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Several models are available for the analysis of pooled time-series cross-section (TSCS) data, defined as "repeated observations on fixed units" (Beck and Katz 1995). In this paper, we run the following models: (1) a completely pooled model, (2) fixed effects models, and (3) multi-level/hierarchical linear models. To illustrate these models, we use a Generalized Least Squares (GLS) estimator with cross-section weights and panel-corrected standard errors (with EViews 8) on the cross-national homicide trends data of forty countries from 1950 to 2005, which we source from published research (Messner et al. 2011). We describe and discuss the similarities and differences between the models, and what information each can contribute to help answer substantive research questions. We conclude with a discussion of how the models we present may help to mitigate validity threats inherent in pooled time-series cross-section data analysis.

The analysis of pooled time-series cross-section (TSCS) data has become increasingly popular in the social sciences. For example, Adolph, Butler, and Wilson (2005) found that the number of political science articles in journals indexed in JSTOR using "time-series-cross-section" terminology increased explosively in the late 1980s, and that roughly two hundred studies published between 1996 and 2000 used time-series-cross-sectional data. Similarly, we find in EBSCO that the number of scholarly (peer-reviewed) journal articles that include the term "time-series-cross-section" in their abstract increased from four in the 1980s to fourteen in the 1990s and ninety-nine between 2000 and 2014.

Pooled TSCS data consist of "repeated observations on fixed units" (Beck and Katz 1995, 634). Thus, the total number of observations equals the number of cross-sections (I) multiplied by the number of time points (T). For example, our data from Messner et al. (2011) include 2,240 observations (i.e., IxT country-years), covering forty countries (I) for fifty-six years (T) between 1950 and 2005. An Ordinary Least Squares (OLS) regression is not appropriate for this type of data because time-series observations are clustered within countries, inducing correlation among observations (Snijders and Bosker 2011). This violates the assumption of independence of observations, which is required for unbiased estimation of variances and standard errors in OLS regression.

The past several decades have witnessed several approaches to the correlated observations problem in analysis of pooled TSCS data.[1] Researchers, however, have to decide which approaches are appropriate for their research by checking whether the underlying assumptions are appropriate for their theories and data. Otherwise, they risk invalid parameter estimates, incorrect standard errors, and/or wrong type-I and type-II error rates. In other words, their findings may simply be wrong.

In this paper, we present a sequence of nested models to make explicit and test the assumptions that underlie each

1 Adolph, Butler, and Wilson (2005) compare: (1) a pooled regression by least squares (Beck and Katz 1995), (2) the Beck-Katz method with fixed effects, (3) an instrument of the lagged level of the dependent variable (Anderson and Hsiao 1981, 1982), and (4) Generalized Methods of Moments (GMM) estimation (Wawro 2002). See also Snijders and Bosker (2011, 197–202).

model. We chose the models based on theory, prior research, and the structure of the data. We start the sequence with a baseline model, the simplest model with the most restrictive assumptions. We specify subsequent models by relaxing these restrictions, and testing whether the restrictions are supported by the data. In this manner, we demonstrate a step-by-step approach to the analysis of TSCS data and illustrate a methodology for exploiting the properties of this data structure. In the first section of this paper, we describe and discuss several TSCS models, their similarities and differences, and what information each can contribute to help answer substantive research questions. In the second section, we illustrate these models with data on cross-national homicide trends from Messner et al. (2011). In the last section, we summarize our analysis and discuss methodological and theoretical implications for the analysis of pooled TSCS data.

1. Models

Several models are available for the analysis of pooled TSCS data. These include completely pooled models in which all observations – all cross-sections and all repeated observations – are assumed to be equivalent. That is, the pooled data are assumed to be a random sample from a population observed over time; and the data-generating process is assumed to be the same for all cross-sections. Fixed-effects models acknowledge that cross-sections and/or time periods may differ in unknown ways. This is known as unobserved variable bias. In pooled data this may also result in unequal variances, or *heteroskedasticity*, at the cross-section and/or time level. Fixed-effects models incorporate these departures from randomness by including a dummy variable for each cross-section and/or each time period, and Random-Effects models account for between-cross-section and/or between-time differences using parameters of a probability distribution.

Multilevel/Hierarchical Linear Models (MLM/HLM) model the nesting structure of the pooled data, whether time-periods are nested within cross-sections or cross-sections are nested within time periods. This is a fundamental ambiguity of time-series cross-section data and models. We can conceptualize them (1) as i cross-sections observed at each of t time-periods, or (2) as t time-periods observed for each of i cross-sections. The former are typically referred to as repeated cross-sections, when surveys on random samples are repeated over time; the latter as time-series cross-section. But these terms describe the perceptions and decisions of the researcher, rather than inherent properties of the data. Our interest and focus is on time – how the past affects the future. Throughout this paper we conceptualize the processes we model as t time-periods observed for each of i cross-sections. Our focus is on time-series within cross-sections, the parameters of the time-series, and similarities and differences of those parameters between cross-sections.

1.1. Completely Pooled Model

A completely pooled model can be expressed as:

$$(1)\ Y_{ti} = \alpha + \Sigma_k \beta_k X_{kti} + \varepsilon_{ti}$$

where $i = 1, 2, 3, \ldots$ I indexes cross-section; $t = 1, 2, 3, \ldots$ T indexes time; and $k = 0, 1, 2, 3 \ldots$ K indexes independent variables. Y_{ti} is a vector of the dependent variable that varies over cross-section and time; X_{kti} are the k independent variables that vary over cross-section and time; β_k are the coefficients on the k independent variables; and ε_{ti} are the stochastic errors that vary over cross-section and time.

There are several important aspects of this model. The total number of observations is N=IxT: one observation for each cross-section for each time unit. These are pooled into one homogenous data matrix with no structural distinctions with respect to cross-section or time. The data matrix can be transposed without affecting its statistical properties. β_k will be the same whether the data represent differences over time or between cross-sections (Beck and Katz 2004, fn. 4). As with individual-level survey data, observations are assumed to be equivalent and can thus be combined to estimate the effects of X on Y. The data are assumed to be homogenous, but this assumption is based on sampling design: If all observations are randomly sampled from the same population, they are in fact equivalent. Thus, Beck and Katz (2004) note the importance of homogeneity in the decision to pool. This is a critical assumption if the completely pooled model is used to make inferences about the population of cross-sections over time.

1.2. Fixed-/Random-Effects Model

Whether or not random samples of cross-sections over time are feasible depends on the substantive issues being investigated. Random sampling of observations of elementary schools over several years or even decades poses no insurmountable problems; but similar sampling of observations on market democracies does. The population of such countries is small; and those with consistent over-time observations are smaller still. Hence, we cannot rely on randomization to eliminate cross-national differences, and cross-sections are not equivalent. Time units may not be either, because historical events (such as recession, war, and international trade conventions) are unique and differentiate some historical periods from others. Therefore, we can expect both cross-sectional and over-time heterogeneity in the pooled data.

Furthermore, non-experimental research cannot control all factors that might impact the substantive issues under investigation. These unobserved and/or unmeasured variables are included in the stochastic error ε_{ti} and, if they are correlated with any independent variable, will induce a correlation between the error and the independent variable. This induced correlation will bias all of the parameter estimates. Fixed- and random-effects models statistically control for unobserved/unmeasured differences between cross-sections and/or over time. A generic fixed- and/or random-effects model is written as:

(2) $Y_{ti} = \alpha_i + \delta_t + \Sigma_k \beta_k X_{kti} + \varepsilon_{ti}$

where α_i is the cross-section effects, a vector of dummy variables indicating cross-section *i* (fixed effects), or a draw from a probability distribution (random effects); δ_t is a vector of dummy variables indicating time *t* (fixed effects) or a draw from a probability distribution (random effects); X_{kti} are the *k* independent variables that vary over cross-section and time; β_k are the respective coefficients indicating the effect of X_k on Y; and ε_{ti} are the stochastic errors that vary over both cross-section and time. It is important to note that, although model parameters may vary over cross-sections and/or time, they may be fixed or random. This is a choice the analyst makes. Fixed effects are estimated as fixed values – for example, a separate intercept for each cross-section or time period. Random effects are estimated as moments of a probability distribution (typically the normal distribution). Estimation of the former uses I-1 (or T-1) degrees of freedom. Estimation of the latter, if (as is typical) a normal distribution is assumed, uses two degrees of freedom (one each for the mean and the standard deviation). We prefer to follow Longford (1993) and call parameters that vary over cross-section and/or time variable-parameters, which may be fixed or random.[2]

The model of equation 2 estimates the effect of X_{ti} on Y_{ti} net of α_i and δ_t, that is, net of the effects of X_i on Y_i and X_t on Y_t. The only variation remaining in these data are cross-section effects that differ over time or time effects that are different for different cross-sections. Thus, the only effects X can have on Y are X_i on Y_j ($i \neq j$) and X_t on Y_s ($t \neq s$). All variation among cross-sections, regardless of functional form, is absorbed by α_i; and all variation over time, regardless of functional form, is absorbed by δ_t. The only way to model these differences within the context of this equation 2 is by including the interaction of α_i with δ_t. But this interaction absorbs all variation in the data and can provide no useful substantive information: All data points are fitted. Thus, one may not include the interaction between cross-section fixed effects and time fixed effects.

Substantively, the previous paragraph means that X_k can affect Y only if X_k varies across both cross-section and time, and that the over-time effect of X_k must then differ among cross-sections (or the cross-section effect must differ over time). But this cannot be accomplished within the context of equation 2. What equation 2 can accomplish is simply to test the following hypothesis: Is there an effect of X_k on Y net of stable cross-sectional differences in Y and net of temporal differences in Y that are constant among cross-sections? H_0: β_k=0. Rejection of this null hypothesis tells us that X_k does affect Y *and* that this effect differs among cross-sections. Rejection of this null, how-

2 Estimating both cross-section and time fixed effects uses (I-1)+(T-1) degrees of freedom. Estimating both cross-section and time random effects uses four degrees of freedom.

ever, tells us neither what these differences are nor why they occur.

1.3. Multilevel/Hierarchical Linear Model (MLM/HLM)

How the MLM/HLM is written depends on the nesting structure of the data, whether time-periods are nested within cross-sections or cross-sections are nested within time periods. As noted, our interest is in the former, in line with virtually all of the pooled time-series cross-section literature (for an exception, see DiPrete and Grusky 1990).

In general, there are i=1, 2, 3, … I cross-sections. Each cross-section contains data for t=1, 2, 3, … T time periods. Interest is on the relationship between a set of independent variables X_{kti} and a dependent variable Y_{ti}. Suppose that relationship can be written for the i^{th} cross-section:

(3) $Y_{ti} = \alpha_i + \delta_t + \Sigma_k \beta_{ki} X_{kti} + u_{ti}$

Except for the additional *'i'* subscript on β_k, equation 3 is identical to equation 2. Equation 2 is a restricted or constrained version of equation 3 where β_k is constant across cross-sections. That is, the effect of X_k on Y is the same for all cross-sections. This constraint is assumed by all completely pooled and most fixed- and random-effect models. MLM/HLM has merit in that a wider range of models can be estimated, and that a richer set of data generating processes can be tested.[3]

The typical presentations of these models focus on the clustering of observations – on level 1 – within some larger units (geographical, organizational, social, etc.), frequently called *contexts* – or level 2 (see Snijders and Bosker 2011; Raudenbush and Bryk 2002). This implies that observations within contexts tend to be more similar to one another than observations from different contexts. Therefore, if observations are assumed to be independent (as statistical theory does), all variance estimates will be wrong. The impetus for modeling this non-independence is to obtain the correct estimates of variances and covariances. This process is generally applicable to pooled time-series cross-section (where time-series observations are nested within cross-sections) and to repeated cross-sections (where cross-section observations are nested within time periods). We think it more useful to focus on the modeling of the substantive process, rather than the statistical consequences of clustering. The development of MLM/HLM with time clustered within cross-section is the same as the development of MLM/HLM with cross-sections clustered within time (except for the issues involving the difference between time-series and cross-sections). At level 1:

$Y_{ti} = \alpha + \beta X_{ti} + u_{ti}$

This relationship holds for each cross-section *i*, but the model parameters may differ among cross-sections. Thus, the level 2 equations are:[4]

$\alpha = \alpha_i$

$\beta = \beta_i$

Their substitution gives the two-level model with time within cross-section:

$Y_{ti} = \alpha_i + \beta_i X_{ti} + u_{ti}$

This model controls for unobserved variables at level 2 by including the cross-section intercept α_i, which can vary among cross-sections. Cross-section differences in slope β_i are also modeled: The effect of X on Y differs among cross-sections. Here there are two varying parameters. In this case they are fixed effects. There are no stochastic components.

More complex models, such as the random intercept: $\alpha = \alpha_i + \varepsilon_i$ or $\alpha = \alpha_0 + \alpha_1 Z_i + \varepsilon_i$, can, of course, be written. These are random effects because of the stochastic component ε_i. In the second case, Z_i is a level 2 variable that varies among

3 At the limit, each independent variable for each fixed unit has a different effect (β_{ki} is different for every *i*). Beck and Katz (2007) refer to these as unit-specific models: a (perhaps different) model for each cross-section. If unit-specific models include the likely cross-equation correlation structure, these are Seemingly Unrelated Regression models described in econometrics (Judge et al. 1982; Wooldridge 2002).

4 Notation varies among authors. It is important to understand the models and not be wedded to some notation system.

cross-sections, but not time. The cross-section effect in this case is random (because of the stochastic component ε_i) and is a linear function of Z_i.

Consider the following abbreviated model from Messner et al. (2011):

$$\text{(4a) } \mathbf{D(Homrate)}_{ti} = \alpha_i + \delta_t + \beta_1 \mathbf{D(Homrate)}_{t-1,i} + \beta_2 \mathbf{D(Div)}_{ti} + \beta_3 \mathbf{D(LnGDPpc)}_{ti} + u_{ti}$$

This is a fixed-effects model with fixed intercepts for country (α_i) and time (δ_t). These two coefficients control for all variables that vary *only* between countries and between time periods. $\mathbf{D(Homrate)}_{t-1,i}$ is the annual change in the homicide rate of country *i* in year *t-1* (the previous year); $\mathbf{D(Div)}_{ti}$ is the annual change in the divorce rate; and $\mathbf{D(LnGDPpc)}_{ti}$ is the annual change in the log of per-capita income. All variables are measured as annual change because the levels are not stationary (see Raffalovich 1994). The annual changes in divorce rate and per-capita income in country *i* in year *t* have effects β_2 and β_3, respectively. Prediction error for country *i* in year *t* is u_{ti}. If one objects to divorce and/or income affecting homicide contemporaneously, one or both can be lagged by one year (or several) so that they will appear in equation 4a as $\mathbf{D(Div)}_{t-1,i}$ and $\mathbf{(LnGDPpc)}_{t-1,i}$. Note that the effects of change in divorce rate and per-capita income are modeled as constant across countries. In other words, the effect of change in the divorce rate on change in the homicide rate is constrained to be the same for all sampled countries, as is the effect of change in per-capita income. These assumptions of both the completely pooled model (equation 1) and the fixed-/random-effects model (equation 2) may or may not be reasonable, depending on substantive theory and prior research. In any case, they can be tested in the context of MLM/HLM. To do so, estimate equation 4a and also the more general model:

$$\text{(4b) } \mathbf{D(Homrate)}_{ti} = \alpha_i + \delta_t + \beta_{1i} \mathbf{D(Homrate)}_{t-1,i} + \beta_{2i} \mathbf{D(Div)}_{ti} + \beta_{3i} \mathbf{D(LnGDPpc)}_{ti} + u_{ti}$$

Equation 4b differs from equation 4a by the inclusion of the between-country differences in the effects of the lagged homicide rate, the divorce rate, and per-capita income on the country's homicide rate. Twice the difference in the log-likelihood (standard output from MLM/HLM software) has a chi-square distribution with degrees of freedom equal to the difference in the number of model parameters. In contrast to the fixed-/random-effects model (equation 2), the MLM/HLM offers insight into the data generating processes through which exogenous variables affect dependent variables of interest, as well as potential heterogeneity between countries and over time. However, there can be a very large number of parameters to be estimated in equation 4b; and thus some constraints are typically necessary in practice, for example, homogeneity for subsets of countries. Tests for these constraints are widely available (for example, Snijders and Bosker 2011). In the following, we will illustrate these observations using data from Messner et al. (2011).

2. Analysis

All data are from Messner et al. (2011). For the examples below, we excerpted the following variables:

- The national homicide rate $\mathbf{Homrate}_{ti}$, its one-year lag $\mathbf{Homrate}_{t-1,i}$, and the annual change $\mathbf{D(Homrate)}_{ti} = (\mathbf{Homrate}_{ti} - \mathbf{Homrate}_{t-1,i})$;
- Annual change in the homicide rate the previous year $\mathbf{D(Homrate)}_{t-1,i}$;
- Annual change in the national divorce rate $\mathbf{D(Div)}_{ti}$;
- Annual change in the log of national per-capita income $\mathbf{D(LnGDPpc)}_{ti}$.

We then used these data to estimate equations 1 through 4b in models 1 through 4d (see Tables 1 and 2). All estimates were produced using EViews-8, a widely used econometrics program. Other software will produce comparable results.

All models have the following form:

$$\text{(4a) } \mathbf{D(Homrate)}_{ti} = \alpha_i + \delta_t + \beta_1 \mathbf{D(Homrate)}_{t-1,i} + \beta_2 \mathbf{D(Div)}_{ti} + \beta_3 \mathbf{D(LnGDPpc)}_{ti} + u_{ti}$$

The various models are distinguished from one another by the application or removal of constraints on model parameters. For example, model 1, the completely pooled model, constrains $\alpha_i = \alpha$ (no unmeasured time-constant country effects) and $\delta_t = \delta$ (no unmeasured country-constant time effects), and also constrains the effects of prior change in the homicide rate, annual change in divorce rate, and annual change in per-capita income on the annual change in the homicide rate to be the same for all countries. Equation 4b removes the constraint that $\beta_{1i} = \beta_1$, $\beta_{2i} = \beta_2$, and $\beta_{3i} = \beta_3$.

$$\text{(4b) } \mathbf{D(Homrate)}_{ti} = \alpha_i + \delta_t + \beta_{1i}\,\mathbf{D(Homrate)}_{t\text{-}1,i} + \beta_{2i}\,\mathbf{D(Div)}_{ti} + \beta_{3i}\,\mathbf{D(LnGDPpc)}_{ti} + u_{ti}$$

There are N=1,478 country-year observations in these data.[5] The data are *unbalanced*, that is, the number of within-country observations varies among the forty countries. Not surprisingly, more observations are available for the United States (55) and Western Europe (median=53), than for Latin America and the Caribbean (median=44), Eastern and Southern Europe (median=34), and Asia/Other (median=25). We use a Feasible Generalized Least Squares (FGLS) estimator, with country weights and panel-corrected standard errors (PCSE). This estimator weights cases by the inverse of country-specific error variance. More precise estimates are weighted more heavily. The countries with more valid and reliable data are thus weighted more heavily. This might bias parameter estimates towards the more *developed* countries, as opposed to Latin America, Asia, and Africa. We test for regional differences later in the analysis.

Table 1: Pooled GLS estimations (cross-section weights, PCSE)

Dependent variable: D(Homrate)							
Independent variable	1	2a	2b	2c	3a	3b	3c
Common Effects							
Constant	0.0024	−0.0273 ***					
	(.008)	(.008)					
$D(Homrate)_{t-1}$	−0.2587 ***	−0.2696 ***	−0.2703 ***	−0.2803 ***	CSSE[a]	−0.2764 ***	−0.2770 ***
	(.029)	(.030)	(.030)	(.030)		(.030)	(.030)
D(Div)	0.1046 **	0.1035 **	0.0899 **	0.0896 **	0.0916 **	CSSE[a]	0.0843 *
	(.032)	(.032)	(.034)	(.034)	(.034)		(.034)
D(lnGDPpc)	−0.5761 **	−0.5198 **	−0.7996 **	−0.7153 **	−0.7000 **	−0.7805 **	CSSE[a]
	(.190)	(.201)	(.245)	(.262)	(.252)	(.251)	
Cross-section fixed effects							
Constant		no print		no print			
$D(Homrate)_{t-1}$					no print		
D(Div)						no print	
D(lnGDPpc)							no print
Time fixed effects							
Constant			no print	no print	no print	no print	no print
N	1,478	1,478	1,478	1,478	1,478	1,478	1,478
R^2	0.0847	0.1055	0.1442	0.1666	0.2046	0.1743	0.1763
Log likelihood	−617.8251	−600.8200	−575.5559	−555.9083	−522.3551	−550.3864	−546.1144
Log likelihood ratio test							
Restricted model		1	1	1	2b	2b	2b
$-2 \times (LL_R - LL_U)$		34.0102	84.5384	123.8336	106.4016	50.3390	58.8830
Df		39	52	91	39	39	39
P-value		0.6966	0.0029	0.0126	0.0000	0.1055	0.0214

[a] The variable is included as cross-section fixed effects. For the test of between-country differences, see the log likelihood ratio test in the bottom rows.
*** $p < .001$; ** $p < .01$; * $p < .05$

5 Messner et al. (2011) report N = 1,129~1,285, depending on the specific models. We excerpted fewer variables from these models, and thus lost fewer cases to listwise deletion.

When examining estimates, we should keep in mind that the completely pooled model (model 1) assumes that countries are identical in all unmeasured respects. Estimates are presented in the first column of Table 1. Except for the intercept (where the null is not rejected), the null hypothesis of no effect can be rejected at p<.01 for lagged change in the homicide rate, change in the divorce rate, and change in per-capita income. The intercept in a model of change represents the rate of change of the dependent variable (in this case the homicide rate) when the independent variables are zero. Here, that means the rate of change of the homicide rate when there is no change in prior homicide rate, divorce, or per-capita income. Model 1 tells us that if these variables do not change, the homicide rate does not change, except for random variation in the error term, because the intercept is not significantly different from zero.

Prior change in the homicide rate represents the accumulation of the effects of all determinants of current change through *t-1* (the past history of the process). It is included in these models because, following Messner et al. (2011), homicide rates are a function of historical patterns, rather than independently distributed through time. Interpretation of the coefficients of lagged dependent variables depends on how their past is believed to affect their future. We interpret neither the sign nor the magnitude of these effects, and refer to them as the effects of *history*.[6]

The effects of the changes in divorce rate and per-capita income are expected (Messner et al. 2011). There may be other variables – stable between-country differences and/or global changes that affect all countries – that we are unaware of or unable to measure. The independent variables in model 1 vary over both country and time, so may be correlated with these unmeasured variables. If so, the results of model 1 are biased and our inferences may therefore be wrong. Models 2a through 2c control for these unmeasured variables. Results are presented in the second, third, and fourth columns of Table 1. Model 2a in the second column includes country fixed effects; model 2b in the third column includes time fixed effects; and model 2c in the fourth column includes both country and time fixed effects. Our interest is not in the fixed-effects per se, but in the impact of their inclusion on the effects of the independent variables. Also, because there are so many fixed effects (forty countries and fifty-three years) we do not report them in this paper.[7] We do report the results of likelihood ratio tests in the bottom panel of Table 1.This test compares the log-likelihoods (LL) of two nested models: the unrestricted model and the restricted one. The models are identical except that the restricted model imposes a set of restrictions on parameters of the unrestricted model. Model 2a, for example, includes all parameters from model 1, as well as forty cross-section fixed effects (country-specific intercepts). In this example, model 2a is unrestricted. Model 1 imposes the restriction that thirty-nine of these effects are zero (and the remaining effect is the one intercept, which is not restricted). The restricted model is indicated in the bottom panel of Table 1, along with the log-likelihoods of both models, the chi-square statistic of -2 times the difference of log-likelihoods, and the degrees of freedom (the number of restrictions). In the second column we see that the restricted model is model 1, and that minus twice the difference in log-likelihoods is 34.0102, distributed as chi-square with 39 degrees of freedom and associated probability of .6966. We therefore fail to reject the null hypothesis that country fixed effects are jointly zero. Intercepts are the same for all countries.[8]

Time fixed effects (model 2b in the third column), on the other hand, are significantly different from one another: Intercepts vary significantly over time (p=.0029). Comparing the coefficients of lagged homicide, divorce, and per-capita income in models 1 and 2b, we see that they are the same sign, but slightly different in magnitude. These differences, however, are small relative to their standard errors. Substantively, findings from model 2b are the same as from model 1.

6 Estimates of models with lagged dependent variables will be biased if error terms are autocorrelated. Correlograms and Ljung-Box test statistics (Granger and Newbold 1986) indicate no significant autocorrelation.

7 These estimates are available on request.

8 We also estimated random between-country effects. Consistent with our findings for fixed-effects, random effects were zero.

The results show that the independent variables, which vary over both country and time, have significant effects net of country and time. This implies that the association between independent and dependent variables involves country-time interaction. We cannot estimate a country-time interaction within the context of model 2 because this $\alpha_i\delta_t$ interaction consumes all degrees of freedom and perfectly fits all data points. We can, however, fit a model where the over-time effect differs among countries. This cannot be done within the context of model 2 because just as with model 1, model 2 constrains model parameters to be the same for all countries. We have noted that model 2b has many parameters (53 just for the time fixed effects), and model 2c has an additional 40, preventing inclusion of either in Table 1. If we relax the assumption that the effects of independent variables are the same for all countries, the number of parameters to be estimated increases. For each of the three variables in the model, we would need an additional 39 coefficient estimates and estimated standard errors. Are all of these necessary to adequately represent the data generating processes? For current purposes, we define "adequate" as the absence of both redundant and omitted variables. A variable is redundant if the null hypothesis of no effect is not rejected. A variable is omitted if, despite theoretical and/or empirical evidence of its importance, it is not included in the estimated model. So redundancy is data-based and omission is theory-based. Fixed-effects were included, for example, because both the theoretical and statistical cases for inclusion were strong. To evaluate redundancy, we estimated a model with fixed effects (models 2a to 2c) and without them (model 1), and compared the likelihood ratios. We found that country fixed effects were redundant, but time fixed-effects were not.

Model 3 relaxes the assumption that the effects of the lagged change in homicide rate, change in the divorce rate, and change in per-capita income are the same for all countries. These results are presented in column 5 for lagged homicide (model 3a), column 6 for divorce (model 3b), and column 7 for per-capita income (model 3c). We do not present estimates for each country:[9] there are 40 different coefficients for each variable. We do present the results of hypothesis tests in the lower panel of Table 1. Like the models with fixed-country and fixed-year effects, these are also likelihood ratio tests. The null hypothesis is that the 40 country-specific coefficients are redundant; the alternative is that some (one or more) are not redundant. The variable being tested is indicated by "CSSE" (Cross-Section Specific Estimate) in the top panel. These are the between-country differences in this effect. The same null hypotheses are tested for between-country differences in the effect of change in divorce and in per-capita income. For these two variables, the null of no difference is not rejected: divorce at the .1 level and per-capita income at the .01 level.[10]

9 These are available on request.

10 The null would be rejected at the .05 level; and we would conclude that there were between-country differences in the effect of the change in per-capita income on the change in homicide. But consider that we are testing many hypotheses. At the .05 significance level, one out of twenty null hypotheses would be wrongly rejected (see Raffalovich et al. 2008). Therefore, we employ a more stringent threshold.

Table 2: Pooled GLS estimations (cross-section weights, PCSE) with regional dummies and interactions

Dependent variable: D(Homrate)					
Independent variable	2b	4a	4b	4c	4d
Common Effects					
D(Homrate)$_{t-1}$	−0.2703 ***	−0.2705 ***	−0.4142 ***	−0.2683 ***	−0.2698 ***
	(.030)	(.030)	(.049)	(.030)	(.030)
D(Div)	0.0899 **	0.0903 **	0.0951 **	0.0124	0.0916 **
	(.034)	(.034)	(.034)	(.057)	(.034)
D(lnGDPpc)	−0.7996 **	−0.7085 **	−0.6707 **	−0.6959 **	−0.0961
	(.245)	(.257)	(.254)	(.262)	(.458)
Northern Europe		*Reference*	*Reference*	*Reference*	*Reference*
Anglo America, UK, Oceania		0.0095	0.0074	0.0061	0.0101
		(.017)	(.017)	(.017)	(.023)
x D(Homrate)$_{t-1}$			0.1356		
			(.075)		
x D(Div)				0.0815	
				(.074)	
x D(lnGDPpc)					−0.0223
					(.737)
Latin America and Caribbean		−0.0243	−0.0270	−0.0284	−0.0114
		(.063)	(.063)	(.064)	(.067)
x D(Homrate)$_{t-1}$			0.1575 *		
			(.070)		
x D(Div)				0.1128	
				(.288)	
x D(lnGDPpc)					−0.5968
					(1.452)
E/S Europe		0.0064	0.0056	−0.0004	0.0297
		(.014)	(.014)	(.014)	(.019)
x D(Homrate)$_{t-1}$			0.2467 ***		
			(.068)		
x D(Div)				0.2054 *	
				(.096)	
x D(lnGDPpc)					−1.0075
					(.580)
Asia and Other		−0.0142	−0.0093	−0.0203	0.0149
		(.018)	(.017)	(.019)	(.025)
x D(Homrate)$_{t-1}$			0.3922 ***		
			(.110)		
x D(Div)				0.2044	
				(.254)	
x D(lnGDPpc)					−0.9905
					(.589)
Time fixed effects					
Constant	no print	no print	no print	no print	no print
N	1,478	1,478	1,478	1,478	1,478
R^2	0.1442	0.1450	0.1584	0.1484	0.1478
Log likelihood	−575.5559	−574.3539	−563.9780	−571.7798	−571.4492
Log likelihood ratio test					
Restricted model		2b	2b	2b	2b
$-2 \times (LL_R - LL_U)$		2.4040	23.1558	7.5522	8.2134
Df		4	8	8	8
P-value		0.6619	0.0032	0.4784	0.4129

*** $p < .001$; ** $p < .01$; * $p < .05$
Note: For the regions and countries, see Appendix.

Ideally, we would want the estimates of the between-country differences (in divorce, for example) so that we could investigate the reasons for these differences, obtain the measures of potential explanations, and test the hypotheses regarding these explanations. For each variable, however, there are 40 parameters to be estimated; for three variables, there are 120; period fixed effects add another 53. Estimating 173 parameters with almost 1,500 cases is not an insurmountable problem. Interpreting those estimates may well be, however, unless they describe a very simple pattern (for example monotonic). To reduce the complexity of this process, we aggregated the forty countries into the five regions defined by Messner et al. (2011). Thus, we further test in models 4a through 4d the regional differences, because regional differences may be more important than between-country differences.

Regional differences in the effects of lagged change in homicide, change in divorce, and change in per-capita income are estimated in Table 2 (see models 4a to 4d). Model 4a tests whether there are regional differences in the rate of change of homicide, net of divorce rates and per-capita income. Earlier, we found no between-country differences; so the finding of no regional differences (Chi-square with 4 df = 2.404, p>.5) is not surprising. Models 3a to 3c tested between-country differences in the effects of change in lagged homicide, divorce, and per-capita income on homicide change. Only the lagged dependent variable – *history* – was found to differ in effect among countries (model 3a, p<.01). Because there were forty countries in these data, and thus forty coefficients for the effect of the lagged dependent variable, specific country differences were hard to interpret, especially in the absence of strong theory and specific hypotheses. Model 4 is a little easier to interpret. First, as we expect from model 3a, only the effect of lagged homicide in model 4b displays regional differences (p<.001). Second, those differences are between Northern Europe (the reference region) and both Eastern/Southern Europe and Asia/Other. Why the historical patterns of homicide would have differential regional impact on annual change in contemporary homicide rates is a topic for future research.

3. Discussion

In this paper we have presented and discussed several models for the analysis of pooled time-series cross-section data, then illustrated these models with data from published research on homicide rates in a sample of forty countries over an average of more than forty years per country. Throughout, our focus has been on within-country time-series and between-country differences in time-series parameters.

The models we discuss range from completely pooled to regionally disaggregated. Completely pooled models require that sampled cross-sections be drawn from a population of equivalent cross-sections so that parameters do not vary among cross-sections and data can be combined to yield more precise estimates of common coefficients. This applies to measured cross-sectional differences, unmeasured cross-sectional differences (error variances and covariances), and to time-series processes within cross-sections. The advantage of pooling is this combining of information. More cross-sections in a sample means larger sample sizes; larger samples have smaller sampling error; and smaller sampling error means more precise parameter estimates. The major threats to validity are that the pooled cross-sections are not from the same population and that causal processes differ among cross-sections. The ability to avoid these threats depends, of course, on sampling design (thus attention to methodology must be emphasized); but frequently researchers rely on secondary data in which case sampling design is not under their control. In the absence of random sampling, a difficult achievement in many research contexts, cross-section and time homogeneity should not simply be assumed. Instead, researchers should test these assumptions. We suggest the likelihood-ratio test within the context of a fixed-effects multilevel statistical model as one useful method for testing heterogeneity in pooled models (see Snijders and Bosker 2011; Raudenbush and Bryk 2002). We do not discuss random effects models other than to note that they are an alternative to fixed-effects models in controlling for unmeasured heterogeneity. We are skeptical of these models because inference is to the population from which the data are sampled, whereas inference in fixed-effects models is conditional on the data in the sample. With TSCS data, the population is vaguely defined and sampling is typically by convenience. Also, random-effects models estimate moments of probability distributions, and this requires comparatively large samples to obtain reasonable estimates (but see Beck and Katz 2007). Fixed-effects models are more appropriate for the TSCS data analyzed here (Beck 2001).

Using the data from Messner et al. (2011), we first estimate a simple model (model 1) that assumes homogeneity with respect to cross-section and time. To rule out cross-sectional or over-time heterogeneity in the pooled data, which may be correlated with unobserved variables, we test this assumption. Thus, in model 2a through 2c, we relax this homogeneity assumption and include country fixed-effects (model 2a), time fixed-effects (model 2b), and both country and time fixed-effects (model 2c). The likelihood-ratio tests (between model 1 and models 2a to 2c) show that the time fixed-effects significantly improve the model. Thus, the later models (models 3a to 4d) include time fixed-effects, with model 2b serving as the restricted model for their likelihood-ratio tests. Next, we test the assumption that the effects of the predetermined variables are the same across the countries. The likelihood-ratio tests (between model 2b and models 3a to 3c) show that the country-specific effects of the lagged dependent variable differ among countries, whereas the effects of the exogenous variables are the same. Therefore, model 3a is the most appropriate of models 1 through 3c.

Model 3a tells us that countries differ in the impact of historical patterns of homicide on current patterns. Interpretation of these differences is problematic because of the large number of estimated parameters, and thus the large number of comparisons that need to be made. We simplify this task by aggregating countries into geographic regions, then testing for regional differences (model 4a to 4d). We find that, like models 3a to 3c, only the impact of historical patterns of homicide differs among regions. Specifically, the Eastern and Southern European region differs from the Anglo-American and Northern European regions, as does Asia and Other in this respect. The Latin America and Caribbean region does not differ from the reference region.

The sequence of models presented and illustrated suggest three important conclusions. First, country effects are redundant, net of change in divorce rate, change in per-capita income, and historical patterns of homicide rate change: The sample is homogenous with respect to stable between-country characteristics. Second, net of those same variables, time-effects are not redundant: The sample is not homogenous with respect to stable over-time differences, and statistical analysis of the pooled data must control for over-time heterogeneity. Third, the significance of independent variables that vary over both country and time implies country-time interaction. The effects of one or more independent variables must differ among countries. The sample is not homogenous with respect to causal processes. Statistical analysis of the pooled data must control for heterogeneity of causal processes.

The analysis of the homicide data shows that change in the divorce rate has a positive impact on the change in homicide rate, and that change in per-capita income has a negative effect. These findings are consistent with the research literature on homicide (for a literature review, see Messner et al. 2011). In addition, we find that these effects are constant across the countries in our data. This substantively important information is not obtainable from the analysis of the completely pooled or fixed-effects models of equation 1 or 2. Also unobtainable from completely pooled or fixed-effect models is the substantively important finding that the impact of history is not constant but varies among countries and regions. Methodologically, this information regarding homogeneity and heterogeneity is critically important to recognize and counter the threats that heterogeneity poses to validity. Heterogeneity is inherent in TSCS data, but not self-evident. Researchers must be diligent and test it.

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Appendix: Regions and Countries

1. Anglo-America/U.K./Oceania (5)

Canada, United States, United Kingdom, Australia, New Zealand

2. Latin America and Caribbean (9)

Costa Rica, Mexico, Trinidad and Tobago, Uruguay, Venezuela, Dominican Republic, El Salvador, Nicaragua, Panama

3. Eastern/Southern Europe (12)

Austria, Bulgaria, France, Greece, Hungary, Italy, Poland, Portugal, Spain, Switzerland, Czech Republic, Estonia

4. Northern Europe (9)

Denmark, Finland, Germany, Iceland, Netherlands, Norway, Sweden, Belgium, Luxembour

5. Asia and Other (5)

Israel, Mauritius, Singapore, Japan, Thailand

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