

Nuclear dimension and n -comparison

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Abstract. It is shown that if a C^* -algebra has nuclear dimension n then its Cuntz semigroup has the property of n -comparison. It then follows from results by Ortega, Perera and Rørdam that σ -unital C^* -algebras of finite nuclear dimension (and even of nuclear dimension at most ω) are stable if and only if they have no nonzero unital quotients and no nonzero bounded traces.

1. INTRODUCTION

In [8], Winter and Zacharias define nuclear dimension for C^* -algebras. This is a form of noncommutative dimension which directly generalizes the covering dimension of topological spaces. Finite nuclear dimension is especially relevant to the classification of C^* -algebras. The simple C^* -algebras of finite nuclear dimension have been proposed as a likely class of C^* -algebras for which Elliott's classification in terms of K-theory and traces holds true.

In the main result of this paper it is shown that the Cuntz semigroup of a C^* -algebra of finite nuclear dimension n satisfies the n -comparison property. For $n = 0$, this property is the same as almost unperforation in the Cuntz semigroup. For arbitrary n , it is reminiscent of the comparability between vector bundles whose fibrewise dimensions differ sufficiently relative to the dimension of the base space.

The n -comparison property for the Cuntz semigroup was first considered by Toms and Winter (see [7, Lemma 6.1]). They showed that n -comparison holds under the more restrictive assumption that the C^* -algebra is simple unital of decomposition rank n (the decomposition rank bounds the nuclear dimension, and, unlike nuclear dimension, is infinite for UCT Kirchberg algebras). The n -comparison property was subsequently studied, and more precisely defined, by Ortega, Perera and Rørdam in [6]. These authors obtained a simple criterion of stability for σ -unital C^* -algebras with n -comparison in their Cuntz semigroup: the C^* -algebra is stable if and only if it has no nonzero unital quotients and no nonzero bounded 2-quasitraces. By Theorem 1.3 below, this stability criterion

applies to all C^* -algebras of finite nuclear dimension. It then follows that σ -unital C^* -algebras of finite nuclear dimension have the corona factorization property.

Let us recall the definition of nuclear dimension given in [8].

Definition 1.1. *The C^* -algebra A has nuclear dimension n if n is the smallest natural number for which there exist nets of completely positive contractive (henceforth abbreviated as c.p.c.) maps*

$$\psi_\lambda^i: A \rightarrow F_\lambda^i \text{ and } \phi_\lambda^i: F_\lambda^i \rightarrow A,$$

with $i = 0, 1, \dots, n$, $\lambda \in \Lambda$, and F_λ^i finite dimensional C^* -algebras for all i and λ , such that

(i) ϕ_λ^i is an order 0 map (i.e., preserves orthogonality) for all i and λ ,

(ii) $\lim_\lambda \sum_{i=1}^n \phi_\lambda^i \psi_\lambda^i(a) = a$ for all $a \in A$.

If no such n exists then A has infinite nuclear dimension.

Let us recall the definition given in [6] of the n -comparison property of an ordered semigroup. For x, y elements of an ordered semigroup S , let us write $x \leq_s y$ if $(k+1)x \leq ky$ for some $k \in \mathbb{N}$.

Definition 1.2. *The ordered semigroup S has the n -comparison property if $x \leq_s y_i$ for $x, y_i \in S$ and $i = 0, 1, \dots, n$ implies $x \leq \sum_{i=0}^n y_i$.*

Let $\text{Cu}(A)$ denote the stabilized Cuntz semigroup of the C^* -algebra A (i.e., the semigroup $W(A \otimes \mathcal{K})$; see [2]).

It is shown in Lemma 2.1 below that for $\text{Cu}(A)$ the n -comparison property can be reformulated as follows: if $[a], [b_i] \in \text{Cu}(A)$, with $i = 0, 1, \dots, n$, satisfy that for each i there is $\varepsilon_i > 0$ such that $d_\tau(a) \leq (1 - \varepsilon_i)d_\tau(b_i)$ for all dimension functions d_τ induced by lower semicontinuous 2-quasitraces, then $[a] \leq \sum_{i=0}^n [b_i]$. It is this formulation of the n -comparison property that is used by Toms and Winter in [7], and that may potentially have the most applications.

Theorem 1.3. *If A has nuclear dimension n then $\text{Cu}(A)$ has the n -comparison property.*

The following section is dedicated to the proof of Theorem 1.3. The last section discusses the application of Theorem 1.3, and of a variation on it that relates to ω -comparison, to establish the stability of C^* -algebras of finite (or at most ω) nuclear dimension.

2. PROOF OF THEOREM 1.3

Let us start by proving that the property of n -comparison may be formulated using comparison by lower semicontinuous 2-quasitraces in place of the relation \leq_s . This result, however, will not be needed in the proof of Theorem 1.3.

For $[a], [b] \in \text{Cu}(A)$, elements of the Cuntz semigroup of A , let us write $[a] <_\tau [b]$ if there is $\varepsilon > 0$ such that $d_\tau(a) \leq (1 - \varepsilon)d_\tau(b)$ for all dimension functions d_τ induced by lower semicontinuous 2-quasitraces $\tau: A^+ \rightarrow [0, \infty]$

(see [3, Sec. 4]). We do not assume that the 2-quasitraces are necessarily finite on a dense subset of A^+ .

Lemma 2.1. *The ordered semigroup $\text{Cu}(A)$ has the n -comparison property if and only if for $[a], [b_i] \in \text{Cu}(A)$, with $i = 0, 1, \dots, n$, $[a] <_\tau [b_i]$ for all i implies that $[a] \leq \sum_{i=0}^n [b_i]$.*

Proof. It is clear that if $[a] \leq_s [b]$ then $[a] <_\tau [b]$. Thus, n -comparison is implied by the property stated in the lemma. Suppose that we have n -comparison in $\text{Cu}(A)$. Let $[a], [b_i] \in \text{Cu}(A)$, with $i = 0, 1, \dots, n$, be such that $[a] <_\tau [b_i]$ for all i . That is, there exist $\varepsilon_i > 0$ such that

$$(1) \quad d_\tau(a) \leq (1 - \varepsilon_i)d_\tau(b_i)$$

for all i . Let us show that $[(a - \varepsilon)_+] \leq_s [b_i]$ for all $\varepsilon > 0$ and all i .

Let $I_i \subseteq A \otimes \mathcal{K}$ denote the closed two-sided ideal generated by b_i and τ_{I_i} the trace that is 0 on I_i^+ and ∞ elsewhere. Setting $\tau = \tau_{I_i}$ in (1) we get that $a \in I_i$. This implies that $(a - \varepsilon)_+ = \sum_{j=1}^{k_i} c_j^* b_i c_j$, which in turn yields $[(a - \varepsilon)_+] \leq_{k_i} [b_i]$.

Let $\lambda: \text{Cu}(A) \rightarrow [0, \infty]$ be additive and order preserving. Let us define $\tilde{\lambda}: \text{Cu}(A) \rightarrow [0, \infty]$ by

$$\tilde{\lambda}([c]) := \sup_{\delta > 0} \lambda([(c - \delta)_+]).$$

It is clear from the definition of $\tilde{\lambda}$ that $\lambda([(c - \delta)_+]) \leq \tilde{\lambda}([c]) \leq \lambda([c])$ for all $[c] \in \text{Cu}(A)$ and $\delta > 0$. By [3, Prop. 4.2 and Lemma 4.7], there exists a lower semicontinuous 2-quasitrace τ such that $\tilde{\lambda}([c]) = d_\tau(c)$ for all $c \in (A \otimes \mathcal{K})^+$. Thus,

$$\lambda([(a - \varepsilon)_+]) \leq \tilde{\lambda}([a]) \leq (1 - \varepsilon_i)\tilde{\lambda}([b_i]) \leq (1 - \varepsilon_i)\lambda([b_i])$$

for all i . That is, $\lambda([(a - \varepsilon)_+]) \leq (1 - \varepsilon_i)\lambda([b_i])$ for any λ that is additive and order preserving. We conclude by [6, Prop. 2.1] that $[(a - \varepsilon)_+] \leq_s [b_i]$ for all i . Since $\text{Cu}(A)$ has n -comparison, $[(a - \varepsilon)_+] \leq \sum_{i=0}^n [b_i]$. Passing on the left to the supremum with respect to $\varepsilon > 0$ we get that $[a] \leq \sum_{i=0}^n [b_i]$. \square

Given an upward directed set Λ and a family of C^* -algebras $(A_\lambda)_{\lambda \in \Lambda}$, let $\bigoplus_\lambda A_\lambda$ denote the C^* -algebra of nets (x_λ) such that $\|x_\lambda\| \rightarrow 0$ and $\prod_\lambda A_\lambda$ the C^* -algebra of nets of uniformly bounded norm. For A a C^* -algebra, let A_Λ denote the C^* -algebra $\prod_\lambda A / \bigoplus_\lambda A$ and $\iota: A \rightarrow A_\Lambda$ the diagonal embedding of A into A_Λ .

Notation convention. Given a homomorphism or c.p.c. order 0 map $\phi: A \rightarrow B$, we shall also denote by ϕ the map $\phi \otimes \text{id}$ from $A \otimes \mathcal{K}$ to $B \otimes \mathcal{K}$.

In [8, Prop. 3.2] Winter and Zacharias show that if A has nuclear dimension n then the maps ψ_λ^i in the definition of nuclear dimension may be chosen to be asymptotically of order 0, i.e., such that the induced maps $\psi^i: A \rightarrow \prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i$ have order 0 for $i = 0, 1, \dots, n$. We get the following proposition as a result of this.

Proposition 2.2. *If A has nuclear dimension n then for $i = 0, 1, \dots, n$ there are c.p.c. order 0 maps $\psi^i: A \rightarrow \prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i$ and $\phi^i: \prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i \rightarrow A_\Lambda$ such that*

$$(2) \quad \iota = \sum_{i=0}^n \phi^i \psi^i,$$

where $\iota: A \rightarrow A_\Lambda$ is the diagonal embedding of A into A_Λ .

Proof. As pointed out in the previous paragraph, by [8, Prop. 3.2] the maps ψ_λ^i in the definition of nuclear dimension may be chosen so that the induced maps ψ^i are of order 0.

The equation (2) is a consequence of Definition 1.1 (ii).

Let us show that the maps $\phi^i: \prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i \rightarrow A_\Lambda$, induced by the order 0 maps ϕ_λ^i , also have order 0. It is clear that $(\phi_\lambda^i): \prod_\lambda F_\lambda^i \rightarrow \prod_\lambda A$ has order 0. Hence, $\alpha \circ (\phi_\lambda^i): \prod_\lambda F_\lambda^i \rightarrow A_\Lambda$, where α is the quotient onto A_Λ , has order 0. We will be done once we show that if $\phi: C \rightarrow D$ is a c.p.c. map of order 0 and $\phi|_I = 0$ for some closed two-sided ideal I , then the induced map $\tilde{\phi}: C/I \rightarrow D$ has order 0. By [9, Cor. 4.1], there is a *-homomorphism $\pi: C \otimes C_0(0, 1] \rightarrow D$ such that $\phi(c) = \pi(c \otimes t)$ for all $c \in C$. From $\pi(I \otimes t) = 0$ we get that $\pi(I \otimes C_0(0, 1]) = 0$. Thus, π induces a *-homomorphism $\tilde{\pi}: C/I \otimes C_0(0, 1] \rightarrow D$. Since $\tilde{\phi}(c) = \tilde{\pi}(c \otimes t)$ for all $c \in C/I$, $\tilde{\phi}$ has order 0. \square

An ordered semigroup S is called unperforated if $kx \leq ky$ for $x, y \in S$ and $k \in \mathbb{N}$ implies $x \leq y$.

In the following two lemmas the index λ ranges through an upward directed set Λ .

Lemma 2.3. (i) *If $\text{Cu}(A)$ is unperforated then so is $\text{Cu}(A/I)$ for any closed two-sided ideal I .*

(ii) *If A_λ are C^* -algebras such that $\text{Cu}(A_\lambda)$ is unperforated for all λ then so are $\text{Cu}(\prod_\lambda A_\lambda)$ and $\text{Cu}(\prod_\lambda A_\lambda / \bigoplus_\lambda A_\lambda)$.*

Proof. (i) Let $[\tilde{a}], [\tilde{b}] \in \text{Cu}(A/I)$ be such that $k[\tilde{a}] \leq k[\tilde{b}]$ for some $k \in \mathbb{N}$. Then for $[a], [b] \in \text{Cu}(A)$ lifts of $[\tilde{a}]$ and $[\tilde{b}]$ we have

$$k[a] \leq k[b] + [i] \leq k([b] + [i])$$

for some $i \in (I \otimes \mathcal{K})^+$ (by [1, Prop. 1]). Since $\text{Cu}(A)$ is unperforated, we have $[a] \leq [b] + [i]$, and passing to $\text{Cu}(A/I)$ we get that $[\tilde{a}] \leq [\tilde{b}]$.

(ii) Let $(a_\lambda)_\lambda, (b_\lambda)_\lambda \in (\prod_\lambda A_\lambda) \otimes \mathcal{K} \subseteq \prod_\lambda (A_\lambda \otimes \mathcal{K})$ be positive elements of norm at most 1 such that $k[(a_\lambda)_\lambda] \leq k[(b_\lambda)_\lambda]$ for some k . Let $\varepsilon > 0$. Then there is $\delta > 0$ such that

$$k[((a_\lambda - \varepsilon)_+)_\lambda] \leq k[((b_\lambda - \delta)_+)_\lambda].$$

Hence, $k[(a_\lambda - \varepsilon)_+] \leq k[(b_\lambda - \delta)_+]$ for all λ . Since $\text{Cu}(A_\lambda)$ is unperforated, $[(a_\lambda - \varepsilon)_+] \leq [(b_\lambda - \delta)_+]$. Let $x_\lambda \in A_\lambda \otimes \mathcal{K}$ be such that

$$(a_\lambda - 2\varepsilon)_+ = x_\lambda^* x_\lambda \text{ and } x_\lambda x_\lambda^* \in \text{her}((b_\lambda - \delta)_+).$$

Since $\|x_\lambda\|^2 \leq \|a_\lambda\| \leq 1$, we have $(x_\lambda)_\lambda \in \prod_\lambda (A_\lambda \otimes \mathcal{K})$. We also have $x_\lambda x_\lambda^* \leq \frac{1}{5} b_\lambda$, whence

$$((a_\lambda - 2\varepsilon)_+)_\lambda = (x_\lambda)_\lambda^* (x_\lambda)_\lambda \text{ and } (x_\lambda)_\lambda (x_\lambda)_\lambda^* \in \text{her}((b_\lambda)_\lambda).$$

But $(\prod_\lambda A_\lambda) \otimes \mathcal{K}$ sits as a closed hereditary subalgebra of $\prod_\lambda (A_\lambda \otimes \mathcal{K})$. Therefore, $(x_\lambda)_\lambda \in (\prod_\lambda A_\lambda) \otimes \mathcal{K}$ and so $[(a_\lambda - 2\varepsilon)_+]_\lambda \leq [(b_\lambda)_\lambda]$. Since ε may be arbitrarily small, we have $[(a_\lambda)_\lambda] \leq [(b_\lambda)_\lambda]$ as desired. \square

Lemma 2.4. *Let $a, b \in (A \otimes \mathcal{K})^+$. If $\text{Cu}(\iota)([a]) \leq \text{Cu}(\iota)([b])$ in $\text{Cu}(A_\lambda)$, then $[a] \leq [b]$.*

Proof. Let $\varepsilon > 0$. Since $[\iota(a)] \leq [\iota(b)]$, there is $d \in A_\lambda \otimes \mathcal{K}$ such that $d^* \iota(b) d = \iota(a - \varepsilon)_+$. That is, there are $d_\lambda \in A_\lambda \otimes \mathcal{K}$ such that $d_\lambda^* b d_\lambda \rightarrow (a - \varepsilon)_+$. Thus, $[(a - \varepsilon)_+] \leq [b]$. Since $\varepsilon > 0$ may be arbitrarily small, we have $[a] \leq [b]$. \square

Remark 2.5. In proving Theorem 1.3, a stronger property than n -comparison will be shown to hold for C^* -algebras of nuclear dimension n : if $x, y_i \in \text{Cu}(A)$ with $i = 0, 1, \dots, n$ satisfy that $k_i x \leq k_i y_i$ for some $k_i \in \mathbb{N}$ and all i , then $x \leq \sum_{i=0}^n y_i$. This property, unlike n -comparison, does not seem to have a formulation in terms of comparison by lower semicontinuous 2-quasitraces.

Proof of Theorem 1.3. Suppose that there are $k_i \in \mathbb{N}$ such that $k_i [a] \leq k_i [b_i]$ for $i = 0, 1, \dots, n$. Since c.p.c. order 0 maps preserve Cuntz comparison (by [9, Cor. 4.5]), we have that $k_i [\psi^i(a)] \leq k_i [\psi^i(b_i)]$ for all i . Since the Cuntz semigroup of finite dimensional algebras is unperforated, we have by Lemma 2.3 that the Cuntz semigroup of $\prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i$ is unperforated. Thus, $[\psi^i(a)] \leq [\psi^i(b_i)]$. The maps ϕ^i preserve Cuntz equivalence (since they are c.p.c. of order 0), whence

$$[\phi^i \psi^i(a)] \leq [\phi^i \psi^i(b_i)] \leq \left[\sum_{j=0}^n \phi^j \psi^j(b_i) \right] = [\iota(b_i)].$$

So,

$$[\iota(a)] = \left[\sum_{i=0}^n \phi^i \psi^i(a) \right] \leq \sum_{i=0}^n [\phi^i \psi^i(a)] \leq \sum_{i=0}^n [\iota(b_i)].$$

By Lemma 2.4, this implies that $[a] \leq \sum_{i=0}^n [b_i]$. \square

3. STABILITY OF C^* -ALGEBRAS

A stable C^* -algebra has no nonzero unital quotients and no nonzero bounded 2-quasitraces (see (i) \Rightarrow (ii) of [6, Prop. 4.7]). In [6, Prop. 4.7] Ortega, Perera and Rørddam show that the converse is true provided that the C^* -algebra is σ -unital and its Cuntz semigroup has the n -comparison property. This, combined with Theorem 1.3 and the fact that for exact C^* -algebras bounded 2-quasitraces are traces, implies that a σ -unital C^* -algebra of finite nuclear dimension is stable if and only if it has no nonzero unital quotients and no nonzero bounded traces. Ortega, Perera, and Rørddam also show that ω -comparison, a weakening of n -comparison, suffices to obtain the same stability criterion.

Definition 3.1. (c.f. [6, Def. 2.11]) Let S be an ordered semigroup closed under passage to suprema of increasing sequences. Then S has the ω -comparison property if $x \leq_s y_i$ for $x, y_i \in S$ and $i = 0, 1, \dots$ implies $x \leq \sum_{i=0}^{\infty} y_i$.

Remark 3.2. The definition of ω -comparison given above differs slightly from the definition given in [6]. Nevertheless, both definitions agree for ordered semigroups in the category \mathbf{Cu} introduced in [2], and therefore, also for ordered semigroups arising as Cuntz semigroups of C^* -algebras.

A notion of nuclear dimension at most ω may be modelled after the statement of Proposition 2.2.

Definition 3.3. Let us say that a C^* -algebra A has nuclear dimension at most ω if for $i = 0, 1, 2, \dots$ there are nets of c.p.c. maps $\psi_\lambda^i: A \rightarrow F_\lambda^i$ and $\phi_\lambda^i: F_\lambda^i \rightarrow A$, with F_λ^i finite dimensional C^* -algebras and $\lambda \in \Lambda$, such that

(ii) the induced maps $\psi^i: A \rightarrow \prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i$ and $\phi^i: \prod_\lambda F_\lambda^i / \bigoplus_\lambda F_\lambda^i \rightarrow A_\Lambda$ are c.p.c. order 0,

(iii) $\iota(a) = \sum_{i=0}^{\infty} \phi^i \psi^i(a)$ for all $a \in A$ (where the series on the right side is understood to be convergent in the norm topology).

For example, if the C^* -algebras $(A_i)_{i=0}^{\infty}$ all have finite nuclear dimension, then $\bigoplus_{i=0}^{\infty} A_i$ has nuclear dimension at most ω . It is not clear whether the assumption that the maps ψ_λ^i be asymptotically order 0 may be dropped in Definition 3.3 (and then proved), or if the other results on finite nuclear dimension proved in [8] also hold for nuclear dimension at most ω . In particular, is it true that the property of nuclear dimension at most ω passes to closed hereditary subalgebras?

The proof of Theorem 1.3 goes through, mutatis mutandis, for nuclear dimension at most ω . We thus have

Theorem 3.4. If A has nuclear dimension at most ω then $\text{Cu}(A)$ has the ω -comparison property.

Combined with the results of [6], Theorem 3.4 yields the following corollary, which improves on [5, Thm. 0.1], [6, Cor. 4.9] and [6, Cor. 5.12].

Corollary 3.5. Let A be a C^* -algebra of nuclear dimension at most ω and let $B \subseteq A \otimes \mathcal{K}$ be hereditary and σ -unital. Then B is stable if and only if it has no nonzero unital quotients and no nonzero bounded traces. B has the corona factorization property.

Proof. By Theorem 3.4, $\text{Cu}(A)$ has the ω -comparison property. Since $\text{Cu}(B)$ is an ordered subsemigroup of $\text{Cu}(A)$ (i.e., the order and addition operation on $\text{Cu}(B)$ agree with the ones induced by its inclusion in $\text{Cu}(A)$) the ω -comparison property holds in $\text{Cu}(B)$, too. Hence, by [6, Prop. 4.7], B is stable if and only if it has no nonzero unital quotients and no nonzero bounded traces. (In the hypotheses of [6, Prop. 4.7] the C^* -algebra A is assumed to be separable. A closer look into the proof of this result reveals that it suffices to assume that the hereditary subalgebra $B \subseteq A \otimes \mathcal{K}$ be σ -unital. This justifies the application of [6, Prop.4.7] made here.)

Let $C \subseteq B \otimes \mathcal{K}$ be full and hereditary, and suppose that $M_n(C)$ is stable. Then $M_n(C)$, and consequently C , cannot have nonzero unital quotients or bounded traces. Thus C is stable. This shows that B has the corona factorization property (see [4, Thm. 4.2]). \square

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