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On transverse triangulations

Aleksey Zinger

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Abstract. We show that every smooth manifold admits a smooth triangulation transverse to a given smooth map. This removes the properness assumption on the smooth map used in an essential way in Scharlemann's construction [6].

1. Introduction

For $l \in \mathbb{Z}^{\geq 0}$, let $\Delta^l \subset \mathbb{R}^l$ denote the standard l-simplex. If $|K| \subset \mathbb{R}^N$ is a geometric realization of a simplicial complex K in the sense of [5, Sec. 3], for each l-simplex σ of K there is an injective linear map[1] $\iota_\sigma : \Delta^l \longrightarrow |K|$ taking Δ^l to $|\sigma|$. If X is a smooth manifold, a topological embedding $\mu : \Delta^l \longrightarrow X$ is a *smooth embedding* if there exist an open neighborhood Δ^l_μ of Δ^l in $\mathbb{R}^l$ and a smooth embedding $\tilde{\mu} : \Delta^l_\mu \longrightarrow X$ so that $\tilde{\mu}|_{\Delta^l} = \mu$. A *triangulation* of a smooth manifold X is a pair $T = (K, \eta)$ consisting of a simplicial complex and a homeomorphism $\eta : |K| \longrightarrow X$ such that

$$\eta \circ \iota_\sigma : \Delta^l \longrightarrow X$$

is a smooth embedding for every l-simplex σ in K and $l \in \mathbb{Z}^{\geq 0}$. If $T = (K, \eta)$ is a triangulation of X and $\psi : X \longrightarrow X$ is a diffeomorphism, then $\psi_* T = (K, \psi \circ \eta)$ is also a triangulation of X.

Theorem 1.1. *If X, Y are smooth manifolds and $h : Y \longrightarrow X$ is a smooth map, there exists a triangulation (K, η) of X such that h is transverse to $\eta|_{\mathrm{Int}\,\sigma}$ for every simplex $\sigma \in K$.*

This theorem is stated in [8] as Lemma 2.3 and described as an obvious fact. As pointed out to the author by Matthias Kreck, Scharlemann [6] proves Theorem 1.1 under the assumption that the smooth map h is proper, and his argument makes use of this assumption in an essential way. For the purposes of [8], a transverse C^1-triangulation would suffice, and the existence of a such

[1] i.e. ι_σ takes the vertices of Δ^l to the vertices of $|\sigma|$ and is linear between them, as in [8, Footnote 5]

triangulation is fairly evident from the point of view of the Sard-Smale Theorem [7, (1.3)]. On the other hand, according to Matthias Kreck, the existence of smooth transverse triangulations without the properness assumption is related to subtle issues arising from the topology of stratifolds [2]. In this note we give a detailed proof of Theorem 1.1 as stated above, using Sard's theorem [3, Section 2].

2. Outline of the proof of Theorem 1.1

If K is a simplicial complex, we denote by $\mathrm{sd}\, K$ the barycentric subdivision of K. For any nonnegative integer l, let K_l be *the l-th skeleton of K*, i.e. the subcomplex of K consisting of the simplices in K of dimension at most l. If σ is a simplex in a simplicial complex K with geometric realization $|K|$, let

$$\mathrm{St}(\sigma, K) = \bigcup_{\sigma\subset\sigma'} \mathrm{Int}\,\sigma'$$

be the *star of σ in K*, as in [5, Sec. 62], and $b_\sigma \in \mathrm{sd}\, K$ the barycenter of σ. The main step in the proof of Theorem 1.1 is the following observation.

Proposition 2.1. *Let $h : Y \longrightarrow X$ be a smooth map between smooth manifolds. If (K, η) is a triangulation of X and σ is an l-simplex in K, there exists a diffeomorphism $\psi_\sigma : X \longrightarrow X$ restricting to the identity outside of $\eta(\mathrm{St}(b_\sigma, \mathrm{sd}\, K))$ so that $\psi_\sigma \circ \eta|_{\mathrm{Int}\,\sigma}$ is transverse to h.*

If σ and σ' are two distinct simplices in K of the same dimension l,

$$\mathrm{St}(b_\sigma, \mathrm{sd}\, K) \cap \mathrm{St}(b_{\sigma'}, \mathrm{sd}\, K) = \varnothing. \tag{1}$$

Since ψ_σ is the identity outside of $\eta(\mathrm{St}(b_\sigma, \mathrm{sd}\, K))$ and the collection $\{\mathrm{St}(b_\sigma, \mathrm{sd}\, K)\}$ is locally finite, the composition $\psi_l : X \longrightarrow X$ of all diffeomorphisms $\psi_\sigma : X \longrightarrow X$ taken over all l-simplices σ in K is a well-defined diffeomorphism[2] of X. Since $\psi_l \circ \eta|_{|\sigma|} = \psi_\sigma \circ \eta|_{|\sigma|}$ for every l-simplex σ in K, we obtain the following conclusion from Proposition 2.1.

Corollary 2.2. *Let $h : Y \longrightarrow X$ be a smooth map between smooth manifolds. If (K, η) is a triangulation of X, for every $l = 0, 1, \ldots, \dim X$, there exists a diffeomorphism $\psi_l : X \longrightarrow X$ restricting to the identity on $\eta(|K_{l-1}|)$ so that $\psi_l \circ \eta|_{\mathrm{Int}\,\sigma}$ is transverse to h for every l-simplex σ in K.*

This corollary implies Theorem 1.1. By [4, Chap. II], X admits a triangulation (K, η_{-1}). By induction and Corollary 2.2, for each $l = 0, 1, \ldots, \dim X - 1$ there exists a triangulation $(K, \eta_l) = (K, \psi_l \circ \eta_{l-1})$ of X which is transverse to h on every open simplex in K of dimension at most l.

[2] The locally finite property implies that the composition of these diffeomorphisms in any order is a diffeomorphism; by (1), these diffeomorphisms commute and so the composition is independent of the order.

3. Proof of Proposition 2.1

Lemma 3.1. *For every $l \in \mathbb{Z}^+$, there exists a smooth function $\rho_l : \mathbb{R}^l \longrightarrow \bar{\mathbb{R}}^+$ such that*

$$\rho_l^{-1}(\mathbb{R}^+) = \mathrm{Int}\,\Delta^l.$$

Proof. Let $\rho : \mathbb{R} \longrightarrow \mathbb{R}$ be the smooth function given by

$$\rho(r) = \begin{cases} \mathrm{e}^{-1/r}, & \text{if } r > 0, \\ 0, & \text{if } r \le 0. \end{cases}$$

The smooth function $\rho_l : \mathbb{R}^l \longrightarrow \mathbb{R}$ given by

$$\rho_l(t_1, \dots, t_l) = \rho\Big(1 - \sum_{i=1}^{i=l} t_i\Big) \cdot \prod_{i=1}^{i=l} \rho(t_i)$$

then has the desired property. □

Lemma 3.2. *Let (K, η) be a triangulation of a smooth manifold X and σ an l-simplex in K. If*

$$\tilde{\mu}_\sigma : \Delta^l_\sigma \times \mathbb{R}^{m-l} \longrightarrow U_\sigma \subset X$$

is a diffeomorphism onto an open neighborhood U_σ of $\eta(|\sigma|)$ in X such that $\tilde{\mu}_\sigma(t, 0) = \eta(\iota_\sigma(t))$ for all $t \in \Delta_\sigma$, there exists $c_\sigma \in \mathbb{R}^+$ such that

$$\big\{(t, v) \in (\mathrm{Int}\,\Delta^l) \times \mathbb{R}^{m-l} \mid |v| \le c_\sigma \rho_l(t)\big\} \subset \tilde{\mu}_\sigma^{-1}\big(\eta(\mathrm{St}(b_\sigma, \mathrm{sd}\,K))\big).$$

Proof. It is sufficient to show[3] that there exists $c_\sigma > 0$ such that

$$\big\{(t, v) \in (\mathrm{Int}\,\Delta^l) \times \mathbb{R}^{m-l} \mid |v| \le c_\sigma \rho_l(t)\big\} \subset \tilde{\mu}_\sigma^{-1}\big(\eta(\mathrm{St}(\sigma, K))\big).$$

We assume that $0 < l < m$. Suppose $(t_p, v_p) \in (\mathrm{Int}\,\Delta^l) \times (\mathbb{R}^{m-l} - 0)$ is a sequence such that

$$(t_p, v_p) \notin \tilde{\mu}_\sigma^{-1}\big(\eta(\mathrm{St}(\sigma, K))\big), \qquad |v_p| \le \frac{1}{p}\rho_l(t_p). \tag{2}$$

Since $\eta(\mathrm{St}(\sigma, K))$ is an open neighborhood of $\eta(\mathrm{Int}\,\sigma)$ in X, by shrinking v_p and passing to a subsequence we can assume that

$$(t_p, v_p) \in \tilde{\mu}_\sigma^{-1}\big(\eta(|\tau'|)\big) \subset \tilde{\mu}_\sigma^{-1}\big(\eta(|\tau|)\big) \tag{3}$$

for an m-simplex τ in K and a face τ' of τ so that $\sigma \not\subset \tau'$, $\tau' \not\subset \sigma$, and $\sigma \subset \tau$. Let $\iota_\tau : \Delta^m \longrightarrow |K|$ be an injective linear map taking Δ^m to $|\tau|$ so that

$$\begin{aligned} \iota_\tau^{-1}(|\sigma|) &= \Delta^m \cap \mathbb{R}^l \times 0 \subset \mathbb{R}^l \times \mathbb{R}^{m-l}, \\ \iota_\tau^{-1}(|\tau'|) &= \Delta^m \cap 0 \times \mathbb{R}^{m-1} \subset \mathbb{R}^1 \times \mathbb{R}^{m-1}. \end{aligned} \tag{4}$$

Choose a smooth embedding $\mu_\tau : \Delta^m_\tau \longrightarrow X$ from an open neighborhood of Δ^m in $\mathbb{R}^m$ such that $\mu_\tau|_{\Delta^m} = \eta \circ \iota_\tau$. Let ϕ be the first component of the diffeomorphism

$$\mu_\tau^{-1} \circ \tilde{\mu}_\sigma : \tilde{\mu}_\sigma^{-1}\big(\mu_\tau(\Delta^m_\tau)\big) \longrightarrow \mu_\tau^{-1}\big(\tilde{\mu}_\sigma(\Delta^l_\sigma \times \mathbb{R}^{m-l})\big) \subset \mathbb{R}^1 \times \mathbb{R}^{m-1}.$$

[3] If K' is the subdivision of K obtained by adding the vertices $b_{\sigma'}$ with $\sigma' \supsetneq \sigma$, then $\mathrm{St}(b_\sigma, \mathrm{sd}\,K) = \mathrm{St}(\sigma, K')$.

By (3), the second assumption in (4), the continuity of $\mathrm{d}\phi$, and the compactness of Δ^l,

$$\big|\phi(t_p,0)\big| = \big|\phi(t_p,0)-\phi(t_p,v_p)\big| \le C|v_p| \qquad \forall\ p, \tag{5}$$

for some $C>0$. On the other hand, by the first assumption in (4), the vanishing of ρ_l on $\mathrm{Bd}\,\Delta^l$, the continuity of $\mathrm{d}\rho_l$, and the compactness of Δ^l,

$$\big|\rho_l(t_p)\big| \le C\big|\phi(t_p,0)\big| \qquad \forall\ p, \tag{6}$$

for some $C>0$. The second assumption in (2), (5), and (6) give a contradiction for $p>C^2$. □

Lemma 3.3. *Let $h: Y \longrightarrow X$ be a smooth map between smooth manifolds, (K,η) a triangulation of X, σ an l-simplex in K, and*

$$\tilde{\mu}_\sigma : \Delta^l_\sigma \times \mathbb{R}^{m-l} \longrightarrow U_\sigma \subset X$$

a diffeomorphism onto an open neighborhood U_σ of $\eta(|\sigma|)$ in X such that $\tilde{\mu}_\sigma(t,0)=\eta(\iota_\sigma(t))$ for all $t\in\Delta_\sigma$. For every $\epsilon>0$, there exists $s_\sigma \in C^\infty(\mathrm{Int}\,\Delta^l;\mathbb{R}^{m-l})$ so that the map

$$\tilde{\mu}_\sigma \circ (\mathrm{id}, s_\sigma) : \mathrm{Int}\,\Delta^l \longrightarrow X \tag{7}$$

is transverse to h,

$$\begin{aligned} \big|s_\sigma(t)\big| < \epsilon^2\rho_l(t) \quad &\forall\ t\in \mathrm{Int}\,\Delta^l,\\ \lim_{t\longrightarrow \mathrm{Bd}\,\Delta^l} \rho_l(t)^{-i}\big|\nabla^j s_\sigma(t)\big| = 0 \quad &\forall\ i,j\in\mathbb{Z}^{\ge 0}, \end{aligned} \tag{8}$$

where $\nabla^j s_\sigma$ is the multilinear functional determined by the j-th derivatives of s_σ.

Proof. The smooth map

$$\phi: \mathrm{Int}\,\Delta^l \times \mathbb{R}^{m-l} \longrightarrow X, \qquad \phi(t,v) = \tilde{\mu}_\sigma\big(t, \mathrm{e}^{-1/\rho_l(t)}v\big),$$

is a diffeomorphism onto an open neighborhood U'_σ of $\eta(\mathrm{Int}\,\sigma)$ in X. The smooth map (7) with $s_\sigma = \mathrm{e}^{-1/\rho_l(t)}v$ is transverse to h if and only if $v\in\mathbb{R}^{m-l}$ is a regular value of the smooth map

$$\pi_2\circ\phi^{-1}\circ h : h^{-1}(U'_\sigma) \longrightarrow \mathbb{R}^{m-l},$$

where $\pi_2 : \mathrm{Int}\,\Delta^l\times\mathbb{R}^{m-l} \longrightarrow \mathbb{R}^{m-l}$ is the projection onto the second component. By Sard's Theorem, the set of such regular values is dense in $\mathbb{R}^{m-l}$. Thus, the map (7) with $s_\sigma = \mathrm{e}^{-1/\rho_l(t)}v$ is transverse to h for some $v\in\mathbb{R}^{m-l}$ with $|v|<\epsilon^2$. The second statement in (8) follows from $\rho_l|_{\mathrm{Bd}\,\Delta^l}=0$. □

Corollary 3.4. *Let $h: Y\longrightarrow X$ be a smooth map between smooth manifolds, (K,η) a triangulation of X, σ an l-simplex in K, and*

$$\tilde{\mu}_\sigma : \Delta^l_\sigma \times \mathbb{R}^{m-l} \longrightarrow U_\sigma \subset X$$

a diffeomorphism onto an open neighborhood U_σ of $\eta(|\sigma|)$ in X such that $\tilde{\mu}_\sigma(t,0)=\eta(\iota_\sigma(t))$ for all $t\in\Delta_\sigma$. For every $\epsilon>0$, there exists a diffeomorphism ψ'_σ of $\Delta^l_\sigma\times\mathbb{R}^{m-l}$ restricting to the identity outside of

$$\big\{(t,v)\in(\operatorname{Int}\Delta^l)\times\mathbb{R}^{m-l}:\ |v|\le\epsilon\rho_l(t)\big\}$$

so that the map $\tilde{\mu}_\sigma\circ\psi'_\sigma|_{\operatorname{Int}\Delta^l\times 0}$ is transverse to h.

Proof. Choose $\beta\in C^\infty(\mathbb{R};[0,1])$ so that

$$\beta(r)=\begin{cases}1, & \text{if } r\le\frac{1}{2},\\ 0, & \text{if } r\ge 1.\end{cases}$$

Let $C_\beta=\sup_{r\in\mathbb{R}}|\beta'(r)|$. With s_σ as provided by Lemma 3.3, define

$$\psi'_\sigma:\Delta^l_\sigma\times\mathbb{R}^{m-l}\longrightarrow\Delta^l_\sigma\times\mathbb{R}^{m-l}\qquad\text{by}$$

$$\psi'_\sigma(t,v)=\begin{cases}\Big(t,v+\beta\Big(\frac{|v|}{\epsilon\rho_l(t)}\Big)s_\sigma(t)\Big), & \text{if } t\in\operatorname{Int}\Delta^l,\\ (t,v), & \text{if } t\notin\operatorname{Int}\Delta^l.\end{cases}$$

The restriction of this map to $(\operatorname{Int}\Delta^l)\times\mathbb{R}^{m-l}$ is smooth and its Jacobian is

$$\mathcal{J}\psi'_\sigma\big|_{(t,v)}=\begin{pmatrix}\mathbb{I}_l & 0\\ (\mathcal{J}\psi'_\sigma\big|_{(t,v)})_{2,1} & \mathbb{I}_{m-l}+\beta'\Big(\frac{|v|}{\epsilon\rho_l(t)}\Big)\frac{s_\sigma(t)}{\epsilon\rho_l(t)}\frac{v^{tr}}{|v|}\end{pmatrix},\tag{9}$$

where

$$(\mathcal{J}\psi'_\sigma\big|_{(t,v)})_{2,1}=\beta\left(\frac{|v|}{\epsilon\rho_l(t)}\right)\nabla s_\sigma(t)-\beta'\left(\frac{|v|}{\epsilon\rho_l(t)}\right)\frac{|v|}{\epsilon\rho_l(t)}\frac{s_\sigma(t)}{\rho_l(t)}\nabla\rho_l.$$

By the first property in (8), this matrix is non-singular if $\epsilon<1/C_\beta$. If W is any linear subspace of $\mathbb{R}^{m-l}$ containing $s_\sigma(t)$,

$$\psi'_\sigma(t\times W)\subset t\times W,\qquad \psi'_\sigma(t,v)=(t,v)\quad\forall\ v\in W\text{ such that }|v|\ge\epsilon\rho_l(t).$$

Thus, ψ'_σ is a bijection on $t\times W$, a diffeomorphism on $(\operatorname{Int}\Delta^l)\times\mathbb{R}^{m-l}$, and a bijection on $\Delta^l_\sigma\times\mathbb{R}^{m-l}$.

Since $\beta(r)=0$ for $r\ge 1$, $\psi'_\sigma(t,v)=(t,v)$ unless $t\in\operatorname{Int}\Delta^l$ and $|v|<\epsilon\rho_l(t)$. It remains to show that ψ'_σ is smooth along

$$\overline{\big\{(t,v)\in(\operatorname{Int}\Delta^l)\times\mathbb{R}^{m-l}:\ |v|\le\epsilon\rho_l(t)\big\}}-(\operatorname{Int}\Delta^l)\times\mathbb{R}^{m-l}=(\operatorname{Bd}\Delta^l)\times 0.$$

Since $|s_\sigma(t)|\longrightarrow 0$ as $t\longrightarrow\operatorname{Bd}\Delta^l$ by the first property in (8), ψ'_σ is continuous at all $(t,0)\in(\operatorname{Bd}\Delta^l)\times 0$. By the first property in (8), ψ'_σ is also differentiable at all $(t,0)\in(\operatorname{Bd}\Delta^l)\times 0$, with the Jacobian equal to $\mathbb{I}_m$. By (9) and the compactness of Δ^l,

$$\big|\mathcal{J}\psi'_\sigma|_{(t,v)}-\mathbb{I}_m\big|\le C\big(|\nabla s_\sigma(t)|+\rho(t)^{-1}|s_\sigma(t)|\big)\quad\forall\ (t,v)\in(\operatorname{Int}\Delta^l)\times\mathbb{R}^{m-l}$$

for some $C>0$. So $\mathcal{J}\psi'_\sigma$ is continuous at $(t,0)$ by the second statement in (8), as well as differentiable, with the differential of $\mathcal{J}\psi'_\sigma$ at $(t,0)$ equal to 0. For $i \ge 2$, the i-th derivatives of the second component of ψ'_σ at $(t,v) \in (\mathrm{Int}\,\Delta^l) \times \mathbb{R}^{m-l}$ are linear combinations of the terms

$$\beta^{\langle i_1 \rangle}\left(\frac{|v|}{\epsilon\rho_l(t)}\right) \cdot \left(\frac{|v|}{\epsilon\rho_l(t)}\right)^{i_1} \cdot \prod_{k=1}^{k=j_1}\left(\frac{\nabla^{p_k}\rho_l}{\rho_l(t)}\right) \cdot \frac{v_J}{|v|^{2j_2}} \cdot \nabla^{i_2} s_\sigma(t),$$

where $i_1, i_2, j_1, j_2 \in \mathbb{Z}^{\ge 0}$ and $p_1, \ldots, p_{j_1} \in \mathbb{Z}^+$ are such that

$$i_1 + (p_1 + p_2 + \ldots + p_{j_1} - j_1) + i_2 = i, \qquad j_1 + j_2 \le i_1,$$

and v_J is a j_2-fold product of components of v. Such a term is nonzero only if $\epsilon\rho_l(t)/2 < |v| < \epsilon\rho_l(t)$ or $i_1 = 0$ and $|v| < \epsilon\rho_l(t)$. Thus, the i-th derivatives of ψ'_σ at $(t,v) \in (\mathrm{Int}\,\Delta^l) \times \mathbb{R}^{m-l}$ are bounded by

$$C_i \sum_{i_1+i_2 \le i} \rho_l(t)^{-i_1} \left|\nabla^{i_2} s_\sigma(t)\right|$$

for some constant $C_i > 0$. By the second statement in (8), the last expression approaches 0 as $t \longrightarrow \mathrm{Bd}\,\Delta^l$ and does so faster than ρ_l. It follows that ψ'_σ is smooth at all $(t,0) \in (\mathrm{Bd}\,\Delta^l) \times 0$. □

Proof of Proposition 2.1. Let Δ^l_σ be a contractible open neighborhood of Δ^l in $\mathbb{R}^l$ and $\mu_\sigma : \Delta^l_\sigma \longrightarrow X$ a smooth embedding so that $\mu_\sigma|_{\Delta^l} = \eta \circ \iota_\sigma$. By the Tubular Neighborhood Theorem [1, (12.11)], there exist an open neighborhood U_σ of $\mu_\sigma(\Delta^l_\sigma)$ in X and a diffeomorphism[4]

$$\tilde{\mu}_\sigma : \Delta^l_\sigma \times \mathbb{R}^{m-l} \longrightarrow U_\sigma \qquad \text{such that} \qquad \tilde{\mu}_\sigma(t,0) = \mu_\sigma(t)\ \forall\, t \in \Delta^l_\sigma.$$

Let $c_\sigma > 0$ be as in Lemma 3.2 and ψ'_σ as in Corollary 3.4 with $\epsilon = c_\sigma$. The diffeomorphism

$$\psi_\sigma = \tilde{\mu}_\sigma \circ \psi'_\sigma \circ \tilde{\mu}_\sigma^{-1} : U_\sigma \longrightarrow U_\sigma$$

is then the identity on $U_\sigma - \mathrm{St}(b_\sigma, \mathrm{sd}\,K)$. Since ψ_σ is also the identity outside of a compact subset of U_σ, it extends by identity to a diffeomorphism on all of X. □

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References

[1] T. Bröcker and K. Jänich, *Introduction to differential topology*, Translated from the German by C. B. Thomas and M. J. Thomas. Cambridge University Press, Cambridge-New York, 1982. MR0674117 (83i:58001)

[2] M. Kreck, *Differential algebraic topology*, Graduate Studies in Mathematics, 110, Amer. Math. Soc., Providence, RI, 2010. MR2641092 (2011i:55001)

[3] J. W. Milnor, *Topology from the differentiable viewpoint*, Based on notes by David W. Weaver. Revised reprint of the 1965 original. Princeton Landmarks in Mathematics. Princeton University Press, Princeton, NJ, 1997. MR1487640 (98h:57051)

[4] Since Δ^l_σ is contractible, the normal bundle to the embedding μ_σ is trivial.

[4] J. R. Munkres, *Elementary differential topology*, Lectures given at Massachusetts Institute of Technology, Fall, 1961. Revised edition. Annals of Mathematics Studies, No. 54 Princeton Univ. Press, Princeton, NJ, 1966. MR0198479 (33 #6637)

[5] J. R. Munkres, *Elements of algebraic topology*, Addison-Wesley, Menlo Park, CA, 1984. MR0755006 (85m:55001)

[6] M. Scharlemann, Transverse Whitehead triangulations, Pacific J. Math. **80** (1979), no. 1, 245–251. MR0534713 (80k:57047)

[7] S. Smale, An infinite dimensional version of Sard's theorem, Amer. J. Math. **87** (1965), 861–866. MR0185604 (32 #3067)

[8] A. Zinger, Pseudocycles and integral homology, Trans. Amer. Math. Soc. **360** (2008), no. 5, 2741–2765. MR2373332 (2008k:57060)

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Aleksey Zinger
Department of Mathematics, SUNY,
Stony Brook, NY 11794-3651, USA
E-mail: azinger@math.sunysb.edu
URL: http://www.math.sunysb.edu/∼azinger