Essays on Regional Labour Mobility

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Contents

Introduction

Various countries suffer from population aggregation and segregation, leading to the formation of both economically strong and weak regions at the same time. An unbalanced distribution of the population with respect to characteristics such as age, gender, income or education has far-reaching social, political and economic consequences. The resulting concentration of poverty in isolated districts or regions is in no one's interest. One of the fundamental determinants of this demographic change is the movement of people across regions, and therefore, this forms the focus of the present thesis. Already in the 19th century, Ernest Ravenstein (1885) postulated the main determinants of migration in his article *The Laws of Migration*. Over the past decades, the number of studies examining the determinants and consequences of migration has grown rapidly. Specifically, a variety of possible determinants have been investigated. These can be related to individuals or to regions. Examples of the latter are unemployment rates, job opportunities or local amenities. The consequences of migration are important to answer the following two questions: First, what are the benefits of relocating for the individual himself and second, what are the impacts on others? A comprehensive overview is provided by Greenwood (1997) and Etzo (2008), who together describe hundreds of papers and allocate them to the literature in categories.

Migration flows can be analysed from two classical perspectives. On the one hand, there is the micro-level approach, which focuses on the behaviour of each individual. The aim is to find factors that exert a personal impact on the decision to migrate, which in turn takes place when the net benefits exceed the costs.[1] On the other hand, there is the macro-level approach, which focuses on regions as a whole. Aggregated flows are the point of interest rather than individual decisions. In both approaches, employment states play an important role when investigating migration, since it is no secret that migration can act as an adjustment mechanism for regional unemployment disparities (Mundell (1961), Greenwood (1975) or Blanchard and Katz (1992)). Nevertheless, many

[1]This includes physical and psychological costs.

countries have strong and persistent differences in unemployment rates, which can be observed in Spain, Italy or Germany, for example. In order to study regional differences and to propose some policies, a first step is to construct a multiregional model with endogenous labour mobility, which is rarely done in the literature. So far, existing models consist of only two regions or consider an arbitrary number of identical identified regions. This thesis contributes to the literature by filling that gap. For both the micro and the macro approaches, models are presented that allow migration between all possible regions and employment states.

Despite its great importance in recent years, international migration is not considered in this paper. The reasons are manifold. On the one hand, immigrants from other countries can have other incentives to leave their country. In addition to voluntary migration due to better working conditions, there is also forced migration by refugees. On the other hand, migration to another country is subject to greater barriers. Work permits and different languages are two obvious examples. The study of these aspects leads to another research topic. However, the main reason for omitting international migration is that international migration has to be considered as an exogenous factor for the economy, which is not consistent with the present research approach.

One of the main difficulties so far in studying internal migration has been the lack of availability, quality and comparability of data. Collecting data at the micro level for a significant period of time is a costly process that only a few developed countries have done. Focussing on Germany[2], the first paper of this thesis emphasises and visualises regional labour mobility between 1995 and 2017, discusses such characteristics as unemployment rates, price levels or population densities in 2016 and provides an advanced gravity model to identify the determinants of internal migration. Specifically, this means analysing about two percent of the labour force across 328 different regions (NUTS 3) on a monthly basis. The focus is particularly on the distinction between employed and unemployed workers and shows which group migrates when and where, both from the origin and destination perspective. The use of a gravity model is a common procedure

[2]The analysis is based on the extensive data set of the Institute for Employment Research (IAB).

in the literature (see Greenwood and Hunt (2003)) and can be summarised by

$$M_{ij,t} = \beta_0 + \beta_1 D_{ij} + \beta_2 P_{i,t} + \beta_3 P_{j,t} + \sum_{s=1} \Big(\gamma_s X_{s,i,t} + \delta_s X_{s,j,t} \Big) + \varepsilon_{ij,t},$$

where $M_{ij,t}$ is the absolute number of migrants from region i to region j in period t, D_{ij} refers to the geographical distance, $P_{i,t}$ and $P_{j,t}$ are the population sizes, and $X_{s,i,t}$ and $X_{s,j,t}$ are other commonly included variables such as unemployment rates or wages. In contrast to the widely used estimation of a log transformation, the Poisson pseudo maximum likelihood (PPML) method suggested by Santos Silva and Tenreyro (2006) is used, which prevents, for example, the elimination of observations where $M_{ij,t}$ is equal to zero.

The second paper of this thesis examines the proceedings from the micro perspective. At its core, the paper is based on the migration model of Sjaastad (1962). In this model, economic disparities can be adjusted through labour migration, as each worker maximizes his expected lifetime utility by rationally choosing his place of residence. The worker's lifetime utility in region i can be expressed in simplified terms as

$$V_i(x_t) = F_i(x_t) + \beta \mathbb{E} \left(\max_j \big(V_i(x_{t+1}) + p_j \big(V_j(x_{t+1}) - V_i(x_{t+1}) - \kappa_{ij} \big) \big) \right),$$

where F_i is the utility of the current period, β is the discount parameter, p_j is the probability of finding a job in region j, V_j is the lifetime utility in region j and κ_{ij} is the total cost of moving from region i to region j. The expectation is taken to represent the uncertainty of productivity for the next period, which, to be precise, is a conditional expectation given the current state x_t. This recursive form of the equation is a possible representation of a Bellman equation. Using well-known fixed-point theorems (see Stokey et al. (1989)), it is shown that for any number of regions, the set of equations has a unique and efficient solution, in the sense that the agents' value functions and their corresponding policy functions do not depend on the distribution of workers across space and employment states. This property is introduced by Menzio and Shi (2009), who call it a *Block Recursive Equilibrium.*

Concerning the macroeconomic perspective of this thesis, New Keynesian models, which have been part of the international macroeconomic mainstream for decades, cannot be ignored. Surprisingly, standard models do not consider movements across regions (see Blanchard and Gali (2010) or Clarida et al. (2002)). Building on Hauser (2014) and House et al. (2018), the third paper presents a multiregional *Dynamic Stochastic General Equilibrium* (DSGE) model which contains the standard elements of the underlying literature. In particular, for the hiring process, a simple search and matching framework in the spirit of Diamond (1982), Mortensen (1982) and Pissarides (1985) is used to describe labour mobility. The financial market, including inflation, price-setting and wage bargaining, is included in order to complete the model.

The extensions in the second as well as in the third paper provide a theoretical foundation. Of particular interest could be the effects on migration and, consequently, its role as an adjustment mechanism, e.g., for regional unemployment rates. For this reason, a simple implementation of a productivity shock has already been conducted in both approaches, but both models suffer from calibration issues. The distinction between regions creates a large set of bilateral parameters that describe the transition costs of moving from region i to region j and vice versa. Naive approaches of reducing these parameters, such as introducing a function that depends on different variables such as distance, real estate prices, regional GDP, etc., lead to an under-determined system of equations. A unique identification is therefore not possible. Additionally, using historical migration rates for the estimation raises the problem that the number of observed movements between two regions decreases or even equals zero as the number of regions increases. Therefore, deriving a suitable estimation for transition costs remains a task for future research.

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Regional Labour Migration - Stylized Facts for Germany

Abstract

We present stylized facts of the local German labour markets. Using a large German administrative dataset and newly available regional price level data, we study workers' biographies at the local level. Huge regional variation is documented in: unemployment rates and nominal as well as real wages. We show that unemployment rates in East and West Germany have converged substantially 30 years after reunification, but the real wage gap still persists. We describe and model monthly worker flows across 328 regions (NUTS3). Counter-intuitively, unemployed workers in depressed regions are less likely to move to a new working place in another region than unemployed workers in prosperous regions. The most (and increasingly) mobile group are unemployed workers in dense and active regions.

Keywords: labour mobility; business cycle fluctuations; regional disparities

1 Introduction

The finding that globalization does not only have positive effects everywhere but may indeed hurt some regions, has triggered a lot of research on local labour markets both in theoretical and empirical economics. Local shocks affect workers in different ways, for instance, by changes in the transition rates between labour market statuses. Local labour market conditions are also related to the incentive to migrate between regions within the home state. While international migration regularly leads to heated debates, internal migration flows within states receive much less attention. International migration is more prominent in the public debate as it is sometimes claimed that it might have a negative impact on national security or labour markets, in particular for unskilled jobs. International migration is also more perceptible due to language barriers or cultural differences. But a careful investigation of internal migration flows also provides interesting insights. As in most other countries, internal migration is also an important phenomenon in Germany. The specific German history, especially the large and persistent regional economic division – the "Poor East" lagging behind the "Wealthy West" – makes it a particularly interesting object to study.

This paper's main contribution is descriptive. We give a comprehensive account of different aspects of local labour markets in Germany between 1995 and 2017 from which we distill "stylized facts". The regional resolution is (roughly) at the level of "Landkreise" or NUTS 3. Our primary data source is the sample of integrated labour market biographies (SIAB) containing information about a 2% random sample for 328 regions. The data set allows insights into trends and patterns of regional migration and local transition rates between labour market statuses across Germany. Adding newly available regional price levels (Weinand and von Auer, 2019), we compute regional real wages and investigate their relationship with other local variables.

Stylized facts about regional worker migration are interesting for several reasons. First, they facilitate drawing a broad-brush picture of the historical development as excessive complexities are suppressed. They may also help put the economic situation into a wider sociological or political context. Second, stylized facts enable us to assess the plausibility of economic theories without formal statistical testing. In general, economic theories are not capable to explain the observations in all details. Statistical hypothesis tests are thus bound to reject virtually any theoretical model if the number of observations is large. In contrast, by comparing model implications to the stylized facts one can judge if the model is useful for the problem at hand. For example, a plain economic theory would suggest that regions with high unemployment levels experience more net emigration of unemployed workers than regions with fewer unemployed. However, this is in contrast to what the stylized facts reveal. Hence, there is need to develop other, more sophisticated economic theories, e.g. postulating persistent differences in unemployment levels even in equilibrium (as in e.g. Molho, 1995 or Aragon et al., 2003). Third, the stylized facts are relevant for policy makers. Government efforts to reduce regional unemployment differentials are pointless when the economy is already in equilibrium. If, however, the differentials are transitory and reflect an ongoing convergence process, there is scope for government intervention to speed up convergence.

Thirty years after the fall of the wall between East and West Germany, migration from the East to the West continues, causing fears that East Germany will remain poorer and less innovative. Glorius (2010) and Hunt (2000) point out that East Germany suffers from brain drain. While a certain degree of mobility of skilled workers is desirable, East Germany lacked immigration of a similar scope until recently. Up to now, East Germany has lost more than 2m residents, approximately 10% of the population of the former German Democratic Republic. The claim that unemployment differentials will level out in the long run is supported by the stylized fact that the gap between East and West unemployment rates is (slowly) narrowing. The related literature focuses mainly on aggregate disparities between East and West Germany. Smolny (2011) and Heiland (2004) are among the first contributions investigating state-level data (NUTS 1). They point out that there is not only a gap between East and West Germany but also between North and South Germany. They argue that interregional migration should be investigated more thoroughly. Bauer et al. (2019) analyse internal migration behaviour in Germany. They find labour market variables to have explanatory power for internal migration. In particular, migration flows of younger cohorts are attracted to urban areas. The remainder of the paper is structured as follows: The next section provides information about the SIAB dataset and the data source for the regional price levels. Section 3 presents aggregated stylized facts without a fine-grained regional disaggregation. In section 4, we derive regional stylized facts. A descriptive regression model of regional mobility is outlined in section 5. Section 6 concludes.

2 The Data

Our analysis primarily relies on the Sample of Integrated Labour Market Biographies (SIAB) provided by the Institute for Employment Research (IAB).[1] The SIAB is a 2% random sample of integrated labour market biographies in Germany from 1975 to 2017 which consists of all German residents who (i) have jobs that are subject to social security, (ii) are in marginal part-time employment, (iii) receive unemployment or social benefits, (iv) are registered as job seekers, or (v) participate in employment or training programs. Civil servants and self-employed workers are not included in the dataset.[2] In total, the dataset covers about 80% of the German labour force.

Each spell in the dataset provides daily information on individuals' employment status, region of work and average daily (pre-tax) wage or social security benefits, as well as the socio-economic variables gender, age, education and nationality. The SIAB's main data source is the employment history (Beschäftigten-Historik, BeH) collected by the Employment Agency for administrative purposes. It is an individual-level dataset covering all workers liable to social security. The data originate from the mandatory German notification procedure for social security, which compels all employers to keep the social security agencies informed about their employees. Some spells in the dataset suffer from overlapping notifications that can occur when data are merged from different adminis-

[1] A detailed description of the SIAB dataset is provided in Ganzer et al. (2017).

[2] Caliendo and Uhlendorff (2008) find that only 3% of the unemployed workers and only 1% of the employed workers enter the state of self-employment in Germany annually.

trative sources. For instance, a worker who receives unemployment benefits may have a part-time job, or an employed worker can lose his second job and become part-time employed. Further, the dataset has a structural break in 2005/2006 after the labour market ("Hartz") reforms had been implemented. From that point on, workers receiving unemployment aid were no longer reported. To avoid inconsistent observations, we only consider spells where the employment status is either "employee liable to social security contributions" (defined as: employed) or "receives unemployment benefits" (defined as: unemployed).

The main advantages of the SIAB dataset are the high data quality and its fine regional resolution. For each spell, employed workers are assigned to one of 328 different regions. The regions mostly correspond to "Landkreise" (NUTS 3), some regions have been merged by the data provider to avoid too small numbers of observations (see the appendix of Antoni et al., 2019). There is no regional information for the unemployed. We impute their region as follows: If unemployed workers were previously employed, we assume that they still live in the region of their last job. For unemployed workers who have never had a job before, we assign the region of their next observable job. Workers without any regional information at all are irrelevant for our study and are excluded.[3] Finally, since data for East Germany are nonexistent before the reunification and rather unreliable until 1995, we focus on the years 1995 until 2017. Our final dataset includes more than 30 million spells.

The wages reported in the SIAB are nominal. In order to derive regional real wages, one needs a regional price index. Weinand and von Auer (2020) is the first study that calculated regional prices based on official data of the Federal Statistical Office. Their methodology is data driven and does not need any restrictive assumptions on the regional patterns of prices. The regional price index is, however, only available for May 2016. A dynamic perspective on real regional wages is unfortunately not yet possible.

[3]This reduces the number of spells by about 4%.

3 Aggregated stylized facts

In this section, we present stylized facts about labour market quantities in Germany without a fine-grained regional disaggregation. We consider aggregated unemployment rates, earnings, transition rates between employment and unemployment as well as migration rates for all German regions together.

3.1 Unemployment and earnings

We proceed to compare the monthly unemployment rates implied by the SIAB dataset to the unemployment rates reported in official statistics. To calculate the unemployment rate, we divide the number of unemployed workers by the number of all workers in the SIAB sample *on the first day of each month.* This unemployment rate definition obviously results in a lower rate than the official statistics (see figure 1, top panel). However, the difference between the two rates is stable over time which means that the historical developments are mirrored (e.g. high unemployment in 1997/1998 and around 2005, a short peak after the financial crisis in 2009/2010).

The bottom panel of figure 1 depicts the development of mean real earnings. Nominal earnings have been inflation adjusted by the nation-wide CPI reported by the Federal Statistical Office. The overall mean (black line) declines until 1998, rises slowly up to about 2010 and increases more strongly since then. There is no notable drop during the financial crises. We classify the regions into three equally large categories: rural, suburban and urban regions.[4] The mean real earnings of the three categories are shown by the coloured lines in figure 1 (since regional price indices are not yet available as time series, rural, suburban and urban earnings have been deflated by the nation-wide price index). Evidently, earnings are lowest in rural regions and highest in urban regions. Suburbian earnings are close to the overall average. The gap between the three categories is roughly constant over time. There are large differences in the level of daily wages.

[4]The three groups are distinguished by their population density. If the number of inhabitants per square kilometre is 157.88 or less, the region is categorized as rural. If the density is between 157.88 and 367.01 it is a suburban region. Regions with densities of more than 367.01 are defined as urban.

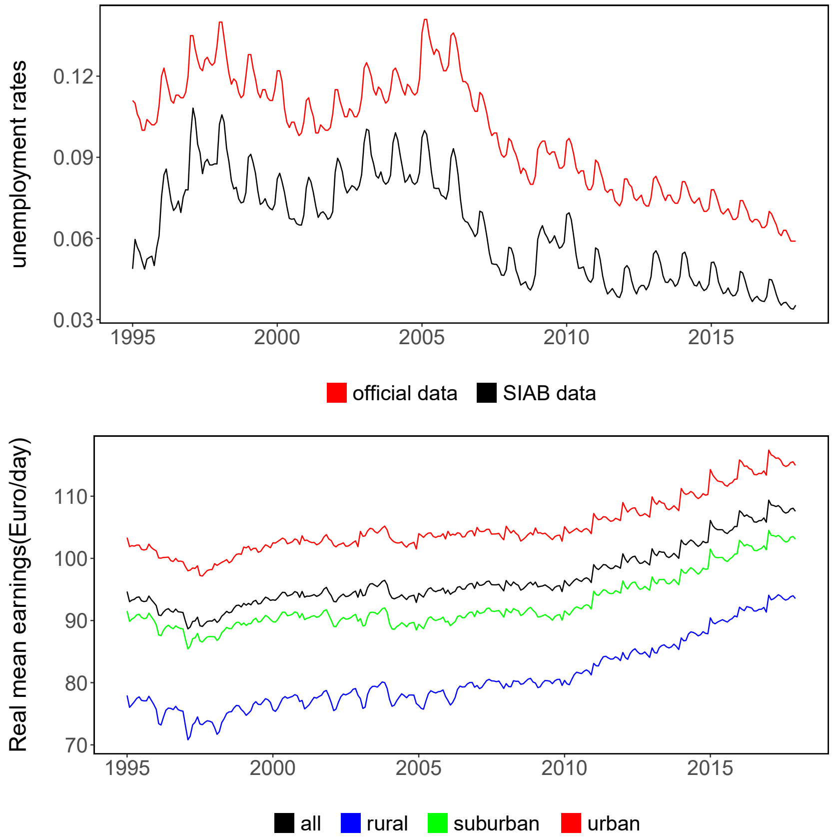

Figure 1: Top panel: Monthly unemployment rates. The red line shows the official unemployment rates, the black line shows the unemployment rates calculated from the SIAB data as described in the text. Bottom panel: Mean real earnings for urban (red), suburban (green) and rural (blue) areas, the overall mean real earnings are shown by the black line.

Workers in rural regions earn on average 20 Euro less per day than workers in urban regions.

3.2 Employment status transition rates

There are three relevant employment status transitions: An unemployed worker can find a job and enter the employment status (job finding transition rate, UE); an employed worker can leave his current job for a new job (job to job rate, EE); an employed worker can enter the unemployment pool (separation rate, EU).

We compute the transition rates by dividing the number of transitions in month t by the number of workers in the origin status in month $t-1$. A transition is defined as a change in the status from the first day of month $t-1$ to the first day of month t. This definition ignores transitions within months. For example, a short unemployment spell that began after the first of a month and did not last until the next month, is not counted. Nordmeier (2012) has shown that a monthly measurement underestimates the number of transitions by roughly 10%. However, as it is common to use monthly estimates in most studies, we follow the literature. Figure 2 displays seasonally adjusted worker flows[5] during the sample period from 1995 to 2017.

The monthly UE flow rate varies greatly over this period around an average of 8.06%. At the beginning of the sample period it drops from nearly 10% by more than 4 percentage points and in some months it even falls below 5%. After the labour market ("Hartz") reforms[6] in 2005, the UE rate increases again to over 10% with a small trough in 2009 during the world financial crisis.

The monthly job to job transition rate EE has a mean of 0.88%. While the time series is always close to its mean, it exhibits several positive outliers in the beginning of the observation period and several negative outliers before the structural break in 2005. Taking into account that the average tenure of young workers (see Rhein and Stüber, 2014) became shorter over the last decades, it is surprising that we cannot observe an increasing trend for the EE rate.

The separation rate EU is quite low over the entire time period. About 0.59% of workers transit from their job into unemployment each month. The EU time series shows that there were three recession periods with relatively high separation rates, namely around 1997, 2005 and 2009.

[5]Because of a change in the reporting system, we computed the deseasonalization separately for the years before and after 2005.

[6]The German government restructured the federal employment agency to enhance the matching process of unemployed workers to jobs. Especially, the Hartz IV reform reduced unemployment benefits substantially and abolished long-term unemployment benefits. For more details see Hartung, Jung and Kuhn (2018).

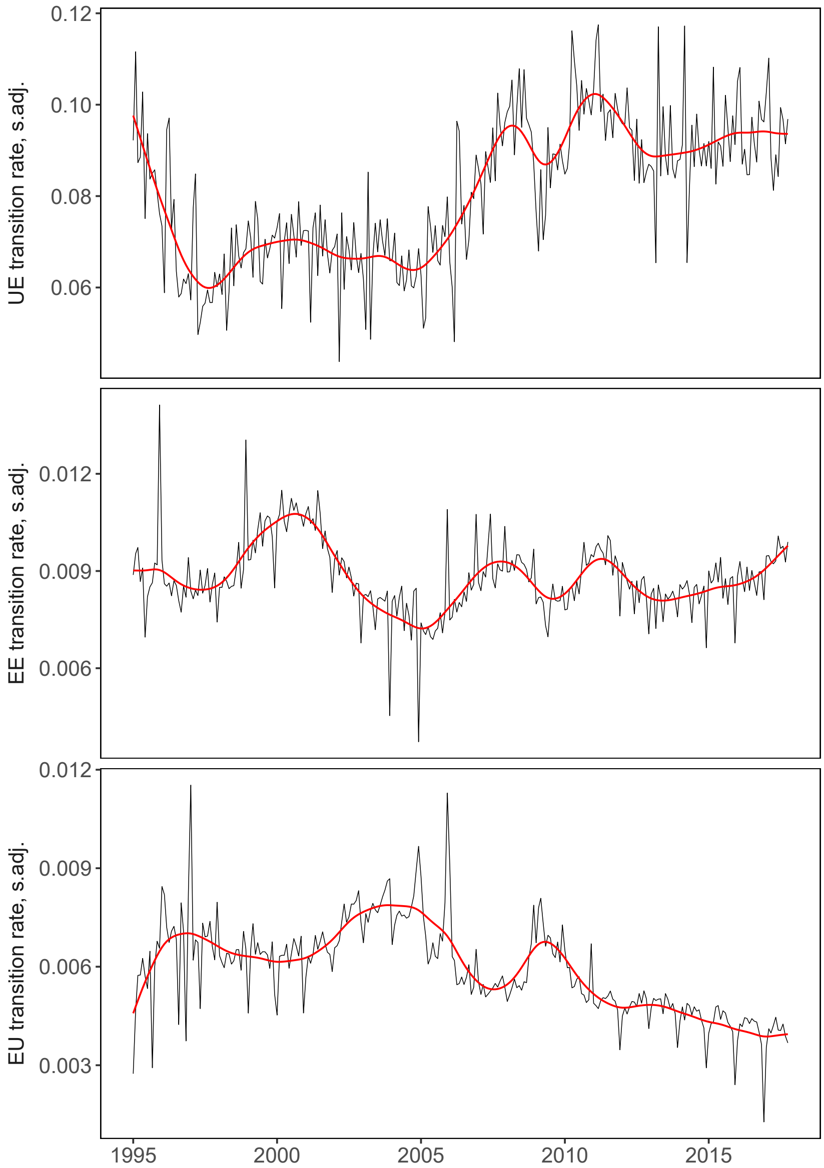

Figure 2: Monthly employment status transition rates. Top panel: unemployment to employment transition rate UE; middle panel: job to job transition rate EE; bottom panel: employment to unemployment separation rate EU; all series are seasonally adjusted, the red lines are smoothing splines.

Business cycle fluctuations are associated with large swings in the labour market conditions (see Shimer, 2005). The larger probability to lose a job and the smaller probability to find a job in a recession is obviously one of the major costs of an economic downturn. Contrary to common expectations our time series do not indicate that the business cycle volatility is lower in Germany than in the U.S. A comparison of both countries (see Shimer, 2005 and Fujita and Nakajima, 2016) shows that business cycle volatility in the German labour market is approximately twice as large as in the U.S. At the same time the level of worker flows is substantially lower[7] because German firms tend to rely on longer-term relationships with their employees. When the economy is hit by a persistent shock, this has a larger effect on firms' present values under long term relationships. If workers enjoy high job protection, expected future productivity is relevant, i.e. the higher the level of job protection, the more the present value of a firm is affected.

3.3 Migration rates

We now consider aggregate measures of workers' movements across regions within Germany. We define a worker as a mover if his or her region of work at the first of month t differs from the first of month $t-1$. Our definition of "mover" does not look at the place of living. If workers move their place of work from one region to another without moving their place of living they are nevertheless counted as movers. Figure 3, top panel, shows the aggregated time series of moving rates from 1995 to 2017. On average, around 0.56% of workers move to a workplace in another region each month. We observe a steep increase in the beginning of the period, which leads to the maximum of nearly 0.75% at the end of the last century. Then followed a decrease to 0.5% in 2005. During the remaining time period, the proportion of movers fluctuates around its mean. For employed workers the average share of movers is only 0.42%, see the middle panel in figure 3. It varies with a similar amplitude as the total population over the sample period.

The fraction of unemployed movers, depicted in the bottom panel of figure 3, is close to 2% until 2005 and then experiences a steep increase up to 4.5% with a small and short

[7] More details can be found in Menzio and Shi (2011) or Jung and Kuhn (2014).

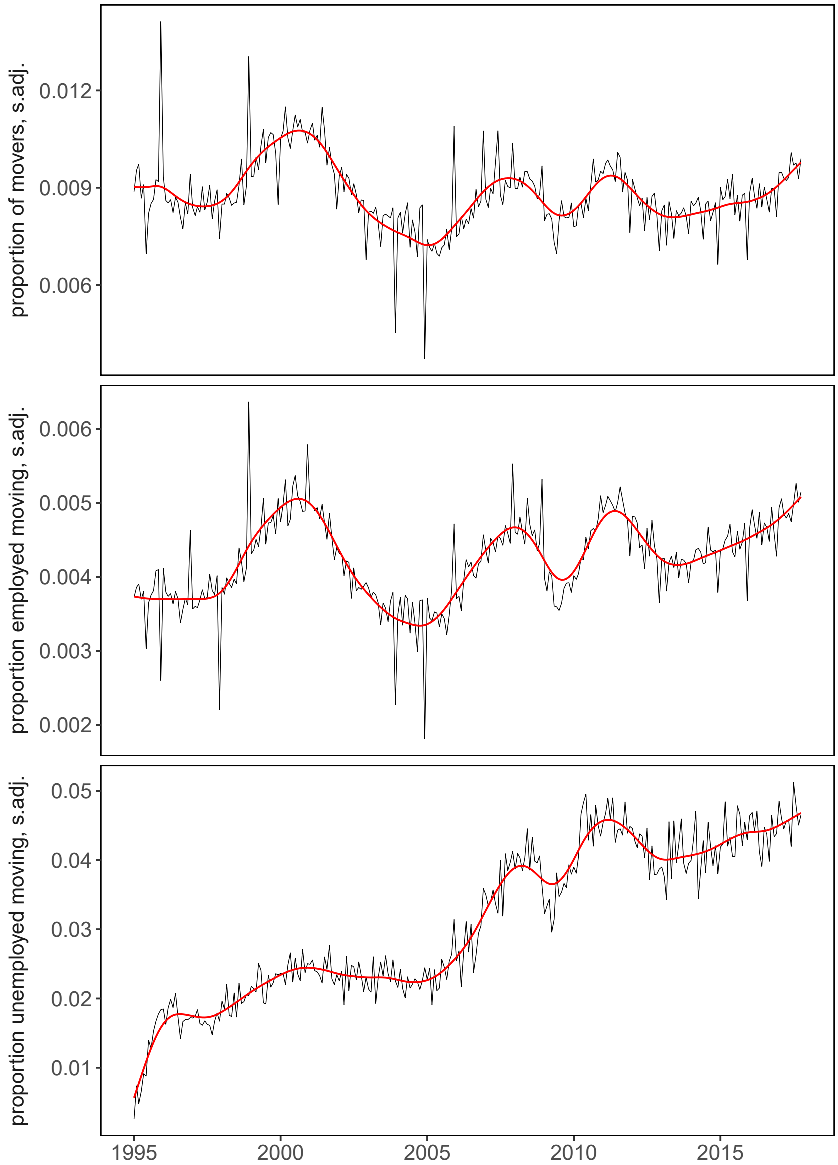

Figure 3: Monthly moving rates. Top panel: proportion of all workers that move in a given month; middle panel: proportion of all employed workers that move; bottom panel: proportion of all unemployed workers that move; all series are seasonally adjusted, the red lines are a smoothing splines.

trough in 2009. The moving rates decline in times of high unemployment rates (1998, 2005 and 2009).

To summarize, the main stylized facts about aggregated labour market quantities are: significant changes after the "Hartz" reforms and the financial crisis; high volatilities in the transition rates; the job finding rate recovered from an intermediately low; a very remarkable upward trend in the moving rate of unemployed workers.

4 Regional stylized facts

4.1 Regional characteristics

In this section we describe the pattern of regional population densities, price levels (overall and housing) and nominal wages. Information about the population distribution is drawn from the German Federal Agency for Cartography and Geodesy. Germany has only few large cities, and only four of them have a population over 1 million: Berlin, Hamburg, Munich and Cologne. On the other hand, Germany has a large number of medium sized and small cities. In total there are currently 82 cities with a population of more than 100,000 inhabitants. These cities account for approximately 2/3 of the population. However, Figure 4 (top left) also demonstrates that there are agglomeration areas around the urban centres Hamburg, Frankfurt, Stuttgart, Munich and Dortmund. The latter is only the 9th largest city, but its metropolitan area is home to over 5 million people ("Ruhr area"). The northern parts in East Germany as well as large parts in Bavaria are rural, sparsely populated areas. The same holds for the border zones between Thuringia and Hessen, or Brandenburg and Saxony. After the reunification most areas in the former German Democratic Republic experienced a dramatic fall in number of residents during the 1990s. Exceptions are the urban regions of Berlin, Dresden and Leipzig. Currently, 17% of the German population live in East Germany.

Regional differences in nominal wages do not always reflect differences in purchasing power. To this end, regional prices have to be taken into account. While the prices of many products are more or less the same all over Germany, this is not true for one of the

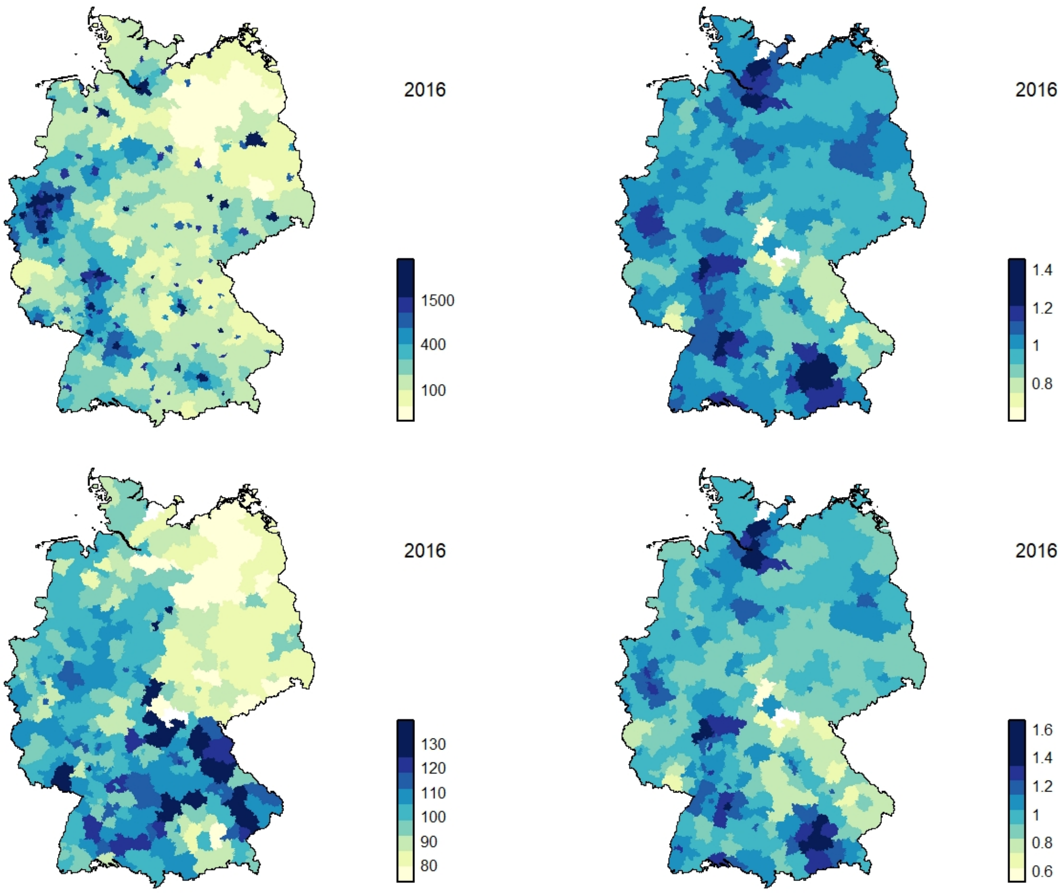

Figure 4: Regional data: population density 2016 (top left), overall price levels 2016 (top right), real wages and benefits 2016 (bottom left), housing price index 2016 (bottom right).

most important shares of the consumption basket: housing. The level of housing costs is far higher in urban areas than in rural ones. There are hardly any recent empirical studies that look at regional price differences in Germany. A study that is based on the official price measurements of the federal and state-level statistical offices is Weinand and von Auer (2020). Their price data set[8] from May 2016 includes housing, services and goods, and is normalized by the population weighted average price level, i.e. the population weighted mean is 1. The authors find that price levels are largely driven by

[8]Price levels for the regions Plön and Hildburghausen are not available.

housing costs and to a much lesser degree by the prices of goods and services.

The plot of regional price levels indicates that regions with a denser population tend to have higher price levels. Another important factor for local prices are spill-overs from neighbouring countries. Luxembourg and Switzerland have substantially higher price levels than Germany has. To avoid these costs, some expenditures are shifted to the German border regions which leads to a price increase there. Furthermore, it is not surprising that figure 4 also reveals inner-German spillover effects. The price levels tend to be higher in the neighbourhood of expensive regions than in the neighbourhood of inexpensive regions. The positive spatial correlation is mainly driven by housing. Overall, the price level in the most expensive region, Munich, is about 27% higher than in the cheapest region.

4.2 Nominal wages and benefits

Figure 5 shows the level of average daily gross wages or benefits for selected years between 1995 and 2017. Wages exceeding the contribution ceiling for statutory pension insurance are only reported up to this limit. Hence, the data are right censored. The contribution ceiling is time-varying and roughly twice as large as the overall average daily wage. Wage differences are especially large between East and West. Between 1995 and 2017 full-time employees in Germany's western states earn a daily average from 84.8 to 95.9 euros, their colleagues in East Germany earn about 30% less, i.e. from 56.5 to 72.3 euros. While the gap has been narrowing in recent years it is still substantial. Of course individual wages do not only depend on the region of work but also on many other factors such as the employees' experience, their qualification and the industrial sector. For example, two cities with very high wages are Wolfsburg and Ingolstadt (with less than 140,000 inhabitants) which are the headquarters for Volkswagen and Audi. Other high-wage industries in Germany are pharmaceuticals, banking and aviation. As a result we observe high salaries in Munich, Frankfurt and Hamburg. The highest wages at the state level are paid in Hesse and Baden-Württemberg. The annual "wage atlas" of Bierbach (multiple years) which analyses more than 490,000 observations confirms this

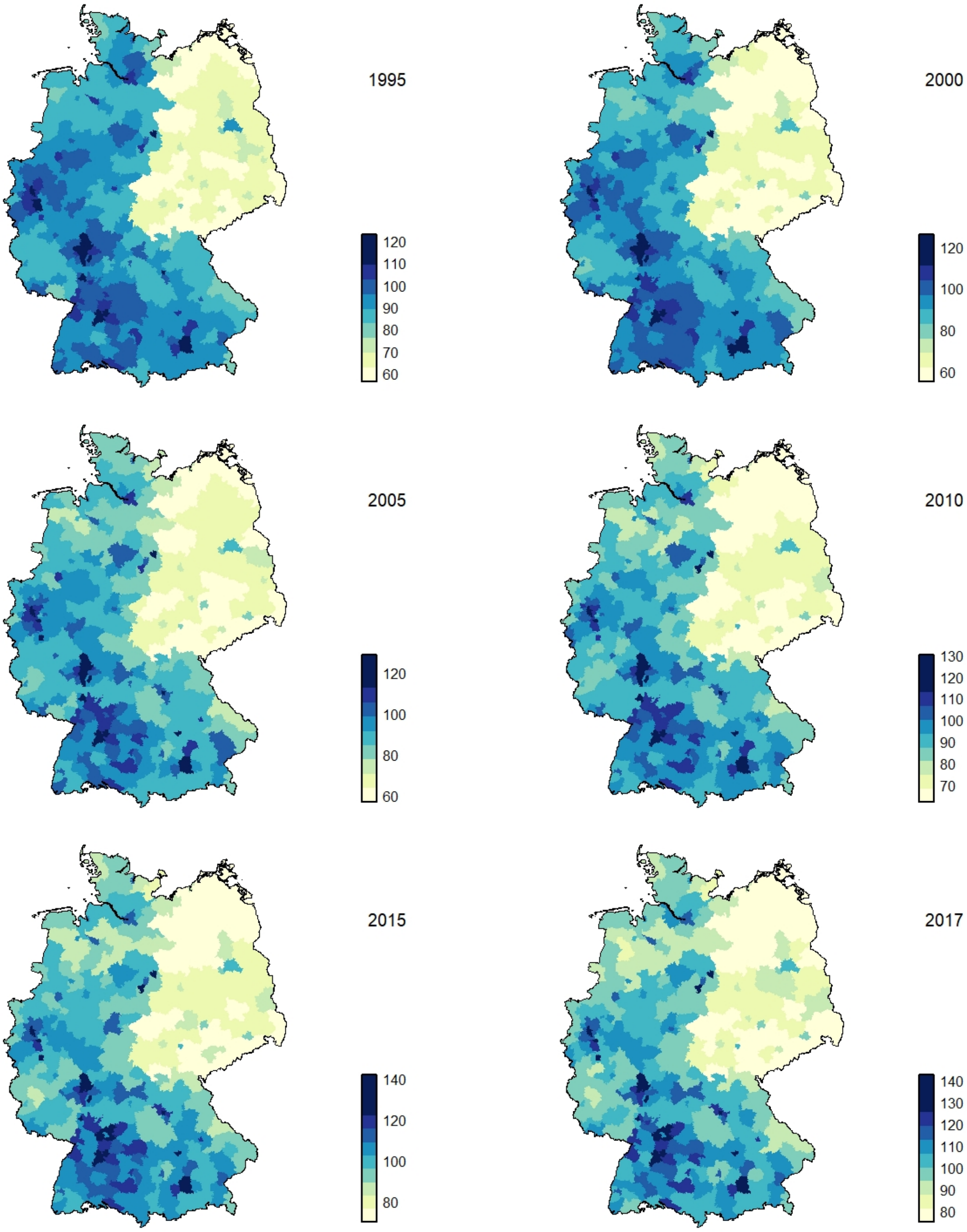

Figure 5: Regional wages and benefits. Note that the colour scale differs between the years.

statement. It reports that wages in Hesse and Baden-Württemberg are 14.1% and 8.6% higher than the nationwide average.

4.3 Unemployment rates

Concerning unemployment rates (see figure 6), there is substantial regional variation. High unemployment rates persist primarily in the East where the average unemployment rate between 1995 and 2017 amounted to 9.9% with a maximum of nearly 20% in the late 1990s.[9] Medium levels of unemployment rates exist in the northern and central West German Federal States including Schleswig-Holstein, Lower Saxony, North Rhine-Westphalia, Hesse, Rhineland-Palatinate and Saarland. Very low rates are observed in Southern Germany, i.e. Bavaria and Baden-Württemberg where the average unemployment rate is 5.2% with a minimum of 3.3% in Biberach.

Looking at the evolution over time, the unemployment rate has been falling sharply all over Germany. In 2017 no East German region had an unemployment rate of more than 8%, some regions in the West even reached full employment (with unemployment rates of less than 2%). The large and persistent differences between East and West Germany can be considered as a consequence of the long lasting German division. However, the differences in unemployment rates are clearly less persistent than the differences in the wage level. The very clear distinction in the unemployment level between East and West that was visible in 1995, is hardly discernible any longer in 2017.

4.4 Migration rates

Figure 7 displays net migration rates for selected years between 1995 and 2017. An advantage of the subdivision into 328 relatively small regions is that only movements within a region remain undetected. A coarser regional division (e.g. by NUTS 1, "Bundesland") would result in much lower rates since most migration occurs over shorter distances (see also section 5). In general, we observe low net migration rates for all regions, with the exception of Munich in 2005. A common finding in the literature is

[9] These numbers are lower than the official unemployment rates as explained in section 3.1.

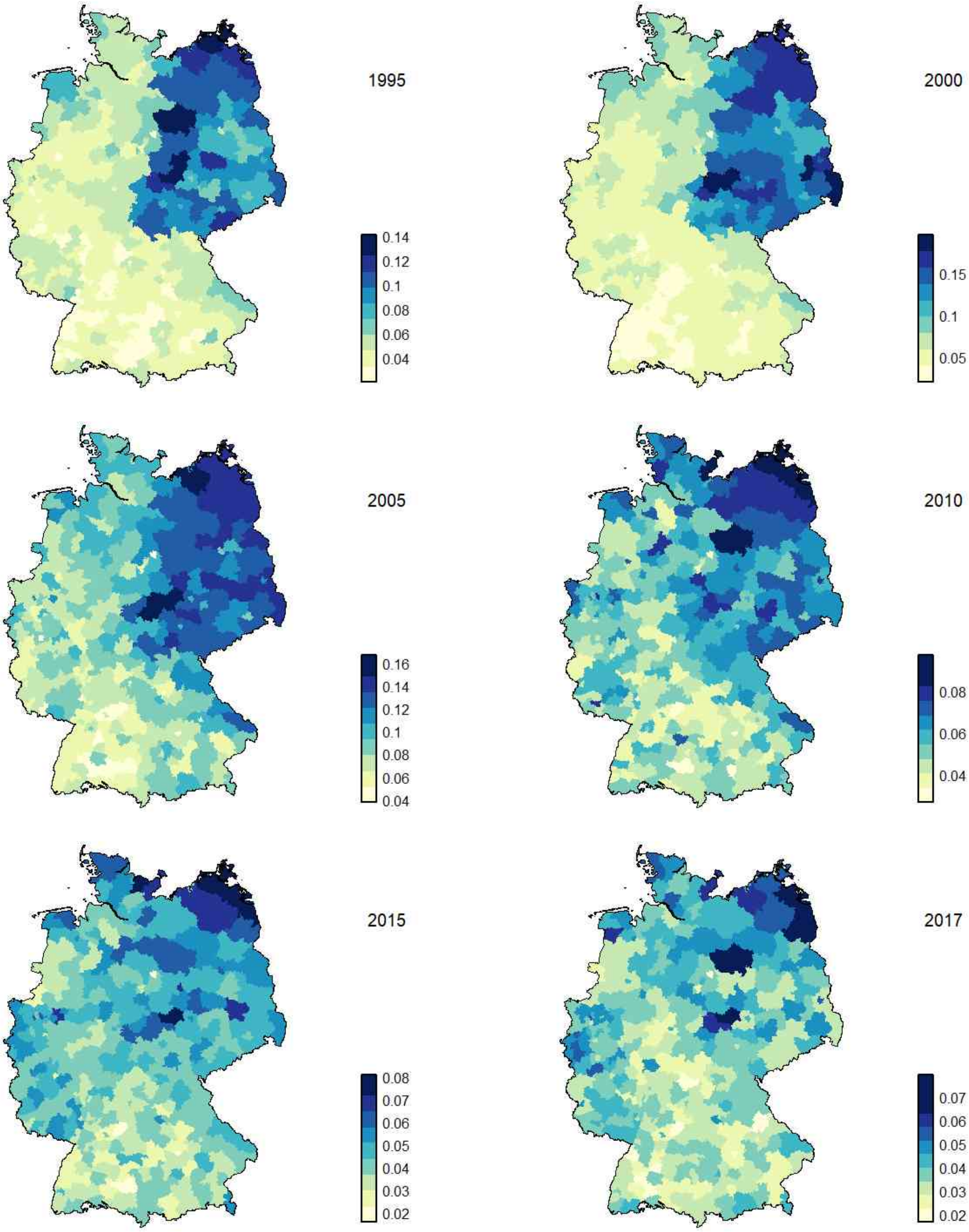

Figure 6: Regional unemployment rates. Note that the colour scale differs between the years.

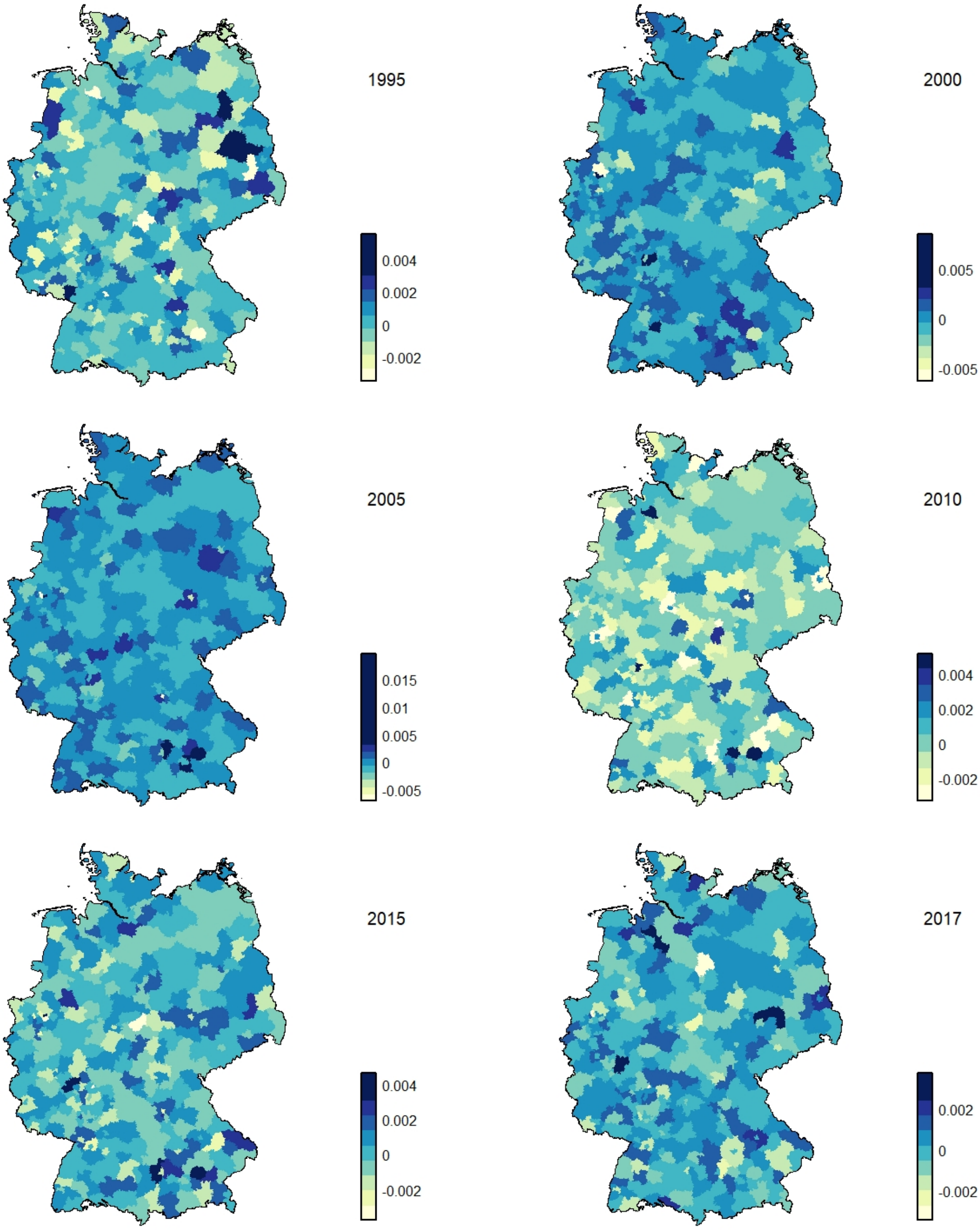

Figure 7: Regional net migration. Note that the colour scale differs between the years.

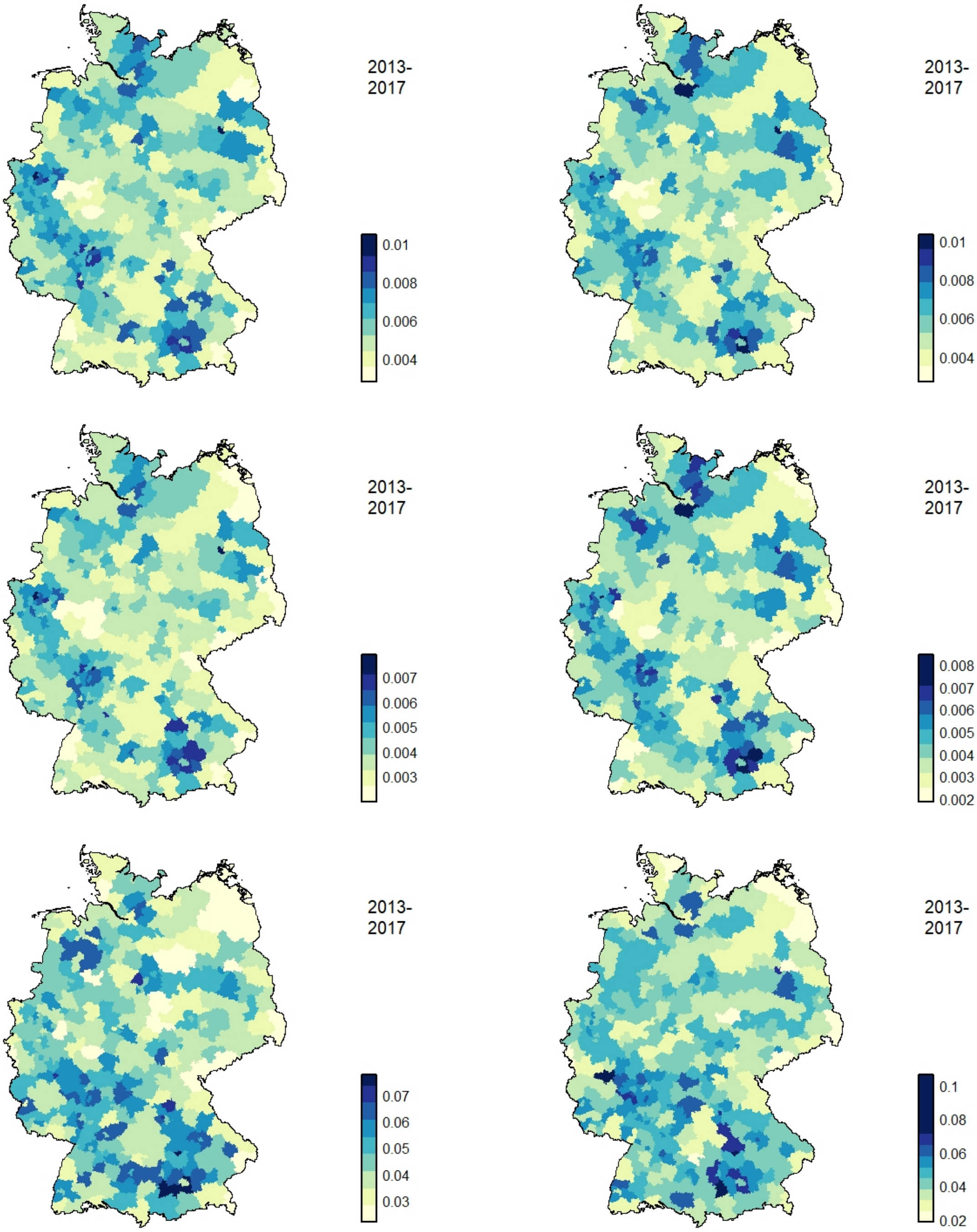

Figure 8: Regional movements (out of/into regions): all workers (top), employed (middle), unemployed (bottom). The rates are averaged over the years 2013-2017.

that migration flows from East to West Germany are substantially larger than in the other direction. Our calculations confirm a relatively low and decreasing net migration from East to West over time. While our data indicate negative net migration in 2017 for East Germany (-0.0034%), Bangel et al. (2019) computed a positive net migration for the first time since reunification.[10]

Further, we plot migration rates between emigrants (left) and immigrants (right) in figure 8, because net values can hide the extent of mobility between regions. The rates are averaged over 2013 to 2017. The regions with the lowest immigration rates are Saxony, Saxony-Anhalt and Mecklenburg-West Pomerania. Areas around metropolises such as Munich, Frankfurt or Hamburg experience the highest immigration rates. This development reflects a recent urbanisation trend. Looking at the differences between employed and unemployed workers, we find that employed workers are less likely to move and their movements are more concentrated into (sub)urban areas.

4.5 Relation between unemployment and other variables

We proceed to take a closer look at the relation between regional unemployment and other variables. Figure 9 shows the relationship between regional unemployment rates and population density; worker flows; movements; and wages (averaged over the years 2013 to 2017). The density tends to be negatively correlated with unemployment rates in sparsely populated regions. More precisely, doubling the density is associated with a decrease of the unemployment rate by nearly 1.5 percentage points for a population density of up to 200 inhabitants per square kilometre. For higher densities this association vanishes. For example, we observe high unemployment rates in the Ruhr area, represented by Dortmund, while there are equally densely populated cities across the country with low unemployment rates.

For employed workers we do not find a connection between moving rates and regional unemployment, but there is a notable negative relationship for unemployed workers:

[10]This is probably caused by a different calculation of the migration rate. We consider the place of work, while Bangel et al. (2019) consider data about the place of residence.

The higher the regional unemployment rate, the smaller the fraction of unemployed workers leaving that region. This is in conflict with the neoclassical adjustment theory that workers migrate from high to low unemployment regions. In particular, regions in East Germany might suffer from this effect. In figure 9, we Vorpommern-Greifswald is marked as a representative region for the disadvantaged East. Berlin and Munich are located below the regression line, which is probably due to migration between districts within their regions. Positive outliers might be explained by geographical proximity to economically powerful centres. An outstanding example is Erding, a satellite city located 31 km north of Munich. Many workers living in Erding tend to search for a job in Munich.

For the job finding rate (UE) and the job to job rate (EE) the scatter plots do not show any clear pattern, but the separation rate (EU) exhibits an approximately linear relationship to the unemployment rate. This association is presumably amplified by the fact that, in our calculations, workers who lose their job, and unemployed workers, by definition, do not move unless they find a new job.

Finally, we investigate the relationship between unemployment and earnings and benefits. Since unemployed workers in high-unemployment regions are less likely to move and tend to be less skilled, wages are negatively correlated with the unemployment rate. This effect is supported by the additional bargaining power of firms in case of larger unemployment pools. Taking a closer look at the wages of movers, we observe substantially higher wages for immigrants than for emigrants, but they are still below the region's average wage level.[11]

5 Regression modelling

The objective of this section is to describe the relation of interregional migration flows and other variables between 1995 and 2017 by regression techniques. Based on all pairwise migration flows between the 328 regions, we estimate the role of demographic,

[11] On average wages for immigrants are 40.1% higher than for emigrants, while they are 9.6% lower than the average wage in their new working region.

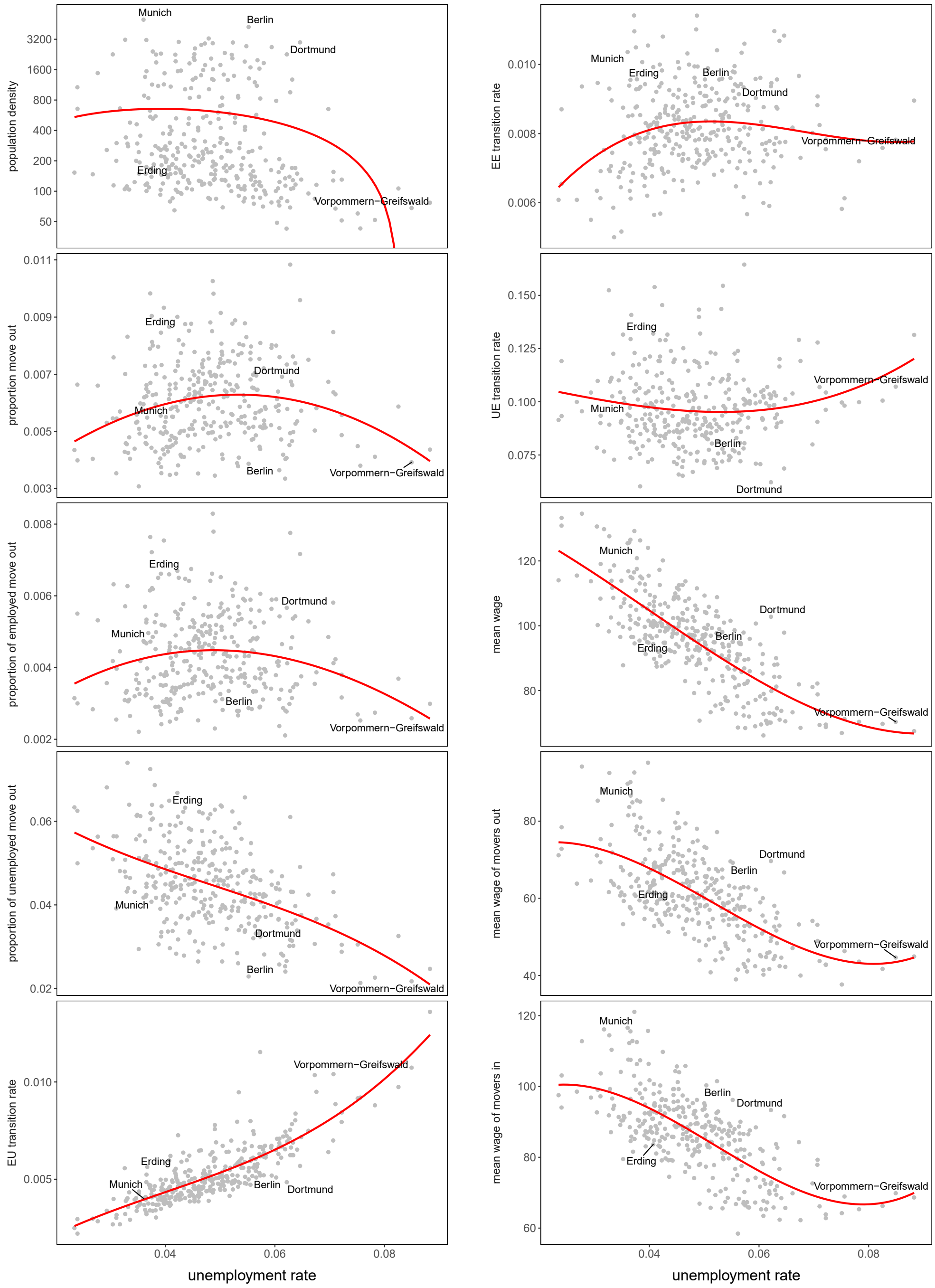

Figure 9: Relationship between unemployment and other economic variables (population density, proportion of workers moving out, proportion of employed workers moving out, proportion of unemployed workers moving out, EU transition rate, EE transition rate, UE transition rate, mean wage, mean wage of movers out, mean wage of movers in).

geographic and economic factors for regional mobility. The regression model is

$$E(M_{ij,t}|\mathcal{F}_t) = \exp\Bigg(\alpha_i^{orig} + \alpha_j^{dest} + \alpha_t^{time} + \beta_1 \ln D_{ij} + \beta_2 \ln P_{i,t} + \beta_3 \ln P_{j,t} + \sum_{s=1}^{n} \gamma_s \ln X_{s,i,t} + \delta_s \ln X_{s,j,t}\Bigg) \quad (1)$$

where $M_{ij,t}$ is the absolute number of migrants from origin region i to destination region j in year t, D_{ij} is the geographical distance between region i and j (calculated as the beeline between the centre points in metres), $P_{i,t}$ and $P_{j,t}$ are the population sizes in regions i and j in year t, and $X_{s,i,t}$ and $X_{s,j,t}$ are lists of economic variables in region i and j, respectively, that could act as push or pull factors for interregional migration. Specifically, these factors include the wage level and the unemployment rate. Finally, the model controls for regional fixed effects of the origin (α_i^{orig}) and the destination (α_j^{dest}). Time fixed effects are denoted as α_t^{time}. The information set $\mathcal{F}_t$ in the conditional expectation includes all regressors.

We estimate (1) using the Poisson pseudo maximum likelihood (PPML) method suggested by Santos Silva and Tenreyro (2006) for estimating gravity models of trade flows. Compared to the more common log-transformation of (1) this approach has two advantages. First, it avoids the inconsistency caused by correlation between the regressors and the error term when there is heteroskedasticity which is linked to regressors. Second, taking the logarithm of the dependent variable $M_{ij,t}$ is not possible for zero observations. In our case, excluding zero observations would reduce the number of observations by more than 90%. In addition to the loss of information, omitting zero observations can induce bias if they are non-random.

The scatterplots in figure 9 inform us that the migration behaviour of the unemployed may differ considerably from the overall population. Therefore, we estimate separate regression models of the migration flows (i) for all workers, and (ii) for the unemployed.

5.1 All workers

The regression estimates for all workers are reported in table 1. As already demonstrated in figures 4, 5 and 6, there is considerable heterogeneity in economic and labour market conditions across Germany's regions. Most coefficients in our base-line model (column 2 in table 1) are in line with extant studies, see Bauer et al., 2019, or Liu, 2018. The distance coefficient is highly significantly negative, indicating that a longer distance lowers the number of migrants with an estimated elasticity of -1.81. Population sizes have, as expected, a positive impact on the absolute number of migrants. Both coefficients have almost the same size. Densely populated regions are subject to both higher in- and out-migration but the elasticities are significantly smaller than unity. Workers in regions with high unemployment rates are more likely to move to a region with fewer unemployed workers to increase their chances of finding better jobs. Counter-intuitively, higher wages in the destination region are associated with lower migration.

In column (3) and (4), we split the sample according to whether the origin region is in East or West Germany and compare the different effects of economic and labour market conditions on internal migration in the two subsamples. The estimation result suggests that distance tends to be a more dampening factor for workers in East Germany. They are less likely to move over long distances. In the East, regional migration mainly takes place between larger cities and their hinterland. The values of some coefficients are nearly twice as large as for the West German subsample, indicating that population size and unemployment rates have a greater impact on the incentive to move for workers in East Germany. While interregional mobility of West German workers is less motivated by these factors, wages in the destination region seem to be have an additional slow-down effect on migration decisions in West Germany.

Next, we compare the migration behaviour of men and women (columns 5 and 6). The effects of distance and population size are similar for both genders and coincide with the estimated value of the joint regression (in column 2). Concerning the unemployment rate we observe a notable difference. The migration decision of men is more strongly linked to the unemployment rates than that of women. This is true both for the unemployment rate in the origin region and in the destination region. Wages are only a significant factor to stay or to leave for men.

Bauer et al. (2019) investigate internal migration of different age groups between 2008 and 2014. In contrast to our study they consider the place of residence rather than the place of work. Their analysis of determinants of internal migration is based on data from various sources (e.g. information on the number of internal migrants by the German population registers). Bauer et al. (2019) exclude individuals with foreign nationalities as they believe that the behaviour of international migrants might be systematically different from the behaviour of natives. Overall, our results coincide with their estimates for the region of origin. The estimates concerning the destination region differ. While their estimated coefficient for the population size in the destination is close to zero and not significant, we observe a positive impact with an elasticity of more than 0.8. In our study, the elasticity of the wage level in the destination region is significantly negative whereas Bauer et al. (2019) find a positive association.

Table 1: Gravity Model of Internal Migration - estimated by PPML; the dependend variable is the (absolute) number of migrants between the origin and destination region.

Variable	**Total** (1)	**Total** (2)	**West** (3)	**East** (4)	**Men** (5)	**Women** (6)
distance	-1.5690***	-1.8137***	-1.7981***	-2.0934***	-1.7961***	-1.8567***
	(0.0070)	(0.0038)	(0.0042)	(0.0100)	(0.0040)	(0.0056)
population_o	0.9985***	0.8769***	0.6669***	0.9992***	0.8840***	0.6762***
	(0.0061)	(0.0529)	(0.0857)	(0.0986)	(0.0520)	(0.0586)
population_d	0.9974***	0.8293***	0.6415***	1.1629***	0.8480***	0.7058***
	(0.0064)	(0.0554)	(0.0712)	(0.0788)	(0.0541)	(0.0598)
unemployment_o	0.1953***	0.1168***	0.0815***	0.2772***	0.1231***	0.0828***
	(0.0199)	(0.0249)	(0.0280)	(0.0698)	(0.0206)	(0.0260)
unemployment_d	-0.0764***	-0.2462***	-0.2256***	-0.3535***	-0.1823***	-0.1246***
	(0.0197)	(0.0252)	(0.0267)	(0.0508)	(0.0203)	(0.0272)
wages_o	-0.0645	-0.6810***	-0.2495	0.0859	-0.9185***	-0.2683
	(0.0401)	(0.1669)	(0.1988)	(0.3996)	(0.1564)	(0.1759)
wages_d	-0.1581***	-0.8830***	-0.6557***	-0.2805	-1.1465***	-0.0600
	(0.0402)	(0.1652)	(0.1830)	(0.3500)	(0.1548)	(0.1731)
N	2,466,888	2,466,888	2,015,628	451,260	2,466,888	2,466,888
R^2	0.1202	0.6879	0.6952	0.6912	0.6462	0.6312
Regional FE	no	yes	yes	yes	yes	yes
Time FE	no	yes	yes	yes	yes	yes

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

5.2 Unemployed workers

Motivated by the surprising shape of the scatterplot "proportion of unemployed moving out" (in figure 9) where the proportion of unemployed workers moving out of regions decreases in the unemployment rate, we re-estimate the previous regression model for the subset of unemployed workers. The regression estimates are reported in table 2. An important factor that keeps unemployed workers from migrating is still distance. In this regression, population sizes are the absolute numbers of unemployed workers. The elasticities with respect to population sizes are larger than unity but not significantly so. As to the outcomes for unemployment rates the picture changes starkly: Higher unemployment rates in the origin region lead to *fewer* unemployed migrants implying that migration fails to act as an adjustment process for differences in the unemployment rate. Even though wages in the destination region appear to be a significant factor, its sign is negative. Unemployed workers are less likely to move into regions with high wages.

Focusing on the differences between East and West Germany (columns 3 and 4), we see again that distance seems to be a more important factor for unemployed workers in East Germany. The elasticities of population sizes (i.e. the total number of unemployed workers in the origin and destination regions) are nearly twice as large as in West Germany. Keeping all other factors constant, doubling the number of unemployed workers in the origin region is associated with roughly a quadrupling of the number of migrants.

Comparing the migration behaviour of men and women (columns 5 and 6) we find that the coefficients of distance and population sizes are as expected and not much different from the joint case (in column 2). In contrast to the set of all workers, the coefficients for unemployment rates are smaller in absolute value for unemployed men than for unemployed women. As to wages, a high level in the origin region is associated with more men but fewer women moving out. Remarkably the coefficient of the destination region's wage level is highly negative for women.

As a concluding remark it is important to keep in mind that all results should be regarded as correlations rather than causal effects, as some regressors might suffer from endogeneity. For example, it is obviously conceivable that migration has an effect on unemployment rates and wages, leading to simultaneous causality.

Table 2: Gravity Model of Internal Migration - estimated by PPML; the dependend variable is the (absolute) number of unemployed migrants between the origin and destination region.

Variable	**Total** (1)	**Total** (2)	**West** (3)	**East** (4)	**Men** (5)	**Women** (6)
distance	-1.5606***	-1.7743***	-1.7472***	-2.2554***	-1.7939***	-1.7402***
	(0.0107)	(0.0232)	(0.0273)	(0.0572)	(0.0027)	(0.0362)
population_o	1.1011***	1.2534***	1.0326***	2.6637***	1.2961***	0.9173**
	(0.0247)	(0.2409)	(0.3934)	(0.5736)	(0.2624)	(0.3618)
population_d	1.0844***	1.3080***	1.1427***	2.1238***	1.3640***	1.4565***
	(0.0223)	(0.2466)	(0.3228)	(0.4547)	(0.2600)	(0.3737)
unemployment_o	-0.3342***	-0.7997**	-0.7165*	-0.6350	-0.6086**	-0.9826**
	(0.0912)	(0.2926)	(0.3980)	(0.6797)	(0.2932)	(0.4400)
unemployment_d	-0.3781***	-1.2286***	-0.8470**	-2.4836***	-1.1600***	-1.8255***
	(0.0877)	(0.3111)	(0.3823)	(0.6438)	(0.2967)	(0.4600)
wages_o	-0.3051*	1.0488	1.9392*	-1.7369	2.5125***	-2.7885**
	(0.1650)	(0.8957)	(1.0892)	(2.1151)	(0.9535)	(1.1319)
wages_d	0.5168***	-2.4881**	-1.3578	-5.2339***	-1.0861	-5.0247***
	(0.1635)	(0.8969)	(1.0823)	(1.7694)	(0.9537)	(1.1535)
N	2,466,888	2,466,888	2,015,628	451,260	2,466,888	2,466,888
R^2	0.9901	0.9908	0.9912	0.9908	0.9933	0.9963
Regional FE	no	yes	yes	yes	yes	yes
Time FE	no	yes	yes	yes	yes	yes

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

6 Conclusion

This paper provides a comprehensive descriptive picture of Germany's regional labour mobility and highlights substantial dynamics over the period 1995 to 2017. Most extant papers have focused on net migration or state level (NUTS 1) patterns for East and West Germany. Using data from the Institute for Employment Research, our analysis overcomes the limits imposed by state level boundaries that have impeded a fine-grained description of labour migration flows. We investigate interregional migration between 328 (NUTS 3) regions and find that both emigration and immigration mostly take place in more urban and economically prosperous regions. On average, the fraction of movements among unemployed workers is around five times higher than that of employed workers. On the aggregated level, the amount of movements rises and falls procyclicly.

Even 30 years after the fall of the wall there are not only persistently large differences in unemployment rates, population density, wages and education levels between East and West Germany, but there are also notable differences between smaller regions. According to plain economic theory, migration acts as a compensator for such disparities, but our results do not show an adjustment trend. In fact, we find that the opposite seems to hold: for unemployed workers emigration has a clearly negative relationship with the unemployment rate.

Finally, our regression results confirm our observations which are consistent with many findings of the existing literature. Distance and population are the most important regressors for migration. Even if – or simply because – family and friends have a big impact on the decision to migrate, most movements are directed to the next bigger city in the surrounding area. Economic variables such as unemployment and wages play in general a much smaller role.

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Directed Search on the Job in Local Labour Markets

Abstract

This paper presents a model of directed search on the job within an economy in which workers are free to choose their place of residence. All movements between regions and employment states are endogenous, resulting in a high-dimensional set of variables. The aim is to prove the existence, uniqueness and efficiency of a Block Recursive Equilibrium (BRE) in which the agent's value functions, their corresponding policy functions and the market tightness functions do not depend on the distribution of workers across regions and employment states. To do this, it is shown that the allocation of the social planner coincide with the BRE. In the empirical part, an example is used to present possible applications and to discuss the strengths and weaknesses of the model.

Keywords: directed search, on the job search, BRE

1 Introduction

In this paper, I provide a labour market model with internal migration. More specifically, all agents (workers and firms) are free to choose their place of residence, based on a finite number of regions. My contribution is embedded in the directed search literature (see Moen (1997) or Acemoglu and Shimer (1999)), meaning that workers know the terms of trade before they apply for a job. The motivation for adding space to the analysis is evident. So far, the corresponding literature has focused on studying transitions between employment states (employed or unemployed) with the aim of explaining fluctuations in unemployment rates. However, this ignores persistent regional differences. In addition, a large part of job-to-job transitions is associated with a relocation of workers, which has an implicit impact on the unemployment rate.

The pioneering work of Menzio and Shi (2007, 2009 and 2011) provides a model of directed search in which transition rates between employment states are endogenous. They showed that there is a unique and socially efficient allocation in which value accruing to agents and policy functions do not depend on the distribution of workers across employment states when the contracts between workers and firms are complete.[1] This allocation is called a Block Recursive Equilibrium (BRE). I take their contribution to the literature and add the component of local labour markets. Every worker, employed or unemployed, who searches for a job is faced with transition costs. Because of the directed search process, workers in low-productivity employment states search for vacancies which offer only a low utility increase, but are easy to find. These jobs require less human capital and are more likely to be located in the worker's surrounding area. Workers in high-productivity employment states search for vacancies which offer high benefits, but are difficult to find. These jobs are rare and spread all over the country. In combination, this leads to different transition costs depending on the region of origin, the destination region, and the current level of productivity. Since I adopt the concept of complete contracts, which guarantees that a match between a worker and a firm does

[1] A detailed description of complete contracts is given in the next section.

not leave any gains from trade unexploited, I can show that my model also has a unique and efficient BRE.

The history of directed search literature is still young. 30 years ago Montgomery (1991) and Peters (1991) developed the first models of directed search. These were followed by papers in which transitions between employment and unemployment are endogenous and also allow for searching while on the job (see Delacroix and Shi (2006) or Li and Tian (2013)). The property of block recursivity is introduced for the first time in Shi (2009). He assumed that contracts are incomplete[2], which does not lead to a unique and efficient solution. Recently, several other papers have been published that include this property. Li and Weng (2017), based on Menzio and Shi (2011), created a continuous time model with Gaussian learning[3], which satisfies the block recursivity. Kaas (2020) discusses more general model frameworks, and Schaal (2017) exploits block recursivity to provide an exhaustive and quantitative analysis for labour markets with firm dynamics and time-varying idiosyncratic productivities.

My work opens a novel field in the directed search literature. Theoretically, the model is able to explain regional worker stocks by individual movements. Specifically, I distinguish between movements from employed to unemployed, called the separation rate (EU rate), from unemployed to employed, called the job finding rate (UE rate), and from one employer to another, called the job-to-job rate (EE rate). When looking at aggregate values for the entire economy, the above rates are averaged, and potential regional fluctuations are missed. Intervention at the country level could lead to undesirable results, whereas adjustments at the regional level are appropriate. In practice, the application of the model requires some specifications. Both agents will develop an affection for certain regions, based on their economic power. Unfortunately, these so-called local amenities are difficult to measure and to evaluate. This leads to the current state of research that direct search models are only suitable for describing regional labour markets if local amenities can be measured adequately.

[2] Wages are set as a function of idiosyncratic productivity or simply drawn at random. In both cases, they are not affected by external offers to workers.

[3] A worker-firm pair understands its quality over the duration of the match.

The paper is organized as follows. The next section describes the physical model of the economy and the contractual environment. In section 3, I formulate the worker's and the firm's problem. Section 4 presents the definition of a BRE and the social planner's problem. I show that the social planner's value function is a contraction, and that the generated allocation satisfies the conditions of a BRE. Section 5 provides and critically discusses an example, based on data from the Institute for Employment Research. Section 6 concludes.

2 The Model

Let $\mathcal{N}(c) \geq 2$ be an integer. Consider a labour market which is separated in $C = \{1, \ldots, \mathcal{N}(c)\}$ different regions with a continuum of infinitely living workers that is normalized to one and a continuum of firms with a positive measure. Each worker, whether employed or unemployed, has a periodical utility function $v(\cdot)$, where $v : \mathbb{R} \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly increasing, weakly concave function with $v'(\cdot) \in [\underline{v}', \bar{v}']$. While employed workers receive a wage w, unemployed workers receives a benefit $b > 0$ from home production or leisure. Each worker maximizes expected lifetime utility by discounting the future at rate $\beta \in (0, 1)$.

Time is discrete and has an infinite horizon. At the beginning of the period, the state space of the economy is described by the vector $(y, u, g) = \psi \in \Psi$. The first element y is the aggregate component of labour productivity, describing the business cycle and is realised by the elements $Y = \{y_1, y_2, \ldots, y_{\mathcal{N}(y)}\}$, where $\underline{y} = y_1 \leq y_2 \leq \cdots \leq y_{\mathcal{N}(y)} = \bar{y}$ and $\mathcal{N}(y) \geq 2$ is an integer. The second element $u : C \rightarrow [0, 1]$ is a function that describes the measure of unemployed workers in region $c \in C$. Finally, g is a function $C \times Z \rightarrow [0, 1]$ that describes the measure of employed workers in region $c \in C$ with idiosyncratic productivity $z \in Z = \{z_1, z_2, \ldots, z_{N(z)}\}$, where $\underline{z} = z_1 \leq z_2 \leq \cdots \leq z_{N(z)} = \bar{z}$ and $N(z) \geq 2$ is an integer. As described above, population is normalized to one, which means $\sum_c \big(u(c) + \sum_z g(c, z)\big) = 1$. An employed worker is endowed with a unit of labour and transforms it into $y + z$ units of output. Each firm maximizes the expected lifetime profits by discounting the future at rate $\beta \in (0,1)$.

The search process between workers and firms is directed. Firms offer contracts to attract workers and compete with each other. Therefore, for each region, the labour market is separated into submarkets denoted by the expected lifetime utility $x \in X = [\underline{x}, \bar{x}]$ with $\underline{x} < v(b)/(1-\beta)$ and $\bar{x} > v(\bar{y}+\bar{z})/(1-\beta)$ that firms promise to workers. Workers are free to choose in which submarket they search for a job, but they can visit only one submarket at a time. In submarket (x, c) a match of a firm and a worker depends on the ratio between the number of vacant jobs and the number of workers searching for jobs in this submarket and is denoted by $\theta(x, c, \psi)$. This is widely known as the tightness of a submarket. In the remaining paper, I use the notation (x, r) for describing distinct submarkets. In this respect, I introduce the function $r : C \times Z \times \Psi \to C$ that determines the region in which workers search for a new job based on the state of the economy, their idiosyncratic productivity and their current residence. To keep it short, I call it the restriction.

The timing within periods is divided into four stages, namely separation, searching, matching and production. Figure 10 provides an illustration. In the first stage, an employed worker of type z in region c becomes unemployed with probability $d(c, z, \psi) \in \{\delta, 1\}$, where $\delta \in (0,1)$ is exogenous job destruction. If the productivity of a match is lower than in unemployment, the worker will decide to separate from the firm, that is $d = 1$. Otherwise it holds that $d = \delta$.

The second stage, searching, gives firms the opportunity to create vacancies in any submarket at a cost of $k \geq 0$ for each period. The searching process for workers is directed, depends on their employment history (explained in detail later) and takes time. Therefore, only a fraction of workers are able to do so. If a worker is unemployed at the beginning of the period, his search efficiency is $\lambda_u \in [0,1]$. Employed workers who did not lose their job in the first stage are less efficient in searching for a new job. Their efficiency is denoted by $\lambda_e \in [0,1]$. Workers who lost their job during the separation stage can start searching for a new job only in the next period.

In the third stage, the matching process between workers and firms is governed by a

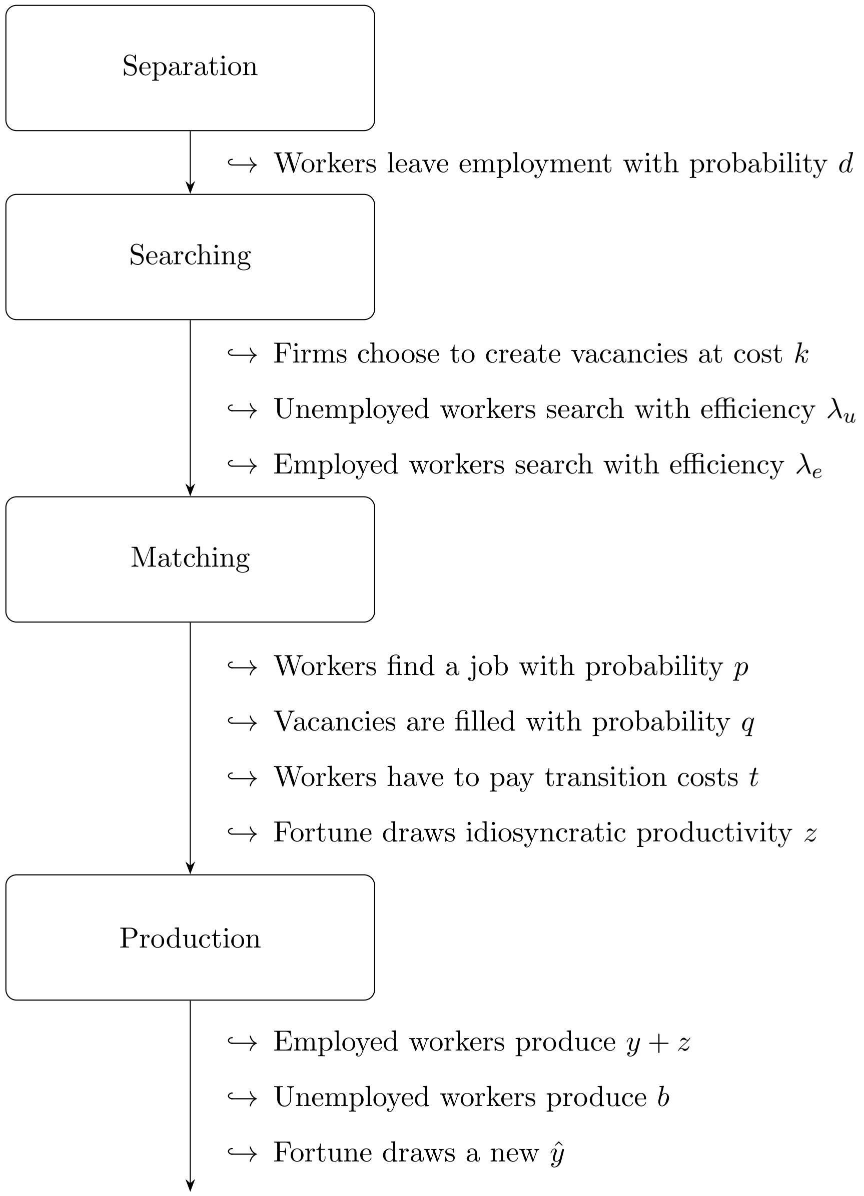

Figure 10: Timing

matching function. Let ψ denote the current state of the economy an $\theta(x, r, \psi)$ denote the tightness in submarket (x, r). In this submarket, workers find a job with probability $p(\theta(x, r, \psi))$, where $p : \mathbb{R}^+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function with $p(0) = 0$. Similarly, I define $q(\theta(x, r, \psi))$ as the probability that a vacancy is filled with a worker, where $q : \mathbb{R}^+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly decreasing, convex function with $q(\theta) = p(\theta)/\theta$, and $q(0) = 1$. When a firm and a worker create a match in this submarket, the worker receives an employment contract that promises lifetime utility x. At the same time, the worker is faced with costs $t(c, z, r)$, where $t : C \times \{Z, b\} \times C \rightarrow \mathbb{R}$ is the transition costs of moving from $(c, \{z, b\})$ to r. At the end of the stage, the worker is endowed with an idiosyncratic productivity $z \in Z$, where z is drawn from the probability function $f_r(z) : \; Z \rightarrow [0, 1]$ independently of the previous characteristics of the worker.

In the last stage, an employed worker produces $y + z$ units of output and receives the wage w which is determined by the employment contract. Since the model does not contain a government, the unemployment benefit $b \in (0, \bar{y} + \bar{z})$ can be interpreted as either home production or leisure. The period ends with a new realisation of the aggregated productivity $\hat{y}$ which follows a Markov process and is drawn from the probability function $\phi_{\hat{y}} : Y \times Y \rightarrow [0, 1]$. To simplify the notation, I omit time subscripts and use the caret to identify the following period.

In this paper, I assume that employment contracts are complete. That is, both agents have all available information, so that the joint value of a match, the sum of the worker's lifetime utility and the firm's lifetime profits, is maximized. Important information for a complete contract are the wage, w, the separation probability, d, and the submarket where the worker can search for a new job, (x, r), which all depend on the idiosyncratic productivity, z, of the worker and the current state of the economy, ψ. Menzio and Shi (2009) show that complete contracts are bilaterally efficient. Therefore, I do not need to describe the individual utility or profit functions of workers and firms and can restrict the model to the joint value of a match. In particular, the model does not need a wage-bargaining process.

3 Agents' problems

3.1 Lifetime Utility

Consider an unemployed worker in region c. The lifetime utility of this worker at the beginning of the production stage is given by

$$V_u(c,\psi) = v(b) + \beta \mathbb{E} \max_{x,r} \big\{ V_u(c,\hat{\psi}) + \lambda_u R_u\big(x,r,V_u(c,\hat{\psi}),c,\hat{\psi}\big)\big\}, \tag{2}$$

where $\mathbb{E}$ denotes the conditional expectation on $\hat{\psi}$ and R_u is defined as

$$R_u\big(x,r,V,c,\psi\big) = p(\theta(x,r,\psi))(x - V - t(c,b,r)). \tag{3}$$

The lifetime utility of an unemployed worker consists of the current home production or leisure b, evaluated by v, plus the discounted expected value of being unemployed at the beginning of the next period. The probability of finding a job in submarket (x,r) is given by $\lambda_u p(\theta(x,r,\hat{\psi}))$. In this case the worker receives utility x and has to pay transition costs $t(c,b,r)$. If the worker stays unemployed, he receives the utility of being unemployed, depending on the state of the economy in the next period $V_u(c,\hat{\psi})$. Finally, I denote $(x_u(c,\psi), r_u(c,\psi))$ as the solutions to the maximization problem in 2. Note that the optimal solution does not necessarily contain the highest $x \in X$ among all restrictions $r \in C$, but contains the highest weighted utility level.

Now consider an employed z-worker (a worker with idiosyncratic productivity z) in region c. The joint lifetime utility of the worker and the firm at the beginning of the production stage is given by

$$\begin{aligned} V_e(c,z,\psi) = y + z + \beta \mathbb{E} \max_{d,x,r} \big\{ & d(c,z,\hat{\psi}) V_u(c,\hat{\psi}) + (1 - d(c,z,\hat{\psi})) V_e(c,z,\hat{\psi}) \\ & + (1 - d(c,z,\hat{\psi}))\lambda_e R_e(x,r,V_e(c,z,\hat{\psi}),c,z,\hat{\psi})\big\}, \end{aligned} \tag{4}$$

where R_e is defined as

$$R_e\big(x,r,V,c,z,\psi\big) = p(\theta(x,r,\psi))(x - V - t(c,z,r)).$$

The joint lifetime utility consists of the current production $y + z$ and the discounted expected value of being employed at the beginning of the next period. In the first stage, the match is separated with probability $d(c, z, \hat{\psi})$, leading to a worker's lifetime utility of $V_u(c, \hat{\psi})$ and a firm's profit of 0. The worker finds a new job in submarket (x, r) with probability $(1 - d(c, z, \hat{\psi}))\lambda_e p(\theta(x, r, \hat{\psi}))$. In this case, the lifetime utility of the worker becomes x minus the transition costs $t(c, b, r)$, while the previous firm's profit is 0. Otherwise, the match between the worker and the firm continues, and they receive the utility $V_e(c, z, \hat{\psi})$. As above denote $(d_e(c, z, \psi), x_e(c, z, \psi), r_e(c, z, \psi))$ as the solutions to the maximization problems in 4.

3.2 Free entry

Firms operate under the same conditions and have labour as their only input. In each period, they are free to choose whether to create vacancies at a cost k. Hence, the total vacancy cost of hiring exactly one worker in submarket (x, c) is given by $k/q(\theta(x, c, \psi))$. When a match is created, the firm transfers the promised utility x to the worker. Taken together, the firm aims to minimize the hiring costs

$$k^* = \min_{x,c} \left\{ \frac{k}{q(\theta(x, c, \psi))} + x \right\}.$$

This condition can only be used for submarkets that are actively visited. To summarize for all submarkets, I can write the complementary slackness condition

$$\theta(x, c, \psi) \left[\frac{k}{q(\theta(x, c, \psi))} + x - k^* \right] = 0.$$

In other words, submarkets either minimize the hiring cost, k^*, or remain unvisited, that is $\theta(x, c, \psi) = 0$. The value of a filled vacancy in submarket (x, c) is given by the expected joint lifetime utility $\sum_z V_e(c, z, \psi) f_c(z)$. Firms will post vacancies as long as the expected value exceeds the hiring cost k^*, leading to the market tightness condition

$$q(\theta(x, c, \psi)) \Big[\sum_{z \in Z} V_e(c, z, \psi) f_c(z) - x \Big] = k \tag{5}$$

or $\theta(x, c, \psi) = 0$. Later, I will show that the free-entry condition is necessary to guarantee the existence of a BRE.

3.3 Laws of Motion

Using the optimal solutions for workers and firms from equations 2 and 4, I can describe the evolution of employment states over time. To do so, consider an unemployed worker and an employed z-worker in region c. Let $\theta_u(c,\psi)$ denote $\theta(x_u(c,\psi), r_u(c,\psi), \psi)$ and let $\theta_e(c,z,\psi)$ denote $\theta(x_e(c,z,\psi), r_e(c,z,\psi), \psi)$. In the next period, the unemployed worker finds a z-job with probability $\lambda_u p(\theta_u(c,\psi)) f_{r_u(c,\psi)}(z)$, while he remains unemployed with probability $1 - \lambda_u p(\theta_u(c,\psi))$. The employed z-worker loses the job with probability $d(c,z,\psi)$. Therefore, the law of motion for unemployed workers is

$$\hat{u}(c) = u(c)\big(1 - \lambda_u p(\theta_u(c,\psi))\big) + \sum_{z\in Z} d(c,z,\psi) g(c,z). \tag{6}$$

The probability of an employed z-worker finding a new job with idiosyncratic productivity $\tilde{z}$ is $\big(1 - d(c,z,\psi)\big)\lambda_e p\big(\theta_e(c,z,\psi)\big) f_{r_e(c,z,\psi)}(\tilde{z})$ and he remains in the current employment position with probability $\big(1 - d(c,z,\psi)\big)\big((1 - \lambda_e p(\theta_e(c,z,\psi))\big)$. Hence, the law of motion for employed workers is given by

$$\hat{g}(c,z) = h(c,\psi) f_c(z) + (1 - d(c,z,\psi))(1 - \lambda_e p(\theta_e(c,z,\psi))) g(c,z), \tag{7}$$

where $h(c,\psi)$ are hired workers in region c and is given by

$$\begin{aligned} h(c,\psi) = \sum_{c'\in C} \Big(& u(c')\lambda_u p(\theta_u(c',\psi)) \mathbb{1}_{\{r_u(c',\psi)=c\}} \\ & + \sum_{z\in Z} (1 - d(c',z,\psi)) \lambda_e p(\theta_e(c',z,\psi)) g(c',z) \mathbb{1}_{\{r_e(c',z,\psi)=c\}} \Big). \end{aligned}$$

The function h uses the indicator function $\mathbb{1}_{\{.\}}$ to ensure that only those workers (employed and unemployed) are counted who find a job in region c.

4 The Block Recursive Equilibrium

4.1 The Definition of a BRE

In this section, I introduce the key element of this paper, a block recursive equilibrium. To do this, I first define a recursive equilibrium.

Definition 4.1. *A recursive equilibrium consists of a market tightness function* $\theta: X \times C \times \Psi \to \mathbb{R}_+$*, an unemployment value function* $V_u: C \times \Psi \to \mathbb{R}$ *with policy functions* $(x_u, r_u): C \times \Psi \to X \times C$*, a joint value function for the firm-worker match* $V_e: C \times Z \times \Psi \to \mathbb{R}$ *with policy functions* $d: C \times Z \times \Psi \to [\delta, 1]$ *and* $(x_e, r_e): C \times Z \times \Psi \to X \times C$*. These functions satisfy the following conditions:*

I) $\theta(x, c, \psi)$ *satisfies* 5 *for all* $(x, c, \psi) \in X \times C \times \Psi$*,*

II) $V_u(c, \psi)$ *satisfies* 2 *for all* $(c, \psi) \in C \times \Psi$ *and* (x_u, r_u) *are the associated policy functions,*

III) $V_e(c, z, \psi)$ *satisfies* 4 *for all* $(c, z, \psi) \in C \times Z \times \Psi$ *and* d *and* (x_e, r_e) *are the associated policy functions,*

IV) $\hat{u}(c)$ *and* $\hat{g}(c, z)$ *satisfy* 6 *and* 7 *for all* $(c, \psi) \in C \times \Psi$*.*

The solution of a recursive equilibrium is complex, since it has to be solved by means of a set of equations in which the functions depend on the entire distribution of workers across regions and employment states. This distribution has a large dimension and therefore, the problem is difficult to solve, even numerically. The aim is to find a recursive equilibrium in which the value and policy functions depend on a manageable set of variables such that the problem disappears. To this end, I identify groups of workers who have the same intention, resulting in a block recursive equilibrium.

Definition 4.2. *A block recursive equilibrium (BRE) is a recursive equilibrium such that the market tightness function and the value functions with their corresponding policy functions depend of the aggregate state of the economy* ψ*, only through the aggregate component of productivity* y*, and not through the entire distribution of workers across regions and employment states* (u, g)*.*

To prove the existence, uniqueness and efficiency of a BRE, I build on the proofs of Menzio and Shi (2009, 2011) and show that the distinction between regions preserves the block recursive structure. To this end, I define the problem of the social planner (a benevolent god). It turns out that the BRE and the solution of the social planner coincide.

4.2 Social Planner's Problem

The social planner has complete information about the state of the economy ψ. During the period, he chooses the destruction probability $d : C \times Z \to \{\delta, 1\}$ for all z-workers in each region, the restrictions $r_u : C \to C$ and $r_e : C \times Z \to C$ where unemployed or employed workers can search for a new job, and the tightnesses $\theta_u : C \to \mathbb{R}_+$ and $\theta_e : C \times Z \to \mathbb{R}_+$ in order to specify the submarket where workers can search. This already determines the distribution of workers across regions and employment states for the next period. The aim of the social planner is to maximize total output, while future production is discounted at the rate β. Then, the social planner's problem is given by the following Bellman equation

$$\begin{aligned}
W(\psi) &= \max_{d,\theta_u,\theta_e,r_u,r_e} F(d,\theta_u,\theta_e,r_u,r_e|\psi) + \beta \mathbb{E} W(\hat{\psi}) \\
\text{s.t.} \qquad \hat{u}(c) &= u(c)\big(1-\lambda_u p(\theta_u(c))\big) + \sum_{z\in Z} d(c,z) g(c,z) \\
\hat{g}(c,z) &= h(c) f_c(z) + (1-d(c,z))(1-\lambda_e p(\theta_e(c,z))) g(c,z) \qquad (8) \\
h(c) &= \sum_{c'\in C} \Big(u(c')\lambda_u p(\theta_u(c')) \mathbb{1}_{\{r_u(c')=c\}} \\
&\quad + \sum_{z\in Z} (1-d(c',z))\lambda_e p(\theta_e(c',z)) g(c',z) \mathbb{1}_{\{r_e(c',z)=c\}} \Big),
\end{aligned}$$

where F is the aggregated output of the current period and given by

$$\begin{aligned}
F(d,\theta_u,\theta_e,r_u,r_e|\psi) = \sum_{c\in C} \Bigg(& \hat{u}(c) v(b) + \sum_{z\in Z} (y+z) \hat{g}(c,z) \\
& - k\Big(\lambda_u u(c)\theta_u(c) + \lambda_e \sum_{z\in Z} (1-d(c,z)) g(c,z) \theta_e(c,z)\Big) \\
& - t\big(c,b,r_u(c)\big)\big(\lambda_u u(c) p(\theta_u(c))\big) \\
& - \sum_{z\in Z} t\big(c,z,r_e(c,z)\big)\big(\lambda_e (1-d(c,z)) g(c,z) p(\theta_e(c,z))\big) \Bigg).
\end{aligned}$$

The problem depends on the aggregated productivity y, the measure of unemployed workers, and the measure of employed workers. A further differentiation is not only made between idiosyncratic productivity, but also regarding the region in which the workers live. Hence, the social planner's value function has many dimensions. The next theorem simplifies the problem and yields a unique solution for the social planner's problem.

Theorem 4.3. *I) The planner's value function $W : \Psi \to \mathbb{R}$ is the unique solution to the functional equation* 8.

II) The planner's value function is linear in c, u and g. That is, there exist functions $W_u : C \times Y \to \mathbb{R}$ and $W_e : C \times Z \times Y \to \mathbb{R}$ such that

$$W(\psi) = \sum_{c \in C} \Big(W_u(c, y)u(c) + \sum_{z \in Z} W_e(c, z, y)g(c, z) \Big).$$

III) The social planner's allocation $(d^*, \theta_u^*, \theta_e^*, r_u^*, r_e^*)$ associated with 8 *depend on ψ only through y.*

Proof. See appendix. □

The social planner's problem can be divided into $\mathcal{N}(c)\big(\mathcal{N}(z)+1\big)$ problems. He chooses for each region and for each employment state, a restriction, a tightness and a destruction probability to maximize total output. The block recursivity property is ensured through a directed search process. If the search process were random, the planner chooses the same tightness for all workers in the economy. Therefore, the problem cannot be divided into $\mathcal{N}(c)\big(\mathcal{N}(z)+1\big)$ problems, and the solution will depend on ψ not only on the aggregated productivity y, but also on the distribution of workers across regions c and employment states u, g.

The efficient choice for the restriction $r_u^*(c)$ that determines the region where unemployed workers from region c search for a new job maximizes

$$\max_{r_u^*(c)} \; - t(c, b, r_u^*(c)) + \sum_{z \in Z} z f_{r_u^*(c)}(z) + \beta \mathbb{E} W_e(r_u^*(c), z, \hat{y}) f_{r_u^*(c)}(z). \tag{9}$$

Similarly, it is the efficient choice for the restriction $r_e^*(c,z)$ that determines the region in which employed z-workers from region c search for a new job maximizes

$$\max_{r_e^*(c,z)} \; -t(c,z,r_e^*(c,z)) + \sum_{z'\in Z} z' f_{r_e^*(c,z)}(z') + \beta \mathbb{E} W_e(r_e^*(c,z), z', \hat{y}) f_{r_e^*(c,z)}(z'). \tag{10}$$

The maximands in 9 and 10 are composed of three terms. The first are the transition costs that have to be paid, when a worker moves to region $r_u^*(c)$ or $r_e^*(c,z)$, respectively. The second terms are the expected idiosyncratic productivities in these regions, and the third terms are the values generated by future periods.

The efficient choice for the tightness $\theta_u^*(c)$ that determines the ratio of vacancies to unemployed workers from region c is such that

$$\begin{aligned} k \geq p'(\theta_u^*(c))\Big[y - v(b) - t(c,b,r_u^*(c)) + \sum_{z\in Z} \Big(z f_{r_u^*(c)}(z) \\ + \beta \mathbb{E}\big(W_e(r_u^*(c), z, \hat{y}) f_{r_u^*(c)}(z) - W_u(c,\hat{y})\big)\Big)\Big] \end{aligned} \tag{11}$$

and $\theta_u^*(c) = 0$ with complementary slackness. Similarly, the efficient choice for the tightness $\theta_e^*(c,z)$ that determines the ratio of vacancies to employed z-workers from region c is such that

$$\begin{aligned} k \geq p'(\theta_e^*(c,z))\Big[-z - t(c,z,r_e^*(c,z)) + \sum_{z'\in Z} \Big(z' f_{r_e^*(c,z)}(z') \\ + \beta \mathbb{E}\big(W_e(r_e^*(c,z), z', \hat{y}) f_{r_e^*(c,z)}(z') - W_e(c,z,\hat{y})\big)\Big)\Big] \end{aligned} \tag{12}$$

and $\theta_e^*(c,z) = 0$ with complementary slackness. The costs for a vacancy are equal across regions and do not depend on the submarket or on the group of workers who should be attracted. Therefore, on the left-hand side of 11 and 12, is the marginal cost of increasing the corresponding tightness. On the right-hand side of both cases is the marginal benefit of increasing the tightness. The first term $p'(\theta)$ is the derivative of p and hence the marginal increase in the probability of finding a job. The second term is the value generated by creating a new match. In the first case, an unemployed worker produces $y + \mathbb{E}Z$ and loses b. In the second case, the worker produces $y + \mathbb{E}Z$ and the old production $y + z$ is terminated. The remaining terms represent transition costs and differences in future production.

The efficient choice of the destruction probability for matches with idiosyncratic productivity z in region c, $d^*(c,z)$, is 1, if

$$\begin{aligned}
&v(b) + \beta \mathbb{E} W_u(c,\hat{y}) > -\lambda_e \big(k\theta_e^*(c,z) + p(\theta_e^*(c,z))t(c,z,r_e^*(c,z))\big) \\
&+ \big(1-\lambda_e p(\theta_e^*(c,z))\big)\big(y + z + \beta \mathbb{E} W_e(c,z,\hat{y})\big) \\
&+ \lambda_e p(\theta_e^*(c,z))\Big(y + \sum_{z' \in Z} \Big(z' f_{r_e^*(c,z)}(z') + \beta \mathbb{E} W_e(r_e^*(c,z), z', \hat{y}) f_{r_e^*(c,z)}(z')\Big)\Big)
\end{aligned} \tag{13}$$

and $d^*(c,z) = \delta$ otherwise. On the left-hand side is the value of an unemployed worker who cannot search for a new job in the current period. This is equal to to value of an employed worker who lost his job. On the right-hand side is the value of a z-worker who survived the separation stage. His value consists of the value generated by the current match and the opportunity to search for a new job. If the left-hand side is greater than the right hand side, the match will be destroyed with probability 1. Otherwise, there is an exogenous probability of δ that the match is destroyed.

4.3 Existence, Efficiency and Uniqueness

Now it is time to establish the paper's main result.

Theorem 4.4. *I) There is a unique BRE.*
II) The BRE is socially efficient in the sense that

a) $\theta(x_u(c,y), r_u(c,y), y) = \theta_u^*(c,y)$ *and* $r_u(c,y) = r_u^*(c,y)$

b) $d(c,z,y) = d^*(c,z,y)$

c) $\theta(x_e(c,z,y), r_e(c,z,y), y) = \theta_e^*(c,z,y)$ *and* $r_e(c,z,y) = r_e^*(c,z,y)$

Proof. See appendix. □

The search and matching literature generally requires information about worker heterogeneity in order to determine the market tightness. This can be a challenging task, when the distribution of workers across the economy is large. The previous result pins down this problem to a smaller set of state variables. To obtain the block recursivity, I mainly use two properties of the model. The first assumption is that the search process is managed in a direct manner. Specifically, a poorly qualified worker in an economically weak region will search for a job that offers a small surplus and is therefore likely to be found, while a highly qualified worker in an economically strong region will search for job that offers a large surplus and is therefore less likely to get it. Hence, both agents, workers and firms, know where they are applying for a new job or which group of workers they are attracting. When the search process is random, all workers will search in the same submarket and the decision as to whether a worker accepts the offer depends on his current productivity. Hence, the firm's expected value from filling a vacancy depends on the distribution of workers across regions and employment states.

The second assumption that ensures block recursivity is given by the free-entry condition. This guarantees that the expected value of a vacancy is equal to the cost of creating it. In other words, the value of a match depends on the probability of creating a match and on the surplus of the match. The former is a function of the market tightness, but does not depend ot the distribution of workers in the economy.

5 A Calibrated Example

5.1 Data

For the analysis in the next section, I use the Sample of Integrated labour Market Biographies (SIAB), which is an employment panel representing 2% of integrated labour market biographies in Germany from 1975 to 2017. This sample covers all residents who contribute to the German social security system. Hence, civil servants and self-employed workers are not included im the sample, resulting in a 80% replication of the German labour force. A detailed description of the dataset is provided in Antoni et al. (2019). The SIAB dataset provides information about the working place of each employed worker. To have a consistent regional assignment, I assume that unemployed workers remain in the region of their last job. If this information is not available, the first observed region in the future is taken, resulting in a sample with no missing values in the regional assignment. Regions in the dataset can mostly be identified as "Landkreise" (NUTS 3) and only a few regions are merged by the data provider to protect the worker's data privacy. In total, the dataset distinguishes between 328 different regions.[4] The employment history of each worker is given on a daily basis. To obtain monthly unemployment rates, I use the first day of each month as a reference date. All Migration rates between two regions are computed in the same way. Finally, data for East Germany are not available before the reunification and are rather unreliable until 1995. Therefore, I restrict my analysis to the years 1995 until 1997. The final sample includes more than 30 million observations. A more extensive analysis is given by Trede and Zimmermann (2020).

5.2 Calibration Strategy

For the model described above, this section provides a calibration to the German economy, mainly using techniques adapted from Menzio and Shi (2009 and 2011). To do this, it is necessary to choose values for the household's preferences (b, β), for the search process $(\lambda_u, \lambda_e, p, \delta)$ and for the production sector (k, f). Table 3 summarizes the calibration outcomes.

First of all, I choose a period length of one month and set $\beta = 0.95^{1/12}$, such that the annual interest rate is 5%. Inspired by the German federal states, I decided to separate Germany into sixteen different regions. There are several reasons for doing so. First, a finer subdivision results in very few observations for each migration rate, which even leads in several cases to no movements being observed between two regions. This is not surprising, considering that there are regions with low economic power that are far away from each other. In this case, higher volatilities also occur, which is inappropriate for my calibration. Second, the computation time increases quadratically with the number of regions. Using all 328 available regions would increase the computation time by several month, maybe even over a year.

I set λ_u equal to 0.636, which implies that only the fraction of short-term unemployed

[4] For calibration purposes, I will aggregate these regions into states.

workers have a chance of finding a job. This is motivated by the fact that Germany has always had a high and constant number of long-term unemployed workers. It is common in the literature (see Shimer (2005) or Mortensen and Nagypál (2007)) to define the job-finding probability function as $p(\theta) = \min\{1, \theta^{\gamma}\}$ with $\gamma \in (0, 1)$. To be more precise, I set the elasticity γ equal to 0.6, which is used in Menzio, Telyukova and Visschers (2005).[5] The remaining parameters are chosen such that statistical moments in the model are close to the corresponding values in the data. Starting with the vacancy cost k, the search efficiency for employed workers λ_e and the exogenous destruction probability δ, I consider the job-finding rate (UE), the job-to-job rate (EE) and the separation rate (EU). In the model, the UE rate is given by $h_{ue} = \sum_c \lambda_u p(\theta_u(c))u(c)/u$, the EE rate by $h_{ee} = \sum_c \sum_z (1 - d(c, z))\lambda_e p(\theta_e(c, z))g(c, z)/(1 - u)$ and the EU rate by $h_{eu} = \sum_c \sum_z d(c, z)g(c, z)/(1 - u)$. In the data, I compute monthly transition rates by counting the affected individuals and dividing them by the employed or unemployed population, respectively. Their averages are used to obtain the desired values. I receive $k = 13.956$, $\delta = 0.0057$ and $\lambda_e = 0.5750$. To calibrate the home productivity b, I calculate the productivity ratio between unemployed and employed workers, which is given by $b/(y + \sum_c \sum_z zg(c, z))$. When productivities, or even wages and benefits, need to be estimated, it is a common assumption to choose b such that this ratio is close to two-thirds (see Christoffel et al. (2009)). I use the ratio estimated by Hall and Milgrom (2008) of 0.71, which yields b equal to 0.7962.

The idiosyncratic component of productivity is given by a 300 point approximation of a Weibull distribution with mean μ, shape α and scale σ. This allows a wide range of possible specifications. The shape of a Weibull distribution is similar to a exponential distribution for $\alpha = 1$, a Rayleigh distribution for $\alpha = 2$, a normal distribution for $\alpha = 4$ and a skewed normal distribution for $\alpha = 10$. For the sake of simplicity, I restrict the possible outcomes to these four values. The remaining procedure is as follows: I set $\mu = 0$ and estimate the other parameters by minimizing the distance between the distribution of workers across tenure in the model and in the data. The underlying idea

[5] For an elasticity of 0.3 and 0.7 they found no significant differences regarding the benchmark model.

is that the probability of a worker remaining in a match increases with the idiosyncratic productivity. Therefore, the distribution of idiosyncratic productivity affects the duration of matches, and hence the tenure. Figure 11 shows the distributions for the model and the data. The result of $\alpha = 4$ and $\sigma = 0.5795$ imply that the productivity of a match in the 90th percentile of the distribution is more than double the productivity of a match in the 10th percentile.

Finally, the transition costs have to be determined, which is the crucial part for the choice of work place. It is logical to assume that workers, whether employed or unemployed, have to bear some costs when they move from one region to another. Here, distances and house prices are primarily the driving factors. Adding local amenities such as natural environment, laws and culture makes things more difficult. Depending on the current region, local amenities other regions may even offset the cost of migration. The good news is that there is a wide set of feasible choices which do not affect the other estimates. So far, there is no suitable way to estimate this factor. To provide an example for the model, I implement a random normal distributed effect for $t(c, z, r)$ for all $(c, z, r) \in C \times \{Z, b\} \times C$. At the end of this section, I provide a detailed discussion.

5.3 Productivity Shocks

I investigate the behaviour of the economy in response to productivity shocks, which is a common procedure for replicating the business cycle. For this purpose, I compute the Block Recursive Equilibrium of the calibrated model, with aggregate productivity y equal to its average realization. For the next and all subsequent periods, the aggregate productivity is increased by one percent.

Figure 12 shows the responses for the unemployment rate and for transition rates between employment states (UE, EU and EE). It is obvious that the joint value of a match in response to an aggregate productivity shock benefits more than the value of an unemployed worker. Firms will offer more vacancies to unemployed workers, leading to an increase in the tightness θ_u. Hence, the transitions from unemployment to employment rise. At the same time, some matches with low idiosyncratic productivity can exceed the threshold where it was previously more efficient to break it up than maintain it. The result is that the separation rate declines. The reason for the increase in job to job transitions is less obvious. A match with high idiosyncratic productivity is more likely to be maintained than a match with a low idiosyncratic productivity and therefore may take advantage of the increase. For this reason, the latter group of workers has the incentive to search for new jobs. In combination, the reactions of these three rates result in a decline in the unemployment rate. Specifically, the steady state values for UE, EU, EE and unemployment rate change by $2.4\%, -2.1\%, 1.9\%$ and -4%, respectively. Regional effects depend on transition costs and thus on their amenities. As mentioned at the beginning, the value of a joint match increases, but unemployed workers also benefit indirectly through possible future employment. Both reasons can lead to an upward shift of the threshold at which matches are destroyed. Figure 13 shows that this effect is rare, but possible. All findings mostly coincide with those in Menzio and Shi (2011). The main difference is that the German labour market needs more time to reach the new equilibrium, which can be explained by a less flexible market. Migration effects between regions cannot be interpreted, because the transition costs were randomized.

When I restrict the productivity shock to a single region, this leads to a huge migration wave. Since all agents have complete market information, the highest revenue would be generated when a substantial number of firms and workers relocate to the observed region. Even if the rigidity of the economy does not lead to an immediate redistribution of the population, there would be a continuous migration towards the shocked region. In terms of numbers, this means that each year, there is an increase in the size of the initial population. Another way to observe the huge impact on that single region is to look at the aggregated transition rates between employment states. This region would be responsible for changes in transition rates that have similar magnitudes as in the case where the entire economy faces a productivity shock.

5.4 Discussion

The previous section has shown that the model reaches its limits when migration behaviour is to be analysed and no suitable determination of transition costs can be made. In the following part, I present a feasible approach to solving this problem, which, however, raises other problems. Transition costs are driven by many factors. The gain and loss of local amenities, as well as distances, are an obvious factor when referring to transition costs, but employment states also play a major role. Highly productive workers look for jobs that are difficult to find and are therefore more willing to migrate to a distant region. Making the transition costs dependent on the employment state, the current place of work and the designated region, leads to a function that is difficult to control. With only 16 regions and a 300 point approximation of the idiosyncratic productivity, this results in 23,189,040 individual transition costs. Since it is also not possible to measure local amenities, it is necessary to find an approximation as a characteristic number that represents local amenities for each region. Housing prices, unemployment rates and regional GDP are just a few possible, and at the same time measurable candidates. This, together with the introduction of a possible continuous relationship between the idiosyncratic productivity and the transition costs, which is a tricky task, could significantly reduce the number of parameters. With access to bilateral migration rates between regions[6], one approach could be to chose the remaining parameters such that the migration rates in the model and in the data coincide. Unfortunately, such a radical reduction of the parameters is associated with an overdetermined system of equations that has no solution. The assumptions of directed search and complete contracts, which are necessary assumptions for establishing a BRE, give firms and workers full information about the market. Therefore, given a small set of parameters that identify local amenities and a functional relationship between idiosyncratic productivity and transition costs, there will be a single state in a single region that represents the perfect submarket. Since both agents have the incentive to maximize their lifetime utility, they relocate themselves to the perfect submarket, which contradicts the approach that migration rates coincide.

[6] One way to compute these rates from the SIAB dataset is given in Trede and Zimmermann (2020)

Overall, this leads to the conclusion that directed search models are not yet applicable in practice.

6 Conclusion

This paper extends Menzio and Shi's (2009, 2011 and 2015) model of directed search on the job, by including the component of space, which is divided into an arbitrary but finite number of regions. This allows for the study of transitions not only between employment states (unemployment and employment with idiosyncratic productivity z), but also across space. I proofed the existence of a unique and efficient recursive equilibrium with the property that the distribution of workers across employment states and regions does not affect the agents' value and policy functions. This ensures that the model can be solved outside the steady state and can be used to investigate labour dynamics over the business cycle.

The theoretical part is followed up by an example based on the German economy. For the calibration, I used values from the literature as well as my own calculations, for which I matched transition rates between employment states in the model and in the data (SIAB-7519). To complete the example, I added random transition costs for each combination of regions and employment states. I found that aggregate productivity shocks primarily create vacancies for unemployed workers and those in a low-productivity segment. Independent of the realisation of transition costs, a 1% increase in productivity reduces the unemployment rate by up to 4% in the long run. Since transition rates are chosen at random, changes in migration rates cannot be interpreted. When productivity rises in a single region, both workers and firm have the incentive to relocate themselves to the shocked region, which results in a huge immigration wave. Therefore, directed search models cannot be used to study regional migration, unless there is an appropriate specification for the transition costs.

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A Calibration Outcome

Parameter	Mnemonic	Value	Target/Source
Discount factor	β	0.996	5% annual interest rate
home productivity	b	0.7962	Hall and Millgrom
off the job search	λ_u	0.636	short time unemployment
on the job search	λ_e	0.5750	EE rate
vacancy cost	k	13.956	UE rate
exogenous destruction	δ	0.0057	EU rate
elasticity	γ	0.6	Menzio and Shi (2011)
avg idiosyncratic productivity	μ	0	Menzio and Shi (2011)
shape idiosyncratic productivity	α	4	tenure distribution
scale idiosyncratic productivity	σ	0.5795	tenure distribution

Table 3: Benchmark calibration

B Plots

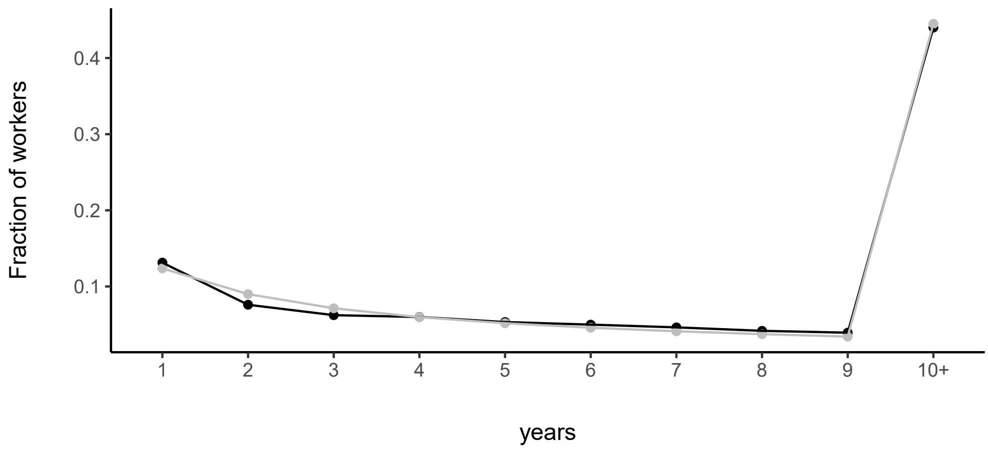

Figure 11: distribution of workers across tenure length (years) in the data (black line) and in the model (grey line).

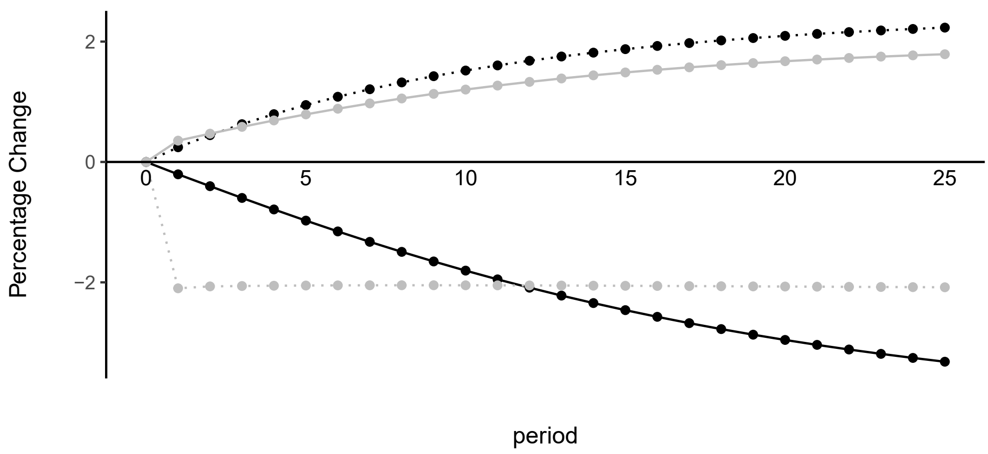

Figure 12: Percentage change of the UE rate (dashed black line), the EU rate (dashed grey line), the EE rate (solid grey line) and the unemployment rate (solid black line) in response to a 1 percent increase in y.

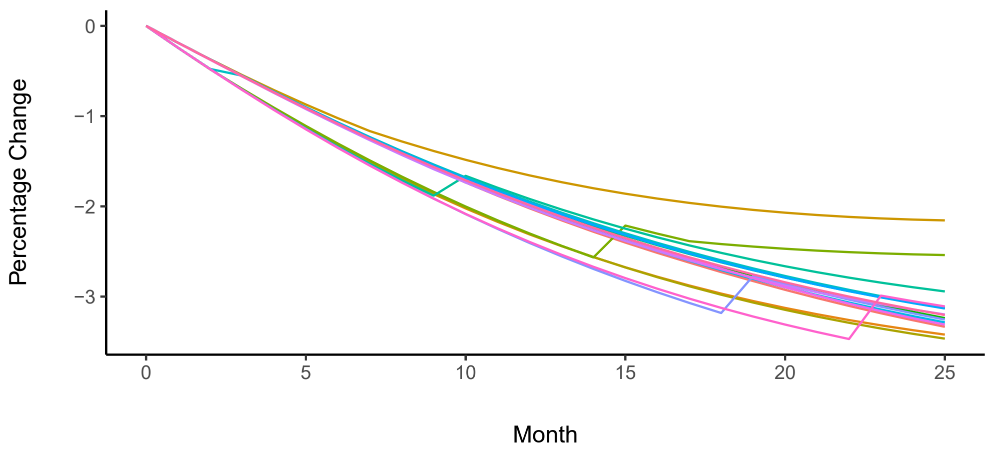

Figure 13: Percentage change of the regional unemployment rates in response to a 1 percent increase in y (16 arbitrary regions).

C Proofs

Proof of Theorem 4.3

I) I define $C(\Psi)$ as the set of bounded and continuous functions $\varphi : \Psi \to \mathbb{R}$ and I define the operator T on $C(\Psi)$ by

$$\begin{aligned}
(T\varphi)(\psi) &= \max_{d,\theta_u,\theta_e,r_u,r_e} F(d,\theta_u,\theta_e,r_u,r_e|\psi) + \beta\mathbb{E}(\varphi(\hat{\psi})) \\
\text{s.t.} \quad \hat{u}(c) &= u(c)\big(1-\lambda_u p(\theta_u(c))\big) + \sum_{z\in Z} d(c,z)g(c,z) \\
\hat{g}(c,z) &= h(c)f_c(z) + (1-d(c,z))(1-\lambda_e p(\theta_e(c,z)))g(c,z) \qquad \text{(C.1)} \\
h(c) &= \sum_{c'\in C}\Big(u(c')\lambda_u p(\theta_u(c'))\mathbb{1}_{\{r_u(c')=c\}} \\
&\quad + \sum_{z\in Z}(1-d(c',z))\lambda_e p(\theta_e(c',z))g(c',z)\mathbb{1}_{\{r_e(c',z)=c\}}\Big).
\end{aligned}$$

The aim is to show that the operator T is a contraction that maps all $\varphi \in C(\Psi)$ into itself. To do this, let $\varphi \in C(\Psi)$ be an arbitrary function. Since φ is bounded, there exists $\underline{\varphi}$ and $\bar{\varphi}$ with $\underline{\varphi} \le \varphi(\psi) \le \bar{\varphi}$ for all $\psi \in \Psi$. Hence, $T\varphi$ is bounded by $\min\{b, y_1 + z_1\} + \beta\underline{\varphi}$ and $\max\{b, y_{N(y)}+z_{N(z)}\}+\beta\bar{\varphi}$. Regardless of the restriction r_u (or r_e) it holds that the market tightness function θ is bounded by 0 and $q^{-1}(k/(\sum_z V(r_u(c),z)-x)) \le q^{-1}(k/(\bar{x}-\underline{x})) = \bar{\theta}$. Therefore, the maximization problem can be reduced to a problem with $\theta_u : C \to [0,\bar{\theta}]$ and $\theta_e : C \times Z \to [0,\bar{\theta}]$. Now, the set of feasible choices for $(d, r_u, r_e, \theta_u, \theta_e)$ is closed and bounded, which is equivalent to compact. Since the maximand is continuous in $(d, r_u, r_e, \theta_u, \theta_e)$ and ψ, the theorem of the maximum (see theorem 3.6 in Stokey et al. (1989)) implies that $T\varphi$ is continuous in φ. Hence, T maps $C(\Psi)$ into itself.

To use Blackwell's sufficient conditions for a contraction (see theorem 3.3 in Stokey et al. (1989)), it is necessary to show that T satisfies the monotonicity and discounting condition. Both conditions are obviously fulfilled, when the operator is inserted. It follows that T has a unique fixed point $\varphi^* \in C(\Psi)$. Finally, for all $\psi \in \Psi$ it holds $\lim_{t\to\infty}\beta^t\varphi^*(\psi) = 0$. This implies that the fixed point φ^* is equal to the planner's value function W.

II) I define the subset $L(\Psi) \subset C(\Psi)$ as the set of bounded and continuous functions which are linear in the measure of unemployed and employed workers. In other words, there exist two functions $\varphi_u : C \times Y \to \mathbb{R}$ and $\varphi_e : C \times Z \times Y \to \mathbb{R}$ such that

$$\varphi(\psi) = \sum_{c \in C} \Big(\varphi_u(c, y) u(c) + \sum_{z \in Z} \varphi_e(c, z, y) g(c, z) \Big).$$

The aim is again to show that the operator T maps all $\varphi \in L(\Psi)$ into itself. To do this, let $\varphi \in L(\Psi)$ be an arbitrary function. By applying the operator T and substituting the constraints, I obtain

$$(T\varphi)(\psi) = \sum_{c \in C} \Big(\tilde{\varphi}_u(c, y) u(c) + \sum_{z \in Z} \tilde{\varphi}_e(c, z, y) g(c, z) \Big), \tag{C.2}$$

where $\tilde{\varphi}_u(c, y)$ is given by

$$\begin{aligned} \tilde{\varphi}_u(c, y) = \max_{\theta_u, r_u} \Big[& - \lambda_u \Big(k\theta_u + p(\theta_u) t(c, b, r_u) \Big) \\ & + \Big(1 - \lambda_u p(\theta_u) \Big) \Big(v(b) + \beta \mathbb{E} \varphi_u(c, \hat{y}) \Big) \\ & + \lambda_u p(\theta_u) \Big(y + \sum_{z \in Z} \Big(z f_{r_u}(z) + \beta \mathbb{E} \varphi_e(r_u, z, \hat{y}) f_{r_u}(z) \Big) \Big) \Big] \end{aligned} \tag{C.3}$$

and $\tilde{\varphi}_e(c, z, y)$ is given by

$$\begin{aligned} & \tilde{\varphi}_e(c, z, y) = \max_{d, \theta_e, r_e} \Big[d(v(b) + \beta \mathbb{E} \varphi_u(c, \hat{y})) \\ & - \big(1 - d\big) \lambda_e \big(k\theta_e + p(\theta_e) t(c, z, r_e) \big) \\ & + \big(1 - d\big) \big(1 - \lambda_e p(\theta_e)\big) \big(y + z + \beta \mathbb{E} \varphi_e(c, z, \hat{y}) \big) \\ & + \big(1 - d\big) \lambda_e p(\theta_e) \Big(y + \sum_{z' \in Z} \Big(z' f_{r_e}(z') + \beta \mathbb{E} \varphi_e(r_e, z', \hat{y}) f_{r_e}(z') \Big) \Big) \Big]. \end{aligned} \tag{C.4}$$

Note, because the maximum is taken within the sums, the functions $d, r_u, r_e, \theta_u, \theta_e$ do not have to be dependent on c and z anymore. Since φ is an arbitrary function in $L(\Psi)$, $C.2$ implies that T maps $L(\Psi)$ into itself. By definition, $L(\Psi)$ is a closed subset of $C(\Psi)$ (as a linear decomposition). Therefore, all conditions for corollary 1 in theorem 3.2 in Stokey et al. (1989) are fulfilled, which implies $W \in L(\Psi)$.

III) The social planner's allocation (θ_u^*, r_u^*) solve $C.3$ and (d^*, θ_e^*, r_e^*) solve $C.4$. Since both equations do not depend on (u, g), the planner's allocation depend on ψ only through y.

Proof of Theorem 5.4

I) I divide the proof into five steps. I show the existence by deriving an operator T that is a contraction and show that the equilibrium functions from the fixed point of T satisfy all equilibrium conditions.

Step 1: I start to unify the notation for V_u and V_e. To do this, I define the function $V : \{0,1\} \times C \times Z \times \Psi \to \mathbb{R}$ as $V(0,c,z,y) = V_u(c,\psi)$ and $V(1,c,z,\psi) = V_e(c,z,\psi)$ for all $(c,z,\psi) \in C \times Z \times \Psi$. Now the equilibrium conditions 2 and 4 can be rewritten as

$$\begin{aligned} V(a,c,z,\psi) =& a\Big(y + z + \beta \mathbb{E} \max_{d,x,r}\Big\{d(c,z)V(0,c,z,\hat{\psi}) + (1-d(c,z))V(1,c,z,\hat{\psi}) \\ &+ \big(1-d(c,z)\big)\lambda_e p\big(\theta(x,r,\hat{\psi})\big)\big(x - V(1,c,z,\hat{\psi}) - t(c,z,r)\big)\Big\}\Big) \\ &+ (1-a)\Big(v(b) + \beta \mathbb{E} \max_{x,r}\Big\{V(0,c,z,\hat{\psi}) \\ &+ \lambda_u p\big(\theta(x,r,\hat{\psi})\big)\big(x - V(0,c,z,\hat{\psi}) - t(c,b,r)\big)\Big\}\Big). \end{aligned} \tag{C.5}$$

Step 2: Now, I express the value offered in submarket x as a function of the tightness θ, the restriction r and the aggregate state of economy ψ by $x(\theta,r,\psi)$. From the market tightness condition 5 it follows

$$x(\theta,r,\psi) = \sum_{z \in Z} V(1,r,z,\psi) f_r(z) - \frac{k}{q(\theta)}. \tag{C.6}$$

Submarkets with $\theta(x,r,\psi) = 0$ are irrelevant because the probability of meeting a vacancy in these submarkets is zero. Hence, let $x(\theta,r,\psi)$ be given by $C.6$ in all submarkets with tightness $\theta(x,r,\psi) = \theta \geq 0$.

Step 3: In the general value function C.5, the promise value x can be replaced with

$x(\theta, r, \psi)$ and the tightness $\theta(x, r, \psi)$ with θ by using equation C.6. I obtain

$$\begin{aligned} V(a, c, z, \psi) = a\Big(y + z + \beta\mathbb{E} \max_{d,\theta,r}\Big\{ & d(c, z)V(0, c, z, \hat{\psi}) - (1 - d(c, z))\lambda_e k\theta \\ & + \big(1 - d(c, z)\big)\big(1 - \lambda_e p(\theta)\big)V(1, c, z, \hat{\psi}) \\ & + \big(1 - d(c, z)\big)\lambda_e p(\theta)\Big(\sum_{z' \in Z} V(1, r, z, \hat{\psi})f_r(z) - t(c, z, r)\Big)\Big\}\Big) \\ & + (1 - a)\Big(v(b) + \beta\mathbb{E} \max_{\theta,r}\Big\{ - k\lambda_u\theta + \big(1 - \lambda_u p(\theta)\big)V(0, c, z, \hat{\psi}) \\ & + \lambda_u p(\theta)\Big(\sum_{z' \in Z} V(1, r, z, \hat{\psi})f_r(z) - t(c, b, r)\Big)\Big\}\Big). \end{aligned} \tag{C.7}$$

Step 4: In the fourth step I make use of the Blackwell's fixed point theorem (see theorem 3.3 in Stokey et al. (1989)). Therefore, I define $\Omega = \{0, 1\} \times C \times Z \times \Psi$ and $C(\Omega)$ as the space of bounded continuous functions $\varphi : \Omega \to \mathbb{R}$. Finally, $T : C(\Omega) \to C(\Omega)$ denotes the operator associated with $C.7$. It has to be shown that T satisfies i) monotonicity, i.e. for $\varphi, \varphi' \in C(\Omega)$ with $\varphi \leq \varphi'$ it holds $T(\varphi) \leq T(\varphi')$ and ii) discounting, i.e. for $\varphi \in C(\Omega)$ and $\varepsilon > 0$ it holds $T(\varphi + \varepsilon) \leq T(\varphi) + \beta\varepsilon$, where $\beta \in (0, 1)$. To prove the monotonicity condition, it is sufficient to note that the relevant parts are non negative. The discounting condition follows directly from the linearity of T. Therefore, T is a contraction and has a unique fixed point V. Further, the fixed point depends on ψ only through y. It follows $V(a, c, z, \psi) = V(a, c, z, y)$.

Step 5: The last step is to show the uniqueness of the policy functions $(\theta, x_u, r_u, d, x_e, r_e)$ and their independence from (u, g). Since $V(a, c, z, \psi)$ depends on ψ only through y, I can rewrite the equilibrium condition 5 as

$$q(\theta(x, r, \psi))\Big(\sum_{z \in Z} V_e(r, z, y)f_r(z) - x\Big) = k$$

or $\theta(x, r, \psi) = 0$, when the submarket is not visited. Hence, $\theta(x, r, \psi)$ is unique and depends on ψ only through y. It follows that $\theta(x, r, \psi) = \theta(x, r, y)$. Now, the equilibrium condition 2 can be rewritten as

$$V_u(c, y) = v(b) + \beta\mathbb{E} \max_{x,r}\big\{V_u(c, \hat{y}) + \lambda_u p\big(\theta(x, r, \hat{y})\big)\big(x - V_u(c, \hat{y}) - t(c, b, r)\big)\big\}. \tag{C.8}$$

Since the maximization problem above depends on $\hat{\psi}$ only through $\hat{y}$, the corresponding policy functions $(x_u(c,\hat{\psi}), r_u(c,\hat{\psi}))$ depend on $\hat{\psi}$ only through $\hat{y}$. It follows that $(x_u(c,\hat{\psi}), r_u(c,\hat{\psi})) = (x_u(c,\hat{y}), r_u(c,\hat{y}))$. Using the same argument as in $C.8$ yields

$$\begin{aligned} V_e(c,z,y) = y + z + \beta\mathbb{E} \max_{d,x,r} \big\{ & d(c,z)V_u(c,\hat{y}) + \big(1 - d(c,z)\big)V_e(c,z,\hat{y}) \\ & + \big(1 - d(c,z)\big)\lambda_e p\big(\theta(x,r,\hat{y})\big)\big(x - V_e(c,z,\hat{y}) - t(c,z,r)\big)\big\}. \end{aligned}$$

Again, the maximization problem above depends on $\hat{\psi}$ only through $\hat{y}$. Hence, the corresponding policy functions $(d(c,\hat{\psi}), x_e(c,\hat{\psi}), r_e(c,\hat{\psi}))$ depend on $\hat{\psi}$ only through $\hat{y}$.

II) To establish the equivalence between the equilibrium and the planner's allocation, consider the linear decompositions

$$\begin{aligned} W_u(c,y) = \max_{\theta_u, r_u} \Big[& -\lambda_u\Big(k\theta_u + p(\theta_u)t(c,b,r_u)\Big) \\ & + \big(1 - \lambda_u p(\theta_u)\big)\big(v(b) + \beta\mathbb{E}W_u(c,\hat{y})\big) \\ & + \lambda_u p(\theta_u)\Big(y + \sum_{z\in Z}\Big(z f_{r_u}(z) + \beta\mathbb{E}W_e(r_u,z,\hat{y})f_{r_u}(z)\Big)\Big)\Big] \end{aligned} \tag{C.9}$$

and

$$\begin{aligned} W_e(c,z,y) = \max_{d,\theta_e,r_e} \Big[& d\big(v(b) + \beta\mathbb{E}W_u(c,\hat{y})\big) \\ & - \big(1-d\big)\lambda_e\big(k\theta_e + p(\theta_e)t(c,z,,r_e)\big) \\ & + \big(1-d\big)\big(1 - \lambda_e p(\theta_e)\big)\big(y + z + \beta\mathbb{E}W_e(c,z,\hat{y})\big) \\ & + \big(1-d\big)\lambda_e p(\theta_e)\Big(y + \sum_{z'\in Z}\Big(z' f_{r_e}(z') + \beta\mathbb{E}W_e(r_e,z',\hat{y})f_{r_e}(z')\Big)\Big)\Big]. \end{aligned} \tag{C.10}$$

Now it is easy to see that $C.7$ is satisfied by $W'(a,c,z,y)$ which is defined as $W(0,c,z,y) = v(b) + \beta\mathbb{E}W_u(c,\hat{y})$ and $W(1,c,z,y) = y + z + \beta\mathbb{E}W_e(c,z,\hat{y})$. Part I) shows that V is the unique solution to $C.7$. It follows that $V_u(c,y) = b + \beta\mathbb{E}W_u(c,\hat{y})$ and $V_e(c,z,y) = y + z + \beta\mathbb{E}W_e(c,z,\hat{y})$. Hence, both allocation coincide.

Labour Mobility - A Multiregional New Keynesian Model

Abstract

In the literature, it is difficult to find an answer to the question of how economic shocks within an economy react to regional disparities. In the partial equilibrium framework, many studies contain only two opposing regions or are focused on regions identified as identical, due to a lack of data availability. Moreover, the development of a general equilibrium framework using Dynamic Stochastic General Equilibrium (DSGE) models is focused on the national level. Only a few researchers investigate disaggregated models, because introducing a dimension of space into the DSGE framework makes the model much more complicated and hard to solve. Economic shocks in the real world are not only correlated in time, but also across space. The aim of this paper is to develop a benchmark model that shows how local shocks lead to movements in population.

Keywords: labour mobility; technology shocks; business cycles; DSGE

1 Introduction

Labour mobility has long been considered as an important adjustment to economic disparities. The intuition is that migration may reduce the effects of region specific shocks, when conventional stabilization mechanisms are not available. This can happen, for example, in a country with many regions in which monetary policy cannot be a valid instrument to compensate for regional shocks. This commonly believed statement goes back to Mundell (1961), who argued that labour mobility is a necessary precondition for an optimum currency area. Focusing on Europe, the literature justifies persistent inequalities mainly by imaginary borders and barriers such as language, the burden of bureaucracy and culture. Surprisingly, large regional differences are also present within many countries where internal migration could in fact become a simple and effective adjustment mechanism. Widely discussed examples include East and West Germany, Northern and Southern Italy or the metropolitan region of Paris. Regional labour mobility between more than two regions is accorded less attention in most of the literature explaining fluctuations in macroeconomic variables.

The contribution of this paper is mainly theoretical. It summarizes existing labour market modelling approaches and extends them by dividing the economy into an unrestricted number of regions. To do this, I develop a multiregional dynamic stochastic general equilibrium (DSGE) model with all key features of the commonly cited literature (see Blanchard and Gali (2010), Clarida et al. (2002) or Kiguchi and Mountford (2019)). The key point in my specification is that I establish a DSGE model of interregional labour mobility in which households choose their work location and where employment is determined through a search and matching framework. Up to now, the literature has provided models that restrict the economy to two countries, known as open economy DSGE models. In this respect, the paper of Gali (2008) is recommended as a seminal paper. Papers that do not restrict their economy to a pair of regions have another drawback. They treat each separated region as identical, such that initial regional disparities in population, unemployment, etc. are not captured (see Kim (2019) or Holden and Swarbrick (2014)).

Of the existing literature, my paper is most closely related to Hauser (2014) and Hauser and Seneca (2019) who, as far as I know, are the first to use a DSGE model for a regional interpretation. Based on data of the United States they show that labour mobility can close inefficiency gaps in regional labour markets, even if households do not fully reap all the benefits of migration. Another paper that is used as a basis for this present work is given by House et al. (2018). They examine the response of labour mobility through variations in unemployment rates in Europe and the United States, and find that cyclical reactions in the United States are three times larger than in Europe. With their model, they simulate higher labour mobility for Europe, leading to a reduction in the regional dispersion of unemployment and GDP per capita. Both papers are in fact based on the work of Fahri and Werning (2014), who investigate the impact of migrants on stayers by means of internal and external demand shortfalls. While stayers do not benefit from an outflow and the associated internal imbalances, they gain a positive impact on welfare when imbalances are external. I adopt and extend their models to explain the behaviour of internal migration within several regions, both in theory and quantitatively, using a calibration with disaggregated data of the German economy, as well as common parameters obtained from the literature.

The remainder of the paper is structured as follows. The next section describes the theoretical model. Section 3 discusses the calibration and Section 4 presents impulse response functions with respect to the impact of productivity and amenity shocks. Section 5 concludes and provides an outlook for the future.

2 The Model

The framework is a variation of a dynamic New Keynesian Model. The main difference is that I allow an arbitrary number of regions to be included in the model. All key elements such as preferences, technology and prices have the same functional form across regions. Therefore, most of the model description can be reduced to the perspective of a single region, although this does not imply that all regions are identified identically. A summary of all necessary equations are listed in the appendix.

2.1 Households

Time is discrete and has an infinite horizon. The population is distributed across the entire economy in $i = 1, \dots, \mathcal{N}$ regions. Region i's number of households consist of a fixed mass of infinitely living members, N^i. For the sake of simplicity, I restrict my model to migration and do not include commuting, which implies that all household members can only work in their city of residence. The migration rate of region i's household members that live in region j at time t is denoted by $n_{ij,t}$. Subscripts in this paper can always be read as from origin i (first) to destination j (second) at time t (third).

The values in the model are defined in per capita terms. To obtain absolute values of these terms for any region, they have to be multiplied by the population of that region. The current population of region i, denoted by $N_{i,t}$, consists of the shares of households from all regions that migrate to region i, that is

$$N_{i,t} = \sum_{j=1}^{\mathcal{N}} n_{ji,t} N^j.$$

Labour can only be supplied in the current city of residence. I denote labour supply per household member of household i living in region j by $L^s_{ij,t}$. Therefore, total labour supply in region i, $L^s_{i,t}$, can be calculated as

$$L^s_{i,t} N_{i,t} = \sum_{j=1}^{\mathcal{N}} n_{ji,t} L^s_{ji,t} N^j.$$

The consumption of household members i living in j is defined in a similar way, such that they consume the final and non-tradeable good only from their current residence and is denoted by $C_{ij,t}$.[1] Then, aggregated consumption in region i, $C_{i,t}$, comprises the

[1] A CES (constant elasticity of substitution) aggregator of domestic and imported goods would be another widely used specification (see Hauser [13]).

individual consumption of households that immigrated from all other regions, that is

$$C_{i,t}N_{i,t} = \sum_{j=1}^{\mathcal{N}} n_{ji,t}C_{ji,t}N^j.$$

As usual, consumption increases and supplying labour decreases the utility of household members. In addition, they receive a time invariant utility gain or loss, interpreted as local specific amenities. I take into account all things that are either difficult to measure or to evaluate. This could capture differences in climate, natural resources, infrastructure, laws or simply culture, to give a few examples. I further assume that households from various regions can be differently affected by local amenities. This can be justified by the following example. While a rainy or cold region can be less attractive in general, it would be easier for households living in a similar climate, most likely a neighbouring region, to immigrate. In particular, geographical distance can be covered this way, which I regard as an important factor. On the other hand, I assume that all members of a household are faced the same amenities, when they consider to migrate. All these properties are summarized in amenities and denoted by A_{ij} for household members i living in j. For their own region, I set A_{ii} equal to zero. Up to now, all members of a household have the same incentive to migrate. This would lead to a concentration of the population in a single region. To prevent this, I assume that the average utility gain per migrant decreases, when the share of emigrants increases. This is formalized by $-\frac{\ln(n_{ij,t})}{\gamma}$, which is decreasing in $n_{ij,t}$ for $\gamma > 0$. Taken together, the expected lifetime utility can be written as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{\mathcal{N}} n_{ij,t}\mathcal{U}_{ij,t} + \sum_{j \neq i} n_{ij,t} \left(A_{ij} - \frac{\ln n_{ij,t}}{\gamma} \right) \right], \tag{2.1}$$

where

$$\mathcal{U}_{ij,t} = \left(\frac{C_{ij,t}}{1-\sigma} \right)^{1-\sigma} - \left(\frac{L^s_{ij,t}}{1+\varphi} \right)^{1+\varphi}. \tag{2.2}$$

Here $0 < \beta < 1$ is the common discount factor which households use to discount future utility, $\mathbb{E}_0$ denotes the expectation of future values of $\mathcal{U}_{ij,t}$ based on information available at time zero, σ is the inverse of the elasticity of intertemporal substitution, and φ is the inverse of the Frisch elasticity of labour supply. The infinity horizon assumption, which simplifies the mathematical analysis of business cycles, is usually justified in terms of transmission of goods across generations.

Households choose consumption, relative labour supply and their place of residence, in order to maximize their expected lifetime utility. Equation 2.2 is a common specification in the literature and known as the CRRA (constant relative risk aversion) utility function, in which slight changes might be possible due to the calibration of parameters. At this point it is worth mentioning that I distinguish between labour supply and migration. Alternatively, I could assume that workers automatically supply their labour in their current region and follow, for example, Hauser and Seneca (2019). The sequence of budget constraints is given by

$$\sum_{j=1}^{\mathcal{N}} P_{j,t} n_{ij,t} C_{ij,t} + \mathbb{E}_t \mathcal{Q}^i_{t,t+1} D_{i,t+1} = D_{i,t} + \sum_{j=1}^{\mathcal{N}} n_{ij,t} W^h_{j,t} L^s_{ij,t} + T_{i,t}, \tag{2.3}$$

where $P_{j,t}$ is the price of a good in region j, $D_{i,t+1}$ denotes the nominal payoff in period $t+1$ of the portfolio purchased at the end of period t, $W^h_{j,t}$ is the nominal wage of a household member working in region j, $T_{i,t}$ are lump-sum profits/taxes and $Q^i_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs.

A fundamental characteristic of all household members is their place of residence. They live either in their region of origin or choose to live abroad. Hence, the shares of workers sums up to one, that is

$$\sum_{j=1}^{\mathcal{N}} n_{ij,t} = 1. \tag{2.4}$$

Maximizing the expected lifetime utility with respect to the budget constraint 2.3 and the sum-up constraint 2.4 yields a set of first-order conditions for consumption, labour supply, location choice and bond holdings. The first-order condition for the location choice $n_{ij,t}$ is given by

$$\begin{aligned} &\mathcal{U}_{ij,t} - \mathcal{U}_{ii,t} + A_{ij} - \frac{1}{\gamma}\Big(\ln n_{ij,t} + 1\Big) \\ &= \left(C_{ij,t} - \frac{W^h_{j,t}}{P_{j,t}} L^s_{ij,t} + T_{j,t}\right)(C_{ij,t})^{-\sigma} - \left(C_{ii,t} - \frac{W^h_{i,t}}{P_{i,t}} L^s_{ii,t} + T_{i,t}\right)(C_{ii,t})^{-\sigma}. \end{aligned}$$

The left-hand side describes the utility gain for moving a household member from region i to region j (remember that local amenities are set to zero for the region of origin). The right-hand side describes the additional costs. If an additional household member moves from region i to region j, the budget constraint is affected by the shift in consumption expenditures, as well as labour income and tax payments from region i to region j.

The labour supplied in region j by a household member born in region i is given by the condition

$$w^h_{j,t} = (C_{ij,t})^{\sigma}(L^s_{ij,t})^{\varphi}.$$

where $w^h_{j,t} = \frac{W^h_{j,t}}{P_{j,t}}$ is the real wage for the households working in region j. The remaining conditions are standard. Let $R_{i,t}$ denote the gross nominal interest rate. Then, I obtain the well known Euler equation

$$\frac{P_{i,t+1}}{P_{i,t}} = \beta R_{i,t}\mathbb{E}_t\left(\frac{C_{ii,t}}{C_{ii,t+1}}\right)^{\sigma},$$

where $(R_{i,t})^{-1} = \mathbb{E}_t\mathcal{Q}^i_{t,t+1}$.

2.2 Firms

This section describes the firms' optimizing behaviour. In my model, the economy consists of two different sectors. First, there is a final goods sector that produce non-tradable final goods used for consumption. Firms in this sector take intermediate goods as the input. Second, there is a perfectly competitive intermediate goods sector that produce the inputs for the final goods. Here, labour is used as the only controllable input. I assume that prices of the intermediate goods are adjusted only infrequently, according to the standard Calvo mechanism.

2.2.1 Intermediate Goods

In each region there is a continuum of monopolistically competitive firms on the unit interval indexed by f. Each firm produces a differentiated good according to a technology function which is linear in labour input

$$Y_{i,t}(f) = Z_{i,t} L^d_{i,t}(f),$$

where $L^d_{i,t}(f)$ is the firm $f's$ labour demand in region i and $Z_{i,t}$ is a region-specific technology level. The version in Plosser (1989) and Prescott (1986) provides a natural basis for discussing the volatility and persistence of productivity. They modelled the stochastic component of productivity as a first-order autoregressive process

$$\log(Z_{i,t}) = \rho_Z \log(Z_{i,t-1}) + \varepsilon_{Z,i,t},$$

where $\rho_Z \in [0,1]$ measures the persistence of the shock and $\varepsilon_{Z,i,t}$ is a white noise process. I adopt this simple framework for my model. A well suited idea is provided by Holden and Swarbrick (2017) who assume that the productivity $Z_{i,t}$ is driven by a permanent component that is not location-specific, Z^P_t and a location-specific component, $Z^T_{i,t}$. Even here, both functions develop according to a first-order autoregressive process, but the white noise process for $Z^P_{i,t}$ is correlated across space. To be more precise, they used the covariance function

$$\operatorname{cov}(\varepsilon_{Z^T,i,t}, \varepsilon_{Z^T,j,t}) = \exp(-\kappa d(i,j)),$$

where $d(i,j)$ is a freely chosen distance function and $\kappa > 0$ controls the persistence.[2] This convenient framework can be considered for future extensions of my model. Up to now, I assume that productivity is not correlated across space.

I describe the labour market in my model through a standard search and matching framework in the spirit of Diamond (1982), Mortensen (1982) and Pissarides (1985). In each period, job seekers, $S_{i,t}$, are matched with offered vacancies $V_{i,t}$. The number of total seekers, $N_{i,t}S_{i,t}$, can be broken down into three different groups: i) Let $U_{i,t}$ be the fraction of unemployed workers in region i at the end of period t. Then, the mass of unemployed workers from the last period, $N_{i,t-1}U_{i,t-1}$, is eligible to search for a new job.

[2]An simple model would be to embed the economy in the plane $[0,1] \times [0,1]$ or in the 2 sphere. In this setting, a corresponding distance function can be based on the euclidean metric.

ii) Workers can lose their job with an exogenous destruction rate δ. Therefore, the mass of workers who have lost their job is given by $\delta N_{i,t-1}L^d_{i,t-1}$. iii) Changes in migration and labour supply can affect the labour pool, leading to differences in the net balance $N_{i,t}L^s_{i,t} - N_{i,t-1}L^s_{i,t-1}$. This can be summarized mathematically as

$$N_{i,t}S_{i,t} = N_{i,t-1}U_{i,t-1} + \delta N_{i,t-1}L^d_{i,t-1} + N_{i,t}L^s_{i,t} - N_{i,t-1}L^s_{i,t-1}.$$

I assume that workers are not able to search for a new job while still being on the current one. The job-matching process between job seekers and firms is established by the standard matching function

$$M_{i,t} = m(S_{i,t}, V_{i,t}) = \phi_i S^{\psi}_{i,t} V^{1-\psi}_{i,t},$$

where ϕ_i is a scale parameter capturing regional matching efficiency, and ψ is the elasticity of the matching function with respect to job seekers. During each period, a job seeker finds a job with the probability

$$f_{i,t} = \frac{m(S_{i,t}, V_{i,t})}{S_{i,t}} = \phi_i \left(\frac{V_{i,t}}{S_{i,t}}\right)^{1-\psi} = \phi_i \vartheta^{1-\psi}_{i,t},$$

and vacancies are filled with the probability

$$g_{i,t} = \frac{m(S_{i,t}, V_{i,t})}{V_{i,t}} = \phi_i \left(\frac{V_{i,t}}{S_{i,t}}\right)^{-\psi} = \phi_i \vartheta^{-\psi}_{i,t}.$$

where $\vartheta = \frac{V}{S}$ is well known as the labour market tightness. Since I can express $f_{i,t}$ by $g_{i,t}$ through $f_{i,t} = g_{i,t}\vartheta_{i,t}$, it follows that $\partial f_{i,t}/\partial\vartheta_{i,t} > 0$ and $\partial g_{i,t}/\partial\vartheta_{i,t} < 0$. In other words, the more vacancies offered, the higher the probability that an unemployed worker will find a job, and the lower the probability that a vacancy will be filled.

Hiring labour is not for free. For each vacancy, a firm has to pay costs, $K_{i,t}$, which is independent of the vacancy and is considered as given by each firm. A commonly used specification in the literature is a linear function of labour market tightness (see Blanchard and Gali (2010)). That is

$$K_{i,t} = BZ_{i,t}\vartheta_{i,t},$$

where $Z_{i,t}$ is the region-specific technology level and B describes the intensity of vacancy costs. Regional productivity is included in the vacancy costs in order to maintain a balanced growth path. Otherwise, vacancy costs become less significant when economic growth rises, resulting in a long term unemployment rate of zero.
Output is produced by workers who are already employed and by newly matched workers. Further, employed workers can lose their job with probability δ. Therefore, the law of motion for employed workers, $L^d_{i,t}$, is given by

$$N_{i,t}L^d_{i,t} = (1-\delta)N_{i,t-1}L^d_{i,t-1} + N_{i,t}M_{i,t}.$$

Finally, job seekers who find a job start to work immediately. Hence, I define the number of unemployed workers at the end of the period, $U_{i,t}$, as the labour force, $L^s_{i,t}$, less the number of workers employed, $L^d_{i,t}$:

$$U_{i,t} = L^s_{i,t} - L^d_{i,t}.$$

2.2.2 Final Goods

Final good producers, sometimes identified as retailers, buy and transform intermediate goods $Y_{i,t}(f)$ into final output goods $Y_{i,t}$ via the Dixit and Stiglitz (1977) form:

$$Y_{i,t} = \left(\int_0^1 Y_{i,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.5}$$

where $\varepsilon > 1$ is the elasticity of substitution between inputs. The final good producer sells $Y_{i,t}$ units of final goods at the nominal price $P_{i,t}$ and takes the prices of intermediate goods $Y_{i,t}(f)$ as given (prices are set in a staggered fashion and will be discussed later in this section). The objective of a final good producer is to maximize profits through

$$P_{i,t}Y_{i,t} - \int_0^1 P_{i,t}(f)Y_{i,t}(f)df$$

subject to the technology function 2.5. Hence, the Lagrangian is

$$\mathcal{L}^P_i = P_{i,t}Y_{i,t} - \int_0^1 P_{i,t}(f)Y_{i,t}(f)df + \Lambda^P_{i,t}\left\{\left(\int_0^1 Y_{i,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right)^{\frac{\varepsilon}{\varepsilon-1}} - Y_{i,t}\right\}.$$

The first-order condition with respect to $Y_{i,t}$ is

$$P_{i,t} = \Lambda^P_{i,t}. \tag{2.6}$$

$\Lambda^P_{i,t}$ is the gain of an additional output unit of region i. Hence, it is equal to the corresponding price $P_{i,t}$. The first-order condition with respect to $Y_{i,t}(f)$ is

$$P_{i,t}(f) = \Lambda^P_{i,t} \frac{\varepsilon}{\varepsilon - 1} \left(\int_0^1 Y_{i,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon - 1}{\varepsilon} Y_{i,t}(f)^{\frac{\varepsilon-1}{\varepsilon}-1}.$$

When I rearrange equation 2.5 to

$$\left(\int_0^1 Y_{i,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right) = Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}$$

and use equation 2.6, I obtain

$$P_{i,t}(f) = P_{i,t} \left(\frac{Y_{i,t}(f)}{Y_{i,t}} \right)^{-\frac{1}{\varepsilon}}.$$

Reordering yields the demand curve

$$Y_{i,t}(f) = \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\varepsilon} Y_{i,t}. \tag{2.7}$$

The aggregate price index for region i is implicitly determined by inserting the demand curve 2.7 into the aggregator 2.5:

$$P_{i,t} = \left(\int_0^1 P_{i,t}(f)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}}. \tag{2.8}$$

2.2.3 Calvo Price Setting

Now, consider the choice of the optimal price conditional on the optimal choice of labour. Since firms have a chance to optimize their price in each period, I can write this as a static problem:

$$\max_{P_{i,t}(f)} P_{i,t}(f) Y_{i,t}(f) - W^f_{i,t}(f) L^d_{i,t}(f) - K_{i,t} V_{i,t}(f)$$

s.t. (2.5) and (2.7).

The nominal prices of the intermediate goods are adjusted only infrequently according to the standard Calvo mechanism (see Calvo 1983). In each period, for any firm f, there is a constant probability $1-\theta,\ 0<\theta<1$ that a firm in region i is able to reoptimize the price $P_{i,t}(f)$ of its good $Y_{i,t}(f)$. The probability is independent of the time a firm last reset its price, leading to an average price duration of $(1-\theta)^{-1}$. Formally:

$$P_{i,t}(f) = \begin{cases} \widetilde{P}_{i,t}(f) & \text{with probability } 1-\theta \\ P_{i,t-1}(f) & \text{with probability } \theta \end{cases}$$

where $\widetilde{P}_{i,t}(f)$ is the reoptimized price in period t. Accordingly, when a firm cannot reoptimize its price for s periods, its price in period $t+s$ is given py $\widetilde{P}_{i,t}(f)$, and stays there until the firm can optimize it again. Hence, the firm's objective in t is to set $\widetilde{P}_{i,t}(f)$ in a way that maximizes expected profits until the firm can reoptimize the price again in some future $t+s$. The probability of keeping the price for s periods constant is given by θ^s. When firms are able to change prices in period t, the Lagrangian of expected discounted sums of nominal profits can be written as

$$\mathcal{L}_i^f = \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^i (P_{i,t+s})^{\varepsilon} Y_{i,t+s} [\widetilde{P}_{i,t}(f)^{1-\varepsilon} - P_{i,t+s} RMC_{i,t+s} \widetilde{P}_{i,t}(f)^{-\varepsilon}] + \dots,$$

where first, $RMC_{i,t}$ denotes the Lagrange multiplier and coincides with firm's real marginal costs; second, I insert the demand curve 2.7 from the final product sector; and third, I focus only on relevant parts of the optimization with respect to $\widetilde{P}_{i,t}(f)$. The first-order condition of maximizing $\mathcal{L}_i^f$ with respect to $\widetilde{P}_{i,t}(f)$ is

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^i (P_{i,t+s})^{\varepsilon} Y_{i,t+s} \left[(1-\varepsilon) \cdot \widetilde{P}_{i,t}(f)^{-\varepsilon} + \varepsilon \cdot P_{i,t+s} \cdot RMC_{i,t+s} \widetilde{P}_{i,t}(f)^{-\varepsilon-1} \right].$$

As $\widetilde{P}_{i,t}(f) > 0$ does not depend on s, I multiply by $\widetilde{P}_{i,t}(f)^{\varepsilon+1}$ and obtain

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^i (P_{i,t+s})^{\varepsilon} Y_{i,t+s} \left[(1-\varepsilon) \cdot \widetilde{P}_{i,t}(f) + \varepsilon \cdot P_{i,t+s} \cdot RMC_{i,t+s} \right].$$

Rearranging yields

$$\widetilde{P}_{i,t}(f) \cdot \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^i (P_{i,t+s})^{\varepsilon} Y_{i,t+s} = \frac{\varepsilon}{\varepsilon-1} \cdot \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^i (P_{i,t+s})^{\varepsilon+1} Y_{i,t+s} RMC_{i,t+s}.$$

Finally, dividing by $(P_{i,t})^{\varepsilon+1}$ gives

$$\underbrace{\frac{\widetilde{P}_{i,t}(f)}{P_{i,t}}}_{\widetilde{p}_{i,t}} \cdot \underbrace{\mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}^i_{t,t+s} \left(\frac{P_{i,t+s}}{P_{i,t}}\right)^{\varepsilon} Y_{i,t+s}}_{\mathfrak{S}^1_{i,t}} = \frac{\varepsilon}{\varepsilon-1} \cdot \underbrace{\mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}^i_{t,t+s} \left(\frac{P_{i,t+s}}{P_{i,t}}\right)^{\varepsilon+1} Y_{i,t+s} RMC_{i,t+s}}_{\mathfrak{S}^2_{i,t}}.$$

Note that all firms that reset prices face the same problem and therefore set the same price, i.e.

$$\widetilde{P}_{i,t}(f) = \widetilde{P}_{i,t}.$$

This is also evident by looking at the infinite sums, as these do not depend on f. I introduce the following notation for the reoptimized relative prices

$$\widetilde{p}_{i,t} := \frac{\widetilde{P}_{i,t}(f)}{P_{i,t}}.$$

The first-order condition can thus be written compactly as

$$\widetilde{p}_{i,t} \mathfrak{S}^1_{i,t} = \frac{\varepsilon}{\varepsilon-1} \cdot \mathfrak{S}^2_{i,t}.$$

Both sums can also be expressed recursively. Details are given in appendix B.

2.2.4 Marginal Costs

Nominal marginal costs are defined in terms of domestic prices

$$MC_{i,t}(f) = RMC_{i,t}(f) P_{i,t}.$$

The real marginal costs of intermediate firm f are, by definition, the partial derivative of real total costs with respect to intermediate output

$$RMC_{i,t}(f) = \frac{\partial \left(w_{i,t} L^d_{i,t}(f) + K_{i,t} V_{i,t}(f) \right)}{\partial Y_{i,t}(f)}.$$

Since intermediate goods have no impact on vacancies, it follows

$$Z_{i,t} RMC_{i,t}(f) = w_{i,t}.$$

Note that all firms in region i are faced with the same costs for labour and all have access to the same production technology. Therefore, marginal costs are identical across firms

$$RMC_{i,t}(f) = RMC_{i,t}.$$

2.3 Value Functions

In my model, workers are matched with intermediate firms through employment agencies. The agencies hire job seekers and try to find a job in the intermediate sector. When they match a firm with a worker, they pay the worker the real wage $w^h_{i,t}$ and receive the payment $w_{i,t}$ from the producing firm. Since an exogenous rate of jobs are destroyed each period with probability $\delta \in (0,1)$, the value of a matched worker for the employment agency is

$$\mathcal{W}_{i,t} = w_{i,t} - w^h_{i,t} + (1-\delta)\mathbb{E}_t \mathcal{Q}^i_{t,t+1} \mathcal{W}_{i,t+1}.$$

Remember that the probability of creating a match between a firm and a job seeker is given by $f_{i,t}$. If the employment agency cannot find a job for a worker, they receive the unemployment benefit b, leading to an expected value of hiring a worker of

$$\mathcal{H}_{i,t} = f_{i,t}\mathcal{W}_{i,t} + (1-f_{i,t})(b - w^h_{i,t}).$$

We now turn to the production perspective. Each period, firms decide to post vacancies $V_{i,t}$ to hire job seekers. In each period, the firm is faced with the cost $K_{i,t}$ to maintain the vacancy. An open vacancy is matched with a job seeker with the probability $g_{i,t}$. When I denote the value of a filled vacancy with $\mathcal{J}$, the value of a open vacancy $\mathcal{V}_{i,t}$ is given by

$$\mathcal{V}_{i,t} = -K_{i,t} + g_{i,t}\mathcal{J}_{i,t} + (1-g_{i,t})\mathbb{E}_t \mathcal{Q}^i_{t,t+1} \mathcal{V}_{i,t+1}.$$

An intermediate production firm sells its product for $\widetilde{p}_{i,t}$ from the final good producers and pays the wage $w_{i,t}$ to the employment agency. Hence, the value of a filled vacancy is given by[3]

$$\mathcal{J}_{i,t} = \widetilde{p}_{i,t} Z_{i,t} - w_{i,t} + (1-\delta)\mathbb{E}_t \mathcal{Q}^i_{t,t+1} \mathcal{J}_{i,t+1} + \delta \mathbb{E}_t \mathcal{Q}^i_{t,t+1} \mathcal{V}_{i,t}.$$

As is generally assumed, I allow vacancies to be created for free, that means in equilibrium, $\mathcal{V}_{i,t} = 0$ must hold. Therefore, the value of a filled vacancy for an intermediate firm simplifies to

$$\mathcal{J}_{i,t} = \widetilde{p}_{i,t} Z_{i,t} - w_{i,t} + (1-\delta)\mathbb{E}_t \mathcal{Q}^i_{t,t+1} \mathcal{J}_{i,t+1}$$

or

$$\mathcal{J}_{i,t} = \frac{K_{i,t}}{g_{i,t}}.$$

2.4 Wage Bargaining

Successful matches of job seekers and firms generate an economic surplus. Following most of the search and matching literature, I split the surplus between workers and firms through a Nash bargaining process. Without further assumptions, movements in productivity lead to large responses in wages. Since it is not realistic that wages will be renegotiated each period, the baseline framework of Diamond, Mortensen and Pissarides does not accurately describe volatilities in unemployment, which is a commonly observed phenomenon. Therefore, it is necessary to include such additional features as wage rigidity or modifications according to the value of being unemployed. I focus on the former and introduce wage rigidity in a backward-looking setting as in Hall (2005). Hence, the actual wage $w_{i,t}$ is given by a weighted average of the previous wage $w_{i,t-1}$ and the bargained wage $\widetilde{w}_{i,t}$, i.e.

$$w_{i,t} = \theta^w w_{i,t-1} + (1-\theta^w)\widetilde{w}_{i,t}. \tag{2.9}$$

This approach is motivated by the Calvo price setting (see Gertler and Trigari (2009)), meaning, for any firm f, there is a constant probability $1-\theta^w$, $0 < \theta^w < 1$ that a

[3] Remember that intermediate goods are produced according to $Y_{i,t}(f) = Z_{i,t} L^d_{i,t}(f)$

firm in region i can renegotiate the wage. The probability is defined independently of when a firm last renegotiated its price, which leads to an average wage duration of $(1-\theta^w)^{-1}$. Since firms that can renegotiate the wage are chosen randomly, the average wage $w_{i,t} = \int_0^1 w_{i,t}(f)df$ evolves according to equation 2.9. Let $\eta \in (0,1)$ denote the worker's bargaining power. The bargained wage, $\widetilde{w}_{i,t}$ is chosen to maximize the Nash product

$$(\mathcal{W}_{i,t}(w_{i,t}) - (b - w^h_{i,t}))^{\eta}(\mathcal{J}_{i,t}(w_{i,t}))^{1-\eta}.$$

The first-order condition for the Nash product with respect to $w_{i,t}$ is given by[4]

$$\eta\mathcal{J}_{i,t}(\widetilde{w}_{i,t}) = (1-\eta)\mathcal{W}_{i,t}(\widetilde{w}_{i,t}).$$

The value of an employed worker for an agency that receives the wage $\widetilde{w}_{i,t}$ can be rewritten as

$$\mathcal{W}_{i,t}(\widetilde{w}_{i,t}) = \widetilde{w}_{i,t} - w_{i,t} + \mathcal{W}_{i,t}(w_{i,t}).$$

Similarly, I have

$$\mathcal{J}_{i,t}(\widetilde{w}_{i,t}) = -\widetilde{w}_{i,t} + w_{i,t} + \mathcal{J}_{i,t}(w_{i,t}).$$

Using bot equations, the bargained wage is given by

$$\widetilde{w}_{i,t} = w_{i,t} + \eta\mathcal{J}_{i,t} - (1-\eta)(\mathcal{W}_{i,t} - (b - w^h_{i,t})).$$

Substituting this into equation 2.9 yields a formula for the current wage

$$\theta^w w_{i,t} = \theta^w w_{i,t-1} + (1-\theta^w)(\eta\mathcal{J}_{i,t} - (1-\eta)(\mathcal{W}_{i,t} - (b - w^h_{i,t}))).$$

[4]For $\chi^{\mathcal{W}}_{i,t} = \frac{\partial \mathcal{W}_{i,t}}{\partial w_{i,t}}$ and $\chi^{\mathcal{J}}_{i,t} = -\frac{\partial \mathcal{J}_{i,t}}{\partial w_{i,t}}$ it holds

$$\chi^{\mathcal{W}}_{i,t} = 1 + (1-\delta)\theta^w \mathbb{E}_t Q^i_{t,t+1}\chi^{\mathcal{W}}_{i,t+1}$$
$$\chi^{\mathcal{J}}_{i,t} = 1 + (1-\delta)\theta^w \mathbb{E}_t Q^i_{t,t+1}\chi^{\mathcal{J}}_{i,t+1}$$

Since the wage $w_{i,t}$ is paid to employment agencies, wage rigidity in $w_{i,t}$ does not lead to substantial responses in the unemployment rate. The solution is to transfer some rigidity to the wage of households $w^h_{i,t}$. For this reason, I assume that the wage of households is set to

$$\theta^w w^h_{i,t} = \theta^w w^h_{i,t-1} + (1 - \theta^w)\mathcal{H}_{i,t}.$$

This wage equation can be interpreted as a result of imperfect competition between employment agencies. When employment agencies can enter the market for free, the wage of households $w^h_{i,t}$ would adjust immediately, and employment agencies would not receive any gain from hiring a job seeker, i.e. $\mathcal{H}_{i,t} = 0$. This holds for $\theta^w = 0$. If $\theta^w > 0$, wages will adjust over time and employment agencies can expect to make some revenue. For the last case, $\theta^w = 1$, wages for households are completely rigid.

2.5 Equilibrium

Private bonds $D_{i,t}$ are in zero net supply on the budget constraints. Note that this condition can only be imposed after taking first-order conditions. It would be invalid to eliminate bonds already in the budget constraint of the households. Even if the net supply of bonds is zero, household savings behaviour in equilibrium still needs to be consistent with the bond market clearing. Aggregate labour is given by

$$L^d_{i,t} = \int_0^1 L^d_{i,t}(f)\ df.$$

The market clearing condition for the final good requires

$$Y_{i,t} = C_{i,t} + K_{i,t}V_{i,t}.$$

To obtain an equilibrium condition for the perspective of the provided production, I have to define $Y^{sum}_{i,t} = \int_0^1 Y_{i,t}(f)df$. Together with the production function as well as the labour market clearing, I obtain

$$Y^{sum}_{i,t} = \int_0^1 Z_{i,t}L^d_{i,t}(f)df = Z_{i,t}L^d_{i,t}.$$

When I define the efficiency distortions $\Delta_{i,t}$ as

$$\Delta_{i,t} = \int_0^1 \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\varepsilon} df$$

and integrate over the demand function 2.7, I obtain

$$Y_{i,t}^{sum} = Y_{i,t} \int_0^1 \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\varepsilon} df = Y_{i,t}\Delta_{i,t}.$$

Equating the expressions for $Y_{i,t}^{sum}$ yields

$$Y_{i,t} = (\Delta_{i,t})^{-1} Z_{i,t} L_{i,t}^d.$$

The law of motion for the efficiency distortion in region i is given by the Calvo mechanism, i.e.

$$\begin{aligned}
\Delta_{i,t} &= \int_0^1 \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\varepsilon} df \\
&= \int_{optimizers} \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\varepsilon} df + \int_{non-optimizers} \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\varepsilon} df \\
&= (1-\theta)(\tilde{p}_{i,t})^{-\varepsilon} + \theta \int_0^1 \left(\frac{P_{i,t-1}(f)}{P_{i,t}} \right)^{-\varepsilon} df \\
&= (1-\theta)(\tilde{p}_{i,t})^{-\varepsilon} + \theta \left(\frac{P_{i,t-1}}{P_{i,t}} \right)^{-\varepsilon} \int_0^1 \left(\frac{P_{i,t-1}(f)}{P_{i,t-1}} \right)^{-\varepsilon} df \\
&= (1-\theta)(\tilde{p}_{i,t})^{-\varepsilon} + \theta (\Pi_{i,t})^{\varepsilon} \Delta_{i,t-1}.
\end{aligned}$$

The law of motion for $\widetilde{p}_{i,t} := \frac{\widetilde{P}_{i,t}(f)}{P_{i,t}}$ can be derived by the aggregate price index in equation 2.8

$$1 = \int_0^1 \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{1-\varepsilon} df$$

and the Calvo mechanism for region i. Both together give

$$
\begin{aligned}
1 &= \int_0^1 \left(\frac{P_{i,t}(f)}{P_{i,t}}\right)^{1-\varepsilon} df \\
&= \int_{optimizers} \left(\frac{P_{i,t}(f)}{P_{i,t}}\right)^{1-\varepsilon} df + \int_{non-optimizers} \left(\frac{P_{i,t}(f)}{P_{i,t}}\right)^{1-\varepsilon} df \\
&= (1-\theta)\left(\frac{\tilde{P}_{i,t}}{P_{i,t}}\right)^{1-\varepsilon} + \theta \int_0^1 \left(\frac{P_{i,t-1}(f)P_{i,t-1}}{P_{i,t}P_{i,t-1}}\right)^{1-\varepsilon} df \\
&= (1-\theta)\left(\tilde{p}_{i,t}\right)^{1-\varepsilon} + \theta \left(\frac{P_{i,t-1}}{P_{i,t}}\right)^{1-\varepsilon} \int_0^1 \left(\frac{P_{i,t-1}(f)}{P_{i,t}}\right)^{1-\varepsilon} df \\
&= (1-\theta)\left(\tilde{p}_{i,t}\right)^{1-\varepsilon} + \theta \left(\Pi_{i,t}\right)^{\varepsilon-1}.
\end{aligned}
$$

2.6 Monetary Policy

I assume that monetary policy is characterized by the following Taylor rule

$$R_{i,t} = R_i \Pi_{i,t}^{\gamma_\pi}.$$

where $\Pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$ and $1/\beta$ is the steady state value for R_i, implied by the Euler equation. Note that due to the Taylor principle, I need $\gamma_\pi > 1$ to have a unique and stable solution.

3 Calibration

I now provide a simple calibration to depict the tractability of the model by replicating the German economy. Therefore, it is necessary to choose specific parameter values for β, σ, φ, γ, A_{ij}, δ, ε, θ, $\phi_i, \psi, \gamma_\pi, \theta^w, b, N^i, \eta, \rho_Z$ and B. In order to obtain a calibration for the German economy (a more sclerotic market than the US), the values are taken as far as possible from German data within the last three decades.[5] Here, both my own calculations and values from the literature were used. Table 4 summarizes the assigned parameter values.

First of all, I choose the model period to be one quarter. In accordance with the federal

[5] Data from previous years cannot be used in order to ensure consistency.

states in Germany, I divide our economy into 16 regions. The common discount factor β is set to 0.99, implying a real annual interest rate about 4%. The intertemporal elasticity of substitution is set to $\sigma = 1$. With this choice, the utility function in specification 2.2 simplifies to a logarithmic function. I follow Christoffel et al. (2009) and choose φ to be equal to 10. This ensures an elasticity of substitution for labour of 0.1.[6] The value of heterogeneity across members' tastes for different locations, $\gamma = 5$, follows the estimation of House et al. (2018).

In the labour market, evidence on price elasticities suggests a gross steady state markup of 10%, which corresponds to $\varepsilon = 11$. For the average contract duration of prices, I choose $\theta = 0.75$, implying an average duration of prices of 12 month. Estimates for the elasticity of matches with respect to unemployment are typically between 0.5 and 0.7 in Europe (see Petrongolo and Pissaridies (2001)). Therefore, $\psi = 0.6$ seems to be an appropriate choice. Unemployment benefits are set such that unemployed workers receive 0.7 of household wages, resulting in b = 0.59. The evidence collected in Christoffel et al. (2009) points to quarterly separation rates of $\delta = 0.06$. B, which determines the level of hiring costs, and is from Blanchard and Gali (2010) and set to $B = 0.12$. The wage rigidity is set to $\theta^w = 0.95$ which is in line with Shimer (2010) and Christoffel and Linzert (2005). It is common in the literature to set the bargaining power equal to the matching function elasticity, so as to fulfil the Hosios condition. Finally, monetary policy follows a standard Taylor rule with a long-run response to inflation of $\gamma_\pi = 1.5$ and I use $\rho_Z = 0.8$ as a realistic value for the degree of persistence.

The remaining values for A_{ij}, N^i and ϕ_i are based on three data sets provided by the Federal Statistical Office of Germany (Dstatis).[7] The first data set contains quarterly numbers of employees subject to social insurance contributions in the 16 federal states between 2008 and 2019. I take the average number for each state to determine N^i. The second set consists of annual cross tabulations for migration flows across states between 1991 and 2019. Together with the number of workers, I can derive an approximation

[6]They justify this rather small value by referring to the study of Evers et al. [8] for the euro area.

[7]Migration flows are available upon request. Remaining data are available at:

https://www-genesis.destatis.de/genesis/online?operation=themes&code=$#abreadcrumb

for a quarterly migration rate between 2008 and 2019. I thus choose the amenities A_{ij} in such a way that the steady state values for migration rates in the model match the values in my calculation. The last data set contains unemployment rates for all states between 1995 and 2019. The matching efficiencies ϕ_i are set such that the model has the same unemployment rates as the average values in the data set.

4 Impulse Response Functions

In this section I investigate the dynamics of the model when a single region is faced with some disturbances. I start by considering a productivity shock, which is a standard procedure in the literature for replicating the business cycle. The focus here is to describe the behaviour of the labour market when the shock occurs. While it is widely known that productivity is one of the main drivers of changes in unemployment rates, the effects on migration are less often discussed. The second shock that I will to consider is less common and caused by a change in amenities. Here, I want to examine whether the shock creates plausible results for the model. Other shocks, such as in monetary policy, labour supply or preferences, are not the subject of this paper. For all results, Dynare version 4.6 is used.

4.1 Productivity Shocks

As usual in the literature, I start by considering an increase in regional productivity. Figure 14 summarizes the responses of population and unemployment to an increase in productivity of one percent of the region concerned. All responses are shown in percentage points and in quarterly terms.

The presence of nominal rigidities prevents the producers from perfectly adjusting prices downward. This leads to a drop in labour demand as the only alternative margin of adjustment. At the same time, more efficient production leads to a higher consumption level. Looking at the optimal decisions for households, it can be seen that higher levels of consumption have a negative impact on labour supply. The latter is a stronger effect, leading to a decrease in unemployment in the region where the shock occurs. As labour supply recovers more quickly from the shock, the change in unemployment rate turns positive before converging to its initial level. All unemployment rates react in a similar way when the shock occurs, with regions with an initially high unemployment rate, as in Bremen, Thuringia or Berlin, being affected slightly more than others. In particular, the initial drop in the most affected region, Bremen, is 3 times larger than in the least affected region, Bavaria. Turning to vacancies, there are two opposing effects. On the one hand, higher output gives firms the opportunity to provide more vacancies. On the other hand, productivity, by definition, affects the costs of vacancies and therefore leads to a decrease. In my model, the second effect outweighs the first, resulting in fewer observed matches. Taken together, this increases the unemployment rate, but this effect plays only a minor role in determining the unemployment rate.

Once again, output rises in the region hit by the shock, as a direct consequence of the increase in productivity which turns into a higher level of consumption. Hence, workers will receive higher utility when they work in this region, creating an incentive to move to or stay in the shocked region. On the other hand, because of the drop in labour demand in the shocked region, workers are optimally sent to work in other regions, since they offer more employment. In my application to Germany the results are similar across states. Migration plays only a minor role when a region is faced with a productivity shock which is evident in responses that are equal to or smaller than $10^{-4}\%$. In particular, Figure 15 provides three examples (Bavaria, Bremen and Hessen) to visualize their bilateral migration rates to other regions. The figure shows that regional productivity shocks mainly effect adjacent regions in addition to the affected region. This effect is especially visible for small states such as Bremen, which is surrounded by Lower Saxony. The occurring immigration wave is five times larger from Lower Saxony than from other states.

4.2 Amenity Shocks

For region i, an increase in local amenities is considered by increasing A_{ji} by one percent for all $j \in \mathcal{N}$. Since the amenities are not based on an autoregressive process, their increase will be constant and continue indefinitely. By construction of the model, this affects the labour market such that workers are more attracted to move to the shocked region by higher amenities. The result is a higher population, ranging from 0.0013 to 0.0035 percent, which is roughly three times larger than for a productivity shock. This leads to an increase in job seekers and, accordingly, to more matches. Initially, the number of matches exceeds the number of new immigrant workers, resulting in a reduction of unemployment. At the same time, the region faces a drop in consumption per capita. Therefore, expenditures from maintaining vacancies will be shifted to consumption, so as to achieve the previous steady state. The unemployment rate will rise immediately and converges slowly to the old level. In the long run, as expected, amenity shocks only affect the choice of work place through immigration, since emigration is not directly affected, again due to the construction of the model (amenities for the home region are zero). This means, in particular, that only a redistribution of the population has taken place. Other variables such as unemployment, consumption or welfare, experience no long-term changes. Looking at the individual states (Bavaria, Bremen and Hessen) as above, it could again be observed that adjacent regions are more affected when local amenities rise.

5 Outlook

I have developed a multiregional New Keynesian DSGE model that includes several standard elements as concave preferences, wage rigidities, staggered prices, as well as internal migration and a search and matching framework that gives rise to unemployment. Accordingly, it is related to a large and rapidly growing literature of DSGE models. The strength of my paper, despite the number of equations, is its simplicity. It is a first step to developing richer and more complex DSGE models with internal migration. So far, I have investigated the behaviour of workers in a simple approximation of the German economy. To this end, I used data at a state level, including size, migration flows and the unemployment rate, in order to calibrate the model. The main finding of this application is that the labour market is rigid and that regional disparities can be compensated for only slightly by migration.

At this point, the model provides many opportunities for further research. In my model, the government is not considered as a participating agent. Examining the competitive behaviour of policy makers could be useful. Further, low migration rates between distant regions can probably be explained mainly by distance. Including this in the model would place less weight on local amenities. Other interesting questions include: To what extent does distance prevent the narrowing of regional disparities? Why are workers barely motivated to migrate? What are the reasons for persistent higher local unemployment rates? What policy could reduce this effect?

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A Calibration Outcome

Parameter	**Mnemonic**	**Value**	**Target/Source**
Discount factor	β	0.99	4% avg. real return
Intertemporal elasticity of substitution	σ	1	House
Labour supply elasticity	φ	10	Christoffel et al
Elasticity of substitution	ε	11	10% price markup
Calvo price stickiness	θ	0.75	12 months exp. duration
Job separation rate	δ	0.06	Christoffel et al.
Efficiency of matches	ϕ_i	0.2-0.7	regional unemployment rate
Elasticity of matches to vacancies	ψ	0.6	Blanchard and Gali
Migration preferences	γ	5	House
Inflation reaction coefficient	γ_π	1.5	Hauser/House
Persistence technology shock	ρ_Z	0.8	Hauser/House
Wage bargaining	η	0.6	Hosios condition
Wage rigidity	θ^w	0.95	Christoffel and Linzert
Unemployment benefit	b	0.59	$0.7 = b/w^h$
Level of hiring costs	B	0.12	Blanchard and Gali
Population	N^i	~10^6	Dstatis
Amenities	A_{ij}	~(-20)	internal migration rates

Table 4: Benchmark calibration

B Derivations

To express $\mathfrak{S}^1_{i,t}$ and $\mathfrak{S}^2_{i,t}$ recursively, I make use of the relationships for the stochastic discount factor. Given that the firms are owned by the households, the nominal stochastic discount factor $\mathcal{Q}^i_{t,t+s}$ between t and $t+s$ is derived from the Euler equation of the households who own the firm, that is[8]

$$\mathbb{E}_t \mathcal{Q}^i_{t,t+s} = \mathbb{E}_t 1/R_{i,t+s} = \mathbb{E}_t \beta^s \frac{\lambda_{i,t+s}}{\lambda_{i,t}} \frac{P_{i,t}}{P_{i,t+s}}.$$

[8] $\lambda_{i,t}$ is the Lagrange multiplier in the optimizing problem for households.

With this expression it obviously holds that

$$\mathcal{Q}_{t,t}^{i} = 1$$

and

$$\mathcal{Q}_{t,t+1+s}^{i} = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \left(\frac{P_{i,t}}{P_{i,t+1}} \right)^{-1} \mathcal{Q}_{t+1,t+1+s}^{i} = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} (\Pi_{i,t+1})^{-1} \mathcal{Q}_{t+1,t+1+s}^{i}.$$

Now, the first recursive sum for region i can be written as

$$\begin{aligned}
\mathfrak{S}_{i,t}^{1} &= \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^{i} \left(\frac{P_{i,t+s}}{P_{i,t}} \right)^{\epsilon} Y_{i,t+s} \\
&= Y_{i,t} + \mathbb{E}_t \sum_{s=1}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^{i} \left(\frac{P_{i,t+s}}{P_{i,t}} \right)^{\epsilon} Y_{i,t+s} \\
&= Y_{i,t} + \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^{s+1} \mathcal{Q}_{t,t+s+1}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t}} \right)^{\epsilon} Y_{i,t+s+1} \\
&= Y_{i,t} + \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^{s+1} \mathcal{Q}_{t,t+s+1}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t+1}} \frac{P_{i,t+1}}{P_{i,t}} \right)^{\epsilon} Y_{i,t+s+1} \\
&= Y_{i,t} + \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^{s+1} \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} (\Pi_{i,t+1})^{-1} \mathcal{Q}_{t,t+s+1}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t+1}} \Pi_{i,t+1} \right)^{\epsilon} Y_{i,t+s+1} \\
&= Y_{i,t} + \theta\beta \mathbb{E}_t \frac{\lambda_{i,t+1}}{\lambda_{i,t}} (\Pi_{i,t+1})^{-1} (\Pi_{i,t+1})^{\epsilon} \underbrace{\mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t+1,t+1+s}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t+1}} \right)^{\epsilon} Y_{i,t+1+s}}_{=\mathfrak{S}_{i,t+s}^{1}}.
\end{aligned}$$

The second recursive sum for region i can be written as

$$\begin{aligned}
\mathfrak{S}_{i,t}^{2} &= \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^{i} \left(\frac{P_{i,t+s}}{P_{i,t}} \right)^{\epsilon+1} Y_{i,t+s} RMC_{i,t+s} \\
&= Y_{i,t} RMC_{i,t} + \mathbb{E}_t \sum_{s=1}^{\infty} (\theta)^s \mathcal{Q}_{t,t+s}^{i} \left(\frac{P_{i,t+s}}{P_{i,t}} \right)^{\epsilon+1} Y_{i,t+s} RMC_{i,t+s} \\
&= Y_{i,t} RMC_{i,t} + \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^{s+1} \mathcal{Q}_{t,t+s+1}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t}} \right)^{\epsilon+1} Y_{i,t+s+1} RMC_{i,t+s+1} \\
&= Y_{i,t} RMC_{i,t} + \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^{s+1} \mathcal{Q}_{t,t+s+1}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t+1}} \frac{P_{i,t+1}}{P_{i,t}} \right)^{\epsilon+1} Y_{i,t+s+1} RMC_{i,t+s+1} \\
&= Y_{i,t} RMC_{i,t} + \mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^{s+1} \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} (\Pi_{i,t+1})^{-1} \mathcal{Q}_{t,t+s+1}^{i} \left(\frac{P_{i,t+s+1}}{P_{i,t+1}} \Pi_{i,t+1} \right)^{\epsilon+1} Y_{i,t+s+1} RMC_{i,t+s+1} \\
&= Y_{i,t} RMC_{i,t} + \theta\beta \mathbb{E}_t \frac{\lambda_{i,t+1}}{\lambda_{i,t}} (\Pi_{i,t+1})^{-1} (\Pi_{i,t+1})^{\epsilon+1} \underbrace{\mathbb{E}_t \sum_{s=0}^{\infty} (\theta)^s \mathcal{Q}_{t+1,t+1+s}^{i} \left(\frac{P_{i,t+1+s}}{P_{i,t+1}} \right)^{\epsilon+1} Y_{i,t+1+s} RMC_{i,t+1+s}}_{=\mathfrak{S}_{i,t+s}^{2}}.
\end{aligned}$$

C Dynare Equations

euler Equation

$$\Pi_{i,t+1} = \beta R_{i,t} \mathbb{E}_t \left(\frac{C_{ii,t}}{C_{ii,t+1}} \right)^{\sigma} \tag{C.1}$$

utility

$$\mathcal{U}_{ij,t} = \left(\frac{C_{ij,t}}{1-\sigma} \right)^{1-\sigma} - \left(\frac{L^s_{ij,t}}{1+\varphi} \right)^{1+\varphi} \tag{C.2}$$

population

$$N_{i,t} = \sum_{j=1}^{\mathcal{N}} n_{ji,t} N^j \tag{C.3}$$

constraint for migration

$$\sum_{j}^{\mathcal{N}} n_{ij,t} = 1 \tag{C.4}$$

total labour supply

$$L^s_{i,t} N_{i,t} = \sum_{j=1}^{\mathcal{N}} n_{ji,t} L^s_{ji,t} N^j \tag{C.5}$$

aggregate consumption

$$C_{i,t} N_{i,t} = \sum_{j=1}^{\mathcal{N}} n_{ji,t} C_{ji,t} N^j \tag{C.6}$$

FOC consumption

$$(C_{ii,t})^{-\sigma} = (C_{ij,t})^{-\sigma} \tag{C.7}$$

location choice

$$\begin{aligned} &\mathcal{U}_{ij,t} - \mathcal{U}_{ii,t} + A_{ij} - \frac{1}{\gamma}\Big(\ln n_{ij,t} + 1\Big) \\ &= \left(C_{ij,t} - \frac{W_{j,t}}{P_{j,t}} L^s_{ij,t} + T_{j,t} \right) (C_{ij,t})^{-\sigma} - \left(C_{ii,t} - \frac{W_{i,t}}{P_{i,t}} L^s_{ii,t} + T_{i,t} \right) (C_{ii,t})^{-\sigma} \end{aligned} \tag{C.8}$$

FOC labour

$$\frac{W_{j,t}}{P_{j,t}} = C^{\sigma}_{ij,t}(L^{s}_{ij,t})^{\varphi} \tag{C.9}$$

evolution of technology

$$\log(G_t) = (1-\rho_G)\log(G) + \rho_G \log(G_{t-1}) + \varepsilon_{G,t} \tag{C.10}$$

$$\log(Z^T_{i,t}) = \rho_Z \log(Z^T_{i,t-1}) + \varepsilon_{Z,i,t} \tag{C.11}$$

$$Z^P_t = G_t Z^P_{t-1} \tag{C.12}$$

$$Z_{i,t} = Z^P_t Z^T_{i,t} \tag{C.13}$$

matching function

$$M_{i,t} = m(S_{i,t}, V_{i,t}) = \phi_0 S^{\phi_1}_{i,t} V^{1-\phi_1}_{i,t} \tag{C.14}$$

vacancies are filled with the probability

$$g_{i,t} = \frac{m(S_{i,t}, V_{i,t})}{V_{i,t}} = \phi_0 \left(\frac{S_{i,t}}{V_{i,t}}\right)^{\phi_1} \tag{C.15}$$

vacancy cost

$$K_{i,t} = BZ_{i,t}\vartheta_{i,t} \tag{C.16}$$

unemployment rate

$$U_{i,t} = L^s_{i,t} - L^d_{i,t} \tag{C.17}$$

marginal value of a filled vacancy

$$\mathcal{J}_{i,t} = \frac{K_{i,t}}{g_{i,t}} \tag{C.18}$$

value of a matched worker for employment agency

$$\mathcal{W}_{i,t} = w_{i,t} - w^h_{i,t} + (1-\delta)\mathbb{E}_t \mathcal{Q}_{t,t+1}\mathcal{W}_{i,t+1}. \tag{C.19}$$

expected value of hiring a worker

$$\mathcal{H}_{i,t} = f_{i,t}\mathcal{W}_{i,t} + (1-f_{i,t})(b - w^h_{i,t}). \tag{C.20}$$

law of motion for employment

$$N_{i,t}L_{i,t}^{d} = (1-\delta)N_{i,t-1}L_{i,t-1}^{d} + N_{i,t}M_{i,t} \tag{C.21}$$

bargained wage

$$\theta^{w}w_{i,t} = \theta^{w}w_{i,t-1} + (1-\theta^{w})(\eta\mathcal{J}_{i,t} - (1-\eta)(\mathcal{W}_{i,t} - (b - w_{i,t}^{h}))). \tag{C.22}$$

household wage rigidity

$$\theta^{w}w_{i,t}^{h} = \theta^{w}w_{i,t-1}^{h} + (1-\theta^{w})\mathcal{H}_{i,t}. \tag{C.23}$$

aggregate demand

$$Y_{i,t} = C_{i,t} + K_{i,t}V_{i,t} \tag{C.24}$$

aggregate supply

$$Y_{i,t} = (\Delta_{i,t})^{-1}Z_{i,t}L_{i,t}^{d} \tag{C.25}$$

real marginal cost

$$Z_{i,t}RMC_{i,t} = \frac{W_{i,t}}{P_{i,t}} \tag{C.26}$$

aggregate price index

$$1 = (1-\theta)\left(\tilde{p}_{i,t}\right)^{1-\varepsilon} + \theta\left(\Pi_{i,t}\right)^{\varepsilon-1} \tag{C.27}$$

efficiency distortion

$$\Delta_{i,t} = (1-\theta)(\tilde{p}_{i,t})^{-\varepsilon} + \theta(\Pi_{i,t})^{\varepsilon}\Delta_{i,t-1} \tag{C.28}$$

nonlinear pricing auxiliary sum 1

$$\mathfrak{S}_{i,t}^{1} = Y_{i,t} + \theta\beta\mathbb{E}_{t}\frac{\lambda_{i,t+s}}{\lambda_{i,t}}\left(\Pi_{i,t+1}\right)^{-1}\left(\Pi_{i,t+1}\right)^{\varepsilon}\mathfrak{S}_{i,t+s}^{1} \tag{C.29}$$

nonlinear pricing auxiliary sum 2

$$\mathfrak{S}_{i,t}^{2} = Y_{i,t}RMC_{i,t} + \theta\beta\mathbb{E}_{t}\frac{\lambda_{i,t+s}}{\lambda_{i,t}}\left(\Pi_{i,t+1}\right)^{-1}\left(\Pi_{i,t+1}\right)^{\varepsilon+1}\mathfrak{S}_{i,t+s}^{2} \tag{C.30}$$

nonlinear pricing

$$\widetilde{p}_{i,t}\mathfrak{S}^1_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \mathfrak{S}^2_{i,t} \tag{C.31}$$

monetary policy rule

$$R_{i,t} = R_i \Pi_{i,t}^{\gamma_\pi} \tag{C.32}$$

D Impulse Response Functions

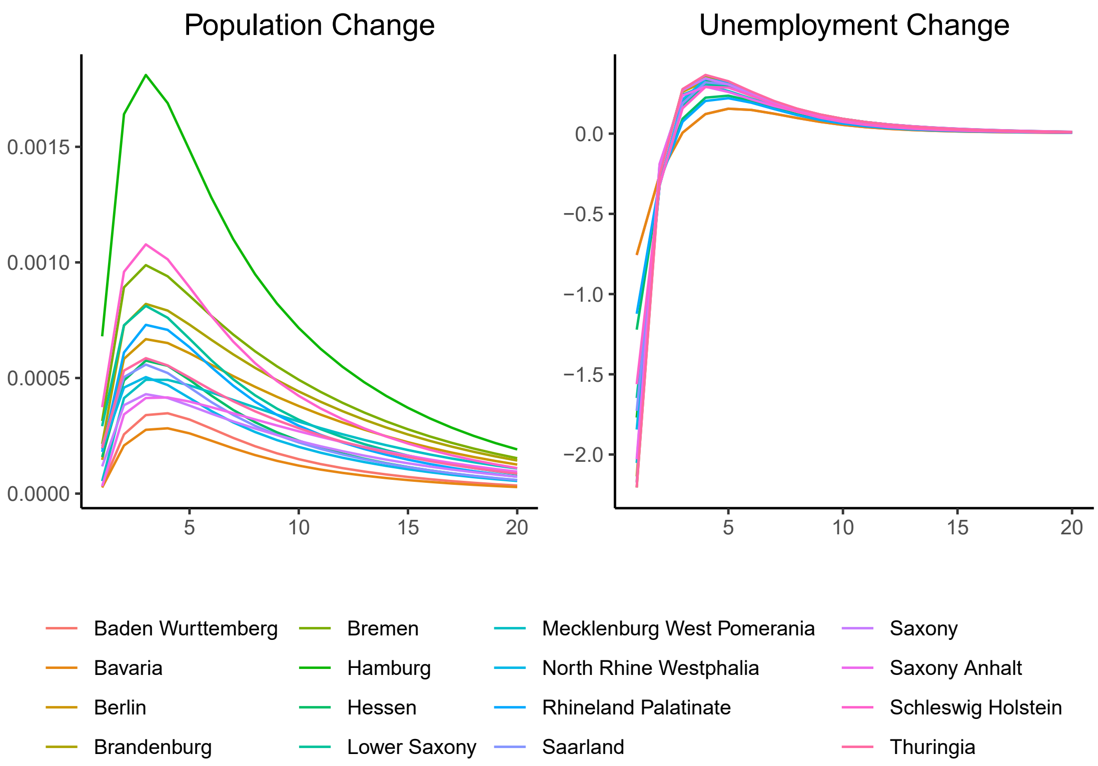

Figure 14: IRF for a 1 percent increase in productivity

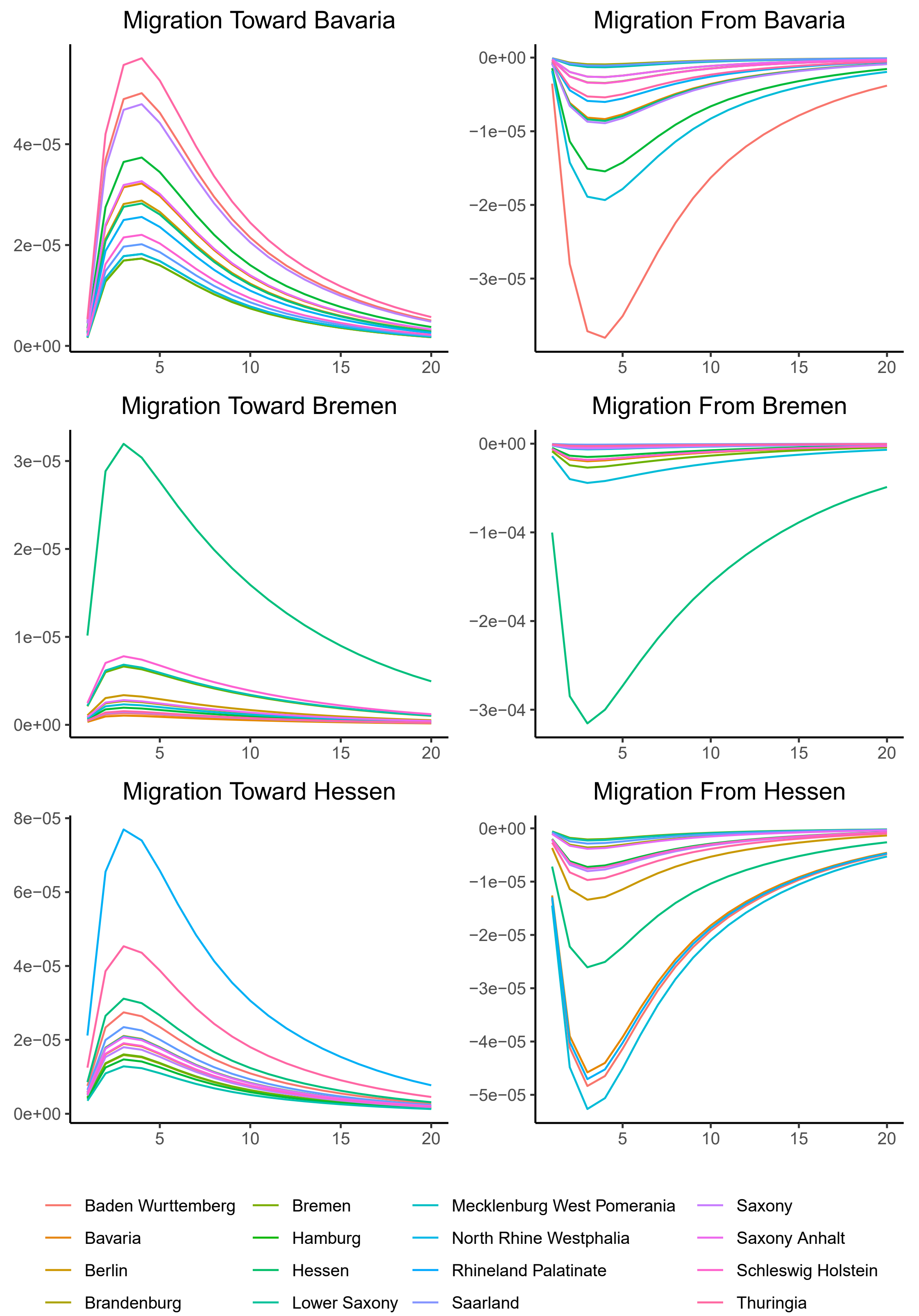

Figure 15: IRF for a 1 percent increase in productivity

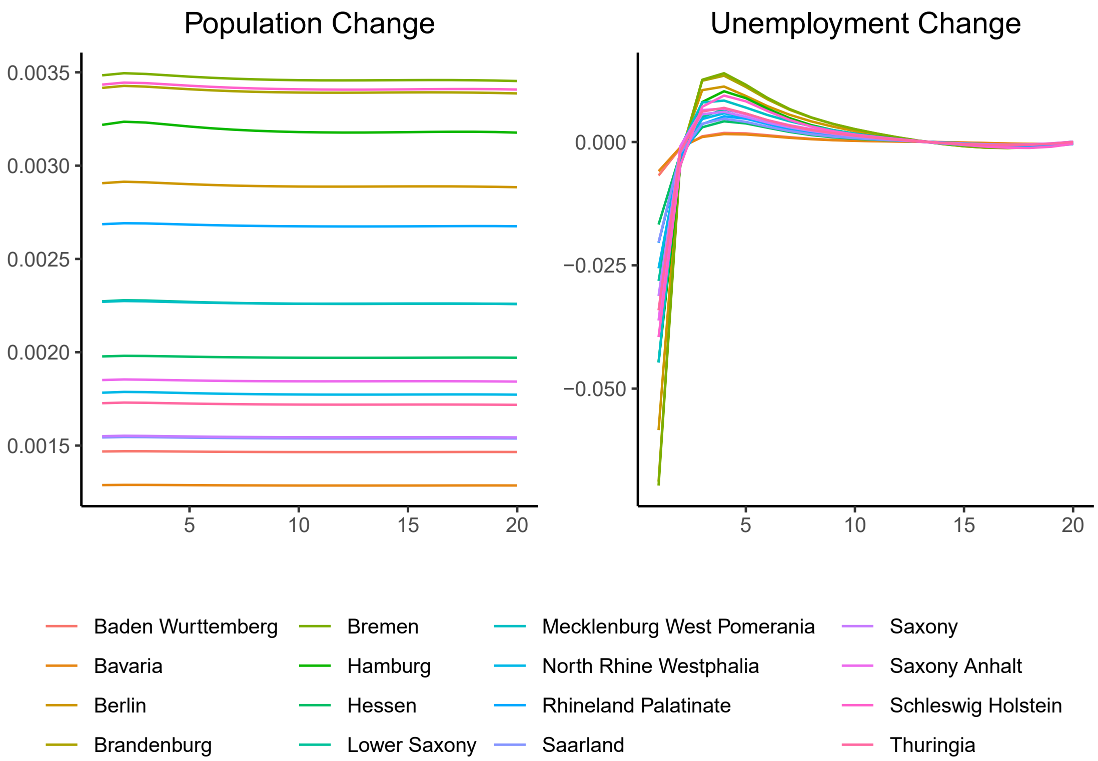

Figure 16: IRF for a 1 percent increase in Amenities

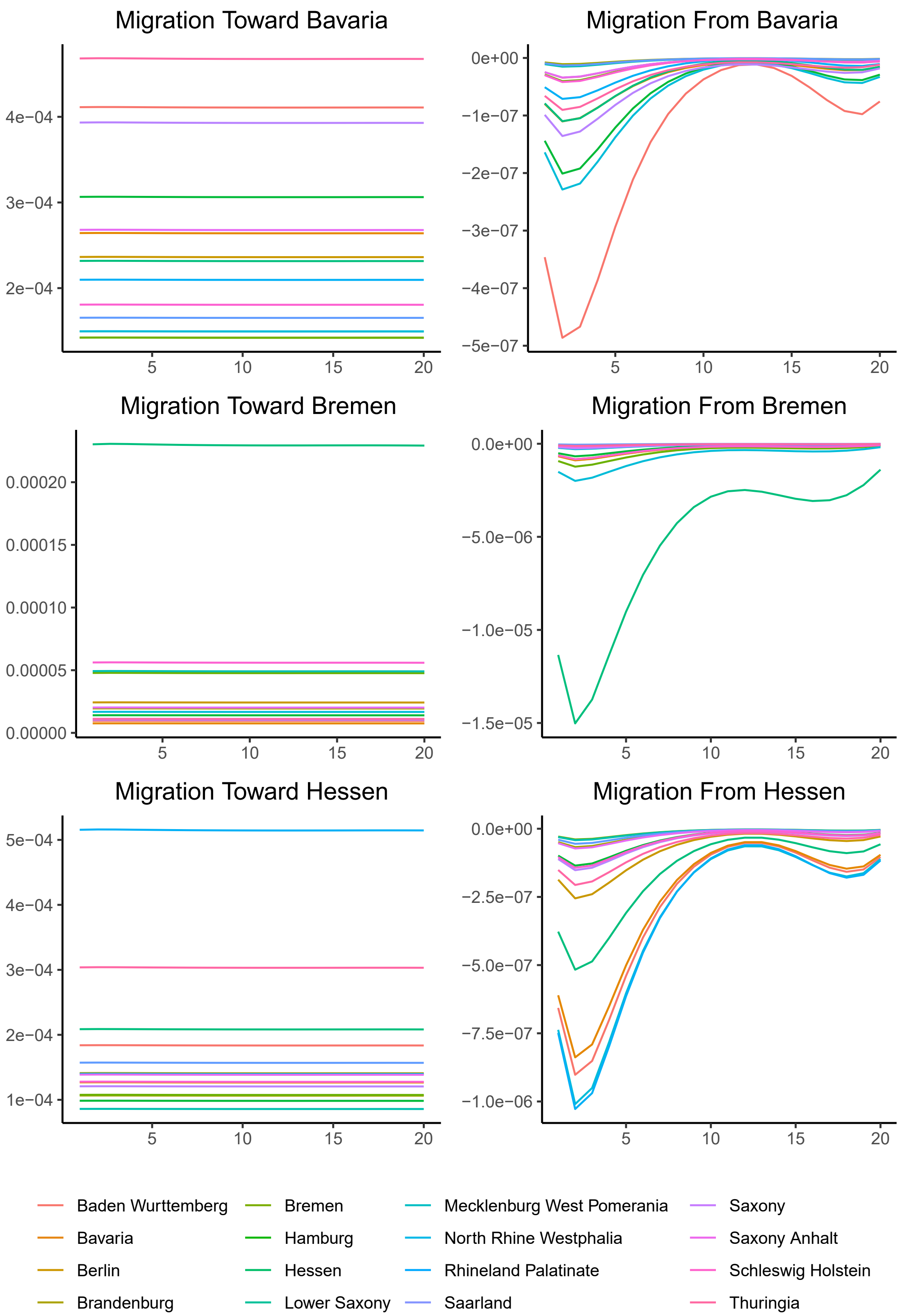

Figure 17: IRF for a 1 percent increase in Amenities