008). In those contributions, human capital accumulation results either from both private and public services, as in Glomm and Ravikumar (1992) and Blankenau and Simpson (2004), or from public spending alone, as in Ni and Wang (1994), in Beauchemin (2001) and in Greiner (2008). In Greiner (2008) an endogenous growth model with human capital is investigated, where education is financed by the government sector. The question how fiscal policy affects human capital formation and economic growth has been studied, assuming that the economy is on a balanced growth path where all variables, including public debt, grow at the same rate.

A large fraction of public expenditures are constituted by expenditures for education and health (Afonso and St. Aubyn, 2005). Tax revenues or public deficits or both of them are used to finance these expenditures on education. As future educated agents will pay back the debt and raise the long-term growth rates, the public deficit to finance the expenditures will not distort the economic environment (Turnovsky and Fisher, 1995). If the national income is weak, unusual events, such as a war or natural disasters, financing public investment, e.g. in education and health, can increase the stock of debt (Daniel et al., 2003). Other reasons for the rise of public debt to finance public investment can be that governments want to avoid market distortion from raising the tax rate (Barro, 1979) or that they follow a Keynesian counter-cyclical fiscal policy implying deficit financing, where the resulting higher debt is not paid back at later stages. If budgetary regimes are introduced that limit the scope for public deficits as in Greiner and Semmler (2000) and in Ghosh and Mourmouras (2004), a deficit financed increase in public investment may lead to a smaller growth rate in the long-run.

With this paper, our aim is to address the following research questions: How does fiscal policy influence long-run growth and the distribution of welfare of differently skilled households, when the government finances educational spending and may run public deficits? How does the transition from a situation with a balanced government budget to one with permanent public deficits affect the households' economic situation? What are the different macroeconomic consequences of the fiscal policies under sustainable debt policy scenarios?

To answer those questions we build an endogenous growth model with publicly funded educational spending, where the government can incur deficits, but, has to stick to the inter-temporal budget constraint. Our model is based on the contribution by Greiner (2016) that is extended by allowing for heterogeneous households, one high-skilled and one low-skilled. Human capital accumulation is the source of ongoing economic growth, making the long-run growth rate an endogenous variable. The government finances educational spending and can run into debt. However, it must stick to the inter-temporal budget constraint that holds for a balanced government budget or for a fiscal policy rule where the primary surplus rises at least linearly with higher public debt. The latter concept is based on the ideas by Bohn (1995). The economic intuition behind it says that if governments run into debt today, they must take corrective actions in the future by increasing the primary surplus. If the government does not act in this way, public debt will not be sustainable, see Greiner and Fincke (2015) for detailed considerations.

In the rest of the paper we proceed as follows. Section 2 introduces the model and presents its structure by describing both households, the productive sector, the government and the process of human capital accumulation. In section 3, we analyze the model dynamics by defining the equilibrium conditions and the balanced growth path under the two different debt policies, balanced government budget and permanent public deficits. In section 4, we study the distribution and welfare effects of the fiscal policies under the two scenarios along the sustainable balanced growth path and also taking into account the transition path. Section 5 summarizes our results.

2 The Structure of the Growth Model

Our economy consists of three sectors: a household sector which receives labor income and income from its saving, a productive sector and the government. We begin with the description of the household sector.

2.1 The household sector

The household sector is composed of two types of households. The first household supplies high-skilled labor, which is employed either in the production of the final good or in the educational sector, while the second household supplies low-skilled labor. We assume that both households behave as immortal families corresponding to finite-lived individuals who are connected via inter-generational transfers that are based on altruism. Thus, although individuals have finite lives each family is considered as a dynasty where the decision maker behaves as if he had an infinite time horizon (Barro and Sala-i Martin, 2004).

The overall number of high-skilled people is composed of a stock of students, S, and of a stock of high-skilled employees, L, who constitute the active high-skilled labor force and produce goods or are hired as teachers. At each point in time a certain number of students, which is determined exogenously, enter the stock of students and a certain number of students become employees. We assume that the number of students becoming employees just equals the number of new students so that the overall stock of students is constant. Further, the number of students becoming employees equals the number of employees leaving the active labor force, so that the active high-skilled labor force and, thus, the total stock of high-skilled labor is constant, too, just like the number of low-skilled labor.

The household sector maximizes the discounted streams of utility arising from per-capita consumption, $C_i(t)$, $i = s, n$, over an infinite time horizon subject to their budget constraints, taking factor prices as given, where the index s (n) denotes high-skilled (low-skilled) labor. The utility function of both households is assumed to be logarithmic,[1] $U(C_i) = \ln C_i$, $i = s, n$, and the households supply labor inelastically.

The maximization problem of the high-skilled household can be written as

$$\max_{C_s} \int_0^\infty e^{-\rho t} \ln C_s \, dt, \tag{1}$$

subject to

$$\dot{K}_s + \dot{B} = r_b(1-\tau_k)B + rK_s(1-\tau_k) + w_s L(1-\tau_s) - \delta K_s - C_s. \tag{2}$$

The parameters $\rho > 0$, $\tau_k, \tau_s \in (0,1)$ and $\delta \in (0,1)$ are the subjective discount rate, the capital and labor income tax rate and the depreciation rate of capital, respectively, and $K_s > 0$ and $C_s > 0$ give the capital stock owned by the high-skilled household and its level of consumption. The variable $B > 0$ denotes public debt that is owned only by the high-skilled household. A no-arbitrage condition requires the return to capital to be equal to the return on government bonds, r_b, implying $r_b = r - \delta/(1-\tau_k)$.

To solve this problem we formulate the current-value Hamiltonian which is written as

$$H_s = \ln C_s + \gamma_1(r(1-\tau_k)A_s + w_s L(1-\tau_s) - \delta A_s - C_s), \tag{3}$$

with γ_1 the shadow-price of capital for household s and $A_s = K_s + B$ total assets. Necessary optimality conditions are given by

$$C_s{}^{-1} = \gamma_1, \tag{4}$$

$$\dot{\gamma}_1 = (\rho+\delta)\gamma_1 - \gamma_1(1-\tau_k)\, r. \tag{5}$$

If the transversality condition $\lim_{t\to\infty} e^{-\rho t} A_s/C_s = 0$ holds, which is fulfilled for a time path on which the assets grow at the same rate as consumption, the necessary conditions are sufficient, too.

[1] In the following we omit the time index t if no ambiguity arises.

The maximization problem of the low-skilled household is given by

$$\max_{C_n} \int_0^\infty e^{-\rho t} \ln C_n \, dt, \tag{6}$$

subject to

$$\dot{K}_n = rK_n(1-\tau_k) + w_n N(1-\tau_n) + T_p - \delta K_n - C_n. \tag{7}$$

The capital stock owned by household two is denoted by $K_n > 0$ and $C_n > 0$ is its consumption. The low-skilled household also saves, but, we assume that it disposes of a smaller capital stock than the household in the first labor market, that is, $K_n < K_s$. Further, it receives transfer payments from the government, T_p, in addition to its market income and it pays a lower labor income tax $\tau_n < \tau_s$.

Again, we formulate the current-value Hamiltonian which is

$$H_n = \ln C_n + \gamma_2(r(1-\tau_k)K_n + w_n N(1-\tau_n) + T_p - \delta K_n - C_n), \tag{8}$$

with γ_2 the shadow-price of capital for the low-skilled household. Necessary optimality conditions are obtained as

$$C_n^{-1} = \gamma_2, \tag{9}$$
$$\dot{\gamma}_2 = (\rho+\delta)\gamma_2 - \gamma_2(1-\tau_k)\, r. \tag{10}$$

These conditions are again sufficient if the transversality condition $\lim_{t\to\infty} e^{-\rho t} K_n/C_n = 0$ is fulfilled.

The growth rates of consumption of the households are obtained from (4)–(5) and (9)–(10) as

$$\frac{\dot{C}_i}{C_i} = -(\rho+\delta) + (1-\tau_k)r, \; i = s, n. \tag{11}$$

Using $C_s + C_n = C$, the growth rate of aggregate consumption is given by

$$\frac{\dot{C}}{C} = \frac{\dot{C}_s}{C_s}\frac{C_s}{C} + \frac{\dot{C}_n}{C_n}\frac{C_n}{C} = (-(\rho+\delta) + (1-\tau_k)r)\left(\frac{C_s}{C} + \frac{C_n}{C}\right), \tag{12}$$

with $C_s/C + C_n/C = 1$.

2.2 The productive sector

The productive sector is represented by one firm which behaves competitively and which maximizes profits. Production of the firm at time t is given by a constant elasticity of substitution (CES) production function as

$$Y(t) = K(t)^{1-\alpha} I_a(t)^{\alpha}, \tag{13}$$

where Y gives output, K denotes physical labor and I_a is defined as

$$I_a = \left(\gamma\,(uh_cL)^{(\sigma-1)/\sigma} + (1-\gamma)\,(\xi h_cN)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}. \tag{14}$$

The variable I_a denotes the total labor input, with L high-skilled labor and N gives labor demand for simple labor. The variable h_c gives per-capita human capital of the high-skilled labor force and the parameter $\xi \in (0,1)$ determines the spill-over effect of high-skilled labor implying that, due to externalities, low-skilled labor benefits to a certain degree from the human capital of high-skilled labor. The coefficient $\alpha \in (0,1)$ gives the elasticity of production with respect to labor, $\sigma > 0$ is the elasticity of substitution between high-skilled and low-skilled labor and $\gamma, (1-\gamma) \in (0,1)$ gives the share of high-skilled and low-skilled labor in production, respectively. The parameter $u \in (0,1)$ gives that share of the high-skilled labor force that is used for the production of final goods.

Static profit maximization determines the wage for the two types of labor as

$$w_s = K^{1-\alpha} h_c^{\alpha} \alpha W^{-1+\alpha\sigma/(\sigma-1)} \gamma u^{-1/\sigma} L^{-1/\sigma} \tag{15}$$

$$w_n = K^{1-\alpha} h_c^{\alpha} \alpha W^{-1+\alpha\sigma/(\sigma-1)} (1-\gamma) \xi^{(\sigma-1)/\sigma} N^{-1/\sigma}, \tag{16}$$

with[2] $W := \gamma\,(uL)^{(\sigma-1)/\sigma} + (1-\gamma)\,(\xi N)^{(\sigma-1)/\sigma}$. Denoting by r the return to capital, profit maximization yields

$$r = (1-\alpha) K^{-\alpha} h_c^{\alpha} W^{\alpha\sigma/(\sigma-1)}. \tag{17}$$

2.3 The government and human capital accumulation

Human capital in our economy is produced in the schooling sector where an exogenously given number of students is educated. As mentioned above, the government hires the fraction $(1-u)$ of the high-skilled labor force as teachers. Additionally, the government

[2]Note that $w_s > (<) w_n$ holds for $\gamma (N/L)^{1/\sigma} u^{-1/\sigma} \xi^{(1-\sigma)/\sigma} / (1-\gamma) > (<) 1$.

uses public resources for education in the schooling sector, like expenditures for books and other teaching material, which is an input in the process of human capital formation, too. Thus, the input in the schooling sector is composed of teachers and of schooling expenditures and we assume decreasing returns to scale to each input but constant returns to both inputs. The evolution of per capita human capital, then, is a function of teachers per student and of expenditures per student.

It should be noted that human capital, which is embodied in students, becomes available to the whole active high-skilled labor force in the economy, once students become employees, and in part to the low-skilled labor force, too. The reason for this assumption is to be seen in spill-over effects of knowledge, which leads to a diffusion of knowledge among the labor force. At first sight, this seems to be a strong assumption. But if one takes into account that in reality newly hired employees interact with existing staff and both learn from each other, this assumption becomes comprehensible.

As concerns the production function for human capital formation we assume a Cobb–Douglas specification. The differential equation describing the change in human per capita capital can be written as

$$\dot{h}_c = \epsilon\left((1-u)h_c L\right)^{\phi} I_e^{1-\phi}/S, \tag{18}$$

with $\epsilon > 0$ a parameter reflecting the efficiency of the inputs, S the total number of students and $\phi \in (0,1)$ gives the elasticity of human capital formation with respect to the number of high-skilled people in that sector. Thus, the change in human capital depends on labor input, such as teachers, and on additional public educational expenditures, I_e, such as teaching materials for example. The share of high-skilled labor used for the formation of human capital, $(1-u)$, is exogenously determined by the government that hires high-skilled labor as teachers (Greiner, 2008, 2012, 2016; Greiner and Flaschel, 2009). Finally, the variable S gives the number of students in the economy, as mentioned above.

The government in our economy receives tax revenues from capital and labor income taxation, it then uses for the remuneration of the teachers, for public spending in the schooling sector and for transfer payments to low-skilled labor, T_P, and for unproductive spending, G. In addition, it runs into debt. Thus, the period budget constraint of the government is given by

$$T + \dot{B} = I_e + (1-u)w_s L + T_p + G, \tag{19}$$

with T denoting tax revenue exclusive of the tax revenue resulting from taxing interest payments on government bonds,

$$T = \tau_s w_s L + \tau_n w_n N + \tau_k r K. \tag{20}$$

We resort to the tax yield exclusive of the revenue achieved by taxing the interest receipts from government bonds because we define the primary surplus exclusive of the tax revenue from the interest yield on government debt. The latter implies that the tax yield from interests on public debt is used for the debt service and that the outstanding government debt is discounted with the net interest rate in the inter-temporal budget constraint of the government.

As concerns public consumption, G, we assume that this variable makes a certain part of the tax revenue, i.e. $G = (1 - \kappa)T$ with $\kappa \in (0, 1)$, and the transfer payments, T_p, constitute a certain fraction of the remaining tax revenue, $T_p = t_r \kappa T$, with $0 < t_r \kappa < 1$. Further, we posit that the inter-temporal budget constraint of the government must hold, $\lim_{t\to\infty} e^{(1-\tau_k)r_b t} \cdot B(t) = 0$.

The government in our economy sets the primary surplus, S_p, such that it is a positive function of public debt. The economic rationale behind that assumption is that the debt to GDP ratio becomes a mean-reverting process when the primary surplus rises as public debt increases so that the debt to GDP ratio remains bounded (Bohn, 1998), if the reaction is sufficiently strong and ensures that the inter-temporal budget constraint of the government holds (Greiner and Fincke, 2015). Formally, this relation is described by

$$S_p = \varrho B + \vartheta Y, \tag{21}$$

with $\varrho \in \mathbb{R}_{++}$ giving the reaction of the primary surplus to higher public debt. The parameter ϱ can vary over time, but, must be positive on average. Thus, ϱ can be interpreted as the average reaction coefficient to public debt. It should be pointed out that, in case of a negative government debt, this rule guarantees that the outstanding debt of the private sector relative to GDP does not explode. The parameter $\vartheta \in \mathbb{R}$ may be positive or negative and shows how the government sets the primary surplus as GDP grows. That parameter reflects the discretionary scope of the government. Positive values of ϑ can be seen to characterize a stability-oriented government that raises the primary surplus as GDP increases, while a negative value of that parameter implies that a higher GDP leads to a smaller primary surplus.

Thus, the evolution of public debt is obtained as

$$\dot{B} = (1 - \tau_k) r_b B - S_p = (1 - \tau_k) r_b B - \varrho B - \vartheta Y. \tag{22}$$

As regards the reaction of the primary surplus to a higher debt, we assume that the government fixes the tax rate and adjusts public educational spending.[3] This is motivated by real world observations showing that governments often reduce productive public spending as public debt increases (Heinemann, 2006). Public educational spending, then, can be written as

$$I_e = T - G - T_p - (1 - u) w_s L - \varrho B - \vartheta Y = \kappa(1 - tr)T - (1 - u) w_s L - \varrho B - \vartheta Y. \tag{23}$$

Equation (23) shows that educational spending plus spending for high-skilled labor employed in the education sector equals the tax revenue minus public consumption minus transfer payments minus the primary surplus of the government.

3 Analysis of the model: The balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximizes profits implying that factor prices equal their marginal products (equations (15), (16) and (17)), the households solve (1) subject to (2) and (6) subject to (7), respectively, the budget constraint of the government (22) is fulfilled with public eduational spending given by (23), human capital evolves according to (18) and the limiting transversality conditions hold, given initial conditions with respect to K, B and h_c.

The economy-wide resource constraint in this economy is obtained by combining the budget constraint of the private households, equations (2) and (7), with the budget constraint of the government (19), where we use $K = K_s + K_n$, as

$$\frac{\dot{K}}{K} = (h_c/K)^{\alpha} \left(W^{\sigma/(1-\sigma)}\right)^{\alpha} - \frac{C}{K} - \frac{I_e}{K} - (1 - \kappa)\frac{T}{K} - \delta, \; K(0) = K_0, \tag{24}$$

where I_e is given by (23).

Aggregate consumption evolves according to equation (12) with r given by (17) so that the growth rate of aggregate consumption can be written as

$$\frac{\dot{C}}{C} = (1 - \tau_k)(1 - \alpha)(h_c/K)^{\alpha} \left(W^{\sigma/(1-\sigma)}\right)^{\alpha} - (\rho + \delta), \; C(0). \tag{25}$$

[3] If it adjusted unproductive spending, public debt would be neutral (Ricardo equivalence).

Human capital and public debt, finally, grow at the following rates

$$\frac{\dot{h}_c}{h_c} = \epsilon \left((1-u)h_c L\right)^{\phi} I_e^{1-\phi}/(h_c S),\; h_c(0) = h_{c0} \tag{26}$$

$$\frac{\dot{B}}{B} = (1-\tau_k)r_b - \varrho - \vartheta Y/B,\; B(0) = B_0. \tag{27}$$

Thus, the economy is completely described by equations (24), (25), (26) and (27) plus the limiting transversality conditions of the households and initial conditions with respect to the assets.

A balanced growth path (BGP) is defined as a path on which K, C and h_c grow at the same constant positive rate, that is, $\dot{K}/K = \dot{C}/C = \dot{h}_c/h_c =: g > 0$ holds, with $g = \textit{constant}$ and public debt B grows at the rate g, too, or is constant, i.e. $\dot{B}/B = g$ (permanent deficits) or $\dot{B} = 0$ (balanced budget). To analyze our economy around a BGP we define the new variables $c = C/K$, $h = h_c/K$ and $b = B/K$. Differentiating these variables with respect to time, results in a three dimensional system of differential equations, given by $\dot{c}/c = \dot{C}/C - \dot{K}/K$, $\dot{h}/h = \dot{h}_c/h_c - \dot{K}/K$ and $\dot{b}/b = \dot{B}/B - \dot{K}/K$, which can be written as follows,

$$\dot{c} = c\left((1-\tau_k)(1-\alpha)h^{\alpha}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha} - (\rho+\delta) - \frac{\dot{K}}{K}\right),\; c(0) \tag{28}$$

$$\dot{h} = h\left((\epsilon/S)(1-u)^{\phi} L^{\phi}(I_e/h_c)^{1-\phi} - \frac{\dot{K}}{K}\right),\; h(0) = h_0 \tag{29}$$

$$\begin{aligned}\dot{b} &= b\left((1-\tau_k)(1-\alpha)h^{\alpha}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha} - \delta - \varrho - \vartheta h^{\alpha}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha} b^{-1}\right) \\ &- b\left(\frac{\dot{K}}{K}\right),\; b(0) = b_0,\end{aligned} \tag{30}$$

with I_e/h_c given by

$$\begin{aligned}\frac{I_e}{h_c} &= -\varrho b/h - \vartheta h^{\alpha-1}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha} + h^{\alpha-1}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha}\kappa(1-t_r)\cdot \\ &\left(\tau_n \alpha W^{-1}(1-\gamma)(\xi N)^{(\sigma-1)/\sigma} + \tau_s \alpha W^{-1}\gamma u^{-1/\sigma}L^{(\sigma-1)/\sigma} + \tau_k(1-\alpha)\right)\end{aligned} \tag{31}$$

and $\dot{K}/K$ is given by (24), with T/K obtained from (20) and where we use $I_e/K = h(I_e/h_c)$.

A solution of $\dot{c} = \dot{h} = \dot{b} = 0$ with respect to h, c, b gives a BGP for our model with permanent public deficits and the corresponding ratios $h^{\star}, c^{\star}, b^{\star}$ on the BGP[4]. In the case

[4]The * denotes BGP values and we exclude the economically meaningless BGP $h^{\star} = c^{\star} = 0$.

of a balanced government budget, a solution of $\dot{c} = \dot{h} = 0$ with respect to h, c together with $b^\star = 0$ yields a BGP.

In the following, we present some results we can derive for the analytical model. Proposition 1 gives results as concerns existence, uniqueness and stability of the BGP in the case of a balanced government budget.[5]

Proposition 1 *Assume that the government runs a balanced budget. Then, there exists a unique saddle point stable BGP.*

Proof: See appendix.

This proposition shows that the balanced growth path for this economy is unique and saddle point stable if the government runs a balanced budget. Saddle point stability implies that the initial value of consumption at time zero, $C(0)$, on the stable manifold is uniquely determined, given initial values of physical capital, of human capital and of public debt. Thus, indeterminate equilibrium paths are excluded.

Proposition 1 has shown that the economy with a balanced government budget is saddle point stable. For the case of permanent deficits, however, it turns out that this does not necessarily hold. Before we deal with the latter question we first give a lemma that characterizes the debt to physical capital ratio on the BGP.

Lemma 1 *On the BGP, the debt to physical capital ratio in the economy with permanent deficits is given by*

$$b^\star = \left(\frac{\vartheta}{\rho - \varrho}\right) (h^\star)^\alpha \left(W^{\sigma/(\sigma-1)}\right)^\alpha .$$

Proof: Immediately obtained from $\dot{C}/C = \dot{B}/B$.

From lemma 1 one realizes that $b^\star > 0$ implies either $\vartheta < 0$, $\rho < \varrho$ or $\vartheta > 0$, $\rho > \varrho$. The latter, however, goes along with an explosive public debt to capital ratio, as will be demonstrated in proposition 3.[6]

Given lemma 1 we know how the choice of the fiscal parameters determines the debt ratio in the long-run and we can use this result to analyze the question under which

[5]Propositions 1 and 4 are similar to two results in Greiner (2016), although the model structure there differs from ours. We include them for sake of completeness.

[6]Therefore, in 4.1.2 we set $\vartheta = -0.05$, $\rho = 0.035$ and vary ϱ starting with $\varrho = 0.1$. Then, $b^\star$ does not become negative neither explosive.

conditions a BGP exists when the government does not run a balanced budget. The next proposition 2 gives sufficient conditions for the existence and uniqueness of a BGP in the case of permanent deficits.

Proposition 2 *The conditions $\vartheta < 0$ and $\rho < \varrho$ are sufficient, but, not necessary for the existence of a unique BGP in the case of permanent public deficits.*

Proof: See appendix.

The condition $\vartheta < 0$ states that the level of the primary surplus declines as GDP rises, a fact that is suppported by empirical data (Greiner and Fincke, 2015). Further, a large reaction coefficient, $\varrho > \rho$, means that the government puts a high weight on stabilizing public debt.

Proposition 2 gives sufficient conditions for the existence of a BGP and it turns out that the BGP is expected to be stable under these conditions, as the next proposition 3 demonstrates. However, it is not possible to give exact results as regards stability of a BGP for the analytical model economy with permanent public deficits. But, we can give an outcome assuming that consumption and human and physical capital grow at the same rate. This is the content of proposition 3.

Proposition 3 *Assume that consumption and human and physical capital grow at the same constant rate g. Then, the debt to capital ratio remains bounded if and only if the reaction coefficient ϱ exceeds the difference between the net interest rate on government bonds and the growth rate.*

Proof: See appendix.

Proposition 3 has an intuitive economic interpretation. Public debt grows at the net interest rate minus the reaction coefficient ϱ while physical capital grows at the rate g. Hence, the ratio remains bounded if the reaction coefficient is sufficiently large, meaning that it must exceed the difference between the net interest rate on public debt and the growth rate of physical capital. Thus, the government must set the reaction coefficient sufficiently large in order to prevent the debt to capital ratio from exploding.

The next proposition compares the balanced growth rate of the aggregate economy with a balanced government budget to the balanced growth rate of an economy where the government runs permanent deficits.

Proposition 4 *The long-run growth rate in the economy with a balanced government budget exceeds the one of the economy with permanent public deficits.*

Proof: See appendix.

Proposition 4 demonstrates that the economy with a balanced government budget gives rise to higher aggregate growth compared to the economy with permanent public deficits. The economic rationale behind that outcome is that the primary surplus in the case of permanent deficits must be higher than in the case of balanced government budget to fulfill the inter-temporal budget constraint. This implies that the government has less financial scope such that less resources are available for educational spending so that the balanced growth rate takes a lower value.

4 Distribution and welfare

In this section we want to study how fiscal policy affects long-run growth and the distribution of welfare of the skilled and of the low-skilled household on the BGP and along the transition path. We investigate the transition from a balanced government budget scenario to a permanent public deficit scenario and its influence on the households' welfare.

From the equations of our model, we identify that aggregate consumption evolves according to equation (12) with r given by (17) that contains $h = h_c/K$, so that the growth rate of aggregate consumption on the BGP is given by (25). It becomes clear that these equations are fundamental for the analysis of the different policy rules. Here, we realize that a policy measure that increases $h = h_c/K$ or on the BGP, $h^\star$, leads to a higher growth rate of consumption in the long-run or on the BGP. The policy measures that we will investigate, only influence the growth rate of consumption through $h = h_c/K$, whereas for example τ_k also directly impacts the growth rate of consumption. Rising τ_k can have both a negative direct effect on the long-run growth rate due to $(1 - \tau_k)$ in equation (25) and a positive indirect effect because the tax revenue increases which is also used to finance public educational spending. Therefore, $h^\star$ rises and positively affects the long-run growth rate. Thus, an inverted U-shaped relationship between growth and τ_k emerges. This has been frequently shown for this class of growth models so that we do not go further into the details.

In our analysis we presume that $\theta \in (0, 1)$ gives the constant share of the capital stock of the skilled household relative to the total capital stock, i.e. $K_s = \theta K$. Thus, the budget constraint of the skilled household on the BGP can be written as

$$\frac{\dot{K}_s}{K_s} + \frac{\dot{B}}{B}\frac{B}{K_s} = g\left(1 + \frac{B}{\theta K}\right) = g\left(1 + \frac{b}{\theta}\right). \tag{32}$$

Using the income of the skilled household from (2), we obtain the level of initial consumption of the skilled household relative to capital on the BGP as

$$\frac{C_s(0)}{K_0} = ((1-\tau_k)r - \delta - g)(b+\theta) + (1-\tau_s)h^{\alpha}\alpha W^{-1+\alpha\sigma/(\sigma-1)}\gamma u^{-1/\sigma}L^{(\sigma-1)/\sigma}. \tag{33}$$

Finally, using $C(0) = C_s(0) + C_n(0)$ gives the initial level of consumption of the low-skilled household that allows to compute welfare on the BGP by

$$F_s = \frac{g}{\rho^2} + \frac{\ln C_s(0)}{\rho}, \tag{34}$$

and

$$F_n = \frac{g}{\rho^2} + \frac{\ln C_n(0)}{\rho}. \tag{35}$$

As regards welfare of the overall economy, F, we assume a Bergson welfare function such that it is the sum of the welfares of the high-skilled and of the low-skilled household on the BGP, $F = F_s + F_n$.

We see that the initial level of consumption (of the high-skilled and the low-skilled household), $C(0) = C_s(0) + C_n(0)$, and the long-run growth rate in the economy, g, positively influence welfare in our economy. From equation (33), we can identify how the different fiscal policy rules affect $C_s(0)$ and from this we can investigate how $C_n(0)$ is influenced. We explained above how the growth rate of consumption in the long-run or on the BGP is affected by the policy measures. Theses explanations are needed to understand the effects on welfare, as in this section we analyze how specific policies influence the variables, g, $C_s(0)$, $C_n(0)$ and, therefore, welfare. We would like to describe the economic mechanism behind the implementation of these policy measures and their different effects, especially on welfare, for the whole economy and for the two households.

As the analytical model turns out to become too complex to derive further results, we resort to simulations and, therefore, we continue to analyze our model numerically. As a benchmark for our simulations, we refer for some parameter values to Greiner (2016) and to Greiner and Fincke (2015) and set $\theta = 0.75$. The household's rate of time preference/the subjective discount rate is set to 3.5 percent, i.e., $\rho = 0.035$. We set the other parameter values as follows:

The depreciation rate of capital, $\delta = 0.075$, the capital and labor income tax rates, $\tau_k = 0.15$, $\tau_s = 0.3$, $\tau_n = 0.2$, the parameter to determine the spill-over effect of high-skilled labor implying that, due to externalities, low-skilled labor benefits to a certain degree from the human capital of high-skilled labor, $\xi = 0.15$, the elasticity of production with respect to labor, $\alpha = 0.7$, the share of high-skilled and low-skilled labor in production $\gamma = 0.6$, the share of the high-skilled labor force that is used for the production of final goods, $u = 0.8$, the efficiency of the inputs, $\epsilon = 0.1$, the elasticity of human capital formation with respect to the number of high-skilled people in that sector, $\phi = 0.7$, the total number of students, $S = 1$, the high-skilled labor, $L = 1$ and the low-skilled labor, $N = 1$. Those parameters are left unchanged throughout our simulations.

Then, we analyze the effects resulting from increasing public educational spending and the effects of raising transfer payments by varying the policy parameters κ and t_r, respectively, for the BGB scenario and for the PPD scenario. Further, we study the effects of different debt and deficit policy scenarios that we obtain by setting ϱ and ϑ, correspondingly. In the tables, we report the results of our simulations for a given value of σ in combination with values for the policy parameters and the two scenarios. The long-run growth rate g is indicated, and $b^{\star}$, $h^{\star}$, and $c^{\star}$ are also stated, as well as the welfare of the two households.

Further, we change the structural parameter σ, the elasticity of substitution between high-skilled and low-skilled labor, to investigate if varying this parameter qualitatively affects the results of the policy rules.

The stability of the dynamic system is determined by the eigenvalues of the Jacobian matrix evaluated at the BGP. Taking the initial human capital stock and the initial public debt as given, respectively as pre-determined variables, and if there exists a unique balanced growth path, one positive and two negative eigenvalues[7] of the corresponding Jacobian matrix indicate saddle point stability, implying that there exists a unique value of initial consumption, set at $t = 0$ to $c(0)$, such that the economy is on the stable manifold and converges to the BGP in the long-run. The model is determinate in this case.

[7]Negative (positive) real or complex conjugate with a negative (positive) real part.

4.1 Fiscal policy on the BGP

In this subsection we analyze the distribution of welfare of our model on the BGP for the balanced government budget (BGB) scenario and for the permanent public deficit (PPD) scenario.

4.1.1 Balanced Government Budget

To model the balanced budget rule, we set $\varrho = 0.85 \cdot r$ and $\vartheta \approx 0$.[8] From (22), one immediately realizes that this implies $\dot{B} = 0$. Such a situation is sustainable, and we can even speak of strong sustainability in this case since the government balances its budget. It should be noted that the debt to physical capital ratio asymptotically converges to zero in this case so that the ratio of public debt to physical capital equals zero on the BGP, i.e. $b^\star = 0$ holds.

With the parameter setting stated above with the elasticity of substitution between high-skilled and low-skilled labor, $\sigma = 0.75$, and $\varrho = 0.85 \cdot r$, $\vartheta \approx 0$, we see that for $\kappa = 0.8$ and $t_r = 0.1$ there exists a unique BGP for the balanced budget scenario. We get the following values, $b^\star = 0$, as stated above due to $\dot{B} = 0$, $h^\star = 0.965617$ and $c^\star = 0.325922$. The positive $c^\star$ states that on the BGP for the balanced budget scenario, we have a positive ratio of consumption to capital and a positive $h^\star$ which signifies that on the BGP the ratio of human (per capita) capital to physical capital is positive, as well. The long-run growth rate g for the stated parameter setting with $\sigma = 0.75$, is $g = 0.0129087$.

To analyze stability of the BGP, we calculate the eigenvalues of the Jacobian evaluated at the balanced growth path. We see that the unique balanced growth path is saddle point stable (one positive and two negative eigenvalues), thus, confirming the result of proposition 1. Furthermore, we get the welfare of the skilled and of the low-skilled household on the BGP with $F_s = -41.3568$ and $F_n = -41.2391$. The sum of both welfare values is the overall welfare of the economy $F = -82.5958$. Note that due to setting $K_0 = 1$ we get negative welfare values which does not pose a problem as we compare welfare levels within groups only.

Next, we set $\sigma = 0.25$ and do the same calculations. For the parameter setting stated above, the long-run growth rate g now is $g = 0.0120387$ and the welfare of the skilled and of the low-skilled household on the BGP are $F_s = -91.2104$ and $F_n = -26.4145$. The

[8] We set $\vartheta = 10^{-12}$ to avoid division by 0 in the Jacobian matrix.

total welfare of the economy is $F = -117.625$. Thus, our model yields a lower long-run growth rate g, lower welfare of the skilled household, higher welfare for the low-skilled household and smaller welfare for the aggregate economy, compared to the parameter setting with $\sigma = 0.75$. Thus, we can state that a lower elasticity of substitution between high-skilled and low-skilled labor makes the first worse off while the second benefits. The tables for $\sigma = 0.25$ can be found in the Appendix.

In a next step, we study our model where the government varies public educational spending and transfer payments, respectively, by varying κ and t_r with $\sigma = 0.75$ for the BGB scenario. Table 1 shows how $b^\star$, $h^\star$, $c^\star$, g, the stability of the BGP and welfare of the two households react when the government changes the public education spending coefficient κ. Table 1 demonstrates that $b^\star = 0$ for the values of $0.4 \leq \kappa \leq 1$, due to the balanced budget rule.

We can see that increasing public education spending, modeled by a higher κ, implies a rise of $h^\star$, i.e. higher public education spending implies more human capital in the long-run. A rise of the public education spending signifies that the government reduces public consumption, G, given by $G = (1 - \kappa)T$ with $\kappa \in (0, 1)$ which means that there is less unproductive spending of the government. Thus, a higher κ means that more tax revenue is available for other types of spending such as public educational spending, considering equation (23). This leads to an increase in the long-run growth rate g with $\sigma = 0.75$ and for $0.4 \leq \kappa \leq 1$. The rise in human capital plays a fundamental role in determining g. Further, we can identify that raising κ goes along with a higher $c^\star$. The economic mechanism behind that result is that less public consumption results in more private consumption, too. From equation (24), we see that when increasing the public education spending κ, $c = C/K$ can rise, as well.

It should be pointed out that sustained growth is not possible when public education falls short of a certain threshold, with our parameter values for κ smaller about 0.4. In that case, the government does not invest sufficiently in the formation of human capital.

From proposition 1 we know that the BGB scenario has a unique saddle point stable BGP. This is confirmed by table 1, where the eigenvalues are given (one positive and two negative eigenvalues).

The welfare of the skilled and of the low-skilled household on the BGP and consequently the overall welfare of the economy rise when productive public spending goes up, i.e. κ increases. If the government spends more for education, the skilled household has higher benefits due to an increasing consumption at $t = 0$, $C_s(0)$, and due to a higher long-run growth rate g. The welfare of the low-skilled household on the BGP increases when raising the public education spending κ because $C_n(0)$, the consumption at $t = 0$ for the low-skilled household, and the long-run growth rate, g, rise. Higher education spending leads to benefits for both households and, therefore, it raises welfare of the whole economy. In the Appendix, we find the corresponding table that shows qualitatively the same results for the parameter setting with $\sigma = 0.25$.

Table 1: Public education spending via κ with $\sigma = 0.75$

κ	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1				no BGP				
0.2				no BGP				
0.3				no BGP				
0.4	0	0.886308	0.304596	0.00575199	(+,-,-)	-48.6293	-49.5249	-98.1541
0.5	0	0.922349	0.313351	0.00902708	(+,-,-)	-45.2923	-45.8915	-91.1838
0.6	0	0.941063	0.318436	0.0107125	(+,-,-)	-43.5809	-43.9292	-87.5102
0.7	0	0.954662	0.322453	0.011931	(+,-,-)	-42.3461	-42.4575	-84.8036
0.8	0	0.965617	0.325922	0.0129087	(+,-,-)	-41.3568	-41.2391	-82.5958
0.9	0	0.974913	0.329052	0.0137358	(+,-,-)	-40.5208	-40.1794	-80.7002
1	0	0.983056	0.331948	0.0144583	(+,-,-)	-39.7913	-39.2303	-79.0216

In a next step we study how transfer payments via the coefficient t_r influence the main indicators on the BGP and welfare of the skilled and of the low-skilled household. Table 2 illustrates how $b^\star$, $h^\star$, $c^\star$, g, the stability of the BGP and welfare of the two households react when the government changes the transfer payments coefficient t_r, where $\sigma = 0.75$ is set.

Table 2 shows that $b^\star = 0$ for $0.1 \leq t_r \leq 0.58$, due to the balanced budget rule, which means that we set $\varrho = 0.85 \cdot r$ and $\vartheta \approx 0$. We note that increasing transfer payments modeled by t_r reduces $h^\star$, i.e., higher transfer payments implicates less human capital in the long-run. The ratio of human (per capita) capital to physical capital, $h^\star$, on the BGP decreases with a higher t_r. When analyzing equation (23), we identify that with higher transfer payments the government spends less on public education and that is why $h^\star$ declines. Furthermore, we see that raising transfer payments via the coefficient t_r first means an increase in $c^\star$, but shortly before there is no BGP, $c^\star$ decreases. Higher transfer payments lead to more consumption in the economy on the BGP up to a certain value of t_r. A rise in t_r indicates a decrease in the long-run growth rate g for the stated parameter setting with $\sigma = 0.75$ and for $0.1 \leq t_r \leq 0.58$. The decrease in human capital plays a fundamental role in determining g, see equation (25).

For each value of t_r with the given parameter setting, the eigenvalues of the Jacobian matrix evaluated at the sustainable balanced growth path of the system are calculated and the stabilities of the BGP are indicated. Table 2 demonstrates that for $0.1 \leq t_r \leq 0.58$, the eigenvalues indicate that the economy is saddle point stable (one positive and two negative eigenvalues).

As a last step in this subsection, the welfare of the high-skilled and of the low-skilled household on the BGP and the overall welfare of the economy are analyzed. We see that the welfare of the skilled household on the BGP declines when transfer payments via the coefficient t_r rise. If the government puts more emphasis on transfer payments, then the skilled household has more disadvantages, due to the fact that the high-skilled household does not receive transfer payments and due to the declining long-run growth rate g that is influenced by the decreasing $h^\star$ and the declining consumption at $t = 0$ for the high-skilled household, $C_s(0)$. The low-skilled household obtains the transfer payments and its welfare on the BGP increases when raising the transfer payments modeled by t_r up to a certain value.

Higher transfer payments via the coefficient t_r imply lower growth, g, which ceteris paribus leads to lower welfare for the low-skilled household. However, higher transfer payments have a positive income effect for the low-skilled household and $C_n(0)$, the consumption at $t = 0$ for the low-skilled household, rises which ceteris paribus yields a positive welfare effect. First, for low transfer payments, the positive welfare effect of higher initial consumption for the low-skilled household, $C_n(0)$, dominates, then, for higher transfer payments, the negative effect of lower growth, g, dominates. As a consequence, an inverted U-shaped relationship between transfer payments and welfare for the low-skilled household emerges. Skilled households do not get transfer payments, therefore they experience only the negative welfare effect resulting from a lower long-run growth rate g and the declining consumption at $t = 0$ for the skilled household, $C_s(0)$ plays also an important role. Thus, the skilled household has only disadvantages when increasing transfer payments via the coefficient t_r. A crucial finding is that the overall welfare of the economy, when summing up welfare of both households, decreases with a rise in transfer payments and, thus, brings disadvantages to the whole economy. In the Appendix, we find further tables as the ones described above, with the stated parameter setting for $\sigma = 0.25$. Here, we can identify that qualitatively, we get the same results with $\sigma = 0.25$. Next, we study our model when the government runs permanent public deficits.

Table 2: Transfer payments via t_r with $\sigma = 0.75$

t_r	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0	0.965617	0.325922	0.0129087	(+,-,-)	-41.3568	-41.2391	-82.5958
0.2	0	0.955985	0.333138	0.0120492	(+,-,-)	-42.2264	-40.5459	-82.7722
0.3	0	0.944398	0.339776	0.0110118	(+,-,-)	-43.2774	-40.1235	-83.4009
0.4	0	0.929457	0.345462	0.0096685	(+,-,-)	-44.6405	-40.0984	-84.739
0.5	0	0.90686	0.349074	0.00762438	(+,-,-)	-46.7196	-40.8799	-87.5995
0.55	0	0.886308	0.348461	0.00575199	(+,-,-)	-48.6293	-42.1916	-90.8209
0.58	0	0.849989	0.341969	0.00241089	(+,-,-)	-52.0498	-45.3326	-97.3824
0.6				no BGP				
0.7				no BGP				
0.8				no BGP				
0.9				no BGP				
1				no BGP				

4.1.2 Permanent Public Deficits

In this subsubsection, we allow permanent public deficits where the government debt grows at the same rate as all other endogenous variables in the long run. Since the government sets the primary surplus according to equation (21), it does not play a Ponzi game in this case, but fulfills the intertemporal budget constraint. This situation can be called weak sustainability since it only guarantees that the government does not play a Ponzi game but public debt grows at the same rate as GDP in the long run, i.e., $\frac{\dot{B}}{B} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{h}_c}{h_c} = g$ holds (Greiner and Fincke, 2015). We limit the analysis to the case $\varrho > 0$, in order to not violate the intertemporal budget constraint of the government along a balanced growth path (Greiner and Fincke, 2015). In lemma 1 and its explanations, we can identify how the policy parameters ϱ and ϑ have effects on the debt to physical capital ratio on the BGP. As the model becomes rather complex, such that it is difficult to gain analytical results, we again resort to simulations in order to gain insights into our model economy.

We study how public debt policy via the reaction coefficient ϱ affects the main indicators on the BGP, the stability of the BGP and welfare on the BGP. Table 3 shows how $b^\star$, $h^\star$, $c^\star$, g, the stability of the BGP and welfare of the two households react when the government changes the reaction coefficient ϱ, that means when it changes the weight of stabilizing public debt, where $\sigma = 0.75$, $\vartheta = -0.05$, $\kappa = 0.8$ and $t_r = 0.1$ are set. Of course, $b^\star$ declines as the government puts more weight on stabilizing public debt, that is when it increases the reaction coefficient ϱ.

We can also see that raising the reaction coefficient ϱ implies a decrease in $c^\star$. The economic mechanism behind that result is that a stricter public debt policy leads to a lower debt to physical capital ratio on the BGP. As the high-skilled household owns public debt, a lower public debt ratio implies a negative wealth or income effect for the high-skilled household. This leads to less consumption of that group which induces a lower aggregate consumption share on the BGP, too.

Table 3 shows that raising the reaction coefficient ϱ increases $h^\star$ and, thus, the long-run growth rate g. With a stricter public debt policy, modeled by a higher value for ϱ, the debt to physical capital ratio on the BGP is reduced and we can see from the table that more human capital in the long-run in our model is observed. The reason is that a lower debt to physical capital ratio on the BGP signifies that the government needs less resources for the debt service and, therefore, it has more resources left for public educational spending, I_e. Increasing the reaction coefficient ϱ leads to a rise in the long-run growth rate g for the stated parameter setting with $\sigma = 0.75$.

For each value of ϱ, the eigenvalues of the Jacobian matrix evaluated on the BGP of the system are calculated. In Table 3, we can observe that for $0.15 \leq \varrho \leq 0.2$ the economy is saddle point stable with one pair of complex conjugate eigenvalues with a real negative part and one positive real eigenvalue. For higher values of ϱ, the eigenvalues are again real with one being positive and two negative.

The welfare of the high-skilled household decreases when the reaction coefficient ϱ rises because it owns public debt and with increasing ϱ, the government reacts stronger to public debt. Even though a higher long-run growth rate g ceteris paribus results in higher welfare, the negative wealth effect for the high-skilled household generated through lower public debt implies a lower consumption at $t = 0$ for the high-skilled household, $C_s(0)$, and, thus, lower welfare of the high-skilled household. The latter effect dominates the positive welfare effect of a higher long-run growth rate g. However, the welfare of the low-skilled household on the BGP increases when the reaction coefficient ϱ rises. As the long-run growth rate g and $C_n(0)$, the consumption at $t = 0$ for the low-skilled household, increase when the government puts more weight on stabilizing public debt, there is a rise in welfare of the low-skilled household on the BGP. The government needs less resources for the debt service and spends more for public education and higher transfer payments. The overall welfare of the economy rises when the reaction coefficient ϱ increases, due to the lower debt to physical capital ratio on the BGP which leads to more public education spending and, thus, to higher growth. If the government puts more weight on stabilizing public debt, then the high-skilled household has lower welfare, whereas the low-skilled household has higher welfare and it brings welfare gains for the whole economy. In the Appendix, we find the corresponding table that shows qualitatively the same results for the parameter setting with $\sigma = 0.25$.

Table 3: Public debt policy via ϱ with $\sigma = 0.75$, $\vartheta = -0.05$, $\kappa = 0.8$ and $t_r = 0.1$

ϱ	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0.36702	0.95172	0.33583	0.01167	(+,-,-)	-40.4229	-42.5267	-82.9496
0.15	0.2084	0.95817	0.33164	0.01224	(+, -0.13 ± i)	-40.7696	-41.928,	-82.6976
0.2	0.14552	0.96053	0.32994	0.01245	(+, -0.15 ± i)	-40.9323	-41.7098	-82.6421
0.25	0.11178	0.96175	0.32902	0.01257	(-,+,-)	-41.0249	-41.5966	-82.6215
0.3	0.09074	0.9625	0.32844	0.01263	(-,+,-)	-41.0844	-41.5274	-82.6118
0.4	0.06592	0.96337	0.32776	0.01271	(-,+,-)	-41.1565	-41.4469	-82.6033
0.5	0.05176	0.96386	0.32736	0.01275	(-,+,-)	-41.1984	-41.4015	-82.5999
0.6	0.04261	0.96417	0.32711	0.01278	(-,+,-)	-41.2258	-41.3724	-82.5982
0.7	0.0362	0.96439	0.32693	0.0128	(-,+,-)	-41.2451	-41.3521	-82.5973
0.8	0.03148	0.96455	0.3268	0.01281	(-,+,-)	-41.2595	-41.3372	-82.5967
0.9	0.02784	0.96468	0.3267	0.01283	(-,+,-)	-41.2706	-41.3258	-82.5964
1	0.02496	0.96478	0.32662	0.01283	(-,+,-)	-41.2794	-41.3167	-82.5962

Now, we study how public education spending affects the main indicators on the BGP, its stability and welfare of the skilled and of the low-skilled household under the permanent public deficit scenario. Table 4 demonstrates how $b^\star$, $h^\star$, $c^\star$, g, the stability of the BGP and welfare of the two households react when the government changes the education spending coefficient κ, where $\sigma = 0.75$, $\vartheta = -0.05$, $\varrho = 0.1$ and $t_r = 0.1$ are set. We see that $b^\star$ augments as the government allows permanent public deficits where debt grows at the same rate as all other endogenous variables in the long-run. We identify that increasing public education spending κ leads to a rise of $h^\star$, i.e. higher education spending results in more human capital in the long-run in our model. On the BGP the ratio of human (per capita) capital to physical capital, $h^\star$, increases with a higher κ due to equation (23). In addition, we note that raising education spending via the coefficient κ indicates an increase in $c^\star$. The economic mechanism behind that result is that a rise of κ signifies that the government generates lower public consumption, G, given by $G = (1 - \kappa)T$ with $\kappa \in (0, 1)$ which means that there is less unproductive spending of the government. Lower public consumption implies higher productive public educational spending and higher private consumption in the economy on the BGP, i.e. when increasing the public education spending κ, $c = C/K$ rises as well, see (24). A rise in public education spending κ signifies an increase in the long-run growth rate g for the stated parameter setting with $\sigma = 0.75$ and for $0.6 \leq \kappa \leq 1$. The increase in human capital plays a fundamental role in determining g.

We calculate for each value of κ, the eigenvalues of the Jacobian matrix evaluated at the BGP of the system and the stabilities of the BGP are indicated. In Table 4, we notice that that the economy is saddle point stable with one positive and two negative real eigenvalues and one positive and a pair of complex conjugate eigenvalues with negative real part for $\kappa = 0.5$, respectively.

The results are similar to the ones of the BGB scenario and, thus, we can observe that the welfare of the skilled and of the low-skilled household on the BGP and, consequently, the overall welfare of the economy rise when productive public spending κ rises, as in the BGB scenario. If the government spends more for education, the skilled household has higher welfare due to an increasing consumption at $t = 0$, $C_s(0)$, and a higher long-run growth rate g. The welfare of the low-skilled household on the BGP increases, too, when raising the public education spending κ because $C_n(0)$, the consumption at $t = 0$ for the low-skilled household, and the long-run growth rate, g, rise. As in the BGB scenario, we identify for the PPD that higher public education spending leads to benefits for both households and therefore it brings welfare gains for the whole economy. In the Appendix, we find the table for the parameter setting with $\sigma = 0.25$ showing qualitatively the same results.

Table 4: Public education spending via κ with $\sigma = 0.75$, $\vartheta = -0.05$, $\varrho = 0.1$ and $t_r = 0.1$

κ	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1				no BGP				
0.2				no BGP				
0.3				no BGP				
0.4				no BGP				
0.5	0.34177	0.85959	0.31192	0.0033	(+, -0.08 ± i)	-48.9748	-51.896	-100.871
0.6	0.35744	0.91643	0.32573	0.00849	(+,-,-)	-43.6555	-46.2427	-89.8982
0.65	0.3606	0.92802	0.32887	0.00954	(+,-,-)	-42.5879	-45.0475	-87.6354
0.7	0.3631	0.93723	0.33148	0.01037	(+,-,-)	-41.7444	-44.0821	-85.8265
0.8	0.36702	0.95172	0.33583	0.01167	(+,-,-)	-40.4229	-42.5267	-82.9496
0.9	0.37011	0.96318	0.33953	0.01269	(+,-,-)	-39.3833	-41.2609	-80.6442
1	0.3727	0.97282	0.34282	0.01355	(+,-,-)	-38.5137	-40.1706	-78.6843

To complete our analysis, we investigate how transfer payments influence the main indicators on the BGP, the stability of the BGP and welfare of the skilled and of the low-skilled household on the BGP. In table 5 we again identify how $b^\star$, $h^\star$, $c^\star$, g, the stability of the BGP and welfare of the two households react when the government changes the transfer payments coefficient t_r, where $\sigma = 0.75$, $\vartheta = -0.05$, $\varrho = 0.1$ and $\kappa = 0.8$ are set. We observe that for $0.1 \leq t_r \leq 0.4$, increasing transfer payments modeled by t_r, leads to a decrease of $h^\star$, i.e. higher transfer payments implies less human capital in the long-run. The ratio of human (per capita) capital to physical capital, $h^\star$, on the BGP decreases with a higher value of t_r. When analyzing equation (23), we identify that with higher transfer payments the government spends less on public education and that is the reason why $h^\star$ declines. Table 5 illustrates that $b^\star$ for the values of $0.1 \leq t_r \leq 0.4$ decreases when t_r rises. From lemma 1 we see that the decrease in $b^\star$ can be explained by the decline in $h^\star$. Furthermore, we see that raising transfer payments via the coefficient t_r first means an increase in $c^\star$, but shortly before the BGP does not exist any longer, $c^\star$ decreases. Higher transfer payments lead to more consumption in the economy on the BGP up to a certain t_r-value. Raising t_r results in a decline of the long-run growth rate g for the stated parameter setting with $\sigma = 0.75$ and for $0.1 \leq t_r \leq 0.4$.

For each t_r-value with the given parameter values, the eigenvalues of the Jacobian matrix evaluated at the sustainable balanced growth path of the system are calculated and the stabilities of the BGP are shown. Table 5 displays that for $0.1 \leq t_r \leq 0.4$ the eigenvalues of each t_r with the given parameter setting demonstrate that the economy is saddle point stable (one positive and two negative eigenvalues).

To finish this subsection, we analyze the welfare of the skilled and of the low-skilled household on the BGP and the overall welfare of the economy. We observe that the welfare of the skilled household on the BGP decreases when transfer payments rise. As the high-skilled household does not obtain transfer payments and as the long-run growth rate g declines and the consumption at $t = 0$ for the high-skilled household, $C_s(0)$, declines, the skilled household suffers welfare losses if the government raises transfer payments. Contrary to the high-skilled household, the low-skilled household gets the governmental transfer payments and, therefore, its welfare on the BGP rises when increasing the transfer payments up to a certain value and then it falls again.

For the PPD scenario, we can conclude as well that higher transfer payments imply lower growth, g, which ceteris paribus results in lower welfare for the low-skilled household. However higher transfer payments have a positive income effect for the low-skilled household and $C_n(0)$, the consumption at $t = 0$ for the low-skilled household, rises which generates a positive welfare effect ceteris paribus. First, for low transfer payments, the positive welfare effect of higher initial consumption for the low-skilled household, $C_n(0)$, dominates, then, for higher transfer payments, the negative effect of lower growth, g, dominates. Consequently, an inverted U-shaped relationship between transfer payments and welfare for the low-skilled household emerges. The high-skilled household does not get transfer payments. Therefore, only the negative welfare effect exists which results from a lower long-run growth rate g and from the declining consumption at $t = 0$ for the skilled household, $C_s(0)$. Thus, the skilled household has only disadvantages when increasing transfer payments via the coefficient t_r. It should be noted that the overall welfare of the economy, when summing up welfare of both households, declines with a rise in transfer payments. In the Appendix, we report the outcomes for $\sigma = 0.25$, demonstrating that the results remain unchanged qualitatively. In a next step, we analyze our model along the transition path.

Table 5: Transfer payments via t_r with $\sigma = 0.75$, $\vartheta = -0.05$, $\varrho = 0.1$ and $\kappa = 0.8$

t_r	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0.36702	0.95172	0.33583	0.01167	(+,-,-)	-40.4229	-42.5267	-82.9496
0.2	0.3636	0.93905	0.34216	0.01053	(+,-,-)	-41.5774	-42.1236	-83.701
0.3	0.35895	0.92198	0.34726	0.00899	(+,-,-)	-43.1442	-42.2281	-85.3723
0.4	0.35075	0.89201	0.34893	0.00627	(+,-,-)	-45.9239	-43.6543	-89.5783
0.43	0.34494	0.871	0.34633	0.00435	(+, -0.095 $\pm$ i)	-47.8957	-45.265	-93.1608
0.5				no BGP				
0.6				no BGP				
0.7				no BGP				
0.8				no BGP				
0.9				no BGP				
1				no BGP				

4.2 Welfare effects of switching from the BGB scenario to the PPD scenario

In this subsection, we analyze the welfare effects resulting from a transition from the Balanced Government Budget scenario (BGB scenario) to the Permanent Public Deficits scenario (PPD scenario), where public debt grows at the same rate as all other endogenous variables in the long run, taking into account transition dynamics. To investigate the welfare effects accounting for the transition path, we assume that the economy is originally on the BGP in the BGB scenario when the government decides to change to the new PPD scenario from $t = 0$ onwards. Therefore, we set ϱ and ϑ as in section 4.1. In section 4.1, we set $\varrho = 0.85 \cdot r$ and $\vartheta \approx 0$ for the BGB scenario and for the PPD scenario we set $\varrho = 0.3$ and $\vartheta = -0.05$ since for these policy parameters and for $\sigma = 0.75$, $\kappa = 0.8$ and $t_r = 0.1$, the model is characterized by a saddle point and the Jacobian matrix with the BGP values from above has two negative real eigenvalues (see Table 3).

To study the effects of a change from one scenario to the other, we examine the solution of the linearized system of equations (28), (29), (30) which is given by

$$b(t) = b^{\star} + C_1 v_{11} e^{(ev_1)t} + C_2 v_{21} e^{(ev_2)t}, \tag{36}$$

$$h(t) = h^{\star} + C_1 v_{12} e^{(ev_1)t} + C_2 v_{22} e^{(ev_2)t}, \tag{37}$$

$$c(t) = c^{\star} + C_1 v_{13} e^{(ev_1)t} + C_2 v_{23} e^{(ev_2)t}, \tag{38}$$

with v_{jl} the l-th element of the eigenvector belonging to the negative real eigenvalue ev_j, $j = 1, 2$ and C_j, $j = 1, 2$, are constants determined by the initial conditions $h(0)$ and $b(0)$. Setting $t = 0$ gives C_j, $j = 1, 2$, as a function of $h(0)$ and $b(0)$. Inserting these C_j, $j = 1, 2$, in equation (38) with $t = 0$ gives the unique $c(0)$ on the stable manifold leading to the BGP of the new PPD scenario in the long-run. It is the initial value for c on the stable branch of the saddle point.

We analyze the effects that arise when the government switches from the BGB scenario to the PPD scenario, where the government runs into debt and the debt ratio becomes positive in the long-run. By comparing the welfare values of the two households, we identify which household benefits to a greater degree and if it is welfare increasing for the whole economy.

When the government runs into debt, the resulting debt service leads to a decline of productive public educational spending in the medium- to long-run, implying less human capital accumulation. As a consequence, the marginal product of capital declines which makes the households shift resources from investment to consumption such that the consumption share rises and the investment share declines which reduces the growth rate. The human capital growth rate will decline, too, in the medium- to long-run because of the resources needed for the debt service that cannot be spent in the educational sector. In the short-run, however, the growth rate of human capital will increase since a share of the deficit is used for additional educational spending, whereas the negative effects of the deficit occur only in the medium- to long-run as public debt rises.

To compute the welfare effects, we numerically calculate the following two expression of the households

$$\max_{C_s} \int_0^{\infty} e^{-\rho t} \ln C_s \, dt, \tag{39}$$

and

$$\max_{C_n} \int_0^{\infty} e^{-\rho t} \ln C_n \, dt. \tag{40}$$

We remember that $c(0) = C(0)/K_0$, where we set $K_0 = 1$. We get $c(0)$ from the solution of the system (28), (29), (30), which is the linearized system of the equations (36), (37), (38).

When analyzing the transition, we switch from the balanced growth path of the BGB scenario to the balanced growth path of the PPD scenario. With the parameter setting from above we obtain $C(0) = c(0) = 0.466265$ for the initial aggregate consumption.

Figure 1: c(t) along the transition path with $c(0) = 0.466265$

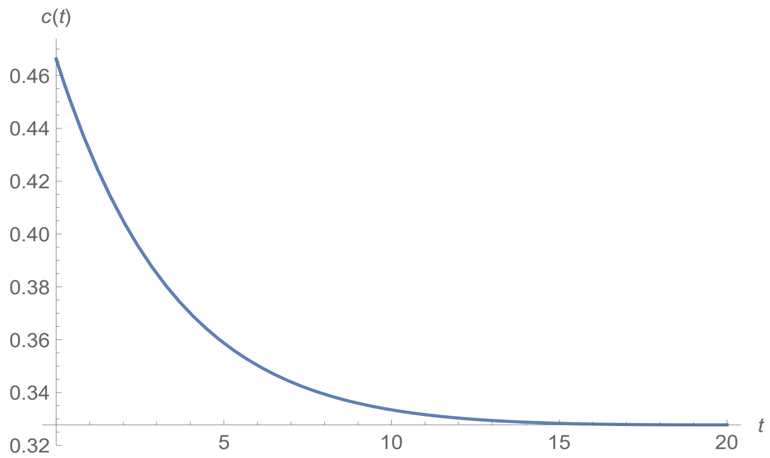


Figure 1 shows $c(t)$ along the transition path. The variable c jumps from the BGP value $c^\star = 0.325922$ in the BGB scenario to $c(0) = 0.466265$, the initial value for c on the stable branch of the saddle point, and converges to the new $c^\star$ of the PPD scenario, $c^\star = 0.328439$. This illustrates that c is a jump variable and the unique $c(0)$ on the stable manifold leads to the BGP of the new PPD scenario in the long-run.

Next, we compute the welfare of the two households resulting from the transition from the BGB scenario to the PPD scenario and the welfare on the BGP in the BGB scenario from 4.1.1 by calculating the two integrals (39) and (40). Thus, we need to compute the initial level of consumption of the skilled household, $C_s(0)$, and of the low-skilled household, $C_n(0)$, after the transition to the PPD scenario at $t = 0$, as well as the total time path of the two variables. To determine the initial levels of consumption, we use the budget constraint of the low-skilled household (7) that yields

$$\frac{C_n(0)}{K_0} = (1-\theta)r(1-\tau_k) + \left(\frac{w_n}{K}\right) N(1-\tau_n) + \frac{T_p}{K} - (1-\theta)\delta - (1-\theta)\, g_K(0), \tag{41}$$

with $g_K(0)$ the growth rate of aggregate capital given by (24), evaluated at $t = 0$, and where we used $K_n = (1 - \theta)K$ so that $\dot{K}_n/K_n = \dot{K}/K$ holds. Again, we set $K_0 = 1$ such that the intial level of consumption of the skilled household is obtained as $C_s(0) = C(0) - C_n(0)$.

Note that the growth rate of capital g_K is evaluated at $t = 0$, implying that the pre-determined variables h and b are equal to their BGP values of the BGB scenario and the jump variable takes the value on the stable manifold leading to the BGP of the PPD scenario, $c = 0.466265$. For our parameter setting with $\sigma = 0.75$, we calculate the initial level of consumption of the low-skilled household after the transition to the PPD scenario which occurs at $t = 0$, with (41) and we get $C_n(0) = 0.204407$ that is higher than the $C_n(0)$ of the BGB scenario. With $C(0) = c(0) = 0.466265$ and $C_n(0)$ after the transition, we calculate the initial level of consumption of the high-skilled household after the transition to the PPD scenario which occurs at $t = 0$ and get $C_s(0) = 0.261857$ which is also higher than the $C_s(0)$-value in the BGB scenario. Then, we numerically solve (11) for both households giving the respective time paths of consumption. Given the time paths of consumption, we can compute welfare by numerically calculating the two integrals (39) and (40). The results are shown in table 6.

Table 6: Welfare in BGB scenario and welfare resulting from a transition to PPD scenario

	BGB scenario	From BGB scenario to PPD scenario
F_s	-41.3568	-28.2135
F_n	-41.2391	-35.2903

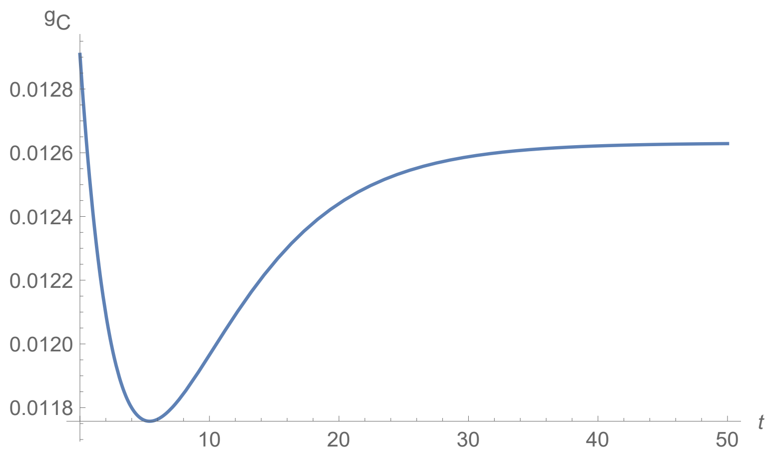
We see that the welfare of both households rises when the government switches from the BGB scenario to the PPD scenario. Further, the percentage increase of welfare of the high-skilled household is higher than the rise in welfare of the low-skilled household, i.e. the high-skilled household benefits more if the government changes its policy from the BGB scenario to the PPD scenario. Along the transition path, the debt to physical capital ratio increases because the government runs permanent deficits. As the high-skilled household owns public debt, we notice that the welfare of this households rises to a stronger degree than that of the low-skilled, when switching from the BGB scenario to the PPD scenario.

To get further insight, we consider figure 2 and see that the consumption growth rate g_C along the transition path first starts at $g_C = g = 0.0129$ of the BGB scenario and after overshooting the long-run growth rate, it converges to the new value $g_C = g = 0.0126$ of the PPD scenario. The long-run growth rate g then is on a slightly lower level than in the BGB scenario. This has a negative effect on the welfare of both households.

However, this is not the only effect that matters for the calculation of the welfare. Another crucial aspect is the level of initial consumption of the high-skilled household after the transition to the PPD scenario at $t = 0$, $C_s(0)$, and $C_n(0)$, the consumption at $t = 0$ of the low-skilled household after the transition. As pointed out above, the initial level of consumption of the high-skilled household, $C_s(0) = 0.261857$, is higher than the one of the low-skilled household, $C_n(0) = 0.204407$, which is the reason why the welfare of the high-skilled household increases to a stronger degree compared to the one of the low-skilled household. The decline of the consumption growth rate g_C affects both households equally and, therefore, only the difference between the $C_n(0)$ and $C_s(0)$ values matters for the difference in welfare changes.

The higher welfare effect for the high-skilled household can be explained by the positive income effect through issuing government debt such that $C_s(0)$ rises. The positive welfare effect of a higher initial level of consumption for the high-skilled household dominates the negative effect of lower consumption growth, g_C, that does not play an important role for the high-skilled household as initial consumption is much higher. Higher consumption for the high-skilled household leads to higher welfare for that household. The initial consumption levels, $C_n(0)$, $C_s(0)$, for both households are higher than in the BGB scenario, which compensates the effect of a decreasing consumption growth rate g_C and leads for the low-skilled to increasing welfare along the transition path, too, in comparison with the BGB scenario. Since the welfare of both households rises, the overall welfare of the economy also increases. For an extended robustness analysis, we analyze the model with a changed parameter setting that can be found in the Appendix, and we get the same outcome from a qualitative point of view which confirms the results from above.

Figure 2: Consumption growth rate g_C along the transition path



5 Conclusion

This paper contributes an endogenous growth model with publicly funded human capital accumulation and public debt, based on Greiner (2016), where we additionally allow for heterogeneous households. As in that paper, it turned out that the economy with a balanced government budget is characterized by a unique saddle point stable long-run growth path, with a growth rate that exceeds the one of the economy with permanent deficits. In addition, we derived a condition for the existence of a balanced growth path in the case of permanent public deficits and we could show that the debt ratio remains bounded if and only if the government reacts to higher public debt such that the reaction coefficients exceeds the difference between the net interest rate and the growth rate at the margin, a result that has not yet been derived in Greiner (2016) and that makes sense from an economic point of view.

As regards distribution and welfare we investigated the impact of fiscal policy on the long-run growth and welfare of the skilled and of the low-skilled household on the BGP under the sustainable debt policy scenarios and along the transition path. Under the BGB scenario we studied the effect of a change of public education spending on welfare of the skilled and of the low-skilled household on the BGP. When raising public educational spending, the welfare of the skilled and of the low-skilled household on the BGP rise and, consequently, the overall welfare of the economy, too, due to a higher initial consumption of both households and due to a higher long-run growth rate g. For the BGB scenario we analyzed how transfer payments via the coefficient t_r impacts welfare of the skilled and of the low-skilled household on the BGP. The welfare of the skilled household on the BGP declines when transfer payments rise because it does not receive transfer payments and the long-run growth rate declines. For the low-skilled household an inverted U-shaped relationship between transfer payments and welfare emerges. First, welfare rises because of higher consumption as a result of more transfer payments then, however, the negative effects of a lower growth rate dominates such that welfare declines. The overall welfare of the economy decreases with a rise in transfer payments.

In a next step, we analyzed for the PPD scenario how public debt policy affects the welfare on the BGP. When the reaction coefficient rises, implying a higher response of the government to rising debt, the welfare of the high-skilled household decreases because it owns public debt. Lower public debt implies a lower BGP consumption at $t = 0$ for the high-skilled household and, thus, ceteris paribus lower welfare. The latter effect dominates the positive welfare effect of a higher long-run growth rate g. The welfare of the low-skilled household on the BGP increases when the reaction coefficient rises. When the government puts more weight on stabilizing public debt, the long-run growth rate g and the initial BGP consumption at $t = 0$, increase. The government needs less resources for the debt service and has more for public education spending and more transfer payments are possible, too. The overall welfare of the economy rises when the reaction coefficient increases due to the higher long-run growth rate g.

For the PPD scenario, we studied the impact of varying educational spending on welfare of the skilled and of the low-skilled household on the BGP. The results are similar to the ones of the BGB scenario and so we identify for the PPD that higher public education spending leads to benefits for both households and therefore it brings welfare gains for the whole economy. Then, we again investigated under the PPD scenario the effect of transfer payments on welfare of both households on the BGP. The welfare of the skilled household on the BGP decreases when transfer payments rise for the same explanations as under the BGB scenario. As above, for the low-skilled household again an inverted U-shaped relationship between transfer payments and welfare for the low-skilled household emerges. The welfare of the whole economy declines as well when transfer payments rise.

In a final step, we analyzed the welfare effects along the transition path from the BGB scenario to the PPD scenario for both households and compared them to the welfare values in the BGB scenario. We identified that the highest welfare for a specific parameter setting is achieved for the high-skilled household by switching from the BGB scenario to the PPD scenario, as the high-skilled household owns public debt. The long-run growth rate g and the consumption at $t = 0$ for the high- and low-skilled household after the transition to the PPD scenario play an important role here. Higher initial levels of consumption for the households raise their welfare such that the overall welfare of the economy rises, too, when the government changes from the BGB scenario to the PPD scenario.

As regards policy recommendation, it depends on which goal the government pursues. If it wants to maximize growth, the recommendation is obvious. In that case the government should run a balanced budget and use most of its spending for investment in human capital, i.e. for educational spending. When the government intends to maximize welfare, things become more complicated. In the very long-run, i.e. when transition dynamics are neglected, high educational spending and low or zero debt ratios yield highest welfare. When the government intends to minimize the difference between high-skilled and low-skilled households it should realize low debt ratios because only the household that holds public debt experiences a positive income effect resulting from interest payments on debt. Further, care must be taken when the transfer payments are increased to raise welfare of the low-skilled household. This holds because more transfers can reduce the welfare of that household, when the transfers are already high, because higher transfers reduce the growth rate and, thus, welfare ceteris paribus. When transition dynamics are accounted for, however, a transition from a balanced government budget to permanent deficits, that implies lower long-run growth, can go along with welfare gains for both households because the rise of initial consumption is valued higher than consumption losses in the future.

This model has its limitations as it does not consider certain factors such as trade imbalances, migration, prices and exchange rates as the focus of this paper is on the distribution of welfare and, therefore, we have a clear motivation. Thus, policy recommendations should be made with care and not only be based on a single paper. Rather, the results of several studies should be taken into account and this paper is just one of them. An integration of for example monetary policy or migration in this model would be interesting to investigate current politico-economic questions and challenges. We leave this open for further research in future.

Appendix

Proof of Proposition 1

To prove this proposition we note that a balanced budget is obtained by setting $\varrho = (1-\tau_k)r_b$ and $\vartheta = 0$ and that it implies $b^\star = 0$. Using this, one immediately sees from (31) that $\partial(I_e/h_c)/\partial h < 0$ such that $\partial(\dot{h}_c/h_c)/\partial h < 0$ results. Further, from (25) it is easily seen that $\partial(\dot{C}/C)/\partial h > 0$ holds. This proves the uniqueness of the BGP. Since any positive $h^\star$ implies a positive balanced growth rate, existence of the BGP is proven, too.

To show saddle point stability, we compute the Jacobian matrix J evaluated at the rest point $\{c^\star, h^\star, 0\}$. The latter is given by,

$$J = \begin{bmatrix} c^\star & \partial\dot{c}/\partial h & \partial\dot{c}/\partial b \\ h^\star & \partial\dot{h}/\partial h & \partial\dot{h}/\partial b \\ 0 & 0 & -g \end{bmatrix},$$

where we used $\dot{K}/K = g$. The eigenvalues of that matrix are given by $\lambda_1 = -g < 0$ and by $\lambda_{2,3} = \left(tr(J_1) \pm \sqrt{tr(J_1)^2 - 4\,detJ_1}\right)/2$, with tr and det the trace and the determinant of the matrix J_1, respectively, with,

$$J_1 = \begin{bmatrix} c^\star & \partial\dot{c}/\partial h \\ h^\star & \partial\dot{h}/\partial h \end{bmatrix}.$$

The determinant of J_1 is given by,

$$\begin{aligned} detJ_1 &= c^\star(\partial\dot{h}/\partial h) - h^\star(\partial\dot{c}/\partial h) \\ &= c^\star h^\star \left\{ \left[\partial(\dot{h}_c/h_c)/\partial h - \partial(\dot{K}/K)/\partial h\right] - \left[\partial(\dot{C}/C)/\partial h - \partial(\dot{K}/K)/\partial h\right] \right\} \\ &= c^\star h^\star \left(\partial(\dot{h}_c/h_c)/\partial h - \partial(\dot{C}/C)/\partial h\right) < 0 \end{aligned}$$

because of $\partial(\dot{h}_c/h_c)/\partial h < 0$ and $\partial(\dot{C}/C)/\partial h > 0$. Thus, $\lambda_2 < 0$, $\lambda_3 > 0$. □

Proof of Proposition 2

To prove that proposition, we use $b = \vartheta h^\alpha \left(W^{\sigma/(1-\sigma)}\right)^\alpha / (\rho - \varrho)$ from lemma 1. Inserting that in I_e/h_c leads to

$$\frac{I_e}{h_c} = -\frac{\varrho\vartheta h^{\alpha-1}\left(W^{\sigma/(1-\sigma)}\right)^\alpha}{\rho - \varrho} - \vartheta h^{\alpha-1}\left(W^{\sigma/(1-\sigma)}\right)^\alpha + h^{\alpha-1}\left(W^{\sigma/(1-\sigma)}\right)^\alpha \cdot \Omega,$$

with

$$\Omega = \kappa(1-t_r)\left(\tau_n \alpha W^{-1}(1-\gamma)(\xi N)^{(\sigma-1)/\sigma} + \tau_s \alpha W^{-1}\gamma u^{-1/\sigma} L^{(\sigma-1)/\sigma} + \tau_k(1-\alpha)\right) > 0.$$

Differentiating I_e/h_c with respect to h yields,

$$\frac{\partial(I_e/h_c)}{\partial h} = (\alpha-1)h^{\alpha-2}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha}\left[(-\vartheta)\left(1+\frac{\varrho}{\rho-\varrho}\right)+\Omega\right] \quad \text{(A.42)}$$

Equation (A.42) shows that $\vartheta < 0$, $\rho < \varrho$ is sufficient for $\partial(I_e/h_c)/\partial h < 0$ and, thus, for $\partial(\dot{h}_c/h_c)/\partial h < 0$. Since $\partial(\dot{C}/C)/\partial h > 0$, this proves the proposition. □

Proof of Proposition 3

The growth rate of public debt relative to physical capital is given by (30) as,

$$\frac{\dot{b}}{b} = r(1-\tau_k) - \delta - \varrho - \vartheta\left(\frac{Y}{K}\right)\left(\frac{K}{B}\right) - \frac{\dot{K}}{K}. \quad \text{(A.43)}$$

Using $g = \dot{K}/K = \dot{C}/C$, equation (A.43) can be rewritten as,

$$\dot{b} = b\left(\rho - \varrho\right) - \vartheta\left(\frac{Y}{K}\right). \quad \text{(A.44)}$$

When h_c and K grow at the same rate g, output Y grows at the rate g, too, such that Y/K is constant. Then, equation (A.44) shows that b remains bounded if and only if $\rho < \varrho$ holds. Using $\rho = r_b(1-\tau_k) - g$ proves the proposition. □

Proof of Proposition 4

To prove that proposition we first note that the $\dot{C}/C$ curve in the economy with a balanced budget is the same as in the economy with permanent deficits and that $\partial(\dot{C}/C)/\partial h > 0$ holds.

Further, I_e/h_c is given by (31) and $\varrho b + \vartheta h^{\alpha}\left(W^{\sigma/(1-\sigma)}\right)^{\alpha} = S_p/K > 0$ holds. This implies that the $\dot{h}_c/h_c$ curve in the case of a balanced budget is always above the $\dot{h}_c/h_c$ curve in the case of permanent deficits.

In addition, $\partial(\dot{h}_c/h_c)/\partial h < 0$ holds such that $h^{\star}$ in the economy with permanent deficits falls short of $h^{\star}$ in the case of permanent deficits. Since a larger $h^{\star}$ goes along with a higher balanced growth rate, this implies that the long-run balanced growth rate in the economy with a balanced government budget exceeds the one of the economy with permanent deficits. □

Robustness of the numerical results

Table A1: Public education spending via κ with $\sigma = 0.25$

κ	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0	1.60564	0.291507	0.00630902	(+,-,-)	-96.0203	-33.0618	-129.082
0.2	0	1.63679	0.295841	0.0078838	(+,-,-)	-94.6983	-31.3124	-126.011
0.3	0	1.6578	0.299049	0.00894095	(+,-,-)	-93.8108	-30.1106	-123.921
0.4	0	1.67421	0.30175	0.00976393	(+,-,-)	-93.12	-29.1564	-122.276
0.5	0	1.6879	0.304155	0.0104487	(+,-,-)	-92.5452	-28.3482	-120.893
0.6	0	1.69976	0.306362	0.0110406	(+,-,-)	-92.0483	-27.6381	-119.686
0.7	0	1.7103	0.308426	0.0115654	(+,-,-)	-91.6078	-26.9991	-118.607
0.8	0	1.71982	0.310382	0.0120387	(+,-,-)	-91.2104	-26.4145	-117.625
0.9	0	1.72854	0.312252	0.0124714	(+,-,-)	-90.8472	-25.8729	-116.72
1	0	1.7366	0.314054	0.012871	(+,-,-)	-90.5119	-25.36643	-115.878

Table A2: Transfer payments via t_r with $\sigma = 0.25$

t_r	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0	1.71982	0.310382	0.0120387	(+,-,-)	-91.2104	-26.4145	-117.625
0.2	0	1.7114	0.316542	0.0116203	(+,-,-)	-91.5617	-26.1361	-117.698
0.3	0	1.7022	0.32256	0.0111624	(+,-,-)	-91.9461	-25.9168	-117.863
0.4	0	1.69203	0.328403	0.0106548	(+,-,-)	-92.3722	-25.7667	-118.139
0.5	0	1.68056	0.334025	0.010082	(+,-,-)	-92.853	-25.7014	-118.554
0.6	0	1.66733	0.33935	0.00941916	(+,-,-)	-93.4094	-25.7464	-119.156
0.7	0	1.65145	0.344248	0.00862208	(+,-,-)	-94.0785	-25.9479	-120.026
0.8	0	1.6311	0.348445	0.00759706	(+,-,-)	-94.939	-26.4045	-121.344
0.9	0	1.60084	0.351143	0.00606548	(+,-,-)	-96.2247	-27.4113	-123.636
1				no BGP				

Table A3: Public debt policy via ϱ with $\sigma = 0.25$, $\vartheta = -0.05$, $\kappa = 0.8$ and $t_r = 0.1$

ϱ	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0.365774	1.70404	0.321413	0.0112541	(+,-,-)	-81.4525	-27.2336	-108.686
0.15	0.20735	1.71121	0.316673	0.0116108	(+, -0.13 ± i)	-85.21	-26.861	-112.071
0.2	0.144676	1.7139	0.314781	0.0117444	(+, -0.15 ± i)	-86.8758	-26.7215	-113.597
0.3	0.0901647	1.71617	0.313129	0.0118576	(-,+,-)	-88.4224	-26.6035	-115.026
0.4	0.065489	1.71719	0.312378	0.0119078	(-,+,-)	-89.1553	-26.551	-115.706
0.5	0.0514174	1.71776	0.31195	0.0119363	(-,+,-)	-89.583	-26.5214	-116.104
0.6	0.0423233	1.71813	0.311673	0.0119546	(-,+,-)	-89.8633	-26.5023	-116.366
0.7	0.0359626	1.71838	0.311479	0.0119673	(-,+,-)	-90.0613	-26.489	-116.55
0.8	0.0312641	1.71857	0.311336	0.0119767	(-,+,-)	-90.2085	-26.4792	-116.688
0.9	0.0276513	1.71872	0.311226	0.0119839	(-,+,-)	-90.3223	-26.4717	-116.794
1	0.0247871	1.71883	0.311138	0.0119896	(-,+,-)	-90.4129	-26.4657	-116.879

Table A4: Public education spending via κ with $\sigma = 0.25$, $\vartheta = -0.05$, $\varrho = 0.1$ and $t_r = 0.1$

κ	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	no BGP							
0.2	0.345428	1.57027	0.300465	0.00450951	(+,-,-)	-87.5582	-34.8691	-122.427
0.3	0.353032	1.61988	0.307163	0.00703015	(+,-,-)	-85.2748	-32.1181	-117.393
0.4	0.356978	1.6458	0.311069	0.00833808	(+,-,-)	-84.0907	-30.6515	-114.742
0.5	0.359836	1.66466	0.31415	0.00928558	(+,-,-)	-83.2332	-29.5661	-112.799
0.6	0.362132	1.67986	0.316808	0.0100467	(+,-,-)	-82.5446	-28.6777	-111.222
0.7	0.364075	1.69275	0.319202	0.0106909	(+,-,-)	-81.9618	-27.9127	-109.875
0.8	0.365774	1.70404	0.321413	0.0112541	(+,-,-)	-81.4525	-27.2336	-108.686
0.9	0.367291	1.71415	0.32349	0.011757	(+,-,-)	-80.9977	-26.618	-107.616
1	0.368668	1.72334	0.325464	0.0122133	(+,-,-)	-80.58517	-26.0519	-106.637

Table A5: Transfer payments via t_r with $\sigma = 0.25$, $\vartheta = -0.05$, $\varrho = 0.1$ and $\kappa = 0.8$

t_r	$b^\star$	$h^\star$	$c^\star$	g	Stability	F_s	F_n	F
0.1	0.365774	1.70404	0.321413	0.0112541	(+,-,-)	-81.4525	-27.2336	-108.686
0.2	0.364275	1.69407	0.327293	0.010757	(+,-,-)	-81.902	-27.0381	-108.94
0.3	0.362589	1.68289	0.332962	0.0101983	(+,-,-)	-82.4074	-26.9252	-109.333
0.4	0.36065	1.67005	0.338353	0.00955547	(+,-,-)	-82.989	-26.9179	-109.907
0.5	0.358339	1.65478	0.343348	0.00878927	(+,-,-)	-83.6824	-27.057	-110.739
0.6	0.355415	1.63552	0.347714	0.00782014	(+,-,-)	-84.5596	-27.4259	-111.986
0.7	0.351228	1.60807	0.350831	0.00643196	(+,-,-)	-85.8166	-28.2533	-114.07
0.8	0.340671	1.53946	0.347935	0.00293235	(+, -0.09 ± i)	-88.9878	-31.3366	-120.324
0.9				no BGP				
1				no BGP				

Table A6: Welfare in BGB scenario and welfare resulting from a transition to PPD scenario

	BGB scenario	From BGB scenario to PPD scenario ($\varrho = 0.1$)
F_s	-41.3568	5.8097
F_n	-41.2391	-15.9577

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