Essays on the Performance and Comparability of the German Pension System

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Introduction

Demographic change, especially population aging, challenges the funding of the German mandatory pension system through an increasing ratio of retirees to workers. The mandatory part of the German pension system is structured as an unfunded pay-as-you-go (PAYG) system and provides the basic or main part of the pension benefits for each retiree. The basic concept of every PAYG pension system is to finance pension benefits in year t by the contributions made in the same year. Hence, contribution income is intended to balance the pension expenditures. An increasing pensioner/contributor ratio will thus put financial pressure on either the contributors (through higher contributions) or the retirees (through lower pension benefits), if the basic concept of a PAYG pension system is intended to be maintained. However, a gap between pension expenditures and contribution income of the German mandatory pension system has existed since the 1960s and even increased since then (see Deutsche Rentenversicherung Bund 2012). Consequently, national finances bridge the gap between contribution income and pension expenditures with federal subsidies, which puts financial pressure on taxpayers. In order to deal with demographic change and to relieve pressure on the mandatory pension system, several reforms have been made, such as introducing the Riester scheme, which subsidizes households that save privately for retirement.[1] However, due to the pending retirement of the baby-boom generation, the effect of an aging population

[1]See the German Retirement Savings Act (AVmG) for details.

on the funding of benefits of the German pension system is becoming even greater. Hence, discussions on the solvency and sustainability of the German pension system are becoming more and more frequent. Therefore, it is important to take a look at the development of both the expenditure and income sides of the German mandatory pension system, especially considering the fundamental pension reforms which had, and still have an impact on both retirees and contributors, as well as national finances and hence taxpayers. Furthermore, since the tax treatment of savers and retirees is central to investigating the development of the Riester scheme, and hence the voluntary part of the German pension system, the Riester scheme is investigated regarding the subsidies it provides, as well as its feature of generating additional fiscal revenue. Moreover, since there are a variety of differently structured pension systems around the world, comparisons with other countries' pension systems, through assessing the similarities between them, enables detecting determinants that drive this similarity and may thus help to improve a nation's own pension system.

Accordingly, evaluating the performance of pension systems is constantly subject to academic as well as political debate, especially with regard to demographic and economic changes. Performance is often measured by the ratio of retirement income to pre-retirement income, usually referred to as the replacement rate or pension level. However, measuring the performance of a pension system using only the replacement rate is not particularly accurate, since it focuses only on the retirees. Rather, it is important to evaluate both the replacement rate and contributions into a given pension system. Hence, irrespective of whether a pension system is fully funded and privately organized, or unfunded and publicly organized, it should be evaluated in terms of both the development of contributions and the benefits received by retirees. Therefore, performance can also be viewed as the behavior, capability or potential of a pension system, or even as simply the development of benefits and contributions. Furthermore, evaluating the comparability of the German pension system, by assessing its similarity to other nations' pension systems, is important in order to support or even improve the local pension system. Since each country's government attempts to adjust the national pension system to demographics and economic developments, so as to ensure that it remains sustainable and solvent, comparisons with other countries may help in making improvements. Therefore, identifying economic, demographic and institutional factors that impact on the similarity of a country's pension system to others, becomes more and more important and useful.

Hence, the first objective of this thesis is to find ways to decrease the financing gap of the mandatory pension system, in order to support its solvency and sustainability, by providing new insights into its future performance, and to show that the tax treatment of Riester savings and pension benefits in general not only costs money, but also generates fiscal revenue, which results in lower governmental net expenditures. The second objective is to introduce a new measure for comparing and assessing the similarity between pension systems of different countries, in order to detect determinants of similarity between pension systems. This may ultimately enable a closer look at particular pension systems, so as to support or even improve one's own system with regard to its solvency and sustainability.

The remainder of this thesis is structured as follows. In order to identify ways of dealing with the impact of demographic change on the performance of the German mandatory pension system, Chapter 2 investigates whether a variable demographic factor can help to lower the financing gap of the German mandatory pension system, and hence the federal subsidies required to close this gap. This can be regarded as an analysis of the effect of the Pension Insurance Sustainability Act, which introduced the demographic factor to the calculation formula of pension benefits of the German mandatory pension system in 2005 and set it to 0.25. This factor is now made variable in order to demonstrate that this may decrease the financing gap of the mandatory pension system without putting excessive financial pressure on the retirees and contributors. This is done by simulating and comparing the development of the expenditures and income of the German mandatory pension system under different assumptions regarding the average gross income of employees, population growth, and the development of the contribution rate in the mandatory pension systems, for both a constant and a variable demographic factor. Besides the fact that the financing gap will increase until 2050, irrespective of the scenario and the assumptions imposed on the demographic factor, it turns out that a variable demographic factor (in contrast to a constant one of 0.25) decreases the financing gap in each scenario, by burdening either the retirees or the contributors or even both. Hence, a more flexible calculation of benefits (approximated by a variable demographic factor) would support the pension system's solvency and sustainability, without excessively burdening retirees and contributors.

Furthermore, Chapter 3 investigates the impact of the regulations of both the German Retirement Savings Act (AVmG, including its supplement, the AVmeG) and the German Retirement Income Act (AltEinkG) on the German pension system. The most important aspect of these two Acts is that both affect national finances, but have been developed and adopted completely independently of each other. Whereas the AVmG has been promising subsidies to Riester savers since 2002, the AltEinkG eventually introduced deferred taxation for pension benefits, including those obtained from Riester savings, in 2005. Consequently, part of the Riester subsidies become subject to income tax when savings are finally converted into pension benefits. The consequent effect is two-sided. On the one hand, tax revenue is required to finance Riester subsidies, which burdens the national finances. On the other hand, pension benefits obtained from Riester savings increase the level of retirement income. Due to progressive income taxation, individual average income tax rates will also increase. This leads to increased tax payments from Riester retirees and consequently leads to additional fiscal revenue. To investigate whether subsidies granted in year t regarding Riester savings may be financed entirely by the additional fiscal revenue due to deferred taxation of (Riester) pension benefits in year t, the so-called *tax-pay-as-you-go* (*TaxGo*) model is constructed in this chapter. This model analyzes Riester subsidies (considered as governmental expenditures) in relation to additional fiscal revenue obtained from the taxation of (Riester) pension benefits. Under different assumptions regarding the average gross income of employees and the development of the share of Riester savers, future

Riester subsidies and additional fiscal revenue are simulated, leading to the result, that Riester subsidies can be financed entirely by the additional fiscal revenue in the 2040s, at least on a yearly basis. Hence, the Riester scheme, as well as deferred taxation of pension benefits and the tax treatment of Riester savers and pensioners in general, not only *costs* money due to subsidizing Riester savers. Additional fiscal revenue is also generated, resulting in lower governmental net expenditures.

Chapter 4 investigates the similarity of the German pension system to other OECD country pension systems. Accordingly, the concept of vector similarity is applied to compare two countries' pension systems with each other. This concept determines the similarity of two vectors (which consist of several pension system representatives) by calculating the cosine of the angle between them. However, vector similarity offers some advantages over common comparison concepts regarding pension systems. Since comparisons are usually made by building composite indices that include a large number of different variables which describe a pension system, or even build sub-indices, the results are not replicable. By contrast, the approach in this thesis only uses three indicators that determine a pension system. Moreover, using the concept of vector similarity places pension systems in relation to each other without the intention of forming a ranking. Hence, the only normative part of this analysis is the choice of the three indicators, namely the gross and net replacement rate and the pension funds' assets as a percentage of gross domestic product (GDP). These indicators not only describe isolated parts of the pension system, but the use of the gross and net replacement rates yields information on the tax treatment of retirees. Additionally, the use of pension funds' assets yields information on the financing structure of the pension system. Under consideration of various demographic and economic variables, as well as institutional features of pension systems, it turns out that similarities between pension systems are driven mainly by demographic and institutional factors. This knowledge about the dependency of the similarity on several demographic and institutional factors contributes to our understanding of pension systems and its workings, and may help support the solvency and sustainability of any pension system. Finally, Chapter 5 concludes.

References

Deutsche Rentenversicherung Bund (2012), Rentenversicherung in Zeitreihen, DRV-Schrift, 22, 1-334.

Chapter 2

The Financing Gap of the German Mandatory Pension System: Simulations and Solutions

Abstract

Population aging challenges pay-as-you-go pension systems. Solving the associated funding problem constantly motivates reform processes. In addition to an aging population, specific regulations of the German public pension system lead to an increasing financial burden for the national finances. To ensure sustainable pension funding, the calculation formula of the German public pension system is investigated in this paper. It will be shown that there are two alterable parameters which are not presently optimally used regarding the funding of public pensions. Simulations show that a variable demographic factor for calculating public pensions can reduce the burden of the national finances.

JEL: H55, J18

Keywords: Public pensions, Population aging, Funding, Simulation.

2.1 Introduction

The funding of pension benefits of the German mandatory pension system (*GRV*) is discussed frequently by politicians as well as economists. The main questions are, first, whether the mandatory pension system will be able to guarantee a sufficient level of pension benefits in the long run, and second, how to provide the necessary funding. Several regulations of Article 154 of the Sixth Book of the German Social Security Code (SGB VI) seem to guarantee a particular pension level (after social security contributions but before tax) from the mandatory pension system, as well as a maximum contribution rate. However, the funding of pension benefits with due regard to these regulations remains the subject of political and academic debate.

Public pension systems that are structured on a pay-as-you-go basis, are financed by contributions from the insured persons. Pension benefits are simultaneously funded by these contributions. An aging population fundamentally challenges this pay-as-you-go system. Various results may follow. First, if the amount of contributions decreases due to a decreasing number of contributors, pension benefits per capita will inevitably decrease if the contribution rate is kept constant. Second, if the pension level is to be kept at the same level, an increasing number of retirees inevitably leads to higher contributions per capita. This effect is amplified if the number of contributors also decreases. Third, if both the pension level and the contribution rate are not allowed to change, national finances need to bridge the gap that arises if the ratio of retirees to contributors increases. According to Deutsche Rentenversicherung Bund (2012), a gap between total expenditures and total income (without considering federal subsidies) of the German mandatory pension system has existed for decades. Thus, the government has been supporting the pension system with federal subsidies. In 1960, these subsidies amounted to 2.1 billion Euros and increased consistently by 2011 to about 58.9 billion Euros. An increasing financing gap consequently increases the amount of federal subsidies and thus, government expenses which are financed by tax payments from the entire population. Hence, an increasing financing gap eventually either increases the tax burden of the population if other government expenditures will prevail to the same extent, or it leads to decreasing government investments in other areas, such as education or national security, if taxes do not increase. However, since sustainable financing of the German mandatory pension system is essential to perpetuating the pay-as-you-go structure of the pension system, national finances are required to support its solvency.

To evaluate the financing gap, a closer look is taken at the present pension formula in Germany and especially the associated pension adjustment formula, that calculates the current pension value, which in turn is an important element of calculating pension benefits. Since the future effects of population aging on the funding of the German mandatory pension system are central to the further analysis, the so-called 'demographic factor' deserves closer attention. This factor is currently defined as a constant value and is set to 0.25, meaning that pensioners will bear 25% of the fiscal burden caused by demographic change. A value of 0.25 was justified, considering certain forecasts regarding the development of several pension system determinants, in order to maintain the regulations of Article 154 SGB VI regarding the pension level (after social security contributions but before tax) and the contribution rate.[1]

The aim of this paper is to show how a variable demographic factor may help to minimize the cumulated financing gap from 2012 to 2050, under different assumptions concerning the gross pension level (in the remainder referred to as 'pension level') and the contribution rate. The analysis in this paper will show that a constant demographic factor of 0.25 leads to significantly larger government subsidies to the pension system. Given the complexity of the pension formula and the pension adjustment formula, as well as the interdependency of target variables, a simulation model will be developed, under consideration of various assumptions concerning the economic and demographic development of Germany, in order to investigate the effect of a variable demographic factor on the financing gap. Note that the presented simulations contain aggregate effects and are conducted to shed light on the effect of a variable demographic factor in the first place. The objective is not to

[1]See also the Pension Insurance Sustainability Act (RV-Nachhaltigkeitsgesetz) for details.

present accurate and realistic forecasts of pension expenditures and developments in contribution rates and pension levels. For accurate simulations see, for example, Holthausen et al. (2012), who use a detailed simulation model that considers individual effects and work histories. However, the following analysis distinguishes between several scenarios which are explained in Section 2.5.

Furthermore, the analysis in this paper is a modified version of the analysis in the one by Bollacke (2014). The underlying income simulation for the basic simulation is different to that in the paper of 2014. Moreover, the so-called 'support rate', which was part of the analysis by Bollacke (2014), is dropped. In addition, the standard pension levels, which were presented both for the former East and West German federal states are now presented only for the former West German federal states as an approximation for entire Germany. This is justified by the fact that the current pension value for the former East federal states is intended to equal that for the former West federal states by 2025 and could be done by making a few changes in calculating the pension benefits and hence, the pension level. However, the basic as well as the alternative simulations are presented in more detail.

The remainder of the paper is structured as follows. Section 2.2 provides a short literature review. Section 2.3 presents the current pension formula and the included pension adjustment formula. Particular attention is directed to the coherence between the demographic factor and the contribution rate. Section 2.4 introduces the simulation model and shows developments of the future average gross income of the employees, the number of employees, the number of pensioners and the pensioner/contributor ratio. Section 2.5 defines the objective function that puts future income of the *GRV* in relation to its expenditures. To find out, whether a variable demographic factor may help to minimize the financing gap of the mandatory pension system, simulations of the cumulated financing gap, the average pension level and the average contribution rate are presented for both a constant and a variable demographic factor. Section 2.6 describes policy implications. Section 2.7 concludes.

2.2 Literature Review

Schmähl (1976) investigated the pension formula in Germany which was valid at that time. He depicted the coherence between those factors that determine the individual pension level and also showed how a low wage could be compensated for by a longer working period.

Keyfitz (1985) analyzed the effect of demographic change on a pay-as-you-go pension system in the USA. He concluded that the implicit interest rate of such a system is negative for persons born after 2000. The results hold true for both a 'fixed-pensions' system (retirees are guaranteed a particular pension level for which contributors have to pay) and a 'fixed-contributions' system (contributors pay a fixed amount of money which is then distributed among the retirees). Furthermore, he found that the difference in implicit interest rates between the cohorts is smaller in a 'fixed-contributions' system than a 'fixed-pensions' system. However, this result was criticized by Lapkoff (1991) who argued that Keyfitz's results only apply when looking at the year 1980, because Keyfitz took no account of existing retirees.

Börsch-Supan (2000) investigated the effects of demographic change on the German mandatory pension system. He stated that a reduction in the pension level will not automatically lead to a sustainable funding of the *GRV*. Since a reduced pension level is followed by more retirees who have to rely on basic security at old-age, it would merely shift the financial burden from one area to another. Börsch-Supan also argued that an increase in the legal retirement age may have negative effects on the labor market, and thus on the pension system. A higher legal retirement age leads to a higher effective retirement age and therefore to a greater supply of workers. This oversupply of labor cannot be absorbed entirely by the labor market and will thus lead to unemployment.

In another paper, Börsch-Supan (2004) investigated the effects of demographic change on the entire economy, but especially on the labor market and on the pensioner/contributor ratio until 2050. He predicts an increase of the pensioner/contributor ratio from about 80% to 100% and a working population of about 31 million by 2050.

Betzelt and Fachinger (2004) addressed the problem of self-employed people who are not compulsorily insured in the *GRV*. These people could encounter poverty in old age. This poverty may be reduced if the self-employed are compulsorily insured in the *GRV*.

Moreover, the financial basis of the *GRV* could be supported by payments from the self-employed. This would decrease the burden on the national finances in two ways. First, if the self-employed contribute to the *GRV*, more contributions would result, leading to less supporting payments by the government to the *GRV*. Second, if the self-employed are compulsorily insured, they would not face as much poverty in old age as before, and thus not have to rely on basic security at old-age.

The pension reform in Germany, as a reaction to the anticipated effects of demographic change on the mandatory pension system is the subject of Börsch-Supan et al. (2007). They investigate how the pension reform and especially the 'demographic factor' may lead to constant contribution rates.

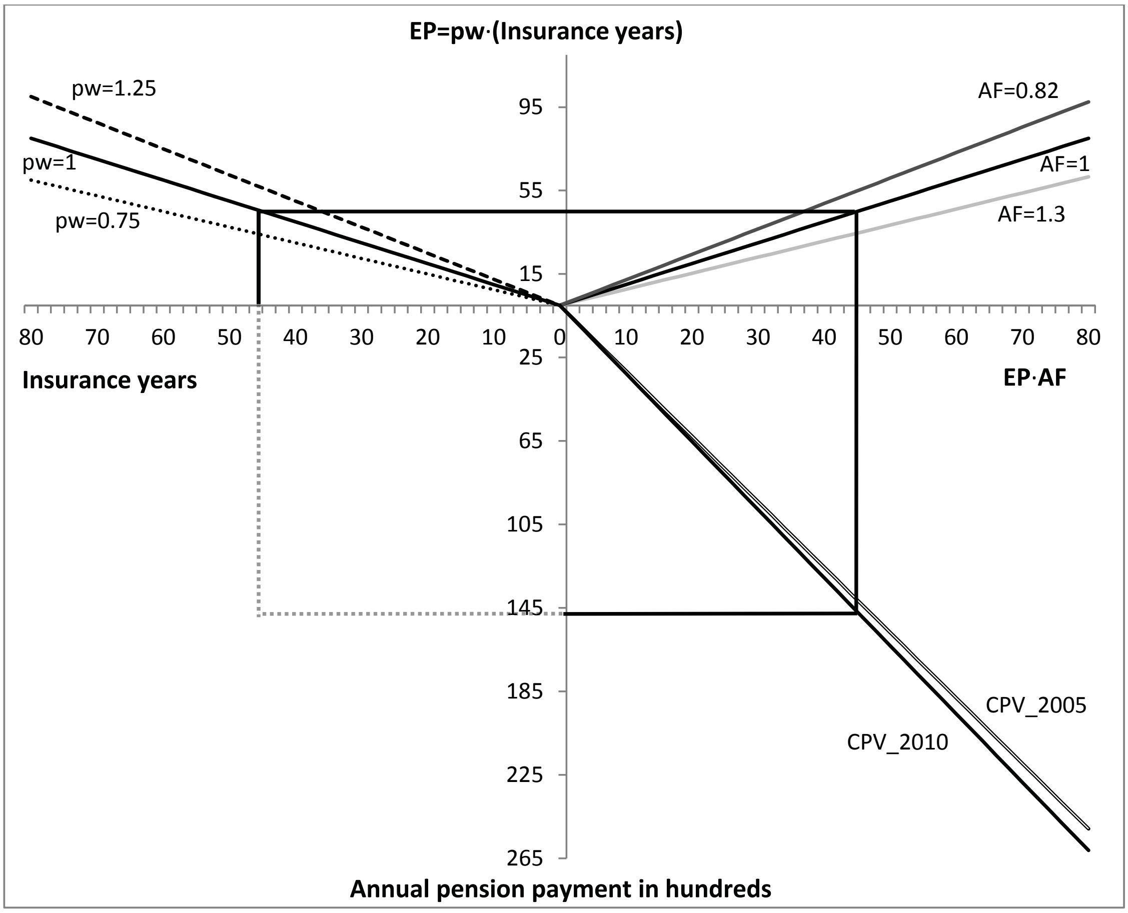
Finally, Gasche and Kluth (2012) compare different pension adjustment formulas and conclude that the current adjustment formula is better than its reputation. Furthermore, they simulate the development of the future pension level and future contribution rates under the assumption of a demographic factor of $\alpha = 1$, in comparison to the current demographic factor of $\alpha = 0.25$. However, an endogenous and therefore variable demographic factor in order to minimize the cumulated financing gap of the *GRV* has not yet been taken into account and is thus central to the further analysis.

2.3 Fundamentals

2.3.1 The Pension Formula

The so-called 'pension formula' in Germany is the calculation principle for the monthly pension benefits of all those entering retirement. Based on personal 'earning points' (EP), the 'pension type factor' (PTF), the 'age factor' (AF) and the 'current pension value' (CPV), the pension formula calculates each individual's

Figure 2.1: Illustration of pension determinants



Source: Own depiction based on Deutsche Rentenversicherung Bund (2012) and Article 77 SGB VI.
Notes: *pw* is the personal income level in relation to average gross income. The rectangle shows how the 'standard pension' in 2010 is calculated.

personal retirement income. According to Article 64 SGB VI, the pension formula is defined as:

$$Pension_{monthly} = EP \cdot PTF \cdot AF \cdot CPV \tag{2.1}$$

The age factor (AF) is unity if a person enters retirement on reaching the legal retirement age. This factor decreases the earlier a person enters retirement and in turn increases, the later a person enters retirement. The pension type factor (PTF) is unity for an old-age pension and, for example, 0.5 if a person claims a pension in the event of partial invalidity. The number of personal earning points (EP) is dependent on a person's individual contributions. A person obtains exactly one earning point in the assessment year, if he or she earned the exact average gross income that is subject to obligatory contributions to the mandatory pension system. The current pension value (CPV) amounted to 27.20 Euros in 2010 in the former West German federal states. It resulted in a monthly gross pension benefit of about 1224 Euros or an annual gross pension benefit of about 14688 Euros, if a retiree received the 'standard pension' (old-age pension at the legal retirement age with 45 earning points). These retirees are also called 'benchmark' pensioners. See Figure 2.1 for the coherence between the personal gross income, the insurance years, the age factor and the current pension value. Thus, the rectangle shows the annual pension benefit of a person receiving the standard pension. Note that only old-age pensions are considered. Hence, the pension type factor is set to unity. The personal income level (pw) shows the effect on the annual pension benefit if a person earned either 75%, 100% or 125% of the average gross income. If a person claims a pension five years *before* reaching the legal retirement age, an age factor of 0.82 is applied. On the other hand, an age factor of 1.3 is applied if a person claims a pension five years *after* reaching the legal retirement age (see Article 77 SGB VI for details). The current pension value is calculated by the 'pension adjustment formula' presented in the next section.

2.3.2 The Pension Adjustment Formula

According to Article 68 (5) SGB VI and Deutsche Rentenversicherung Bund (2012), the pension adjustment formula is defined by:

$$CPV_t = CPV_{t-1} \cdot \frac{GI_{t-1}}{GI_{t-2} \cdot \frac{\frac{GI_{t-2}}{GI_{t-3}}}{\frac{mGI_{t-2}}{mGI_{t-3}}}} \cdot \frac{1 - OAP_{t-1} - CR_{t-1}}{1 - OAP_{t-2} - CR_{t-2}} \cdot [(1 - \frac{PCR_{t-1}}{PCR_{t-2}}) \cdot \alpha + 1] \quad (2.2)$$

In equation (2.2), CPV defines the current pension value, GI defines the average gross income, mGI defines the average gross income that is subject to obligatory contributions to the mandatory pension system[2], OAP defines a factor for state-

[2] GI and mGI differ due to the maximum income threshold of the mandatory pension system. See Articles 159, 162 163 SGB VI and Annex 2 SGB VI. More on that in Section 2.4.

promoted old age provision,[3] CR defines the contribution rate to the mandatory pension system, PCR defines the pensioner/contributor ratio and α the demographic factor.

Since CR and α are the only two determinants in the pension adjustment formula that could be influenced directly by the government, particular attention is paid to them. Note that the change in the contribution rate to the mandatory pension system is actually dependent on the sustainability reserve of the pension system according to Article 158 SGB VI.[4] However, an increasing contribution rate (CR) has a negative effect on the development of the current pension value. Moreover, the demographic factor (α) is strongly dependent on the development of the pensioner/contributor ratio (PCR). Assuming the current demographic factor of $\alpha = 0.25$, an increase in the pensioner/contributor ratio from 0.50 to 0.51 leads to $[(1-\frac{0.51}{0.50})\cdot 0.25+1] = 0.995$. Increasing the demographic factor under these assumptions would decrease the current pension value. A decreasing pensioner/contributor ratio from 0.50 to 0.49 results in $[(1-\frac{0.49}{0.50})\cdot 0.25+1] = 1.005$. Increasing the demographic factor under this assumption would increase the current pension value and thus, the pension benefits if all other pension determinants are kept constant. Hence, the demographic factor and especially the coherence between the demographic factor and the contribution rate require a closer look.

2.3.3 The Coherence Between the Demographic Factor and the Contribution Rate

The coherence between α and CR is depicted by their isoquant (see Schmähl 1976 for additional information). Each point on the isoquant shows a combination of α and CR that leads to the same current pension value. The isoquant is determined by rearranging the pension adjustment formula in equation (2.2) to CR_{t-1}. With

[3]This factor simulates the financial burden of all employees due to state-promoted private retirement savings. It has been set to 0.04 since 2012. See Article 68 (5) SGB VI for details.

[4]See Article 216 SGB VI for further information on the sustainability reserve.

$(1+\gamma_{t-1}) = \frac{GI_{t-1}}{GI_{t-2}}$, $(1+\gamma_{t-2}) = \frac{GI_{t-2}}{GI_{t-3}}$, $(1+\beta_{t-2}) = \frac{mGI_{t-2}}{mGI_{t-3}}$, $(1+\theta_t) = \frac{CPV_t}{CPV_{t-1}}$ and $q_{t-1} = \frac{PCR_{t-1}}{PCR_{t-2}} - 1$, this yields:

$$CR_{t-1} = (1 - OAP_{t-1}) - \frac{(1 - OAP_{t-2} - CR_{t-2}) \cdot (1+\gamma_{t-2}) \cdot (1+\theta_t)}{(1+\gamma_{t-1}) \cdot (1+\beta_{t-2}) \cdot (1 - q_{t-1} \cdot \alpha)} \tag{2.3}$$

In equation (2.3), $(1+\theta)$ defines the growth rate of the current pension value, $(1+\gamma)$ defines the growth rate of average gross income, $(1+\beta)$ defines the growth rate of average gross income that is subject to obligatory contributions to the mandatory pension system, and q defines the rate of change of the pensioner/contributor ratio. Assuming that $(1+\gamma_{t-2}) = (1+\beta_{t-2})$, equation (2.3) simplifies to:

$$CR_{t-1} = (1 - OAP_{t-1}) - \frac{(1 - OAP_{t-2} - CR_{t-2}) \cdot (1+\theta_t)}{(1+\gamma_{t-1}) \cdot (1 - q_{t-1} \cdot \alpha)} \tag{2.4}$$

Assuming that CR_{t-1} and α can be seen as input factors to produce a particular or desired growth rate of the current pension value, which results in a 'fixed-pensions' system, the marginal rate of substitution of equation (2.4) can be determined. Thus, calculating the first derivative of equation (2.4) with respect to α shows how CR_{t-1} needs to react to an increasing demographic factor, so as to produce a particular growth rate of the current pension value. Equation (2.5) shows the derivative.

$$\frac{\partial CR_{t-1}}{\partial \alpha} = -\frac{q_{t-1} \cdot (1 - OAP_{t-2} - CR_{t-2}) \cdot (1+\theta_t)}{(1+\gamma_{t-1}) \cdot (1 - q_{t-1} \cdot \alpha)^2} \tag{2.5}$$

It is evident that $(1+\theta_t) > 0$, $(1+\gamma_{t-1}) > 0$ and $(1 - OAP_{t-2} - CR_{t-2}) > 0$. Furthermore, it is assumed that $q_{t-1} < 1$.[5] Therefore, an increase of α leads to a decreasing CR_{t-1} if the pensioner/contributor ratio increases ($q_{t-1} > 0$). To determine whether the isoquant in equation (2.5) is convex or concave, the second derivative of CR_{t-1} with respect to α is calculated. This yields:

$$\frac{\partial^2 CR_{t-1}}{\partial \alpha^2} = -\frac{2 \cdot q_{t-1}^2 \cdot (1 - OAP_{t-2} - CR_{t-2}) \cdot (1+\theta_t)}{(1+\gamma_{t-1}) \cdot (1 - q_{t-1} \cdot \alpha)^3} \tag{2.6}$$

[5] Only an increase in the pensioner/contributor ratio of more than 100% in one year would lead to $q_{t-1} \geq 1$.

Under the same assumptions as before, equation (2.6) is negative and therefore concave. The above calculations and considerations, in which a particular growth rate of the current pension value is to be reached, is called a 'fixed-pensions' system. In such a system, contribution rates are calculated in such a way that a particular minimum pension level or a particular current pension value results. This is considered in the simulations in Section 2.5. On the other hand, 'fixed-contributions' systems keep the contribution rates constant and calculate pension benefits for the retirees under consideration of other factors, such as income and the pensioner/contributor ratio. In contrast to a 'fixed-pensions' system, the growth rate of the current pension value is not fixed, but rather calculated by means of the accumulated contributions. Hence, the growth rate of the current pension value can be depicted as a function of the demographic factor. In this way, it is possible to investigate how the current pension value reacts to a varying demographic factor. Since 'fixed contributions' means that contribution rates are kept constant, the term $\frac{1-OAP_{t-1}-CR_{t-1}}{1-OAP_{t-2}-CR_{t-2}} = 1$ can be dropped. Furthermore, $(1+\gamma_{t-2}) = (1+\beta_{t-2})$ is still assumed. Rearranging equation (2.2) then yields:

$$(1+\theta_t)(\alpha) = (1+\gamma_{t-1}) \cdot [(1-(1+q_{t-1})) \cdot \alpha + 1] \tag{2.7}$$

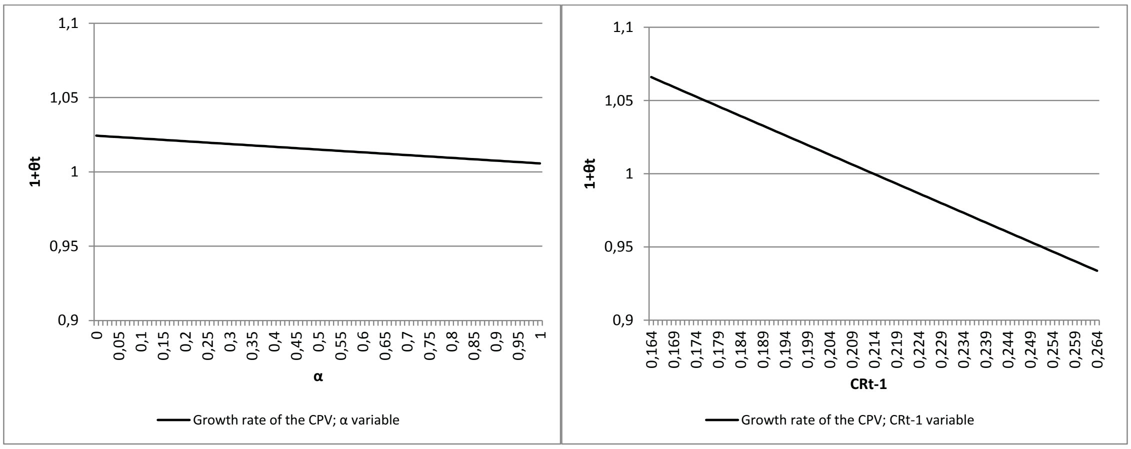
Calculating the first and the second derivative yields:

$$\frac{\partial(1+\theta_t)(\alpha)}{\partial\alpha} = -(1+\gamma_{t-1}) \cdot q_{t-1} \tag{2.8}$$

$$\frac{\partial^2(1+\theta_t)(\alpha)}{\partial\alpha^2} = 0 \tag{2.9}$$

Considering the rate of change of the pensioner/contributor ratio for 2011 with $q_{t-1} = 0.0182$ (see Deutsche Rentenversicherung Bund 2012) the first derivative presented in equation (2.8) is negative. Hence, an increasing demographic factor results in a decreasing growth rate of the current pension value. The second derivative presented in equation (2.9) shows that this development is linear. See the left part of Figure 2.2.

Figure 2.2: Development of the growth rate of the current pension value for a varying α (left part) and a varying CR_{t-1} (right part) for 2011



Source: Own depiction and calculation based on Deutsche Rentenversicherung Bund (2012).

If neither a 'fixed-pensions' nor a 'fixed-contributions' pension system is assumed and furthermore, a constant demographic factor of $\alpha = 0.25$ is assumed, all combinations of the growth rate of the current pension value and the contribution rate can be achieved (see the right part of Figure 2.2). This is shown by rearranging equation (2.4) and setting the growth rate of the current pension value as a function of CR_{t-1}. This yields:

$$(1+\theta_t)(CR_{t-1}) = \frac{(1 - OAP_{t-1} - CR_{t-1})}{(1 - OAP_{t-2} - CR_{t-2})} \cdot (1+\gamma_{t-1}) \cdot (1 - q_{t-1} \cdot \alpha) \qquad (2.10)$$

Calculating the first and second derivative of equation (2.10) leads to:

$$\frac{\partial(1+\theta_t)(CR_{t-1})}{\partial CR_{t-1}} = -\frac{(1+\gamma_{t-1}) \cdot (1 - q_{t-1} \cdot \alpha)}{(1 - OAP_{t-2} - CR_{t-2})} \qquad (2.11)$$

$$\frac{\partial^2(1+\theta_t)(CR_{t-1})}{\partial CR_{t-1}^2} = 0 \qquad (2.12)$$

Comparing the left and the right part of Figure 2.2 demonstrates that an increasing contribution rate has a stronger effect on the development of the growth rate of the current pension value than an increasing demographic factor. However, as shown in equation (2.5), the contribution rate is negatively affected by the demographic factor. Increasing the demographic factor will indeed lead to a decreasing growth rate of the current pension value, but it also leads to a decreasing contribution rate. This in turn increases the growth rate of the current pension value. Which effect is stronger depends on the development of gross income and the population or the pensioner/contributor ratio, respectively. This will be simulated in Section 2.5. However, it has become clear that the coherence between the contribution rate, CR_{t-1}, and the demographic factor, α, is central to the investigations concerning the income from contributions and the expenditures of the German mandatory pension system. Hence, the next section explains the simulation model which provides the foundation for further analysis.

2.4 Simulation of the Income and Expenditures of the GRV

2.4.1 Simulation Model

In order to investigate how demographic change and especially an aging population affects the funding of the German mandatory pension system, simulations of future contributions and future expenditures of the *GRV* are required. Again, the presented simulation model is constructed to present aggregate effects and shed light on the effect of a variable demographic factor on the financing gap in the first place. Since the earnings of all people will be indexed to the mean, i.e. all workers earn the average gross income, while all pensioners receive standard pensions, the model may not be able to present detailed simulations of pension expenditures, contribution rates and pension levels. However, different assumptions concerning the developments of economic and demographic determinants, presented in different simulations, enable to make almost general statements regarding the effect of a variable demographic factor on the financing gap. Other pension simulations and simulation models have been used in the past. For example, Wilke (2004) introduced and explained the MEA-PENSIM model to simulate different aspects of the German mandatory pension system. This model is able to consider contributions to health care and the long-term care of retirees. Moreover, it considers cohort-specific earning points to determine pension benefits and the federal subsidies that are paid to support the funding of the *GRV*. Börsch-Supan et al. (2010) used this model to analyze the dependency of the *GRV* on several economic factors, such as the business cycle. In order to do that, the authors used age-specific earnings profiles based on calculations by Fitzenberger et al. (2001). However, an appropriate simulation model for the purpose of this paper is presented below. It is, at least in some parts, heavily simplified compared to the MEA-PENSIM model. For example, cohort-specific earnings point are not incorporated. Rather use is made of the so-called 'benchmark' pensioner. Since the focus is on long-term, aggregate developments rather than short-term, detailed developments of individuals, this approach seems to be most suitable for the purpose of this paper. However, a more detailed wage simulation in comparison to Nagl and Vandrei (2013), who assumed an increase in wages of 2% per year, is incorporated into the model. Nevertheless, their approach is considered in an alternative simulation.

The *income of the GRV* can be separated roughly into five groups: the contribution income, federal subsidies, investment income, refundings and other income (see Deutsche Rentenversicherung Bund 2012). The following investigation only considers the contribution income, given the fact that, in a perfect world, old-age pension benefits are financed entirely by employee contributions, when the pension system is structured as a pay-as-you-go system. To simplify the simulation of the contribution income, the number of employees is used to approximate the number of contributors. The data are from Deutsche Rentenversicherung Bund (2012). The number of employees is then multiplied by the actual contribution rate of the *GRV* and the actual average gross income of the employees. Since the gross income that is subject to obligatory contributions of the mandatory pension system is about 93.5% of the actual average gross income in the period considered, this factor is incorporated into the calculations.[6] Thus, the contribution income is defined by:

$$I_t = 0.935 \cdot w_t \cdot EM_t \cdot CR_t \quad \forall t \in \{2012, ..., 2050\} \tag{2.13}$$

[6]The fact that the average income that is subject to obligatory contributions is lower than the statistical average income is due to the maximum income threshold of the mandatory pension system. Earnings above the threshold are not subject to contributions. See Articles 159, 162 163 SGB VI and Annex 2 SGB VI.

In equation (2.13), I defines the contribution income, w the average gross income of the employees, EM the number of the employees and CR the contribution rate.

The *average gross income*, w, is simulated in the following way. The rates of change of gross incomes from 1991 to 2011 are used and assumed to repeat after 2011. The calculation is based on data from Deutsche Rentenversicherung Bund (2012) and is defined by:

$$w_t = w_{t-1} \cdot \frac{w_{t-20}}{w_{t-21}} \quad \forall t \in \{2012, ..., 2050\} \tag{2.14}$$

With regard to equation (2.2), $w_t = GI_t$ and $(0.935 \cdot w_t) = mGI_t$. The *number of employees*, EM, is defined by:

$$EM_t = LF_t \cdot (1 - UR_t) \cdot 0.9 \quad \forall t \in \{2012, ..., 2050\} \tag{2.15}$$

In equation (2.15), LF defines the labor force which is approximated by all persons between 26 and 63, according to the population forecast 1-W2 of the German Federal Statistical Office (see Statistisches Bundesamt 2009).[7] Furthermore, UR defines the unemployment rate according to Deutsche Rentenversicherung Bund (2012). The unemployment rates between 1991 and 2011 are taken and assumed to repeat after 2011. Therefore, $LF_t \cdot (1 - UR_t)$ defines the working population. This is multiplied by 0.9 which is, according to Deutsche Rentenversicherung Bund (2012), the average share of employees to the working population between 1991 and 2011.

The *expenditures of the GRV*, PE, can also be separated into different categories. For example pension expenditures, expenses incurred for parenting and those for

[7]The upper age limit is justified by the effective retirement age of 63.5 (see Deutsche Rentenversicherung Bund 2012). The lower age limit then gives the best fit to the actual labor force. More detailed approximations of the labor force may also be useful, as in Holthausen et al. (2012). They use all persons between 15 and 71 multiplied by their age-specific labor force participation rate. The results of their simulation are similar to those in this paper. Since the earnings of all people are indexed to the mean income and are not dependent on age or work history, the use of age-specific labor force participation rates is redundant. The development of those working, in comparison to the pensioners, is rather important, due to demographic change. This is displayed sufficiently by the approximation of the labor force and the employees made in this paper.

old-age health care (see Deutsche Rentenversicherung Bund 2012).[8] In the following analysis, the focus will be on all expenditures of the *GRV* that are not extraneous benefits and thus need to be financed by contributions from those in work. These are referred to as 'pension expenditures' in the remainder of the paper and are simply calculated by multiplying the number of 'benchmark pensioners' (subsequently referred to as 'pensioners') with the 'standard pension' (separated into former West and East German federal states). However, pension expenditures are defined by:

$$PE_t = Pwest_t \cdot SPwest_t + Peast_t \cdot SPeast_t \quad \forall t \in \{2012, ..., 2050\} \tag{2.16}$$

In equation (2.16), PE defines the pension expenditures and P the number of pensioners in the former West and East German federal states, respectively.[9] This is simulated by all those between 64 and 100 and above, according to the population forecast 1-W2 of the German Federal Statistical Office (see Statistisches Bundesamt 2009), based on the average effective retirement age of 63.5 years according to Deutsche Rentenversicherung Bund (2012).[10] Moreover, SP defines the respective standard pensions.

The *standard pension*, SP, is calculated using equation (2.2) multiplied by 45 (number of earning points for a benchmark pensioner). Multiplying by 12 yields the annual standard pension.[11] It is therefore defined by:

$$SPwest_t = CPVwest_t \cdot 45 \cdot 12 \quad \forall t \in \{2012, ..., 2050\} \tag{2.17}$$

[8] According to Article 213 SGB VI, the government provides specific subsidies for extraneous benefits paid by the *GRV*. However, these are not incorporated in the following calculations.

[9] According to the German Federal Statistical Office (see Statistisches Bundesamt 2009) about 83% of all people in Germany lived in the former West German federal states between 1991 and 2008. This will increase to an average of about 86% until 2050, which will be considered in calculating the expenditures.

[10] First of all, it can be assumed that all those entering retirement have earned benefit entitlements from the mandatory pension system in the course of their working lives. This means that each person has at least worked for five years as an employee. Second, for ease of calculation, it is further assumed that on average, all pensioners enter retirement at the average effective retirement age and have earned 45 earnings points during their working lives.

[11] Note that for the calculation of the current pension value, it is assumed that the growth rates of gross income and gross income that is subject to obligatory contributions to the mandatory pension system are equal.

and

$$SPeast_t = CPVeast_t \cdot 45 \cdot 12 \quad \forall t \in \{2012, ..., 2050\} \tag{2.18}$$

Since the current pension value differs for the former West and East German federal states, different standard pensions result. The calculation of the standard pension for either former West or East German federal states is based on the growth rates of their respective gross income (see Deutsche Rentenversicherung Bund 2012 and Article 68 SGB VI). It is assumed that the growth rates of the state groups is the same. Note, that in order to calculate CPV_t, the value of the previous year, CPV_{t-1} is required. The value of 2011 is obtained from Deutsche Rentenversicherung Bund (2012). However, as required for by statute, the current pension value for the former East German federal states is intended to equal that for the former West German federal states in 2025.[12] Therefore, $CPVeast$ is expressed as a ratio of $CPVwest$. According to Deutsche Rentenversicherung Bund (2012) $CPVeast$ was 88.8% of $CPVwest$ in 2012 and will increase subsequently until 2025 reaching a value of 100% times $CPVwest$.

The current pension value is dependent on the pensioner/contributor ratio, PCR, defined by:

$$PCR_t = \frac{P_t}{EM_t} \quad \forall t \in \{2012, ..., 2050\} \tag{2.19}$$

Furthermore, the standard pension level, PL, is defined by:

$$PL_t = \frac{SPwest_t}{w_t} \quad \forall t \in \{2012, ..., 2050\} \tag{2.20}$$

The pension level is thus the standard pension in the former West German federal states divided by the average gross income of unified Germany. Although the standard pension level should be calculated by putting the standard pension of the former West German federal states in relation to the respective average gross income of the insured people, the pension level that is calculated in equation (2.20)

[12] See the 'Rentenüberleitungs-Abschlussgesetz' for details.

may not be perfectly accurate. This comes from the fact that the average gross income in the former West German federal states is slightly higher than in unified Germany, and that the average income of the insured people is also slightly higher than the average income of all employees in Germany. However, due to the fact that standard pensions will be equal in the state groups by 2025, this approach will become more accurate beginning in 2025. Note that the pension level is rather used to generate an additional scenario and not intended to provide a realistic forecast.

Finally, the *financing gap*, FG, and the *cumulated financing gap*, GAP_t, are defined simply by:

$$FG_t = PE_t - I_t \quad \forall t \in \{2012, ..., 2050\} \tag{2.21}$$

$$GAP_t = \sum_t FG_t = \sum_t (PE_t - I_t) \quad \forall t \in \{2012, ..., 2050\} \tag{2.22}$$

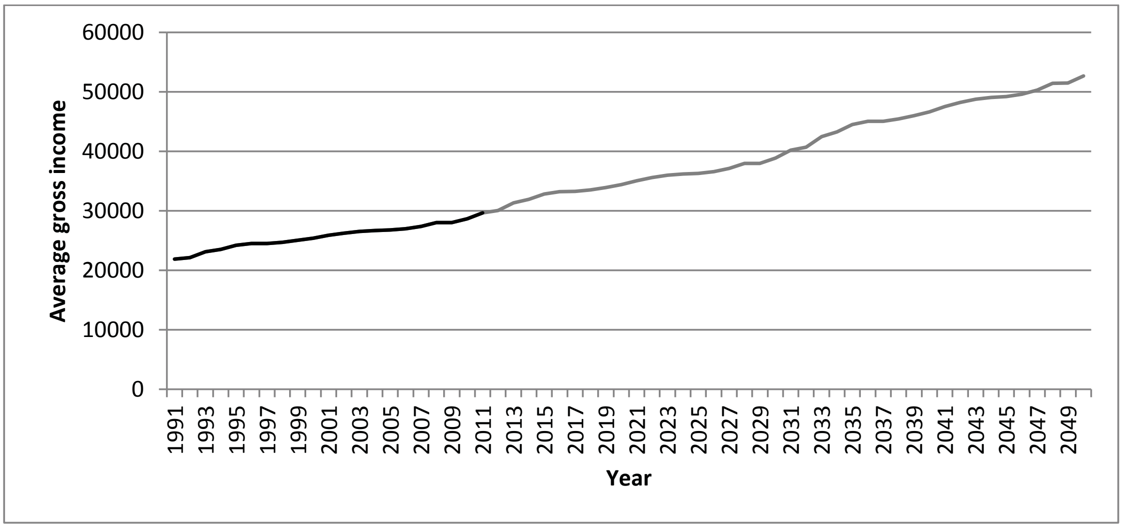
2.4.2 Future Developments

The future developments of the income and expenditures of the GRV are heavily dependent on the development of the contribution rate and the demographic factor. Since the development of the contribution rate and the demographic factor, under different assumptions concerning the demographic factor, is subject to the following analysis, the future development of the income and expenditures of the GRV cannot be described entirely at this point.

However, the development of the *gross income* of employees, the *number of employees*, the *number of pensioners* and the *pensioner/contributor ratio* can be described. Note, that the simulations presented in Figures 2.3 to 2.6 are based on equation (2.14), the assumption that the actual retirement age is 64 and on the population forecast 1-W2 of the German Federal Statistical Office (see Statistisches Bundesamt 2009).

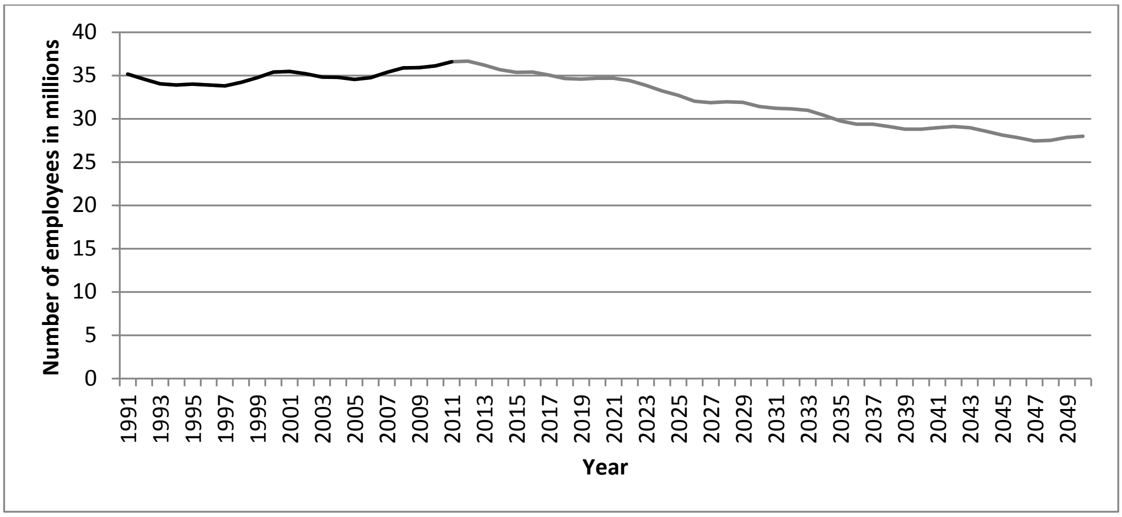
Figure 2.3 shows the actual and the simulated average gross income in Germany, based on data from Deutsche Rentenversicherung Bund (2012). The black line shows the actual values between 1991 and 2011. The gray line shows the simulation

Figure 2.3: Actual and simulated average gross income of the employees; 1991-2050



Source: Own depiction and calculation based on Deutsche Rentenversicherung Bund (2012).
Notes: The actual average gross income is depicted as the black line. The simulation begins in 2012 and is depicted as the gray line.

Figure 2.4: Actual and simulated number of employees; 1991-2050

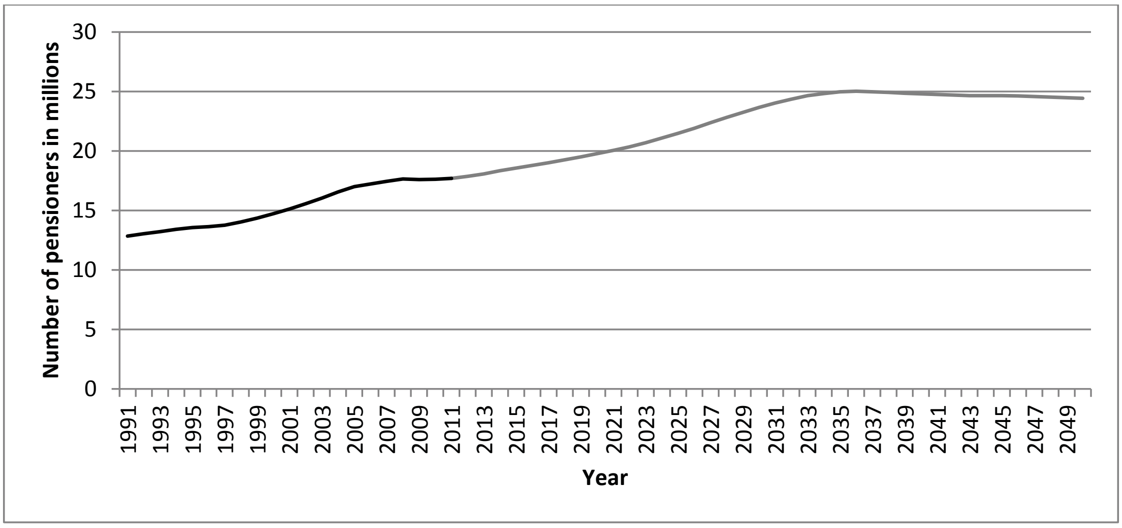


Source: Own depiction and calculation based on Deutsche Rentenversicherung Bund (2012) and Statistisches Bundesamt (2009).
Notes: The actual number of employees is depicted as the black line. The simulation begins in 2012 and is depicted as the gray line.

between 2012 and 2050. It can be seen that the average gross income increases until 2050, reaching a value of more than 50,000 Euros per year.

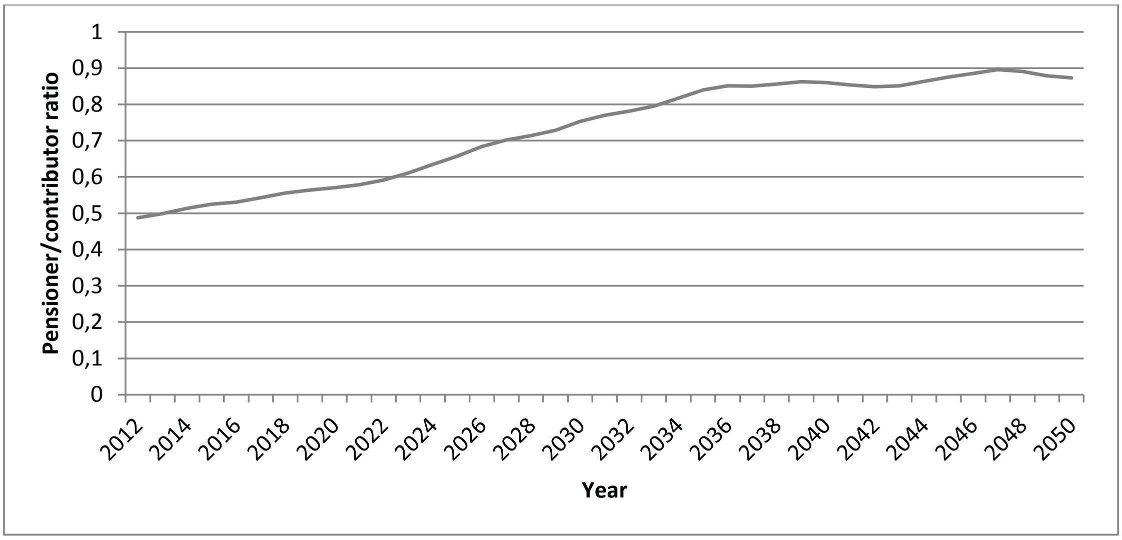
The actual and simulated number of employees is depicted in Figure 2.4. The data are based on Deutsche Rentenversicherung Bund (2012) and Statistisches Bundesamt (2009). Again, the black line shows the actual values between 1991 and 2011,

Figure 2.5: Actual and simulated number of pensioners; 1991-2050



Source: Own depiction and calculation based on Deutsche Rentenversicherung Bund (2012) and Statistisches Bundesamt (2009).
Notes: The actual number of pensioners is depicted as the black line. The simulation begins in 2012 and is depicted as the gray line.

Figure 2.6: Simulated pensioner/contributor ratio; 2012-2050



Source: Own depiction and calculation based on Deutsche Rentenversicherung Bund (2012) and Statistisches Bundesamt (2009).

while the gray line shows the simulated values between 2012 and 2050. It can be seen that the number of employees increases until 2012, with a number of more than 36 million employees. After 2012, the number of employees decreases to about 28 million in 2050.

Figure 2.5 shows the actual and simulated number of pensioners. The data are also based on Deutsche Rentenversicherung Bund (2012) and Statistisches Bundesamt (2009). Again, the black line shows the actual values between 1991 and 2011, while the gray line shows the simulation between 2012 and 2050. The number of pensioners increases until 2035, reaching a value of about 25 million pensioners. After 2035, it decreases slowly until 2050, to a value of about 24 million pensioners.

Finally, Figure 2.6 shows the simulated pensioner/contributor ratio between 2012 and 2050. The simulation is based on the simulated numbers of employees (as an approximation for the contributors) and pensioners. The ratio increases from almost 0.5 in 2012 to about 0.87 in 2050, meaning that by then, 87 pensioners must be financed by 100 employees.

2.5 Minimizing the Cumulated Financing Gap

2.5.1 Objective Function and Proceeding

In this section, the cumulated financing gap (GAP) in 2050 is minimized using the Microsoft Excel Solver. This tool is based on a nonlinear optimization procedure called the 'Generalized Reduced Gradient Algorithm' (GRG) (see Lasdon et al. 1978).[13] Under consideration of equations (2.2), (2.4) and (2.13) to (2.19), the objective function is defined by:

$$\min_{\alpha,CR} GAP_{2050} = \sum_{2012}^{2050} FG_t = \sum_{2012}^{2050} (PE_t - I_t),$$

$$\begin{aligned} \text{s.t.}\ \ & 0 \leq \alpha_t \leq 1 && \forall t \in \{2012, ..., 2050\} \\ & 0 \leq CR_t \leq 0.20 && \forall t \in \{2012, ..., 2020\} \\ & 0 \leq CR_t \leq 0.22 && \forall t \in \{2021, ..., 2030\} \\ & 0 \leq CR_t \leq 0.25 && \forall t \in \{2031, ..., 2050\} \end{aligned} \tag{2.23}$$

Thus, the cumulated financing gap is minimized by varying the contribution rate (CR) and the demographic factor (α), which is now dependent on the time, t. Note

[13]See Fylstra et al. (1998) for the implementation in the Microsoft Excel Solver.

again that the change in the contribution rate to the mandatory pension system is actually dependent on the sustainability reserve of the pension system according to Article 158 SGB VI.[14] Since a different target function, see equation (2.23), forms the basis for the following analysis, these regulations remain unconsidered. As shown in the above equation, the calculations are subject to several assumptions. First of all, the demographic factor, α, takes on values between zero (pensioners do not carry any burden caused by an aging population) and unity (pensioners carry the entire burden of an aging population). Furthermore, it is assumed that the contribution rate to the mandatory pension system will not exceed 20% of a person's income until 2020 and will not exceed 22% of a person's income until 2030. These rates are required by law according to Article 154 SGB VI. According to simulations by Holthausen et al. (2012), who predict a contribution rate of about 25% in 2050 in their basic scenario, it is further assumed that the contribution rate will not exceed 25% until 2050. The investigations in this section are based entirely on the assumption that the effective retirement age is 64. The appendix shows the results if an effective retirement age of 65 is assumed.

Furthermore, the first scenario (*Fixed Contributions*) keeps the contribution rates into the *GRV* constant at their maximum permissible level. That is, 20% until 2020, 22% until 2030 and 25% until 2050. Under these assumptions, the resulting pension levels and the financing gap under consideration of population aging will be calculated. For the purpose of this paper, it is not possible to calculate contribution rates as provided for by statute. Article 158 SGB VI states that the change in contribution rates is dependent on the sustainability reserve and thus on the development of the federal subsidies. Since federal subsidies are endogenous and dependent on the contribution rate and the demographic factor in the further investigations, contribution rates will be kept at their maximum permissible level.

The second scenario (*Fixed Pensions GR*) calculates contribution rates that need to be paid if the cumulated financing gap must be minimized and the current pension value is required to increase by 1% each year.

[14] See Article 216 SGB VI for further information on the sustainability reserve.

The third scenario (*Fixed Pensions PL*) also calculates contribution rates, but under the assumption that a minimum pension level of 43% is required.[15] Furthermore, different assumptions regarding the development of the income and the population are made. Finally, all scenarios consider the development of the financing gap, the pension level and the contribution rate, assuming both a variable and a fixed demographic factor of 0.25. However, the 'safeguard-clause', which states that the current pension value may not decrease, even though economic or demographic developments would lead the pension adjustment formula to calculate a decreasing value, remains unconsidered in the *Fixed Pensions PL* scenario, since a lower limit is already set to the pension level.[16]

2.5.2 Basic Simulations

The results presented in this subsection are based on the wage simulation shown in equation (2.14). Moreover, an effective retirement age of 64 is assumed. The tables show the cumulated financing gap (*GAP*) in 2050 in trillions of Euros, the average pension level and the average contribution rate between 2012 and 2050 as a percentage of gross income, both for a constant demographic factor of 0.25 and a variable demographic factor. The figures show the respective developments only for the *Fixed Pensions (PL)* scenario, since this is the only scenario that sets upper limits to the contribution rates as well as lower limits to the pension level.

[15] According to Article 154 SGB VI, the standard pension level after social security contributions but before tax ('safety level') should be at least 46% until 2020 and 43% until 2030. It is calculated by dividing the standard pension after the share of social security contributions that has to be paid by retirees (partial amount of health care and full amount of long-term care) before tax, by the average income of the insured people after the share of social security contributions that has to be paid by the employees (partial health care, pension insurance, unemployment insurance and long-term care as well as an average amount of additional old-age provision) before tax. The problem in calculating this level is, that earnings of civil servants are included in the calculation of the average income of the employees. But civil servants do not have to pay social security contributions. This distorts the average income after social security contributions of the employees. Moreover, a change in contribution rates would affect the safety level. Hence, for the ease of calculation, the following analysis assumes the minimum level for the gross pension level. This will, with almost certainty, guarantee the compliance with the requirement of Article 154 SGB VI. According to Deutsche Rentenversicherung (2012) a gross pension level of 46% led to a safety level of 50.1% in 2011. However, these regulations are simplified to a pension level of 43% in the following analysis, since otherwise, the applied algorithm does not yield any solutions.

[16] See Article 68a SGB VI for further information on the safeguard clause, and Articles 68a and 255e SGB VI for further information on the calculation of the current pension value.

Table 2.1: Results for 2012-2050; Basic; 1-W2

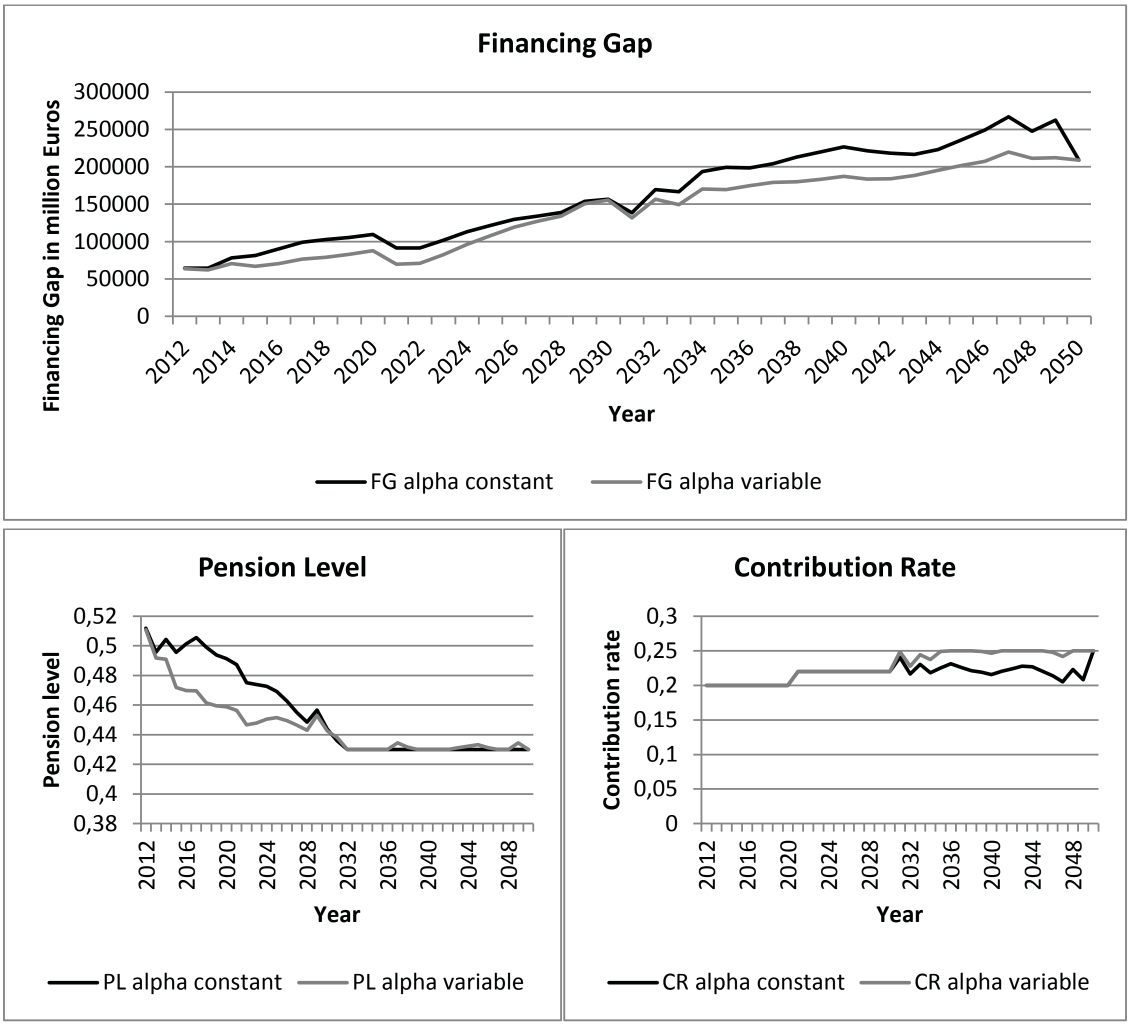
	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α constant	5.26	8.87	6.31
α variable	2.63	6.15	5.47
Pension Level (avg.) (in percent of gross income)			
α constant	44.70	47.72	45.51
α variable	38.17	45.92	44.61
Contribution Rate (avg.) (in percent of gross income)			
α constant	23.08	18.01	21.70
α variable	23.08	22.32	22.92

Notes: Own calculations.

First, Table 2.1 shows the results when simulations are based on the population forecast 1-W2 of the German Federal Statistical Office (see Statistisches Bundesamt 2009). It can be seen that the cumulated financing gap and the average pension level are lower when the demographic factor may vary in each scenario. Since in the *Fixed Contributions* scenario, the contribution rate is kept at the maximum permissible level, pensioners have to bear the entire burden of a lower financing gap.

In this scenario, the cumulated financing gap is about 2.63 trillion Euros in 2050, when the demographic factor is variable, compared to 5.26 trillion Euros when the demographic factor is 0.25. The average pension level is then 38.17% compared to 44.70%. The average contribution rate is the same at 23.08%. In the *Fixed Pensions GR* scenario, the cumulated financing gap is 6.15 trillion Euros (α variable) compared to 8.87 trillion Euros (α constant). In this scenario, both pensioners and contributors have to bear the financial burden. The average pension level is 45.92% compared to 47.72%, while the average contribution rate is 22.32%, compared to 18.01%. The results are similar in the *Fixed Pensions PL* scenario. The cumulated financing gap amounts to 5.47 trillion Euros (α variable), compared to 6.31 trillion

Figure 2.7: Financing gap, pension level and contribution rate; Fixed-Pensions (PL); Basic; 1-W2; 2012-2050



Source: Own depiction and calculation.

Euros (α constant). The average pension level between 2012 and 2050 is 44.61%, compared to 45.51%, while the average contribution rate is 22.92% compared to 21.70%.

In addition to Table 2.1, Figure 2.7 shows the development of the financing gap, the pension level and the contribution rate between 2012 and 2050 for the *Fixed Pensions PL* scenario, both for a variable and a constant demographic factor. All in all, the financing gap increases from about 64 billion Euros in 2012 to about 208 billion Euros in 2050. It is also obvious that over almost the entire time span, the financing gap is larger when a constant demographic factor is assumed than when a variable one is assumed. This holds true especially between 2030 and 2050, due to the fact that beginning in 2030, the baby-boom generation will almost entirely have entered retirement.[17]

The gross pension level is about 51% in 2012, both for a variable and a constant demographic factor. From 2032 until 2050, it is 43% if α is constant, and 43% with a few swings when α is variable. A huge difference between the pension levels can be seen between 2012 and 2032. During that period, the pension level is significantly lower when α is variable. This might be the reason for the financing gap being lower for a variable demographic factor during that period. Since the contribution rate is at the maximum permissible level almost each year until 2030, for both a constant and a variable demographic factor, pensioners bear the entire burden of a lower financing gap until 2030. The contribution rate is significantly higher when α is variable between 2030 and 2050, and reaches its permitted maximum almost each year, beginning in 2035. Since the pension levels do not really differ during that period, it can be concluded that contributors bear the entire burden of a lower financing gap between 2030 and 2050.

Table 2.2 shows the results when the simulations are based on the population forecast 1-W1 of the German Federal Statistical Office (see Statistisches Bundesamt 2009). Compared to the population forecast 1-W2, it assumes a migration balance of 100,000 instead of 200,000.

However, the cumulated financing gap and the average pension level are still lower when a variable demographic factor is assumed. In the *Fixed Contributions* scenario, the cumulated financing gap is about 2.88 trillion Euros in 2050 when the demographic factor is variable, compared to 5.44 trillion Euros when the demographic factor is 0.25. This is similar to the results presented in Table 2.1 and only slightly higher. The average pension level is then 37.96% compared to 44.40%. In the *Fixed Pensions GR* scenario, the cumulated financing gap is 6.45 trillion Euros (α variable) compared to 9.33 trillion Euros (α constant). These values are also slightly higher than those shown in Table 2.1. Again, in this scenario, both

[17] See also Nagl and Vandrei (2013).

Table 2.2: Results for 2012-2050; Basic; 1-W1

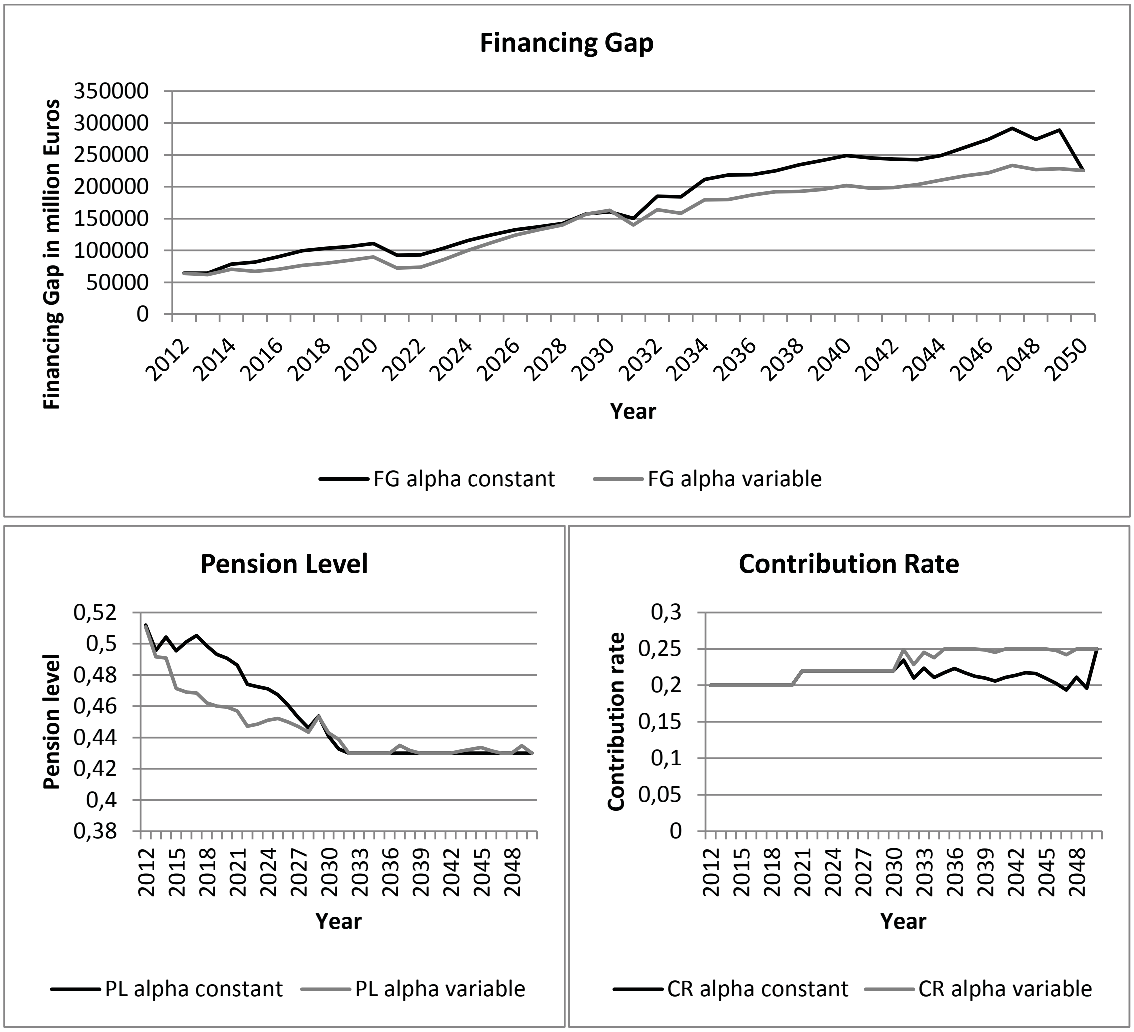
	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	5.44	9.33	6.77
α *variable*	2.88	6.45	5.78
Pension Level (avg.) (in percent of gross income)			
α *constant*	44.40	47.71	45.44
α *variable*	37.96	45.90	44.63
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	17.45	21.25
α *variable*	23.08	22.32	22.93

Notes: Own calculations.

pensioners and contributors have to bear the financial burden. The average pension level is 45.90%, compared to 47.71%, while the average contribution rate is 22.32% compared to 17.45%. As was already the case for the results in Table 2.1, the results are similar in the *Fixed Pensions PL* scenario. The cumulated financing gap is 5.78 trillion Euros (α variable), compared to 6.77 trillion Euros (α constant). The average pension level between 2012 and 2050 is 44.63% compared to 45.44%, while the average contribution rate is 22.93% compared to 21.25%. Hence, it seems that a slightly different population forecast does not substantially affect the cumulated financing gap and the average pension level. Thus, the results of all three scenarios, for both the population forecast 1-W2 and 1-W1, show that a variable demographic factor leads to a lower cumulated financing gap in 2050, to the disadvantage of both pensioners (lower pension level) and contributors (higher contribution rate).

In addition to Table 2.2, Figure 2.8 shows the development of the financing gap, the pension level and the contribution rate between 2012 and 2050 for the *Fixed Pensions PL* scenario, both for a variable and a constant demographic factor. The financing gap increases from about 64 billion Euros in 2012 to about 225 billion Euros in 2050. As for the population forecast 1-W2, it is also obvious that the

Figure 2.8: Financing gap, pension level and contribution rate; Fixed-Pensions (PL); Basic; 1-W1; 2012-2050



Source: Own depiction and calculation.

financing gap is larger when a constant demographic factor is assumed than with a variable one. The gross pension level is about 51% in 2012, both for a variable and a constant demographic factor. From 2030 until 2050 it is 43% if α is constant and 43% with a few swings when α is variable. The greatest difference between the pension levels can again be seen between 2012 and 2032. When α is variable, the pension level is significantly lower than when α is constant. Again, the contribution rate is at the maximum permissible level almost each year until 2030 for both a constant and a variable demographic factor. Hence, pensioners bear the entire burden of a lower financing gap until 2030. The contribution rate is significantly higher when α is variable between 2030 and 2050. Since the pension levels do not really differ during that period, the entire burden of a lower financing gap devolves on the contributors.

2.5.3 Alternative Income Simulations

Growth Rate: 2%

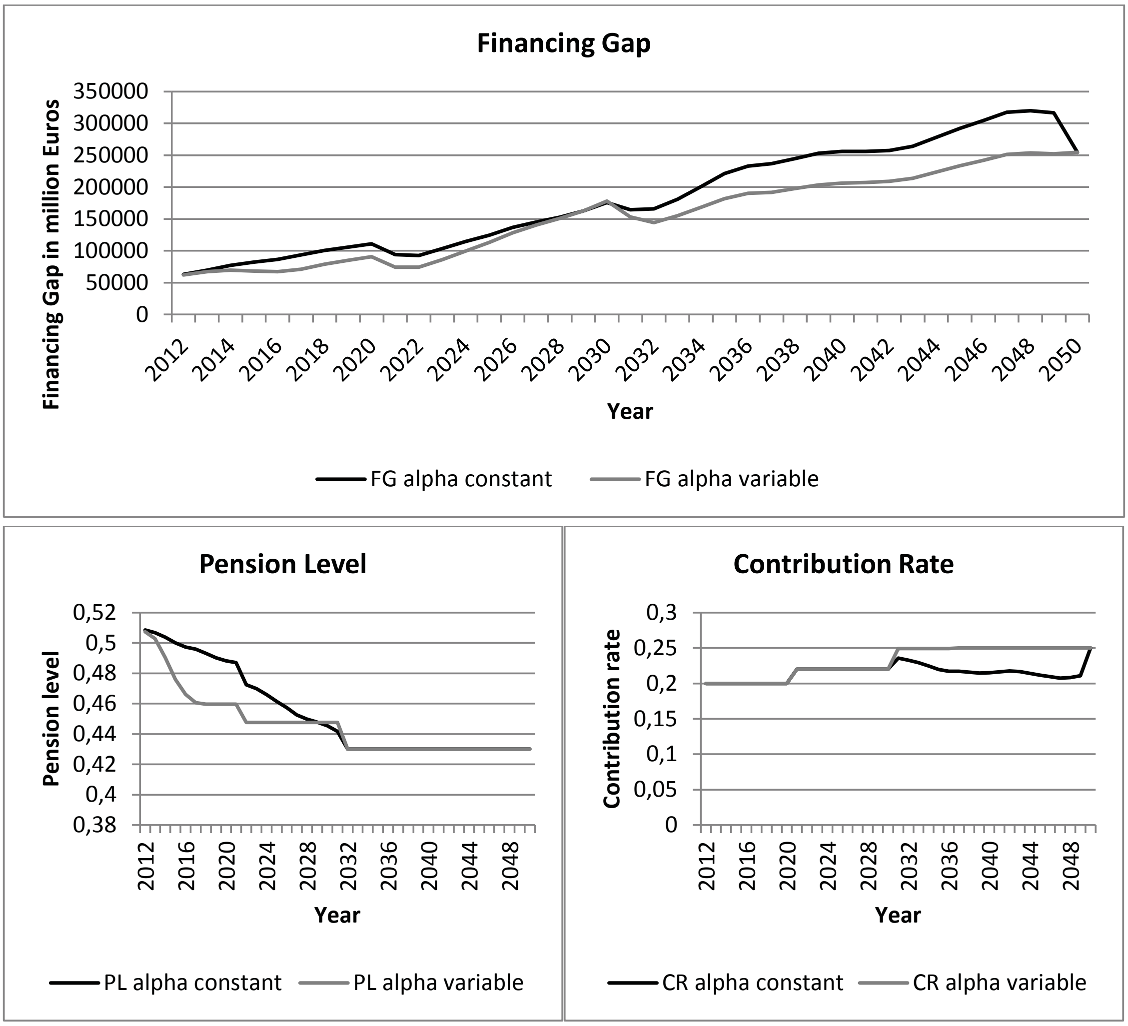
Compared to the income simulation in equation (2.14), on which the results presented in the previous subsection are based, this subsection assumes an annual growth rate of average gross income of 2%, as in Nagl and Vandrei (2013). Table 2.3 shows the results based on the population forecast 1-W2. These are similar to those in the basic simulations. For all three scenarios, the cumulated financing gap is lower when a variable demographic factor is assumed than when it is kept constant at 0.25. The same applied for the average pension level between 2012 and 2050. It is slightly lower when the demographic factor is variable, which means that pensioners have to bear at least part of the lower financial burden caused by a variable demographic factor. Except for the *Fixed Contributions* scenario, the contribution rate is higher when the demographic factor is variable, meaning that

Table 2.3: Results for 2012-2050; Income 2%; 1-W2

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α constant	2.02	6.09	7.11
α variable	1.87	5.02	6.00
Pension Level (avg.) (in percent of gross income)			
α constant	36.29	44.78	45.40
α variable	36.02	42.59	44.58
Contribution Rate (avg.) (in percent of gross income)			
α constant	23.08	22.58	21.50
α variable	23.08	22.76	23.06

Notes: Own calculations.

Figure 2.9: Financing gap, pension level and contribution rate; Fixed-Pensions (PL); Income 2%; 1-W2; 2012-2050



Source: Own depiction and calculation.

the contributors also have to bear part of the lower financial burden caused by a variable demographic factor.

Figure 2.9 shows the development of the financing gap, the pension level and the contribution rate between 2012 and 2050. As in the basic simulations, the financing gap increases from more than 62 billion Euros in 2012 to more than 254 billion Euros in 2050. The increase is due to a constant growth rate of gross income of 2%, slightly steeper than in the basic simulations. However, the financing gap is lower when the demographic factor is variable than when it is constant, especially after 2030, when the baby-boom generation will have almost entirely entered retirement.

Furthermore, the average pension level is about 51% in 2012, decreasing to 43% in 2032 for both a variable and a constant demographic factor. It stays at 43% until 2050. The contribution rate is at its maximum permissible level until 2030. When the demographic factor is made variable, it increases to about 25% from 2030 until 2050. It is lower at about 22% in most of the years when the demographic factor is kept constant. Hence, both pensioners and contributors have to bear part of the lower financial burden caused by a variable demographic factor.

Table 2.4 presents the results when the population forecast 1-W1 is assumed. Again, the cumulated financing gap in 2050 and the average pension level between 2012 and 2050 is lower when a variable demographic factor is assumed. Except for the *Fixed Contributions* scenario, the contribution rate is higher when the demographic factor is variable. Thus, both pensioners and contributors have to bear the financial burden of a lower financing gap caused by a variable demographic factor.

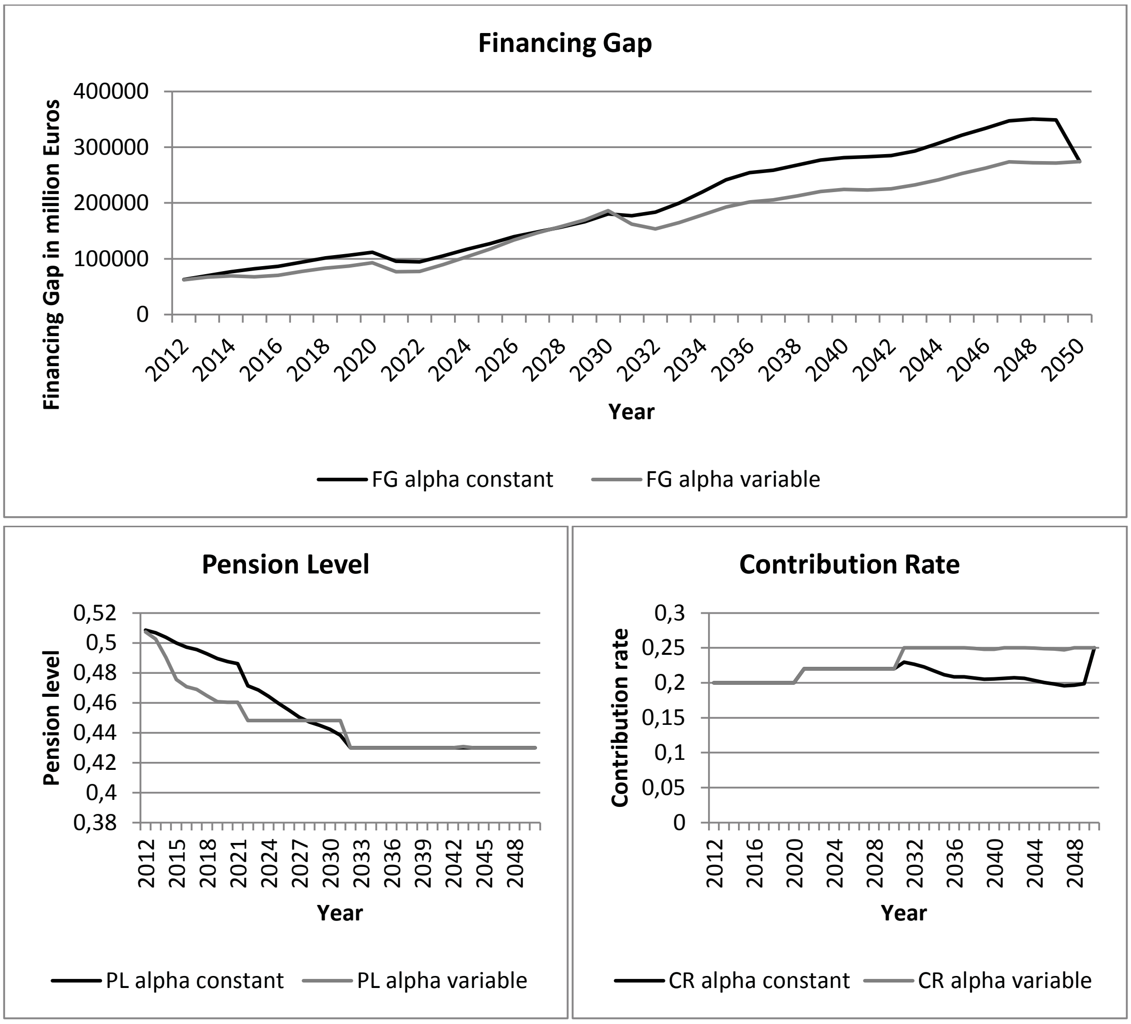
Figure 2.10 shows the development of the financing gap, the pension level and the contribution rate between 2012 and 2050. Due to an exponential increase in average gross incomes and a lower migration balance, the financing gap increases

Table 2.4: Results for 2012-2050; Income 2%; 1-W1

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α constant	5.94	6.40	7.63
α variable	2.20	5.36	6.38
Pension Level (avg.) (in percent of gross income)			
α constant	44.17	44.56	45.33
α variable	35.88	42.56	44.65
Contribution Rate (avg.) (in percent of gross income)			
α constant	23.08	22.43	21.04
α variable	23.08	22.77	23.05

Notes: Own calculations.

Figure 2.10: Financing gap, pension level and contribution rate; Fixed-Pensions (PL); Income 2%; 1-W1; 2012-2050



Source: Own depiction and calculation.

from about 63 billion Euros in 2012 to about 274 billion Euros in 2050. However, the financing gap is lower when the demographic factor is variable than when it is constant, again especially after 2030 when the baby-boom generation has almost entirely entered retirement. Moreover, the pension level is still about 51% in 2012, decreasing to 43% in 2032. It stays at 43% until 2050. Again, the contribution rate is at its maximum permissible level until about 2030. If the demographic factor is made variable, it increases to about 25% from 2030 until 2050. It is lower at about 21% in each year, except for 2050, when the demographic factor is kept constant.

Thus, both pensioners and contributors have to bear part of the lower financial burden caused by a variable demographic factor.

Growth Rate: 1%

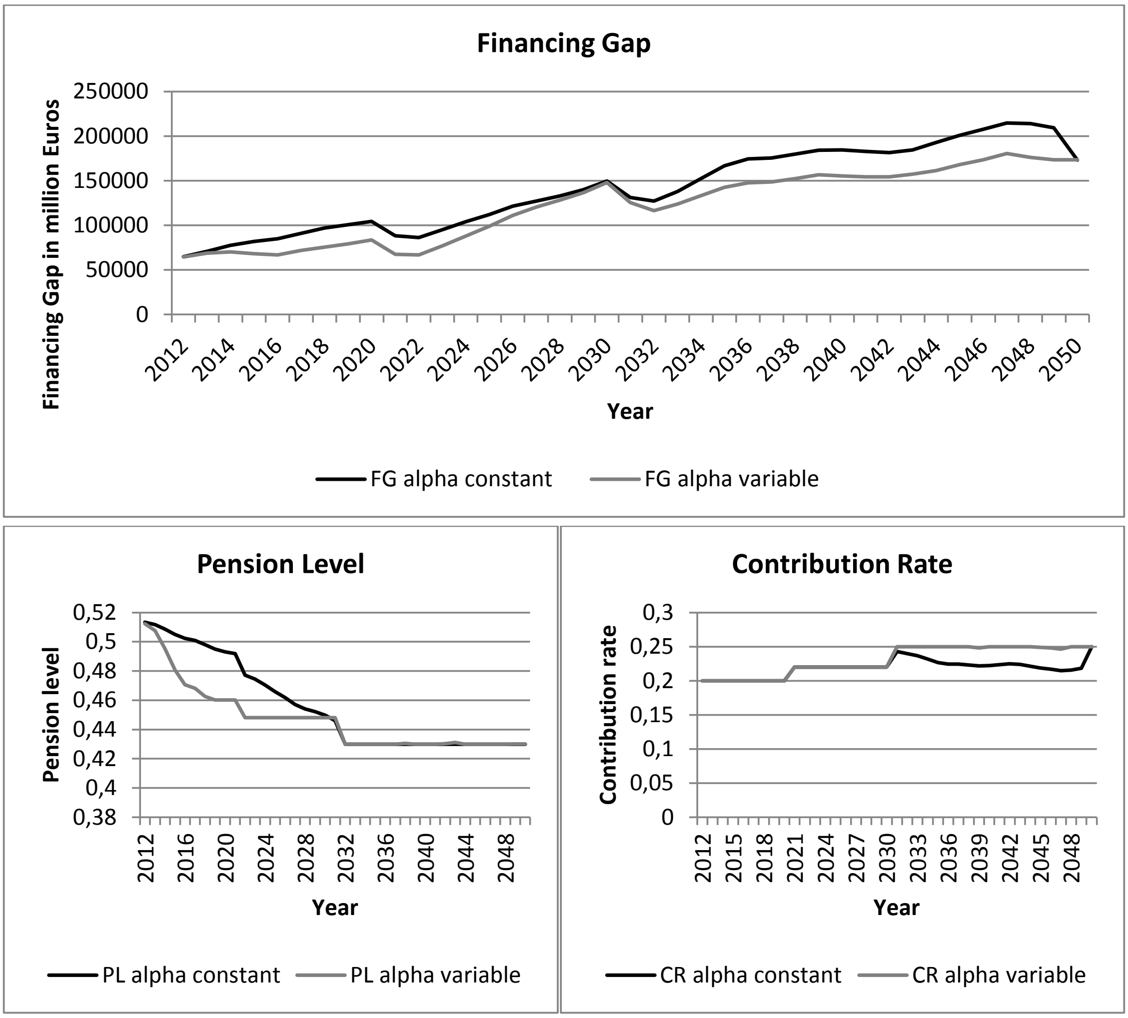
The results in this section are based on an income simulation that assumes an annual growth rate of 1% from 2012 until 2050. Table 2.5 presents the results if population forecast 1-W2 is assumed. For the most part, it shows similar results to those above. The cumulated financing gap in 2050 is, in each scenario, lower when the demographic factor is variable than when it is kept constant at 0.25. Whereas the average pension level between 2012 and 2050 was also clearly lower in each scenario of the previous simulations when a variable demographic factor was assumed, it no longer holds true for the *Fixed Pensions GR* scenario in this simulation. The average pension level is 51.35% for a constant demographic factor and 51.24% for a variable demographic factor. Hence, the contributors have to bear most of the burden of a lower financing gap in this scenario. The average contribution rate between 2012 and 2050 is 20.38% compared to 12.28%. This can be explained by the fact that pensions are assumed to increase at least by

Table 2.5: Results for 2012-2050; Income 1%; 1-W2

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	3.82	11.54	5.51
α *variable*	3.70	8.12	4.76
Pension Level (avg.) (in percent of gross income)			
α *constant*	42.26	51.35	45.64
α *variable*	41.94	51.24	44.70
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	12.28	21.85
α *variable*	23.08	20.38	23.05

Notes: Own calculations.

Figure 2.11: Financing gap, pension level and contribution rate; Fixed-Pensions (PL); Income 1%; 1-W2; 2012-2050



Source: Own depiction and calculation.

the same rate as gross income. Thus, to minimize the cumulated financing gap in 2050, the algorithm tries to keep the growth rate of the pension benefits at its minimum permissible level, which is 1%, leading the pension benefits to increase at the same rate as the average gross income, in almost each year. This results in the contributors bearing almost the entire burden of a lower financing gap caused by a variable demographic factor.

Figure 2.11 shows the development of the financing gap, the pension level and the contribution rate in detail. Due to an smaller increase of average gross incomes than assumed in the previous section, the financing gap increases from more than 64 billion Euros in 2012 to only about 173 billion Euros in 2050. However, the financing gap is lower when the demographic factor is variable than when it is constant, again especially after 2030 when the baby-boom generation has almost entirely entered retirement. Additionally, the pension level exceeds 51% in 2012, decreasing to 43% in 2032. It stays at about 43% until 2050. Again, the contribution rate is at its maximum permissible level until about 2030. If the demographic factor is made variable, it increases to about 25% from 2030 until 2050. It is lower at about 22% in each year, except for 2050, when the demographic factor is kept constant. Thus, both pensioners and contributors have to bear part of the lower financial burden caused by a variable demographic factor.

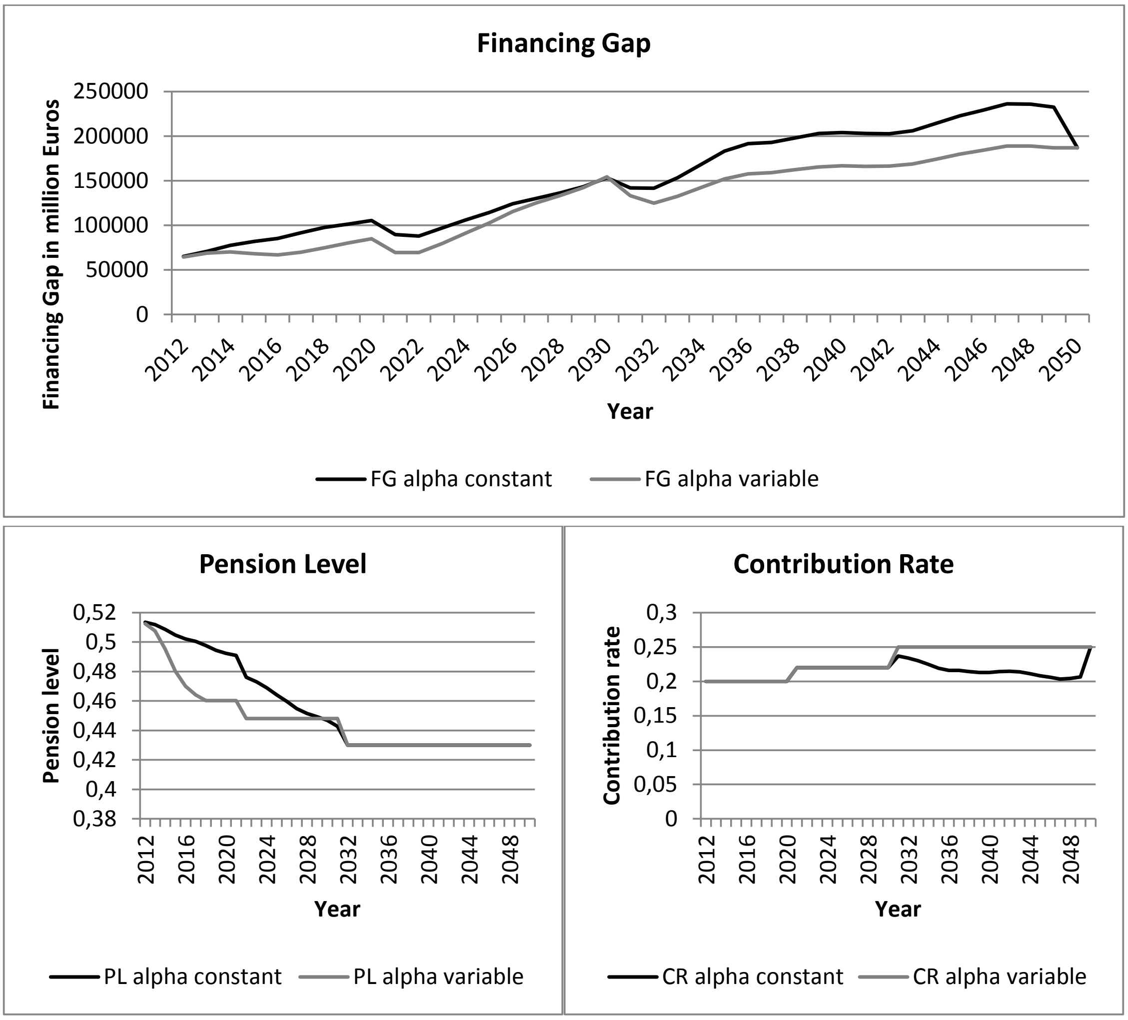
Finally, Table 2.6 presents the results for the population forecast 1-W1. The central statement is again unaffected by different assumptions concerning the migration balance. The cumulated financing gap in 2050 is still lower in each scenario when a variable demographic factor is assumed. Since the gross income is still assumed to increase by 1% each year, the average pension level is again 51.35% compared to 51.24% in the *Fixed Pensions GR* scenario. Hence, the contributors have to bear

Table 2.6: Results for 2012-2050; Income 1%; 1-W1

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	4.87	11.84	5.90
α *variable*	3.95	8.37	5.02
Pension Level (avg.) (in percent of gross income)			
α *constant*	44.61	51.35	45.57
α *variable*	41.86	51.24	44.67
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	11.62	21.40
α *variable*	23.08	20.26	23.08

Notes: Own calculations.

Figure 2.12: Financing gap, pension level and contribution rate; Fixed-Pensions (PL); Income 1%; 1-W1; 2012-2050



Source: Own depiction and calculation.

almost the entire financial burden of a lower financing gap caused by a variable demographic factor.

Figure 2.12 shows similar results to those in the previous simulations. The financing gap increases until 2050 to almost 187 billion Euros. The pension level and the contribution rate react almost entirely as in the previous simulations.

Compared to the basic simulations presented in Section 2.5.2, the results, and hence the central statement of this paper is unaffected by different assumptions regarding the growth rate of gross incomes or the population forecast. Both population forecasts and all three income scenarios lead to the same result. The cumulated financing gap in 2050 is lower when a variable demographic factor instead of a constant demographic factor of 0.25 is applied. The lower financing gap is funded both by pensioners (through lower pension levels) and contributors (through higher contribution rates). The only exception is the *Fixed Pensions GR* scenario, in which average gross income increases at the same rate as the pension benefits. This leads to the financial burden of a lower financing gap caused by a variable demographic factor being almost entirely borne by the contributors. In addition, at least in the *Fixed Pensions PL* scenario, upper limits for the contribution rate as well as lower limits for the pension level are set, which can be viewed as an approximation to the regulations of Article 154 SGB VI. Furthermore, it appears that the baby-boom generation has a strong effect on the development of the financing gap, pension level and contribution rate. Especially when looking at the development of the financing gap in Figures 2.7 to 2.12, it becomes clear that the slope of the gap between 2020 and 2030 is steeper than between 2012 and 2019 or 2031 and 2050, respectively. This is due to the baby-boom generation entering retirement.

To evaluate the effects of a higher effective retirement age, the Appendix (Tables 2.7 - 2.12) shows the results when people enter retirement at 65 instead of 64. Both population forecasts and all three income scenarios are considered.

2.6 Policy Implications

The previous section shows that a variable demographic factor may help to reduce the financial burden of national finances regarding the mandatory pension system. The lower financing gap only emerges, because financial pressure is shifted away from national finances to either contributors or pensioners or both. Hence, to maintain a sustainable funding of the German mandatory pension system in accordance with a minimized cumulated financing gap, the pensioners need to live with slightly lower pensions, and contributors need to pay slightly higher contributions. But, applying the idea of a variable demographic factor is harder than it seems. Since the current pension value in t is always dependent on the current pension value in $t-1$, the demographic factor not only affects the current pension value in t, but as a consequence, automatically affects the current pension value in every further period (see equation (2.2)). Thus, it is not possible to react immediately to demographic and economic changes by altering the demographic factor. Therefore, it is necessary to define a target year, which is 2050 in this paper. Hence, a period of time was defined for which all necessary demographic and economic determinants have been simulated. This provides an opportunity to calculate the optimal path of α, so as to minimize the cumulated financing gap for different scenarios. The current demographic factor of $\alpha = 0.25$ was set using almost the identical idea. Again, it was justified under consideration of particular forecasts regarding the development of several pension system determinants, in order to maintain the regulations of Article 154 SGB VI regarding the pension level (after social security contributions but before tax) and contribution rate.[18]

However, the calculation algorithm used in this paper sets the demographic factor to $\alpha = 0$ almost each year, especially after 2020 when the baby-boom generation starts entering retirement. This holds true for each scenario, both population forecasts and all three income simulations. Thus, the pensioner/contributor ratio has, for most years, no direct influence on the development of the current pension value. Yet, the number of both pensioners and contributors has an impact, since they affect both the level of the entire pension benefits and contributions. However, it seems that a positive demographic factor would result in an excessively low current pension value, so that the regulations of Article 154 SGB VI possibly cannot be maintained. The algorithm almost entirely uses the contribution rate to influence the current pension value, so as to minimize the cumulated financing gap. Furthermore, there is also evidence that a constant demographic factor of $\alpha = 0.25$ could result in a pension level that is even lower than 40%. Therefore, the current calculation of pensions is not able to guarantee the regulations of Article 154 SGB VI in the long run with certainty. Hence, reforming the German mandatory pension system by making it more flexible and enabling it to react to demographic and economic shocks is inevitable.

[18] See again the Pension Insurance Sustainability Act (RV-Nachhaltigkeitsgesetz) for details.

2.7 Conclusions

This paper investigates whether a variable demographic factor, in comparison to a constant demographic factor of $\alpha = 0.25$, could reduce the financing gap of the German mandatory pension system. The financing gap is defined as the gap between the expenditures and contribution income of the *GRV*. Based on a simple simulation model, the cumulated financing gap in 2050 was minimized, by altering the contribution rate and/or the demographic factor, under different assumptions regarding the development of the population and the average gross income. Furthermore, various different scenarios were analyzed which compared the development of the financing gap, assuming either a constant or a variable demographic factor.

It has been shown that in each scenario and each simulation a variable demographic factor (in contrast to a constant one of 0.25) may reduce the financing gap of the *GRV*. The reduction of the financing gap is, however, funded by both pensioners (through lower pension levels) and contributors (through higher contribution rates). It has also been shown that between about 2020 and 2030, the slope of the financing gap is steeper than between 2012 and 2019, and 2031 and 2050. This is due to the baby-boom generation entering retirement. Moreover, beginning in about 2030, the financing gap is significantly lower using a variable demographic factor than when using a constant one, which can be explained by the baby-boom generation having almost entirely entered retirement. Hence, the financial burden of national finances, especially after 2020, needs to be funded either by higher contribution rates, lower pension levels or higher federal subsidies. However, higher federal subsidies automatically lead to either higher taxes (which burdens both contributors and pensioners as well as any other taxpayer) or to decreasing governmental investments in other areas.

Nevertheless, the simulations in this paper showed that, due to an aging population and the baby-boom generation entering retirement, the pension system will simultaneously face increasing expenditures and decreasing contribution income. The resulting financing gap can only be closed by using federal subsidies, although these can be reduced by making the demographic factor variable. But as already mentioned in the previous section, this is more difficult than it seems. However, reforming the pension system to make it more flexible is necessary, in order to at least guarantee the regulations of Article 154 SGB VI. Aside from that, the analysis in this paper did not consider private pension savings. Additional provision for old-age may increase the individual pension level and hence, could reduce the individual pension gap (see Börsch-Supan and Gasche 2010).

Appendix

The following Tables present the results when an effective retirement age of 65 is assumed. It can be seen that the central statement is not affected by this assumption. The financing gap is still lower when a variable demographic factor is assumed. In addition, both pensioners and contributors are burdened financially via lower pension levels and higher contribution rates.

Table 2.7: Results for 2012-2050; Basic; 1-W2; effective retirement age 65

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	4.48	8.17	5.10
α *variable*	1.65	5.29	4.43
Pension Level (avg.) (in percent of gross income)			
α *constant*	45.56	48.64	46.03
α *variable*	38.15	46.52	44.72
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	17.98	22.28
α *variable*	23.08	22.33	22.94

Notes: Own calculations.

Table 2.8: Results for 2012-2050; Basic; 1-W1; effective retirement age 65

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α constant	4.69	8.64	5.59
α variable	1.97	5.60	4.79
Pension Level (avg.) (in percent of gross income)			
α constant	45.24	48.63	45.95
α variable	38.06	46.52	44.84
Contribution Rate (avg.) (in percent of gross income)			
α constant	23.08	17.43	21.86
α variable	23.08	22.33	22.93

Notes: Own calculations.

Table 2.9: Results for 2012-2050; Income 2%; 1-W2; effective retirement age 65

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α constant	1.58	5.27	5.76
α variable	0.89	4.01	4.86
Pension Level (avg.) (in percent of gross income)			
α constant	37.83	45.64	45.90
α variable	36.14	42.92	44.70
Contribution Rate (avg.) (in percent of gross income)			
α constant	23.08	22.56	22.10
α variable	23.08	22.79	23.08

Notes: Own calculations.

Table 2.10: Results for 2012-2050; Income 2%; 1-W1; effective retirement age 65

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	5.11	5.58	6.31
α *variable*	1.24	4.37	5.31
Pension Level (avg.) (in percent of gross income)			
α *constant*	45.02	45.40	45.83
α *variable*	36.00	42.90	45.01
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	22.44	21.66
α *variable*	23.08	22.81	23.08

Notes: Own calculations.

Table 2.11: Results for 2012-2050; Income 1%; 1-W2; effective retirement age 65

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	3.37	10.71	4.45
α *variable*	2.80	7.05	3.82
Pension Level (avg.) (in percent of gross income)			
α *constant*	43.83	52.10	46.17
α *variable*	41.95	51.24	44.76
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	12.62	22.41
α *variable*	23.08	20.52	23.08

Notes: Own calculations.

Table 2.12: Results for 2012-2050; Income 1%; 1-W1; effective retirement age 65

	Fixed Contributions	*Fixed Pensions (GR)*	*Fixed Pensions (PL)*
GAP in 2050 (in trillion Euros)			
α *constant*	4.19	11.02	4.86
α *variable*	3.06	7.31	4.10
Pension Level (avg.) (in percent of gross income)			
α *constant*	45.46	52.10	46.09
α *variable*	41.86	51.24	44.76
Contribution Rate (avg.) (in percent of gross income)			
α *constant*	23.08	11.98	22.00
α *variable*	23.08	20.42	23.08

Notes: Own calculations.

References

Betzelt, S., U. Fachinger (2004), Selbständige – arm im Alter? Für eine Absicherung Selbständiger in der GRV, Wirtschaftsdienst, 6, 379-386.

Bollacke, S. (2014), Die Finanzierungslücke der Gesetzlichen Rentenversicherung in Deutschland - Eine Simulationsstudie zu Lösungsmöglichkeiten, Zeitschrift für Wirtschaftspolitik, 63 (2), 128-159.

Börsch-Supan, A. (2000), A Model under Siege: A Case Study of the German Retirement Insurance System, The Economic Journal, 110, No. 461, F24-F45.

Börsch-Supan, A. (2004), Gesamtwirtschaftliche Folgen des demografischen Wandels, MEA-Diskussionspapier, 51-2004, 1-20.

Börsch-Supan, A., A. Reil-Held, C. Wilke (2007), How an Unfunded Pension System looks like Defined Benefits but works like Defined Contributions: The German Pension Reform, Espina, A., Estado de Bienestar y competitividad - La experiencia europea, Fundación Carolina, Madrid, 319-347.

Börsch-Supan, A., M. Gasche, C. Wilke (2010), Konjunkturabhängigkeit der Gesetzlichen Rentenversicherung am Beispiel der aktuellen Finanz- und Wirtschaftskrise, Zeitschrift für Wirtschaftspolitik, 59, No. 3, 298-328.

Börsch-Supan, A., M. Gasche (2010), Kann die Riester-Rente die Rentenlücke in der gesetzlichen Rente schließen?, MEA-Diskussionspapier, 201-2010, 1-36.

Deutsche Rentenversicherung Bund (2012), Rentenversicherung in Zeitreihen, DRV-Schrift, 22, 1-334.

Fitzenberger, B., R. Hujer, T. E. MaCurdy, R. Schnabel (2001), Testing for uniform wage trends in West Germany: A cohort analysis using quantile regressions for censored data, Empirical Economics, 26, 41-86.

Fylstra, D., L. Lasdon, J. Watson, A. Waren (1998), Design and Use of the Microsoft Excel Solver, INTERFACES, 28, No. 5, 29-55.

Gasche, M., S. Kluth (2012), Dynamisierung der Rente: Was ist die beste Rentenanpassungsformel?, Zeitschrift für Wirtschaftspolitik, 61, No. 1, 3-45.

Holthausen, A., J. Rausch, C. Wilke (2012), MEA-PENSIM 2.0: Weiterentwicklung eines Rentensimulationsmodells, Konzeption und ausgewählte Anwendungen, MEA-Diskussionspapier, 254-2012, 1-46.

Keyfitz, N. (1985), The Demographics of Unfunded Pensions, European Journal of Population, 1, No. 1, 5-30.

Lasdon, L. S., A.D. Waren, A. Jain, M. Ratner (1978), Design and Testing of a Generalized Reduced Gradient Code for Nonlinear Programming, ACM Transactions on Mathematical Software, 4, No. 1, 34-50.

Lapkoff, S. F. (1991), A Research Note on Keyfitz's 'The Demographics of Unfunded Pensions', European Journal of Population, Vol. 7, No. 2, S. 159-169.

Nagl, W., L. Vandrei (2013), Der Beitragssatz zur Gesetzlichen Rentenversicherung im demographischen Wandel – Die Babyboomer gehen in Rente, ifo Dresden, 04/2013, 30-33.

Schmähl, W. (1976), Graphische Darstellung und Interpretation der 'Rentenformel' in der Bundesrepublik Deutschland, Finanzarchiv, 35, 310-321.

Statistisches Bundesamt (2009), Bevölkerung Deutschlands bis 2060 - Ergebnisse der 12. koordinierten Bevölkerungsvorausberechnung, Statistisches Bundesamt, Wiesbaden.

Wilke, C. (2004), Ein Simulationsmodell des Rentenversicherungssystems: Konzeption und ausgewählte Anwendungen von MEA-PENSIM, MEA-Diskussionspapier, 48-2004, 1-33.

Chapter 3

Subsidized Private Pension Savings and the Deferred Taxation of Retirement Income: Is the German Riester Scheme Self-Financing?

Abstract

In order to incentivize people to save privately for retirement, the German government introduced individual retirement accounts with voluntary but tax deductible contributions: The Riester Scheme. Using the example of Germany, this paper investigates whether the Riester scheme is self-financing on both a yearly and cumulative basis. In order to do that, simulations of future government expenditures on Riester subsidies will be provided and compared with the simulated additional fiscal revenue from deferred taxation of Riester pension payments. The results show that, on a yearly basis, government expenditures on Riester subsidies can be covered by the additional fiscal revenue obtained from deferred taxation of Riester pension payments until 2050.

JEL: H24, H55, J18

Keywords: Private savings, Retirement, Riester scheme, Subsidies, Deferred taxation.

3.1 Introduction

Demographic change and especially an aging population challenge pay-as-you-go pension systems because of an increasing ratio of retirees to employees. This either leads to rising contributions to the mandatory pension system or to declining pension replacement rates (or a combination of both). Closing the gap between pre-retirement income and retirement income without increasing mandatory social security contributions for the working population has become a policy issue for many countries in the last couple of years (see Antolin and Whitehouse 2009). In order to close the gap between pre-retirement and retirement income *without* increasing future mandatory social security contributions for the working population, many countries have tried to incentivize people to save privately for retirement by introducing individual retirement accounts in which the contributions are voluntary but tax deductible (see OECD 2017). Consequently, the return on investment is tax exempt but the entire savings are taxed when converted into pension benefits. However, this paper focuses on the German program design and its future development. The German government introduced so-called Riester contracts, as part of the German Retirement Savings Act (AVmG) in 2001, that subsidize households when saving privately for retirement. Since 2002, households have been either granted a basic subsidy or tax allowances (in combination with possible child subsidies) to incentivize private retirement provision by means of a Riester contract.[1] Due to the fact that these subsidies are tax-financed, the German government will face additional financial costs in the future, when continuing to subsidize private retirement savings.

The focus of research is still mostly on the amount of subsidization of Riester contracts. However, the deferred taxation of (Riester) pension payments has rarely been investigated.[2] This was developed and adopted as part of the German Retirement Income Act (AltEinkG), which came into force in 2005. Although the AVmG and the AltEinkG have been adopted independently of each other, they both affect

[1] See Börsch-Supan et al. (2012) for an overview of the design of the Riester scheme.

[2] See Article 10(1) point 2b, 10a and Article 22 of the German Income Tax Act (EStG) for further information on deferred taxation of (Riester) pension payments.

the fiscal budget balance of the German government, when it comes to subsidization of private retirement savings by means of Riester contracts.

Hence, the purpose of this paper is to determine whether there is a point in time at which the Riester scheme becomes self-financing on a yearly (and) or a cumulative basis. Since government-promoted private retirement savings lead to Riester pension payments, retirement income increases. Consequently, retirees will face a greater tax burden (due to both higher incomes and higher average income tax rates). It is therefore important to investigate whether the additional fiscal revenue due to deferred taxation of (Riester) pension payments may balance the Riester subsidies. This is done by simulating future government expenditures on Riester subsidies from 2010 until 2050, under certain assumptions concerning the income distribution among the Riester savers.[3] It is then analyzed whether deferred taxation (and hence, the simulated additional fiscal revenue) of these Riester pension payments (under specific assumptions concerning tax rates) can finance the subsidies on an annual basis. This can be called a *tax-pay-as-you-go* (*TaxGo*) system, i.e. that subsidies granted in year t are financed by the additional fiscal revenue from deferred taxation of (Riester) pension payments in year t. Note that taxes are usually not earmarked but rather financial charges used to fund various public expenditures. Hence, the additional fiscal revenue is not literally used to finance subsidies. Although the additional fiscal revenue from deferred taxation of Riester pensions is required to finance all public expenditures, this calculation is conducted for illustration purposes in order to show that the Riester scheme entails not only government expenditures, but also generates tax revenues. Furthermore, it is considered whether the additional cumulated fiscal revenue from deferred taxation of Riester pension payments may exceed the cumulated governmental expenditures on Riester subsidies in 2050. If this is the case, the Riester scheme is self-financing on a cumulative basis. However, the analysis in this paper is based almost entirely on the one in the paper by Bollacke (2016).

[3]Note that data is only available until 2009 because it is generated by individual income tax returns which can be provided retrospectively for the last four years. Thus, simulations will begin in 2010, except for the gross income for which data until 2013 is available.

The remainder of the paper is structured as follows. Section 3.2 gives a short literature review. Section 3.3 presents some general information on the Riester scheme and Section 3.4 explains the *TaxGo* model. Section 3.5 contains simulations of the total governmental expenditures for subsidizing private savings and the additional fiscal revenue from deferred taxation under various assumptions. Finally, it analyzes whether the Riester scheme is or could be self-financing. Section 3.6 concludes.

3.2 Literature Review

According to the microeconomic analysis of the Riester scheme by Prinz et al. (2003), the subsidization of Riester savings through basic subsidies causes windfall gains during the saving period. This is supported by Corneo et al. (2010) and Pfarr and Schneider (2010), who conclude that many households simply reallocated some of their private savings from non-subsidized contracts into subsidized ones what obviously increases public expenditures.

There are also various other aspects of the German Riester scheme that have already been investigated. Kiesewetter (2001) analyzed whether concluding Riester contracts has any advantages over other private pension contracts. In general, he found that, regarding revenues, Riester contracts are indeed advantageous, particularly if not misused.[4] In contrast, Schröder et al. (2007), Corneo et al. (2009), as well as Pfarr and Schneider (2011, 2013), investigated the effect of Riester contracts on the savings behavior of private households and especially of low income households. They came to the conclusion that Riester contracts and thus, government-promoted private retirement savings are dependent on the individual income and do not incentivize low-income households to conclude such contracts. Hagen and Kleinlein (2011) conclude that Riester revenues are usually very small. This leads to the presumption that there is no incentive effect on (low-income) households. Furthermore, the Riester scheme does not seem to significantly mobilize people to save privately for their retirement at all. A recent study by Corneo et al. (2015)

[4] The government wants Riester pension payments to be paid as a life annuity. If someone wanted the savings to be paid as a lump-sum, the Riester subsidies would be considered misused, which would lead to the repayment of subsidies (see Article 93 EStG).

investigated the distributional effects of the Riester scheme and found that 38% of the subsidies accrue to the top 20% income households, but only 7.3% to the bottom 20%. In addition, Bucher-Koenen and Lusardi (2011) found a positive effect of financial knowledge on retirement planning and also that financial literacy is lacking among those with a low level of education and a low income. Furthermore, Börsch-Supan et al. (2007) looked at the microeconomic effects of the Riester scheme and its effect on the savings behavior of households. They concluded that it is too early to draw any definitive conclusions, but that evidence from the USA suggests that there will be further growth in the acceptance of Riester pensions. In addition, Börsch-Supan and Gasche (2010) investigated whether Riester pension payments could close the pension gap of households.[5] They found that this depends heavily on several factors, such as the share of income that is saved and the length of the saving period.

However, while recent analysis concentrate on the microeconomic effects of the Riester scheme, the following analysis focuses on the fiscal and governmental impacts of subsidizing private retirement savings which later, when converted into pension benefits, become subject to deferred income taxation.

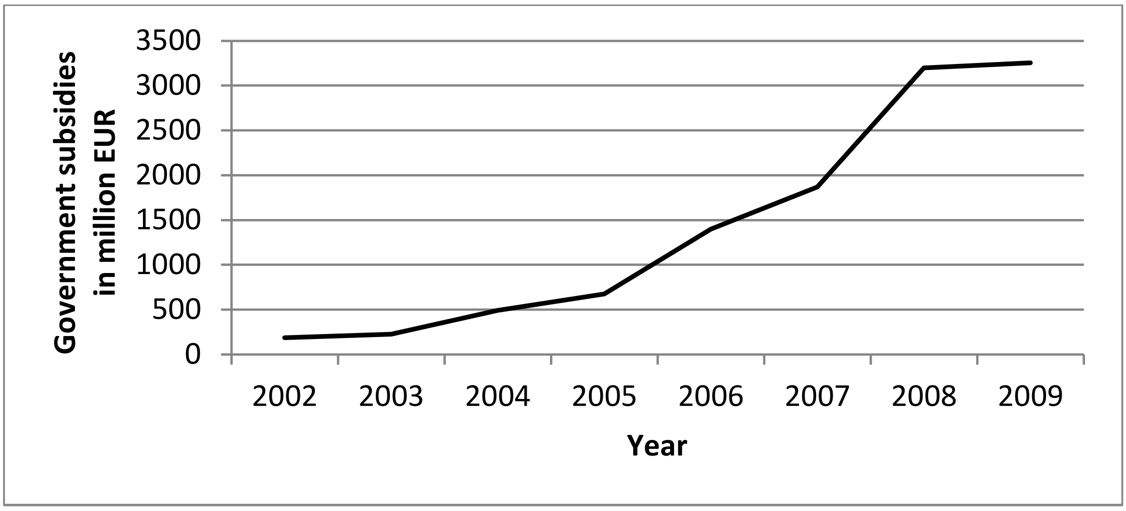
3.3 The Riester Scheme

Given the complex design of the Riester scheme, this section focuses on the most important facts concerning the objectives, subsidies and eligibility criteria of Riester contracts. As part of the pension reform, the German government introduced Riester contracts in 2001 to subsidize households by means of a basic subsidy, child subsidy and/or tax allowances, in the context of saving privately for retirement. This incentive to provide for retirement was introduced to close the pension gap that will arise due to a reform of the German mandatory pension system, because of the aging population. This reform, which became effective in 2005,[6] reduces the pension payments of the mandatory pension system, by means of the so-called *old-*

[5]The pension gap explains the gap between the last pre-retirement income and the retirement income.

[6]See Article 68 of the Sixth Book of the German Social Security Code (SGB VI).

Figure 3.1: Government subsidies for Riester contracts; 2002-2009



Source: Statistisches Bundesamt (2009a) and Gerber (2010, 2011a, 2011b, 2012, 2013).

age provision factor and can also be reduced by the so-called *sustainability factor*. Therefore, the intention was (concerning the eligibility criteria) to entitle all households that are affected by the adjustments made to the mandatory pension system to conclude Riester contracts. However, the full basic subsidy, which has been 154 Euros per year since 2008, is only granted if Riester savers put at least 4% of their previous year's gross income into a Riester contract on a regular basis. The same holds true for child subsidies. Child subsidies[7] are granted for each eligible child (for details on the design of the Riester scheme see Börsch-Supan et al. 2012).[8] If less than 4% of the income is saved, the basic and the child subsidy decrease.[9]

Figure 3.1 shows the government subsidies for Riester contracts from 2002 to 2009. Starting with expenditures of about 185 million Euros in 2002, the subsidies rose to 3255 million Euros by 2009.[10] This means that in 2009, the German government granted subsidies of more than 17 times the 2002 level. This increase in government expenditures on private retirement savings is of course partly due to an increasing number of people signing Riester contracts.

[7]185 Euros per year for each child. 300 Euros for each child born after 2008. See Article 85 EStG.

[8]See Article 68 SGB VI and Articles 84, 85, 86 EStG for further information.

[9]Note that since 2005, at least 60 Euros per year have to be saved in order to be entitled to a subsidy. See Article 86 EStG for further information.

[10]See Statistisches Bundesamt (2009a) and Gerber (2010, 2011a, 2011b, 2012, 2013).

However, although the development of these government subsidies displays a slightly convex shape, the development of the number of Riester savers and Riester contracts is concave. Nonetheless, the steeper increase in government subsidies beginning in 2005 can be explained partially by an increasing number of Riester contracts being concluded during that period. This can in turn be explained by a simplification of the design of the Riester scheme in that year, including simplifications regarding the application for basic subsidies (for details, again see Börsch-Supan et al. 2012). In addition, the steeper increase in government subsidies in 2005 can also be explained by institutional factors. Specifically, in 2002, the maximum contribution rate to Riester contracts was 1% of gross income, for which a basic subsidy of 38 Euros was paid. This maximum contribution rate increased over of the years. Consequently, the reason for the flatter development between 2008 and 2009 may be due the contribution rate for which Riester contracts are subsidized reached its final maximum of 4% of an individuals' gross income in 2008. However, apart from this descriptive view on the Riester scheme, the following analysis does not focus on the absolute amounts of money. Rather, it focuses on the self-financing aspect, so that the absolute amounts are not of interest, which makes it unnecessary to discount the simulated amounts of money.

3.4 The TaxGo Model

Pay-as-you-go systems are usually used for mandatory pension systems, for which the contributions in year t are devoted to financing the pension payments in year t. The *TaxGo* model presented here applies this concept to state-promoted private retirement savings. Since Riester pension payments are added to the pension payments of the mandatory pension system, the overall income of the relevant retirees increases. Since pension payments are taxed at the pensioners' individual income tax rate for the full duration of retirement, fiscal revenues increase due to higher retirement incomes and increasing marginal income tax rates.

The fundamental equation of the *TaxGo* model is given by:

$$G_t = T_t^A \tag{3.1}$$

This implies that in year t, government expenditures on Riester subsidies (G) equal the additional fiscal revenue (T^A).

With FG denoting the financial gap that arises from subtracting the additional fiscal revenue from the governmental expenditures on Riester subsidies, and GAP as the cumulative of FG, that year t^* is to be found for which equation (3.1) holds true, i.e., for which

$$FG_t = G_t - T_t^A \leq 0 \qquad \forall\ t \geq t^* \tag{3.2}$$

If, for all years after t^*, it is true that $FG_t \leq 0$, the GAP in equation (3.3) also reaches its maximum in year t^*:

$$GAP_t = \sum_{t \geq 2010} FG_t. \tag{3.3}$$

To calculate *government expenditures on Riester subsidies*, G, and thus the left side of the *TaxGo* model for each year, some further assumptions are necessary. Overall government subsidies per year are calculated by multiplying the number of Riester savers, R, by government subsidies per Riester saver, g. Accordingly, total government subsidies G per year can be written as:

$$G_t = g_t \cdot R_t \qquad \forall\ t \in \{2010, ..., 2050\} \tag{3.4}$$

Government subsidies per Riester saver (including child subsidies), g, are given by:

$$g_t = max\Big\{0.8 \cdot w_t \cdot \theta_t^n - \big[0.8 \cdot w_t - min\{a \cdot w_{t-1}, 2100^{t-2015}\}\big] \cdot \theta_t^r,$$
$$min\{1, \frac{a}{0.04}\} \cdot (154 + CS_t) \cdot max\{1, 1.01^{t-2015}\}\Big\} \quad (3.5)$$

$$\forall \; t \in \{2010, ..., 2050\}$$

In equation (3.5), w denotes the average gross income which is assumed for all employees in this model, θ^n the personal income tax rate for all persons with no Riester contract, θ^r the personal income tax rate for all persons with a Riester contract[11] and a, the savings rate for all persons for Riester contracts. The factor 0.8 is included to approximate the taxable income (gross income less social security contributions).

Hence, the left-hand side of the big curved brackets considers government subsidies per Riester saver as a tax allowance and the right-hand side as a basic and child subsidy. The full basic subsidy is set equal to 154 Euros per Riester saver entitled to receive subsidies since 2008.[12] To account for inflation, this amount is (as well as the child subsides) assumed to increase by 1% each year, beginning in 2016. Moreover, if a person saves more than 4% of the gross income, only the full basic and child subsidy is granted. If a person saves less than 4%, the share of subsidy decreases accordingly. As long as the tax allowance exceeds the granted basic and child subsidy, the tax allowance is applied as the actual subsidy per Riester saver. This procedure is called a 'most-favored test' or 'most favorable tax treatment'.[13] Note that private savings are only subsidized up to a savings level of 2100 Euros per year.[14] Furthermore, this ceiling is also assumed to increase by 1% each year from 2016 onwards.

[11]See Article 32a EStG.
[12]See Article 84 EStG for details.
[13]In German: 'Günstigerprüfung'. See § 10a EStG for details.
[14]See Article 10a EStG for details.

The average gross income of the employees is calculated in the following way. The rates of change of the gross incomes from 1994 to 2013 are taken and assumed to repeat after 2013. The calculation is based on data by Deutsche Rentenversicherung Bund (2014) and is defined by:

$$w_t = w_{t-1} \cdot \frac{w_{t-20}}{w_{t-21}} \qquad \forall\ t \in \{2014, ..., 2050\} \tag{3.6}$$

Moreover, the child subsidy, CS, granted for each Riester saver is given by:

$$CS_t = 0.7 \cdot c_t \cdot max\{1, 1.01^{t-2015}\} \qquad \forall\ t \in \{2010, ..., 2050\} \tag{3.7}$$

with $c_t = \begin{cases} 185 \text{ Euros}, & \text{for children born before 2008} \\ 300 \text{ Euros}, & \text{for children born since 2008} \end{cases}$

Equation (3.7) includes the factor 0.7, which is assumed to be the average number of children per Riester saver.[15] This is finally multiplied by the per child subsidy, c.[16] To account for inflation, this amount is assumed to increase by 1% each year, beginning in 2016. It is further assumed that, on average, parents receive child benefits for children up to the age of 20. Furthermore, the age of children is assumed to be equally distributed among all children in each year. For instance, in 2010, there are (3/20) children born after January 1, 2008 (for which child subsidies of 300 Euros each are paid) and (17/20) children born before 2008 (for which child subsidies of 185 Euros each are paid).

The number of people with Riester contracts, R, is forecasted initially by multiplying the age-group-specific population P_i by the age-group-specific share of Riester savers r_i in year 2009. Hence, $R_{i,2009} = P_{i,2009} \cdot r_{i,2009}$. The specific share of Riester savers in year 2009 is calculated by the number of the Riester savers in the particular age group (see Gerber 2013), divided by the population of the same age group.[17] Furthermore, t denotes the time period, i the respective age group with

[15] Since the average number of children is about 1.4 per woman in 2012 and it is assumed that half of all Riester savers are female, each Riester saver has 0.7 children on average. See Pötzsch (2012) for details.

[16] See Article 85 EStG.

[17] For the following simulations, the population is taken from the population forecast 1-W2 of the German Federal Statistical Office. See Statistisches Bundesamt (2009b) for details.

the numbers 1 to 6 (for the age groups 15-20, 21-30, 31-40, 41-50, 51-60 and 61-65) and r_i the age-group-specific share of Riester savers.

Hence, with $R_{1,t} = P_{1,t} \cdot r_{1,2009}$, the future number of Riester savers per age groups 2-6 and year is given by:

$$R_{i,t} = \Big[R_{i,t-1} + \big(\frac{1}{s_{i-1}} \cdot R_{i-1,t-1} - \frac{1}{s_i} \cdot R_{i,t-1}\big)\Big] \cdot n \tag{3.8}$$

$$\forall\ t \in \{2010, ..., 2050\},\ \ i \in \{2, ..., 6\}$$

with s as the age-span of the particular age group. Moreover, the growth parameter n is assumed to account for people either dying or making their Riester contracts non-contributory ($n = 0.97$), concluding Riester contracts ($n = 1.03$) or maybe both ($n = 1$).[18] Note, that although migration is generally considered in the 1-W2 population forecast of the German Federal Statistical Office, the recent increase in immigration is not considered explicitly in this model. However, the growth parameter n is able to capture effects that migration might have on the development of future Riester subsidies and Riester tax revenues.[19] Thus, the Riester savers of age group i in year t equal the Riester savers of age group i in year $(t-1)$, plus the share of Riester savers of age group $(i-1)$ in year $(t-1)$ that is now entering age group i. The share of Riester savers of age group i in year $(t-1)$, now entering age group $(i+1)$, is subtracted.

With x_i^h as the upper age bound and x_i^l as the lower age bound of age group i, the age-span of the particular age group in each year, s_i, is defined by:

$$s_i = \left[x_i^h - (x_i^l - 1)\right] \qquad \forall\ i \in \{1, ..., 6\} \tag{3.9}$$

It is also assumed that ages within the respective age groups are distributed equally. Thus, either one-tenth, one-sixth or one-fifth of the Riester savers (depend-

[18] Canceling Riester contracts usually leads to the repayment of subsidies (see Article 93 EStG). This can be avoided by making Riester contracts non-contributory. The latter is considered in what follows.

[19] See Berger et al. (2016) for cross-country differences in the contribution of future migration to old-age financing.

ing on the age group) leave or enter, respectively, age group i each year. Finally, the total number of Riester savers in year t is simply:

$$R_t = \sum_{i=1}^{6} R_{i,t} \qquad \forall\ t \in \{2010, ..., 2050\},\ \ i \in \{1, ..., 6\} \tag{3.10}$$

i.e., the sum of Riester savers over six age groups.

The *additional fiscal revenue*, T^A, that the government receives is defined by the fiscal revenue that arises from deferred taxation of both benefits obtained from the mandatory pension system as well as from the Riester scheme, in contrast to only taxing benefits from the mandatory pension system. First, fiscal revenue after taxing the retirement income, *including* Riester pension payments, is to be defined (T^R). Second, the fiscal revenue after taxing the retirement income *without* Riester pension payments is calculated (T^M). The difference between T^R and T^M yields the additional fiscal revenue from deferred taxation of (Riester) pension payments (T^A):

$$T_t^A = T_t^R - T_t^M. \qquad \forall\ t \in \{2010, ..., 2050\} \tag{3.11}$$

with

$$T_t^R = \sum_k T_{t,k}^R \qquad \forall\ t \in \{2010, ..., 2050\},\ \ k \in \{8, ..., (t-2002)\} \tag{3.12}$$

and

$$T_{t,k}^R = RP_{t,k} \cdot \left[(w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t + l_k^R \cdot w_t) \cdot \theta_{t,k}^h \right] \qquad \forall\ t \in \{2010, ..., 2050\},\ \ k \in \{8, ..., (t-2002)\} \tag{3.13}$$

i.e., the fiscal revenue from deferred taxation of retirement income, which includes payments of the mandatory and Riester pension system. In equation (3.13), $RP_{t,k}$ denotes the Riester pensioners in year t that saved for the length of $k \in \{8, ..., (t-2002)\} \ \forall \ t \in \{2010, ..., 2050\}$ years. Moreover, w denotes the average gross income and l^M the gross pension replacement rate[20] when receiving only pension payments from the mandatory pension system which is assumed to be equal for all Riester pension receivers. Furthermore, l^R denotes the additional gross pension replacement rate when receiving Riester pension payments and θ^h the personal income tax rate when receiving both mandatory and Riester pension payments which differs according to l^R and thus, k.[21] Finally, τ denotes the average percentage of taxable (mandatory) retirement income.

Moreover,

$$T_t^M = RP_t \cdot (w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t \cdot \theta_t^l) \quad \forall \ t \in \{2010, ..., 2050\} \tag{3.14}$$

denotes the fiscal revenue from deferred taxation of retirement income that only includes payments of the mandatory pension system, with θ^l as the personal income tax rate when only receiving mandatory pension benefits.

[20]Note that the term *replacement rate* refers to the *standard replacement rate*. This rate is set to 46% until 2020 and to 43% until 2050. According to Article 154 SGB VI, the standard replacement rate (or pension level) after social security contributions but before tax ('safety level') should be at least 46% until 2020 and 43% until 2030. As already explained in Chapter 2.5.1, it is calculated by dividing the standard pension benefits after the share of social security contributions that has to be paid by retirees (partial amount of health care and full amount of long-term care) before tax, by the average income of the insured people after the share of social security contributions that has to be paid by the employees (partial health care, pension insurance, unemployment insurance and long-term care as well as an average amount of additional old-age provision) before tax. The problem in calculating this level is, that earnings of civil servants are included in the calculation of the average income of the employees. But civil servants do not have to pay social security contributions. This distorts the average income after social security contributions of the employees. Moreover, a change in contribution rates would affect the safety level. Hence, for the ease of calculation, the following analysis assumes the minimum level for the gross replacement rate. This will, with almost certainty, guarantee the compliance with the requirement of Article 154 SGB VI. According to Deutsche Rentenversicherung (2014) a gross replacement rate of 46% led to a safety level of 50.1% in 2011.

[21]See Article 32a EStG.

Finally,

$$T_t^A = \sum_k \left\{ RP_{t,k} \cdot \left[(w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t + l_k^R \cdot w_t) \cdot \theta_{t,k}^h \right] \right\}$$
$$- RP_t \cdot (w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t \cdot \theta_t^l) \qquad (3.15)$$

$$\forall\ t \in \{2010, ..., 2050\},\ \ k \in \{8, ..., (t - 2002)\}$$

is the additional fiscal revenue from deferred taxation of (Riester) pension payments.

In the equations above, RP represents the number of Riester pensioners, which can be calculated by

$$RP_t = RP_{t-1} + RP_t^{new} - RP_{t-b}^{new}, \quad \forall\ t \in \{2010, ..., 2050\} \qquad (3.16)$$

Furthermore,

$$RP_{t,k}^{new} = (0.2 \cdot R_{6,t-1}) \cdot \frac{1}{t - 2009}, \qquad (3.17)$$

$$\forall\ t \in \{2010, ..., 2050\},\ \ k \in \{8, ..., (t - 2002)\}$$

Hence,

$$RP_t^{new} = \sum_{k=8}^{t-2002} RP_{t,k}^{new}, \quad \forall\ t \in \{2010, ..., 2050\} \qquad (3.18)$$

Due to the fact that approximately 20% of Riester savers in the age group 61-65 in year t are Riester pensioners in year $t+1$, this is included in equation (3.17). The fraction $\frac{1}{t-2009}$ accounts for the share of new retirees in year t that saved for k years. However, k denotes the number of years for which Riester receivers have made payments to the Riester scheme. This is assumed to be equally distributed for all *new* retirees in year t. As an example for 2011: $RP_{2011,8}^{new} = (0.2 \cdot R_{6,2010}) \cdot \frac{1}{2}$ and $RP_{2011,9}^{new} = (0.2 \cdot R_{6,2010}) \cdot \frac{1}{2}$. Thus, half of the *new* retirees in 2011 saved for eight years and half of them for nine years. Hence, all retirees in 2010 saved for eight years. Consequently, for all Riester savers becoming retirees in year t, it is possible to have saved privately for retirement for the length of k years. Moreover, equation (3.16) shows the simulation of the total number of Riester pensioners in year t that saved for k years. With b as the average time span for which individuals receive pension payments, the last part of equation (3.16) accounts for mortality. Factor b is set to 20 years.[22] Assuming that all Riester pensioners are 66 when entering retirement, those who retire in 2010 will for example die in 2030. In effect, it is supposed that on average, all pensioners live to 85. Consequently, no Riester pensioners die before 2030. It is also assumed that there are no Riester pensioners in 2009. Thus, all Riester pensioners in 2010 have just retired.

Finally,

$$RP_{t,k} = RP_{t-1,k} + RP_{t,k}^{new} - RP_{t-b,k}^{new} \qquad \forall \; t \in \{2010, ..., 2050\}, \;\; k \in \{8, ..., (t-2002)\} \tag{3.19}$$

To calculate the additional gross pension replacement rate when receiving Riester pension payments, l^R, the following assumptions are made: Let A_t denote the accumulated amount of capital in year t for a person who saved since 2002. With an average rate of return of 4.5%[23] per year and assumed costs for Riester contracts of 11.9% of the amount saved[24], A_t is given by:

$$A_t = \left[A_{t-1} + (1-0.119) \cdot a \cdot w_{t-1}\right] \cdot 1.045 \qquad \forall \; t \in \{2003, ..., 2050\} \tag{3.20}$$

In equation (3.20), a denotes the savings rate for Riester contracts which holds for all persons with a Riester contract. Hence, in 2002, no accumulated capital is available since savings start in 2002. In relation to the assumed average period, b, over which pensioners receive retirement income and the average gross income,

[22] According to data from the German Federal Pension Fund, the average time span for which retired persons receive pension payments was 19.3 years in 2013 (see Deutsche Rentenversicherung Bund 2014).

[23] This is the average rate of return from life insurance policies from 2002 to 2013 in Germany. See Gesamtverband der Deutschen Versicherungswirtschaft (2014) for details.

[24] This is the average cost-rate of Riester contracts according to Gasche et al. (2013).

w, the additional gross pension replacement rate for receiving Riester pensions, denoted by l^R, can be calculated by:

$$l_k^R = \frac{A_t}{b \cdot w_t} \qquad \forall k \in \{8, ..., 48\}, \;\; t = k + 2002 \tag{3.21}$$

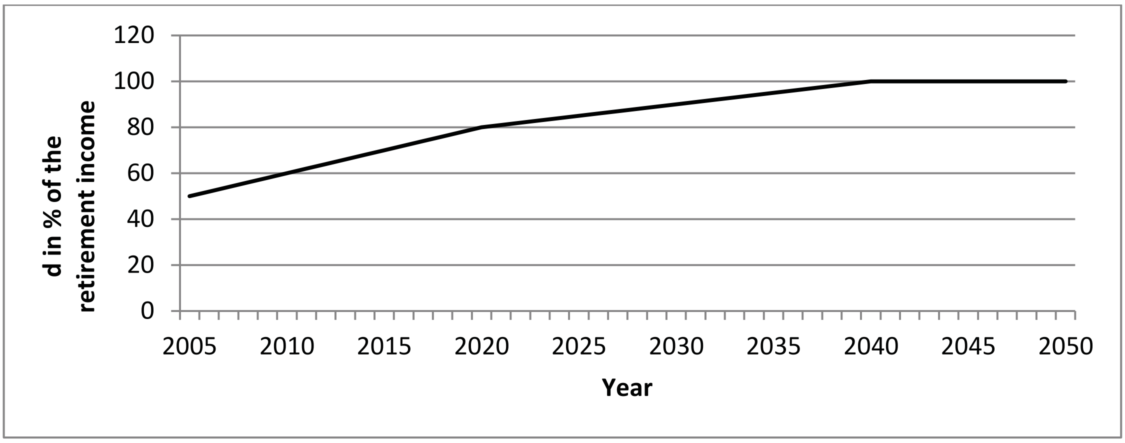
Thereby calculated additional gross pension replacement rates for the length of the saving period, k, are therefore (for ease of calculation) approximated by the calculation of the accumulated amount of capital beginning in 2002, and hence assumed to be independent of the time of entering retirement and only dependent on k.

Moreover, the smaller k, the lower the additional gross pension replacement rate when receiving Riester pension payments. For instance, if individuals have saved privately by means of Riester contract since 2002, it is assumed they they first enter retirement in 2010 and thus, saved until 2009. As a protection against inflation, the Riester pension payments are assumed to increase each year by the same rate as the average income. This is justified by the fact, that the remaining capital, that has not yet been transformed into a pension benefit, still generates interest. Thus, a Riester receiver saving for k years receives l_k^R for the entire length of retirement. Note that the tax-free allowance, and the associated limit for entering higher marginal tax rates will increase by 1% each year, starting in 2016. Some operands in the tax tariff change accordingly. The average percentage of taxable (mandatory) retirement income, τ, accounts for the stepwise deferred taxation used for mandatory pension payments.

Figure 3.2 shows the share of taxable retirement income, d, for those entering retirement in the respective year. Note that Riester pension payments are *entirely* subject to the income tax[25], whereas the full mandatory retirement income will become subject to income taxation for those entering retirement in 2040 and later. Beginning with a taxation rate of 50% of retirement income in 2005, the share

[25] See Article 10a EStG.

Figure 3.2: Percentage of the taxable mandatory retirement income; 2005-2050



Source: Based on Article 22 point 1aa EStG.

of taxable (mandatory) retirement income will increase by two percentage points each year until 2020, when 80% of the (mandatory) retirement income will become taxable. This share will increase by one percentage point each year until 2040, finally reaching 100%.[26]

Existing pensioners are treated in the following way. As an example: A person entering retirement in 2005 with a gross pension of 10,000 Euros has a non-taxable income of 5,000 Euros. These 5,000 Euros will be the person's personal allowance for the rest of his life.

Thus, the average percentage of taxable mandatory retirement income over all Riester pensioners, τ, is defined by:

$$\tau_t = \begin{cases} 1 - \frac{\sum_{j=2010}^{t} \left(w_j \cdot l_j^M \cdot 0.9 \cdot f_j \cdot \frac{RP_j^{new}}{RP_t} \right)}{w_t \cdot l_t^M \cdot 0.9}, & \text{for } t < 2030 \\ 1 - \frac{\sum_{j=t-19}^{t} \left(w_j \cdot l_j^M \cdot 0.9 \cdot f_j \cdot \frac{RP_j^{new}}{RP_t} \right)}{w_t \cdot l_t^M \cdot 0.9}, & \text{for } t \geq 2030 \end{cases} \quad \forall \ t \in \{2010, ..., 2050\} \tag{3.22}$$

Note that retirees still have to pay for health insurance and long-term care insurance. This is approximately 10% of the gross pension payments of the mandatory pension system. Riester pension payments are beyond the scope of national insurance and subject only to income tax. Thus, pension payments are multiplied by 0.9

[26]See Article 22 point 1aa EStG.

to approximate the taxable income of pensioners.[27] With $f = 1 - d$, the term in brackets is the personal allowance of those Riester pensioners entering retirement in year j. The sum of the personal allowances over j divided by gross pension payments after social security contributions yields the average tax free allowance over all $j's$. For $t \geq 2030$, $j = t - 19$ since it is assumed that the Riester pensioners live for 20 years in retirement. Hence, in year 2030, the sum in equation (3.22) starts at year 2011.

Equations (3.2) and(3.3) can finally be transformed into:

$$\begin{aligned} FG_t &= g_t \cdot R_t \\ &- \Big(\sum_k \big\{ RP_{t,k} \cdot \big[(w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t + l_k^R \cdot w_t) \cdot \theta_{t,k}^h \big] \big\} \\ &- RP_t \cdot (w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t \cdot \theta_t^l) \Big) \leq 0 \quad \forall \ t \geq t^* \end{aligned} \tag{3.23}$$

$$\text{with } t \in \{2010, ..., 2050\}, \ \ k \in \{8, ..., (t - 2002)\}$$

and

$$\begin{aligned} GAP_t = \sum_{t=2010}^{2050} \Big[& g_t \cdot R_t \\ & - \Big(\sum_k \big\{ RP_{t,k} \cdot \big[(w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t + l_k^R \cdot w_t) \cdot \theta_{t,k}^h \big] \big\} \\ & - RP_t \cdot (w_t \cdot l_t^M \cdot 0.9 \cdot \tau_t \cdot \theta_t^l) \Big) \Big] + \sum_{t=2002}^{2009} G_t \end{aligned} \tag{3.24}$$

$$\forall t \in \{2010, ..., 2050\}, \ \ k \in \{8, ..., (t - 2002)\}$$

Again, the aim is to find that year t^* for which FG is no longer positive and GAP reaches its maximum. In equation (3.24), the government subsidies of the years 2002 to 2009 also need to be included since these subsidies have already been granted. Ignoring these would distort further results.

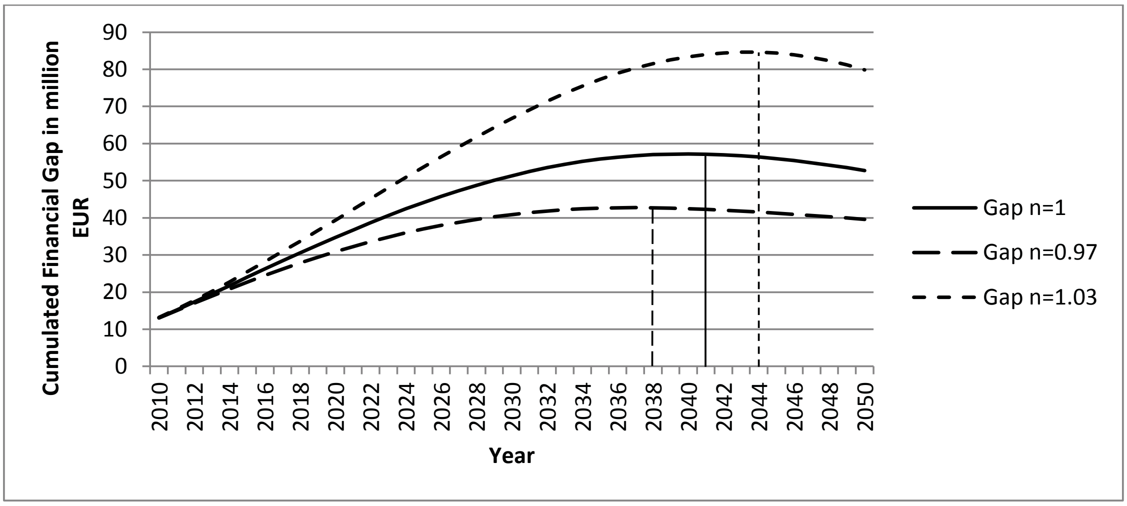
[27] See Article 228, 241, 247 and 249a of the Fifth Book of the German Social Security Code (SGB V) and Article 20 of the Eleventh Book of the German Social Security Code (SGB XI).

3.5 Simulations

3.5.1 Scenario I

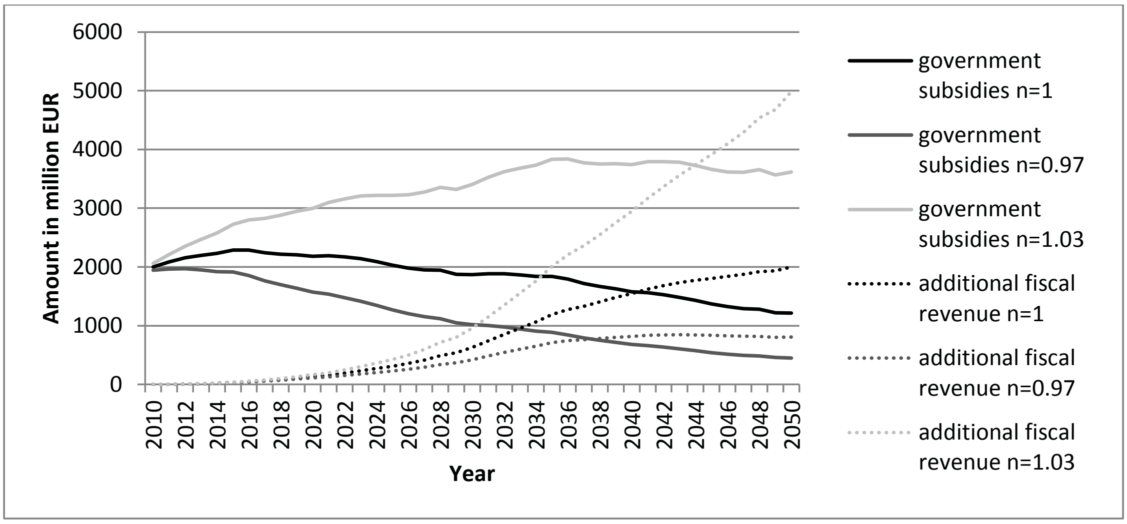
This section provides simulations of the total government subsidies that are spent on Riester savers, G, and on the additional fiscal revenue the government receives from deferred taxation of pension payments, T^A, for the years 2010 to 2050.

Figure 3.3: Cumulated financial gap and point in time of self-financing; 4% savings rate



Source: Own calculation.

Figure 3.4: Government subsidies and additional fiscal revenue by year; 4% savings rate



Source: Own calculation.

These two values are compared and put in relation to each other for each year. The same is done for the cumulated values. Note that all following analysis assume individual income taxation and the average income, w, for all Riester savers.[28] The break-even points for the three assumed growth parameters, n, are shown graphically in Figure 3.3 for the case of a 4% savings rate ($a = 4\%$). This is exactly the share of income that employees are required to put into a Riester contract to be entitled to the full government subsidy (see Börsch-Supan et al. 2012). The vertical lines mark the corresponding years in which the cumulated financial gap reaches its maximum and, therefore, the years in which the Riester scheme starts to finance itself on a yearly basis. Assuming a growth parameter of $n = 0.97$, it is self-financing from 2038 onwards and from 2041 onwards, if a growth parameter of $n = 1$ is assumed. If a growth parameter of $n = 1.03$ is assumed, the Riester scheme is self-financing from 2044 onwards. Since the cumulated financial gap is still positive in 2050 for all assumed growth parameters, the Riester scheme is not self-financing on a cumulated basis. Figure 3.4 shows the government subsidies and the additional fiscal revenue by year. It also indicates that the government subsidies can be covered by the additional fiscal revenue in 2038 ($n = 0.97$), year 2041 ($n = 1$) and year 2044 ($n = 1.03$).

3.5.2 Scenario II

To investigate the effect of the progressiveness of the national income tax in Germany, Scenario II analyzes whether the Riester scheme is self-financing when different income groups are assumed (see Gerber 2013). Table 3.1 summarizes the assumptions.

It is further assumed, that, in consequence of their lower income, the last two income groups only save 0.6 or 0.4 times a savings rate, a, of 4%. This of course affects the government subsidies, since these two income groups consequently are only eligible to receive 0.6 and 0.4 times, respectively, the basic and child subsidy. Finally, since there is an upper limit for individual savings that are subsidized,

[28] Incorporating tax splitting for married couples, as in Article 32a (5) EStG, does not significantly change the points in time when the Riester scheme becomes self-financing.

Table 3.1: Assumptions concerning the different income groups; applies to all years and all age groups

	% of Riester savers	% of average income	Savings rate in %	Subsidy per Riester saver
Group A	5	200	2.8	tax allowance
Group B	25	160	3.3	tax allowance
Group C	30	100	4	tax allowance
Group D	20	60	2.4	$[(0.6 \cdot (154 + CS_t) \cdot 1.01^{t-2015})$Euros]
Group E	20	40	1.6	$[(0.4 \cdot (154 + CS_t) \cdot 1.01^{t-2015})$Euros]

Notes: Own calculation based on Gerber (2013). CS=child subsidy per Riester saver.

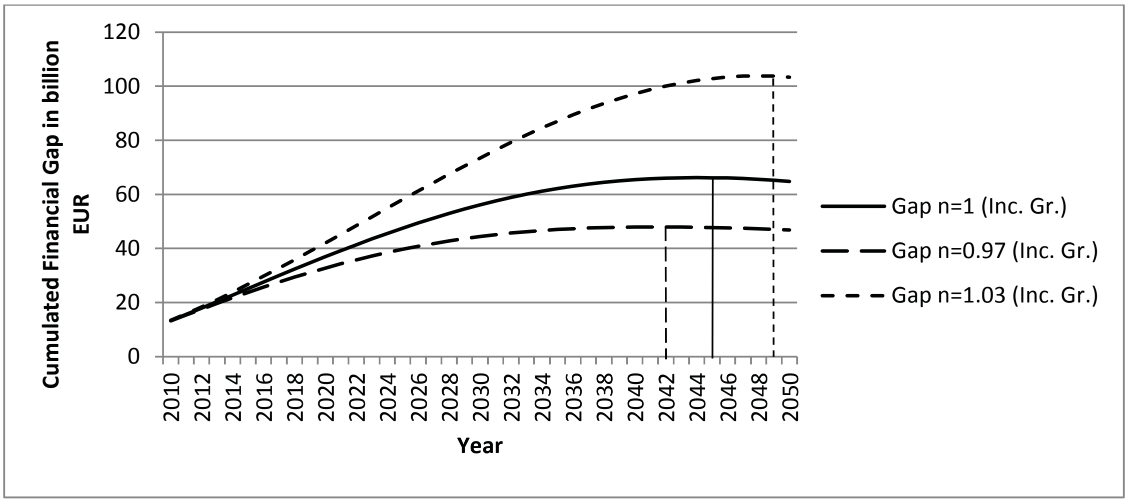
income groups A and B only save the upper limit (2100 Euros per year). This in turn affects their savings rate which consequently affects their Riester pension payments and thus, alters the results. Nevertheless, the most-favored test, as presented in equation (3.5), is still applied for each group.

Figure 3.5 presents the results for all assumed growth parameters, n, in relation to the savings rate of 4%. As in the previous analysis, the vertical lines mark the break-even points and, thus, the self-financing year if different income groups are assumed. In this case, the Riester scheme is self-financing from 2045 onwards for $n = 1$, and 2042 onwards for $n = 0.97$. If a growth parameter of $n = 1.03$ is assumed, the Riester scheme is self-financing from 2049 onwards. Again, since the cumulated financial gap is still positive in 2050, the Riester scheme is not self-financing on a cumulated basis.

Figure 3.6 presents the governmental expenditures for Riester subsidies and the additional fiscal revenue from deferred taxation of (Riester) pension payments by year, and shows at which points in time the Riester scheme starts to finance itself on a yearly basis for Scenario II.

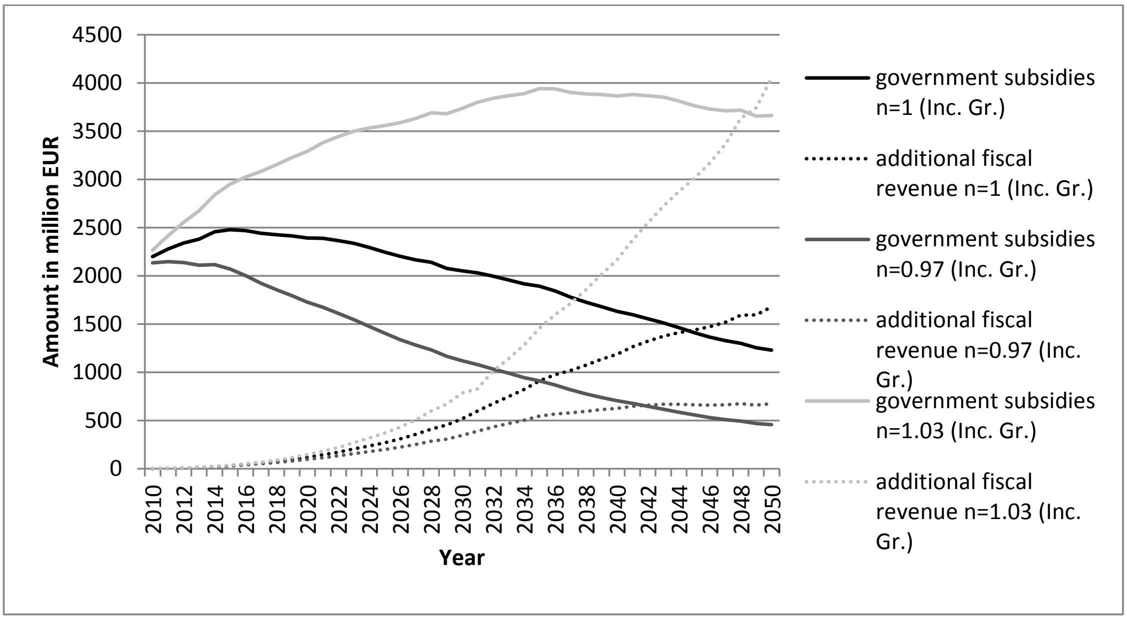
Finally, Table 3.2 summarizes the percentages of government subsidies that can be covered by the additional fiscal revenue on a yearly and on a cumulated basis for both scenarios. It shows that the Riester scheme is self-financing on a yearly basis and that more than 110% of government subsidies can be covered by additional fiscal revenue until 2050 for all assumed growth parameters. The cumulated values

Figure 3.5: Cumulated financial gap and point in time of self-financing; different income groups (Inc. Gr.)



Source: Own calculation.

Figure 3.6: Government subsidies and additional fiscal revenue by year; different income groups (Inc. Gr.)



Source: Own calculation.

for a 4% savings rate (Scenario I) show that between 31% and about 45% of the cumulated government subsidies can be covered by the cumulated additional fiscal revenue in 2050. Assuming different income groups (Scenario II), Table 3.2 shows that 29.68% ($n = 1$), 23.69% ($n = 0.97$) and 33.24% ($n = 1.03$) of the cumulated government subsidies can be covered by the additional fiscal revenue in 2050.

Table 3.2: Percentage of government subsidies that can be covered by additional fiscal revenue; yearly and cumulated; both scenarios; 2010-2050

	2010	2020	2030	2040	2050
			Scenario I		
n=1 yearly	0	6.30	33.63	97.91	164.88
n=1 cumulated	0	1.48	7.42	21.81	39.31
n=0.97 yearly	0	7.10	41.18	120.02	178.97
n=0.97 cumulated	0	1.42	6.86	18.93	31.49
n=1.03 yearly	0	5.66	28.11	78.80	137.91
n=1.03 cumulated	0	1.54	7.91	24.04	45.53
			Scenario II		
n=1 yearly	0	5.02	25.25	72.83	136.43
n=1 cumulated	0	1.26	5.93	16.24	29.68
n=0.97 yearly	0	5.65	30.84	88.87	147.61
n=0.97 cumulated	0	1.21	5.51	14.16	23.69
n=1.03 yearly	0	4.50	21.03	56.08	110.52
n=1.03 cumulated	0	1.30	6.26	17.05	33.24

Notes: Own calculation.

Although the assumptions concerning the income distribution, so as to account for the progressive national income tax, slightly change the previous results, the general statement regarding the self-financing of the Riester scheme remains unaffected.

3.5.3 Further Considerations

Another situation that needs to be considered is one with a different number of Riester savers. The simulation of the Riester savers in this paper is based on data of Statistisches Bundesamt (2009a) and Gerber (2010, 2011a, 2011b, 2012, 2013) which in turn are based on data of persons that provided information on their Riester contracts in their income tax returns. However, there are other extrapolations that also include persons that applied for basic and child subsidies, but did not give any information on their Riester contracts in their income tax return. These extrapolations show a different number of Riester savers; one that is about 1.5 times larger than assumed in Sections 3.4 and 3.5.[29] Conducting the analysis with such

[29]The extrapolations can be found in Statistisches Bundesamt (2009a), Gerber (2010, 2011a, 2011b, 2012, 2013) and Kruse and Scherbarth (2015).

a larger number of Riester savers yields higher governmental expenditures on the one hand, but on the other hand, also leads to higher additional fiscal revenue. Obviously, this is due to a larger number of Riester savers, leading to a larger number of Riester savers entering retirement and thus receiving Riester pension payments. Although the number of Riester savers is sometimes reported to be about 1.5 times higher in each year than assumed above, it is not known whether it is equally distributed among the six age groups. Assuming that the number of Riester savers is 1.5 times higher in *each* of the six age groups leads to exactly the same results, regarding the percentages of the government subsidies that can be covered by the additional fiscal revenue, as presented before, since both the Riester pensioners and the Riester savers are multiplied by 1.5 in each year. Under the assumption that there are no Riester savers in 2009 and that all Riester savers in 2010 have just entered retirement (again see Section 3.4), it is clear that a change in the number of Riester savers by some constant factor does not affect the relation between T^A and G. Finally, a change in the absolute number of Riester savers does not alter the previous results, as long as the relationship between T^A and G remains unchanged.

Moreover, although the analysis presented in this paper assumes individual income taxation for all Riester savers and Riester receivers, incorporating tax splitting for married couples does not significantly alter the results. Assuming that each Riester saver earns the average income, w, and is married to a non-Riester saver who also does not have any income,[30] the Riester scheme would become self-financing on a yearly basis in the 2040s. Note that these assumptions represent an extreme case, since not all Riester savers are married to non-Riester savers who do not have any income. Thus, the real effect of incorporating tax splitting for married couples would not significantly affect the points in time when the Riester scheme becomes self-financing. Additionally, this paper also does not address some institutional features of the Riester scheme (as the possibility to withdraw up to 30% of the Riester savings from the contract at the point of retirement and the effect of people

[30] This is the income combination at which the splitting gain reaches its maximum.

receiving basic security at old age), and neglects the fact that individuals exhibit a life-cycle earnings profile. However, the results obtained from this simulation model give some insights into the future development of the expenditures and revenues of the German Riester scheme. Further work on the Riester scheme could address the neglected points. Especially including a life-cycle earnings profile would shed light on the effect of the potential different savings behavior of individuals during their working life on the Riester subsidies and the consequent additional fiscal revenue obtained from deferred taxation of (Riester) pension payments. However, in the simulation model presented in this paper, this is extremely hard to handle because it is not possible to track individuals without putting further assumptions on their behavior.

3.6 Conclusions

Generally, drawing final conclusions on the performance of the Riester scheme still seems premature. This paper investigates future government expenditures on the German Riester scheme and uses simulations to investigate whether the Riester scheme is potentially self-financing at some point in time, both on a yearly and a cumulative basis. First of all, the Riester scheme is not only a system in which money is spent on the subsidization of households that save privately for retirement. Considering that Riester pension payments are subject to deferred taxation, the fiscal revenue also increases, since those pension payments increase the retirement income and consequently the individual average income tax rates. Thus, the net expenditures (expenditures less additional fiscal revenue) of the Riester scheme decrease.

At a first glance, it seems that the financial gap could sum up to more than 100 billion Euros in 2050. However, due to the demographic change it seems that for all $t \geq t^*$ there are enough Riester receivers, compared to Riester savers, that the cumulated financial gap starts to shrink due to higher total additional income tax payments of the Riester receivers, even before 2050. In this regard, it is evident that (in both scenarios investigated and for each assumed growth parameter) financing governmental subsidies in the context of Riester contracts, with the additional fiscal revenue from deferred taxation of (Riester) pension payments, is possible until 2050, at least on a yearly basis. However, the Riester scheme will still not finance itself on a cumulated basis. Less than 46% of the cumulated government subsidies can be covered by the additional fiscal revenue from deferred taxation of (Riester) pension payments until 2050. Furthermore, considering different income groups with regard to progressive income taxation also does not affect the general result. Moreover, a larger number of Riester savers, as explained in Section 3.5.3, does not change the results regarding the year from which the Riester scheme is self-financing, since both government expenditures and additional fiscal revenue from deferred taxation of pension payments are multiplied, and thus increase by the same factor. Furthermore, incorporating tax splitting for married couples does not significantly change the results.

Opposing Hagen and Kleinlein (2011), who suggest to possibly stop subsidizing private retirement savings to support the mandatory pension system, this paper reveals that there are not only governmental expenditures regarding Riester contracts. The analysis in this paper has been shown that the Riester scheme also generates tax revenues and is able to finance itself until 2050, at least on a yearly basis (depending on the scenario and the assumed growth parameter, n). This means, that there is a break-even point for some $t^* \leq 2050$. Consequently, the additional fiscal revenue obtained from deferred taxation of (Riester) pension payments exceeds the total government expenditures for Riester subsidies after that year t^* and decreases the cumulated financial gap in the long run. Hence, since the Riester scheme not only costs money by means of federal subsidies, but also generates fiscal revenue, governmental net expenditures will decrease.

However, although this paper deals with the Riester scheme in particular, its results and especially its research approach are applicable to other pension schemes that subsidize private pension savings which are in turn subject to deferred taxation when converted into retirement income.

References

Antolin, P., E. Whitehouse (2009), Filling the Pension Gap: Coverage and Value of Voluntary Retirement Savings, OECD Social, Employment and Migration Working Papers No. 69, 1-30.

Berger, J., T. Davoine, P. Schuster, L. Strohner (2016), Cross-country differences in the contribution of future migration to old-age financing, International Tax and Public Finance, 23, 1160-1184.

Bollacke, S. (2016), Subsidized Private Pension Savings and the Deferred Taxation of Retirement Income: Is the German Riester Scheme Self-Financing?, Available at SSRN: http://ssrn.com/abstract=2744672.

Börsch-Supan, A., A. Reil-Held, D. Schunk (2007), The Savings Behaviour of German Households: First Experience with State Promoted Private Pensions, MEA Discussion Paper, 136, 1-40.

Börsch-Supan, A., M. Gasche (2010), Kann die Riester-Rente die Rentenlücke in der Gesetzlichen Rente schließen?, MEA Discussion Paper, 201, 1-36.

Börsch-Supan, A., M. Coppola, A. Reil-Held (2012), Riester Pensions in Germany: Design, Dynamics, Targetting Success and Crowding-In, NBER Working Paper, 18014, 1-30.

Bucher-Koenen, T., A. Lusardi (2011), Financial literacy and retirement planning in Germany, Journal of Pension Economics and Finance, 10, 565-584.

Corneo, G., M. Keese, C. Schröder (2009), The Riester Scheme and Private Savings: An Empirical Analysis based on the German SOEP, Schmollers Jahrbuch, 129, 321-332.

Corneo, G., M. Keese, C. Schröder (2010), The Effect of Saving Subsidies on Household Saving - Evidence from Germany, Ruhr Economic Papers, 170, 1-48.

Corneo, G., C. Schröder, J. König (2015), Distributional Effects of Subsidizing Retirement Savings Accounts: Evidence from Germany, Freie Universität Berlin - School of Business & Economics Discussion Paper, 2015/18, 1-34.

Deutsche Rentenversicherung Bund (2014), Rentenversicherung in Zeitreihen, DRV-Schrift, 22, 1-334.

Gasche, M., T. Bucher-Koenen, M. Haupt, S. Angstmann (2013), Die Kosten der Riester-Rente im Vergleich, MEA Discussion Paper, 269-2013, 1-62.

Gerber, U. (2010), Staatliche Förderung der Riester-Rente 2005, Statistisches Bundesamt, Wiesbaden.

Gerber, U. (2011a), Staatliche Förderung der Riester-Rente 2006, Statistisches Bundesamt, Wiesbaden.

Gerber, U. (2011b), Staatliche Förderung der Riester-Rente 2007, Statistisches Bundesamt, Wiesbaden.

Gerber, U. (2012), Staatliche Förderung der Riester-Rente 2008, Statistisches Bundesamt, Wiesbaden.

Gerber, U. (2013), Staatliche Förderung der Riester-Rente 2009. Statistisches Bundesamt, Wiesbaden.

Gesamtverband der Deutschen Versicherungswirtschaft (2014), Die deutsche Lebensversicherung in Zahlen 2014, Gesamtverband der Deutschen Versicherungswirtschaft e.V..

Hagen, K., A. Kleinlein (2011), Zehn Jahre Riester-Rente: Kein Grund zum Feiern, DIW Wochenbericht, 47, 3-14.

Kiesewetter, D. (2001), Für wen lohnt sich die Riester-Rente?, Tübinger Diskussionsbeiträge, 224, 1-27.

Kruse, E., A. Scherbarth (2015), Förderung der Riester-Rente durch Zulagen und Sonderausgabenabzug: Mehr als 10,8 Millionen geförderte Personen im Beitragsjahr 2011, Rentenversicherung Aktuell, 3/2015, 55-65.

OECD (2017), Tax treatment of retirement savings in private pension plans across OECD countries, OECD Project On Financial Incentives And Retirement Savings, Policy Brief No. 1.

Pfarr, C., U. Schneider (2010), Mitnahmeeffekte im Rahmen der Riester-Rente: Eine empirische Analyse, In B. A. Kuchinke, T. Sundmacher, J. Zerth (Ed.), Wettbewerb und Gesundheitskapital, DIBOGS-Beiträge zur Gesundheitsökonomie und Sozialpolitik, 4, 92–106, Ilmenau: Universitäts-Verlag Ilmenau.

Pfarr, C., U. Schneider (2011), Anreizeffekte und Angebotsinduzierung im Rahmen der Riester-Rente: Eine empirische Analyse geschlechts- und sozialisationsbedingter Unterschiede, Perspektiven der Wirtschaftspolitik, 12 (1), 27-46.

Pfarr, C., U. Schneider (2013), Choosing between subsidized or unsubsidized private pension schemes: evidence from German panel data, Journal of Pension Economics and Finance, 12, 62-91.

Pötzsch, O. (2012), Geburten in Deutschland, Statistisches Bundesamt, Wiesbaden.

Prinz, A., A. Reichardt, K. Wrohlich (2003), Mikroökonomische Analyse der 'Riester-Rente', Wirtschaftswissenschaftliches Studium, 32 (7), 399 – 405.

Schröder, C., M. Keese, G. Corneo (2007), Erhöht die Riester-Förderung die Sparneigung von Geringverdienern?, Economics working paper, Christian-Albrechts-Universität Kiel, Department of Economics, 30.

Statistisches Bundesamt (2009a), Staatliche Förderung der Riester-Rente 2004, Statistisches Bundesamt, Wiesbaden.

Statistisches Bundesamt (2009b), Bevölkerung Deutschlands bis 2060 - Ergebnisse der 12. koordinierten Bevölkerungsvorausberechnung, Statistisches Bundesamt, Wiesbaden.

Chapter 4

Vector Similarity as a Measure for Comparing Pension Systems: An Application to OECD Countries

Abstract

The purpose of this paper is to compare pension systems across OECD countries to assess their similarity by using the concept of vector similarity. Furthermore, differences between pension systems are explained in a cross-section panel estimation. After constructing one vector for each country that represents its pension system, vector similarity is calculated for them with reference to Germany. Employing vector similarity as the dependent variable, a cross-section panel estimation is conducted, using several institutional, demographic and economic factors as explanatory variables. It turns out that differences between pension systems can be explained rather by demographic and institutional variables than economic ones.

JEL: H55

Keywords: Vector similarity, Pension systems, Panel estimation.

4.1 Introduction

This paper uses vector similarity as a new approach for comparing pension systems. The aim is to express similarities between pension systems in one value, without intending to form a ranking. These similarities are explained subsequently in a cross-section panel model using institutional, demographic and economic impact factors as explanatory variables. In addition to the obvious demographic and economic impacts on the level of several pension system indicators (e.g. a higher effective retirement age often leads to higher pension benefits), this approach sheds light on the question of whether differences between pension systems may be driven by demographic and economic variables and are not only a product of institutional features.

Demographic change and financial pressure are currently challenging governments and other pension system managing institutions. Since it may help for reforming and improving the national pension system, comparisons with other pension systems are becoming more important than ever. For example, the net pension replacement rate of the German mandatory pension system decreased from about 72% in 2004 to 50% in 2014. In the United States, the net pension replacement rate decreased from about 51% in 2004 to about 45% in 2014 (see OECD 2005, 2007, 2009, 2011, 2013, 2015). Pension funds' assets as a percentage of gross domestic product (GDP) increased during that time span in both countries. Alongside with institutional features of a pension system, these trends are mainly driven by several economic and especially demographic factors, such as the old-age dependency ratio. This increased from about 30% in 2004 to about 35% in 2014 in Germany and from almost 21% in 2004 to 24% in 2014 in the United States (see OECD 2016b). However, since particular proceedings that work for the pension system of country A may not automatically work in country B, knowledge about possible similarities between pension systems, expressed in one value in the first place, is required in order to detect features that drive these similarities.

Existing comparisons of pension systems are mainly carried out by building composite indices that use a variety of different pension system characteristics with the aim of ranking countries by their pension systems (e.g. see Allianz Global Investors 2014 and Mercer 2011, 2012, 2013, 2014, 2015). The problem is that this approach is a rather normative one, since it constructs a ranking that results in stating that some pension systems are better than others, irrespective of the underlying economic and demographic situation. Furthermore, since they use a large number of pension system indicators, put a weighting on these indicators or build sub-indices, the results of the comparisons are difficult to replicate.

However, comparing pension systems to assess their similarity does not require a normative analysis. Thus, the objectives of this paper are to construct an unweighted vector for each country and year that includes only a few expressive factors which represent the pension system; to compare these vectors across countries so as to assess their similarity by using the concept of vector similarity; and to explain similarities in a cross-section panel estimation, using demographic and economic variables, in addition to institutional features, as explanatory ones. The analysis in this paper is based on the one by Bollacke (2016).

The remainder of the paper is structured as follows. Section 4.2 gives a short overview of the main institutional features of pension systems. Section 4.3 explains vector similarity, its calculation and advantages. Section 4.4 explains the choice of the data used for the construction of the pension systems' vectors. Moreover, vector similarity is calculated and first results are presented. Section 4.5 presents the panel regression model, the estimation methods and the estimation results. Section 4.6 concludes.

4.2 Core Features of Pension Systems

Although each pension system in the world is unique, there are some cornerstones that all pension systems have in common. They all grant benefits to retirees and they need contributions to finance the benefits. But pointing out all the differences between all OECD countries concerning pension systems would be beyond the scope of this paper.[1] However, the classification of pension systems by institutional fea-

[1] See OECD (2005, 2007, 2009, 2011, 2013, 2015) for details.

tures is made here for the mandatory part of the pension system. The voluntary part is considered indirectly in the analysis in sections 4.4 and 4.5.

First of all, almost every pension system is composed of different tiers, basically, a mandatory and a voluntary part. The mandatory part can be separated into an 'adequacy part' (usually called the 'first tier') and an 'earnings-related part', which is usually called 'second tier'. The second tier can be separated into *defined contribution* (DC) and *defined benefit* (DB) systems. Moreover, pension systems can be financed on a public or a private basis or on a mixture of both. That is, a pension system is either organized by the government (public basis; usually pay-as-you-go pension systems) or the insured individuals pay a particular amount of money to an insurance company (private basis). Most of the privately organized pension systems are defined contribution systems in which contributions are paid into an individual account. The accumulated amount of money plus the investment return is then later transformed into retirement income. There is a special case called 'notional defined contributions' (NDC), in which workers contribute to an individual account with a 'notional' rate of return. That means, that the rate of return is not the market rate of return, but only exists for the managing institution, in this case: the government.

On the other hand, most of the publicly organized pension systems are defined benefit systems in which the retirement income depends on a specific calculation formula which takes account of individual earnings and the number of years of contribution. However, there is also a special case of a pension system, located somewhere between defined benefit and defined contribution, called *Point* systems. In those systems, for example in Germany, workers earn points depending on their earnings. When retiring, these points are multiplied by a pension-point value and thus, converted into a retirement income (see OECD 2015).

The 'adequacy part' of a pension system offers a minimum or basic pension to the pensioners. Basic pensions may be residence-based or contribution-based. That means, that a full basic pension is guaranteed after either a particular period of time of residence in the country or a particular time of contribution to the country's pen-

Table 4.1: Institutional features of pension systems of OECD 30 countries; 2014

	Public	Private	Basic/Minimum
Australia		DC	B
Austria	DB		-
Belgium	DB		M
Canada	DB		B
Czech Republic	DB		B/M
Denmark		DC	B
Finland	DB		B
France	DB+Points		M
Germany	Points		-
Greece	DB		B
Hungary	DB		M
Iceland		DB	B
Ireland	-	-	B
Italy	NDC		M
Japan	DB		B
Korea	DB		-
Luxembourg	DB		B/M
Mexico		DC	M
Netherlands		DB	B
New Zealand	-	-	B
Norway	NDC	DC	B
Poland	NDC		M
Portugal	DB		M
Slovak Republic	Points	DC	-
Spain	DB		M
Sweden	NDC	DC	B
Switzerland	DB	DB	M
Turkey	DB		M
United Kingdom	DB		B
United States	DB		-

Source: Own depiction based on OECD (2015).

Notes: DB: Defined benefits; DC: Defined contributions; NDC: Notional defined contributions; B: Basic pension; M: Minimum pension; B/M: Basic and minimum pension.

sion system, irrespective of the amount of contributions. Minimum pensions on the other hand guarantee a top-up to the individual pension benefits. If a worker cannot reach the minimum pension threshold with his or her contributions, the minimum pension fills this gap. However, a particular period of time of contribution to the pension system is required to be eligible for the minimum pension. The 'voluntary part', which is usually called the 'third tier', is the additional, voluntary and usually privately organized old-age provision. Again, this is considered indirectly in the analysis in sections 4.4 and 4.5.

Table 4.1 presents the basic institutional features of the pension systems of the OECD 30 members for the year 2014.[2] Ireland and New Zealand cannot be classified entirely due to lack of information. These countries are nevertheless included in the comparison in section 4.4. However, most of the considered countries are structured as a mandatory public defined benefits system, while there are also mandatory private defined benefits systems. Furthermore, most of the countries offer either a minimum or a basic pension to the pensioners.

4.3 Vector Similarity

Vector similarity is a method for comparing two vectors with each other (see Jones and Furnas 1987, Busch 1998). The similarity between two vectors can be determined by the cosine of the angle between them. Let γ depict the angle between vectors $\mathbf{A}$ and $\mathbf{B}$. The cosine of γ is then given by:

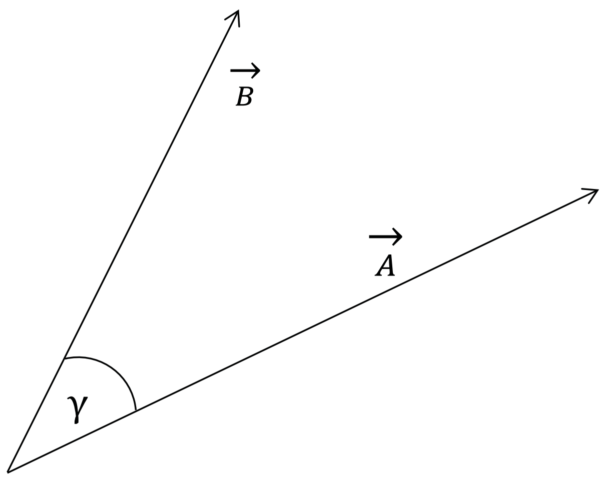
$$cos(\gamma) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2} \sqrt{\sum_{i=1}^{n} B_i^2}} \tag{4.1}$$

In equation 4.1, A_i and B_i depict the components of the vectors $\mathbf{A}$ and $\mathbf{B}$. Hence, the cosine of an angle is calculated using the dot product and the magnitude of two vectors $\mathbf{A}$ and $\mathbf{B}$. Furthermore, the cosine of an angle is defined by domain values between -1 and 1. However, all components that are used in this paper are larger than zero. Thus, the cosine of the angle between two vectors is within the range zero and unity, with zero indicating complete dissimilarity, and unity, perfect similarity.

Figure 4.1 illustrates two differing vectors, $\mathbf{A}$ and $\mathbf{B}$, and the included angle γ. For $0° < \gamma < 90°$, the vector similarity is between 0 and 1. If γ is very small, the vectors $\mathbf{A}$ and $\mathbf{B}$ almost point in the same direction. Whatever the magnitude, the

[2] Only countries that are OECD members since 2004 are included.

Figure 4.1: Illustration of two vectors, **A** and **B**, with angle γ and different magnitudes



Notes: Own depiction.

vector similarity is, hence, close to 1. If γ is 0 or 90 degrees, respectively, the vector similarity is unity or zero, respectively.

This procedure has not yet been applied to comparisons in social security analysis. In addition, it has rarely been applied in economics at all. There are a few studies in which vector similarity is used in terms of structural change and economic growth (see e.g. Dobrescu 2011, Noseleit 2013). However, there are some advantages of this measure, which could make it a useful tool for social security analysis and particularly for the comparison of pension systems. For example its ease of calculation, its reliance on ratios rather than absolute values and the fact that it is not intended to form a ranking. Furthermore, the vector components require neither normalization nor positive values. Regardless of the number of components and the magnitudes of the vectors, the domain of the cosine of an angle of the two vectors is always between -1 and 1, as long as the two vectors consist of the same number of components.

Alternatively, one could use the Pearson correlation coefficient as an appropriate measure for comparing two vectors, since it also has some of the advantages mentioned above. However, the following example shows that it is not appropriate for assessing similarities. Let $X = [35\,45\,60]$ and $Y = [70\,90\,120]$ be vectors with three components representing two different pension systems. It is obvious that the components of vector Y are double the size of the components of vector X. They are perfectly identical when compared by vector similarity. The Pearson correlation coefficient also yields a perfect correlation between these two vectors and is also easy to calculate. But, using the Pearson correlation coefficient is inappropriate for the objective of this paper, since *correlation* is not the same as *similarity*. It is a rather special case in which vector similarity and the Pearson correlation coefficient yield both perfect similarity and correlation, respectively. If a third vector $Z = [70\,80\,95]$ is introduced, it is clear that all components of vector Z are 35 points higher than for vector X. While the Pearson correlation coefficient still shows perfect correlation, vector similarity does not (due to a steeper slope of vector X). Furthermore, it is obvious that X and Z have a perfectly positive linear relationship. However, since the first component of Z is double the size of the first component of X, but the other two components are not double the size, the vectors X and Z cannot be considered as perfectly similar. Hence, the Pearson correlation coefficient is an inappropriate measure for similarity studies. Vector similarity takes account of the ratios of the components and is used in the following analysis.[3]

4.4 Comparison Concept and Measurement

A pension system can roughly be depicted as a concept, using institutional features, that transforms demographic and economic inputs into outcomes. Input variables, such as the old-age dependency ratio and the GDP per capita, determine the outcomes, measured, for example, by the gross pension replacement rate of a country's pension system. Outcomes are often used to represent a pension system's quality. However, gross pension replacement rates of the mandatory pension system are not able to say anything about the pension system as a whole, since only retirees

[3]See Van Eck and Waltman (2008) for further discussion of the problems associated with the Pearson correlation coefficient for similarity studies.

are represented when using only gross pension replacement rates. Moreover, the use of gross pension replacement rates cannot answer the questions of whether a pension system is funded or unfunded, how pensioners are treated regarding income tax and social security contributions or whether it has a strong voluntary part or not. Complementing the gross pension replacement rate by the net pension replacement rate indirectly includes the tax and social security treatment of pensioners. Adding pension funds' assets, measured as a percentage of the GDP, may reveal whether a pension system is a funded or unfunded one or at least, if there is a strong funded part (which may be voluntary).

Hence, the indicators for forming the pension system vectors are:

(1) the gross pension replacement rate

(2) the net (after social security contributions and tax) pension replacement rate

(3) the pension funds' assets as a percentage of gross domestic product (GDP).

The gross pension replacement rate is the level of retirement income relative to earnings during the working period. It is calculated using rules that apply in a respective year and provides a theoretical value for people entering the labor market in that year and work until the legal retirement age. Nevertheless, it gives insights into the current condition of national pension systems. However, this indicator is used relative to average earners. Hence, it equals the relative pension level of average earners.[4] Note that the pension replacement rate is measured as the ratio of the benefits to individual lifetime average earnings for a full career and not as the ratio to final earnings. A full career is defined as the time span from entering the labor market at 20 years and working until the legal retirement age.[5] Furthermore, the net pension replacement rate is the level of net pension benefits relative to net pre-retirement earnings. This indicator is included to capture differences between the social security and tax systems of the different countries. Moreover, pension

[4] There are countries that provide higher/lower pension replacement rates for low-income/high-income workers. See, for example, OECD 2015.

[5] See again, for example, OECD 2015.

Table 4.2: Indicators used for representing the pension system by country; 2014

	GRR	NRR	PFA
Australia	44.5	58.0	102.2
Austria	78.1	91.6	5.7
Belgium	46.6	60.9	5.0
Canada	36.7	47.9	70.8
Czech Republic	49.0	63.8	7.3
Denmark	67.8	66.4	42.1
Finland	55.8	63.5	48.7
France	55.4	67.7	0.4
Germany	**37.5**	**50.0**	**6.1**
Greece	66.7	72.9	0.1
Hungary	58.7	89.6	4.0
Iceland	69.2	76.7	141.2
Ireland	34.7	42.2	52.3
Italy	69.5	79.7	6.0
Japan	35.1	40.4	29.2
Korea	39.3	45.0	6.0
Luxembourg	76.8	88.6	2.1
Mexico	25.5	28.4	14.7
Netherlands	90.5	95.7	148.7
New Zealand	40.1	43.0	18.8
Norway	49.8	60.2	8.1
Poland	43.1	52.8	18.2
Portugal	73.8	89.5	8.9
Slovak Republic	62.1	80.6	9.8
Spain	82.1	89.5	8.8
Sweden	56.0	55.8	9.1
Switzerland	40.2	46.9	113.4
Turkey	75.7	104.8	4.8
United Kingdom	29.7	38.3	99.6
United States	35.2	44.8	83.2

Source: OECD (2015, 2016c).
Notes: GRR: Gross pension replacement rate; NRR: Net pension replacement rate;
PFA: Pension funds' assets as a percentage of GDP.

funds' assets as a percentage of GDP are also included, since this indicator may reveal whether a pension system is, at least partly, funded or unfunded. The higher the pension funds' assets, the more likely the pension system has a funded part.

Building a vector and calculating the vector similarity from these three indicators is obviously easier than using a large number of indicators and building sub-indices to compare pension systems or to compare every single feature of pension systems. The indicators by country for the year 2014 are presented in Table 4.2. There a large differences in both the gross and net pension replacement rates as well as in the pension funds' assets. However, for each country, except for Denmark and Sweden, the net pension replacement is higher than the gross pension replacement rate, which indicates a redistribution of incomes. Since the ratio of the net pension replacement rate to the gross pension replacement rate differs slightly across countries, this might be a first reason for differences between pension systems. But the pension funds' assets differ heavily across countries, even more than the pension replacement rates. For example, the Netherlands have pension funds' assets of 148.7% of GDP compared to 0.1% in Greece. Germany also has relatively low pension funds' assets of 6.1% of GDP. Hence, this indicator may be the main reason for differences between pension systems.

The vector created out of these indicators will be constructed for each country and year, and subsequently compared using vector similarity. The corresponding data is from OECD publications and datasets. Vectors are constructed for the 30 OECD members between 2004 and 2014.[6] Since the aim of this paper is to compare countries in different years and not the development of a country's pension system over time, the choice of a reference vector is fundamental. In the following analysis, all results are calculated with respect to the German pension system. Thus, vector **A**, as in equation (4.1), is defined as the vector for Germany. Hence, vector similarity close to 1 means that the pension system is similar to the German pension system in terms of ratios between the gross and net pension replacement rates and the pension funds' assets. It is possible that a country with a vector similarity of almost 1 has half of the pension replacement rates and half of the pension funds' assets compared to Germany. But it is still a similar pension system since it seems like that it treats pensioners similarly in terms of taxation and is also an unfunded system and has a weak voluntary part.[7] A low vector similarity means that the pension system is not very similar to the German one. It could

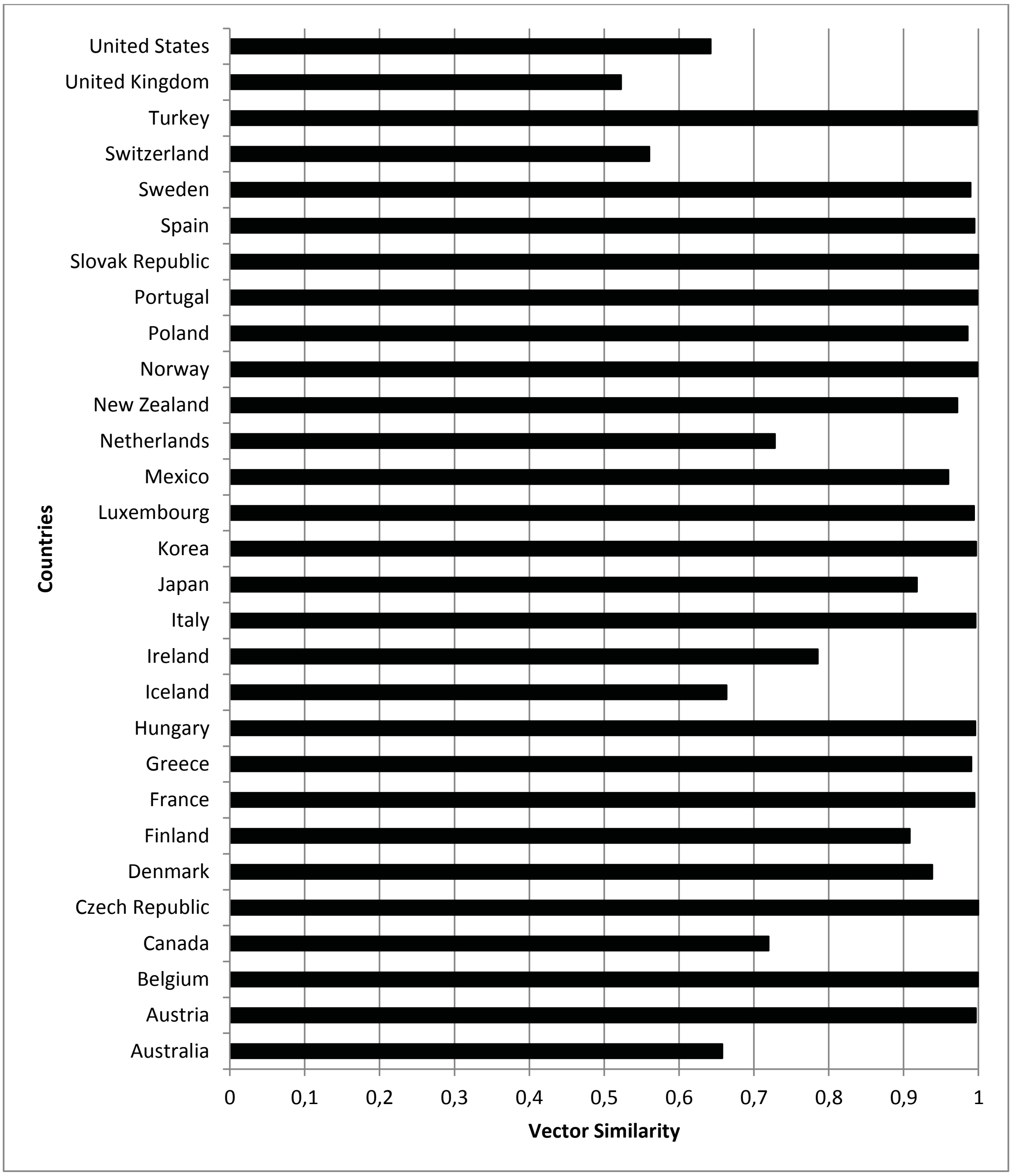
[6]See OECD (2005, 2007, 2009, 2011, 2013, 2015, 2016c).

[7]Note, that Germany has relatively low pension funds' assets. See OECD (2016c).

be possible that the tax treatment of pensioners is different compared to Germany, resulting in a relatively higher or lower net pension replacement rate compared to the gross pension replacement rate. On the other hand, if the relation between the pension replacement rate is similar to the one in Germany, it must have something to do with the pension funds' assets. Those have to be either significantly lower or higher, meaning that the system is an unfunded one with an even weaker voluntary part, or is a funded system combined with a possible strong voluntary part. It might also be an unfunded system with a very strong voluntary part. Nevertheless, it then has a differently structured pension system compared to Germany. The results for the year 2014 are presented in Figure 4.2. There are some countries that reveal a substantial difference from Germany, such as the United States, the United Kingdom, Switzerland, the Netherlands, Ireland, Iceland, Canada and Australia. As it was supposed, these countries have the highest share of pension funds' assets of more than 50% of GDP. On the other hand, countries like the Slovak Republic, Italy, Greece, France, the Czech Republic and Belgium, for example, reveal a high similarity to the German pension system.

Figure 4.3 presents the development of pension system vector similarity with respect to the German pension system over time for countries with a vector similarity exceeding 0.95. Furthermore, it is subdivided into countries that have a mandatory public DB pension system, plus New Zealand (top of Figure 4.3) and the remainder countries (bottom of Figure 4.3). Although each country in the top figure yields a vector similarity to Germany exceeding 0.98 in 2004, vector similarity increases until 2014 with a value of 0.99 and above for each country. The bottom of Figure 4.3 also shows countries that are very similar to Germany regarding the pension system. The difference is that they are not structured as mandatory public DB pension systems. However, they all have a vector similarity exceeding 0.96. In addition, there is no clear development trend. For example, Mexico has a value larger than 0.99 in 2004 that decreases to about 0.96 in 2014. On the other hand, the vector similarity to the German pension system increases for all other countries except for Poland.

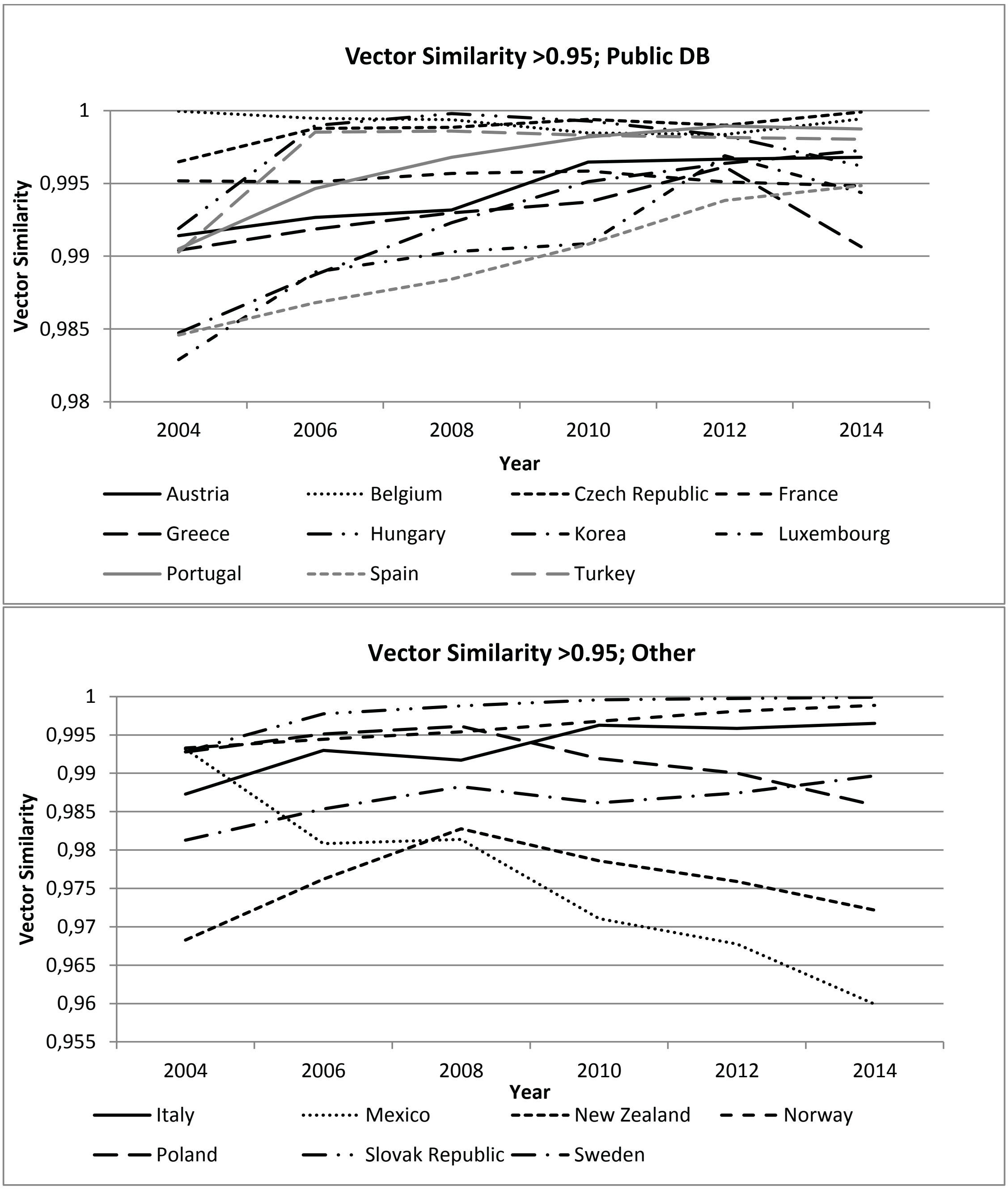
Figure 4.2: Vector Similarity of pension systems with respect to the German pension system; 2014



Source: Own calculation based on OECD (2015, 2016c).

Figure 4.4 shows the development of vector similarity of pension systems with respect to the German pension system over time for countries that exhibit a vector similarity of less than 0.95. This figure is subdivided into countries that have a private mandatory pension system plus Ireland (top of Figure 4.4) and other (bottom

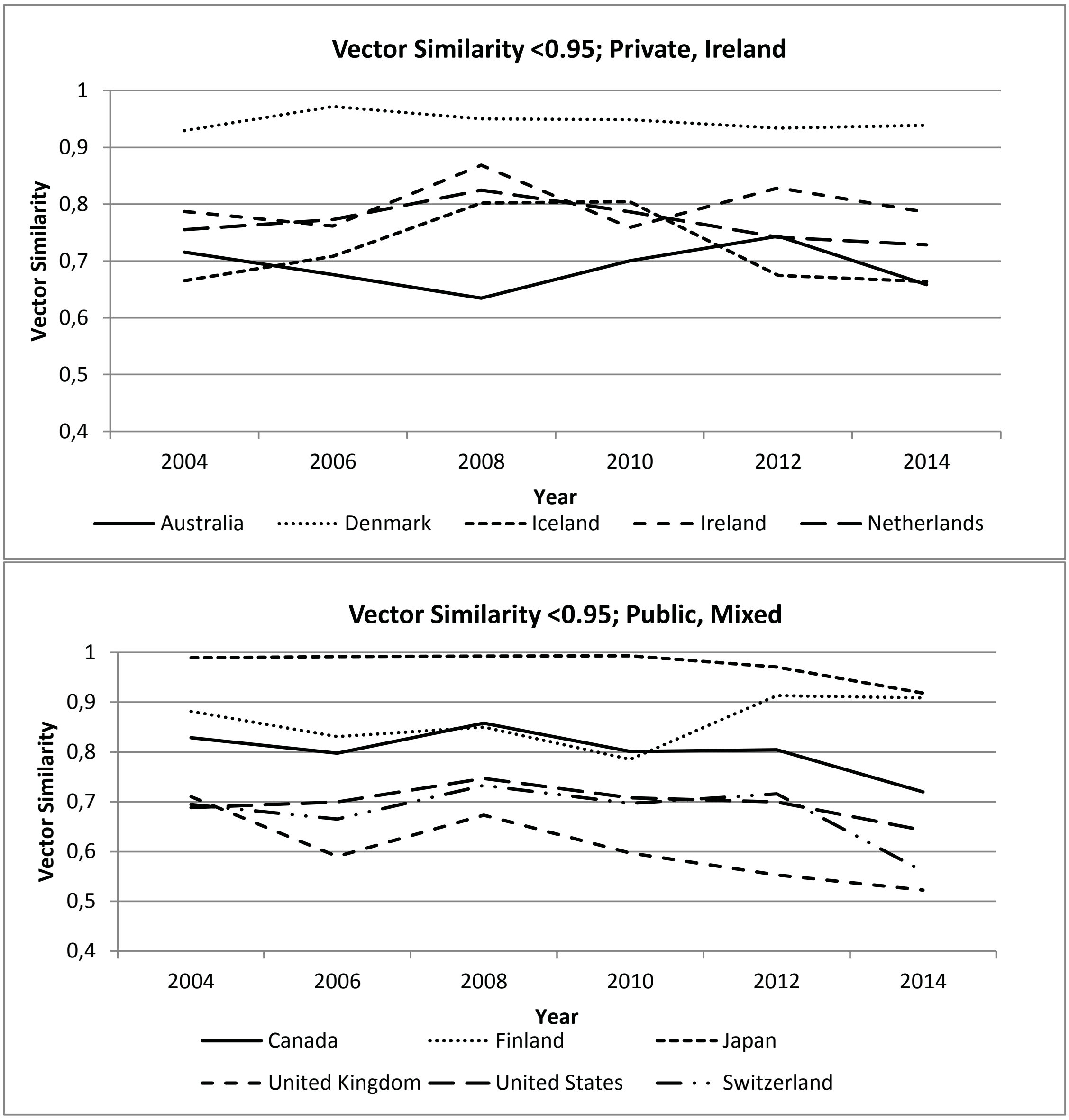
Figure 4.3: Vector Similarity of pension systems with respect to the German pension system over time; Vector Similarity >0.95



Source: Own calculation based on OECD (2005, 2007, 2009, 2011, 2013, 2015, 2016c).

of Figure 4.4). In addition to the fact that the presented countries are considerably less similar to Germany than the countries presented in the previous figure, they also seem to have greater volatility. This applies to both the private mandatory pension systems (top of Figure 4.4) and the other pension system structures (bottom

Figure 4.4: Vector Similarity of pension systems with respect to the German pension system over time; Vector Similarity <0.95



Source: Own calculation based on OECD (2005, 2007, 2009, 2011, 2013, 2015, 2016c).

of Figure 4.4). For example, the United Kingdom has a vector similarity of about 0.7 in 2004, which decreases to about 0.52 in 2014. Note, that although Denmark and Japan have a vector similarity that is greater than 0.95 in some years, they are considered here since they have a vector similarity of less than 0.95 in 2014.

The two figures presented above reveal that the level and development of vector similarity cannot be explained completely by institutional features, such as Public/Private or DB/DC, of a pension system. There must be other influences affecting the similarity of a country's pension system to the German one. The next section analyzes which factors drive the abovementioned differences and similarities.

4.5 Empirical Analysis of Pension System Similarities

4.5.1 Variable Selection

A variety of demographic and economic variables that have an impact on pension systems is used as explanatory variables in the following analysis. To explain those, a simple depiction of a pension system and its workings is presented in Figure 4.5.

Figure 4.5 depicts demography, the economy and institutional features as inputs to the pension system. Those have then an impact on the pension system, measured by the 'representatives', which are the gross and net pension replacement rate and the pension funds' assets as a percentage of GDP. Demographic inputs are the fertility rate (FR, see OECD 2014) and the old-age dependency ratio (OA), measured as the number of those aged 65 and above as the share of those between 20 and 64 (see OECD 2016b). GDP per capita in USD (gdp, constant prices, 2010 PPPs divided by 10,000, see OECD 2016e) and the public gross debt as a percentage of GDP ($GDebt$, see International Monetary Fund 2014) are used as economic inputs. Furthermore, the legal retirement age, LR, (for pensioners entering retirement in the considered year, see OECD 2005, 2007, 2009, 2011, 2013, 2015) is considered as an institutional input. Moreover, the effective retirement age (ER, see OECD 2016a) is also considered as an input. This seems odd since the retirement may also

Figure 4.5: Inputs to and representatives of pension systems

$$\underbrace{\begin{pmatrix} \text{Demography} \\ \text{Economy} \\ \text{Institutional Features} \end{pmatrix}}_{\textit{Inputs}} \rightarrow \underbrace{\begin{pmatrix} \text{Gross replacement rate} \\ \text{Net replacement rate} \\ \text{Pension funds' assets} \end{pmatrix}}_{\textit{Representatives of Pension System}}$$

Notes: Own depiction.

be considered as an output of a pension system. For example, the effective retirement age obviously affects the gross pension replacement rate, since earlier/later retirement leads to lower/higher pension benefits. On the other hand, an expected high gross pension replacement rate could incentivize people to retire earlier, since the marginal utility of retirement may exceed the marginal costs, i.e. the loss of corresponding benefits. However, it is considered as an input since the choice of entering retirement is also influenced by expectations about the pension benefits and personal situations, such as happiness at the working place and health issues. Hence, the effective retirement is considered as both a demographic and an economic input as well as an institutional one. Finally, the employment rate of those aged 65 and above ($Emp65$, measured as the number of employed people aged 65 and above, divided by the population of those aged 65 and above, see OECD 2016d) can be considered both as a demographic (due to increasing longevity) and an economic input. The choice of inputs is justified by their effects on the performance of pension systems. Especially for pay-as-you-go pension systems, the effects of the inputs on the benefits, and hence the pension replacement rates, of a pension system are evident. As for example shown by Cipriani (2014), population aging and hence, an increasing old-age dependency ratio, leads to decreasing pension benefits. Furthermore, an increasing legal retirement age and also an increasing effective retirement age, which is positively correlated with the employment rate of those aged 65 and above, has an impact on the old-age dependency ratio and hence, results in lower pension benefits. Moreover, Cigno (2006) and Cremer et al. (2006) found that decreasing fertility leads to a decreasing number of workers and hence, results in 'a decentralized equilibrium outcome with too few children' (Cremer et al. 2006). The effect of the GDP per capita on the pension system is as follows. A high GDP per capita results in more savings or higher saving rates. This is found, for example, by Masson et al. (1998). However, they also found a squared effect, which means, that savings rates may also decrease for relatively high incomes. Nevertheless, it is evident that GDP per capita impacts on savings rates and is thus likely to have an impact on pension funds' assets. In addition, the debt-to-GDP ratio may have a negative impact on the pension replacement rates. This assumption comes from the fact, that, for example in Germany, the government has to subsidize the pension system to guarantee particular pension replacement rates. High public debts may lead governments to reduce federal subsidies which in turn may affect pension benefits and thus, pension replacement rates. But also demographic changes, such as changes in the fertility rate and the old-age dependency ratio, have an impact on capital markets and thus on the funded part of pension systems. As found, for example, by Bakshi and Chen (1994) and Börsch-Supan and Winter (2001), an aging population changes household savings behavior and will thus impact on capital markets. This affects individual's investment returns and hence, the benefits. Finally, the gross pension replacement rate, the net pension replacement rate and the pension funds' assets are used to represent a pension system in a whole, since these indicators capture different parts of a pension system, such as the taxation of pensioners and the funding structure. Table 4.3 presents the input variables by country and some descriptive statistics for 2014.

Compared to the other countries presented in Table 4.3, Germany has a relatively low fertility rate (about 75% of the countries have a higher or equal fertility rate), a low effective retirement age (more than 50% of the countries have a higher effective retirement age) and a low legal retirement age (more than 50% of the countries have a higher legal retirement age). This is the reason why it is likely that the variables FR, ER and LR may have a negative effect on the similarity to the German pension system, since a higher effective retirement age, fertility rate or legal retirement age affects the benefits of a pension system and thus the vector similarity. On the other hand, Germany has a relatively high old-age dependency ratio (more than 75% of the countries have a lower old-age dependency ratio). Hence, the variable OA is likely to have a positive effect on the similarity to the German pension system. Germany also has a relatively high GDP per capita (more than 50% of the countries have a lower GDP per capita). Hence, gdp should have a negative effect on the vector similarity. Since Germany has a relatively low debt-to-GDP ratio (more than 50% of the countries have a higher debt-to-GDP ratio), the public gross debt

Table 4.3: Demographic and economic input variables by country in absolute terms and descriptive statistics; 2014

	OA	ER	LR	FR	gdp	Emp65	GDebt
Australia	24.50	65.35	67.00	1.89	4.41	12.05	30.83
Austria	30.10	62.25	65.00	1.44	4.28	5.22	79.09
Belgium	31.00	59.95	65.00	1.87	3.96	2.29	99.83
Canada	25.10	64.49	67.00	1.63	4.21	12.88	87.38
Czech Republic	27.10	63.34	68.00	1.49	2.76	4.91	49.21
Denmark	31.50	62.97	67.00	1.88	4.20	7.08	45.60
Finland	34.00	61.86	65.00	1.87	3.75	10.10	60.16
France	31.90	59.37	63.00	2.02	3.64	2.33	95.76
Germany	**35.00**	**62.70**	**65.00**	**1.39**	**4.25**	**5.77**	**74.55**
Greece	32.90	61.60	62.00	1.51	2.43	2.53	174.70
Hungary	27.80	62.60	65.00	1.26	2.31	3.15	79.10
Iceland	22.00	69.41	67.00	2.20	4.10	36.25	91.75
Italy	35.90	61.42	67.00	1.41	3.24	3.65	134.51
Japan	45.70	69.29	65.00	1.39	3.49	20.78	243.52
Korea	19.00	72.90	65.00	1.23	3.37	31.27	37.98
Luxembourg	23.10	61.89	60.00	1.63	8.48	4.03	24.13
Netherlands	29.50	62.89	67.00	1.80	4.46	7.32	75.03
Norway	27.40	65.17	67.00	1.95	6.00	19.25	29.52
Poland	23.00	62.09	67.00	1.38	2.30	4.74	49.51
Portugal	31.10	67.02	66.00	1.37	2.57	11.73	126.69
Slovak Republic	20.00	61.09	67.00	1.40	2.65	1.89	58.62
Spain	29.00	62.22	65.00	1.38	3.12	1.63	98.81
Sweden	34.10	65.19	65.00	1.98	4.28	16.53	41.48
Switzerland	29.00	66.05	65.00	1.54	5.24	11.65	48.11
United Kingdom	30.30	64.13	68.00	1.98	3.78	10.03	91.50
United States	24.00	65.86	67.00	1.93	5.06	17.73	105.70
0.25 percentile	24.65	61.94	65.00	1.39	3.15	3.75	48.39
0.50 percentile	29.25	62.93	65.50	1.59	3.87	7.20	77.06
0.75 percentile	31.80	65.31	67.00	1.89	4.28	12.67	98.04
Mean	29.00	63.97	65.65	1.65	3.94	10.26	82.04

Source: International Monetary Fund (2014)
and OECD (2014, 2015, 2016a, 2016b, 2016d, 2016e).

as a percentage of GDP is likely to have a negative impact on the similarity to the German pension system. Moreover, the level of employment of people aged 65 and above is expected to have a negative impact on vector similarity, since Germany has a relatively low employment rate of people aged 65 and above (more than 50% of the countries have a higher employment rate of people aged 65 and above).

4.5.2 Regressions

The Basic Model

A multiple linear regression model is constructed using vector similarity (VS), as calculated in sections 4.3 and 4.4, as the dependent variable. Since there is data for several countries for six years, a panel estimation can be employed.

Thus, the regression model reads:

$$\begin{aligned} VS_{it} = \alpha + \beta_1 \cdot OA^G_{it} + \beta_2 \cdot LR^G_{it} + \beta_3 \cdot ER^G_{it} + \beta_4 \cdot FR^G_{it} \\ + \beta_5 \cdot ln(gdp^G)_{it} + \beta_6 \cdot Emp65^G_{it} + \beta_7 \cdot GDebt^G_{it} + \varepsilon_{it} \end{aligned} \tag{4.2}$$

In equation (4.2), VS is the Vector Similarity of the pension system of country i at time t to the German pension system. For the following analysis, the value of all explanatory variables is relative to Germany, for example,

$FR^G_{it} = \frac{(Fertility\ Rate)_{it}}{(Fertility\ Rate)_{Germany,t}}$.

The same transformation applies for all explanatory variables. The resulting variables show the relationship of country $i's$ indicator to the respective German indicator. This transformation makes interpretations of the slope coefficients easier and enables comparing the magnitude of the effects.

Hence, OA^G depicts the old-age dependency ratio, LR^G the legal retirement age, ER^G the effective retirement age, FR^G the fertility rate, $ln(gdp^G)$ the logarithmized GDP per capita in USD (constant prices, 2010 PPPs), $Emp65^G$ the employment rate of people aged 65+ and $GDebt^G$ the public gross debt in % of the GDP relative to Germany. Finally, β_k depicts the slope coefficients, α the constant term and ε the error term. OECD member countries for 2004 are included in the estimations. Note that Ireland and New Zealand are dropped due to a lack of data. Moreover, Mexico and Turkey are dropped because they are not high-income economies according to the Worldbank definition and might thus distort the results. Furthermore, data is available for 2004, 2006, 2008, 2010, 2012 and 2014. Hence, 25 OECD countries are included in the estimations for six years. The pooled correlation matrix of the explanatory variables and the dependent variable is presented in Table 4.4.

Table 4.4: Correlation matrix

	VS	OA^G	LR^G	ER^G	FR^G	$ln(gdp^G)$	$Emp65^G$	$GDebt^G$
VS	1.0000							
OA^G	0.2014	1.0000						
LR^G	-0.3766	0.0402	1.0000					
ER^G	-0.2796	-0.1150	0.1902	1.0000				
FR^G	-0.4975	-0.0102	0.2777	0.0146	1.0000			
$ln(gdp^G)$	-0.3163	0.0407	0.1840	0.0138	0.5549	1.0000		
$Emp65^G$	-0.2351	-0.2476	0.1956	0.8471	0.1411	0.0721	1.0000	
$GDebt^G$	0.1185	0.6560	0.0337	0.1680	-0.1932	-0.2227	0.0491	1.0000

Notes: Pearson correlation coefficient. Pooled data. $n = 150$.

First of all, the correlation coefficient has the expected sign when correlating the explanatory variables with the dependent variable. However, the correlation is not large enough to suggest a strong relationship. Second, the correlation between the effective retirement age and the employment of people aged 65 and above is about 0.85. This collinearity may cause problems regarding the regression analysis, since those variables can be explained almost entirely by one another. This dependency is quite obvious. Certainly, the higher the share of people working over the age of 65, the higher the effective retirement age.

To solve the collinearity problem, the following auxiliary regression is run:

$$ER_{it}^G = \delta_0 + \delta_1 \cdot Emp65_{it}^G + u_{it} \tag{4.3}$$

with δ_0 as the constant term, δ_1 as the slope coefficient and u as the error term.

After estimating equation (4.3) with ordinary least squares, the errors are saved as a new variable called $ERadj$. This includes the share of the effective retirement age that is not explained by a linear relationship with the employment of people aged 65 and above. The effective retirement age is substituted by $ERadj$ in what follows. Because of the panel structure of equation (4.2), it is possible to run a fixed-effects regression. However, since the aim of this paper is not to detect effects over time, but to compare different countries according to their pension system, it is appropriate to run a time fixed-effects regression. The results are presented in Table 4.5.

Table 4.5: Regression results for equation (4.2)

	Pooled	Time Fixed Effects
Old-age dependency ratio	0.313***	0.383***
	(0.0864)	(0.0904)
Legal retirement age	-1.120***	-0.978**
	(0.334)	(0.399)
Effective retirement age (adjusted)	-1.263***	-1.640***
	(0.312)	(0.346)
Fertility rate	-0.302***	-0.343***
	(0.0524)	(0.0548)
ln(gdp)	-0.0366	-0.0321
	(0.0327)	(0.0327)
Employment by population 65+	-0.00165	0.00125
	(0.00417)	(0.00435)
Public debt in % of GDP	-0.0336	-0.0469**
	(0.0223)	(0.0229)
Constant	2.157***	2.017***
	(0.324)	(0.387)
F-stat	15.94	16.93
Adjusted (within) R2	0.4124	0.4638
N	150	150

Notes: Results from estimation of equation (4.2). Dependent variable: Vector Similarity (VS). Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

First, all exogenous variables have the expected sign. Second, since the time fixed effects are not statistically significant (not reported), it is possible to run a pooled-OLS estimation. In both estimations, $ln(gdp)$ and the employment of people aged 65 and above, are insignificant. In the pooled-OLS estimation, the debt-to-GDP ratio is also insignificant, whereas it is significant at the 5% significance level in the time fixed effects estimation. All other explanatory variables are significant at the 1% significance level. Although the results of the pooled-OLS estimation do not differ substantially from those of the time fixed effects estimation, the coefficient of $ERadj$ is the most affected. Moreover, the old-age dependency ratio is the only explanatory variable that has a positive effect on the similarity to the German pension system. The higher country $i's$ old-age dependency ratio, the more similar it is to the German pension system. All other variables (legal retirement age, (adjusted) effective retirement age, fertility rate) have a negative effect on such similarity.

Since all explanatory variables are used in relation to Germany, conclusions about the magnitudes of the effects can be drawn. If each variable were increased by one percentage point, the (adjusted) effective retirement age would have the largest effect on the similarity to the German pension system. The effect is almost five times larger than that of an increase in the fertility rate. Moreover, GDP per capita is insignificant in both estimations. This may be due to the fact that only high-income OECD countries are included in the estimations. The variation in $ln(gdp)$ might not to be sufficiently large to exert a statistically significant impact on vector similarity. Although GDP per capita, the employment of people aged 65 and above, as well as the debt-to-GDP ration do not have a statistical significant effect on vector similarity (except for $GDebt$ in the time fixed effects estimation), all other input variables (old-age dependency ratio, legal retirement age, (adjusted) effective retirement age, fertility rate) are significant at the 1% significance level and have the expected sign. Hence, it seems that the similarity to the German pension system is mainly driven by demographic variables.

The Extended Model

To investigate whether further institutional features, as those explained in section 4.2, have an impact on the similarities between pension systems, equation (4.2) is modified. Some descriptive statistics of the three used pension system indicators and the vector similarity, classified by institutional features, are presented in Tables 4.6 - 4.8.

Table 4.6 shows the results of pension systems that are entirely publicly organized and other. Both the average gross pension replacement rate and the average net pension replacement rate are lower in entirely publicly organized pension systems. The same holds true for the 0.5 percentile. Also the average pension funds' assets are lower compared to otherwise organized pension systems. However, although the ratio of the gross pension replacement rate to the net pension replacement rate

Table 4.6: Descriptive statistics for pension system indicators; entirely publicly organized or not; all years

	Public	Other
Replacement rate (gross)		
Mean	56.82	63.86
0.5 Percentile	53.30	59.30
Standard deviation	18.41	14.43
Replacement rate (net)		
Mean	68.05	74.54
0.5 Percentile	64.70	69.70
Standard deviation	18.94	16.02
Pension funds' assets in % of GDP		
Mean	20.92	56.15
0.5 Percentile	5.4	42.10
Standard deviation	29.12	49.82
Vector similarity		
Mean	0.9293	0.8663
0.5 Percentile	0.9923	0.9487
Standard deviation	0.1227	0.1424
N	95	55

Source: Own calculation based on OECD (2005, 2007, 2009, 2011, 2013, 2014, 2015, 2016c).

is similar, irrespective of the organization structure, the pension funds' assets are significantly higher when pension systems are not publicly organized. This may be the reason for the vector similarity being lower for those countries.

Table 4.7 presents the results classified by the fact whether a mandatory pension system is an entirely defined contribution system or other. There is almost no difference in the means and the 0.5 percentiles for the pension replacement rates, especially not for the ratio between the gross and the net pension replacement rate. There is a small difference in the average pension funds' assets which is likely to be the reason for the vector similarity being slightly different. Hence, the fact that a pension system is DC could have a positive impact on the vector similarity.

Finally, Table 4.8 presents the results when pension systems are classified by the fact whether they offer basic and/or minimum pensions or not. Both the average and 0.5 percentile of the gross and net pension replacement rate are slightly lower for countries that do offer basic and/or minimum pensions. But the ratio between them is almost identical. However, the pension funds' assets are lower for countries

Table 4.7: Descriptive statistics for pension system indicators; entirely defined contributions or not; all years

	DC	**Other**
Replacement rate (gross)		
Mean	59.55	59.36
0.5 Percentile	58.7	55.30
Standard deviation	11.19	18.92
Replacement rate (net)		
Mean	69.56	70.71
0.5 Percentile	69.30	66.60
Standard deviation	11.42	19.84
Pension funds' assets in % of GDP		
Mean	26.30	36.22
0.5 Percentile	9.25	8.40
Standard deviation	31.44	44.09
Vector similarity		
Mean	0.9367	0.8966
0.5 Percentile	0.9909	0.9905
Standard deviation	0.1138	0.1377
N	36	114

Source: Own calculation based on OECD (2005, 2007, 2009, 2011, 2013, 2014, 2015, 2016c).

that offer basic and/or minimum pension than for those that do not. This holds true for both the mean and the 0.5 percentile. Again, this might be the reason for the vector similarity being higher for the countries that do offer basic and/or minimum pensions. Hence, the fact that a country offers these types of pensions could also affect the vector similarity to the German pension system.

To estimate the effect of the institutional features of pension systems, the following, extended regression equation is estimated.

$$\begin{aligned} VS_{it} = \alpha &+ \beta_1 \cdot OA^G_{it} + \beta_2 \cdot LR^G_{it} + \beta_3 \cdot ERadj^G_{it} + \beta_4 \cdot FR^G_{it} \\ &+ \beta_5 \cdot ln(gdp^G)_{it} + \beta_6 \cdot Emp65^G_{it} + \beta_7 \cdot GDebt^G_{it} \\ &+ \beta_8 \cdot Public_{it} + \beta_9 \cdot DC_{it} + \beta_{10} \cdot MB_{it} \\ &+ \beta_{11} \cdot PublicDC_{it} + \beta_{12} \cdot DCMB_{it} + \beta_{13} \cdot PublicMB_{it} \\ &+ \beta_{14} \cdot PublicDCMB_{it} + \varepsilon_{it} \end{aligned} \tag{4.4}$$

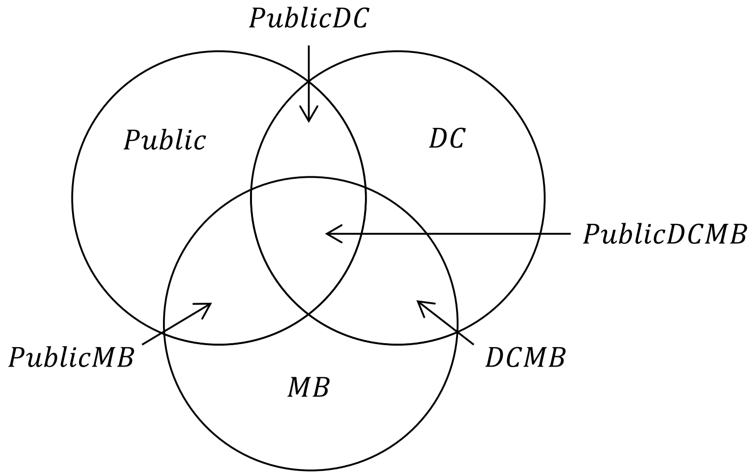
Table 4.8: Descriptive statistics for pension system indicators; minimum and/or basic pension or none; all years

	MB	**Other**
Replacement rate (gross)		
Mean	58.83	61.60
0.5 Percentile	56.00	64.80
Standard deviation	17.40	14.22
Replacement rate (net)		
Mean	69.87	73.74
0.5 Percentile	66.40	75.30
Standard deviation	17.93	18.89
Pension funds' assets in % of GDP		
Mean	32.03	40.77
0.5 Percentile	8.60	9.60
Standard deviation	41.71	40.73
Vector similarity		
Mean	0.9158	0.8694
0.5 Percentile	0.9905	0.9873
Standard deviation	0.1284	0.1470
N	119	31

Source: Own calculation based on OECD (2005, 2007, 2009, 2011, 2013, 2014, 2015, 2016c).

The difference between equation (4.4) and equation (4.2) is that in equation (4.4), several dummy-variables are included. The variable *Public* is a dummy variable, with 1 indicating that the country has a mandatory public pension system and 0 otherwise. The variable *DC* is also a dummy variable, with 1 indicating that the country has defined contribution structure, 0 otherwise. The variable MB is a dummy variable, with 1 indicating that the country offers basic and/or minimum pensions, 0 otherwise. But including these three dummies is not enough. Interaction terms need to be constructed to capture the effects of, for example, a publicly organized pension system that has a defined contributions structure and also offers basic pensions. Therefore, the variables *PublicDC*, $DCMB$, *PublicMB*, *PublicDCMB* are the consequent interaction terms and hence, also dummy variables. Figure 4.6 shows the Venn diagram for the institutional dummy-variables. Ignoring the interaction terms would overestimate the effects of *Public*, *DC* and MB, and may distort the estimation of coefficients since, for example, a pension system, which is publicly organized, has a defined contributions structure and offers a basic pension

Figure 4.6: Venn diagram of institutional dummy-variables



Notes: Own depiction.

would be ascribed to all three dummies simultaneously. Including the interaction terms enables the regression to ascribe this country to $PublicDCMB$.

However, since Germany has a mandatory public pension system, the variable $Public$ is expected to have a positive effect on the similarity. The same holds true for the variable DC. Börsch-Supan et al. (2007) concluded that after several reform processes, the German mandatory pension system moved silently from a DB system to a DC or NDC system. This may explain why DC systems seem to have a higher vector similarity (see again Table 4.7). With regard to Table 4.8 MB may also have a positive effect on vector similarity. However, the inclusion of the interaction terms may lead to different results than expected. Due to the abovementioned endogeneity problem, $ERadj$ is used instead of the effective retirement age. The estimation results of equation (4.4) are presented in Table 4.9.

There is almost no evidence that the additional institutional features have an impact on the vector similarity. Except for the variable $DCMB$. Hence, pension systems that are DC systems and also offer either basic or minimum pensions or both increase the similarity to the German pension system by 0.204 (0.190) in the pooled (fixed effects) estimation. This is statistically significant at the 1% significance level. All other explanatory variables have the same sign as in the previous estimation.

Table 4.9: Regression results for equation (4.4)

	Pooled 2	Time Fixed Effects 2
Public	0.00999	0.0107
	(0.0494)	(0.0499)
DC	-0.0239	-0.0161
	(0.0599)	(0.0604)
MB	-0.0424	-0.0386
	(0.0482)	(0.0487)
PublicDC	0.0283	0.0111
	(0.0795)	(0.0809)
DCMB	0.204***	0.190***
	(0.0655)	(0.0665)
PublicMB	0.0873	0.0831
	(0.0549)	(0.0554)
PublicDCMB	-0.169	-0.158
	(0.107)	(0.109)
Old-age dependency ratio	0.252***	0.299***
	(0.0785)	(0.0841)
Legal retirement age	-1.044***	-0.946**
	(0.306)	(0.367)
Effective retirement age (adjusted)	-1.362***	-1.610***
	(0.303)	(0.343)
Fertility rate	-0.321***	-0.349***
	(0.0490)	(0.0523)
ln(gdp)	-0.000619	0.00105
	(0.0302)	(0.0305)
Employment by population aged 65+	-0.00038	0.00124
	(0.0038)	(0.0040)
Public debt in % of GDP	-0.0123	-0.0200
	(0.0209)	(0.0217)
Constant	2.090***	1.993***
	(0.295)	(0.356)
F-stat	14.07	14.07
Adjusted (within) R2	0.5512	0.6025
N	150	150

Notes: Results from estimation of equation (4.4). Dependent variable: Vector Similarity (VS). Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Moreover, the old-age dependency ratio, the (adjusted) effective retirement age, the legal retirement age, as well as the fertility rate remain significant at the 1% significance level. Again, the economic input variables ($ln(gdp)$ and $GDebt$) do not have a statistically significant effect on vector similarity. Also, the coefficients of the variables do not change significantly when including the dummies. Except for the old-age dependency ratio. The effect of this variable is now 0.252 in the Pooled model and 0.299 in the Fixed-Effects model. Furthermore, since time fixed effects are not significant, the results are not driven by time specific effects, but apply over the entire investigated time span. Hence, the similarity between the pension system of a country to the German pension system is driven mainly by demographic inputs rather than economic inputs. In addition, since the (adjusted) effective retirement age has the strongest impact on vector similarity to Germany, the time of labor market exit seems to be a substantial determinant of such similarity. Finally, most of the institutional dummies do not significantly affect the vector similarity.

4.6 Conclusions

This paper investigates the similarity of pension systems in OECD countries in comparison to the German pension system by using the concept of vector similarity. Thus, pension systems are not compared looking at every single aspect of the system, but by calculating one value for each pension system. To do that, a vector including three pension system indicators, representing the pension system, is built for each country. Those indicators are the gross pension replacement rate and the net pension replacement rate (to capture effects of income taxation and social security contributions of pensioners), and the pension funds' assets as a percentage of GDP (to capture effects of a different funding structure and possible voluntary old-age provision).

Subsequently, vector similarity is calculated in order to asses the similarity to the German pension system as a reference country. Several countries, such as the United States, the United Kingdom, Switzerland, the Netherlands, Ireland, Iceland, Finland, Canada and Australia are quite different from the German pension system, while others (Slovak Republic, Italy, Greece, France, the Czech Republic and Belgium) are very similar. These differences may be due to institutional features such as the funding structure. However, section 4.4 shows evidence that these differences cannot be explained entirely by institutional factors. Thus, since demographic and economic factors affect different parts of pension systems, it may be assumed that these also impact on their similarity.

Therefore, these differences or similarities, respectively, are explained in a panel regression model using demographic and economic input variables as explanatory ones. It is evident that the results are not driven by time fixed effects. Moreover, it turns out that the effective retirement age, the legal retirement age and the fertility rate, have a significantly negative effect on the similarity of the pension systems to the German pension system. Hence, the higher these variables or inputs, the lower the vector similarity to the German pension system. On the other hand, the old-age dependency ratio has a significantly positive effect on the similarity. Therefore, a higher old-age dependency ratio goes along with a higher similarity to the German pension system. Economic variables, such as the GDP per capita, public gross debt as a percentage of GDP and the employment rate of people aged 65 and above (due to the fact that it is largely incorporated in the adjusted effective retirement age after applying the auxiliary regression) do not have a statistically significant effect on vector similarity. Furthermore, since institutional features of a pension system, such as the fact if it is publicly or privately organized, a DB or a DC system or offers a basic and/or minimum pension to the pensioners, seem to be important (see Tables 4.6 - 4.8), dummy variables are included to capture the effect of institutional features on the similarity to the German pension system. It turns out, that aside from the legal retirement age, only the fact that a pension is a DC system and as well offers either a basic or a minimum pension or both has an effect on the similarity to the German pension system. Hence, the similarity of country $i's$ pension system to the German pension system is driven mainly by demographic variables and institutional features.

References

Allianz Global Investors (2014), Pension Sustainability Index 2014, International Pension Papers, No. 1/2014.

Bakshi, G. S., Z. Chen (1994), Baby Boom, Population Aging, and Capital Markets, Journal of Business, 67 (2), 165-202.

Bollacke, S. (2016), Vector Similarity as a Measure for Comparing Pension Systems – An Application to OECD Countries, Available at SSRN: https://ssrn.com/abstract=2793701 or http://dx.doi.org/10.2139/ssrn.2793701.

Börsch-Supan, A. H., J. K. Winter (2001), Population Aging, Savings Behavior and Capital Markets, NBER Working Paper Series, 8561.

Börsch-Supan, A. H., A. Reil-Held, C. B. Wilke (2007), How an Unfunded Pension System looks like Defined Benefits but works like Defined Contributions: The German Pension Reform, Espina, A., Estado de Bienestar y competitividad - La experiencia europea, Fundación Carolina, Madrid, 319-347.

Busch, P. (1998), Orthogonality and Disjointness in Spaces of Measures, Letters in Mathematical Physics, 44, 215–224.

Cigno, A. (2006), A constitutional theory of the family, Journal of Population Economics, 19, 259-283.

Cipriani, G. H. (2014), Population aging and PAYG pensions in the OLG model, Journal of Population Economics, 27, 251-256.

Cremer, H., F. Gahvari, P. Pestieau (2006), Pensions with endogenous and stochastic fertility, Journal of Public Economics, 90, 2303-2321.

Dobrescu, E. (2011), Sectoral Structure and Economic Growth, Romanian Journal of Economic Forecasting, 3/2011, 5-36.

International Monetary Fund (2014), World Economic Outlook Database, April 2014: https://www.imf.org/external/pubs/ft/weo/2014/01/weodata/index.aspx [accessed March 10, 2016].

Jones, W. P., G. W. Furnas (1987), Pictures of Relevance: A Geometric Analysis of Similarity Measures, Journal of the American Society for Information Science, 38(6), 420-442.

Masson, P. R., T. Bayoumi, H. Samiei (1998), International Evidence on the Determinants of Private Savings, The World Bank Economic Review, 12 (3), 483-501.

Mercer (2011), Melbourne Mercer Global Pension Index, Australian Centre for Financial Studies, Melbourne.

Mercer (2012), Melbourne Mercer Global Pension Index, Australian Centre for Financial Studies, Melbourne.

Mercer (2013), Melbourne Mercer Global Pension Index, Australian Centre for Financial Studies, Melbourne.

Mercer (2014), Melbourne Mercer Global Pension Index, Australian Centre for Financial Studies, Melbourne.

Mercer (2015), Melbourne Mercer Global Pension Index, Australian Centre for Financial Studies, Melbourne.

Noseleit, F. (2013), Entrepreneurship, structural change, and economic growth, Journal of Evolutionary Economics, 23, 735-766.

OECD (2005), Pensions at a Glance 2005: Public Policies Across OECD Countries, OECD Publishing.

OECD (2007), Pensions at a Glance 2007: Public Policies Across OECD Countries, OECD Publishing.

OECD (2009), Pensions at a Glance 2009: Retirement-Income Systems in OECD Countries, OECD Publishing.

OECD (2011), Pensions at a Glance 2011: Retirement-Income Systems in OECD Countries and G20 Countries, OECD Publishing.

OECD (2013), Pensions at a Glance 2013: OECD and G20 Indicators, OECD Publishing.

OECD (2014), OECD Factbook 2014: Economic, Environmental and Social Statistics, OECD Publishing.

OECD (2015), Pensions at a Glance 2015: OECD and G20 Indicators, OECD Publishing.

OECD (2016a), OECD Ageing and Employment Policies - Statistics on average effective age of retirement: https://www.oecd.org/els/emp/average-effective-age-of-retirement.htm [accessed March 7, 2016].

OECD (2016b), Dataset: Population: https://stats.oecd.org/Index.aspx?DatasetCode=POP_FIVE_HIST [accessed March 7, 2016].

OECD (2016c), Dataset: Funded Pensions Indicators: https://stats.oecd.org/Index.aspx?DataSetCode=PNNI_NEW [accessed March 8, 2016].

OECD (2016d), Dataset: LFS by sex and age - indicators: https://stats.oecd.org/Index.aspx?DataSetCode=LFS_SEXAGE_I_R [accessed March 8, 2016].

OECD (2016e), Dataset: Level of GDP per capita and productivity: https://stats.oecd.org/index.aspx?DataSetCode=PDB_LV [accessed March 29, 2016].

Van Eck, N., L. Waltman (2008), Appropriate Similarity Measures for Author Cocitation Analysis, Journal of the American Society for Information Science and Technology, 59(10), 1653-1661.

Chapter 5

Conclusion

An aging population, and especially the upcoming retirement of the baby-boom generation, is challenging the funding of the German mandatory pension system. After all, an increasing number of pensioners has to be financed by a decreasing number of contributors. Thus, solutions are needed to support the pension system's solvency and sustainability, in order to perpetuate its pay-as-you-go structure. Accordingly, this thesis provides solution approaches to lower the financing gap of the German mandatory pension system and gives new insights into the future performance of the German Riester scheme. Moreover, the German pension system's similarity to other countries' pension systems is investigated, in order to compare the various systems and to detect determinants of similarity, in order to improve their operations and viability.

However, the first objective of this thesis is to find ways to decrease the financing gap of the mandatory pension system, in order to support its solvency and sustainability. Accordingly, Chapter 2 investigates whether a more flexible calculation of the benefits paid by the mandatory pension system could help lower the financing gap. This is done by simulating the pension expenditures and the contribution income of the mandatory pension system, under different assumptions regarding the average gross income of employees, population growth, and the development of the contribution rate. Subsequently, the demographic factor, which is currently set to 0.25, is made variable, so as to find its optimal path in order to minimize the financing gap. Whereas a constant demographic factor of 0.25 burdens retirees with the effect of an aging population by 25%, a variable demographic factor is able to react to demographic and economic influences and thus spread the burden of demographic change on either contributors or retirees. Since the aim of this Chapter is to show ways to minimize the financing gap (and hence federal subsidies) of the mandatory pension system, the demographic factor itself conforms to demographic and economic changes, so as to keep federal subsidies as low as possible. It turns out that a variable demographic factor (in contrast to a constant one of 0.25) leads to a lower financing gap, and hence to lower federal subsidies, to the disadvantage of both retirees and contributors. Although the contribution rate is higher and the pension level slightly lower in the case of a variable demographic factor compared to a constant one in each scenario, the regulations of Article 154 SGB VI regarding the maximum allowed contribution rate and the minimum pension level are satisfied with certainty, in most of the three scenarios in each simulation. However, the simulations in Chapter 2 show that a variable demographic factor, which is able to react to demographic and economic changes, may help to lower federal subsidies of the German mandatory pension system (and hence shift the financial burden away from taxpayers), in contrast to a constant demographic factor of 0.25. Hence, a variable demographic factor and thus a more flexible calculation of benefits may foster the solvency and sustainability of the German mandatory pension system.

Moreover, to show that the German Riester scheme not only costs money by granting federal subsidies to people who save privately for retirement, but also generates fiscal revenue due to the deferred taxation of pension benefits, Chapter 3 introduces the so-called *tax-pay-as-you-go* (*TaxGo*) model. In order to find out whether the Riester scheme is able to finance itself on both a yearly and cumulated basis, the development of governmental expenditures on the Riester scheme and the additional fiscal revenue obtained from the deferred taxation of (Riester) pension benefits are simulated and placed in relation to each other. As a consequence of the German Retirement Savings Act (AVmG) of 2002, federal subsidies were granted to people who save privately for retirement by means of a Riester contract. In contrast, as a result of the German Retirement Income Act (AltEinkG) of 2005, deferred taxation was introduced for retirement income, including that obtained from the mandatory pension system, as well as from Riester contracts. While the focus of research is still mostly on the amount of federal subsidies that has been, and will be granted to people who save privately for retirement, Chapter 3 investigates the future additional fiscal revenue obtained from deferred taxation of both mandatory pension benefits and Riester scheme benefits. The aim is to find out whether the additional fiscal revenue might balance the Riester subsidies until 2050. Since pension payments from the Riester scheme increase retirement income, which consequently increases the individual average income tax rate, tax payments of those retirees who saved privately for retirement obviously increase. Hence, fiscal revenue obtained from the taxation of people who receive pension benefits only from the mandatory pension system is simulated and subtracted from the fiscal revenue obtained from taxing the same number of people if they had been receiving pension benefits from both the mandatory pension system and the Riester scheme. This additional fiscal revenue is then put in relation to subsidies granted to Riester savers, with the aim of calculating, how much of the subsidies can be funded by additional fiscal revenue until 2050. It turns out that in the 2040s, the Riester scheme will be able to finance itself on a yearly basis, meaning that the additional fiscal revenue exceeds the federal subsidies of Riester savers. Hence, it is shown that the Riester scheme not only costs money by means of federal subsidies, but also generates tax revenue, resulting in lower governmental net expenditures.

The second objective is to introduce a new measure for comparing pension systems of different countries, in order to detect determinants of similarity between pension systems. Thus, Chapter 4 investigates the similarity between the German pension system and other OECD country pension systems. Although there are various comparisons of pension systems already, their evaluations are either based on composite indices consisting of a large number of features of pension systems, or even on sub-indices which are ultimately combined into one index. Consequently, replicating results is almost impossible. To provide an easier and especially a replicable method for comparing pension systems, the analysis in Chapter 4 is based on the concept of vector similarity, which determines the similarity of two vectors by calculating the cosine of the angle between them. In doing so, a vector is built for each country's pension system, consisting of three determinants that characterize a pension system. However, after calculating vector similarity between the German pension system and each pension system of the other OECD countries, vector similarity is used as the dependent variable in a regression analysis, which estimates the effect of particular demographic, economic and institutional factors on the similarity of several OECD pension systems to the German one. It turns out that similarities between pension systems are driven mainly by demographic and institutional factors in contrast to economic ones. Hence, this analysis provides new insights into the dependency of the similarity between country pension systems on several demographic and institutional factors. This contributes to our understanding of pension systems and its workings and helps to support or even improve any pension system, since determinants such as the old-age dependency ratio, retirement age and fertility rate, have been proven to exert a significant impact on pension system similarity.

To conclude, this thesis provides new insights into the future performance of the German pension system, under consideration of population aging and several pension reforms which impact on both retirees and contributors. It turns out that the additional financial burden on the German mandatory pension system, which arises mainly from the baby-boom generation entering retirement, can be placed on both retirees and contributors, as well as taxpayers, by making the pension system more flexible in order to react appropriately to demographic and economic change. Moreover, the Riester scheme does not only *require* money in terms of federal subsidies. It also generates tax income since Riester pension benefits are subject to income taxation, leading to lower governmental net expenditures. Additionally, using the concept of vector similarity provides a new tool for comparing and assessing the similarity of pension systems, in order to detect determinants of such similarity. This may reveal new opportunities for supporting and improving the pension system of any country.